Supernova axion emissivity with $\Delta(1232)$ resonance in heavy baryon chiral perturbation theory

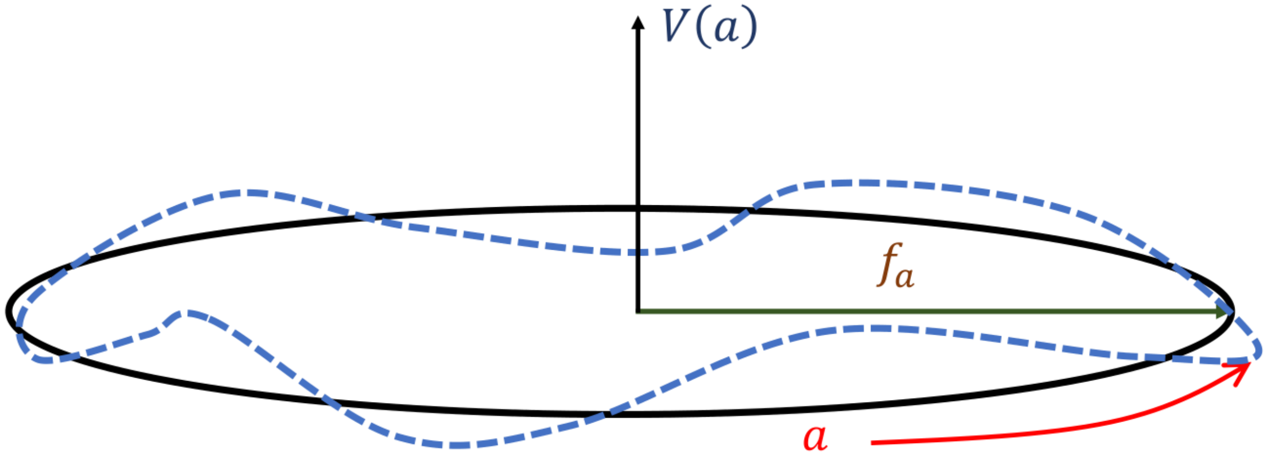
Jongkuk Kim (Chung-Ang University)

PRD 107 (2023) 7, 075002 [arXiv: 2212.01155]

In collaboration with Shu-Yu Ho, Pyungwon Ko, Jae-hyeon Park

Axion and its basic feature

- The axion is a **pseudo Nambu Goldstone boson** which satisfies
  - SM singlet, compact with the period $2\pi f_a$: $a \cong a + 2\pi n f_a, \quad n \in \mathrm{Z}$
  - The axion is well described in a theory with a cut-off $\Lambda \ll f_a$
  - Under the Parity and Time reversal operators: $P: a \to -a, T: a \to -a$
  - Possess a approximate continuous shift symmetry
    - Approximate: All PQ breaking effects are generated at scales well below f_a
  - All interactions at low energies can be described by a/f_a



The strong CP problem in QCD

- In the SM, the gauge symmetry allows P & T violating theta term

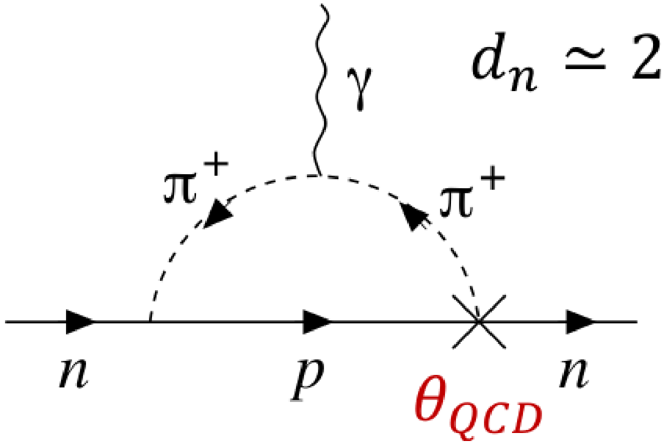
$$\mathcal{L}_\theta = \theta \frac{g_s^2}{32\pi} G^{c\mu\nu} \tilde{G}^c_{\mu\nu}$$

strong CP phase

- The low energy consequence: Nucleon Electric Dipole Moment

electric moment

$$L_{eff} = \bar{N}(i\gamma^\mu D_\mu - m_N)N + \bar{N}\sigma^{\mu\nu}N\left(\mu_N F^{\mu\nu} + d_N \tilde{F}^{\mu\nu}\right) + \cdots \to H_{eff} \ni -\vec{S}_N \cdot \left(\frac{g_N e}{2m_N}\vec{B} + d_N \vec{E}\right)$$



$$d_n \simeq 2.4 \times 10^{-16} \theta_{QCD}\, e\, cm \sim \frac{\theta_{QCD}}{8\pi^2} \frac{e}{m_N}$$

$(\theta_{QCD} = \theta + \mathrm{Arg\,det}(Y_u Y_d))$

$$|d_n^{exp}| < 1.8 \times 10^{-26} e\, cm \to |\theta_{QCD}| < 10^{-10}$$

The strong CP problem in QCD

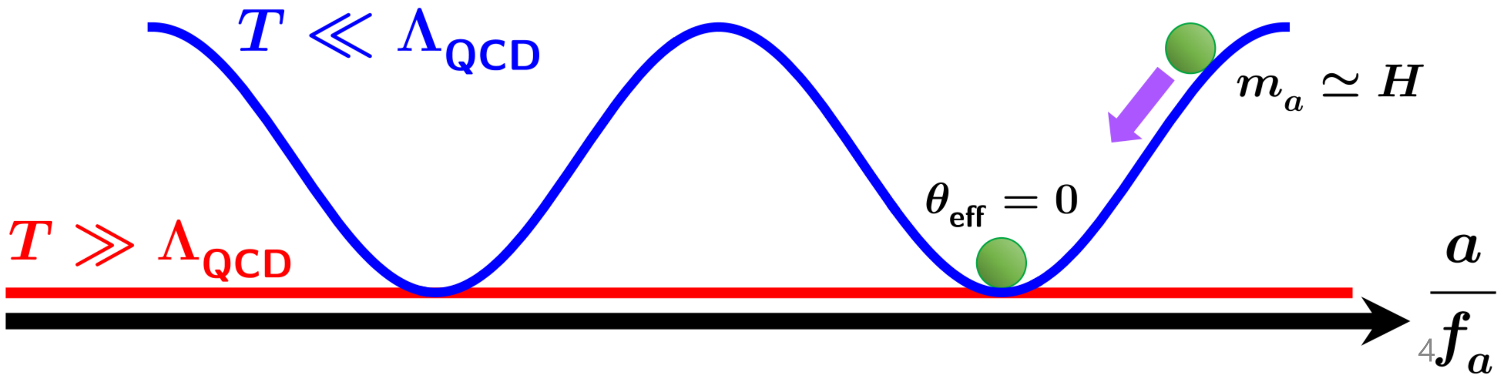
- Strong CP problem
  - Why are P & T violating effects from the strong sector too small?

- The QCD axion is an elegant solution to this problem.
  - A possible candidate of the cold Dark Matter (Coherently oscillating scalar field)

Peccei, Quinn `77, Weinberg `78, Wilczek `78

- Basic idea: Peccei-Quinn (PQ) mechanism
  - Promoting the theta angle to the dynamic field
  - The theta dependent ground state energy becomes the scalar potential of the axion

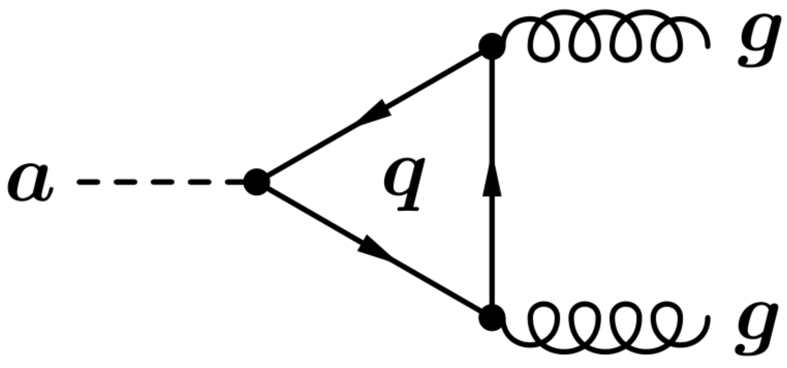
$$\mathcal{L}_\theta = \underbrace{\left[\theta + \frac{a(x)}{f_a}\right]}_{\theta_{\text{eff}}(x)} \frac{g_s^2}{32\pi} G^{c\mu\nu}\widetilde{G}^c_{\mu\nu}$$



Axion interactions with the SM particles

- Axion-gluon interaction

$$\mathcal{L}_{agg} = \frac{g_s^2}{32\pi}\frac{a}{f_a}G^{c\mu\nu}\widetilde{G}^c_{\mu\nu}$$



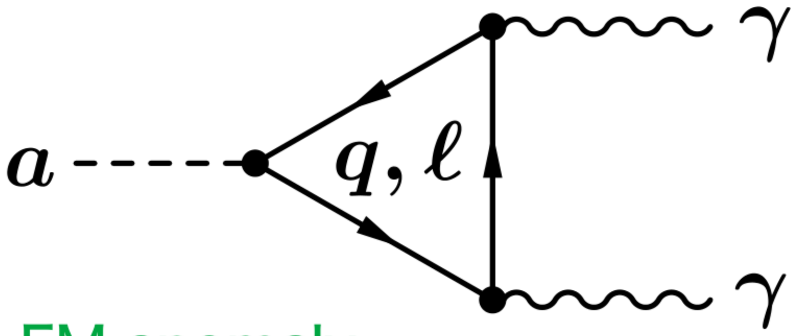
$$f_a = \frac{v_{\rm PQ}}{N_{\rm DW}}$$

$v_{\rm PQ}$ → PQ symmetry breaking scale

$N_{\rm DW}$ → domain wall number

- Axion-photon interaction

$$\mathcal{L}_{a\gamma\gamma} = \frac{g_{a\gamma}}{4}aF^{\mu\nu}\widetilde{F}_{\mu\nu}$$



$$g_{a\gamma} = \frac{\alpha_{\rm EM}}{2\pi f_a}\left(-\frac{2}{3}\frac{m_u+4m_d}{m_u+m_d}+\frac{\mathcal{E}}{\mathcal{N}}\right)$$

$\mathcal{E}$ → EM anomaly

$\mathcal{N}$ → color anomaly

Axion interactions with the SM particles

- Axion-electron interaction

$$\mathcal{L}_{aee} = -iC_{ae}\frac{m_e}{f_a}a\overline{\psi_e}\gamma^5\psi_e \overset{\text{E.O.M. \& I.P.}}{=} C_{ae}\frac{\partial_\mu a}{2f_a}\overline{\psi_e}\gamma^\mu\gamma^5\psi_e$$

C_{ae} : model-dependent coefficient

- Axion-nucleons interaction

$$\mathcal{L}_{aNN} = \sum_{N=p,n} C_{aN}\frac{\partial_\mu a}{2f_a}\overline{\psi_N}\gamma^\mu\gamma^5\psi_N$$

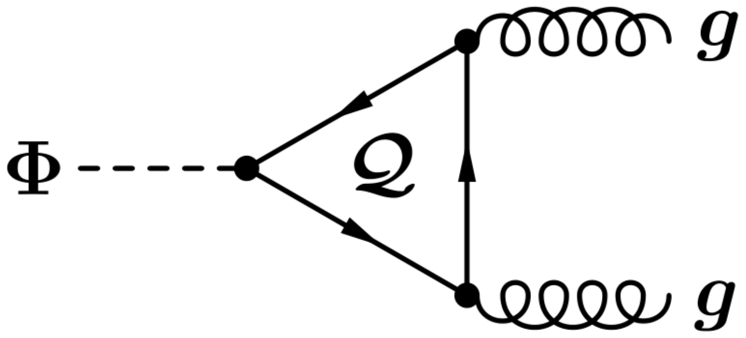
- The axion couples to the SM particles with strength inversely proportional to the decay constant. Hence, the axion **feebly** couples to the SM particles due to the large decay constant.

KSVZ axion model

- The QCD anomaly is realized by introducing a **heavy vector-like quark**

Kim `79,
Shifman, Vainshtein, Zakharov `80

$$\mathcal{Q} = \mathcal{Q}_L + \mathcal{Q}_R \sim (\mathbf{3}, \mathbf{1})_0$$



- Interactions: $y_Q \Phi \overline{\mathcal{Q}_L} \mathcal{Q}_R + \text{h.c.}$

- Under PQ symmetry

$$\Phi \to e^{iq_{\text{PQ}}} \Phi \quad \mathcal{Q}_L \to e^{iq_{\text{PQ}}/2} \mathcal{Q}_L \quad \mathcal{Q}_R \to e^{-iq_{\text{PQ}}/2} \mathcal{Q}_R$$

- Only Φ and $\mathcal{Q}$ have PQ charges: $X_u = X_d = X_s = 0$

(at tree level)

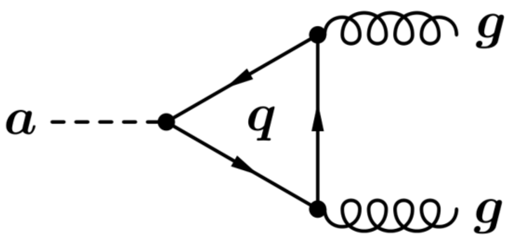
DFSZ axion model

- The QCD anomaly is induced by assuming 2HDM $H_u \& H_d$ couples to the SM quark fields.

Dine, Fischler, Srednicki `81
Zhitnitsky `80

- Interactions: $H_u^\dagger H_d (\Phi^*)^2$ $\overline{Q_L}(\mathcal{Y}_u \widetilde{H}_u U_R + \mathcal{Y}_d H_d D_R) + \mathrm{h.c.}$

- Under PQ symmetry



$$\Phi \to e^{iq_{\mathsf{PQ}}}\Phi \quad H_u \to e^{-iq_{\mathsf{PQ}}} H_u \quad H_d \to e^{iq_{\mathsf{PQ}}} H_d$$

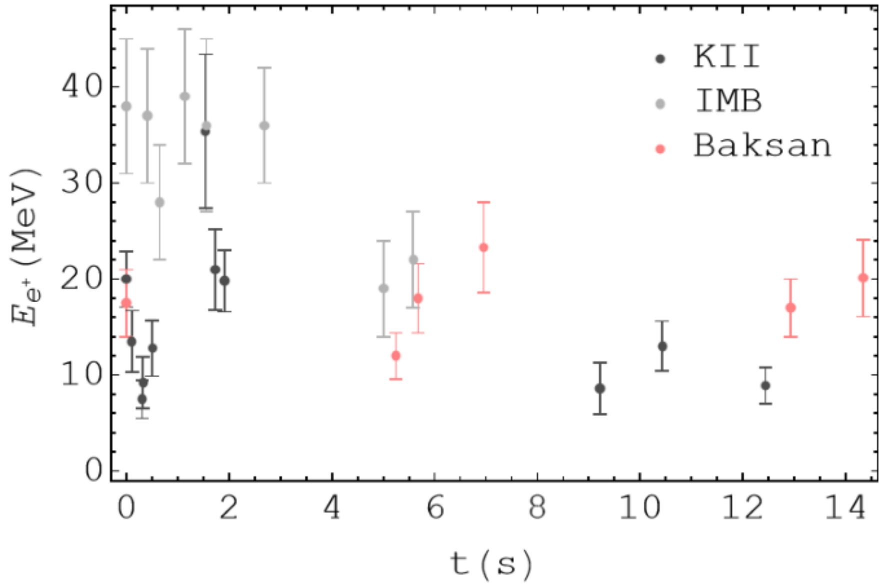
$$Q_L \to Q_L \quad U_R \to e^{-iq_{\mathsf{PQ}}} U_R \quad D_R \to e^{-iq_{\mathsf{PQ}}} D_R$$

- The axion as a **linear combination of the CP-odd scalars** can couple to the SM quarks: $X_u = \frac{\cos^2\beta}{3} \; , \; X_{d,s} = \frac{\sin^2\beta}{3}$ $\tan\beta = \frac{\langle H_u \rangle}{\langle H_d \rangle}$

SN1987A: Neutrino signal

- For massive stars the nuclear fusion produces heavy elements in an onion structure and a degenerate iron core
  - Iron in the core cannot be burnt and the star starts to collapse

- The SN core is an extreme environment
  - Density: $10^{14}\mathrm{g/cm}^3$
  - Temperature: 30MeV
  - Magnetic field: 10^{15}G

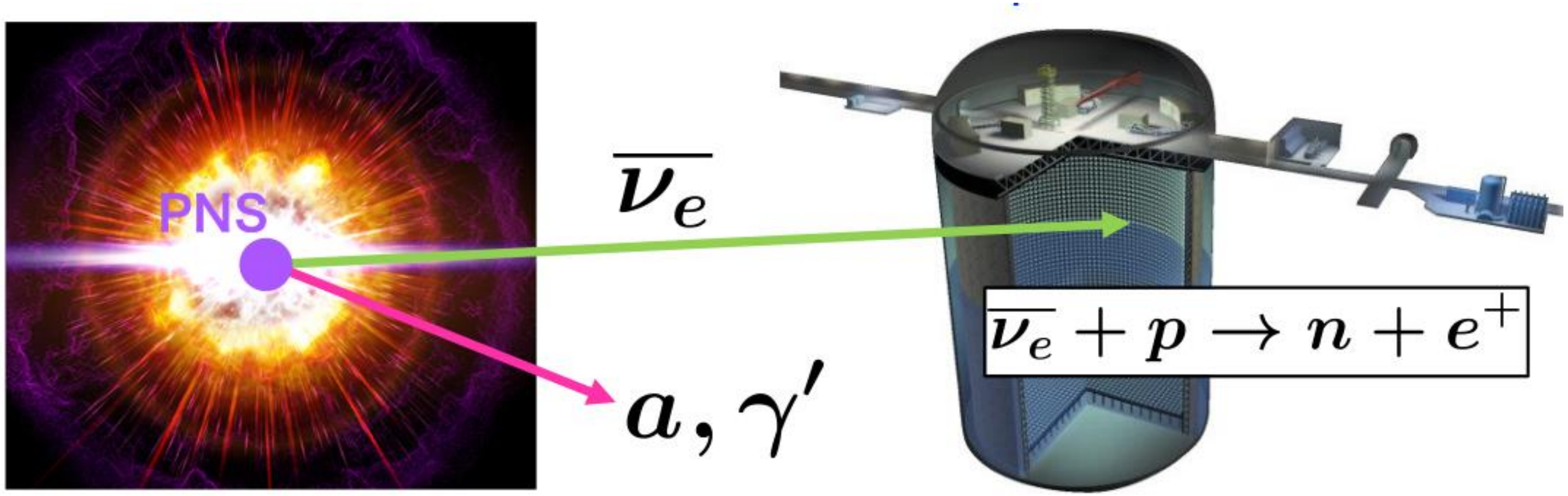


- From the few $\bar{\nu}_e p \to n e^+$ events of SN 1987A we know that
  - ~10^{53}erg emitted as neutrinos with energy $\sim O(15\mathrm{MeV})$ in ~10s

Super-Kamiokande

The energy-loss argument

- The axions can be produced copiously from hot dense celestial objects such as supernova (SN).
  - Stars can produce axions which escape, draining energy from the core

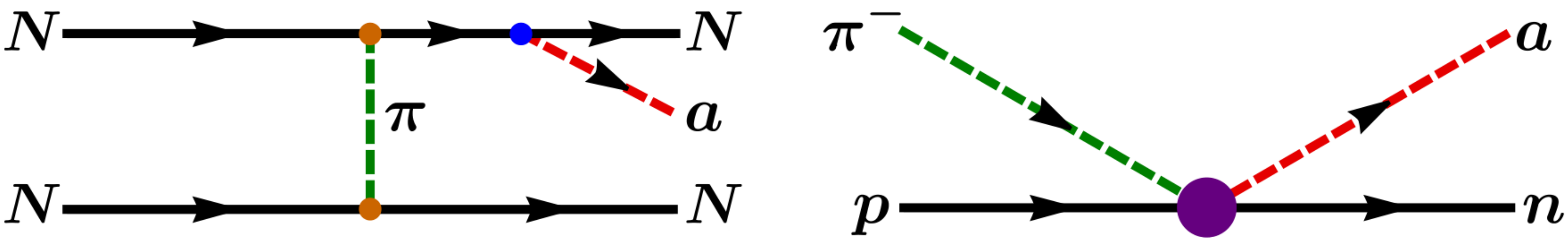


- Axions affect strongly the SN neutrino burst if $L_a > L_\nu \sim 3 \times 10^{52} \mathrm{erg/s}$
  - We can impose stringent bound from $L_a < L_\nu \sim 3 \times 10^{52} \mathrm{erg/s}$

Raffelt `90

Axion emission process from SN

- Two hadronic processes that can create axions inside SN
- **Nucleon-nucleon bremsstrahlung (NNB)** : $NN \to NNa$

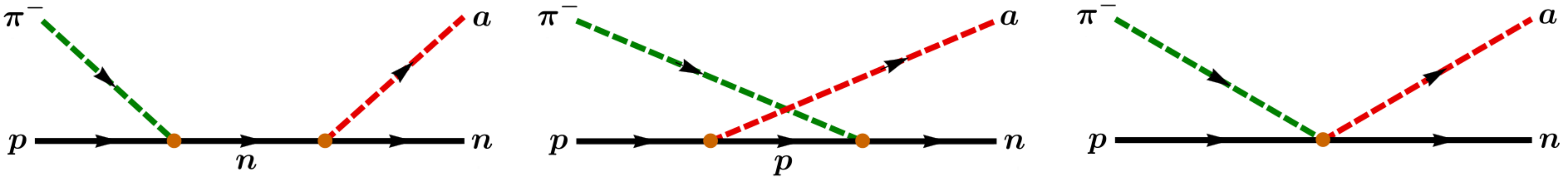


- It has been thought the NNB as the dominant axion emission for a while due to the underestimate of the n_π inside SN.
- **Pion-induced Compton-like scattering (PCS)** : $\pi^- p \to na$
- Recent studies have shown that the PCS dominates over the NNB to be the main source of the axion emission inside SN.

B. Fore and S. Reddy (2020), P. Carenza, et al. (2021), T. Fischer, et al. (2021)

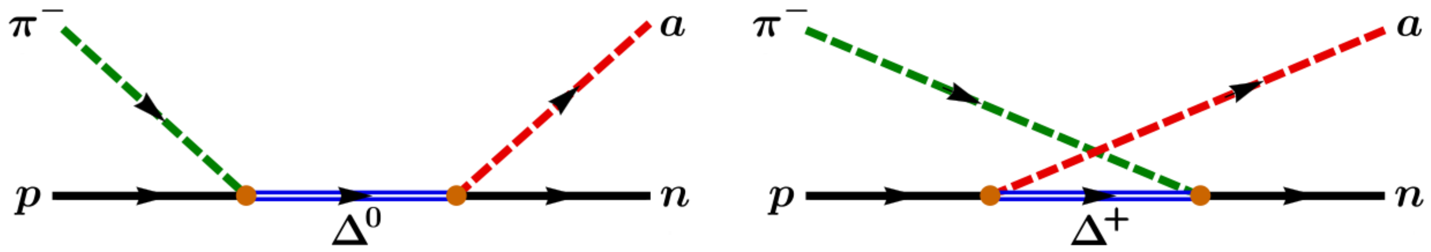
Axion emission process from SN

- We evaluate the supernova axion emission rate including the Δ resonance in the heavy baryon chiral perturbation theory



P. Carenza, B. Fore, M. Giannotti, A. Mirizzi and S. Reddy (2021) K. Choi, H. J. Kim, H. Seong & C. S. Shin (2022)

$|\boldsymbol{k}_\pi| \simeq m_\pi \ll m_p$



In our work

- For T_{SN}~30MeV, $|k_\pi| \approx \sqrt{3m_\pi T_{SN}} \approx m_\pi, \quad E_\pi$~180MeV
  - The $m_{\pi^- p}$ is somewhere in the middle of Δ and N masses.

Heavy Baryon Chiral Perturbation Theory

Jenkins & Manohar `91

- In this formalism, the nucleon is almost on-shell with a nearly unchanged velocity when it exchanges some tiny momentum with the pion

$$p_N^\mu = m_N v^\mu + \delta k_\pi^\mu \qquad v^2 = 1 \qquad m_N/\Lambda_\chi \sim 1 \qquad \delta k_\pi^\mu/\Lambda_\chi \ll 1$$

$$\Lambda_\chi \sim 1\,\mathrm{GeV}$$

- Velocity-dependence baryon field

$$\mathcal{B}_v(x) = e^{i m_B v\cdot x}\mathcal{B}(x) \longrightarrow \overline{\mathcal{B}}(i\partial\!\!\!/ - m_B)\mathcal{B} \to \overline{\mathcal{B}_v} i\partial\!\!\!/ \mathcal{B}_v$$

- The power counting expansion of the effective field theory for pions and baryons can be systematic and well-behaved.
- The algebra of the spin operator formalism can be much simpler than that of the gamma matrix formalism.

Heavy Baryon Chiral Perturbation Theory

- Interactions of meson octet, baryon octet and baryon decuplet

$$\mathcal{L}_{\pi BT} = -i\,\overline{(\mathcal{T}_v^\mu)}_{ijk} v^\rho \mathcal{D}_\rho (\mathcal{T}_{v\mu})_{ijk} + \Delta m_{TB} \overline{(\mathcal{T}_v^\mu)}_{ijk} (\mathcal{T}_{v\mu})_{ijk} + \mathcal{C}\epsilon_{ijk}\left[\overline{(\mathcal{T}_v^\mu)}_{i\ell m} (\mathcal{A}_\mu)_{\ell j} (\mathcal{B}_v)_{mk} + \overline{(\mathcal{B}_v)}_{km} (\mathcal{A}_\mu)_{j\ell} (\mathcal{T}_{v\mu})_{i\ell m}\right] + \cdots$$

- Spin-3/2 Rarita-Schwinger field: $(\mathcal{T}_v^\mu)_{ijk}$

- Under the $SU(3)_L \otimes SU(3)_R$ symmetry, $(\mathcal{T}_v^\mu)_{ijk} \to (\mathcal{U}_H)_{i\ell} (\mathcal{U}_H)_{jm} (\mathcal{U}_H)_{kn} (\mathcal{T}_v^\mu)_{lmn}$

- Representation of the Delta baryon: $(\mathcal{T}_{v\mu})_{112} = \frac{1}{\sqrt{3}} \Delta_{v\mu}^+ \,, \quad (\mathcal{T}_{v\mu})_{122} = \frac{1}{\sqrt{3}} \Delta_{v\mu}^0$

$$\longrightarrow \quad \mathcal{L}_{\pi N\Delta} = \frac{\mathcal{C}}{\sqrt{6} f_\pi}\left(\overline{n_v} \Delta_{v\mu}^+ \partial^\mu \pi^- + \overline{\Delta_{v\mu}^+} n_v \partial^\mu \pi^+ - \overline{p_v} \Delta_{v\mu}^0 \partial^\mu \pi^+ - \overline{\Delta_{v\mu}^0} p_v \partial^\mu \pi^-\right)$$

pion-nucleon-delta interaction

The QCD axion Lagrangian

- $v_{\text{PQ,EW}} \gg T \gg \Lambda_{QCD}$ & at the leading order in a/f_a

$$\mathcal{L}_{aqg} = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{g_s^2}{32\pi^2}\frac{a}{f_a} G^c_{\mu\nu}\widetilde{G}^{c\mu\nu} + \overline{q} i \gamma^\mu \partial_\mu q - \left(\overline{q_L}\mathcal{M}_q q_R + \text{h.c.}\right) + \frac{\partial_\mu a}{2 f_a}\overline{q}\gamma^\mu\gamma^5 \mathcal{X}_q q$$

- Chiral transformation: $q \to \exp\left(-i\gamma^5 \frac{a}{2f_a}\mathcal{Q}_a\right) q \;,\; \langle \mathcal{Q}_a \rangle = 1$

$$\mathcal{L}_{aq} = \frac{1}{2}\partial_\mu a \partial^\mu a + \overline{q} i \gamma^\mu \partial_\mu q + \left\langle \mathcal{M}_a q_R \overline{q_L} + \mathcal{M}_a^\dagger q_L \overline{q_R} \right\rangle + \frac{\partial_\mu a}{f_a}\left\langle (\mathcal{X}_q + \mathcal{Q}_a)\hat{t}^A \right\rangle \mathcal{J}_q^{A\mu}$$

- Next step is to replace the conserved quark currents with the conserved hadron currents in the HBChPT.

The QCD axion Lagrangian

- Axion couplings to pions and baryons: $\mathcal{J}_q^{A\mu} \sim \mathcal{J}_{\text{hadron}}^{A\mu}$

$$\frac{\partial_\mu a}{f_a}\left\langle \left(\mathcal{X}_q + \mathcal{Q}_a\right)\hat{t}^A\right\rangle \mathcal{J}_q^{A\mu} \longrightarrow \mathcal{L}_{a\pi BT} = \frac{\partial_\mu a}{f_a}\left\langle \left(\mathcal{X}_q + \mathcal{Q}_a\right) t^A\right\rangle \mathcal{J}_{\pi BT}^{A\mu}$$

$$\epsilon_{ijk}\overline{\left(\mathcal{T}_v^\mu\right)}_{ijm}\left(\mathcal{B}_v\right)_{mk} = 0$$

- Axion couplings to pions, nucleons:

$$\mathcal{L}_{a\pi N} = \frac{\partial_\mu a}{f_a}\left[C_{ap}\overline{p_v}S_v^\mu p_v + C_{an}\overline{n_v}S_v^\mu n_v + \frac{i}{2f_\pi}C_{a\pi N}\left(\pi^+\overline{p_v}v^\mu n_v - \pi^-\overline{n_v}v^\mu p_v\right)\right]$$

$$C_{ap} = X_u\Delta u + X_d\Delta d + X_s\Delta s + \frac{\Delta u + z\Delta d + w\Delta s}{1+z+w}$$

$$C_{an} = X_d\Delta u + X_u\Delta d + X_s\Delta s + \frac{z\Delta u + \Delta d + w\Delta s}{1+z+w}$$

$$C_{a\pi N} = \frac{1}{\sqrt{2}}\left(X_u - X_d + \frac{1-z}{1+z+w}\right) = \frac{C_{ap} - C_{an}}{\sqrt{2}g_A}$$

nucleon matrix element

$$\langle p|\overline{q}S_v^\mu q|p\rangle = s^\mu \Delta q/2$$

$$\Delta u = 0.847$$

$$\Delta d = -0.407$$

$$\Delta s = -0.035$$

The QCD axion Lagrangian

- Axion couplings to pions and baryons: $\mathcal{J}_q^{A\mu} \sim \mathcal{J}_{\text{hadron}}^{A\mu}$

$$\frac{\partial_\mu a}{f_a}\left\langle\left(\mathcal{X}_q + \mathcal{Q}_a\right)\hat{t}^A\right\rangle \mathcal{J}_q^{A\mu} \longrightarrow \mathcal{L}_{a\pi BT} = \frac{\partial_\mu a}{f_a}\left\langle\left(\mathcal{X}_q + \mathcal{Q}_a\right)t^A\right\rangle \mathcal{J}_{\pi BT}^{A\mu}$$

$$\epsilon_{ijk}\overline{(\mathcal{T}_v^\mu)_{ijm}}(\mathcal{B}_v)_{mk} = 0$$

- Axion couplings to pions, nucleons and Delta baryons:

$$\mathcal{L}_{aN\Delta} = \frac{\partial_\mu a}{2f_a}\left[C_{ap\Delta}\left(\overline{p_v}\Delta_\mu^+ + \overline{\Delta_\mu^+}p_v\right) + C_{an\Delta}\left(\overline{n_v}\Delta_\mu^0 + \overline{\Delta_\mu^0}n_v\right)\right]$$

$$C_{ap\Delta} = C_{an\Delta} \equiv C_{aN\Delta} = -\frac{\mathcal{C}}{\sqrt{3}}\left(X_u - X_d + \frac{1-z}{1+z+w}\right) = -\frac{\sqrt{3}}{2}\left(C_{ap} - C_{an}\right)$$

- Notice that $C_{a\pi N}$ and $C_{a\Delta N}$ are not independent parameters because they can be expressed in terms of $C_{ap} - C_{an}$

The QCD axion Lagrangian

- Numerically, we obtain

$$C_{ap} = \begin{cases} +0.430 & \text{KSVZ model} \\ +0.712 - 0.430\sin^2\beta & \text{DFSZ model} \end{cases}$$

$$C_{an} = \begin{cases} +0.002 & \text{KSVZ model} \\ -0.134 + 0.406\sin^2\beta & \text{DFSZ model} \end{cases}$$

$$C_{a\pi N} = \begin{cases} +0.241 & \text{KSVZ model} \\ +0.477 - 0.471\sin^2\beta & \text{DFSZ model} \end{cases}$$

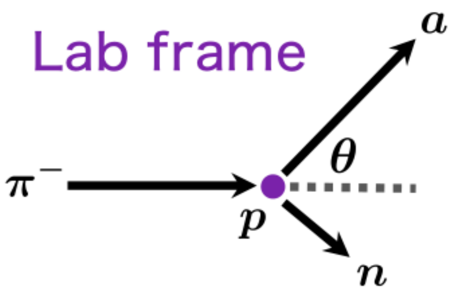
$$C_{aN\Delta} = \begin{cases} -0.370 & \text{KSVZ model} \\ -0.732 + 0.724\sin^2\beta & \text{DFSZ model} \end{cases}$$

Heavy Baryon Chiral Perturbation Theory

- Squared matrix element

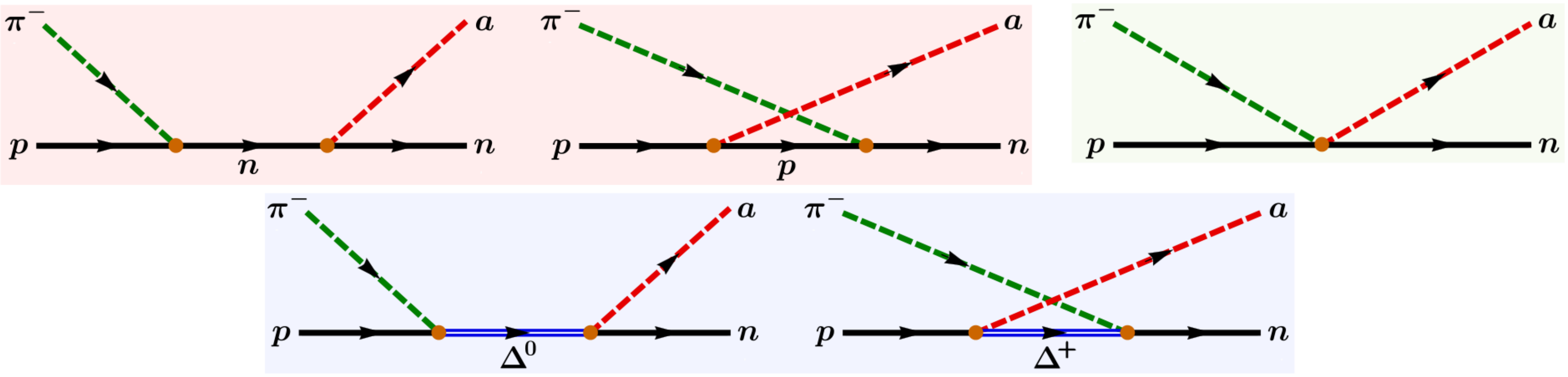
$$\overline{\left|\mathcal{M}_{\pi^- p\to na}\right|^2} = \frac{2m_N^2}{f_\pi^2 f_a^2}\langle P_+\Omega^\dagger P_+\Omega\rangle$$

$$P_+ = \mathrm{diag}(1,1,0,0)$$
$$\Theta = \mathrm{diag}\left(e^{+i\theta}, e^{-i\theta}, e^{+i\theta}, e^{-i\theta}\right)$$



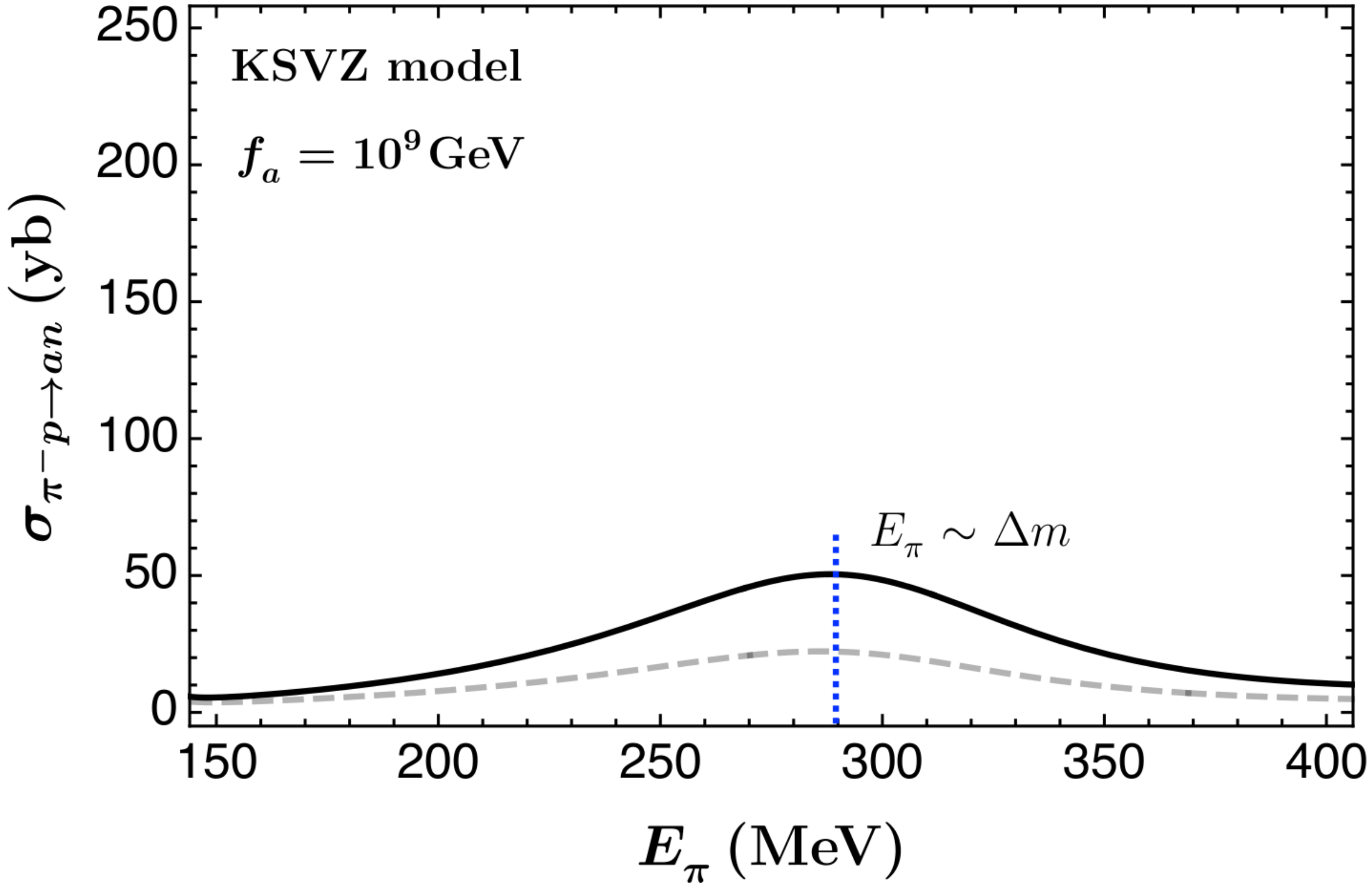
$$\Omega = \frac{\sqrt{2}g_A|\boldsymbol{k}_\pi||\boldsymbol{k}_a|}{4E_\pi}\left(C_{ap}\Theta - C_{an}\Theta^\dagger\right) + \frac{C_{a\pi N}|\boldsymbol{k}_a|}{2}\mathbb{I}_{4\times 4}$$
$$+\frac{\mathcal{C}|\boldsymbol{k}_\pi||\boldsymbol{k}_a|}{6\sqrt{6}}\left[\frac{C_{an\Delta}\left(3\cos\theta\,\mathbb{I}_{4\times4}-\Theta^\dagger\right)}{E_\pi-\Delta m+i\Gamma_\Delta/2}+\frac{C_{ap\Delta}\left(3\cos\theta\,\mathbb{I}_{4\times4}-\Theta\right)}{E_\pi+\Delta m-i\Gamma_\Delta/2}\right]$$

$$\Delta m = m_\Delta - m_N \simeq 293\,\mathrm{MeV}$$
$$\Gamma_\Delta \simeq 117\,\mathrm{MeV}$$

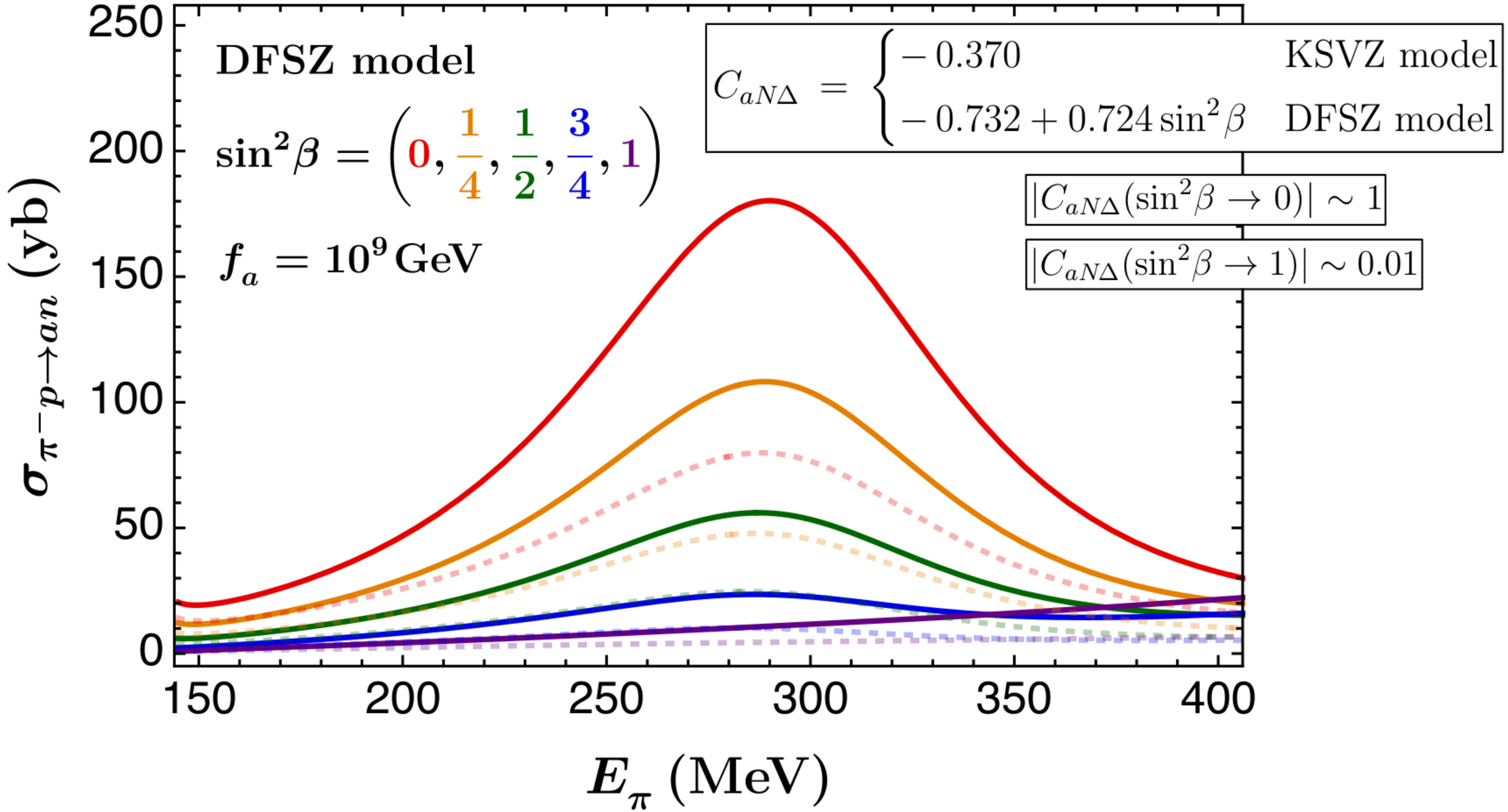


Scattering cross section vs E_π

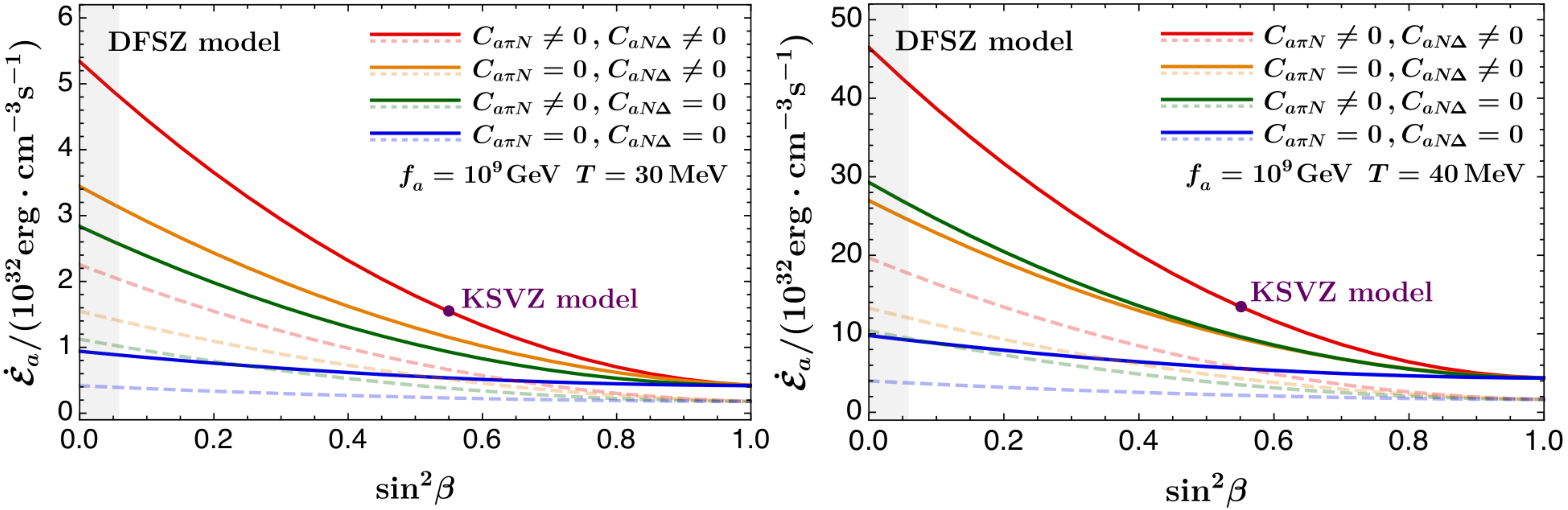
-



Scattering cross section vs E_π

- 

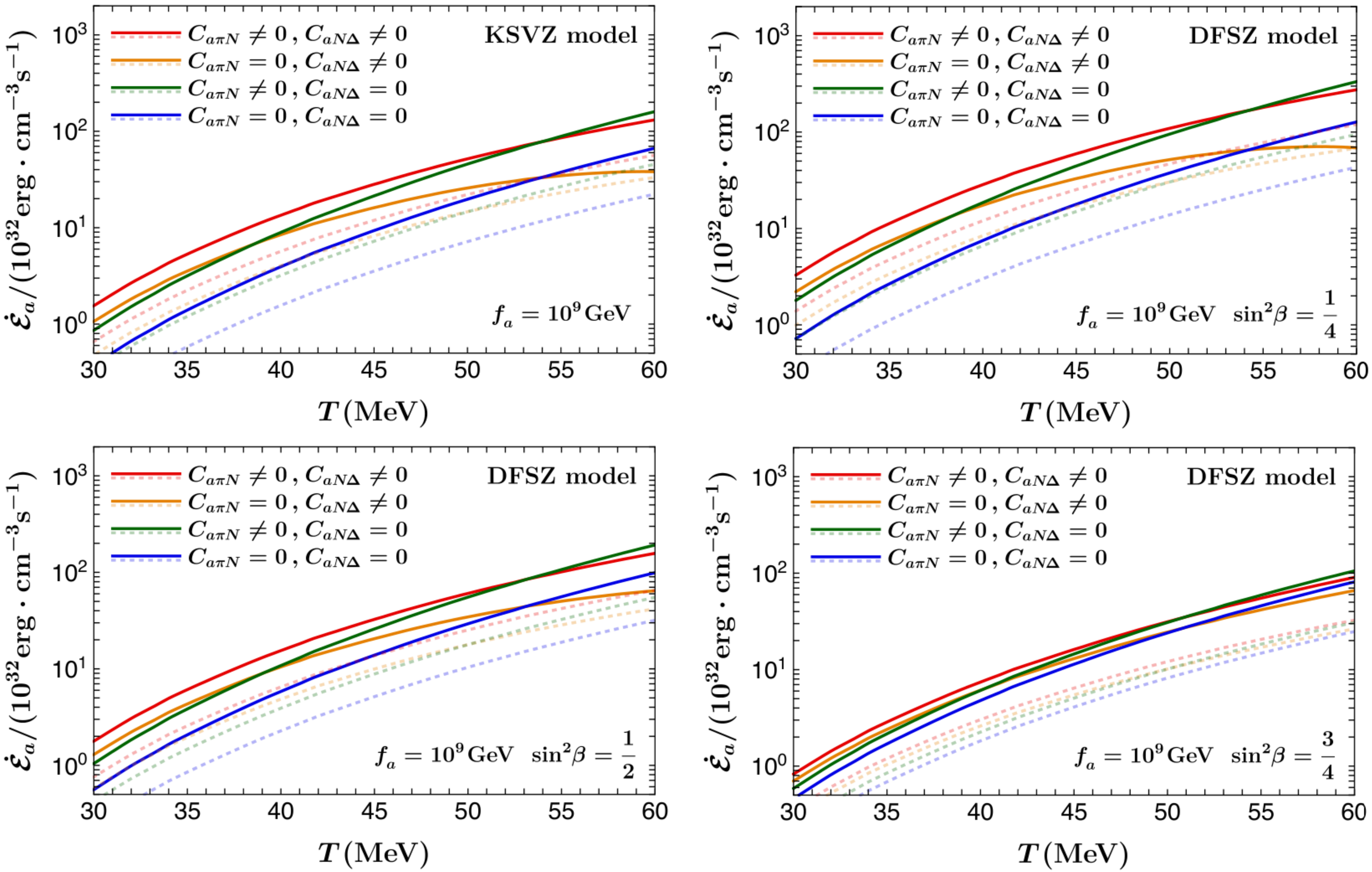
Supernova axion emissivity vs $\sin^2\beta$



- The gray region is ruled out by tree-level Unitarity of fermion scattering
- Supernova axion emissivity can be enhanced at most by a factor of 5 for $\sin\beta \to 0$ compared to the earlier studies

Supernova axion emissivity vs Temperature

-



Conclusion

- We have estimated the supernova axion emissivity with the $\Delta(1232)$ resonance in the HBChPT.

- We have noticed that the supernova axion emissivity was overestimated by in DFSZ and KSVZ models.

- We have shown that the supernova axion emissivity can be enhanced by a factor of ~4 in the KSVZ model and up to a factor of ~5 in the DFSZ model with compared to the case without the $C_{a\pi N}$ and $C_{aN\Delta}$.

- We have found that the Δ resonance can give a destructive contribution to the supernova axion emissivity at high T.