

Light-cone distribution amplitudes of a light baryon in large-momentum effective theory

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ABSTRACT: Momentum distributions of quarks/gluons inside a light baryon in a hard exclusive process are encoded in the light-cone distribution amplitudes (LCDAs). In this work, we point out that the leading twist LCDAs of a light baryon can be obtained through a simulation of a quasi-distribution amplitude calculable on lattice QCD within the framework of the large-momentum effective theory. We calculate the one-loop perturbative contributions to LCDA and quasi-distribution amplitudes and explicitly demonstrate the factorization of quasi-distribution amplitudes at the one-loop level. Based on the perturbative results, we derive the matching kernel in the $\overline{\text{MS}}$ scheme and regularization-invariant momentum-subtraction scheme. Our result provides a first step to obtaining the LCDA from first principle lattice QCD calculations in the future.

KEYWORDS: Effective Field Theories of QCD, Parton Distributions, Factorization, Renormalization Group, Higher-Order Perturbative Calculations

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Contents

1	Introduction	1
2	LCDA at one loop level	2
3	Quasi-DA at one loop level	5
4	Off-shell results	10
5	Matching kernel	15
6	Summary	17
A	Gauge invariance in LCDAs	17
B	Projection and trace formulae	18
C	Expansion by regions and factorization of quasi-DA at one-loop	19

1 Introduction

Light-cone distribution amplitudes (LCDAs) of a light baryon describe the momentum distributions of a quark/gluon in a baryonic system and are a fundamental non-perturbative input in QCD factorization for an exclusive process with a large momentum transfer. An explicit example of this type is weak decays of bottom baryons which are valuable to extract the CKM matrix element in the standard model [1] and to probe new physics beyond the standard model [2]. In addition, in contrast with parton distribution functions that encode the probability density of parton momenta in hadrons, the LCDAs offer a probability amplitude description of the partonic structure of hadrons, from which one can potentially calculate various quark/gluon distributions. Thus the knowledge of LCDAs is also key to understanding the internal structure of light baryons, such as a proton.

Though many progresses have been made in obtaining the LCDAs of a nucleon in the past decades [3–16], most of the available analyses are limited to the few lowest moments of LCDAs. Due to the lack of a complete knowledge of baryon LCDAs, many phenomenological analyses adopt model parameterizations resulting in uncontrollable errors in theoretical predictions for decay branching fractions of heavy baryons [17–19]. Thus, it is highly indispensable to develop a method to calculate the full shape of baryon LCDAs from the first principle of QCD.

Since LCDAs are defined as the correlation functions of lightcone operators inside a hadron, these quantities can not be directly evaluated on the lattice. In 2013, a very

inspiring approach was proposed to circumvent this problem and is now formulated as the large-momentum effective theory (LaMET) [20, 21]. In LaMET, instead of directly calculating light-cone correlations, one can start from equal-time correlations in a large-momentum hadron state, which are known as quasi-distributions. The quasi-distributions share the same infrared properties with lightcone distributions and are connected to PDFs and LCDAs via a matching scheme. Under the framework of LaMET, encouraging results are recently obtained on the lattice and for recent reviews please see refs. [22–24] and many references therein. Based on this approach, results on LCDAs of light mesons can be found in refs. [25–30]. Other methods to extract lightcone PDFs and LCDAs can also be found in refs. [31–34].

In this work, we aim to provide an exploration of the leading twist lightcone distribution amplitude of a light baryon in LaMET. Taking the Λ baryon as an example, we first calculate the one-loop perturbative QCD contributions to LCDAs and quasi-DAs of a light baryon. We demonstrate that these two quantities have the same infrared structure which explicitly validates the factorization at the one-loop level. We also provide an analysis based on expansion by region, which gives direct proof. Based on the one-loop results, we derive the matching kernel. To regularize remnant UV divergences, we also give the matching results in a regularization-invariant momentum-subtraction scheme. Future improvements in lattice realization will be briefly mentioned in the end.

The rest of this paper is organized as follows. In section 2, we present a brief review of the twist-2 LCDAs of a light baryon and the one-loop perturbative results. In section 3, we calculate the contributions to the quasi-DA in the modified minimal subtraction scheme. In section 4, we calculate the one-loop contributions to quasi-DA with the off-shell external states with a RI/MOM subtraction. In section 5, we give the one-loop matching coefficients from quasi-DA to LCDA. A summary is presented in section 6. Some details are provided in the appendix.

2 LCDA at one loop level

In the factorization analysis of heavy-to-light baryonic transition, one is led at leading-twist to the matrix element of a three-quark operator between the vacuum and the baryon state. Taking the Λ baryon which is made of uds as an example, one can see that the LCDA is defined by the non-local light-ray operators

$$\epsilon_{ijk} \langle 0 | u_i^T(t_1 n) \Gamma d_j(t_2 n) s_k(0) | \Lambda \rangle, \tag{2.1}$$

with i, j, k being color indices. T denotes the transpose in the spinor space. Under the assignment of \bar{n} as the light quark flight direction, the three light quarks are separated in the n direction in coordinate space. The two lightcone unit vectors are defined as $n^\mu = (1, 0, 0, -1)/\sqrt{2}$ and $\bar{n}^\mu = (1, 0, 0, 1)/\sqrt{2}$. The covariant derivative is $D_\mu = \partial_\mu - igA_\mu$.

Two pieces of gauge links are not shown in the above formulae

$$\mathcal{W}_{ij}(0, x) = \mathcal{P} \exp \left[ig_s \int_x^0 dt n_\mu A_{ij}^\mu(tn) \right]. \tag{2.2}$$

It is worthwhile pointing out that the above form of the Wilson line is not unique, but a gauge invariant building block, e.g. for a quark field with color i , is

$$Q_i(x) = \mathcal{W}_{ii'}(\infty, x) q_{i'}(x), \quad (2.3)$$

and the piece from 0 to ∞ is omitted in eq. (2.2) since it is irrelevant of LCDA. A proof is included in appendix A.

The collinear twist expansion makes use of the decomposition of the quark field into large and small components (see for example [35])

$$q = \left(\frac{\not{n} \bar{\not{n}}}{2} + \frac{\bar{\not{n}} \not{n}}{2} \right) q. \quad (2.4)$$

The large component is projected out by $\bar{\not{n}} \not{n}$ if quark's flight direction is chosen \bar{n} , i.e. $p^\mu = (p^z, 0, 0, p^z)$ (p is the momentum of baryon). The twist-3 LCDAs are made of three large components, and for Λ baryon one has the explicit form

$$\Phi(x_1, x_2) f_\Lambda u_\Lambda(p) = \int \frac{dt_1 p^+}{2\pi} \int \frac{dt_2 p^+}{2\pi} e^{ix_1 p^+ t_1 + ix_2 p^+ t_2} \epsilon_{ijk} \langle 0 | U_i^T(t_1 n) \Gamma D_j(t_2 n) S_k(0) | \Lambda \rangle, \quad (2.5)$$

where $\Gamma = C \gamma^5 \not{n}$, and $C = i \gamma^2 \gamma^0$. f_Λ is the decay constant for Λ , and $u_\Lambda(p)$ is the Λ spinor. The short-distance coefficient is insensitive to the hadrons, i.e. the UV behavior of LCDAs is irrelevant to the low energy dynamics. In the calculation of LCDAs, one can replace the hadron with a partonic state with the same quantum numbers.

In the following calculation, we replace the hadron state $|\Lambda\rangle$ by three constituent quarks state, i.e. $|\Lambda\rangle \rightarrow |u_a(k_1) d_b(k_2) s_c(k_3)\rangle$. Here $p = k_1 + k_2 + k_3$ is the momentum conservation condition. In this case, the leading twist LCDA is defined as

$$\begin{aligned} \phi(x_1, x_2, \mu) S &= \int \frac{dt_1 p^+}{2\pi} \int \frac{dt_2 p^+}{2\pi} e^{ix_1 p^+ t_1 + ix_2 p^+ t_2} \frac{\epsilon_{ijk} \epsilon_{abc}}{6} \\ &\times \langle 0 | U_i^T(t_1 n) \Gamma D_j(t_2 n) S_k(0) | u_a(k_1) d_b(k_2) s_c(k_3) \rangle, \end{aligned} \quad (2.6)$$

where $p^+ = n \cdot p$, $x_{i,0} = k_i^+ / p^+$, $\sum_{i=1}^3 x_i = 1$, $\sum_{i=1}^3 x_{i,0} = 1$, and all longitudinal momentum fractions carried by baryons satisfy $0 < x_{i,0} < 1$. The normalization factor S can be constructed in terms of the partonic local operator matrix element:

$$S = \frac{\epsilon_{ijk} \epsilon_{abc}}{6} \langle 0 | (U_i)^T(0) \Gamma D_j(0) S_k(0) | u_a(k_1) d_b(k_2) s_c(k_3) \rangle. \quad (2.7)$$

At tree level, we have

$$\begin{aligned} S^{(0)} &= -\frac{\epsilon_{abc} \epsilon_{abc}}{6} \left[u^{s_1}(k_1) \right]^T \Gamma u^{s_2}(k_2) u(k_3) \\ &= 2p^+ u(k_3), \end{aligned} \quad (2.8)$$

where $\left[u^{s_1}(k_1) \right]^T \Gamma u^{s_2}(k_2) = \frac{1}{2} \text{tr} [\not{p} C \gamma^5 \Gamma]$ is employed, and the superscript index ‘(0)’ refer to tree-level result. Here, the quark state is chosen to have the same J^{PC} with the Λ , and the spin average and color average are assumed in this calculation. In appendix B, we provide a detailed explanation of eq. (2.8), and the corresponding trace formalism to derive this convention.

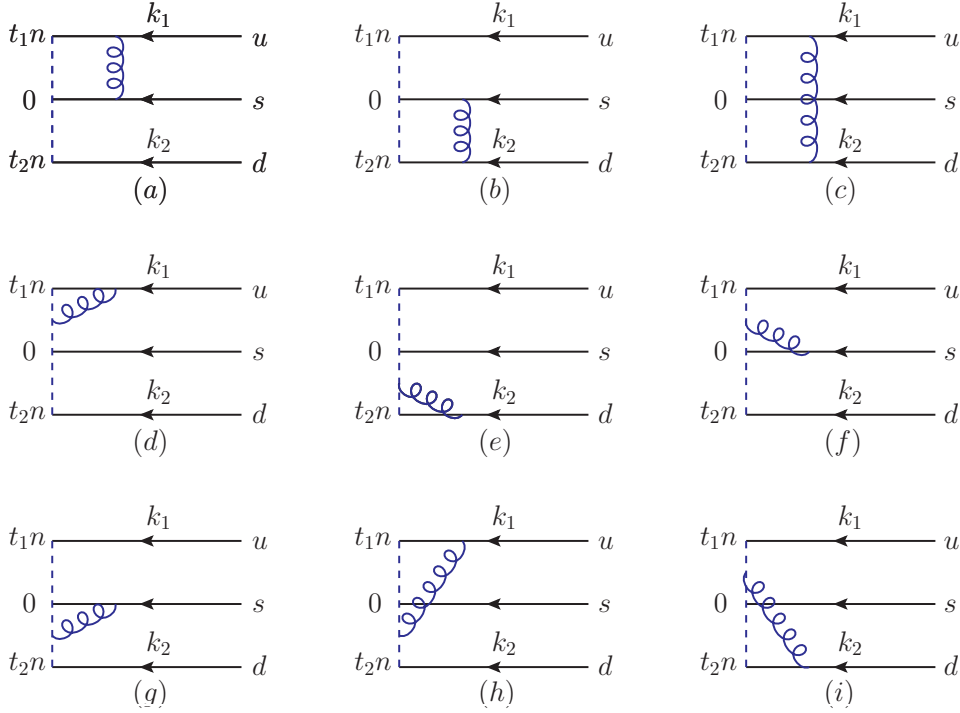


Figure 1. One-loop corrections to LCDAs of a light baryon Λ .

After a bit of algebra, we obtain the result of LCDA at the tree level

$$\begin{aligned} \phi^{(0)}(x_1, x_2, \mu) S^{(0)} &= \int \frac{dt_1 p^+}{2\pi} \int \frac{dt_2 p^+}{2\pi} e^{ip^+(x_1 t_1 + x_2 t_2)} \left[u^{s_1}(k_1) \right]^T \Gamma u^{s_2}(k_2) u(k_s) e^{-ik_1^+ t t_1} e^{-ik_2^+ t t_2} \\ &= \delta(x_1 - x_{1,0}) \delta(x_2 - x_{2,0}) S^{(0)}, \end{aligned} \quad (2.9)$$

which leads to

$$\phi^{(0)}(x_1, x_2, \mu) = \delta(x_1 - x_{1,0}) \delta(x_2 - x_{2,0}). \quad (2.10)$$

At the one-loop order, two gluons can be radiated from (1) a light quark by QCD interaction and/or (2) the Wilson-line. These five objects (three quarks and two pieces of Wilson line) give $C_5^2 = 10$ terms together with five self-energy corrections in total. The diagram from two Wilson lines is zero since $n^2 = 0$ and the rest is displayed in figure 1 (quark self-energy corrections are not shown). We choose the dimensional regularization $D = 4 - 2\epsilon$ to regularize both UV and IR divergences.

We have found that due to the cancellation of UV divergences and IR divergences, all the integrals involved in figure 1 are scaleless. After separating the different divergences and including the normalization, we have the complete result for the one-loop $\overline{\text{MS}}$ renormalized LCDA as

$$\phi(x_1, x_2, \mu) = \delta(x_1 - x_{1,0}) \delta(x_2 - x_{2,0}) - \frac{\alpha_s C_F}{8\pi} \frac{1}{\epsilon_{\text{IR}}} f(x_1, x_2, x_{1,0}, x_{2,0}), \quad (2.11)$$

with

$$\begin{aligned}
 f(x_1, x_2, x_{1,0}, x_{2,0}) = & \left\{ \frac{\delta(x_1 - x_{1,0})\theta(x_1)\theta(\bar{x}_1)\theta(x_2)\theta(\bar{x}_2)}{\bar{x}_1(x_2 - x_{2,0})} \right. \\
 & \times \left[\frac{x_3(x_2 - x_{2,0} + 2\bar{x}_1)}{x_{3,0}}\theta(x_2 - x_{2,0}) + \frac{x_2(x_2 - x_{2,0} - 2\bar{x}_1)}{x_{2,0}}\theta(x_{2,0} - x_2) \right] \\
 & + \frac{\delta(x_3 - x_{3,0})2x_1\theta(x_1)\theta(\bar{x}_1)\theta(x_2)\theta(\bar{x}_2)}{x_{1,0}} \left[\frac{\theta(x_2 - x_{2,0})}{x_1 + x_2} - \frac{\theta(x_{1,0} - x_1)}{x_1 - x_{1,0}} \right] \\
 & \left. + \{x_1 \leftrightarrow x_2, x_{1,0} \leftrightarrow x_{2,0}\} \right\}_{\oplus}, \tag{2.12}
 \end{aligned}$$

where $\bar{x} = 1 - x$. The \oplus denotes

$$[g(x_1, x_2, x_{1,0}, x_{2,0})]_{\oplus} = g(x_1, x_2, x_{1,0}, x_{2,0}) - \delta(x_1 - x_{1,0})\delta(x_2 - x_{2,0}) \times \int dy_1 dy_2 g(y_1, y_2, x_{1,0}, x_{2,0}). \tag{2.13}$$

The renormalized LCDA can be obtained by removing the UV divergence due to the renormalization of the composite operator of LCDA. The dependence of the renormalized LCDA $\Phi(x_1, x_2, \mu)$ on $\ln \mu$ can be obtained from the evolution equation

$$\frac{d\Phi(x_1, x_2, \mu)}{d \ln \mu} = \frac{\alpha_s C_F}{4\pi} \int dy_1 \int dy_2 V \Phi(y_1, y_2, \mu). \tag{2.14}$$

At the one-loop, the evolution kernel V is

$$V = f(x_1, x_2, y_1, y_2). \tag{2.15}$$

This evolution kernel V in eq. (2.14) for baryon LCDA was first studied in ref. [36] and widely applied in various processes. We should also note that the form of eq. (2.14) is similar to the Efremov-Radyushkin-Brodsky-Lepage (ERBL) evolution equation for mesons [37, 38].

3 Quasi-DA at one loop level

In this section, we will introduce an equal-time operator matrix element which is often named as quasi-distribution amplitudes [20]. The quasi-DA for the Λ is defined as

$$\tilde{\Phi}(x_1, x_2, \mu) \tilde{f}_{\Lambda} u_{\Lambda}(p) = \int \frac{dt_1 p^z}{2\pi} \int \frac{dt_2 p^z}{2\pi} e^{ix_1 p^z t_1 + ix_2 p^z t_2} \epsilon_{ijk} \langle 0 | U_i^T(t_1 n_z) \tilde{\Gamma} D_j(t_2 n_z) S_k(0) | \Lambda \rangle, \tag{3.1}$$

where \tilde{f}_{Λ} is the quasi decay constant for Λ .

In a similar way, the corresponding partonic operator matrix element is

$$\begin{aligned}
 \tilde{\phi}(x_1, x_2, \mu) \tilde{S} = & \int \frac{dt_1 p^z}{2\pi} \int \frac{dt_2 p^z}{2\pi} e^{ix_1 p^z t_1 + ix_2 p^z t_2} \frac{\epsilon_{ijk} \epsilon_{abc}}{6} \\
 & \times \langle 0 | (U_i)^T(t_1 n_z) \tilde{\Gamma} D_j(t_2 n_z) S_k(0) | u_a(k_1) d_b(k_2) s_c(k_3) \rangle. \tag{3.2}
 \end{aligned}$$

The $\tilde{\Gamma}$ is the Dirac matrix for the quasi-DA and two popular choices are $\tilde{\Gamma} = C\gamma^5 \not{n}_{\lambda}$ ($\lambda = t$ or z) for quasi-DA. The two choices will give the same results at leading twist and a

brief explanation is given in appendix C. Here $n_t^\mu = (1, 0, 0, 0)$ and $n_z^\mu = (0, 0, 0, -1)$. The corresponding normalization factor \tilde{S} is

$$\tilde{S} = \frac{\epsilon_{ijk}\epsilon_{abc}}{6} \langle 0 | (U_i)^T(0) \tilde{\Gamma} D_j(0) S_k(0) | u_a(k_1) d_b(k_2) s_c(k_3) \rangle. \quad (3.3)$$

At the tree level, we have the matrix element

$$\begin{aligned} \tilde{S}^{(0)} &= -\frac{\epsilon_{abc}\epsilon_{abc}}{6} \left[u^{s_1}(k_1) \right]^T \tilde{\Gamma} u^{s_2}(k_2) u(k_3) \\ &= 2p^z u(k_3), \end{aligned} \quad (3.4)$$

where $\left[u^{s_1}(k_1) \right]^T \tilde{\Gamma} u^{s_2}(k_2) = \frac{1}{2} \text{tr} [\not{p} C \gamma^5 \tilde{\Gamma}]$ is employed. The tree-level result for quasi-DA is

$$\begin{aligned} \tilde{\phi}^{(0)}(x_1, x_2, \mu) \tilde{S}^{(0)} &= p^{z^2} \int \frac{dt_1}{2\pi} \int \frac{dt_2}{2\pi} e^{ix_1 t_1 p^z + ix_2 t_2 p^z} \left[u^{s_1}(k_1) \right]^T \tilde{\Gamma} u^{s_2}(k_2) u(k_3) \\ &\quad \times e^{-ix_1, 0 p^z t_1} e^{-ix_2, 0 p^z t_2} \\ &= \delta(x_1 - x_{1,0}) \delta(x_2 - x_{2,0}) \tilde{S}^{(0)}. \end{aligned} \quad (3.5)$$

Therefore, the normalized quasi-DA at the tree level is

$$\tilde{\phi}^{(0)}(x_1, x_2, \mu) = \delta(x_1 - x_{1,0}) \delta(x_2 - x_{2,0}). \quad (3.6)$$

We can find that the normalized LCDA at the tree-level gives the same result as the quasi-DA. The one-loop diagrams of quasi-DA for baryon Λ are similar to that of the LCDA which are shown in figure 1, except the n direction is changed to n_z . The real diagram for quasi-DA shown in figure 1(a) can be obtained as follows:

$$\begin{aligned} \tilde{\mathcal{M}}^a &= ig^2 \frac{-C_F}{2} p^{z^2} \left(\frac{\mu^2}{e^{\ln(4\pi) - \gamma_E}} \right)^\epsilon \int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2 + i\epsilon} \\ &\quad \times \frac{1}{(k_3 - q)^2 + i\epsilon} \frac{\delta(x_1 p^z - q^z - k_1^z) \delta(x_2 p^z - k_2^z)}{(q + k_1)^2 + i\epsilon} \\ &\quad \times \left[u^{s_1}(k_1) \right]^T \gamma^\mu (\not{q} + \not{k}'_1) \tilde{\Gamma} u^{s_2}(k_2) (\not{k}'_3 - \not{q}) \gamma_\mu u(k_3), \\ &= ig^2 \frac{-C_F}{2} p^{z^2} \left(\frac{\mu^2}{e^{\ln(4\pi) - \gamma_E}} \right)^\epsilon \int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2 + i\epsilon} \\ &\quad \times \frac{1}{(k_3 - q)^2 + i\epsilon} \frac{\delta(x_1 p^z - q^z - k_1^z) \delta(x_2 p^z - k_2^z)}{(q + k_1)^2 + i\epsilon} \\ &\quad \times \left(-\frac{1}{2} \right) \text{tr} (\not{p} \gamma^5 \gamma^\mu (\not{q} + \not{k}'_1) \gamma^5 \not{k}'_\lambda) (\not{k}'_3 - \not{q}) \gamma_\mu u(k_3), \end{aligned} \quad (3.7)$$

where $\tilde{\mathcal{M}}$ denotes $\tilde{\phi}(x_1, x_2, \mu) \tilde{S}$ in short. The last line in the above equation reads

$$-\frac{1}{2} \text{tr} (\not{p} \gamma^5 \gamma^\mu (\not{q} + \not{k}'_1) \gamma^5 \not{k}'_\lambda) (\not{k}'_3 - \not{q}) \gamma_\mu u(k_3) = \left[q^2 - (p \cdot q) \frac{q^\dagger + q^z}{p^z} - q_\perp \not{k}'_\perp \not{k}'_z \right] \tilde{S}^{(0)}, \quad (3.8)$$

and the third term gives zero contribution because the integrand as in eq. (3.7) is odd in q_\perp . This result indicates the equivalence of the two Lorentz structures $\tilde{\Gamma}$.

Finally, the quasi-DA in figure 1(a) can be simplified as

$$\begin{aligned}
 \tilde{\mathcal{M}}^a &= \frac{\alpha_s C_F}{8\pi} \tilde{S}^{(0)} \delta(x_2 - x_{2,0}) \\
 &\times \begin{cases} \left[\frac{x_1 \ln \frac{-x_1}{x_3}}{(1-x_{2,0})x_{3,0}} - \frac{x_1 \ln \frac{-x_1}{x_{1,0}x_{3,0}}}{x_{1,0}x_{3,0}} - \frac{\ln \frac{x_{1,0}-x_1}{x_3}}{x_{3,0}} \right], & x_1 < 0 \\ \left[\frac{x_1 \left(\ln \frac{(x_{1,0}-x_1)x_1}{\mu^2/(2p^z)^2} - 1 \right)}{x_{1,0}x_{3,0}} - \frac{x_1 \left(\ln \frac{x_1 x_3}{\mu^2/(2p^z)^2} - 1 \right)}{(1-x_{2,0})x_{3,0}} - \frac{\ln \frac{x_{1,0}-x_1}{x_3}}{x_{3,0}} - \frac{x_1}{x_{1,0}(1-x_{2,0})} \frac{1}{\epsilon_{\text{IR}}} \right], & 0 < x_1 < x_{1,0} \\ \left[\frac{x_1 \ln \frac{x_1}{x_{1-x_{1,0}}}}{x_{1,0}x_{3,0}} - \frac{x_1 \left(\ln \frac{x_1 x_3}{\mu^2/(2p^z)^2} - 1 \right)}{(1-x_{2,0})x_{3,0}} + \frac{\ln \frac{(x_1-x_{1,0})x_3}{\mu^2/(2p^z)^2} - 1}{x_{3,0}} - \frac{x_3}{x_{3,0}(1-x_{2,0})} \frac{1}{\epsilon_{\text{IR}}} \right], & x_{1,0} < x_1 < 1-x_2 \\ \left[\frac{x_1 \ln \frac{x_1}{x_{1-x_{1,0}}}}{x_{1,0}x_{3,0}} + \frac{x_1 \ln \frac{-x_3}{x_1}}{(1-x_{2,0})x_{3,0}} + \frac{\ln \frac{x_1-x_{1,0}}{-x_3}}{x_{3,0}} \right], & x_1 > 1-x_2 \end{cases}
 \end{aligned} \tag{3.9}$$

For the remaining diagrams, rather than enumerating the calculations in detail, we directly give their results

$$\tilde{\mathcal{M}}^b = \tilde{\mathcal{M}}^a|_{x_2 \leftrightarrow x_1, x_{2,0} \leftrightarrow x_{1,0}}, \tag{3.10}$$

$$\begin{aligned}
 \tilde{\mathcal{M}}^c &= \frac{\alpha_s C_F}{4\pi} \tilde{S}^{(0)} \delta(x_1 + x_2 - x_{1,0} - x_{2,0}) \\
 &\times \begin{cases} \left[\frac{x_1}{x_{1,0}(x_{1,0}+x_{2,0})} \ln \frac{x_2}{-x_1} + \frac{x_{2,0}-x_2}{x_{1,0}x_{2,0}} \ln \frac{x_2-x_{2,0}}{x_2} \right], & x_1 < 0 \\ \left\{ \frac{x_1 \left[\ln \left(\frac{x_1 x_2}{\mu^2/(2p^z)^2} \right) + 1 \right]}{x_{1,0}(x_{1,0}+x_{2,0})} + \frac{(x_2-x_{2,0}) \ln \frac{x_2}{x_2-x_{2,0}}}{x_{1,0}x_{2,0}} - \frac{x_1}{x_{1,0}(x_{1,0}+x_{2,0})} \frac{1}{\epsilon_{\text{IR}}} \right\}, & 0 < x_1 < x_{1,0} \\ \left\{ \frac{x_1 \left[\ln \left(\frac{x_1 x_2}{\mu^2/(2p^z)^2} \right) + 1 \right]}{x_{1,0}(x_{1,0}+x_{2,0})} + \frac{(x_2-x_{2,0}) \left[\ln \frac{x_2(x_{2,0}-x_2)}{\mu^2/(2p^z)^2} + 1 \right]}{x_{1,0}x_{2,0}} - \frac{x_2}{x_{2,0}(x_{1,0}+x_{2,0})} \frac{1}{\epsilon_{\text{IR}}} \right\}, & x_{1,0} < x_1 < x_1+x_2 \\ \left[\frac{x_1}{x_{1,0}(x_{1,0}+x_{2,0})} \ln \frac{x_1}{-x_2} + \frac{x_{2,0}-x_2}{x_{1,0}x_{2,0}} \ln \frac{-x_2}{x_{2,0}-x_2} \right], & x_1 > x_1+x_2 \end{cases}
 \end{aligned} \tag{3.11}$$

$$\begin{aligned}
 \tilde{\mathcal{M}}^d &= \frac{\alpha_s C_F}{4\pi} \tilde{S}^{(0)} \\
 &\times \left\{ \begin{aligned} &\left\{ \delta(x_2 - x_{2,0}) \left[\frac{1}{x_{1,0} - x_1} - \frac{2x_1 \ln \frac{-x_1}{x_{1,0} - x_1}}{x_{1,0}(x_{1,0} - x_1)} \right] \right\}_{\oplus}, & x_1 < 0 \\ &\left\{ \delta(x_2 - x_{2,0}) \left[\frac{1}{x_{1,0} - x_1} - \frac{2x_1 \left[\ln \frac{(x_{1,0} - x_1)x_1}{\mu^2/(2p^z)^2} - 1 \right]}{x_{1,0}(x_1 - x_{1,0})} + \frac{2x_1}{x_{1,0}(x_1 - x_{1,0})} \frac{1}{\epsilon_{\text{IR}}} \right] \right\}_{\oplus}, & 0 < x_1 < x_{1,0} \\ &\left\{ \delta(x_2 - x_{2,0}) \left[\frac{1}{x_1 - x_{1,0}} - \frac{2x_1 \ln \frac{x_1}{x_1 - x_{1,0}}}{x_{1,0}(x_1 - x_{1,0})} \right] \right\}_{\oplus}, & x_1 > x_{1,0} \end{aligned} \right. ,
 \end{aligned} \tag{3.12}$$

$$\tilde{\mathcal{M}}^e = \tilde{\mathcal{M}}^d|_{x_2 \leftrightarrow x_1, x_{2,0} \leftrightarrow x_{1,0}} \tag{3.13}$$

$$\begin{aligned}
 \tilde{\mathcal{M}}^f &= \frac{\alpha_s C_F}{8\pi} \tilde{S}^{(0)} \\
 &\times \left\{ \begin{aligned} &\left\{ \delta(x_2 - x_{2,0}) \left[\frac{1}{x_{1,0} - x_1} + \frac{2x_3 \ln \frac{x_{1,0} - x_1}{x_3}}{x_{3,0}(x_{1,0} - x_1)} \right] \right\}_{\oplus}, & x_1 < x_{1,0} \\ &\left\{ \delta(x_2 - x_{2,0}) \left[\frac{1}{x_1 - x_{1,0}} + \frac{2x_3 \left[\ln \frac{(x_1 - x_{1,0})x_3}{\mu^2/(2p^z)^2} - 1 \right]}{x_{3,0}(x_1 - x_{1,0})} - \frac{2x_3}{x_{3,0}(x_1 - x_{1,0})} \frac{1}{\epsilon_{\text{IR}}} \right] \right\}_{\oplus}, & x_{1,0} < x_1 < 1 - x_2 \\ &\left\{ \delta(x_2 - x_{2,0}) \left[\frac{1}{x_1 - x_{1,0}} + \frac{2x_3 \ln \frac{x_1 - x_{1,0}}{-x_3}}{x_{3,0}(x_1 - x_{1,0})} \right] \right\}_{\oplus}, & x_1 > 1 - x_2 \end{aligned} \right. ,
 \end{aligned} \tag{3.14}$$

$$\tilde{\mathcal{M}}^g = \tilde{\mathcal{M}}^f|_{x_2 \leftrightarrow x_1, x_{2,0} \leftrightarrow x_{1,0}} \tag{3.15}$$

$$\begin{aligned}
 \tilde{\mathcal{M}}^h &= \frac{\alpha_s C_F}{8\pi} \tilde{S}^{(0)} \left[\delta(x_3 - x_{3,0}) - \delta(x_2 - x_{2,0}) \right] \\
 &\times \left\{ \begin{aligned} &\left[\frac{1}{x_{1,0} - x_1} + \frac{2x_1 \ln \frac{x_{1,0} - x_1}{-x_1}}{x_{1,0}(x_{1,0} - x_1)} \right], & x_1 < 0 \\ &\left\{ \frac{1}{x_{1,0} - x_1} - \frac{2x_1 \left[\ln \frac{(x_{1,0} - x_1)x_1}{\mu^2/(2p^z)^2} - 1 \right]}{x_{1,0}(x_1 - x_{1,0})} + \frac{2x_1}{x_{1,0}(x_1 - x_{1,0})} \frac{1}{\epsilon_{\text{IR}}} \right\}, & 0 < x_1 < x_{1,0} \\ &\left[\frac{1}{x_1 - x_{1,0}} + \frac{2x_1 \ln \frac{x_1 - x_{1,0}}{x_1}}{x_{1,0}(x_1 - x_{1,0})} \right], & x_1 > x_{1,0} \end{aligned} \right. ,
 \end{aligned} \tag{3.16}$$

$$\tilde{\mathcal{M}}^i = \tilde{\phi}^h|_{x_2 \leftrightarrow x_1, x_{2,0} \leftrightarrow x_{1,0}} \tag{3.17}$$

In addition, since $n_z^2 \neq 0$, the self-energy diagram of the Wilson line of the quasi-DA also contributes. These two Wilson lines give three terms of one-loop self-energy corrections

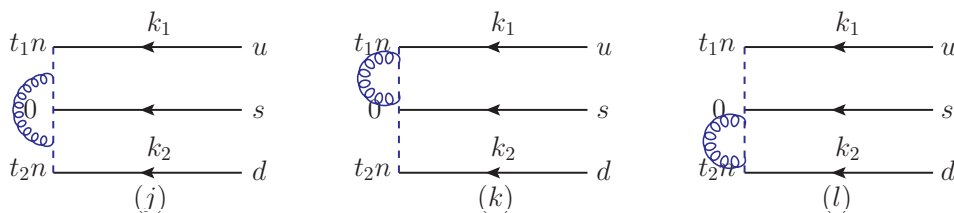


Figure 2. One loop self-energy corrections for quasi-DA.

which are shown in figure 2. Those three self-energy reads

$$\begin{aligned}
 \tilde{\mathcal{M}}^j &= \frac{\alpha_s C_F}{4\pi} \tilde{S}^{(0)} \left[\frac{\delta(x_2 - x_{2,0})}{\sqrt{(x_1 - x_{1,0})^2}} + \frac{\delta(x_1 - x_{1,0})}{\sqrt{(x_2 - x_{2,0})^2}} - \frac{\delta(x_3 - x_{3,0})}{\sqrt{(x_1 - x_{1,0})^2}} \right]_{\oplus}, \\
 \tilde{\mathcal{M}}^k &= -\frac{\alpha_s C_F}{2\pi} \tilde{S}^{(0)} \left[\frac{\delta(x_2 - x_{2,0})}{\sqrt{(x_1 - x_{1,0})^2}} \right]_{\oplus}, \\
 \tilde{\mathcal{M}}^l &= -\frac{\alpha_s C_F}{2\pi} \tilde{S}^{(0)} \left[\frac{\delta(x_1 - x_{1,0})}{\sqrt{(x_2 - x_{2,0})^2}} \right]_{\oplus}.
 \end{aligned} \tag{3.18}$$

According to eq. (3.2)–(3.3), after adding up all the results of one-loop quasi-DA diagrams, we find the normalized quasi-DA as

$$\begin{aligned}
 \tilde{\phi}(x_1, x_2, \mu) &= \delta(x_1 - x_{1,0}) \delta(x_2 - x_{2,0}) \\
 &+ \frac{\alpha_s C_F}{8\pi} \left\{ \left[g_2 \delta(x_2 - x_{2,0}) + g_3 \delta(x_3 - x_{3,0}) + \{x_2 \leftrightarrow x_1, x_{2,0} \leftrightarrow x_{1,0}\} \right] \right\}_{\oplus},
 \end{aligned} \tag{3.19}$$

with

$$g_2 = \begin{cases} \frac{(x_{1,0} + x_1)(x_{3,0} + x_3) \ln\left(-\frac{x_{3,0} - x_3}{x_3}\right) - x_1(2x_{1,0} + x_{3,0} + x_3) \ln\left(-\frac{x_1}{x_3}\right)}{x_{1,0} x_{3,0} (x_{3,0} - x_3)} - \frac{x_1(2x_{1,0} + x_{3,0} + x_3) \ln\left(-\frac{x_1}{x_3}\right)}{x_{1,0} (x_{3,0} - x_3) (x_{1,0} + x_{3,0})}, & x_1 < 0 \\ \frac{x_1(-x_{1,0} + 2x_{2,0} + x_1 - 2)}{(x_1 - x_{1,0}) x_{1,0} (x_{2,0} - 1) \epsilon_{\text{IR}}} + \frac{2x_1 \ln\left(\frac{4x_1(x_3 - x_{3,0})(p^z)^2}{\mu^2}\right)}{x_{1,0} (x_3 - x_{3,0})} + \frac{x_1 \ln\left(\frac{4x_1 x_3 (p^z)^2}{\mu^2}\right)}{x_{1,0} (x_{1,0} + x_{3,0})} \\ + \frac{x_1(-3x_{1,0} - 2x_{3,0} + x_1)}{x_{1,0} (x_3 - x_{3,0}) (x_{1,0} + x_{3,0})} - \frac{((x_3 - x_{3,0})^2 - 2x_3 x_{1,0}) \ln\left(\frac{x_3 - x_{3,0}}{x_3}\right)}{x_{1,0} (x_3 - x_{3,0}) x_{3,0}}, & 0 < x_1 < x_{1,0} \\ \frac{x_3(-x_{1,0} - 2x_{2,0} + x_1 + 2)}{(x_1 - x_{1,0}) (x_{2,0} - 1) x_{3,0} \epsilon_{\text{IR}}} + \frac{2x_3 \ln\left(\frac{4x_3(x_1 - x_{1,0})(p^z)^2}{\mu^2}\right)}{(x_1 - x_{1,0}) x_{3,0}} + \frac{x_3 \ln\left(\frac{4x_1 x_3 (p^z)^2}{\mu^2}\right)}{x_{3,0} (x_{1,0} + x_{3,0})} \\ + \frac{x_3(-2x_{1,0} - 3x_{3,0} + x_3)}{(x_1 - x_{1,0}) x_{3,0} (x_{1,0} + x_{3,0})} - \frac{((x_1 - x_{1,0})^2 - 2x_1 x_{3,0}) \ln\left(\frac{x_1 - x_{1,0}}{x_1}\right)}{(x_1 - x_{1,0}) x_{1,0} x_{3,0}}, & x_{1,0} < x_1 < x_{1,0} + x_{3,0} \\ \frac{(x_{1,0} + x_1)(x_{3,0} + x_3) \ln\left(-\frac{x_{3,0} - x_3}{x_3}\right) - x_1(2x_{1,0} + x_{3,0} + x_3) \ln\left(-\frac{x_1}{x_3}\right)}{x_{1,0} x_{3,0} (x_{3,0} - x_3)} - \frac{x_1(2x_{1,0} + x_{3,0} + x_3) \ln\left(-\frac{x_1}{x_3}\right)}{x_{1,0} (x_{3,0} - x_3) (x_{1,0} + x_{3,0})}, & x_1 > x_{1,0} + x_{3,0} \end{cases}$$

$$g_3 = \begin{cases} \frac{(x_{1,0}x_{2,0}+x_1x_2)\ln\left(\frac{x_2-x_{2,0}}{x_2}\right)}{x_{1,0}(x_2-x_{2,0})x_{2,0}} - \frac{x_1(x_{1,0}+x_2)\ln\left(-\frac{x_1}{x_2}\right)}{x_{1,0}(x_2-x_{2,0})(x_{1,0}+x_{2,0})}, & x_1 < 0 \\ \frac{2x_1(x_{1,0}+x_2)}{(x_1+x_2)(x_1-x_{1,0})x_{1,0}\in\mathbb{R}} + \frac{x_1\ln\left(\frac{4x_1(x_2-x_{2,0})(p^z)^2}{\mu^2}\right)}{x_{1,0}(x_2-x_{2,0})} + \frac{x_1\ln\left(\frac{4x_1x_2(p^z)^2}{\mu^2}\right)}{x_{1,0}(x_{1,0}+x_{2,0})} \\ + \frac{1}{x_1-x_{1,0}} + \frac{2x_1+x_2}{x_{1,0}(x_{1,0}+x_{2,0})} + \frac{(x_{1,0}(x_{2,0}+x_1)-x_1^2)\ln\left(\frac{x_2-x_{2,0}}{x_2}\right)}{x_{1,0}(x_2-x_{2,0})x_{2,0}}, & 0 < x_1 < x_{1,0} \\ \frac{2x_2(x_{2,0}+x_1)}{(x_1+x_2)(x_2-x_{2,0})x_{2,0}\in\mathbb{R}} + \frac{x_2\ln\left(\frac{4x_2(x_1-x_{1,0})(p^z)^2}{\mu^2}\right)}{(x_1-x_{1,0})x_{2,0}} + \frac{x_2\ln\left(\frac{4x_1x_2(p^z)^2}{\mu^2}\right)}{x_{2,0}(x_{1,0}+x_{2,0})} \\ + \frac{1}{x_2-x_{2,0}} + \frac{x_1+2x_2}{x_{2,0}(x_{1,0}+x_{2,0})} + \frac{((x_{1,0}+x_2)x_{2,0}-x_2^2)\ln\left(\frac{x_1-x_{1,0}}{x_1}\right)}{(x_1-x_{1,0})x_{1,0}x_{2,0}}, & x_{1,0} < x_1 < x_{1,0}+x_{2,0} \\ \frac{(x_{1,0}x_{2,0}+x_1x_2)\ln\left(\frac{x_1-x_{1,0}}{x_1}\right)}{(x_1-x_{1,0})x_{1,0}x_{2,0}} - \frac{x_2(x_{2,0}+x_1)\ln\left(-\frac{x_2}{x_1}\right)}{(x_1-x_{1,0})x_{2,0}(x_{1,0}+x_{2,0})}, & x_1 > x_{1,0}+x_{2,0}. \end{cases}$$

There is no divergence for $x_1(x_2) < 0$. The expressions of the quasi-DA $\tilde{\phi}(x_1, x_2, \mu)$ differ by a minus sign between interval $x_1(x_2) < 0$ and interval $x_1(x_2) > 1-x_2(x_1)$. The above results have infrared divergence in the other two intervals. We can find that this infrared divergence is consistent with the infrared divergence in LCDA eq. (2.11), which validate the factorization assumption at one-loop order. A direct demonstration using expansion by region is shown in appendix C.

4 Off-shell results

Compared to the continuum space, the renormalization of lattice operators is a necessary ingredient to obtain physical results from numerical simulations. In the literature, it has been noticed that a regularization invariant momentum subtraction method (RI/MOM) [39] can avoid the use of lattice perturbation theory and allow a non-perturbative determination of the renormalization constants of many composite operators [40–48]. In the following we also provide an analysis of the baryon distribution amplitudes in this scheme.

We calculate the quasi-DA in the space-like $p^2 = -\rho(p^z)^2 < 0$ kinematics region. Figure 1(a) gives

$$\begin{aligned} \tilde{\mathcal{M}}_{\text{OF}}^a &= ig^2 \frac{-C_F}{2} p^z{}^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2+i\epsilon} \frac{1}{(k_3-q)^2+i\epsilon} \frac{\delta(x_1p^z-q^z-k_1^z)\delta(x_2p^z-k_2^z)}{(q+k_1)^2+i\epsilon} \\ &\quad \times \left(-\frac{1}{2}\right) \text{tr}(\not{p}\gamma^5\gamma^\mu(\not{q}+k_1')\gamma^5\gamma^z)(k_3'-\not{q})\gamma_\mu u(k_3). \end{aligned} \quad (4.1)$$

A subtle issue for the off-shell matrix elements is that there are multiple projection ways, and here we adopt a strategy called the minimal projection [48]. Namely, we use the trace formulae technique, and all kinds of Lorentz structures above the spinor part can be projected out:

$$\left(-\frac{1}{2}\right) \text{tr}(\not{p}\gamma^5\gamma^\mu(\not{q}+k_1')\gamma^5\gamma^z)(k_3'-\not{q})\gamma_\mu u(k_3) = (a_1+a_2\not{t}_t\not{t}_z+a_3\not{t}_\perp\not{t}_z+a_4\not{t}_\perp\not{t}_t)\tilde{S}, \quad (4.2)$$

where

$$\begin{aligned}
 a_1 &= x_{1,0} \left((p^z)^2 \rho (x_1 - x_{1,0}) + 2p \cdot q \right) + (p^z)^2 x_1 \rho x_{3,0} + q^2 \\
 a_2 &= \frac{p \cdot q (p^0 x_{3,0} - q^0)}{p^z} - p^z \left\{ x_{1,0} [q^0 (1 - \rho) + p^0 (\rho + 1) x_{3,0}] + x_1 (q^0 - p^0 x_{3,0}) + p^0 x_{1,0}^2 + q^0 x_{3,0} - p^0 x_1^2 \right\} \\
 a_3 &= \frac{p \cdot q (q_\perp - p_\perp x_{3,0})}{p^z} \\
 &\quad + p^z \left\{ x_{1,0} (p_\perp x_{1,0} + q_\perp (1 - \rho)) + x_{3,0} [q_\perp + p_\perp (1 + \rho) x_{1,0}] + x_1 (q_\perp - p_\perp x_{3,0}) - p_\perp x_1^2 \right\} \\
 a_4 &= (x_{1,0} + x_{3,0} + x_1) (p_\perp q^0 - p^0 q_\perp). \tag{4.3}
 \end{aligned}$$

In the on-shell limit, the third and the last term a_3, a_4 disappear after integrating out the momentum q , and the product $\not{p}_t \not{p}_z$ goes to a unit matrix. Therefore, the summation $a_1 + a_2$ captures all terms that lead to UV divergences in the on-shell limit. This is similar to figure 1(b,f,g). The corresponding results are as follows

$$\begin{aligned}
 \tilde{\mathcal{M}}_{\text{OF}}^a &= \frac{\delta(x_2 - x_{2,0}) \alpha_s C_F}{64\pi(1-\rho)^{3/2} x_{1,0} x_{3,0} (x_{1,0} + x_{3,0}) p^z} \tilde{S}^{(0)} \\
 &\times \left\{ \begin{aligned}
 &\ln \left(\frac{((\sqrt{1-\rho}+1)(x_{1,0}+x_{3,0})-2x_1)^2}{((\sqrt{1-\rho}-1)(x_{1,0}+x_{3,0})+2x_1)^2} \right) (\rho(x_{1,0}+x_{3,0})^2 p^0 - 4(\rho-1)x_1(x_{1,0}+x_{3,0})p^z - 4x_1^2 p^0) \\
 &+ \ln \left(\frac{((\sqrt{1-\rho}+1)x_{3,0}+2x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{3,0}-2x_{1,0}+2x_1)^2} \right) \\
 &\times (4(\rho-1)(x_1-x_{1,0})(x_{1,0}+x_{3,0})p^z - p^0(\rho x_{3,0}(2x_{1,0}+x_{3,0})+4x_1 x_{1,0}-4x_1^2)) \\
 &+ \ln \left(\frac{((\sqrt{1-\rho}+1)x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{1,0}+2x_1)^2} \right) \\
 &\times (4x_1(x_{3,0}p^0 + (\rho-1)(x_{1,0}+x_{3,0})p^z) - \rho x_{1,0}(x_{1,0}+2x_{3,0})p^0 + 4x_1^2 p^0), \quad x_1 < 0 \\
 &\ln \left(\frac{((\sqrt{1-\rho}+1)x_{3,0}+2x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{3,0}-2x_{1,0}+2x_1)^2} \right) \\
 &\times (4(\rho-1)(x_1-x_{1,0})(x_{1,0}+x_{3,0})p^z - p^0(\rho x_{3,0}(2x_{1,0}+x_{3,0})+4x_1 x_{1,0}-4x_1^2)) \\
 &+ x_{3,0} p^0 \ln \left(\frac{(\sqrt{1-\rho}+1)^2}{(\sqrt{1-\rho}-1)^2} \right) (\rho x_{3,0} + 4x_1) - \frac{8(1-\rho)^{3/2} x_1 x_{3,0} p^0}{z^2}, \quad 0 < x_1 < x_{1,0} \\
 &x_{1,0} \ln \left(\frac{(\sqrt{1-\rho}+1)^2}{(\sqrt{1-\rho}-1)^2} \right) (p^0(\rho x_{1,0} - 4x_1) - 4(\rho-1)(x_{1,0}+x_{3,0})p^z) + \ln \left(\frac{((\sqrt{1-\rho}+1)x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{1,0}+2x_1)^2} \right) \\
 &(4x_1(-x_{3,0}p^0 - (\rho-1)(x_{1,0}+x_{3,0})p^z) + \rho x_{1,0}(x_{1,0}+2x_{3,0})p^0 - 4x_1^2 p^0) \\
 &- \frac{8(1-\rho)^{3/2} x_{1,0}(x_{1,0}+x_{3,0}-x_1)p^0}{z^2}, \quad x_{1,0} < x_1 < x_{1,0}+x_{3,0} \\
 &\ln \left(\frac{((\sqrt{1-\rho}+1)(x_{1,0}+x_{3,0})-2x_1)^2}{((\sqrt{1-\rho}-1)(x_{1,0}+x_{3,0})+2x_1)^2} \right) (-\rho(x_{1,0}+x_{3,0})^2 p^0 + 4(\rho-1)x_1(x_{1,0}+x_{3,0})p^z + 4x_1^2 p^0) \\
 &+ \ln \left(\frac{((\sqrt{1-\rho}+1)x_{3,0}+2x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{3,0}-2x_{1,0}+2x_1)^2} \right) \\
 &\times (p^0(\rho x_{3,0}(2x_{1,0}+x_{3,0})+4x_1 x_{1,0}-4x_1^2) - 4(\rho-1)(x_1-x_{1,0})(x_{1,0}+x_{3,0})p^z) \\
 &+ \ln \left(\frac{((\sqrt{1-\rho}+1)x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{1,0}+2x_1)^2} \right) \\
 &\times (4x_1(-x_{3,0}p^0 - (\rho-1)(x_{1,0}+x_{3,0})p^z) + \rho x_{1,0}(x_{1,0}+2x_{3,0})p^0 - 4x_1^2 p^0), \quad x_1 > x_{1,0}+x_{3,0}
 \end{aligned} \right.
 \end{aligned}$$

$$\begin{aligned}
 \tilde{\mathcal{M}}_{\text{OF}}^c &= \frac{\delta(x_3 - x_{3,0}) \alpha_s C_F}{8\pi \sqrt{1-\rho} x_{1,0} x_{2,0} (x_{1,0} + x_{2,0})} \tilde{S}^{(0)} \\
 &\times \left\{ \begin{aligned}
 &x_1 x_{2,0} \ln \left(\frac{((\sqrt{1-\rho}+1)x_{1,0} - 2x_1)^2}{((\sqrt{1-\rho}-1)x_{1,0} + 2x_1)^2} \right) \\
 &+ x_{1,0} (x_{1,0} + x_{2,0} - x_1) \ln \left(\frac{((\sqrt{1-\rho}+1)x_{2,0} + 2x_{1,0} - 2x_1)^2}{((\sqrt{1-\rho}-1)x_{2,0} - 2x_{1,0} + 2x_1)^2} \right), \quad x_1 < 0 \\
 &x_1 x_{2,0} \ln \left(\frac{(\sqrt{1-\rho}+1)^2}{(\sqrt{1-\rho}-1)^2} \right) \\
 &+ x_{1,0} (x_{1,0} + x_{2,0} - x_1) \ln \left(\frac{((\sqrt{1-\rho}+1)x_{2,0} + 2x_{1,0} - 2x_1)^2}{((\sqrt{1-\rho}-1)x_{2,0} - 2x_{1,0} + 2x_1)^2} \right), \quad 0 < x_1 < x_{1,0} \\
 &x_{1,0} (x_{1,0} + x_{2,0} - x_1) \ln \left(\frac{(\sqrt{1-\rho}+1)^2}{(\sqrt{1-\rho}-1)^2} \right) \\
 &- x_1 x_{2,0} \ln \left(\frac{((\sqrt{1-\rho}+1)x_{1,0} - 2x_1)^2}{((\sqrt{1-\rho}-1)x_{1,0} + 2x_1)^2} \right), \quad x_{1,0} < x_1 < x_{1,0} + x_{2,0} \\
 &x_1 x_{2,0} \left(-\ln \left(\frac{((\sqrt{1-\rho}+1)x_{1,0} - 2x_1)^2}{((\sqrt{1-\rho}-1)x_{1,0} + 2x_1)^2} \right) \right) \\
 &- x_{1,0} (x_{1,0} + x_{2,0} - x_1) \ln \left(\frac{((\sqrt{1-\rho}+1)x_{2,0} + 2x_{1,0} - 2x_1)^2}{((\sqrt{1-\rho}-1)x_{2,0} - 2x_{1,0} + 2x_1)^2} \right), \quad x_1 > x_{1,0} + x_{2,0}
 \end{aligned} \right. \\
 \tilde{\mathcal{M}}_{\text{OF}}^d &= \frac{\alpha_s C_F}{16\pi \sqrt{1-\rho} x_{1,0}} \tilde{S}^{(0)} \\
 &\times \left\{ \begin{aligned}
 &\left\{ \frac{\delta(x_2 - x_{2,0})}{x_1 - x_{1,0}} \left[(\rho x_{1,0} - 4x_1) \ln \left(\frac{((\sqrt{1-\rho}+1)x_{1,0} - 2x_1)^2}{((\sqrt{1-\rho}-1)x_{1,0} + 2x_1)^2} \right) - 4\sqrt{1-\rho} x_{1,0} \right] \right\}_{\oplus}, \quad x_1 < 0 \\
 &\left\{ \frac{\delta(x_2 - x_{2,0})}{x_1 - x_{1,0}} \left[4\sqrt{1-\rho} (2x_1 - x_{1,0}) - \ln \left(\frac{(\sqrt{1-\rho}+1)^2}{(\sqrt{1-\rho}-1)^2} \right) (4x_1 - \rho x_{1,0}) \right] \right\}_{\oplus}, \quad 0 < x_1 < x_{1,0} \\
 &\left\{ \frac{\delta(x_2 - x_{2,0})}{x_1 - x_{1,0}} \left[4\sqrt{1-\rho} x_{1,0} + (4x_1 - \rho x_{1,0}) \ln \left(\frac{((\sqrt{1-\rho}+1)x_{1,0} - 2x_1)^2}{((\sqrt{1-\rho}-1)x_{1,0} + 2x_1)^2} \right) \right] \right\}_{\oplus}, \quad x_1 > x_{1,0}
 \end{aligned} \right.
 \end{aligned}$$

$$\tilde{\mathcal{M}}_{\text{OF}}^f = \frac{\alpha_s C_F}{32\pi(1-\rho)^{3/2} x_{3,0} p^z} \tilde{S}^{(0)}$$

$$\times \left\{ \begin{aligned} & \left\{ \frac{\delta(x_2-x_{2,0})}{x_1-x_{1,0}} \left[\ln \left(\frac{((\sqrt{1-\rho}+1)x_{3,0}+2x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{3,0}-2x_{1,0}+2x_1)^2} \right) \right. \right. \\ & \times \left. \left. (x_{3,0}(p^0-(\rho-1)(p^0+2p^z)) - 2(x_1-x_{1,0})(p^0-(\rho-1)p^z)) - \frac{4(1-\rho)^{3/2}x_{3,0}p^0}{z^2} \right] \right\}_{\oplus}, \quad x_1 < x_{1,0} \\ & \left\{ \frac{\delta(x_2-x_{2,0})}{x_1-x_{1,0}} \left[\frac{4(1-\rho)^{3/2}(-2x_{1,0}-x_{3,0}+2x_1)p^0}{z^2} - \ln \left(\frac{(\sqrt{1-\rho}+1)^2}{(\sqrt{1-\rho}-1)^2} \right) \right. \right. \\ & \times \left. \left. (2(x_1-x_{1,0})(p^0-(\rho-1)p^z) + x_{3,0}((\rho-2)p^0+2(\rho-1)p^z)) \right] \right\}_{\oplus}, \quad x_{1,0} < x_1 < x_{1,0}+x_{3,0} \\ & \left\{ \frac{\delta(x_2-x_{2,0})}{x_1-x_{1,0}} \left[\ln \left(\frac{((\sqrt{1-\rho}+1)x_{3,0}+2x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{3,0}-2x_{1,0}+2x_1)^2} \right) \right. \right. \\ & \times \left. \left. (2(x_1-x_{1,0})(p^0-(\rho-1)p^z) + x_{3,0}((\rho-2)p^0+2(\rho-1)p^z)) + \frac{4(1-\rho)^{3/2}x_{3,0}p^0}{z^2} \right] \right\}_{\oplus}, \quad x_1 > x_{1,0}+x_{3,0} \end{aligned} \right.$$

$$\tilde{\mathcal{M}}_{\text{OF}}^h = \frac{\alpha_s C_F}{32\pi\sqrt{1-\rho}x_{1,0}} \tilde{S}^{(0)}$$

$$\times \left\{ \begin{aligned} & \left\{ \frac{\delta(x_2-x_{2,0})-\delta(x_3-x_{3,0})}{x_{1,0}-x_1} \left[(\rho x_{1,0}-4x_1) \ln \left(\frac{((\sqrt{1-\rho}+1)x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{1,0}+2x_1)^2} \right) - 4\sqrt{1-\rho}x_{1,0} \right] \right\}_{\oplus}, \quad x_1 < 0 \\ & \left\{ \frac{\delta(x_2-x_{2,0})-\delta(x_3-x_{3,0})}{x_{1,0}-x_1} \left[4\sqrt{1-\rho}(2x_1-x_{1,0}) + \ln \left(\frac{(\sqrt{1-\rho}+1)^2}{(\sqrt{1-\rho}-1)^2} \right) (\rho x_{1,0}-4x_1) \right] \right\}_{\oplus}, \quad 0 < x_1 < x_{1,0} \\ & \left\{ \frac{\delta(x_2-x_{2,0})-\delta(x_3-x_{3,0})}{x_{1,0}-x_1} \left[4\sqrt{1-\rho}x_{1,0} + (4x_1-\rho x_{1,0}) \ln \left(\frac{((\sqrt{1-\rho}+1)x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{1,0}+2x_1)^2} \right) \right] \right\}_{\oplus}, \quad x_1 > x_{1,0} \end{aligned} \right.$$

$$\begin{aligned} \tilde{\mathcal{M}}_{\text{OF}}^b &= \tilde{\mathcal{M}}_{\text{OF}}^a|_{x_2 \leftrightarrow x_1, x_{2,0} \leftrightarrow x_{1,0}}, \\ \tilde{\mathcal{M}}_{\text{OF}}^e &= \tilde{\mathcal{M}}_{\text{OF}}^d|_{x_2 \leftrightarrow x_1, x_{2,0} \leftrightarrow x_{1,0}}, \end{aligned} \tag{4.4}$$

$$\begin{aligned} \tilde{\mathcal{M}}_{\text{OF}}^g &= \tilde{\mathcal{M}}_{\text{OF}}^f|_{x_2 \leftrightarrow x_1, x_{2,0} \leftrightarrow x_{1,0}}, \\ \tilde{\mathcal{M}}_{\text{OF}}^i &= \tilde{\mathcal{M}}_{\text{OF}}^h|_{x_2 \leftrightarrow x_1, x_{2,0} \leftrightarrow x_{1,0}}, \\ \tilde{\mathcal{M}}_{\text{OF}}^{j,k,l} &= \tilde{\mathcal{M}}^{j,k,l}. \end{aligned} \tag{4.5}$$

The subscript ‘‘OF’’ indicates the off-shell case. We have found that the self-energy correction of the Wilson line is independent of whether the momentum of the external leg is on-shell or not. Finally, the off-shell quasi-DA up to one-loop accuracy is given as

$$\begin{aligned} \tilde{\phi}(x_1, x_2, \mu)_{\text{OF}} &= \delta(x_1-x_{1,0})\delta(x_2-x_{2,0}) + \frac{\alpha_s C_F}{8\pi} \\ & \times \left\{ \left[g_2' \delta(x_2-x_{2,0}) + g_3' \delta(x_3-x_{3,0}) + \{x_2 \leftrightarrow x_1, x_{2,0} \leftrightarrow x_{1,0}\} \right] \right\}_{\oplus}, \end{aligned} \tag{4.6}$$

with

$$g_2' = \frac{1}{8p^z(1-\rho)^{5/2}(x_1-x_{1,0})x_{1,0}x_{3,0}(x_{1,0}+x_{3,0})}$$

$$\left\{ \begin{aligned}
 & -8(1-\rho)^{3/2}x_{1,0}x_{3,0}(x_{1,0}+x_{3,0})(p^0+(\rho-1)p^z) - (1-\rho)(x_1-x_{1,0}) \ln \left(\frac{((\sqrt{1-\rho}+1)(x_{1,0}+x_{3,0})-2x_1)^2}{((\sqrt{1-\rho}-1)(x_{1,0}+x_{3,0})+2x_1)^2} \right) \\
 & \times \left[-\rho(x_{1,0}+x_{3,0})^2p^0+4(\rho-1)x_1(x_{1,0}+x_{3,0})p^z+4x_1^2p^0 \right] - (1-\rho) \ln \left(\frac{((\sqrt{1-\rho}+1)x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{1,0}+2x_1)^2} \right) \\
 & \times \left\{ 2(\rho-1)x_{3,0}(x_{1,0}+x_{3,0})(\rho x_{1,0}-4x_1)p^z - (x_1-x_{1,0}) \left[4x_1(x_{3,0}p^0+(\rho-1)(x_{1,0}+x_{3,0})p^z) \right. \right. \\
 & \left. \left. - \rho x_{1,0}(x_{1,0}+2x_{3,0})p^0+4x_1^2p^0 \right] \right\} - (1-\rho) \ln \left(\frac{((\sqrt{1-\rho}+1)x_{3,0}+2x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{3,0}-2x_{1,0}+2x_1)^2} \right) \\
 & \times \left\{ 2x_{1,0}(x_{1,0}+x_{3,0}) \left[x_{3,0}((\rho-2)p^0+2(\rho-1)p^z)+2(x_1-x_{1,0})(p^0-\rho p^z+p^z) \right] - (x_1-x_{1,0}) \right. \\
 & \times \left. \left[4(x_1-x_{1,0})((\rho-1)x_{1,0}p^z+x_1p^0)+2x_{3,0}(2(\rho-1)x_1p^z-x_{1,0}(\rho p^0+2(\rho-1)p^z))-\rho x_{3,0}^2p^0 \right] \right\}, \quad x_1 < 0 \\
 & -8(1-\rho)^{3/2}x_{3,0}(x_{1,0}(x_{1,0}+x_{3,0})(p^0-(1-\rho)p^z)-x_1(x_{1,0}(p^0+2(1-\rho)p^z)+2(1-\rho)x_{3,0}p^z)+x_1^2p^0) \\
 & + (1-\rho)x_{3,0} \ln \left(\frac{(\sqrt{1-\rho}+1)^2}{(\sqrt{1-\rho}-1)^2} \right) \left\{ -\rho x_{1,0} \left[x_{3,0}p^0+2(\rho-1)(x_{1,0}+x_{3,0})p^z \right] + x_1 \left[x_{3,0}(\rho p^0+8(\rho-1)p^z) \right. \right. \\
 & \left. \left. - 4x_{1,0}(p^0-2(\rho-1)p^z) \right] + 4x_1^2p^0 \right\} + (1-\rho) \ln \left(\frac{((\sqrt{1-\rho}+1)x_{3,0}+2x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{3,0}-2x_{1,0}+2x_1)^2} \right) \\
 & \times \left\{ -x_1 \left[2x_{1,0}x_{3,0}((\rho+2)p^0+2(\rho-1)p^z)+\rho x_{3,0}^2p^0+4(\rho-1)x_{1,0}^2p^z \right] - 4x_1^2 \left[x_{1,0}(2p^0-\rho p^z+p^z) - (\rho-1)x_{3,0}p^z \right] \right. \\
 & \left. + x_{1,0} \left[4x_{1,0}x_{3,0}(2p^0-\rho p^z+p^z)+x_{3,0}^2(4(p^0+p^z)-\rho(p^0+4p^z))+4x_{1,0}^2p^0 \right] + 4x_1^3p^0 \right\}, \quad 0 < x_1 < x_{1,0} \\
 & -8(1-\rho)^{3/2}x_{1,0}((x_{1,0}+x_{3,0})(x_{3,0}(p^0+(1-\rho)p^z)+x_{1,0}p^0)-x_1x_{3,0}p^0-x_1^2p^0) - (1-\rho)x_{1,0} \ln \left(\frac{(\sqrt{1-\rho}+1)^2}{(\sqrt{1-\rho}-1)^2} \right) \\
 & \times \left\{ 2x_{1,0}x_{3,0} \left[(\rho-4)p^0+2(\rho-1)p^z \right] + 2x_{3,0}^2 \left[(\rho-2)p^0+2(\rho-1)p^z \right] + (\rho-4)x_{1,0}^2p^0+x_1p^0(4x_{3,0}-\rho x_{1,0})+4x_1^2p^0 \right\} \\
 & + (1-\rho) \ln \left(\frac{((\sqrt{1-\rho}+1)x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{1,0}+2x_1)^2} \right) \left\{ 2(\rho-1)x_{3,0}(x_{1,0}+x_{3,0})(\rho x_{1,0}-4x_1)p^z \right. \\
 & \left. - (x_1-x_{1,0}) \left[4x_1(x_{3,0}p^0+(\rho-1)(x_{1,0}+x_{3,0})p^z) - \rho x_{1,0}(x_{1,0}+2x_{3,0})p^0+4x_1^2p^0 \right] \right\}, \quad x_{1,0} < x_1 < x_{1,0}+x_{3,0} \\
 & 8(1-\rho)^{3/2}x_{1,0}x_{3,0}(x_{1,0}+x_{3,0})(p^0-(1-\rho)p^z) + (1-\rho)(x_1-x_{1,0}) \ln \left(\frac{((\sqrt{1-\rho}+1)(x_{1,0}+x_{3,0})-2x_1)^2}{((\sqrt{1-\rho}-1)(x_{1,0}+x_{3,0})+2x_1)^2} \right) \\
 & \times \left[-\rho(x_{1,0}+x_{3,0})^2p^0+4(\rho-1)x_1(x_{1,0}+x_{3,0})p^z+4x_1^2p^0 \right] + (1-\rho) \ln \left(\frac{((\sqrt{1-\rho}+1)x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{1,0}+2x_1)^2} \right) \\
 & \times \left\{ 2(\rho-1)x_{3,0}(x_{1,0}+x_{3,0})(\rho x_{1,0}-4x_1)p^z - (x_1-x_{1,0}) \left[4x_1(x_{3,0}p^0+(\rho-1)(x_{1,0}+x_{3,0})p^z) \right. \right. \\
 & \left. \left. - \rho x_{1,0}(x_{1,0}+2x_{3,0})p^0+4x_1^2p^0 \right] \right\} + (1-\rho) \ln \left(\frac{((\sqrt{1-\rho}+1)x_{3,0}+2x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{3,0}-2x_{1,0}+2x_1)^2} \right) \left\{ 2x_{1,0}(x_{1,0}+x_{3,0}) \right. \\
 & \times \left[x_{3,0}((\rho-2)p^0+2(\rho-1)p^z)+2(x_1-x_{1,0})(p^0-\rho p^z+p^z) \right] - (x_1-x_{1,0}) \left[4(x_1-x_{1,0})((\rho-1)x_{1,0}p^z+x_1p^0) \right. \\
 & \left. \left. - 2x_{3,0}(x_{1,0}(\rho p^0+2(\rho-1)p^z)-2(\rho-1)x_1p^z)-\rho x_{3,0}^2p^0 \right] \right\}, \quad x_1 > x_{1,0}+x_{3,0}
 \end{aligned} \right.$$

$$g'_3 = \frac{1}{4\sqrt{1-\rho}(x_{1,0}+x_{2,0})x_{1,0}x_{2,0}(x_1-x_{1,0})} \times \left\{ \begin{aligned} & 4x_2(x_1-x_{1,0})x_{1,0} \ln \left(\frac{((\sqrt{1-\rho}+1)x_{2,0}+2x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{2,0}-2x_{1,0}+2x_1)^2} \right) + (x_{1,0}+x_{2,0})x_{1,0}(\rho x_{2,0}-4x_2) \\ & \times \ln \left(\frac{((\sqrt{1-\rho}+1)x_{2,0}-2x_2)^2}{((\sqrt{1-\rho}-1)x_{2,0}+2x_2)^2} \right) + x_{2,0}(\rho x_{1,0}(x_{1,0}+x_{2,0})-4x_1(2x_{1,0}+x_{2,0})+4x_1^2) \\ & \times \ln \left(\frac{((\sqrt{1-\rho}+1)x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{1,0}+2x_1)^2} \right), \quad x_1 < 0 \\ & 8\sqrt{1-\rho}x_1x_{2,0}(x_{1,0}+x_{2,0})+4x_2(x_1-x_{1,0})x_{1,0} \ln \left(\frac{((\sqrt{1-\rho}+1)x_{2,0}+2x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{2,0}-2x_{1,0}+2x_1)^2} \right) \\ & + (x_{1,0}+x_{2,0})x_{1,0}(\rho x_{2,0}-4x_2) \ln \left(\frac{((\sqrt{1-\rho}+1)x_{2,0}-2x_2)^2}{((\sqrt{1-\rho}-1)x_{2,0}+2x_2)^2} \right) \\ & + x_{2,0} \ln \left(\frac{(\sqrt{1-\rho}+1)^2}{(\sqrt{1-\rho}-1)^2} \right) (\rho x_{1,0}(x_{1,0}+x_{2,0})-4x_1(2x_{1,0}+x_{2,0})+4x_1^2), \quad 0 < x_1 < x_{1,0} \\ & -8\sqrt{1-\rho}x_2x_{1,0}(x_{1,0}+x_{2,0})+x_{1,0} \ln \left(\frac{(\sqrt{1-\rho}+1)^2}{(\sqrt{1-\rho}-1)^2} \right) (4x_1(x_2-x_{2,0})-(\rho-4)x_{2,0}(x_{1,0}+x_{2,0})) \\ & -x_{2,0}(\rho x_{1,0}(x_{1,0}+x_{2,0})-4x_1(2x_{1,0}+x_{2,0})+4x_1^2) \ln \left(\frac{((\sqrt{1-\rho}+1)x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{1,0}+2x_1)^2} \right), \quad x_{1,0} < x_1 < x_{1,0}+x_{2,0} \\ & -4(x_1-x_{1,0})x_{1,0}(x_{1,0}+x_{2,0}-x_1) \ln \left(\frac{((\sqrt{1-\rho}+1)x_{2,0}+2x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{2,0}-2x_{1,0}+2x_1)^2} \right) \\ & + \frac{(x_1-x_{1,0})x_{1,0}(x_{1,0}+x_{2,0})(\rho x_{2,0}-4x_2) \ln \left(\frac{((\sqrt{1-\rho}+1)x_{2,0}-2x_2)^2}{((\sqrt{1-\rho}-1)x_{2,0}+2x_2)^2} \right)}{x_2-x_{2,0}} \\ & -x_{2,0}(\rho x_{1,0}(x_{1,0}+x_{2,0})-4x_1(2x_{1,0}+x_{2,0})+4x_1^2) \ln \left(\frac{((\sqrt{1-\rho}+1)x_{1,0}-2x_1)^2}{((\sqrt{1-\rho}-1)x_{1,0}+2x_1)^2} \right), \quad x_1 > x_{1,0}+x_{2,0}. \end{aligned} \right.$$

5 Matching kernel

In the large momentum $p^z \gg \Lambda_{\text{QCD}}$ limit, the quasi observables can be factorized as a convolution of a perturbatively calculable matching coefficient and the corresponding light-cone observable up to power corrections suppressed by $(\frac{1}{x_1 p^z}, \frac{1}{x_2 p^z}, \frac{1}{(1-x_1-x_2)p^z})$. Through this factorization, one can extract light-cone observables from quasi-ones calculated on the lattice. The matching of quasi-DA and LCDA is given as

$$\tilde{\Phi}(x_1, x_2, \mu) = \int dy_1 dy_2 \mathcal{C}(x_1, x_2, y_1, y_2, \mu) \Phi(y_1, y_2, \mu) + \mathcal{O}\left(\frac{1}{x_1 p^z}, \frac{1}{x_2 p^z}, \frac{1}{(1-x_1-x_2)p^z}\right). \quad (5.1)$$

With the results presented in the previous sections, one can easily obtain the matching kernel in the $\overline{\text{MS}}$ scheme up to one-loop level

$$\begin{aligned} \mathcal{C}(x_1, x_2, y_1, y_2, \mu) = & \delta(x_1-y_1)\delta(x_2-y_2) + \frac{\alpha_s C_F}{8\pi} \left[C_2(x_1, x_2, y_1, y_2)\delta(x_2-y_2) \right. \\ & \left. + C_3(x_1, x_2, y_1, y_2)\delta(x_3-y_3) + \{x_1 \leftrightarrow x_2, y_1 \leftrightarrow y_2\} \right]_{\oplus}, \end{aligned} \quad (5.2)$$

where $y_3 = 1 - y_1 - y_2$ and

$$C_2(x_1, x_2, y_1, y_2) = \begin{cases} \frac{(x_1 + y_1)(x_3 + y_3) \ln \frac{y_1 - x_1}{-x_1} - x_3(x_1 + y_1 + 2y_3) \ln \frac{x_3}{-x_1}}{y_1(y_1 - x_1)y_3} - \frac{x_3(x_1 + y_1 + 2y_3) \ln \frac{x_3}{-x_1}}{(y_1 - x_1)y_3(y_1 + y_3)}, & x_1 < 0 \\ \frac{(x_1 - 3y_1 - 2y_3)x_1}{y_1(x_3 - y_3)(y_1 + y_3)} - \frac{[(x_3 - y_3)^2 - 2x_3y_1] \ln \frac{x_3 - y_3}{x_3}}{y_1(x_3 - y_3)y_3} + \frac{2x_1 \ln \frac{4x_1(x_3 - y_3)p_z^2}{\mu^2}}{y_1(x_3 - y_3)} + \frac{x_1 \ln \frac{4x_1x_3p_z^2}{\mu^2}}{y_1(y_1 + y_3)}, & 0 < x_1 < y_1 \\ \frac{(x_3 - 2y_1 - 3y_3)x_3}{y_3(x_1 - y_1)(y_1 + y_3)} - \frac{[(x_1 - y_1)^2 - 2x_1y_3] \ln \frac{x_1 - y_1}{x_1}}{(x_1 - y_1)y_1y_3} + \frac{2x_3 \ln \frac{4x_3(x_1 - y_1)p_z^2}{\mu^2}}{(x_1 - y_1)y_3} + \frac{x_3 \ln \frac{4x_1x_3p_z^2}{\mu^2}}{y_3(y_1 + y_3)}, & y_1 < x_1 < y_1 + y_3 \\ \frac{(x_1 + y_1)(x_3 + y_3) \ln \frac{y_3 - x_3}{-x_3} - x_1(x_3 + 2y_1 + y_3) \ln \frac{x_1}{-x_3}}{y_1y_3(y_3 - x_3)} - \frac{x_1(x_3 + 2y_1 + y_3) \ln \frac{x_1}{-x_3}}{y_1(y_3 - x_3)(y_1 + y_3)}, & x_1 > y_1 + y_3 \end{cases} \quad (5.3)$$

$$C_3(x_1, x_2, y_1, y_2) = \begin{cases} \frac{(x_1x_2 + y_1y_2) \ln \frac{x_2 - y_2}{x_2} - x_1(x_2 + y_1) \ln \frac{-x_1}{x_2}}{y_1(x_2 - y_2)y_2} - \frac{x_1(x_2 + y_1) \ln \frac{-x_1}{x_2}}{y_1(x_2 - y_2)(y_1 + y_2)}, & x_1 < 0 \\ \frac{1}{x_1 - y_1} + \frac{2x_1 + x_2}{y_1(y_1 + y_2)} + \frac{[(x_1 + y_2)y_1 - x_1^2] \ln \frac{x_2 - y_2}{x_2}}{y_1(x_2 - y_2)y_2} + \frac{x_1 \ln \frac{4x_1(x_2 - y_2)p_z^2}{\mu^2}}{y_1(x_2 - y_2)} + \frac{x_1 \ln \frac{4x_1x_2p_z^2}{\mu^2}}{y_1(y_1 + y_2)}, & 0 < x_1 < y_1 \\ \frac{1}{x_2 - y_2} + \frac{x_1 + 2x_2}{y_2(y_1 + y_2)} + \frac{[(x_2 + y_1)y_2 - x_2^2] \ln \frac{x_1 - y_1}{x_1}}{(x_1 - y_1)y_1y_2} + \frac{x_2 \ln \frac{4x_2(x_1 - y_1)p_z^2}{\mu^2}}{(x_1 - y_1)y_2} + \frac{x_2 \ln \frac{4x_1x_2p_z^2}{\mu^2}}{y_2(y_1 + y_2)}, & y_1 < x_1 < y_1 + y_2 \\ \frac{(x_1x_2 + y_1y_2) \ln \frac{x_1 - y_1}{x_1} - x_2(x_1 + y_2) \ln \frac{-x_2}{x_1}}{y_1(x_1 - y_1)y_2} - \frac{x_2(x_1 + y_2) \ln \frac{-x_2}{x_1}}{y_2(x_1 - y_1)(y_1 + y_2)}, & x_1 > y_1 + y_2. \end{cases} \quad (5.4)$$

If we integrate over the physical region of the momentum fraction $y_{1,2}$ of the matching kernel, we find that the integral diverges. In order to eliminate this ultraviolet divergence and renormalize the lattice operators, we need a suitable renormalization of the quasi-DA $\tilde{\phi}(x_1, x_2, \mu)$. In the RI/MOM scheme, this is given as

$$\tilde{\phi}(x_1, x_2, \mu)_{\text{RI/MOM}} = \frac{\tilde{\phi}(x_1, x_2, \mu)}{\tilde{\phi}(x_1, x_2, \mu)_{\text{OF}}}. \quad (5.5)$$

Therefore, the renormalized matching coefficient \mathcal{C} in eq. (5.1) is

$$\begin{aligned} \mathcal{C}^{\mathcal{R}}(x_1, x_2, y_1, y_2, \mu) = & \delta(x_1 - y_1)\delta(x_2 - y_2) + \frac{\alpha_s C_F}{8\pi} \left[C_2'(x_1, x_2, y_1, y_2)\delta(x_2 - y_2) \right. \\ & \left. + C_3'(x_1, x_2, y_1, y_2)\delta(x_3 - y_3) + \{x_1 \leftrightarrow x_2, y_1 \leftrightarrow y_2\} \right]_{\oplus}, \end{aligned} \quad (5.6)$$

where $C'_2 = C_2 - g'_2|_{x_{1,0} \rightarrow y_1, x_{2,0} \rightarrow y_2}$ and $C'_3 = C_3 - g'_3|_{x_{1,0} \rightarrow y_1, x_{2,0} \rightarrow y_2}$. In the RI/MOM scheme, the UV divergence in the quasi-DAs can be removed by the renormalization constant determined nonperturbatively.

It should be emphasized that although in the above calculation, an off-shell result is used to remove the UV divergences in the integration, additional infrared effects are likely to be introduced. Recently, hybrid renormalization and self-renormalization schemes have been adopted to obtain a more coherent result [49–52]. The hybrid renormalization scheme treats the short-distance and long-distance renormalization separately while the self-renormalization scheme aims to extract the linear divergence by the zero-momentum matrix element. An analysis of the renormalization of the baryon quasi-DA in such a scheme is undergoing. More recently, a newly proposed method is also shown in ref. [53].

6 Summary

In this work, we have pointed out that LCDAs of a light baryon can be obtained through a simulation of a quasi-distribution amplitude calculable on lattice QCD under the framework of large-momentum effective theory. We have calculated the one-loop perturbative contributions to LCDA and quasi-distribution amplitudes and explicitly have demonstrated the factorization of quasi-distribution amplitudes at the one-loop level. A direct analysis using expansion by region also verifies the factorizability of quasi-DA. Based on the perturbative results, we have derived the matching kernel.

For the renormalization of quasi-distribution amplitudes, we have adopted the simplest procedure at this stage and subtracted the results with an off-shell parton state as a RI/MOM result. Our result provides a first step to obtaining the LCDA from first principle lattice QCD calculations in the future. An improved renormalization procedure might be performed in the self-renormalization or hybrid approach.

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A Gauge invariance in LCDAs

According to eqs. (2.1)–(2.2), a gauge-invariant form for the LCDA of a light baryon Λ can be constructed as:

$$\epsilon_{ijk} \langle 0 | \mathcal{W}_{i'i'}(\infty, t_1 n) u_{i'}^T(t_1 n) \Gamma \mathcal{W}_{j'j}(\infty, t_2 n) d_{j'}(t_2 n) \mathcal{W}_{kk'}(\infty, 0) s_{k'}(0) | \Lambda \rangle. \quad (\text{A.1})$$

If we focus on the color structure, we can find

$$\begin{aligned}
 & \epsilon_{ijk} \mathcal{W}_{ii'}(\infty, t_1 n) \mathcal{W}_{jj'}(\infty, t_2 n) \mathcal{W}_{kk'}(\infty, 0) \\
 &= \epsilon_{ijk} \mathcal{W}_{il}(\infty, 0) \mathcal{W}_{li'}(0, t_1 n) \mathcal{W}_{jm}(\infty, 0) \mathcal{W}_{mj'}(0, t_2 n) \mathcal{W}_{kk'}(\infty, 0) \\
 &= \mathcal{W}_{li'}(0, t_1 n) \mathcal{W}_{mj'}(0, t_2 n) \epsilon_{lmk'} \det |\mathcal{W}(\infty, 0)|.
 \end{aligned} \tag{A.2}$$

Here we have used the identity, the definition of 3×3 matrix determinant, $\epsilon_{ijk} \mathcal{W}_{il}(\infty, 0) \mathcal{W}_{jm}(\infty, 0) \mathcal{W}_{kk'}(\infty, 0) = \epsilon_{lmk'} \det |\mathcal{W}(\infty, 0)|$. The Wilson line satisfy the property of SU(3) group. Therefore, the gauge-invariant LCDA eq. (A.2) can be also written as

$$\epsilon_{lmk'} \langle 0 | \mathcal{W}_{li'}(0, t_1 n) u_i^T(t_1 n) \Gamma \mathcal{W}_{mj'}(0, t_2 n) d_{j'}(t_2 n) s_{k'}(0) | \Lambda \rangle,$$

or equivalently:

$$\epsilon_{ijk} \langle 0 | \mathcal{W}_{ii'}(0, t_1 n) u_i^T(t_1 n) \Gamma \mathcal{W}_{jj'}(0, t_2 n) d_{j'}(t_2 n) s_k(0) | \Lambda \rangle. \tag{A.3}$$

B Projection and trace formulae

We consider a tree-level matrix element:

$$\begin{aligned}
 & \int \frac{da_1 p^+}{2\pi} \int \frac{da_2 p^+}{2\pi} e^{i(x_1 a_1 + x_2 a_2) p^+} \frac{\epsilon_{ijk} \epsilon_{abc}}{6} \langle 0 | u_i^T(a_1 n) \\
 & \quad \times C \not{n} \gamma^5 d_j(a_2 n) s_k(0) | u_a(x_{1,0} p) d_b(x_{2,0} p) s_c(x_{3,0} p) \rangle \\
 &= \int \frac{da_1 p^+}{2\pi} \int \frac{da_2 p^+}{2\pi} e^{i(x_1 a_1 + x_2 a_2) p^+} \langle 0 | u^T(a_1 n) C \not{n} \gamma^5 d(a_2 n) s(0) \frac{1}{\sqrt{2}} [b_{\uparrow, u}^\dagger(x_{1,0} p) \\
 & \quad \times b_{\downarrow, d}^\dagger(x_{2,0} p) - b_{\downarrow, u}^\dagger(x_{1,0} p) b_{\uparrow, d}^\dagger(x_{2,0} p)] b_s^\dagger(x_{3,0} p) | 0 \rangle \\
 &= -\delta(x_1 - x_{1,0}) \delta(x_2 - x_{2,0}) \frac{1}{\sqrt{2}} \text{Tr} \{ [u_\downarrow(x_{2,0} p) u_\uparrow^T(x_{1,0} p) \\
 & \quad - u_\uparrow(x_{2,0} p) u_\downarrow^T(x_{1,0} p)] C \not{n} \gamma^5 \} u(x_{3,0} p),
 \end{aligned} \tag{B.1}$$

where the arrow \uparrow and \downarrow denote the spin $+1/2$ and $-1/2$ for ud quark pair. Using the spinor

$$u^\uparrow(xp) = \sqrt{x p^z} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad u^\downarrow(xp) = \sqrt{x p^z} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \tag{B.2}$$

under the Dirac representation, one has

$$\frac{u_\downarrow(x_{2,0} p) u_\uparrow^T(x_{1,0} p) - u_\uparrow(x_{2,0} p) u_\downarrow^T(x_{1,0} p)}{\sqrt{2}} = c_1 \frac{1}{2} \not{n} C \gamma^5, \tag{B.3}$$

with the coefficient $c_1 = -\sqrt{2x_{1,0}x_{2,0}}$.

Since this factor c_1 appears both in the evaluation of tree-level and one-loop operator matrix elements, one can neglect this factor. Thus one can employ a tree-level operator

matrix element

$$\begin{aligned}
& \int \frac{da_1 p^+}{2\pi} \int \frac{da_2 p^+}{2\pi} e^{i(x_1 a_1 + x_2 a_2) p^+} \frac{\epsilon_{ijk} \epsilon_{abc}}{6} \langle 0 | u_i^T(a_1 n) \\
& \quad \times C \not{n} \gamma^5 d_j(a_2 n) s_k(0) | u_a(x_{1,0} p) d_b(x_{2,0} p) s_c(x_{3,0} p) \rangle |_{\text{tree}} \\
& = \delta(x_1 - x_{1,0}) \delta(x_2 - x_{2,0}) \frac{1}{2} \text{tr} \{ \gamma^5 \not{p} \not{n} \gamma^5 \} u(x_{3,0} p) \\
& = 2p^+ \delta(x_1 - x_{1,0}) \delta(x_2 - x_{2,0}) u(x_{3,0} p).
\end{aligned} \tag{B.4}$$

The normalization of the LCDA will lead to the local operator matrix element

$$\begin{aligned}
& \int dx_1 \int dx_2 \int \frac{da_1 p^+}{2\pi} \int \frac{da_2 p^+}{2\pi} e^{i(x_1 a_1 + x_2 a_2) p^+} \\
& \quad \times \frac{\epsilon_{ijk} \epsilon_{abc}}{6} \langle 0 | u_i^T(a_1 n) C \not{n} \gamma^5 d_j(a_2 n) s_k(0) | u_a d_b s_c \rangle \\
& = (p^+)^2 \int \frac{da_1}{2\pi} \int \frac{da_2}{2\pi} \delta(a_1 p^+) \delta(a_2 p^+) \frac{\epsilon_{ijk} \epsilon_{abc}}{6} \langle 0 | u_i^T(a_1 n) C \not{n} \gamma^5 d_j(a_2 n) s_k(0) | u_a d_b s_c \rangle \\
& = \frac{\epsilon_{ijk} \epsilon_{abc}}{6} \langle 0 | u_i^T(0) C \not{n} \gamma^5 d_j(0) s_k(0) | u_a d_b s_c \rangle \\
& = S.
\end{aligned} \tag{B.5}$$

Therefore the normalized LCDA at the one-loop accuracy is

$$\begin{aligned}
\phi & = \frac{S^{(0)} \delta(x_1 - x_{1,0}) \delta(x_2 - x_{2,0}) + \sum_i \mathcal{M}^i}{S^{(0)} + S^{(1)}} \\
& = \delta(x_1 - x_{1,0}) \delta(x_2 - x_{2,0}) + \frac{1}{S^{(0)}} \left(\sum_i \mathcal{M}^i - \delta(x_1 - x_{1,0}) \delta(x_2 - x_{2,0}) S^{(1)} \right) \\
& = \delta(x_1 - x_{1,0}) \delta(x_2 - x_{2,0}) + \frac{1}{S^{(0)}} \left(\sum_i \mathcal{M}^i - \delta(x_1 - x_{1,0}) \delta(x_2 - x_{2,0}) \right) \\
& \quad \times \int dy_1 \int dy_2 \sum_i \mathcal{M}^i |_{x_1 \rightarrow y_1, x_2 \rightarrow y_2},
\end{aligned} \tag{B.6}$$

where we adopted the convention for perturbative expansion, and \mathcal{M} denotes $\phi(x_1, x_2, \mu) S$ in short. The normalization of quasi-DA will give a similar form.

C Expansion by regions and factorization of quasi-DA at one-loop

In LaMET, it is conjectured that the quasi-distribution amplitudes can be factorized as a convolution of the LCDAs and a hard kernel. A rigorous proof of quasi PDFs can be elegantly found in refs. [33, 54]. In the following, we adopt the technique of expansion by region [55] for quasi-DA and explicitly demonstrate the factorization of quasi-DA.

In the definition of quasi-DA, one has two popular choices for the Lorentz structures in the interpolating operator: $\tilde{\Gamma} = C \gamma^5 \gamma^z$, and $\tilde{\Gamma} = C \gamma^5 \gamma^t$. We will show that the short-distance results, namely the hard kernel, are the same at the one-loop level in $\overline{\text{MS}}$ scheme.

We will analyze the normalized coefficient $\tilde{\phi}^i|_{(1/0)}$ or $\mathcal{M}^i|_{(1/0)}$:

$$\tilde{\phi}^i = \tilde{\phi}^i|_{(1/0)} \tilde{\mathcal{S}}, \tag{C.1}$$

$$\mathcal{M}^i = \mathcal{M}^i|_{(1/0)} \mathcal{S}. \tag{C.2}$$

In the quasi-DA, there are three potential leading power contributions according to the decomposition of the momentum $q = (q^+, q^-, q_\perp)$,

- Hard mode with $q \sim (1, 1, 1)Q$:

in this region, all the hard kinetic components must be retained. Then, one can find that the magnitude of the amplitude is order one $\mathcal{O}(1)$.

- Collinear mode with $q \sim (Q, \Lambda_{\text{QCD}}^2/Q, \Lambda_{\text{QCD}})$:

in this region, one can find that the amplitude is $\mathcal{O}(1)$, and actually the amplitudes for both structures are reduced to the LCDA:

$$\begin{aligned}
 \tilde{\phi}_{(1/0)}^a|_C &= ig^2 \frac{C_F}{2} p^z \delta(x_2 - x_{2,0}) \int \frac{d^4 q}{(2\pi)^4} \frac{\delta(x_1 p^z - q^z - k_1^z)}{(q+k_1)^2 + i\epsilon} \frac{1}{q^2 + i\epsilon} \frac{q_1^2}{(k_3 - q)^2 + i\epsilon} \Big|_C \\
 &= ig^2 \frac{C_F}{2} \frac{p^+ - p^-}{\sqrt{2}} \delta(x_2 - x_{2,0}) \int \frac{d^4 q}{(2\pi)^4} \frac{\sqrt{2} \delta((x_1 p^+ - q^+ - k_1^+) - (x_1 p^- - q^- - k_1^-))}{(q+k_1)^2 + i\epsilon} \\
 &\quad \frac{1}{q^2 + i\epsilon} \frac{q_1^2}{(k_3 - q)^2 + i\epsilon} \Big|_C \\
 &= ig^2 \frac{C_F}{2} p^+ \delta(x_2 - x_{2,0}) \int \frac{d^4 q}{(2\pi)^4} \frac{\delta(x_1 p^+ - q^+ - k_1^+)}{(q+k_1)^2 + i\epsilon} \frac{1}{q^2 + i\epsilon} \frac{q_1^2}{(k_3 - q)^2 + i\epsilon} \\
 &= \phi_{(1/0)}^a. \tag{C.3}
 \end{aligned}$$

$$\begin{aligned}
 \tilde{\phi}_{(1/0)}^d|_C &= ig^2 C_F p^z \left[\delta(x_2 - x_{2,0}) \int \frac{d^4 q}{(2\pi)^4} \frac{\delta(x_1 p^z - q^z - k_1^z)}{(k_1 + q)^2 + i\epsilon} \frac{1}{q^2 + i\epsilon} \frac{2k_1^z + q^0 + q^z}{q^z} \right]_{\oplus} \Big|_C \\
 &= ig^2 C_F \frac{p^+}{\sqrt{2}} \left[\delta(x_2 - x_{2,0}) \int \frac{d^4 q}{(2\pi)^4} \frac{\delta(\frac{x_1 p^+}{\sqrt{2}} - \frac{q^+ - q^-}{\sqrt{2}} - \frac{k_1^+}{\sqrt{2}})}{(k_1 + q)^2 + i\epsilon} \frac{1}{q^2 + i\epsilon} \frac{2\frac{k_1^+}{\sqrt{2}} + \frac{q^+ + q^-}{\sqrt{2}} + \frac{q^+ - q^-}{\sqrt{2}}}{\frac{q^+ - q^-}{\sqrt{2}}} \right]_{\oplus} \Big|_C \\
 &= ig^2 C_F \frac{p^+}{\sqrt{2}} \left[\delta(x_2 - x_{2,0}) \int \frac{d^4 q}{(2\pi)^4} \frac{\sqrt{2} \delta(x_1 p^+ - q^+ - k_1^+)}{(k_1 + q)^2 + i\epsilon} \frac{1}{q^2 + i\epsilon} \frac{2k_1^+ + q^+ + q^+}{q^+} \right]_{\oplus} \\
 &= ig^2 C_F p^+ \left[\delta(x_2 - x_{2,0}) \int \frac{d^4 q}{(2\pi)^4} \frac{\delta[(x_1 - x_{1,0})p^+ - q^+]}{(k_1 + q)^2 + i\epsilon} \frac{1}{q^2 + i\epsilon} \frac{2(k_1^+ + q^+)}{q^+} \right]_{\oplus} \\
 &= \phi_{(1/0)}^d.
 \end{aligned}$$

- Soft mode $q \sim (\Lambda_{\text{QCD}}, \Lambda_{\text{QCD}}, \Lambda_{\text{QCD}})$:

In this kinematics region, one can find the power of the amplitude is $\mathcal{O}(\Lambda_{\text{QCD}}/Q)$, and namely, this amplitude is suppressed.

This analysis indicates that the amplitude from figure 1(a) and 1(d) are independent of the Lorentz structure, and moreover, we have checked other amplitudes in figure 1. We have found that the one-loop LCDA and quasi-DA for baryon Λ does not contain the soft contributions. The one-loop quasi-DA contain the collinear and hard mode. As anticipated the one-loop LCDA only contains the collinear mode at leading power. As a result, QCD factorization shows that the hard and collinear modes in the quasi-DA can be factorized into a convolution of the hard matching coefficient and the LCDA which only contains collinear modes.

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References

- [1] LHCb collaboration, *Determination of the quark coupling strength $|V_{ub}|$ using baryonic decays*, *Nature Phys.* **11** (2015) 743 [[arXiv:1504.01568](#)] [[INSPIRE](#)].
- [2] LHCb collaboration, *Observation of the decay $\Lambda_b^0 \rightarrow pK^- \mu^+ \mu^-$ and a search for CP violation*, *JHEP* **06** (2017) 108 [[arXiv:1703.00256](#)] [[INSPIRE](#)].
- [3] V.L. Chernyak and I.R. Zhitnitsky, *Nucleon Wave Function and Nucleon Form-Factors in QCD*, *Nucl. Phys. B* **246** (1984) 52 [[INSPIRE](#)].
- [4] I.D. King and C.T. Sachrajda, *Nucleon Wave Functions and QCD Sum Rules*, *Nucl. Phys. B* **279** (1987) 785 [[INSPIRE](#)].
- [5] V.L. Chernyak, A.A. Ogloblin and I.R. Zhitnitsky, *Wave Functions of Octet Baryons*, *Yad. Fiz.* **48** (1988) 1410 [[INSPIRE](#)].
- [6] V.L. Chernyak, A.A. Ogloblin and I.R. Zhitnitsky, *Calculation of Exclusive Processes With Baryons*, *Yad. Fiz.* **48** (1988) 1398 [[INSPIRE](#)].
- [7] V.M. Braun et al., *Light-cone Distribution Amplitudes of the Nucleon and Negative Parity Nucleon Resonances from Lattice QCD*, *Phys. Rev. D* **89** (2014) 094511 [[arXiv:1403.4189](#)] [[INSPIRE](#)].
- [8] RQCD collaboration, *Light-cone distribution amplitudes of octet baryons from lattice QCD*, *Eur. Phys. J. A* **55** (2019) 116 [[arXiv:1903.12590](#)] [[INSPIRE](#)].
- [9] N.G. Stefanis and M. Bergmann, *On the Nucleon distribution amplitude: the Heterotic solution*, *Phys. Rev. D* **47** (1993) R3685 [[hep-ph/9211250](#)] [[INSPIRE](#)].
- [10] J. Bolz and P. Kroll, *Modeling the nucleon wave function from soft and hard processes*, *Z. Phys. A* **356** (1996) 327 [[hep-ph/9603289](#)] [[INSPIRE](#)].
- [11] S. Groote, J.G. Korner and O.I. Yakovlev, *An Analysis of diagonal and nondiagonal QCD sum rules for heavy baryons at next-to-leading order in alpha-s*, *Phys. Rev. D* **56** (1997) 3943 [[hep-ph/9705447](#)] [[INSPIRE](#)].
- [12] V. Braun, R.J. Fries, N. Mahnke and E. Stein, *Higher twist distribution amplitudes of the nucleon in QCD*, *Nucl. Phys. B* **589** (2000) 381 [[hep-ph/0007279](#)] [[INSPIRE](#)].
- [13] V.M. Braun, A. Lenz and M. Wittmann, *Nucleon Form Factors in QCD*, *Phys. Rev. D* **73** (2006) 094019 [[hep-ph/0604050](#)] [[INSPIRE](#)].
- [14] QCDSF collaboration, *Nucleon distribution amplitudes and proton decay matrix elements on the lattice*, *Phys. Rev. D* **79** (2009) 034504 [[arXiv:0811.2712](#)] [[INSPIRE](#)].
- [15] I.V. Anikin, V.M. Braun and N. Offen, *Nucleon Form Factors and Distribution Amplitudes in QCD*, *Phys. Rev. D* **88** (2013) 114021 [[arXiv:1310.1375](#)] [[INSPIRE](#)].
- [16] J.-Y. Kim, H.-C. Kim and M.V. Polyakov, *Light-cone distribution amplitudes of the nucleon and Δ baryon*, *JHEP* **11** (2021) 039 [[arXiv:2110.05889](#)] [[INSPIRE](#)].
- [17] C.-D. Lu et al., *Anatomy of the pQCD Approach to the Baryonic Decays $\Lambda_b \rightarrow p\pi, pK$* , *Phys. Rev. D* **80** (2009) 034011 [[arXiv:0906.1479](#)] [[INSPIRE](#)].
- [18] K.-S. Huang, W. Liu, Y.-L. Shen and F.-S. Yu, *$\Lambda_b \rightarrow p, N^*(1535)$ form factors from QCD light-cone sum rules*, *Eur. Phys. J. C* **83** (2023) 272 [[arXiv:2205.06095](#)] [[INSPIRE](#)].
- [19] J.-J. Han et al., *$\Lambda_b \rightarrow p$ transition form factors in perturbative QCD*, *Eur. Phys. J. C* **82** (2022) 686 [[arXiv:2202.04804](#)] [[INSPIRE](#)].

- [20] X. Ji, *Parton Physics on a Euclidean Lattice*, *Phys. Rev. Lett.* **110** (2013) 262002 [[arXiv:1305.1539](#)] [[INSPIRE](#)].
- [21] X. Ji, *Parton Physics from Large-Momentum Effective Field Theory*, *Sci. China Phys. Mech. Astron.* **57** (2014) 1407 [[arXiv:1404.6680](#)] [[INSPIRE](#)].
- [22] K. Cichy and M. Constantinou, *A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results*, *Adv. High Energy Phys.* **2019** (2019) 3036904 [[arXiv:1811.07248](#)] [[INSPIRE](#)].
- [23] Y. Zhao, *Unraveling high-energy hadron structures with lattice QCD*, *Int. J. Mod. Phys. A* **33** (2019) 1830033 [[arXiv:1812.07192](#)] [[INSPIRE](#)].
- [24] X. Ji et al., *Large-momentum effective theory*, *Rev. Mod. Phys.* **93** (2021) 035005 [[arXiv:2004.03543](#)] [[INSPIRE](#)].
- [25] J.-H. Zhang et al., *Pion Distribution Amplitude from Lattice QCD*, *Phys. Rev. D* **95** (2017) 094514 [[arXiv:1702.00008](#)] [[INSPIRE](#)].
- [26] LP3 collaboration, *Kaon Distribution Amplitude from Lattice QCD and the Flavor $SU(3)$ Symmetry*, *Nucl. Phys. B* **939** (2019) 429 [[arXiv:1712.10025](#)] [[INSPIRE](#)].
- [27] R. Zhang, C. Honkala, H.-W. Lin and J.-W. Chen, *Pion and kaon distribution amplitudes in the continuum limit*, *Phys. Rev. D* **102** (2020) 094519 [[arXiv:2005.13955](#)] [[INSPIRE](#)].
- [28] LATTICE PARTON collaboration, *Distribution Amplitudes of K^* and ϕ at the Physical Pion Mass from Lattice QCD*, *Phys. Rev. Lett.* **127** (2021) 062002 [[arXiv:2011.09788](#)] [[INSPIRE](#)].
- [29] LATTICE PARTON collaboration, *Pion and Kaon Distribution Amplitudes from Lattice QCD*, *Phys. Rev. Lett.* **129** (2022) 132001 [[arXiv:2201.09173](#)] [[INSPIRE](#)].
- [30] X. Gao et al., *Pion distribution amplitude at the physical point using the leading-twist expansion of the quasi-distribution-amplitude matrix element*, *Phys. Rev. D* **106** (2022) 074505 [[arXiv:2206.04084](#)] [[INSPIRE](#)].
- [31] K. Orginos, A. Radyushkin, J. Karpie and S. Zafeiropoulos, *Lattice QCD exploration of parton pseudo-distribution functions*, *Phys. Rev. D* **96** (2017) 094503 [[arXiv:1706.05373](#)] [[INSPIRE](#)].
- [32] A.V. Radyushkin, *Quasi-parton distribution functions, momentum distributions, and pseudo-parton distribution functions*, *Phys. Rev. D* **96** (2017) 034025 [[arXiv:1705.01488](#)] [[INSPIRE](#)].
- [33] Y.-Q. Ma and J.-W. Qiu, *Extracting Parton Distribution Functions from Lattice QCD Calculations*, *Phys. Rev. D* **98** (2018) 074021 [[arXiv:1404.6860](#)] [[INSPIRE](#)].
- [34] Y.-Q. Ma and J.-W. Qiu, *Exploring Partonic Structure of Hadrons Using *ab initio* Lattice QCD Calculations*, *Phys. Rev. Lett.* **120** (2018) 022003 [[arXiv:1709.03018](#)] [[INSPIRE](#)].
- [35] V.M. Braun, G.P. Korchemsky and D. Müller, *The Uses of conformal symmetry in QCD*, *Prog. Part. Nucl. Phys.* **51** (2003) 311 [[hep-ph/0306057](#)] [[INSPIRE](#)].
- [36] M.E. Peskin, *Anomalous Dimensions of Three Quark Operators*, *Phys. Lett. B* **88** (1979) 128 [[INSPIRE](#)].
- [37] G.P. Lepage and S.J. Brodsky, *Exclusive Processes in Quantum Chromodynamics: Evolution Equations for Hadronic Wave Functions and the Form-Factors of Mesons*, *Phys. Lett. B* **87** (1979) 359 [[INSPIRE](#)].

- [38] A.V. Efremov and A.V. Radyushkin, *Factorization and Asymptotical Behavior of Pion Form-Factor in QCD*, *Phys. Lett. B* **94** (1980) 245 [INSPIRE].
- [39] G. Martinelli et al., *A General method for nonperturbative renormalization of lattice operators*, *Nucl. Phys. B* **445** (1995) 81 [hep-lat/9411010] [INSPIRE].
- [40] M. Constantinou and H. Panagopoulos, *Perturbative renormalization of quasi-parton distribution functions*, *Phys. Rev. D* **96** (2017) 054506 [arXiv:1705.11193] [INSPIRE].
- [41] C. Sturm et al., *Renormalization of quark bilinear operators in a momentum-subtraction scheme with a nonexceptional subtraction point*, *Phys. Rev. D* **80** (2009) 014501 [arXiv:0901.2599] [INSPIRE].
- [42] C. Alexandrou et al., *A complete non-perturbative renormalization prescription for quasi-PDFs*, *Nucl. Phys. B* **923** (2017) 394 [arXiv:1706.00265] [INSPIRE].
- [43] I.W. Stewart and Y. Zhao, *Matching the quasiparton distribution in a momentum subtraction scheme*, *Phys. Rev. D* **97** (2018) 054512 [arXiv:1709.04933] [INSPIRE].
- [44] J.-W. Chen et al., *Parton distribution function with nonperturbative renormalization from lattice QCD*, *Phys. Rev. D* **97** (2018) 014505 [arXiv:1706.01295] [INSPIRE].
- [45] LP3 collaboration, *Improved parton distribution functions at the physical pion mass*, *Phys. Rev. D* **98** (2018) 054504 [arXiv:1708.05301] [INSPIRE].
- [46] Y.-S. Liu et al., *Matching the meson quasidistribution amplitude in the RI/MOM scheme*, *Phys. Rev. D* **99** (2019) 094036 [arXiv:1810.10879] [INSPIRE].
- [47] Y.-S. Liu et al., *Matching generalized parton quasidistributions in the RI/MOM scheme*, *Phys. Rev. D* **100** (2019) 034006 [arXiv:1902.00307] [INSPIRE].
- [48] LATTICE PARTON collaboration, *Unpolarized isovector quark distribution function from lattice QCD: A systematic analysis of renormalization and matching*, *Phys. Rev. D* **101** (2020) 034020 [arXiv:1807.06566] [INSPIRE].
- [49] X. Ji et al., *A Hybrid Renormalization Scheme for Quasi Light-Front Correlations in Large-Momentum Effective Theory*, *Nucl. Phys. B* **964** (2021) 115311 [arXiv:2008.03886] [INSPIRE].
- [50] LATTICE PARTON COLLABORATION (LPC) collaboration, *Self-renormalization of quasi-light-front correlators on the lattice*, *Nucl. Phys. B* **969** (2021) 115443 [arXiv:2103.02965] [INSPIRE].
- [51] C.-Y. Chou and J.-W. Chen, *One-loop hybrid renormalization matching kernels for quasiparton distributions*, *Phys. Rev. D* **106** (2022) 014507 [arXiv:2204.08343] [INSPIRE].
- [52] LATTICE PARTON COLLABORATION (LPC) collaboration, *Renormalization of Transverse-Momentum-Dependent Parton Distribution on the Lattice*, *Phys. Rev. Lett.* **129** (2022) 082002 [arXiv:2205.13402] [INSPIRE].
- [53] M. Constantinou and H. Panagopoulos, *Improved renormalization scheme for nonlocal operators*, *Phys. Rev. D* **107** (2023) 014503 [arXiv:2207.09977] [INSPIRE].
- [54] T. Izubuchi et al., *Factorization Theorem Relating Euclidean and Light-Cone Parton Distributions*, *Phys. Rev. D* **98** (2018) 056004 [arXiv:1801.03917] [INSPIRE].
- [55] M. Beneke and V.A. Smirnov, *Asymptotic expansion of Feynman integrals near threshold*, *Nucl. Phys. B* **522** (1998) 321 [hep-ph/9711391] [INSPIRE].