Lightcone and quasi distribution amplitudes for light octet and decuplet baryons

曾军（Jun Zeng)

Institute of Nuclear and Particle Physics/INPAC,
Shanghai Jiao Tong University, P.R. China

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Collaborators: Zhi-Fu Deng, Chao Han, Wei Wang, Jia-Lu Zhang (SJTU)

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Outline

上海交通大学
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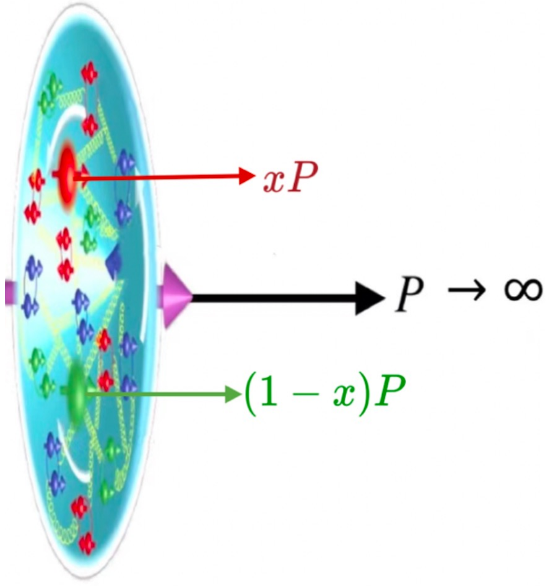
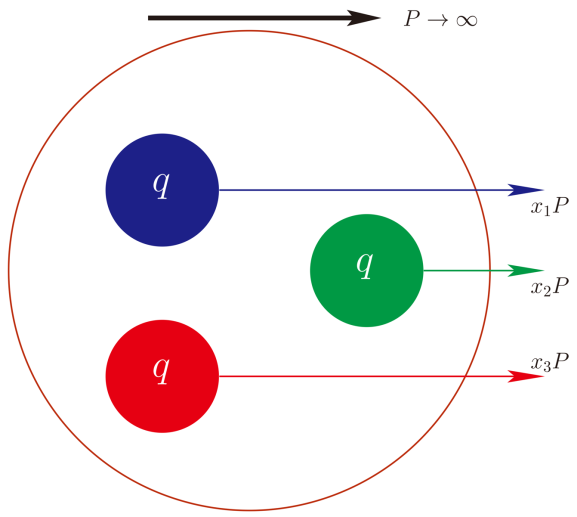
目 录

Background

- Feynman proposed a parton model more than 50 years ago, and hadron structure information has been obtained by fitting a large number of high-energy experimental data.
- The hadron light-cone distribution amplitude (LCDA) is a physical quantity that describes the momentum distribution of all parts of hadrons and reflects the internal structure of hadrons.
- The calculation of the hardron LCDA by the basic theory of the strong interaction has been slow for a long time.

Especially the baryon LCDA!

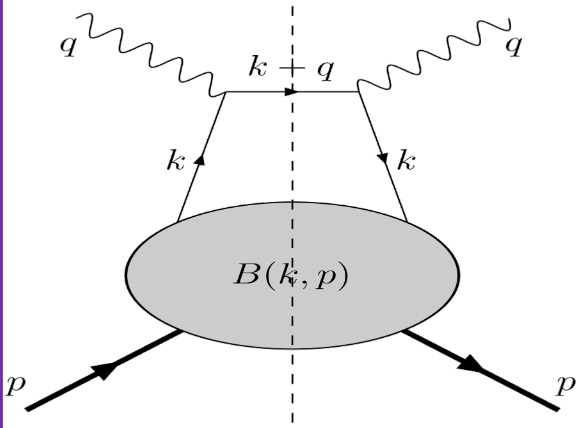




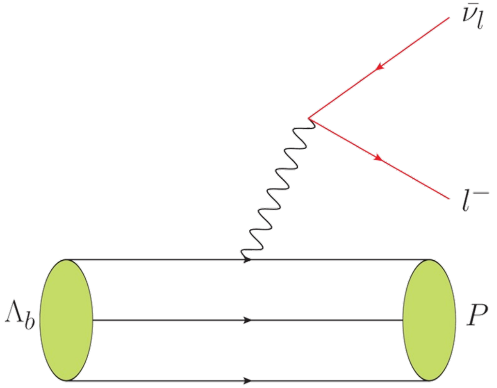
Background

- **Describing Internal Structure:** LCDAs provide detailed information about the distribution of quark and gluons inside hadrons, especially in high-energy collisions.
- **Calculating Hadronic Decay Processes:** They help theoretical physicists predict experimental outcomes and compare them with actual observations, thereby validating or refining existing theories.
- **Non-Perturbative QCD Studies:** Through lattice QCD and other non-perturbative methods, LCDAs can be calculated from first principles, providing deep insights into strong interactions.
- **Exploring New Physics:** In the search for new physics beyond the Standard Model, LCDAs offer a crucial theoretical framework.

LCDAs is very very important！



Inclusive: PDF



Exclusive: LCDAs

Baryon LCDA

Challenging:

1. Nonperturbative: cannot be calculated directly from first principles; must be extracted from experimental data or nonperturbative QCD approaches, such as lattice QCD simulations or phenomenological models.

2. Large number of degrees of freedom: The determination of LCDAs involves accounting for the contributions from all possible quark and gluon configurations within the baryon, leading to a large number of degrees of freedom and complicating the analysis.

3.Complexity of the operator product expansion (OPE): the OPE involves three-quark operators and gluonic operators, leading to a more complex and computationally challenging analysis compared to mesons.

4.Limited experimental data: limited experimental data for baryon structure observables, makes it challenging to constrain baryon light-cone distribution functions directly from the experiment.

Progress on baryon LCDA

Lattice QCD

1. **Nucleon distribution amplitudes and proton decay matrix elements on the lattice(Vladimir M. Braun, 2009) present a calculation of the first few moments of the leading-twist nucleon DA**
2. **Light-cone distribution amplitudes of the nucleon and negative parity nucleon resonances from lattice QCD(Vladimir M. Braun, 2014) calculate moments**
3. **Light-cone distribution amplitudes of octet baryons from lattice QCD (RQCD Collaboration,2019)**

Model

1. Modelling the Nucleon WF from Soft and hard processes(1996) parameterization
2. Nucleon distribution amplitude: The heterotic solution(1993)

   amalgamates features of the Chernyak-Ogloblin-Zhitnitsky model with those of the Cari-Stefanis model
3. Nucleon WF and Form Factors in QCD(1983)

   A model for the nucleon wave function is proposed based on a knowledge of these few first moments.
4. LCDA of the baryon(2021) Chiral quark-soliton model
5. Estimates of the isospin-violating Λb → Σ0φ;Σ0J=ψ decays and the Σ −Λ mixing(2023) COZ model

Progress on baryon LCDA

QCD sum rule

1. Nucleon WFs and QCD sum rules(1987)
2. Higher twist distribution amplitudes of the nucleon in QCD(2000)

   present the first systematic study of higher-twist LCDAs of the nucleon in QCD. Nonperturbative input parameters are estimated from QCD sum rules.
3. $\Lambda_b \to p$ transition form factors in perturbative QCD(2022)

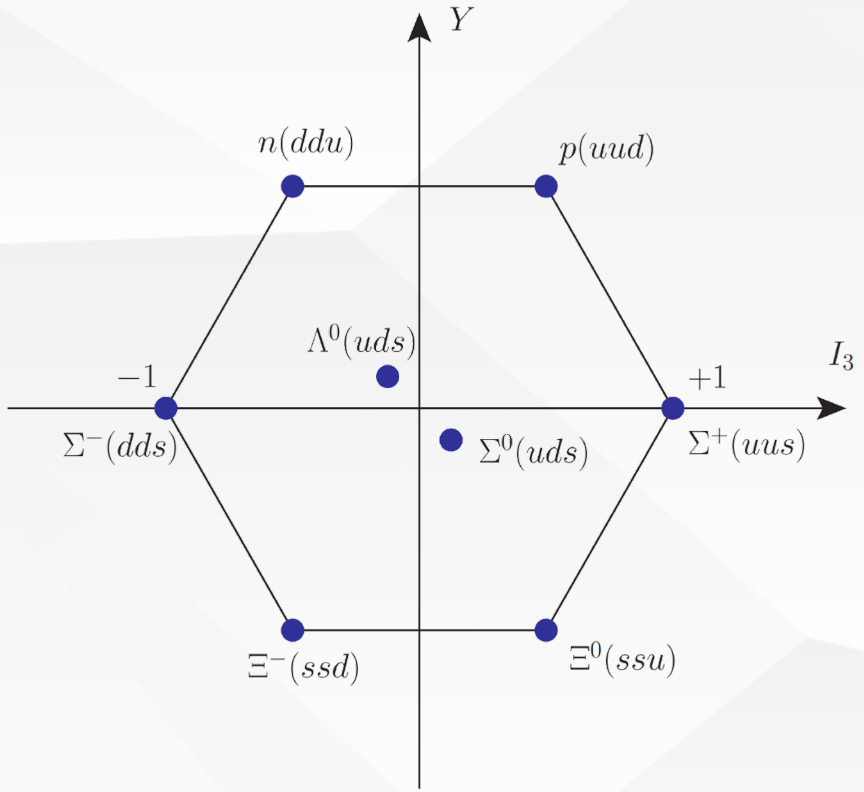
Ligh-cone Sum Rule

1. Nucleon form factors and distribution amplitudes in QCD(2013)

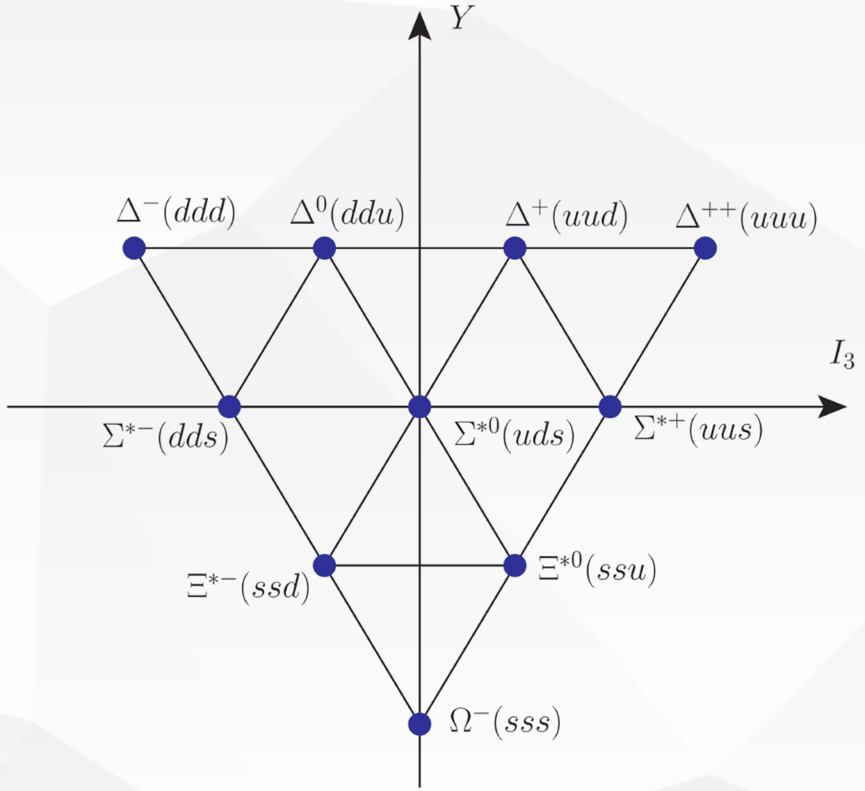
   extracted from the comparison with the experimental data on form factors
2. Wave functions of octet baryons(1989)

   The model wave functions are proposed which fulfill the sum rules requirements.
3. Nucleon form factors in QCD(2006)

2. Baryon LCDA



octet baryon



decuplet baryon

$$\varepsilon^{ijk} \times \langle 0| f_\alpha^{i'}(z_1)\, U_{i'i}(z_1, z_0)\, g_\beta^{j'}(z_2)\, U_{j'j}(z_2, z_0)\, h_\gamma^{k'}(z_3)\, U_{k'k}(z_3, z_0)\, |B(P_B, \lambda)\rangle$$

$$U(x, y) = \mathcal{P} \exp\left[ig \int_0^1 \mathrm{d}t (x-y)_\mu A^\mu(tx + (1-t)y) \right].$$

2. Baryon LCDA

At leading twist, the LCDAs for an octet baryon can be decomposed into three terms as

$$\langle 0 | f_\alpha (z_1 n) g_\beta (z_2 n) h_\gamma (z_3 n) | B(P_B, \lambda)\rangle$$
$$= \frac{1}{4} f_V \left[(\not{P}_B C)_{\alpha\beta} (\gamma_5 u_B)_\gamma \underline{V^B (z_i n \cdot P_B)} + (\not{P}_B \gamma_5 C)_{\alpha\beta} (u_B)_\gamma \underline{A^B (z_i n \cdot P_B)} \right]$$
$$+ \frac{1}{4} f_T (i\sigma_{\mu\nu} P_B^\nu C)_{\alpha\beta} (\gamma^\mu \gamma_5 u_B)_\gamma \underline{T^B (z_i n \cdot P_B)},$$

where $C \equiv i\gamma^2\gamma^0$ is the charge conjugation matrix, u_B stands for the spinor for the baryon and $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$. $f_{V/A/T}$ is the corresponding decay constant for each LCDA. For proton and neutron, $f_T = f_V$ due to the isospin symmetry.

$$\langle 0 | f^T (z_1 n) (C\not{n}) g(z_2 n) h(z_3 n) | B\rangle = -f_V V^B (z_i n \cdot P_B) P_B^+ \gamma_5 u_B,$$
$$\langle 0 | f^T (z_1 n) (C\gamma_5 \not{n}) g(z_2 n) h(z_3 n) | B\rangle = f_V A^B (z_i n \cdot P_B) P_B^+ u_B,$$
$$\langle 0 | f^T (z_1 n) (iC\sigma_{\mu\nu} n^\nu) g(z_2 n) \gamma^\mu h(z_3 n) | B\rangle = 2 f_T T^B (z_i n \cdot P_B) P_B^+ \gamma_5 u_B,$$

with the normalization：

$$V^{B\neq\Lambda}(0,0,0) = T^{B\neq\Lambda}(0,0,0) = 1, \quad A^{B\neq\Lambda}(0,0,0) = 0,$$
$$V^\Lambda(0,0,0) = T^\Lambda(0,0,0) = 0, \quad A^\Lambda(0,0,0) = 1.$$

2. Baryon LCDA

To fully exploit the benefits of SU(3) flavor symmetry it proves convenient to define the following set of DAs:

$$\Phi_{\pm}^{B\neq\Lambda}(x_{123}) = \frac{1}{2}\Big([V-A]^B(x_{123}) \pm [V-A]^B(x_{321})\Big),$$

$$\Pi^{B\neq\Lambda}(x_{123}) = T^B(x_{132}),$$

$$\Phi_{+}^{\Lambda}(x_{123}) = \sqrt{\frac{1}{6}}\Big([V-A]^\Lambda(x_{123}) + [V-A]^\Lambda(x_{321})\Big),$$

$$\Phi_{-}^{\Lambda}(x_{123}) = -\sqrt{\frac{3}{2}}\Big([V-A]^\Lambda(x_{123}) - [V-A]^\Lambda(x_{321})\Big),$$

$$\Pi^{\Lambda}(x_{123}) = \sqrt{6}\,T^\Lambda(x_{132}),$$

asymptotic form：

$$\begin{aligned}
\Phi_+^B &= 120 x_1 x_2 x_3 \left(\varphi_{00}^B \mathcal{P}_{00} + \varphi_{11}^B \mathcal{P}_{11} + \ldots\right),\\
\Phi_-^B &= 120 x_1 x_2 x_3 \left(\varphi_{10}^B \mathcal{P}_{10} + \ldots\right),\\
\Pi^{B\neq\Lambda} &= 120 x_1 x_2 x_3 \left(\pi_{00}^B \mathcal{P}_{00} + \pi_{11}^B \mathcal{P}_{11} + \ldots\right),\\
\Pi^{\Lambda} &= 120 x_1 x_2 x_3 \left(\pi_{10}^\Lambda \mathcal{P}_{10} + \ldots\right).
\end{aligned}$$

2. Baryon LCDA

Similarly, the LCDA for a decuplet baryon can be decomposed into four terms as

$$\begin{aligned}&\langle 0|f_\alpha(z_1 n)g_\beta(z_2 n)h_\gamma(z_3 n)|B(P_B,\lambda)\rangle\\&=\frac{1}{4}\lambda_V\left[(\gamma_\mu C)_{\alpha\beta}\Delta^\mu_\gamma V^B(z_i n\cdot P_B)+(\gamma_\mu\gamma_5 C)_{\alpha\beta}(\gamma_5\Delta^\mu)_\gamma A^B(z_i n\cdot P_B)\right]\\&-\frac{1}{8}\lambda_T(i\sigma_{\mu\nu}C)_{\alpha\beta}(\gamma^\mu\Delta^\nu)_\gamma T^B(z_i n\cdot P_B)-\frac{1}{4}\lambda_\varphi\left[(i\sigma_{\mu\nu}C)_{\alpha\beta}\left(P_B^\mu\Delta^\nu-\frac{1}{2}M_B\gamma^\mu\Delta^\nu\right)_\gamma\varphi^B(z_i n\cdot P_B)\right]\end{aligned}$$

The spin-3/2 vector can be expressed with spinors and polarized vectors:

$$\Delta_{\mu\gamma}(p,\lambda=1/2)=\sqrt{\frac{2}{3}}e^0_\mu(p)u^\uparrow_\gamma(p)+\sqrt{\frac{1}{3}}e^{+1}_\mu(p)u^\downarrow_\gamma(p),\quad \Delta^\mu_\gamma(p,\lambda=3/2)=e^{+1}_\mu(p)u^\uparrow_\gamma(p).$$

Up to leading twist, the light baryon mass can be neglected $M_B\simeq 0$ and the polarized vectors are $e^0_\mu\simeq\bar{n}$; $e^+_\mu=(0,1,-i,0)$; $e^-_\mu=(0,1,i,0)$. $\lambda_{V/A/T/\varphi}$ is the corresponding decay constant for each LCDA. In the decomposition of decuplet baryons, V,A,T correspond to helicity-1/2 state while φ corresponds to helicity-3/2 state.

2. Baryon LCDA

For the decuplet baryons, we have

$$\left\langle 0\left|f^T(z_1n)(C\not{n})g(z_2n)h(z_3n)\right|B\left(P_B,\frac{1}{2}\right)\right\rangle = -\lambda_V V^B(z_in\cdot P_B)\gamma_5(n\cdot\Delta),$$

$$\left\langle 0\left|f^T(z_1n)(C\gamma_5\not{n})g(z_2n)h(z_3n)\right|B\left(P_B,\frac{1}{2}\right)\right\rangle = \lambda_V A^B(z_in\cdot P_B)(n\cdot\Delta),$$

$$\left\langle 0\left|f^T(z_1n)(iC\sigma_{\mu\nu}n^\nu)g(z_2n)\gamma^\mu h(z_3n)\right|B\left(P_B,\frac{1}{2}\right)\right\rangle = -\lambda_T T^B(z_in\cdot P_B)(n\cdot\Delta),$$

$$\left\langle 0\left|f^T(z_1n)(iC\sigma^{\mu\nu}n_\mu)g(z_2n)h(z_3n)\right|B\left(P_B,\frac{3}{2}\right)\right\rangle = -\lambda_\varphi\varphi^B(z_in\cdot P_B)P_B^+\Delta^\nu,$$

with the normalization $V^\Lambda(0,0,0)=T^\Lambda(0,0,0)=\varphi^B(0,0,0)=1, \quad A^\Lambda(0,0,0)=0.$

$$V^B(x_1,x_2,x_3)=V^B(x_2,x_1,x_3),$$
$$T^B(x_1,x_2,x_3)=T^B(x_2,x_1,x_3),$$

$$A^B(x_1,x_2,x_3)=-A^B(x_2,x_1,x_3),$$
$$\varphi^B(x_1,x_2,x_3)=\varphi^B(x_2,x_1,x_3).$$

other than Σ^{*0}

When isospin symmetry is assumed, it also holds for Σ^{*0}.

Further relation:

$$T^B(x_1,x_2,x_3)=[V-A]^B(x_2,x_3,x_1),$$
$$\varphi^B(x_1,x_2,x_3)=\varphi^B(x_2,x_3,x_1)=\varphi^B(x_3,x_1,x_2),$$

which indicates that φ^B is totally symmetric.

2. Baryon LCDA

- LCDAs are defined as the correlation functions of light-cone operators inside a hadron, these quantities can not be directly evaluated on the lattice.
- It is highly indispensable to develop a method to calculate the full shape of baryon LCDAs from the first principle of QCD.
- A very inspiring approach was proposed to circumvent this problem and is now formulated as the ***large-momentum effective theory*** (LaMET).

X. Ji, Phys.Rev.Lett. 110, 262002 (2013)
X. Ji, Sci. China Phys. Mech. Astron. 57, 1407-1412 (2014)
X. Ji, et al., Rev.Mod.Phys. 93 (2021)

2. Baryon quasi distribution amplitudes(quai-DAs)

the definitions of quasi-DAs for octet baryons B: Chernyak, Zhitnitsky (1984)

$$\widetilde{M}_V^B(z_1,z_2,z_3,P_B^z)=\left\langle 0\left|f^T(z_1n_z)(C\gamma^z)g(z_2n_z)h(z_3n_z)\right|B(P_B,\lambda=\frac{1}{2})\right\rangle=-f_V\tilde{V}^B(z_1,z_2,z_3,P_B^z)P_B^z\gamma_5u_B,$$

$$\widetilde{M}_A^B(z_1,z_2,z_3,P_B^z)=\left\langle 0\left|f^T(z_1n_z)(C\gamma_5\gamma^z)g(z_2n_z)h(z_3n_z)\right|B(P_B,\lambda=\frac{1}{2})\right\rangle=f_A\tilde{A}^B(z_1,z_2,z_3,P_B^z)P_B^zu_B,$$

$$\widetilde{M}_T^B(z_1,z_2,z_3,P_B^z)=\left\langle 0\left|f^T(z_1n_z)(\frac{1}{2}C[\gamma^z,\gamma^\mu])g(z_2n_z)\gamma_\mu h(z_3n_z)\right|B(P_B,\lambda=\frac{1}{2})\right\rangle=2f_T\tilde{T}^B(z_1,z_2,z_3,P_B^z)P_B^z\gamma_5u_B,$$

the coordinates are defined as $z_i^\mu=z_in_z^\mu$, where $n_z^\mu=(0,0,0,1)$.

the definitions of quasi-DAs for decuplet baryons B: Farrar, Zhang, Ogloblin, Zhitnitsky (1989)

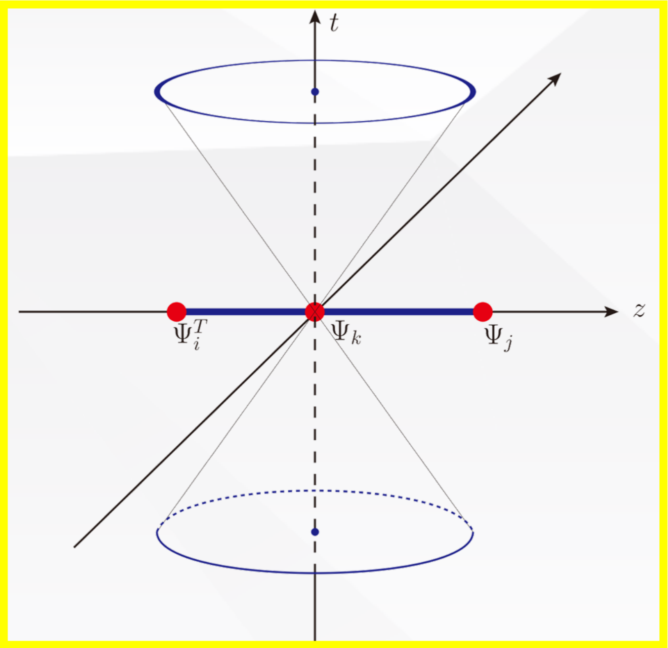
$$\widetilde{M}_V^B(z_1,z_2,z_3,P_B^z)=\left\langle 0\left|(f(z_1n_z))^T(C\gamma^z)g(z_2n_z)h(z_3n_z)\right|B(P_B,\lambda=\frac{1}{2})\right\rangle=-\lambda_V\tilde{V}^B(z_1,z_2,z_3,P_B^z)\gamma_5(n_z\cdot\Delta),$$

$$\widetilde{M}_A^B(z_1,z_2,z_3,P_B^z)=\left\langle 0\left|(f(z_1n_z))^T(C\gamma_5\gamma^z)g(z_2n_z)h(z_3n_z)\right|B(P_B,\lambda=\frac{1}{2})\right\rangle=\lambda_A\tilde{A}^B(z_1,z_2,z_3,P_B^z)(n_z\cdot\Delta),$$

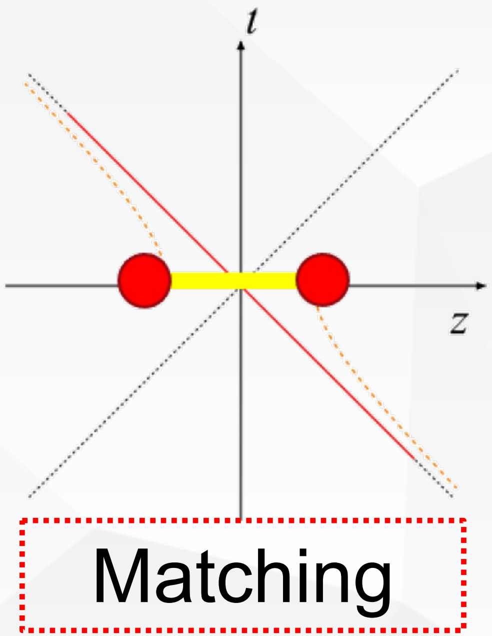
$$\widetilde{M}_T^B(z_1,z_2,z_3,P_B^z)=\left\langle 0\left|(f(z_1n_z))^T(\frac{1}{2}C[\gamma^z,\gamma^\mu])g(z_2n_z)\gamma_\mu h(z_3n_z)\right|B(P_B,\lambda=\frac{1}{2})\right\rangle=-\lambda_T\tilde{T}^B(z_1,z_2,z_3,P_B^z)(n_z\cdot\Delta).$$

$$\widetilde{M}_\varphi^B(z_1,z_2,z_3,P_B^z)=\left\langle 0\left|(f(z_1n_z))^T\left(\frac{1}{2}C[\gamma^\nu,\gamma^z]\right)g(z_2n_z)h(z_3n_z)\right|B(P_B,\lambda=\frac{3}{2})\right\rangle=-\lambda_\varphi\tilde{\varphi}^B(z_1,z_2,z_3,P_B^z)\Delta^\nu.$$

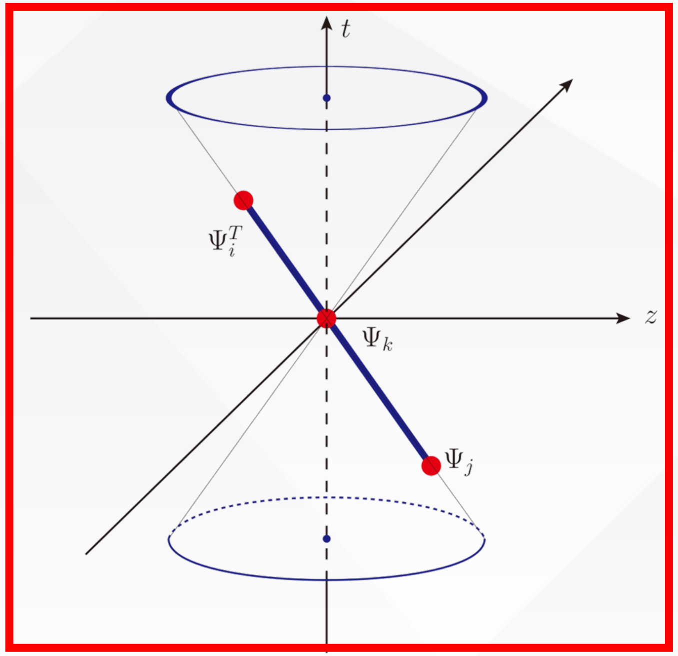
2. Baryon LCDA: Large-momentum effective theory (LaMET)



equal time
quasi distribution



Lorentz boost



light-cone
Light-cone distribution

The LCDA can be extracted by matching the quasi-DA to light cone direction by perturbation.

2. Baryon LCDA

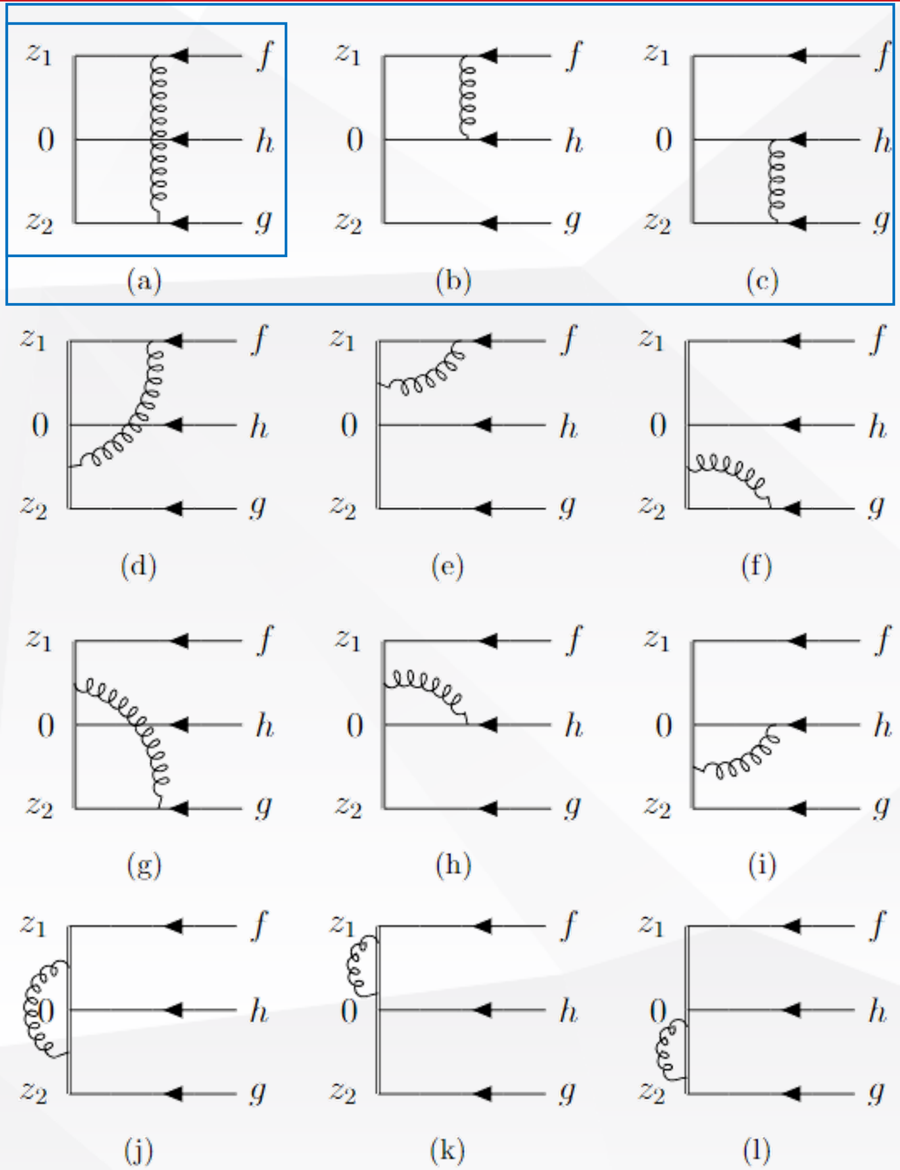
$$\tilde{\Phi}^B\left(x_1, x_2, P_B^z, \mu\right)=\int d y_1 d y_2 \mathcal{C}\left(x_1, x_2, y_1, y_2, P_B^z, \mu\right) \Phi^B\left(y_1, y_2, \mu\right)+\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{\left(x_1 P_B^z\right)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{\left(x_2 P_B^z\right)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{\left(\left(1-x_1-x_2\right) P_B^z\right)^2}\right).$$

Here, P_B^z is the momentum of the hadron along the z direction, $\tilde{\Phi}$ stands for $\tilde{V}$, $\tilde{A}$, $\tilde{T}$ and $\tilde{\varphi}$, the corresponding quasi-DAs of V, A, T and φ, and the $x_3(y_3)$ argument is omitted for simplicity. $\mathcal{C}(x_1, x_2, y_1, y_2, P_B^z, \mu)$, referred to as the hard kernel, compensates for the UV difference between LCDAs and quasi-DAs.

Matching kernel is insensitive to the hadrons, in the calculation of LCDAs one can replace the hadron by the partonic state.

$$\left|B\left(P_B, \lambda\right)\right\rangle \longrightarrow \frac{\epsilon_{abc}}{6}\left|f_a\left(x_1 P\right) g_b\left(x_2 P\right) h_c\left(x_3 P\right)\right\rangle$$

the quark state is chosen to have the same J^{PC} with the baryon.



IR

IR&UV

IR&UV

UV

T wave: No diagram (a).

φ wave: No diagram (a-c)

No Wilson line self-energy in LCDA.

2. Baryon quasi distribution amplitude(quai-DA)

Normalization:

$$\widehat{\mathcal{M}}_{V,A,T,\varphi}(z_1, z_2, z_3 = 0, P^z, \mu) = \frac{\widetilde{\mathcal{M}}_{V,A,T,\varphi}(z_1, z_2, 0, P^z, \mu)}{\widetilde{\mathcal{M}}_{V,A,T,\varphi}(0, 0, 0, P^z, \mu)}$$

hybrid renormalization scheme: Ratio scheme

$$\mathfrak{M}_{V,A,T,\varphi}(z_1, z_2, 0, P^z, \mu) = \frac{\widehat{M}_{V,A,T,\varphi}(z_1, z_2, 0, P^z, \mu)}{\widehat{M}_{V,A,T,\varphi}(z_1, z_2, 0, 0, \mu)}.$$ remove the UV poles.

The renormalized quasi-DAs in momentum space are defined as

$$\tilde{\Psi}_H^{V,A,T,\varphi} = \int_{-\infty}^{+\infty} \frac{P^z d\, z_1}{2\pi} \frac{P^z d\, z_2}{2\pi} \times e^{-ix_1 P^z z_1 - ix_2 P^z z_2} \times \mathfrak{M}_{V,A,T,\varphi}(z_1, z_2, 0, P^z, \mu)$$

SJTU

2. Baryon LCDA

The lightcone correlator can be normalized by dividing it by its zero-momentum matrix element:

$$\mathfrak{I}_{V,A,T,\varphi}(\nu_1,\nu_2,\nu_3,\mu) \equiv \frac{\mathcal{I}_{V,A,T,\varphi}(\nu_1,\nu_2,\nu_3,\mu)}{\mathcal{I}_{V,A,T,\varphi}(0,0,0,\mu)}.$$

The LCDAs with Fock states in momentum space are defined by

$$\Psi^{V,A,T,\varphi}_{\overline{\text{MS}}} = \int_{-\infty}^{+\infty} \frac{n\cdot P d\, z_1}{2\pi}\frac{n\cdot P d\, z_2}{2\pi} \times e^{ix_1 n\cdot P z_1 + ix_2 n\cdot P z_2} \times \mathfrak{I}_{V,A,T,\varphi}\left(z_1 n\cdot P, z_2 n\cdot P, 0, \mu\right)$$

3. Matching

- **Partonic state**

$$\widetilde{\Psi}_H^{V,A,T,\varphi}(x_1, x_2, P^z, \mu) = \int dy_1 dy_2 \mathcal{C}_{V,A,T,\varphi}(x_1, x_2, y_1, y_2, P^z, \mu) \Psi_{\overline{\mathrm{MS}}}^{V,A,T,\varphi}(y_1, y_2, \mu)$$

- **Hadron state**

$$\tilde{\Phi}_H^B(x_1, x_2, P^z, \mu) = \int dy_1 dy_2 \mathcal{C}_{V,A,T,\varphi}(x_1, x_2, y_1, y_2, P^z, \mu) \Phi_{\overline{\mathrm{MS}}}^B(y_1, y_2, \mu)$$

- **Partonic state**

pQCD

pQCD

$$\widetilde{\Psi}_{H}^{V,A,T,\varphi}(x_1,x_2,P^z,\mu)=\int dy_1 dy_2 \mathcal{C}_{V,A,T,\varphi}(x_1,x_2,y_1,y_2,P^z,\mu)\Psi_{\overline{\mathrm{MS}}}^{V,A,T,\varphi}(y_1,y_2,\mu)$$

- **Hadron state**

Target:LCDA

$$\tilde{\Phi}_{H}^{B}(x_1,x_2,P^z,\mu)=\int dy_1 dy_2 \mathcal{C}_{V,A,T,\varphi}(x_1,x_2,y_1,y_2,P^z,\mu)\Phi_{\overline{\mathrm{MS}}}^{B}(y_1,y_2,\mu)$$

Matching

Lattice QCD

3. Matching

$$\mathcal{C}_{V,A}\left(x_1,x_2,y_1,y_2,P^z,\mu\right)=\delta\left(x_1-y_1\right)\delta\left(x_2-y_2\right)+\frac{\alpha_s C_F}{4\pi}C_{1V,A}\left(x_1,x_2,y_1,y_2,P^z,\mu\right)$$
$$+\frac{\alpha_s C_F}{4\pi}\times\left[C_2\left(x_1,x_2,y_1,y_2,P^z,\mu\right)\delta\left(x_2-y_2\right)\right.$$
$$\left.+C_3\left(x_1,x_2,y_1,y_2,P^z,\mu\right)\delta\left(x_3-y_3\right)+\left\{x_1\leftrightarrow x_2,y_1\leftrightarrow y_2\right\}\right]_{\oplus}$$

$$\mathcal{C}_{T}\left(x_1,x_2,y_1,y_2,P^z,\mu\right)=\delta\left(x_1-y_1\right)\delta\left(x_2-y_2\right)+\frac{\alpha_s C_F}{4\pi}C_{1T}\left(x_1,x_2,y_1,y_2,P^z,\mu\right)$$
$$+\frac{\alpha_s C_F}{4\pi}\times\left[C_2\left(x_1,x_2,y_1,y_2,P^z,\mu\right)\delta\left(x_2-y_2\right)\right.$$
$$\left.+(C_3-C_5)\left(x_1,x_2,y_1,y_2,P^z,\mu\right)\delta\left(x_3-y_3\right)+\left\{x_1\leftrightarrow x_2,y_1\leftrightarrow y_2\right\}\right]_{\oplus}$$

$$\mathcal{C}_{\varphi}\left(x_1,x_2,y_1,y_2,P^z,\mu\right)=\delta\left(x_1-y_1\right)\delta\left(x_2-y_2\right)+\frac{\alpha_s C_F}{4\pi}C_{1\varphi}\left(x_1,x_2,y_1,y_2,P^z,\mu\right)$$
$$+\frac{\alpha_s C_F}{4\pi}\times\left[(C_2-C_4)\left(x_1,x_2,y_1,y_2,P^z,\mu\right)\delta\left(x_2-y_2\right)\right.$$
$$\left.+(C_3-C_5)\left(x_1,x_2,y_1,y_2,P^z,\mu\right)\delta\left(x_3-y_3\right)+\left\{x_1\leftrightarrow x_2,y_1\leftrightarrow y_2\right\}\right]_{\oplus}$$

$$\left[g\left(x_1,x_2,y_1,y_2\right)\right]_{\oplus}=g\left(x_1,x_2,y_1,y_2\right)-\delta\left(x_1-y_1\right)\delta\left(x_2-y_2\right)\int dx_1'dx_2'g\left(x_1',x_2',y_1,y_2\right).$$

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4. Expansion by regions

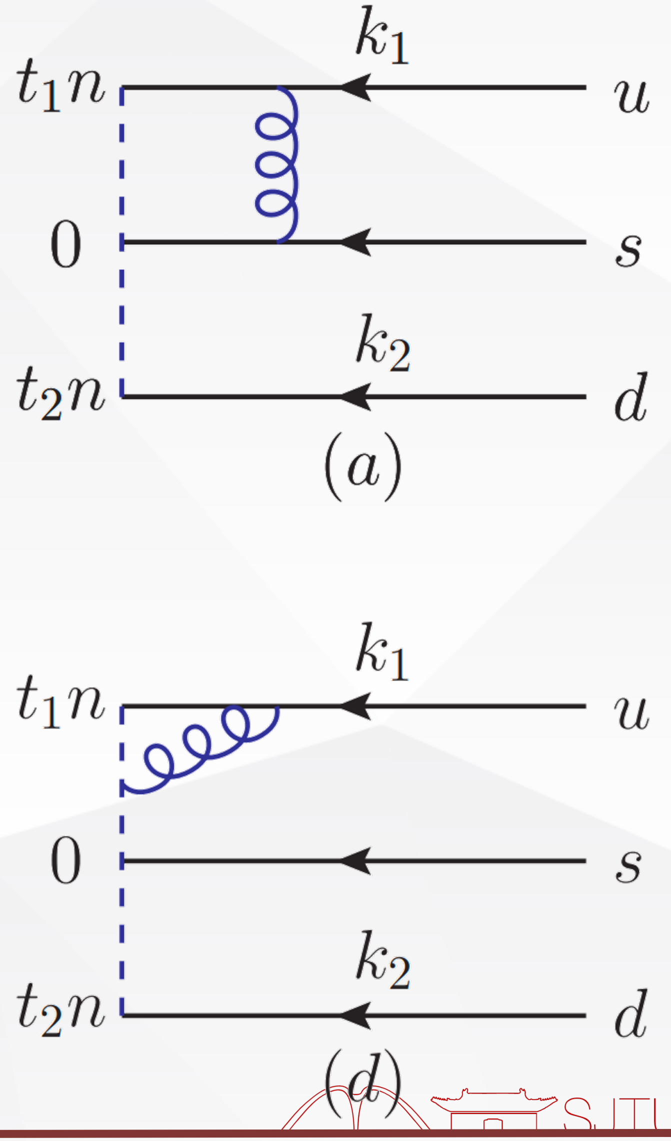
$$q^\mu = (q^+, q^-, q_\perp)$$

Expansion by regions:

- ✓ **Hard:** $q^\mu \sim (Q, Q, Q)$
- ✓ **Collinear:** $q^\mu \sim (Q, \Lambda^2/Q, \Lambda)$
- ✓ **Soft:** $q^\mu \sim (\Lambda, \Lambda, \Lambda)$

$$\tilde{\phi}^a_{(1/0)} = i g^2 \frac{C_F}{2} p^z \delta(x_2 - x_{2,0}) \int \frac{d^4 q}{(2\pi)^4} \frac{\delta(x_1 p^z - q^z - k_1{}^z)}{(q+k_1)^2 + i\epsilon} \frac{1}{q^2 + i\epsilon} \frac{q_\perp^2}{(k_s - q)^2 + i\epsilon}$$

$$\tilde{\phi}^d_{(1/0)}|_C = i g^2 C_F p^z \left[\delta(x_2 - x_{2,0}) \int \frac{d^4 q}{(2\pi)^4} \frac{\delta(x_1 p^z - q^z - k_1{}^z)}{(k_1+q)^2 + i\epsilon} \frac{1}{q^2 + i\epsilon} \frac{2k_1^z + q^0 + q^z}{q^z} \right]_\oplus$$



4. Expansion by regions

$$q^\mu = (q^+, q^-, q_\perp)$$

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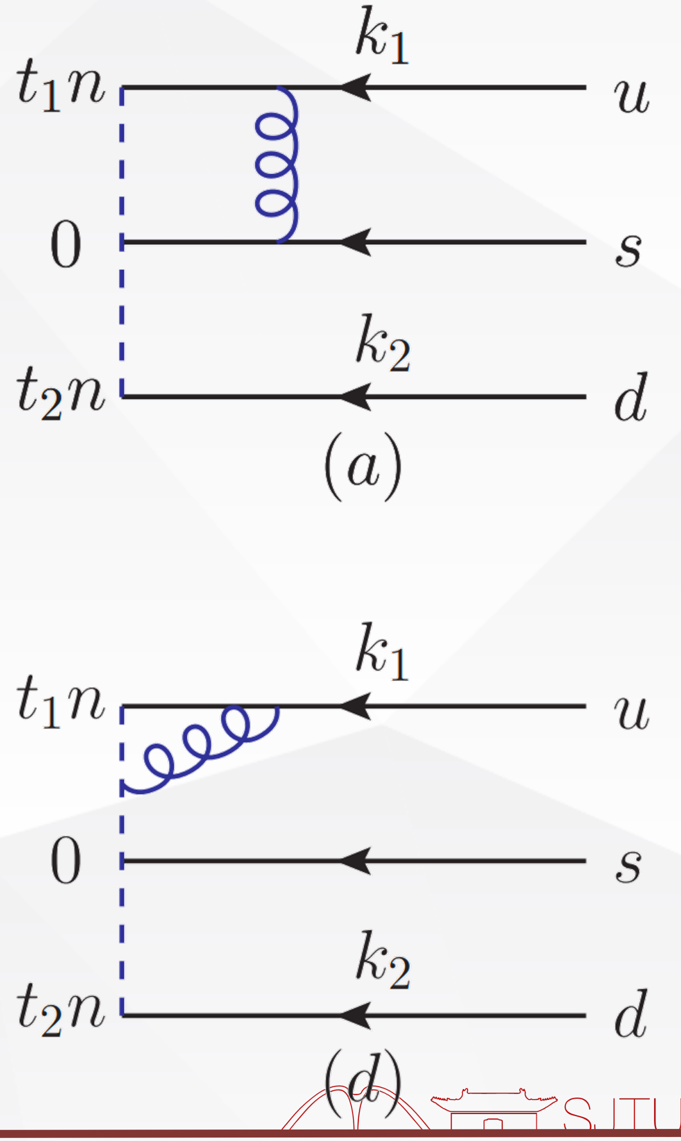
$$\tilde{\phi}^a_{(1/0)} = ig^2 \frac{C_F}{2} p^z \delta(x_2 - x_{2,0}) \int \frac{d^4 q}{(2\pi)^4} \frac{\delta(x_1 p^z - q^z - k_1{}^z)}{(q+k_1)^2 + i\epsilon} \frac{1}{q^2 + i\epsilon} \frac{q_\perp^2}{(k_s - q)^2 + i\epsilon}$$

$$\tilde{\phi}^d_{(1/0)}|_C = ig^2 C_F p^z \left[\delta(x_2 - x_{2,0}) \int \frac{d^4 q}{(2\pi)^4} \frac{\delta(x_1 p^z - q^z - k_1{}^z)}{(k_1+q)^2 + i\epsilon} \frac{1}{q^2 + i\epsilon} \frac{2k_1^z + q^0 + q^z}{q^z} \right]_\oplus$$

Leading Power!



4. Expansion by regions

✓ **Collinear:** $q^\mu \sim (Q, \Lambda^2/Q, \Lambda)$ quasi-DA = LCDA

quasi

$$\tilde{\phi}^a_{(1/0)}|_C = ig^2\frac{C_F}{2}p^z\delta(x_2-x_{2,0})\int\frac{d^4q}{(2\pi)^4}\frac{\delta(x_1p^z-q^z-k_1{}^z)}{(q+k_1)^2+i\epsilon}\frac{1}{q^2+i\epsilon}\frac{q_\perp^2}{(k_3-q)^2+i\epsilon}|_C$$

$$= ig^2\frac{C_F}{2}\frac{p^+-p^-}{\sqrt{2}}\delta(x_2-x_{2,0})\int\frac{d^4q}{(2\pi)^4}\frac{\sqrt{2}\delta((x_1p^+-q^+-k_1{}^+)-(x_1p^--q^--k_1{}^-))}{(q+k_1)^2+i\epsilon}\frac{1}{q^2+i\epsilon}\frac{q_\perp^2}{(k_3-q)^2+i\epsilon}|_C$$

$$= ig^2\frac{C_F}{2}p^+\delta(x_2-x_{2,0})\int\frac{d^4q}{(2\pi)^4}\frac{\delta(x_1p^+-q^+-k_1{}^+)}{(q+k_1)^2+i\epsilon}\frac{1}{q^2+i\epsilon}\frac{q_\perp^2}{(k_3-q)^2+i\epsilon}$$

$$= \phi^a_{(1/0)}.$$ Light-cone

quasi

$$\tilde{\phi}^d_{(1/0)}|_C = ig^2C_Fp^z\left[\delta(x_2-x_{2,0})\int\frac{d^4q}{(2\pi)^4}\frac{\delta(x_1p^z-q^z-k_1{}^z)}{(k_1+q)^2+i\epsilon}\frac{1}{q^2+i\epsilon}\frac{2k_1^z+q^0+q^z}{q^z}\right]_\oplus|_C$$

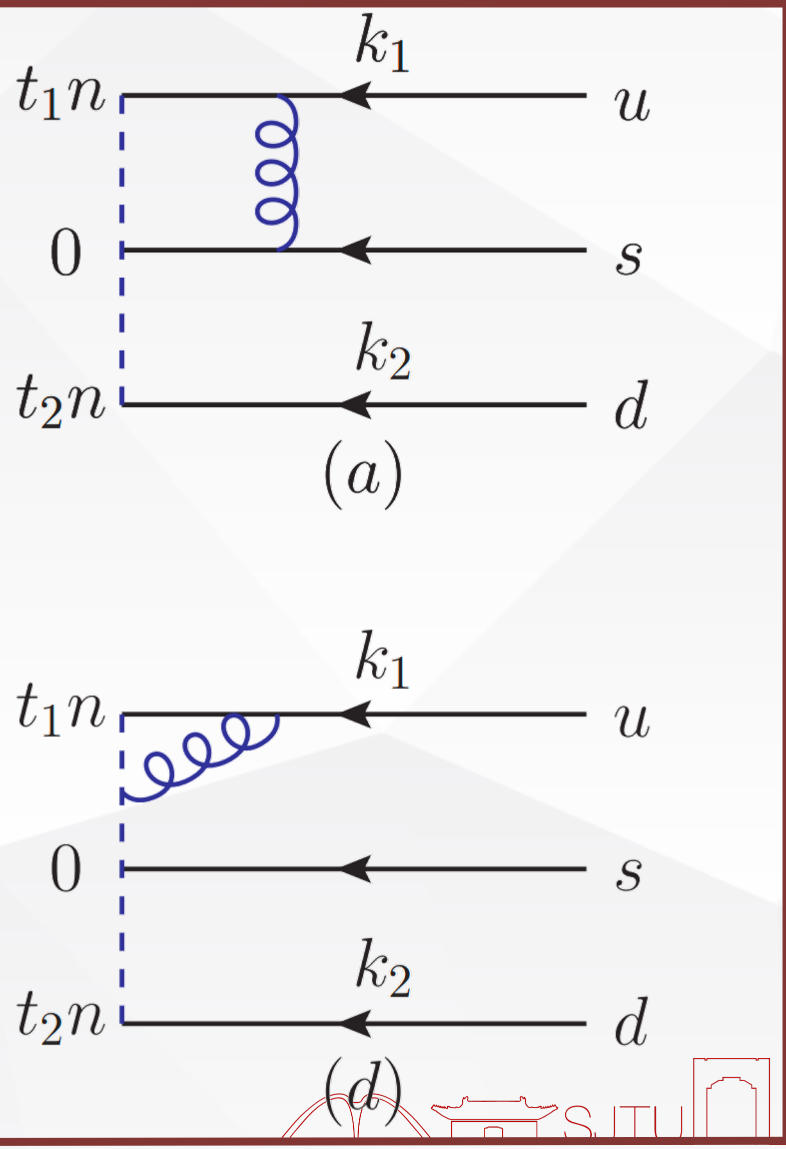
$$= ig^2C_F\frac{p^+}{\sqrt{2}}\left[\delta(x_2-x_{2,0})\int\frac{d^4q}{(2\pi)^4}\frac{\delta(\frac{x_1p^+}{\sqrt{2}}-\frac{q^+-q^-}{\sqrt{2}}-\frac{k_1^+}{\sqrt{2}})}{(k_1+q)^2+i\epsilon}\frac{1}{q^2+i\epsilon}\frac{2\frac{k_1^+}{\sqrt{2}}+\frac{q^++q^-}{\sqrt{2}}+\frac{q^+-q^-}{\sqrt{2}}}{\frac{q^+-q^-}{\sqrt{2}}}\right]_\oplus|_C$$

$$= ig^2C_F\frac{p^+}{\sqrt{2}}\left[\delta(x_2-x_{2,0})\int\frac{d^4q}{(2\pi)^4}\frac{\sqrt{2}\delta(x_1p^+-q^+-k_1^+)}{(k_1+q)^2+i\epsilon}\frac{1}{q^2+i\epsilon}\frac{2k_1^++q^++q^+}{q^+}\right]_\oplus$$

$$= ig^2C_Fp^+\left[\delta(x_2-x_{2,0})\int\frac{d^4q}{(2\pi)^4}\frac{\delta[(x_1-x_{1,0})p^+-q^+]}{(k_1+q)^2+i\epsilon}\frac{1}{q^2+i\epsilon}\frac{2(k_1^++q^+)}{q^+}\right]_\oplus$$

$$= \phi^d_{(1/0)}.$$ Light-cone

Leading Power!



4. Expansion by regions

$$q^\mu = (q^+, q^-, q_\perp)$$

Expansion by regions:

- ✓ **Hard: $q^\mu \sim (Q, Q, Q)$**
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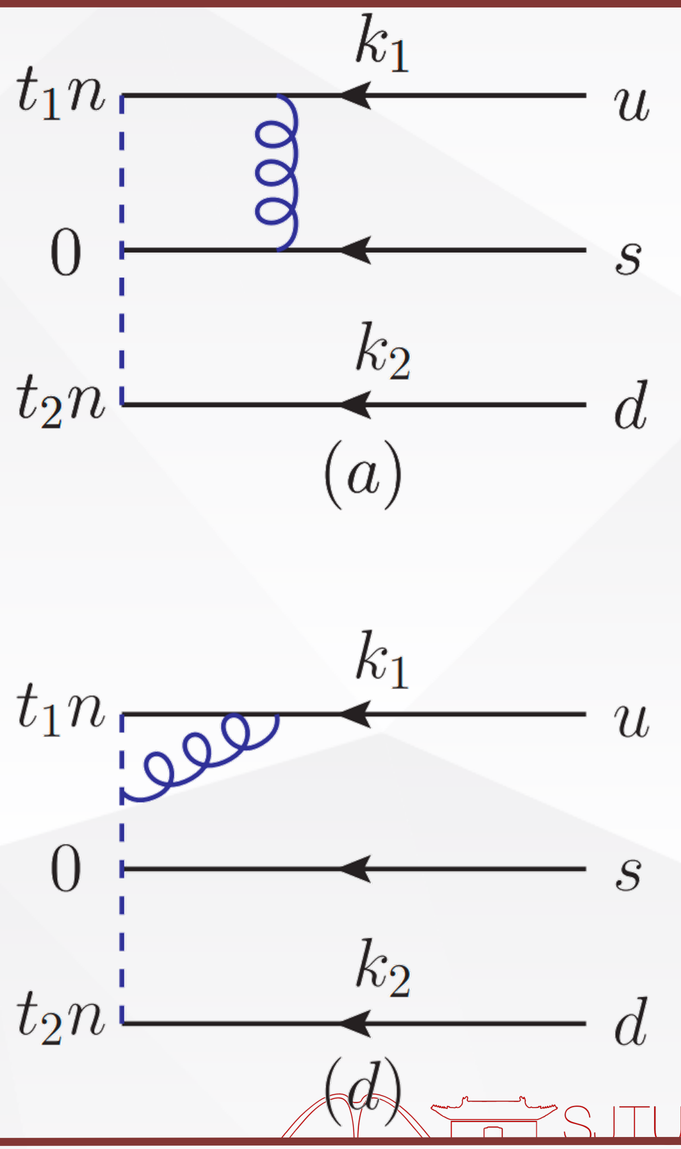
- ✓ **Soft: $q^\mu \sim (\Lambda, \Lambda, \Lambda)$**

$$\tilde{\phi}^a_{(1/0)} = ig^2 \frac{C_F}{2} p^z \delta(x_2 - x_{2,0}) \int \frac{d^4q}{(2\pi)^4} \frac{\delta(x_1 p^z - q^z - k_1{}^z)}{(q+k_1)^2 + i\epsilon} \frac{1}{q^2 + i\epsilon} \frac{q_\perp^2}{(k_s - q)^2 + i\epsilon}$$

$$\tilde{\phi}^d_{(1/0)}|_C = ig^2 C_F p^z \left[\delta(x_2 - x_{2,0}) \int \frac{d^4q}{(2\pi)^4} \frac{\delta(x_1 p^z - q^z - k_1{}^z)}{(k_1+q)^2 + i\epsilon} \frac{1}{q^2 + i\epsilon} \frac{2k_1^z + q^0 + q^z}{q^z} \right]_\oplus$$

$\mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)$ Power suprresed!



4. Expansion by regions

- The one-loop LCDA and quasi-DA for baryon does not contain the soft contributions.
- The one-loop quasi-DA contain the collinear and hard mode.
- QCD factorization shows that the hard and collinear modes in the quasi-DA can be factorized into a convolution of the hard matching coefficient and the LCDA which only contains collinear modes.

Hard Mode + Collinear Mode

Collinear Mode

$$\boxed{\tilde{\Phi}^B\left(x_1, x_2, P_B^z, \mu\right)} = \int dy_1 dy_2 \boxed{\mathcal{C}\left(x_1, x_2, y_1, y_2, P_B^z, \mu\right)} \boxed{\Phi^B\left(y_1, y_2, \mu\right)} + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{(x_1 P_B^z)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(x_2 P_B^z)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{((1-x_1-x_2)\, P_B^z)^2}\right)$$

Hard Mode

5.Summary and outlook

- LCDAs of a light baryon can be obtained through a simulation of a quasi-DA calculable on lattice QCD under the LaMET.

- We have calculated the one-loop perturbative contributions to LCDA and quasi-DA and explicitly have demonstrated the factorization of quasi-DA at the one-loop level.

- Our result provides a first step to obtaining the SU3 baryon LCDA from first principle lattice QCD calculations in the future.

Thanks

Back up

Conformal spin expansion of LCDAs

$$\Phi_R^B(x_1,x_2,x_3,\mu) = 120 x_1 x_2 x_3 \sum_{N=0}^{\infty}\sum_{q=0}^{N} \phi_{N,q}(\mu_0) P_{N,q}(x_1,x_2,x_3) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_{N,q}/\beta_0}$$

They are orthogonal with respect to

$$\int_0^1 [dx] x_1 x_2 x_3 P_{N,q}(x_1,x_2,x_3) P_{N',q'}(x_1,x_2,x_3) = c_{N,q}\delta_{qq}\delta_{NN'}$$

- For octet and decuplet baryons $B \neq \Lambda$, the expansions are

$$V^B(x_1, x_2, x_3, \mu) = 120 x_1 x_2 x_3 \left[\phi_{0,0}^v(\mu) + \phi_{1,1}^v(\mu)(1 - 3x_3) + \ldots\right],$$

$$A^B(x_1, x_2, x_3, \mu) = 120 x_1 x_2 x_3 [\phi_{1,0}^a(\mu)(x_2 - x_1) + \ldots],$$

$$T^B(x_1, x_2, x_3, \mu) = 120 x_1 x_2 x_3 \left[\phi_{0,0}^t(\mu) + \phi_{1,0}^t(\mu)(1 - 3x_3) + \ldots\right],$$

where $\phi_{N,q}^{v/a/t}(\mu)$ is the moment for $V^B/A^B/T^B$ and the ellipsis stands for terms suppressed by higher conformal spins.

- For neutron and proton, due to isospin symmetry, $T^{n/p}(x_1, x_2, x_3, \mu)$ can be expressed as

$$T^{n/p}(x_1, x_2, x_3, \mu) = 120 x_1 x_2 x_3 \left[\phi_{0,0}^v(\mu) + \frac{1}{2}\left(\phi_{1,0}^a - \phi_{11}^v\right)(\mu)(1 - 3x_3) + \ldots\right].$$

- The expansions of LCDAs for Λ are

$$
\begin{aligned}
V^{\Lambda}(x_1,x_2,x_3,\mu) &= 120x_1x_2x_3[\phi^v_{1,0}(\mu)\,(x_2-x_1)+\ldots],\\
A^{\Lambda}(x_1,x_2,x_3,\mu) &= 120x_1x_2x_3[\phi_{0,0}(\mu)^a+\phi^a_{1,1}(\mu)(1-3x_3)+\ldots],\\
T^{\Lambda}(x_1,x_2,x_3,\mu) &= 120x_1x_2x_3[\phi^t_{1,0}(\mu)\,(x_2-x_1)+\ldots].
\end{aligned}
$$

- For the φ^B of decuplet baryons, the expansions are

$$
\varphi^B(x_1,x_2,x_3,\mu) = 120x_1x_2x_3\left[\phi_{0,0}(\mu)+\phi_{1,0}(\mu)\,(1-3x_3)+\ldots\right].
$$

- For the alternative basis defined in eq. (2.12), the corresponding expansions are

$$
\begin{aligned}
\Phi^B_+(x_1,x_2,x_3,\mu) &= 120x_1x_2x_3\left(\varphi^B_{0,0}(\mu)\mathcal{P}_{00}(x_1,x_2,x_3)+\varphi^B_{1,1}(\mu)\mathcal{P}_{11}(x_1,x_2,x_3)+\ldots\right),\\
\Phi^B_-(x_1,x_2,x_3,\mu) &= 120x_1x_2x_3\left(\varphi^B_{1,0}(\mu)\mathcal{P}_{10}(x_1,x_2,x_3)+\ldots\right),\\
\Pi^{B\neq\Lambda}(x_1,x_2,x_3,\mu) &= 120x_1x_2x_3\left(\pi^B_{0,0}(\mu)\mathcal{P}_{00}(x_1,x_2,x_3)+\pi^B_{1,1}(\mu)\mathcal{P}_{11}(x_1,x_2,x_3)+\ldots\right),\\
\Pi^{\Lambda}(x_1,x_2,x_3,\mu) &= 120x_1x_2x_3\left(\pi^A_{1,0}(\mu)\mathcal{P}_{10}(x_1,x_2,x_3)+\ldots\right),
\end{aligned}
$$

where $\varphi^B_{N,q}(\mu)$ and $\pi^B_{N,q}(\mu)$ are the conformal moments defined in the alternative basis. The explicit forms for $\mathcal{P}_{00,10,11}$ are given as:

$$
\mathcal{P}_{00}=1,\quad \mathcal{P}_{10}=21(x_1-x_3),\quad \mathcal{P}_{11}=7(x_1-2x_2+x_3).
$$

Results for quasi-DAs

$$\widetilde{\mathcal{M}}_{V(A)}(z_1,z_2,0,P^z,\mu)=$$
$$\times\left\{1+\frac{\alpha_s C_F}{\pi}\left(\frac{1}{2}L_1^{\rm UV}+\frac{1}{2}L_2^{\rm UV}+\frac{1}{2}L_{12}^{\rm UV}+\frac{3}{2}\right)\right\}\widetilde{\mathcal{M}}_0(z_1,z_2,0,P^z,\mu)$$
$$-\frac{\alpha_s C_F}{8\pi}\int_0^1 d\eta_1\int_0^{1-\eta_1}d\eta_2$$
$$\times\left\{\left(L_1^{\rm IR}-1+\frac{1}{\epsilon_{\rm IR}}\right)\widetilde{\mathcal{M}}_0((1-\eta_1)z_1,z_2,\eta_2 z_1,P^z,\mu)+\left(L_2^{\rm IR}-1+\frac{1}{\epsilon_{\rm IR}}\right)\widetilde{\mathcal{M}}_0(z_1,(1-\eta_1)z_2,\eta_2 z_2,P^z,\mu)\right.$$
$$\left.+2\left(L_{12}^{\rm IR}-3+\frac{1}{\epsilon_{\rm IR}}\right)\widetilde{\mathcal{M}}_0((1-\eta_1)z_1+\eta_1 z_2,(1-\eta_2)z_2+\eta_2 z_1,0,P^z,\mu)\right\}$$
$$-\frac{\alpha_s C_F}{4\pi}\int_0^1 d\eta\times\left\{\widetilde{\mathcal{M}}_0((1-\eta)z_1+\eta z_2,z_2,0,P^z,\mu)\left\{\left(L_{12}^{\rm IR}+1+\frac{1}{\epsilon_{\rm IR}}\right)\left(\frac{1-\eta}{\eta}\right)_+ +2\left(\frac{\ln\eta}{\eta}\right)_+\right\}\right.$$
$$+\widetilde{\mathcal{M}}_0(z_1,(1-\eta)z_2+\eta z_1,0,P^z,\mu)\left\{\left(L_{12}^{\rm IR}+1+\frac{1}{\epsilon_{\rm IR}}\right)\left(\frac{1-\eta}{\eta}\right)_+ +2\left(\frac{\ln\eta}{\eta}\right)_+\right\}$$
$$+\widetilde{\mathcal{M}}_0((1-\eta)z_1,z_2,0,P^z,\mu)\left\{\left(L_1^{\rm IR}+1+\frac{1}{\epsilon_{\rm IR}}\right)\left(\frac{1-\eta}{\eta}\right)_+ +2\left(\frac{\ln\eta}{\eta}\right)_+\right\}$$
$$+\widetilde{\mathcal{M}}_0(z_1,(1-\eta)z_2,0,P^z,\mu)\left\{\left(L_2^{\rm IR}+1+\frac{1}{\epsilon_{\rm IR}}\right)\left(\frac{1-\eta}{\eta}\right)_+ +2\left(\frac{\ln\eta}{\eta}\right)_+\right\}$$
$$-\widetilde{\mathcal{M}}_0(z_1,z_2,\eta z_1,P^z,\mu)\left\{\left(L_1^{\rm IR}+1+\frac{1}{\epsilon_{\rm IR}}\right)\left(\frac{1-\eta}{\eta}\right)_+ +2\left(\frac{\ln\eta}{\eta}\right)_+\right\}$$
$$\left.-\widetilde{\mathcal{M}}_0(z_1,z_2,\eta z_2,P^z,\mu)\left\{\left(L_2^{\rm IR}+1+\frac{1}{\epsilon_{\rm IR}}\right)\left(\frac{1-\eta}{\eta}\right)_+ +2\left(\frac{\ln\eta}{\eta}\right)_+\right\}\right\},$$

$$\widetilde{\mathcal{M}}_{T}(z_1,z_2,0,P^z,\mu)=$$
$$\times\left\{1+\frac{\alpha_s C_F}{\pi}\left(\frac{1}{2}L_1^{\rm UV}+\frac{1}{2}L_2^{\rm UV}+\frac{1}{2}L_{12}^{\rm UV}+\frac{3}{2}\right)\right\}\widetilde{\mathcal{M}}_0(z_1,z_2,0,P^z,\mu)$$
$$-\frac{\alpha_s C_F}{8\pi}\int_0^1 d\eta_1\int_0^{1-\eta_1}d\eta_2$$
$$\times\left\{\left(L_1^{\rm IR}-1+\frac{1}{\epsilon_{\rm IR}}\right)\widetilde{\mathcal{M}}_0((1-\eta_1)z_1,z_2,\eta_2 z_1,P^z,\mu)+\left(L_2^{\rm IR}-1+\frac{1}{\epsilon_{\rm IR}}\right)\widetilde{\mathcal{M}}_0(z_1,(1-\eta_1)z_2,\eta_2 z_2,P^z,\mu)\right\}$$
$$-\frac{\alpha_s C_F}{4\pi}\int_0^1 d\eta\times\left\{\widetilde{\mathcal{M}}_0((1-\eta)z_1+\eta z_2,z_2,0,P^z,\mu)\left\{\left(L_{12}^{\rm IR}+1+\frac{1}{\epsilon_{\rm IR}}\right)\left(\frac{1-\eta}{\eta}\right)_+ +2\left(\frac{\ln\eta}{\eta}\right)_+\right\}\right.$$
$$+\widetilde{\mathcal{M}}_0(z_1,(1-\eta)z_2+\eta z_1,0,P^z,\mu)\left\{\left(L_{12}^{\rm IR}+1+\frac{1}{\epsilon_{\rm IR}}\right)\left(\frac{1-\eta}{\eta}\right)_+ +2\left(\frac{\ln\eta}{\eta}\right)_+\right\}$$
$$+\widetilde{\mathcal{M}}_0((1-\eta)z_1,z_2,0,P^z,\mu)\left\{\left(L_1^{\rm IR}+1+\frac{1}{\epsilon_{\rm IR}}\right)\left(\frac{1-\eta}{\eta}\right)_+ +2\left(\frac{\ln\eta}{\eta}\right)_+\right\}$$
$$+\widetilde{\mathcal{M}}_0(z_1,(1-\eta)z_2,0,P^z,\mu)\left\{\left(L_2^{\rm IR}+1+\frac{1}{\epsilon_{\rm IR}}\right)\left(\frac{1-\eta}{\eta}\right)_+ +2\left(\frac{\ln\eta}{\eta}\right)_+\right\}$$
$$-\widetilde{\mathcal{M}}_0(z_1,z_2,\eta z_1,P^z,\mu)\left\{\left(L_1^{\rm IR}+1+\frac{1}{\epsilon_{\rm IR}}\right)\left(\frac{1-\eta}{\eta}\right)_+ +2\left(\frac{\ln\eta}{\eta}\right)_+\right\}$$
$$\left.-\widetilde{\mathcal{M}}_0(z_1,z_2,\eta z_2,P^z,\mu)\left\{\left(L_2^{\rm IR}+1+\frac{1}{\epsilon_{\rm IR}}\right)\left(\frac{1-\eta}{\eta}\right)_+ +2\left(\frac{\ln\eta}{\eta}\right)_+\right\}\right\},$$

$$\int_0^1 du\,[G(u)]_+\,F(u)=\int_0^1 duG(u)[F(u)-F(0)]$$

$$L_1^{\rm IR,\ UV}=\ln\left(\frac{1}{4}\mu^2_{\rm IR,\ UV}z_1^2e^{2\gamma_E}\right)\qquad L_2^{\rm IR,UV}=\ln\left(\frac{1}{4}\mu^2_{\rm IR,UV}z_2^2e^{2\gamma_E}\right)\qquad L_{12}^{\rm IR,UV}=\ln\left(\frac{1}{4}\mu^2_{\rm IR,UV}(z_1-z_2)^2e^{2\gamma_E}\right)$$

Here $\widetilde{\mathcal{M}}_0$ stands for tree-level matrix element.

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Results for quasi-DAs

$$
\begin{aligned}
&\widetilde{\mathcal{M}}_{\varphi}(z_1,z_2,0,P^z,\mu)=\\
&\times\left\{1+\frac{\alpha_s C_F}{\pi}\left(\frac{1}{2}L_1^{\mathrm{UV}}+\frac{1}{2}L_2^{\mathrm{UV}}+\frac{1}{2}L_{12}^{\mathrm{UV}}+\frac{3}{2}\right)\right\}\widetilde{\mathcal{M}}_0(z_1,z_2,0,P^z,\mu)\\
&-\frac{\alpha_s C_F}{4\pi}\int_0^1 d\eta\times\left\{\widetilde{\mathcal{M}}_0((1-\eta)z_1+\eta z_2,z_2,0,P^z,\mu)\left\{\left(L_{12}^{\mathrm{IR}}+1+\frac{1}{\epsilon_{\mathrm{IR}}}\right)\left(\frac{1-\eta}{\eta}\right)_+ +2\left(\frac{\ln\eta}{\eta}\right)_+\right\}\right.\\
&+\widetilde{\mathcal{M}}_0(z_1,(1-\eta)z_2+\eta z_1,0,P^z,\mu)\left\{\left(L_{12}^{\mathrm{IR}}+1+\frac{1}{\epsilon_{\mathrm{IR}}}\right)\left(\frac{1-\eta}{\eta}\right)_+ +2\left(\frac{\ln\eta}{\eta}\right)_+\right\}\\
&+\widetilde{\mathcal{M}}_0((1-\eta)z_1,z_2,0,P^z,\mu)\left\{\left(L_1^{\mathrm{IR}}+1+\frac{1}{\epsilon_{\mathrm{IR}}}\right)\left(\frac{1-\eta}{\eta}\right)_+ +2\left(\frac{\ln\eta}{\eta}\right)_+\right\}\\
&+\widetilde{\mathcal{M}}_0(z_1,(1-\eta)z_2,0,P^z,\mu)\left\{\left(L_2^{\mathrm{IR}}+1+\frac{1}{\epsilon_{\mathrm{IR}}}\right)\left(\frac{1-\eta}{\eta}\right)_+ +2\left(\frac{\ln\eta}{\eta}\right)_+\right\}\\
&-\widetilde{\mathcal{M}}_0(z_1,z_2,\eta z_1,P^z,\mu)\left\{\left(L_1^{\mathrm{IR}}+1+\frac{1}{\epsilon_{\mathrm{IR}}}\right)\left(\frac{1-\eta}{\eta}\right)_+ +2\left(\frac{\ln\eta}{\eta}\right)_+\right\}\\
&\left.-\widetilde{\mathcal{M}}_0(z_1,z_2,\eta z_2,P^z,\mu)\left\{\left(L_2^{\mathrm{IR}}+1+\frac{1}{\epsilon_{\mathrm{IR}}}\right)\left(\frac{1-\eta}{\eta}\right)_+ +2\left(\frac{\ln\eta}{\eta}\right)_+\right\}\right\}.
\end{aligned}
$$

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Results for LCDAs

$$
\begin{aligned}
&\mathcal{I}_{V(A)}(\nu_1,\nu_2,0,\mu)=\mathcal{I}_0(\nu_1,\nu_2,0,\mu)\\
&-\frac{\alpha_s C_F}{8\pi}\frac{1}{\epsilon_{\rm IR}}\Bigg\{\Bigg[\int_0^1 d\eta_1\int_0^{1-\eta_1}d\eta_2\,[2\mathcal{I}_0((1-\eta_1)\nu_1+\eta_1\nu_2,(1-\eta_2)\nu_2+\eta_2\nu_1,0,\mu)\\
&+\mathcal{I}_0((1-\eta_1)\nu_1,\nu_2,\eta_2\nu_1,\mu)+\mathcal{I}_0(\nu_1,(1-\eta_1)\nu_2,\eta_2\nu_2,\mu)]\\
&+2\int_0^1 d\eta\left(\frac{1-\eta}{\eta}\right)_+\{(\mathcal{I}_0((1-\eta)\nu_1+\eta\nu_2,\nu_2,0,\mu)+\mathcal{I}_0(\nu_1,(1-\eta)\nu_2+\eta\nu_1,0,\mu))\\
&+(\mathcal{I}_0((1-\eta)\nu_1,\nu_2,0,\mu)+\mathcal{I}_0(\nu_1,\nu_2,\eta\nu_1,\mu))+(\mathcal{I}_0(\nu_1,(1-\eta)\nu_2,0,\mu)+\mathcal{I}_0(\nu_1,\nu_2,\eta\nu_2,\mu))\}\Bigg\},\\
&\mathcal{I}_T(\nu_1,\nu_2,0,\mu)=\mathcal{I}_0(\nu_1,\nu_2,0,\mu)\\
&-\frac{\alpha_s C_F}{8\pi}\frac{1}{\epsilon_{\rm IR}}\Bigg\{\Bigg[\int_0^1 d\eta_1\int_0^{1-\eta_1}d\eta_2\,[\mathcal{I}_0((1-\eta_1)\nu_1,\nu_2,\eta_2\nu_1,\mu)+\mathcal{I}_0(z_1,(1-\eta_1)\nu_2,\eta_2\nu_2,\mu)]\\
&+2\int_0^1 d\eta\left(\frac{1-\eta}{\eta}\right)_+\{(\mathcal{I}_0((1-\eta)\nu_1+\eta\nu_2,\nu_2,0,\mu)+\mathcal{I}_0(\nu_1,(1-\eta)\nu_2+\eta\nu_1,0,\mu))\\
&+(\mathcal{I}_0((1-\eta)\nu_1,\nu_2,0,\mu)+\mathcal{I}_0(\nu_1,\nu_2,\eta\nu_1,\mu))+(\mathcal{I}_0(\nu_1,(1-\eta)\nu_2,0,\mu)+\mathcal{I}_0(\nu_1,\nu_2,\eta\nu_2,\mu))\}\Bigg\},\\
&\mathcal{I}_\varphi(\nu_1,\nu_2,0,\mu)=\mathcal{I}_0(\nu_1,\nu_2,0,\mu)\\
&-\frac{\alpha_s C_F}{8\pi}\frac{1}{\epsilon_{\rm IR}}\Bigg\{2\int_0^1 d\eta\left(\frac{1-\eta}{\eta}\right)_+\{(\mathcal{I}_0((1-\eta)\nu_1+\eta\nu_2,\nu_2,0,\mu)+\mathcal{I}_0(\nu_1,(1-\eta)\nu_2+\eta\nu_1,0,\mu))\\
&+(\mathcal{I}_0((1-\eta)\nu_1,\nu_2,0,\mu)+\mathcal{I}_0(\nu_1,\nu_2,\eta\nu_1,\mu))+(\mathcal{I}_0(\nu_1,(1-\eta)\nu_2,0,\mu)+\mathcal{I}_0(\nu_1,\nu_2,\eta\nu_2,\mu))\}\Bigg\},
\end{aligned}
$$

where $\nu_1\equiv z_1 n\cdot P$, $\nu_2\equiv z_2 n\cdot P$, and $\mathcal{I}_0$ stands for tree-level matrix element.

The lightcone correlator can be normalized by dividing it by its zero-momentum matrix element:

$$
\mathfrak{I}_{V,A,T,\varphi}(\nu_1,\nu_2,\nu_3,\mu)\equiv\frac{\mathcal{I}_{V,A,T,\varphi}(\nu_1,\nu_2,\nu_3,\mu)}{\mathcal{I}_{V,A,T,\varphi}(0,0,0,\mu)}.
$$

The LCDAs with Fock states in momentum space are defined by

$$
\begin{aligned}
&\Psi^{V,A,T,\varphi}_{\overline{\rm MS}}\\
&=\int_{-\infty}^{+\infty}\frac{n\cdot P d z_1}{2\pi}\frac{n\cdot P d z_2}{2\pi}\\
&\times e^{ix_1 n\cdot P z_1+ix_2 n\cdot P z_2}\times\mathfrak{I}_{V,A,T,\varphi}(z_1 n\cdot P,z_2 n\cdot P,0,\mu)
\end{aligned}
$$

The coefficient of matching kernel

$$\mathcal{C}_{1V,A}(x_1,x_2,y_1,y_2,P^z,\mu)=2(P^z)^2\Bigg[I_{\rm H}^{V/A}[(x_1-y_1)P^z,(x_2-y_2)P^z]+I_{\rm HSI}^{V/A}[(x_1-y_1)P^z,(x_2-y_2)P^z]$$
$$+I_{\rm HSII}^{V/A}[(x_1-y_1)P^z,(x_2-y_2)P^z]+I_{\rm HSIII}^{V/A}[(x_1-y_1)P^z,(x_2-y_2)P^z]+I_{\rm HSIV}^{V/A}[(x_1-y_1)P^z,(x_2-y_2)P^z]$$
$$+I_{\rm S}^{V/A}[(x_1-y_1)P^z,(x_2-y_2)P^z]+\delta[(x_1-y_1)P^z]\delta[(x_2-y_2)P^z]\left(\frac{5}{2}\ln\left(\frac{\mu^2e^{2\gamma_E}}{4}\right)+4\right)\Bigg]$$

$$\mathcal{C}_{1T}(x_1,x_2,y_1,y_2,P^z,\mu)=2(P^z)^2\Bigg[I_{\rm H}^{T}[(x_1-y_1)P^z,(x_2-y_2)P^z]+I_{\rm HSI}^{T}[(x_1-y_1)P^z,(x_2-y_2)P^z]$$
$$+I_{\rm HSII}^{T}[(x_1-y_1)P^z,(x_2-y_2)P^z]+I_{\rm HSIII}^{T}[(x_1-y_1)P^z,(x_2-y_2)P^z]+I_{\rm HSIV}^{T}[(x_1-y_1)P^z,(x_2-y_2)P^z]$$
$$+I_{\rm S}^{T}[(x_1-y_1)P^z,(x_2-y_2)P^z]+\delta[(x_1-y_1)P^z]\delta[(x_2-y_2)P^z]\left(\frac{9}{4}\ln\left(\frac{\mu^2e^{2\gamma_E}}{4}\right)+\frac{13}{4}\right)\Bigg]$$

$$\mathcal{C}_{1\varphi}(x_1,x_2,y_1,y_2,P^z,\mu)=2(P^z)^2\Bigg[I_{\rm H}^{\varphi}[(x_1-y_1)P^z,(x_2-y_2)P^z]+I_{\rm HSI}^{\varphi}[(x_1-y_1)P^z,(x_2-y_2)P^z]$$
$$+I_{\rm HSII}^{\varphi}[(x_1-y_1)P^z,(x_2-y_2)P^z]+I_{\rm HSIII}^{\varphi}[(x_1-y_1)P^z,(x_2-y_2)P^z]+I_{\rm HSIV}^{\varphi}[(x_1-y_1)P^z,(x_2-y_2)P^z]$$
$$+I_{\rm S}^{\varphi}[(x_1-y_1)P^z,(x_2-y_2)P^z]+\delta[(x_1-y_1)P^z]\delta[(x_2-y_2)P^z]\left(\frac{9}{4}\ln\left(\frac{\mu^2e^{2\gamma_E}}{4}\right)+3\right)\Bigg]$$

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The coefficient of matching kernel

$$\mathcal{C}_{1V,A}(x_1,x_2,y_1,y_2,P^z,\mu)=2(P^z)^2\Bigg[I_{\rm H}^{V/A}[(x_1-y_1)P^z,(x_2-y_2)P^z]+I_{\rm HSI}^{V/A}[(x_1-y_1)P^z,(x_2-y_2)P^z]$$

$$+I_{\rm HSII}^{V/A}[(x_1-y_1)P^z,(x_2-y_2)P^z]+I_{\rm HSIII}^{V/A}[(x_1-y_1)P^z,(x_2-y_2)P^z]+I_{\rm HSIV}^{V/A}[(x_1-y_1)P^z,(x_2-y_2)P^z]$$

$$+I_{\rm S}^{V/A}[(x_1-y_1)P^z,(x_2-y_2)P^z]+\delta[(x_1-y_1)P^z]\delta[(x_2-y_2)P^z]\left(\frac{5}{2}\ln\left(\frac{\mu^2e^{2\gamma_E}}{4}\right)+4\right)\Bigg]$$

$$\mathcal{C}_{1T}(x_1,x_2,y_1,y_2,P^z,\mu)=2(P^z)^2\Bigg[I_{\rm H}^{T}[(x_1-y_1)P^z,(x_2-y_2)P^z]+I_{\rm HSI}^{T}[(x_1-y_1)P^z,(x_2-y_2)P^z]$$

$$+I_{\rm HSII}^{T}[(x_1-y_1)P^z,(x_2-y_2)P^z]+I_{\rm HSIII}^{T}[(x_1-y_1)P^z,(x_2-y_2)P^z]+I_{\rm HSIV}^{T}[(x_1-y_1)P^z,(x_2-y_2)P^z]$$

$$+I_{\rm S}^{T}[(x_1-y_1)P^z,(x_2-y_2)P^z]+\delta[(x_1-y_1)P^z]\delta[(x_2-y_2)P^z]\left(\frac{9}{4}\ln\left(\frac{\mu^2e^{2\gamma_E}}{4}\right)+\frac{13}{4}\right)\Bigg]$$

$$\mathcal{C}_{1\varphi}(x_1,x_2,y_1,y_2,P^z,\mu)=2(P^z)^2\Bigg[I_{\rm H}^{\varphi}[(x_1-y_1)P^z,(x_2-y_2)P^z]+I_{\rm HSI}^{\varphi}[(x_1-y_1)P^z,(x_2-y_2)P^z]$$

$$+I_{\rm HSII}^{\varphi}[(x_1-y_1)P^z,(x_2-y_2)P^z]+I_{\rm HSIII}^{\varphi}[(x_1-y_1)P^z,(x_2-y_2)P^z]+I_{\rm HSIV}^{\varphi}[(x_1-y_1)P^z,(x_2-y_2)P^z]$$

$$+I_{\rm S}^{\varphi}[(x_1-y_1)P^z,(x_2-y_2)P^z]+\delta[(x_1-y_1)P^z]\delta[(x_2-y_2)P^z]\left(\frac{9}{4}\ln\left(\frac{\mu^2e^{2\gamma_E}}{4}\right)+3\right)\Bigg]$$

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