Symmetries of large N QCD

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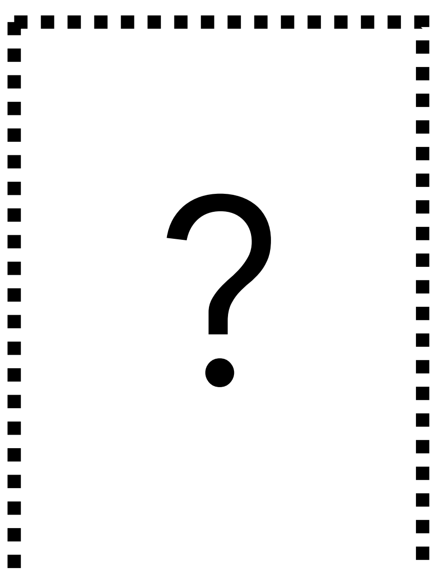
Collaborators:



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The large N limit of QCD is interesting!

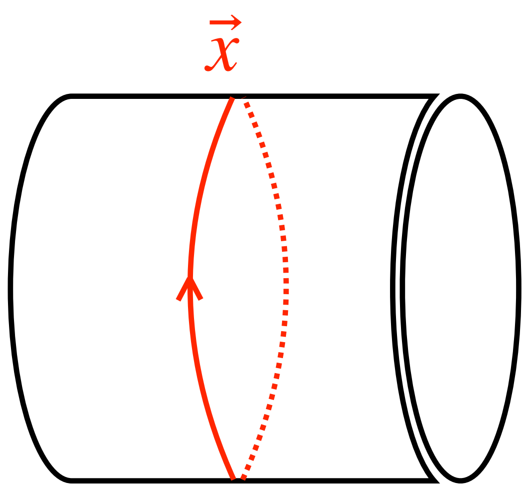
- QCD is always weakly coupled at short distances.
- For any notion of weak coupling at **large** distances, need an 't Hooft large N limit.
    - Number of colors $N \to \infty$ with N_F, Λ_{QCD}, $\lambda = g_{YM}^2 N$, m_q, etc held fixed.
    - As $N \to \infty$, mesons and glueballs become free.
- Physically $N = 3$, but the large N limit gives many qualitative (+ some quantitative) insights into physics of real QCD.
- Improved understanding large N QCD $\Rightarrow$ improved understanding of real QCD!

The question

- Do extra symmetries emerge as $N \to \infty$ in QCD?
  - Sure: $U(1)_A$ symmetry, $SU(2N_f)$ spin-flavor symmetry, …
  - This talk: does large N QCD inherit a 1-form `center' symmetry from large N pure YM theory?
- Why should you care about the answer:
  - Symmetries ⇒ correlation function selection rules.
  - Symmetries ⇒ various labels for phases of a QFT.
  - Symmetries ⇒ anomalies ⇒ `selection rules' on phases of a QFT

SU(N) pure Yang-Mills

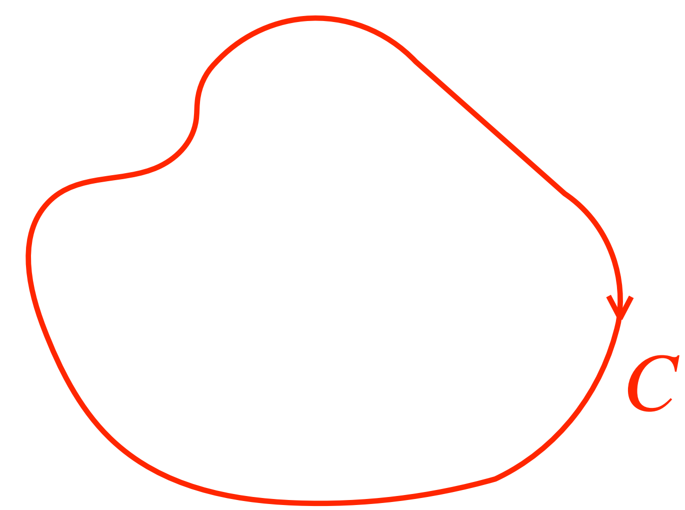
- Wilson line operators in pure YM obey **selection rules**:



$$\left\langle \frac{1}{N} \mathrm{tr}_F \, P(\vec{x}) \right\rangle = 0$$

in any finite volume

Quark free energy diverges



$$\lim_{A(C)\to\infty} \left\langle \frac{1}{N} \mathrm{tr}_F \, W(C) \right\rangle = 0$$

in any scheme

Linear quark-anti-quark potential

- These selection rules encode confinement!

$\mathbb{Z}_N$ 1-form symmetry in YM theory

- Selection rules follow from global symmetries!
- Symmetries that natively act on line operators — such as Wilson loops — are called ``1-form'' symmetries.
  - In QM, symmetry really means having **symmetry generators**.
  - In pure YM theory, Wilson loop selection rules follow from a $\mathbb{Z}_N$ 1-form symmetry.
  - Package of N test quarks isn't confined ⇒``N'' in $\mathbb{Z}_N$
- $\mathbb{Z}_N$ 1-form symmetry is a generalization of the 1980s idea of $\mathbb{Z}_N$ center symmetry.

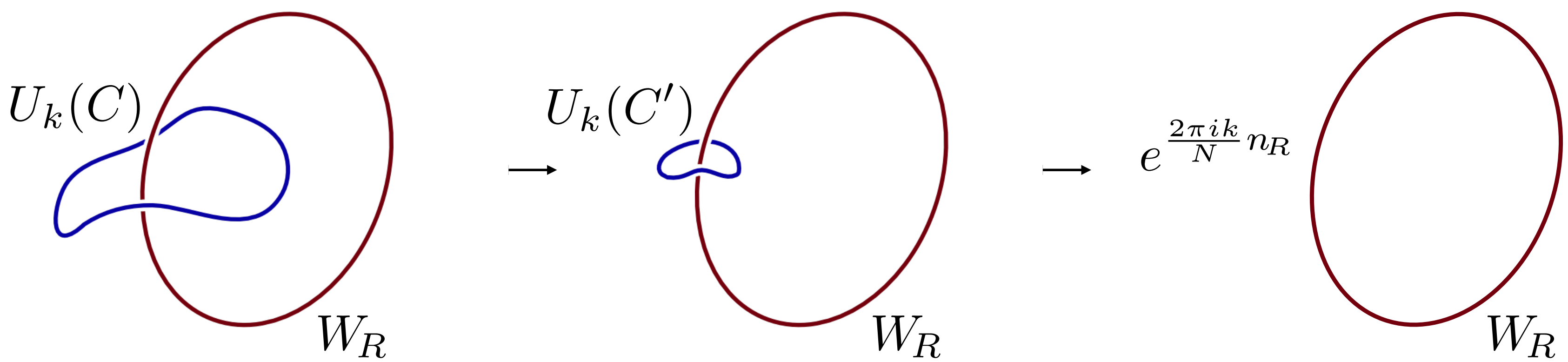
$\mathbb{Z}_N$ 1-form symmetry in YM theory

Gaiotto, Kapustin, Seiberg, Willett 2014

Symmetry generators are N co-dimension-2 topological operators U_k:

$$U_k(M_{d-2})U_n(M_{d-2}) = U_{k+n \bmod N}(M_{d-2})$$

$$\langle U_k(M_{d-2})W_F(C)\rangle = e^{2\pi i \frac{k}{N}\mathrm{Link}(C,M_{d-2})}\langle W_F(C)\rangle$$

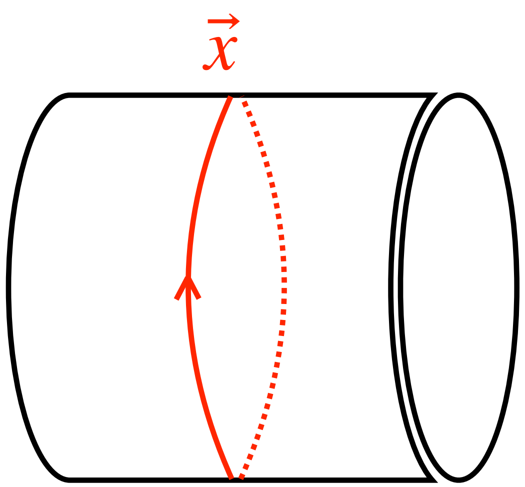


- U_k's are surface operators in 4d, line operators in 3d, and local operators in 2d.
- Invariance under deformation of M_{d-2} $\Leftrightarrow$ charge conservation!

SU(N) QCD at large N

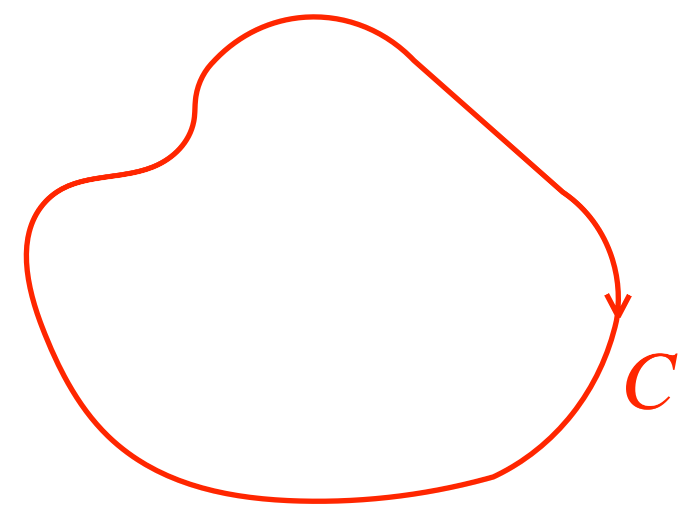
With $N_F \sim \mathcal{O}(1)$ fundamental-rep quark fields, quark loops are suppressed at large N.

- Large N QCD obeys the same selection rules as pure YM:



$$\left\langle \frac{1}{N} \mathrm{tr}_F \, P(\vec{x}) \right\rangle = 0$$

in any finite volume



$$\lim_{A(C)\to\infty} \left\langle \frac{1}{N} \mathrm{tr}_F \, W(C) \right\rangle = 0$$

in any scheme

Confinement in large N QCD

Do Wilson loop selection rules of large N QCD follow from a symmetry?

- Wide-spread belief/guess: at large N, there's a ``$\mathbb{Z}_N$'' 1-form symmetry which explains the selection rules, just as in YM theory.
  - For experts: Here ``$\mathbb{Z}_N$'' = $\mathbb{Q}/\mathbb{Z}$, and not e.g. $\mathbb{Z}$
- Rather bizarrely, this guess is wrong. 🤯
  - Rest of the talk will explain why.

Obstructions to 1-form symmetry

- Two basic issues:
  - Existence of open Wilson lines in large N QCD.
  - Large N quark loop suppression isn't universal.
- Today I'll explain the issue with quark loop suppression in a simple example: 2d scalar QCD on the lattice.
  - Conclusions hold much more generally.

We find that there are no non-trivial topological co-dimension-2 operators in large N QCD with an action on Wilson loops.

⇒ Large N QCD does not have a 1-form \`center' symmetry!

Strategy

- A 1-form symmetry in large N QCD is expected because the symmetry generators U_k and Wilson loop operators are **gluonic color-singlet operators**, which are expected to enjoy quark loop suppression.
- I'll explain calculation of the simplest correlator of U_k.
- The non-trivial U_k at large N have $k \sim \mathcal{O}(N)$. (Otherwise $U_k \to 1$ at large N.)
- Quark loop suppression fails for U_k with $k \sim N$!

2d scalar QCD

- I'll explain how things work in 2d scalar SU(N) QCD on the lattice:

$$Z = \prod_{\ell} \int du_\ell \prod_x \int d\phi_x d\phi_x^\dagger \prod_p e^{-s_{\mathrm{YM}}(u_p)} \prod_\ell e^{-s_{\mathrm{H}}(\phi_x^\dagger u_\ell \phi_{x'})} \prod_x e^{-m^2 \phi_x^\dagger \phi_x}$$

p = plaquettes $\qquad \ell$ = links $\qquad$ x = sites

- Rich enough to share the key features of real QCD, but easy to study explicitly.
- Results go through if we add fermions, take $d \geq 2$, take continuum limit etc, but I'll keep it simple for this talk.

2d scalar QCD

$$Z = \prod_{\ell} \int du_{\ell} \prod_{x} \int d\phi_x d\phi_x^{\dagger} \prod_{p} e^{-s_{\mathrm{YM}}(u_p)} \prod_{\ell} e^{-s_{\mathrm{H}}(\phi_x^{\dagger} u_{\ell} \phi_{x'})} \prod_{x} e^{-m^2 \phi_x^{\dagger} \phi_x}$$

- We used the 'heat kernel' action for s_{YM}:

$$e^{-s_{\mathrm{YM}}(u_p)} = \sum_{\alpha} d_{\alpha} \chi_{\alpha}(u_p) \, e^{-g^2 c_{\alpha} A_p}$$

Migdal 1975,
Menotti+Onofri, 1981
Witten 1992

- For pure YM get continuum results even on coarse lattices
- Equivalent to standard Wilson action + infinite series of improvement terms.
- The hopping term (scalar kinetic term) is

$$s_{\mathrm{H}}(\phi_x^{\dagger} u_{\ell} \phi_{x'}) = -\kappa \, \phi_x^{\dagger} u_{\ell} \phi_{x'} + \mathrm{h.c.}$$

Hopping expansion

- For small κ, matter field integral = sum over Wilson loop insertions $W_F(\Gamma)$ (~quark world-lines). Schematically:

$$\langle \mathcal{O}_{\rm glue} \rangle = \langle \mathcal{O}_{\rm glue} \rangle_0 + \sum_{\text{loops } \Gamma} \left(\frac{\kappa}{m^2} \right)^{L_\Gamma} \langle \mathcal{O}_{\rm glue} \, W_F[\Gamma] \rangle_0 + \cdots$$

- Physically, $\kappa/m^2 \sim 1/(m^2 a^2)$.

- Here we'll focus on $\mathcal{O}_{\rm glue} = W_F(C)$ and $U_k(x)$.

Wilson loop in pure YM

- At 0th order in the hopping expansion, Wilson loop expectation value is, of course, same as in pure YM:

$$\frac{1}{N}\langle W_{\mathrm{F}}(C)\rangle = e^{-\sigma A[C]}\left\{1 + O(\kappa^4)\right\},\ \sigma = \tfrac{1}{2}\lambda + O(1/N)$$

- Area law behavior ⇔ 2d pure YM confines. ✔

Wilson loop in 2d lattice QCD

- First corrections appear at κ^4:

$$\frac{1}{N}\langle W_{\mathrm{F}}(C)\rangle = \frac{1}{N}\langle W_{\mathrm{F}}(C)\rangle_0 \left\{ 1 + A[C]\left(\frac{\kappa}{m^2}\right)^4 \frac{2}{N}\sinh\left(\frac{\lambda}{2}\right) + O(\kappa^6) \right\}$$

- κ^4 term = quark loop, 1/N suppressed as expected.
- Working to higher order, we find a perimeter-law piece:

$$\frac{1}{N}\langle W_{\mathrm{F}}(C)\rangle = e^{-\sigma A[C]} + \frac{1}{N}e^{-\mu L[C]} + \cdots, \;\; \mu = \log m^2/\kappa$$

- If $N \to \infty$ with loop geometry fixed, confinement!
- 2d QCD has everything needed for our puzzle.

1-form symmetry generators $U_k(x)$

- In continuum, $U_k(x)$ viewed as ``defect/Gukov-Witten'' operator.
- On lattice, `thin center-vortex' definition is nicer:

$$U_k(\tilde{x} = \star p) = \exp\left[s_{\rm YM}(u_p) - s_{\rm YM}(\omega^{-k} u_p)\right] \qquad \omega = e^{2\pi i/N}$$

- Can you see why $U_k(\tilde{x})$ is trivial at large N? 😄
- A short calculation can verify that

$$\langle U_k(x) W(C) U_{k'}(x') \rangle_{\rm YM}$$

only cares whether x, x' are inside C or not.

- In YM one finds $\langle U_k(x) \rangle_{\rm YM} = 1$.
  - Experts: yes, could be a phase, but I don't want to talk about universes!
- What happens in QCD at large N?

Expectation value of $U_k(\tilde{x})$

- Approach: shut up and calculate! To $\mathcal{O}(\kappa^8)$, we get

$$\langle U_k(\tilde{x})\rangle = 1 - \left(\frac{\kappa}{m^2}\right)^4 2N\, e^{-g^2 c_{\mathrm{F}}} \left(1 - \cos\left(\frac{2\pi k}{N}\right)\right) + \frac{1}{2}\left[\left(\frac{\kappa}{m^2}\right)^4 2N\, e^{-g^2 c_{\mathrm{F}}} \left(1 - \cos\left(\frac{2\pi k}{N}\right)\right)\right]^2 + \cdots$$

- $c_F \sim N$ is the quadratic Casimir.
- We then proved that this exponentiates:

$$\langle U_k(\tilde{x})\rangle = \exp\left[-\left(\frac{\kappa}{m^2}\right)^4 2N\, e^{-g^2 c_{\mathrm{F}}} \left(1 - \cos\left(\frac{2\pi k}{N}\right)\right) + O(\kappa^6)\right]$$

Expectation value of $U_k(\tilde{x})$

- So $\langle U_k(\tilde{x})\rangle \sim e^{-N}$ for $k \sim N$, and

$$\langle U_k(\tilde{x})\rangle = 0$$

- In YM, result was $\langle U_k(x)\rangle_{\mathrm{YM}} = 1$, so the correction is $\mathcal{O}(1)$!
- How?
    - Things like large N factorization, quark loop suppression hold for `light' operators, which e.g. aren't built from $O(N)$ fields, while $U_k \sim U_1 U_1 \cdots U_1$
    - We need $k \sim N$ to make U_k non-trivial at large N.
    - But once $k \sim N$, normal large N counting **fails**.
- So the $U_k(\tilde{x})$'s don't generate a 1-form symmetry.

Fate of $\mathbb{Z}_N$ 1-form symmetry in large N QCD

- Symmetry generators of pure YM don't survive in large N QCD.
  - Fundamental matter contributions are not suppressed!
- Can one somehow **modify** the $U_k's$ so that they do survive?
  - No, due to the existence of open Wilson lines in large N QCD, which are inconsistent with existence of topological codimension-2 operators.
  - Feel free to ask me about it later!

Outlook

- Wilson loops in large N QCD obey the same selection rules as pure YM.
- But there is no 1-form symmetry at large N. 🤯
    - Could there be generalization of "generalized global symmetry" to explain large N QCD selection rules?
    - If no, we'd have to accept selection rules without symmetries. Seems very strange…
- While I think our results are right, I want them to be wrong.
    - Please push hard on everything I said! 🤓 🤜

Thank you for listening!

Backup: exponentiation of $\langle U_k(\tilde{x})\rangle$

- The heat kernel action on each plaquette can be written as

$$\sum_\alpha d_\alpha \chi_\alpha(u_p)\, e^{-g^2 c_\alpha} = \exp\left[\sum_\alpha \mathrm{Re}\frac{1}{g_\alpha^2}\chi_\alpha(u_p)\right]$$

- Inserting $U_k(\tilde{x})$ = changing weight of one plaquette $p = \star\,\tilde{x}$:

$$g_\alpha^2 \to e^{\frac{2\pi i k}{N} n_\alpha} g_\alpha^2$$

n_α = N-ality

- Clustering arguments then imply

$$\tilde{Z}(g^2, k) = e^{-A\tilde{F}(k)} = e^{-A(F+\frac{1}{A}f(k))} \Rightarrow \langle U_k(\tilde{x})\rangle = e^{-f(k)}$$

where f(k) has a nice 1/N expansion, akin to free energy F

Backup: rescaling

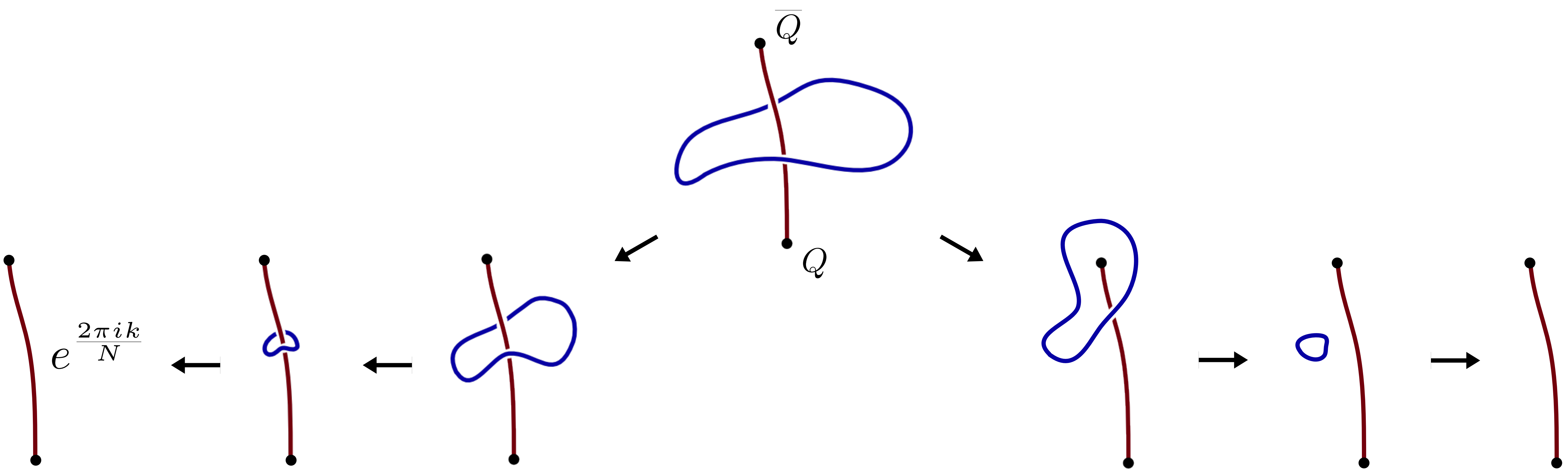
- We could try defining $V_k(x) \equiv \dfrac{U_k(x)}{|\langle U_k(x)\rangle|}$, which is forced to have a unit VEV both in YM and in large N QCD.
  - Immediate conflict with $\mathbb{Z}_N$ fusion rule is avoided.
- But at large N these operators are very singular. Correlators don't satisfy cluster decomposition:
  - $\langle V_k(x)^\dagger V_k(0)\rangle$ with $k \sim N$ diverges for any separation $r \sim N^0$, only decays once $r \gtrsim \sqrt{N}$.
- Can't interpret $V_k(x)$ as generators of a $\mathbb{Z}_N$ 1-form symmetry.

Backup: Endability Issue

Rudelius, Shao 2020

Large N QCD has open Wilson lines: $\left\langle \frac{1}{N}\mathrm{tr}\, \bar{Q}(x) e^{i\int_x^{x'} a} Q(x') \right\rangle = \mathcal{O}(1)$

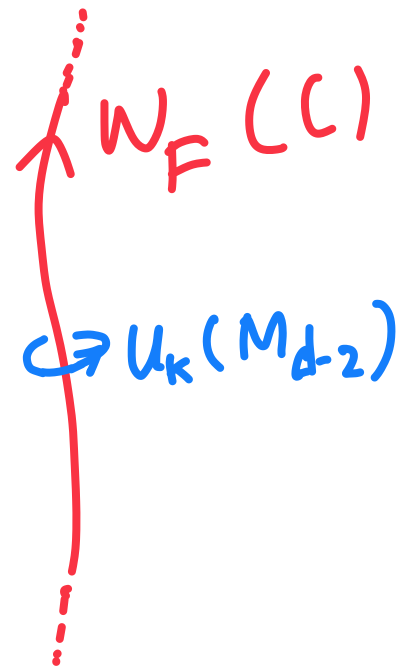
- Suppose it has topological $U_k(M_{d-2})$ operators. Then



- This is inconsistent, so $U_k(M_{d-2})$ cannot be topological operators in large N QCD.

Backup: closed versus open Wilson lines

- Given the assumption that $U_k(M_{d-2})$ is topological, its action on a Wilson line on a curve C can be calculated by "shrinking":



- Data can be obtained from an infinitesimal neighborhood of C - no info on whether C is open or closed!
- So failure of topological property on open Wilson lines implies failure for closed Wilson loops.