Properties of X(3872) from hadronic potentials coupled to quarks

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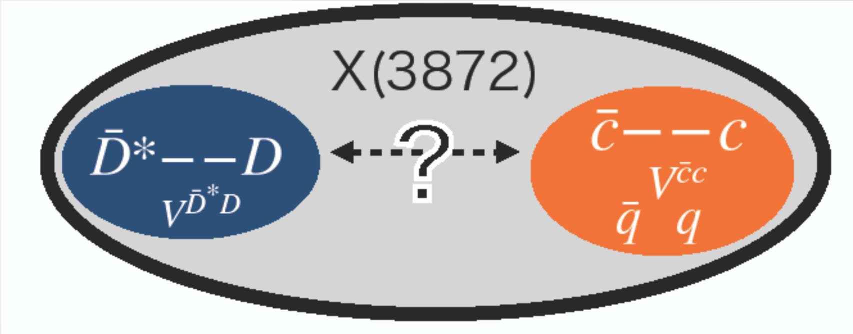
[I. Terashima and T. Hyodo, PhysRevC.108.035204 (2023)]

Exotic hadron $X(3872)$

- There is no restriction by QCD which prohibits the mixing with each d.o.f
  - States with same quantum numbers mix by definition
- Structure of $X(3872)$ [A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, and S. Yasui, PTEP **2016** (2016)]
  - **Mixing with quark and hadron** degrees of freedom
  - Not enough experimental data and lattice QCD results

> - How about a channel coupling between **quark** and **hadron** degrees of freedom like $\mathbf{X(3872)}$?
> - Revealing the internal structure of exotic hadrons by compositeness



1 : Molecule ⟺ Compositeness ⟺ 0 : Elementary

Channel coupling

✓ Formulation according to Feshbach method [H. Feshbach, Ann. Phys. 5, 357 (1958); ibid., 19, 287 (1962)]

■ Hamiltonian H with channel between quark potential V^q and hadron V^h

$$H = \begin{pmatrix} T^q & 0 \\ 0 & T^h + \Delta \end{pmatrix} + \begin{pmatrix} V^q & V^t \\ V^t & V^h \end{pmatrix}$$

T^q, T^h:Kinetic energy

Δ:Threshold energy

V^t:Transition potential

- Schrödinger equation with wave functions of quark and hadron channels $|q\rangle, |h\rangle$

$$H \begin{pmatrix} |q\rangle \\ |h\rangle \end{pmatrix} = E \begin{pmatrix} |q\rangle \\ |h\rangle \end{pmatrix}$$

$$\langle \boldsymbol{r}'_h \mid V^h_{\text{eff}}(E) \mid \boldsymbol{r}_h \rangle = \langle \boldsymbol{r}'_h \mid V^h \mid \boldsymbol{r}_h \rangle + \sum_n \frac{\langle \boldsymbol{r}'_h \mid V^t \mid \phi_n \rangle \langle \phi_n \mid V^t \mid \boldsymbol{r}_h \rangle}{E - E_n}$$

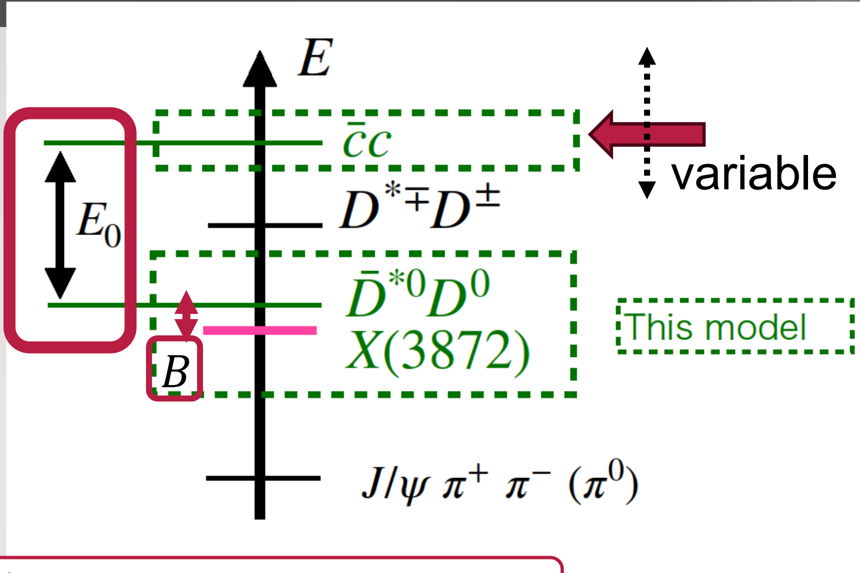
➢ Quark channel contribution. Sum of discrete eigenstates E_n

Formulation of $X(3872)$

◆Quark channel : $\bar{c}c$

$$H = \begin{pmatrix} T^q & 0 \\ 0 & T^h + \Delta \end{pmatrix} + \begin{pmatrix} V^q & V^t \\ V^t & V^h \end{pmatrix}$$

◆Hadron channel : $D^0\,\bar{D}^{*0}$



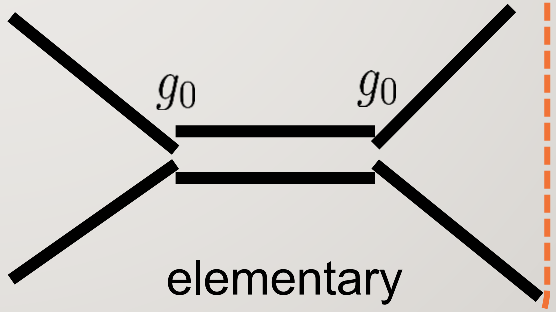
$$\langle \boldsymbol{r}'_h \mid V^t \mid \boldsymbol{r}_h \rangle = \omega(E) \frac{e^{-\mu r}}{r} \frac{e^{-\mu r'}}{r'}$$

✓ Separable
✓ Yukawa
μ: cut-off

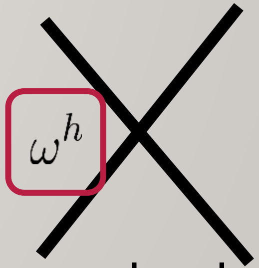
$$V_{\mathrm{eff}}^{\bar{D}^* D}(\boldsymbol{r}, \boldsymbol{r}', E) = [\omega^q(E) + \omega^h] V(\boldsymbol{r}) V(\boldsymbol{r}')$$
$$= \omega(E) \frac{e^{-\mu r}}{r} \frac{e^{-\mu r'}}{r'}$$

$\omega(E)$: Potential strength

$$\omega^q = \frac{g_0^2(B)}{E - E_0}$$



$\omega^h \in \mathbb{R}$



$g_0(B)$: coupling constant

➢ Determine to reproduce mass of $X(3872)$

$$g_0^2(B) = (B + E_0) \cdot (-1/G(E = -B) + \omega^h)$$

$G(E)$ is a loop function

Wave functions ψ

- The wave function $\psi_k(r)$ and the phase shift $\delta(k)$ can be obtained analytically in our formulation

$$\langle \boldsymbol{r}_h' \mid V^t \mid \boldsymbol{r}_h \rangle = \omega(E) \frac{e^{-\mu r}}{r} \frac{e^{-\mu r'}}{r'}$$

- Scattering wave function ψ^s

$$\psi_k^s(r) = \frac{\sin[kr + \delta(k)] - \sin\delta(k) e^{-\mu r}}{kr}$$

$$k\cot\delta(k) = -\frac{\mu[4\pi m\omega(E) + \mu^3]}{8\pi m\omega(E)} + \frac{1}{2\mu}\left[1 - \frac{2\mu^3}{4\pi m\omega(E)}\right]k^2 - \frac{1}{8\pi m\omega(E)}k^4$$

- Bound state wave function ψ^b
- $\mathcal{N}_b$ is a normalization constant

$$\psi_{k=i\kappa}^b(r) = \mathcal{N}_b\left(-\frac{\kappa e^{-\kappa r}}{r} + \frac{\kappa e^{-\mu r}}{r}\right)$$

Formulation：Compositeness 1

- Bound state wave function is normalized as:

$$1 = \int \mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}'\Psi_E^*(\boldsymbol{r}')(\delta(\boldsymbol{r}-\boldsymbol{r}') - \frac{\partial}{\partial E}V(\boldsymbol{r},\boldsymbol{r}',E))\Psi_E(\boldsymbol{r})$$

[Kenta Miyahara and Tetsuo Hyodo. Phys. Rev. C, 93(1):015201, 2016.]

- Definition of compositeness $1 = X_1 + Z_1$
  - Compositeness $X_1 = \int \mathrm{d}\boldsymbol{r}|\Psi_{E=-B}(\boldsymbol{r})|^2$
  - Elementality $Z_1 = -\int \mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}'\Psi_E^*(\boldsymbol{r}')\frac{\partial}{\partial E}V(\boldsymbol{r},\boldsymbol{r}',E)\Psi_E(\boldsymbol{r})$

- Exact X in case of separable and Yukawa potential

$$X_1 = [1 + \frac{g_0^2\kappa\mu(\kappa+\mu)^3}{8\pi m^2(g_0^2 + (-B-E_0)\omega^h)^2}]^{-1}$$

Formulation：Compositeness 2

- L-S equation with loop function $G(E)$ and potential $v(E) = \omega(E)$:

$$T(E) = \frac{1}{\frac{1}{v(E)} - G(E)}$$

- Definition of compositeness $\quad 1 = X_2 + Z_2$
  - Compositeness $\quad X_2 = \frac{G'(E)}{(v^{-1})' + G'(E)}|_{E=-B}$
  - Elementality $\quad Z_2 = \frac{(v^{-1})'}{(v^{-1})' + G'(E)}|_{E=-B}$
- Exact X in case of separable and Yukawa potential

$$X_2 = [1 + 2\pi \frac{g_0^2}{(B+E_0)^2} \frac{\kappa}{\mu(\mu+\kappa)}]^{-1} = X_1$$

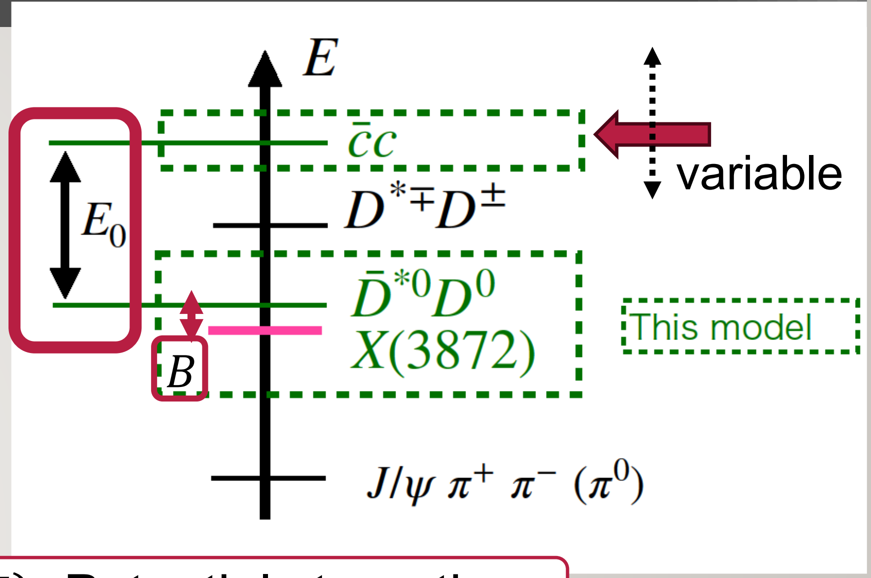
  - Equal to the compositeness by bound state normalization

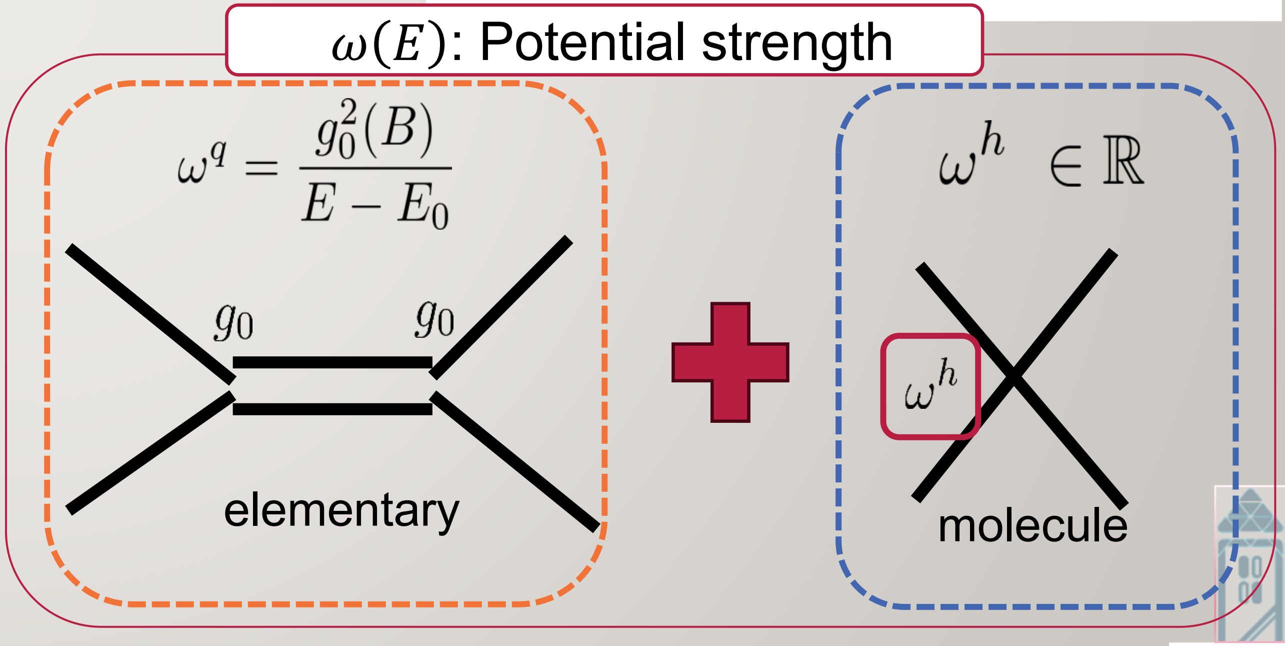
Parameters

- Parameters in this model

Physical observable	Typical value
$\boldsymbol{E_0}$	0.0078 [GeV] ($\chi_{C1}(2P)$)
B	4×10^{-5} [GeV]
μ	0.14 [GeV]
ω^h	0 [dim'less]

$$V_{\rm eff}^{\bar{D}^* D}(\boldsymbol{r},\boldsymbol{r'},E)=\omega(E)\frac{e^{-\mu r}}{r}\frac{e^{-\mu r'}}{r'}$$



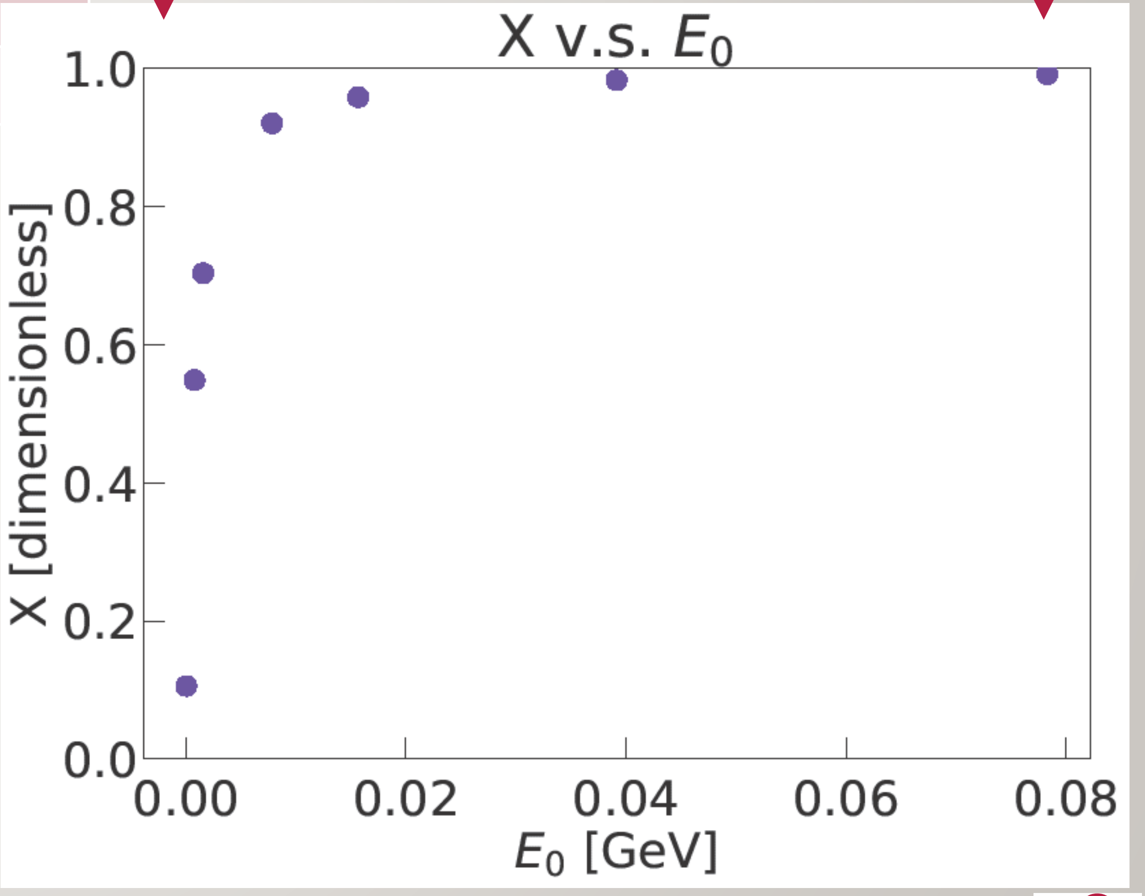


Result : E_0 dependence

Physical observable	Fixed quantity	Typical value
$\boldsymbol{E_0}$	-	0.0078 [GeV] $(\chi_{C1}(2P))$
B	4×10^{-5} [GeV]	4×10^{-5} [GeV]
μ	0.14 [GeV]	0.14 [GeV]
ω^h	0 [dim'less]	0 [dim'less]

$E_0 = B$ limit

quark model



- E_0 Increase ⇒ X Increase

Reason:

- Self energy increases as quark channel energy E_0 is far from the bare mass

Memo:

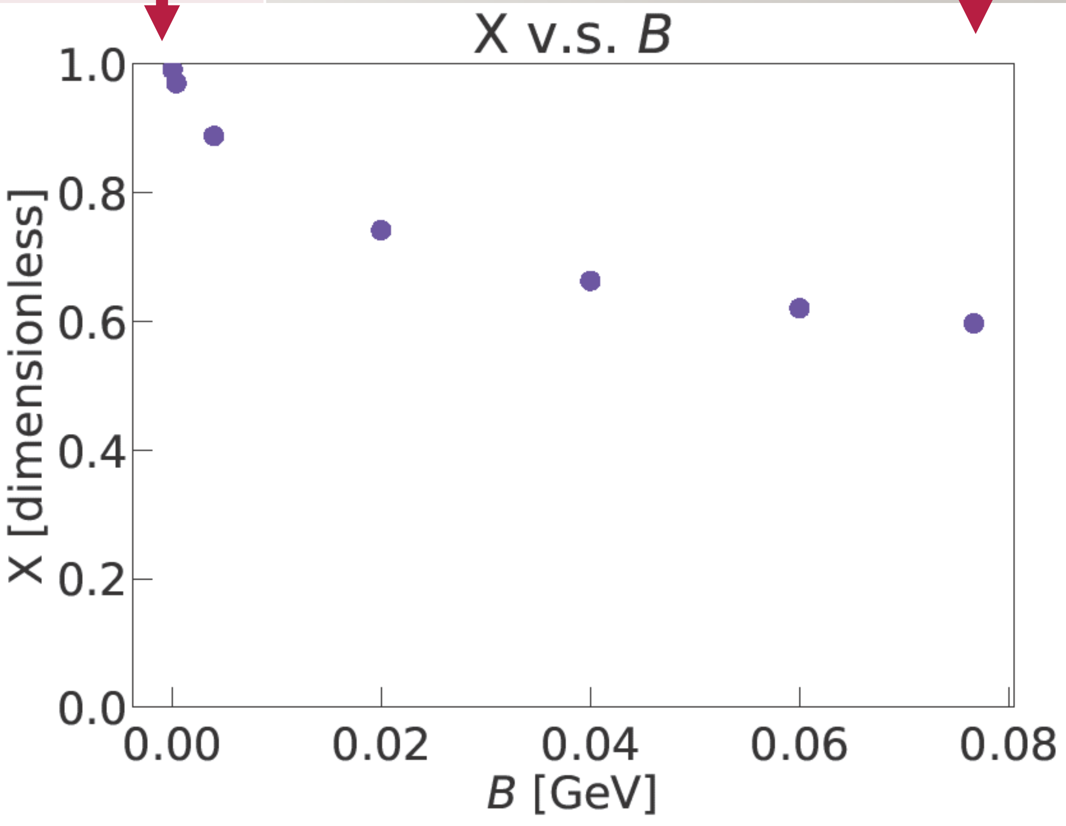
✓Changes are **huge** in $X(3872)$

Result : B dependence

Physical observable	Fixed quantity	Typical value
E_0	0.0078 [GeV] ($\chi_{C1}(2P)$)	0.0078 [GeV] ($\chi_{C1}(2P)$)
$\boldsymbol{B}$	-	4×10^{-5} [GeV]
μ	0.14 [GeV]	0.14 [GeV]
ω^h	0 [dim'less]	0 [dim'less]

upper B limit by Yukawa potential (depends on μ)

$B = 4 \cdot 10^{-5}$



- B Increase ⟹ X Decrease

Reason:

✓ $X \to 1$ ($B \to 0$): Completely composite in the weak-binding limit

Memo:

✓ X does not have strong dependence on B in $X(3872)$

Result : μ dependence

Physical observable	Fixed quantity	Typical value
E_0	0.0078 [GeV] ($\chi_{C1}(2P)$)	0.0078 [GeV] ($\chi_{C1}(2P)$)
B	4×10^{-5} [GeV]	4×10^{-5} [GeV]
$\boldsymbol{\mu}$	-	0.14 [GeV]
ω^h	0 [dim'less]	0 [dim'less]

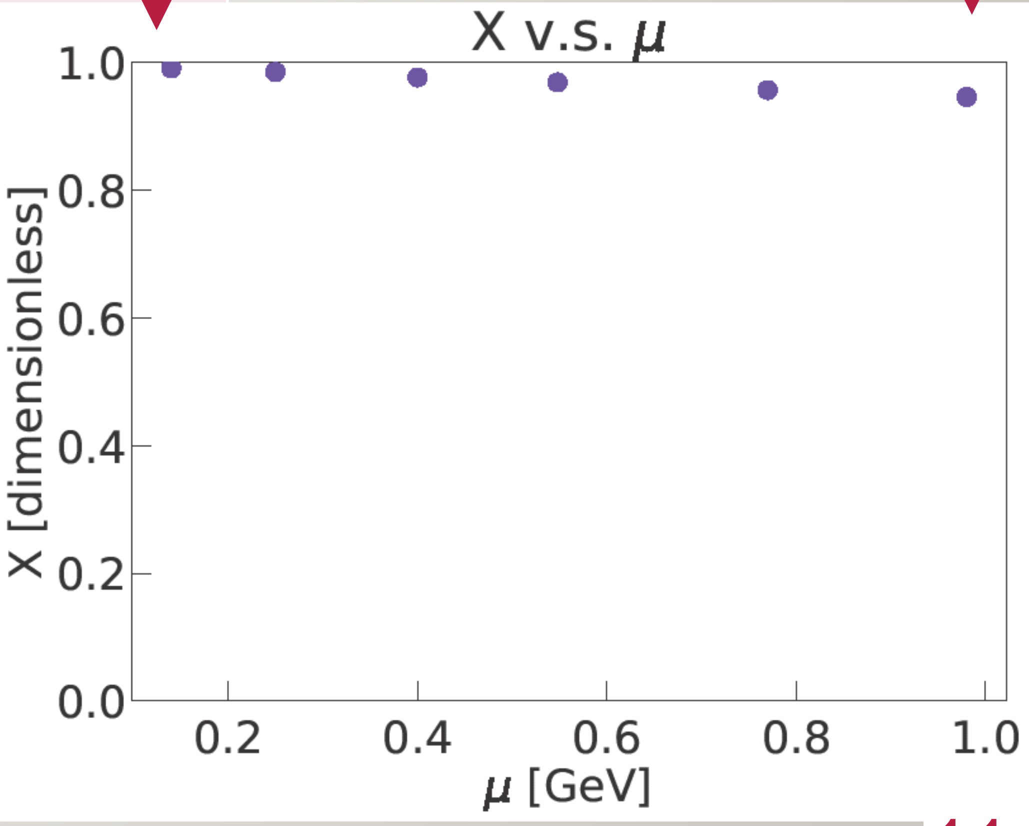
- μ Increase ⟹ X Decrease

Memo:

✓X does not have strong dependence on B in $X(3872)$

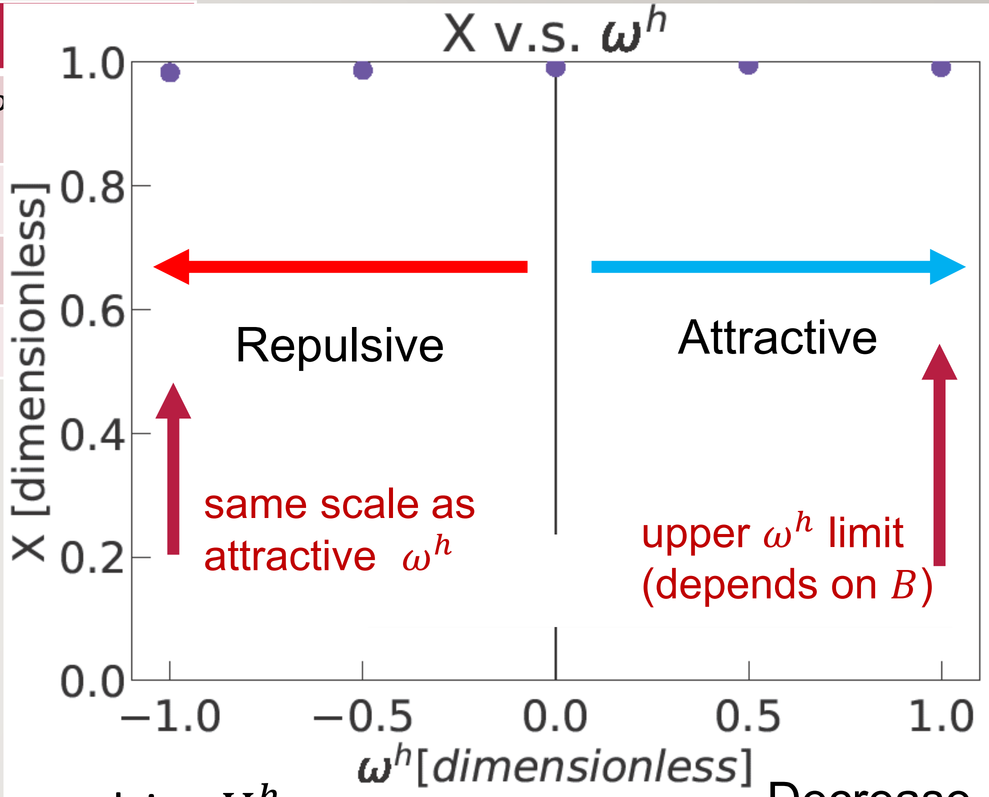
1 GeV
(typical scale of hadron)

mass of π
(lightest exchanging meson)

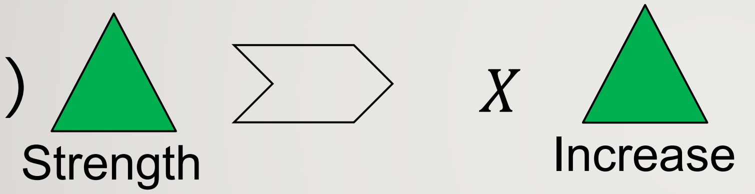


Result : ω^h dependence

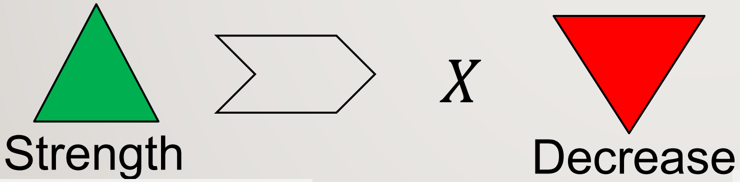
Physical observable	Fixed quantity	Typical value
E_0	0.0078 [GeV] ($\chi_{C1}(2P)$)	0.0078 [GeV] ($\chi_{C1}(2P$
B	4×10^{-5} [GeV]	4×10^{-5} [GeV]
μ	0.14 [GeV]	0.14 [GeV]
$\boldsymbol{\omega^h}$	-	0 [dim'less]



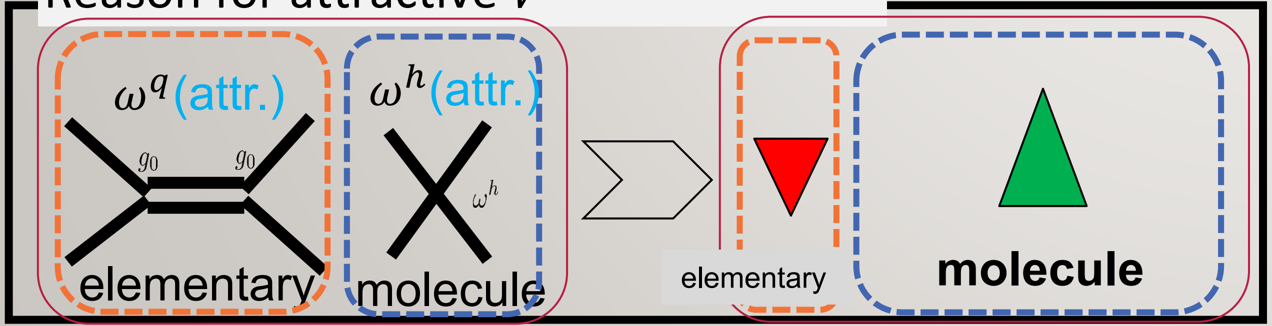
- ω^h(attractive)



- ω^h(repulsive)

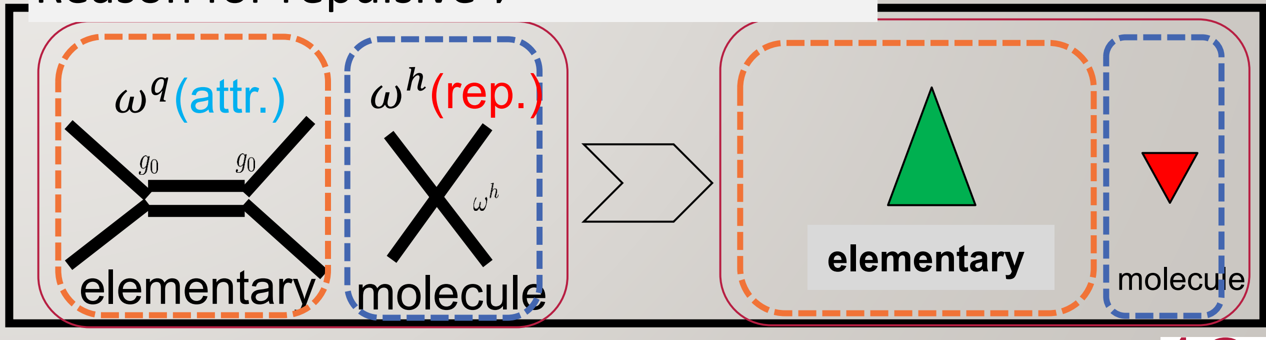


Reason for attractive V^h



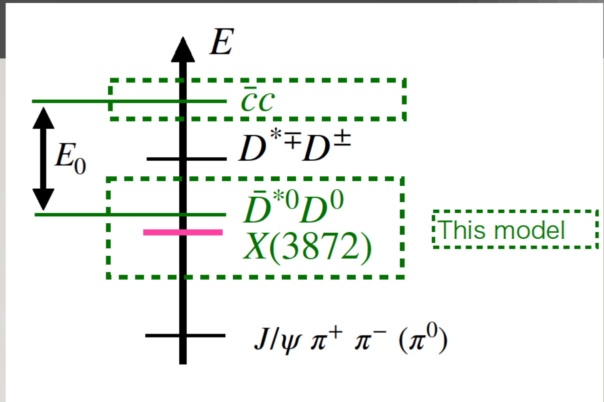
Reason for repulsive V^h

Decrease



Summary

[I. Terashima and T. Hyodo, PhysRevC.108.035204 (2023)]

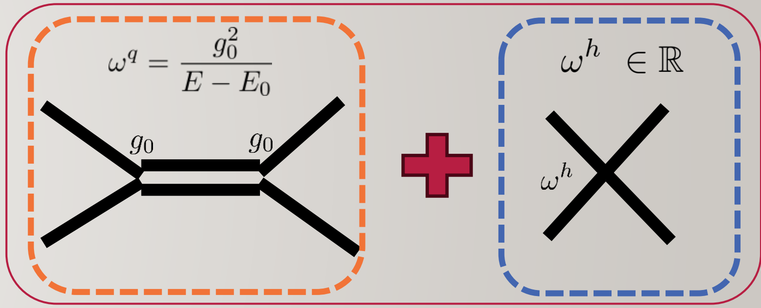


- Channel coupling between $c\bar{c}$ and $D\bar{D}^*$ in $X(3872)$

$$H = \begin{pmatrix} T^{c\bar{c}} & 0 \\ 0 & T^{\bar{D}^*D} + \Delta \end{pmatrix} + \begin{pmatrix} V^{c\bar{c}} & V^t \\ V^t & V^{\bar{D}^*D} \end{pmatrix}$$

- Effective potential with explicit V^q and V^h

$$V_{\text{eff}}^{\bar{D}^*D}(\boldsymbol{r}, \boldsymbol{r}', E) = [\omega^q(E) + \omega^h(E)] V(\boldsymbol{r}) V(\boldsymbol{r}')$$



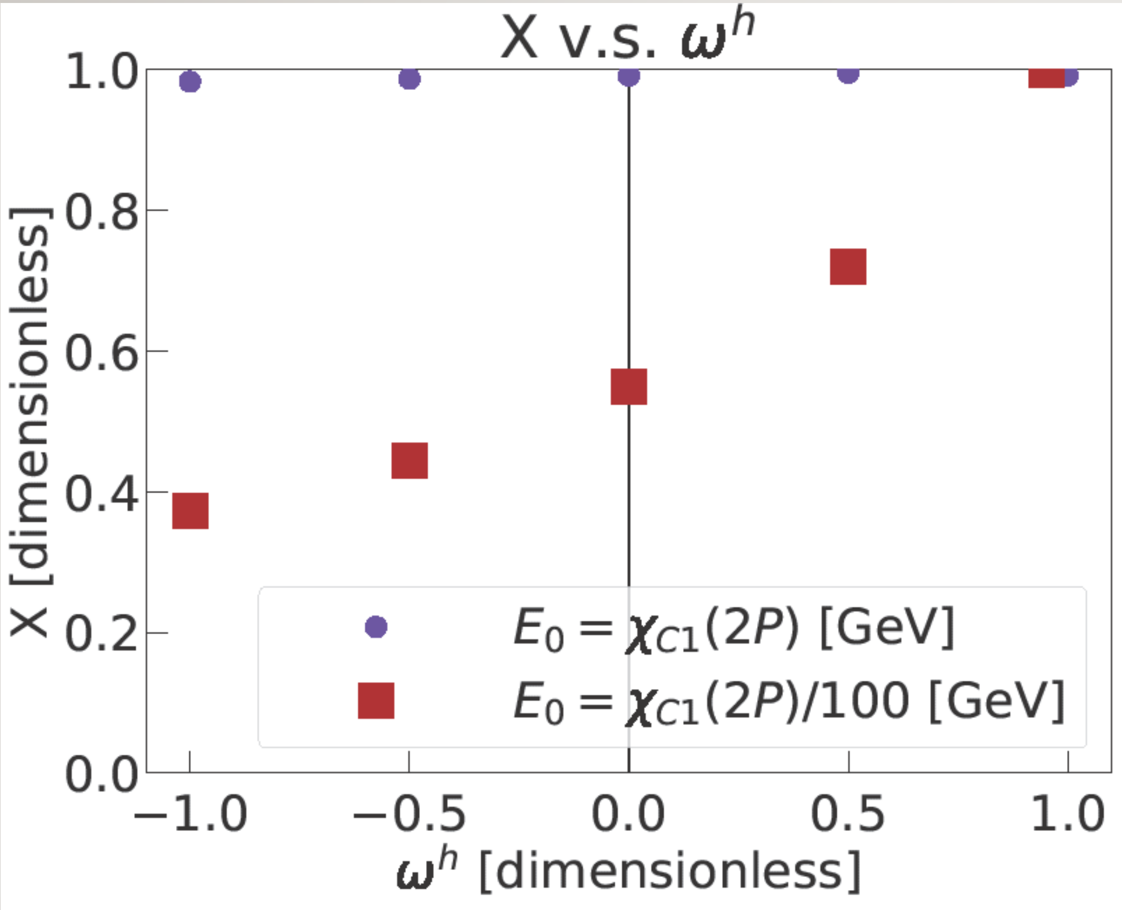
- Compositeness X in analytical form

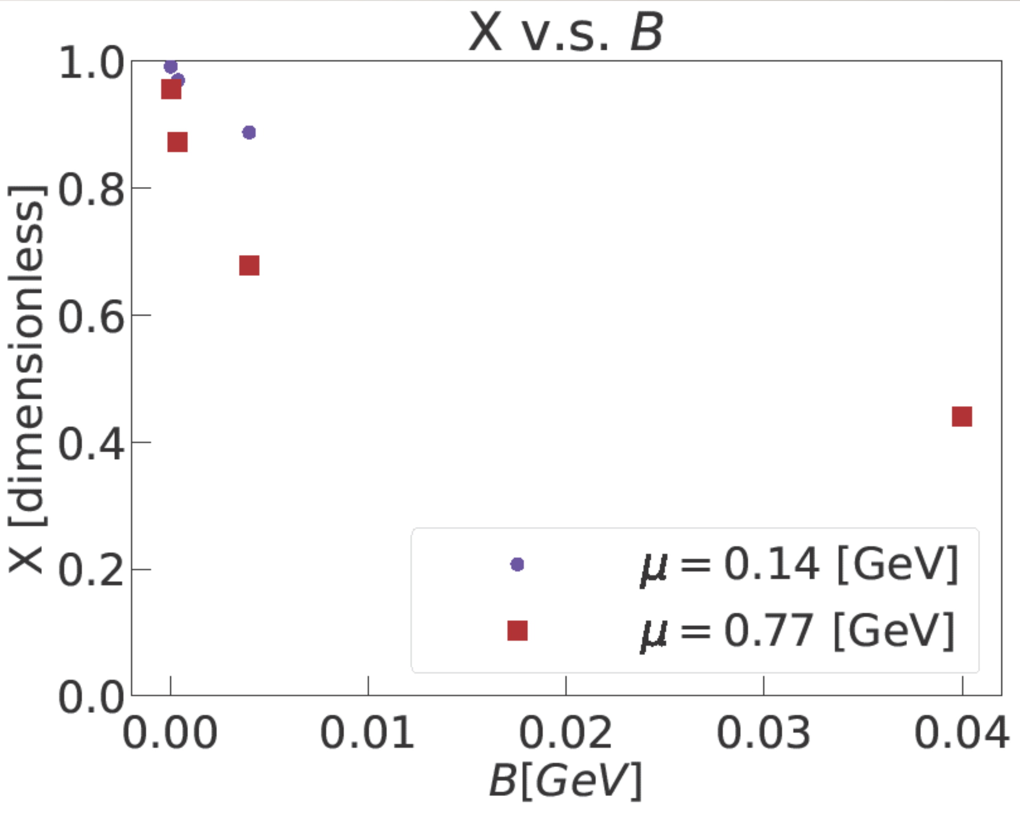
$$X = \left[1 + \frac{g_0^2 \kappa \mu (\kappa + \mu)^3}{8\pi m^2 (g_0^2 + (E - E_0)\omega^h)^2}\right]^{-1} = \left[1 + 2\pi \frac{g_0^2}{(B + E_0)^2} \frac{\kappa}{\mu(\mu + \kappa)}\right]^{-1}$$

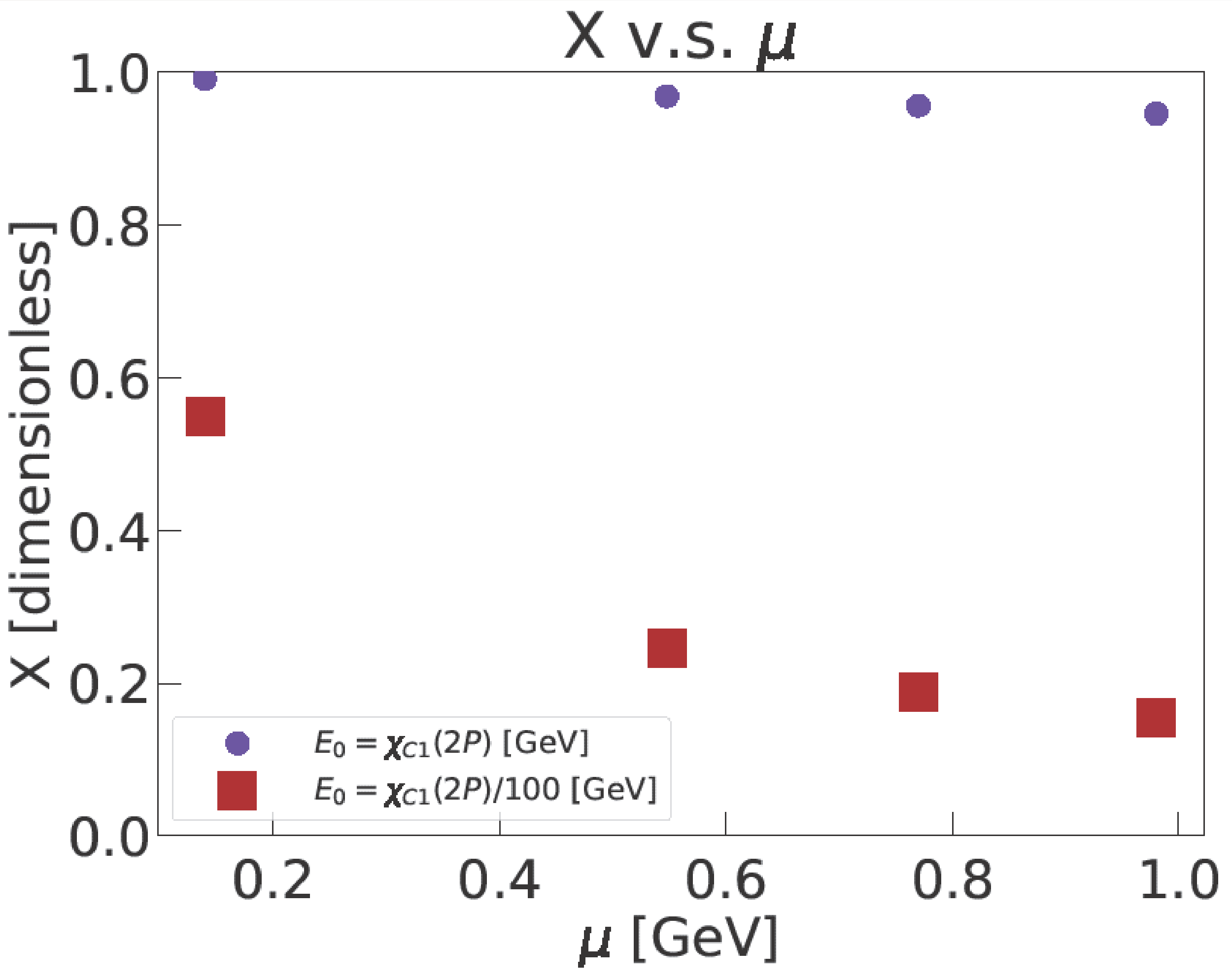
- Parameter dependences for compositeness

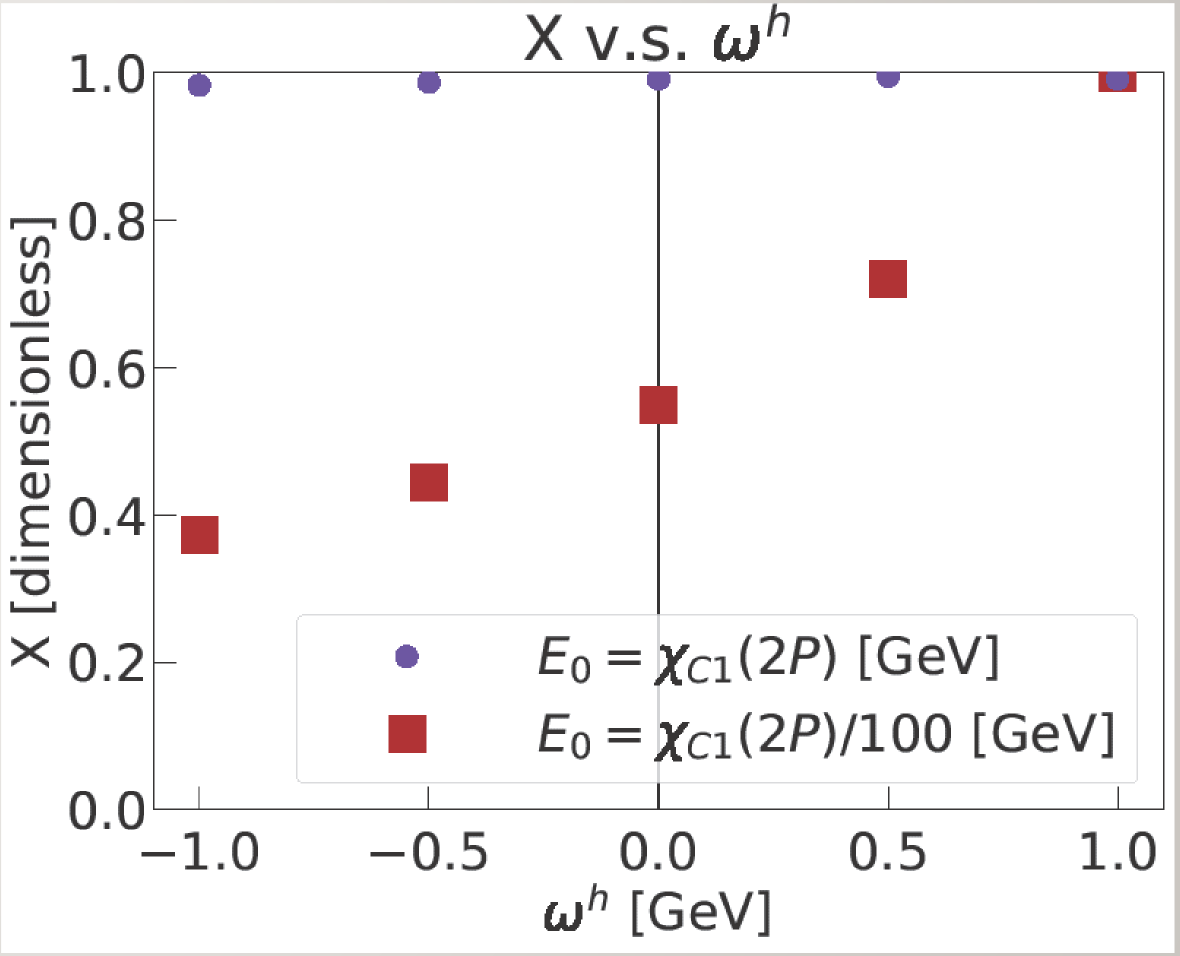
Physical observable	Correlation to compositeness
E_0 (quark channel energy)	**Positive (large)**
B (binding energy of $X(3872)$)	Negative (small)
μ (cut-off of Yukawa potential)	Negative (small)
$\omega^h_{attr.}$ (attractive hadron-ch. potential)	Positive (small)
$\omega^h_{rep.}$ (repulsive hadron-ch. potential)	Negative (small)

◁ Dominant ! in our typical scaling


X v.s. ω^h
X [dimensionless]
ω^h [dimensionless]
$E_0 = \chi_{C1}(2P)$ [GeV]
$E_0 = \chi_{C1}(2P)/100$ [GeV]
1.0
0.8
0.6
0.4
0.2
0.0
−1.0
−0.5
0.0
0.5
1.0


X v.s. B
X [dimensionless]
B[GeV]
$\mu = 0.14$ [GeV]
$\mu = 0.77$ [GeV]
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.01
0.02
0.03
0.04


X v.s. μ
X [dimensionless]
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0
μ [GeV]
$E_0 = \chi_{C1}(2P)$ [GeV]
$E_0 = \chi_{C1}(2P)/100$ [GeV]


X v.s. ω^h
X [dimensionless]
1.0
0.8
0.6
0.4
0.2
0.0
−1.0
−0.5
0.0
0.5
1.0
ω^h [GeV]
$E_0 = \chi_{C1}(2P)$ [GeV]
$E_0 = \chi_{C1}(2P)/100$ [GeV]

Effective potential

■Eliminate quark channel to obtain an effective Hamiltonian of $H^h_{\rm eff}(E)$ hadron channel

with, $H^h_{\rm eff}(E)\,|h\rangle = E\,|h\rangle$, $V^h_{\rm eff}(E)$

$$H^h_{\rm eff}(E) = T^h + \Delta^h + V^h + V^t G^q(E) V^t$$

✓ No approximation

✓ G_q is the Green function of quark channel

$$G_q(E) = (E - (T^q + V^q))^{-1}$$

➢ Quark channel contribution by coupled channels

- Coordinate representation with initial relative coordinate $\boldsymbol{r}$ and final $\boldsymbol{r}'$

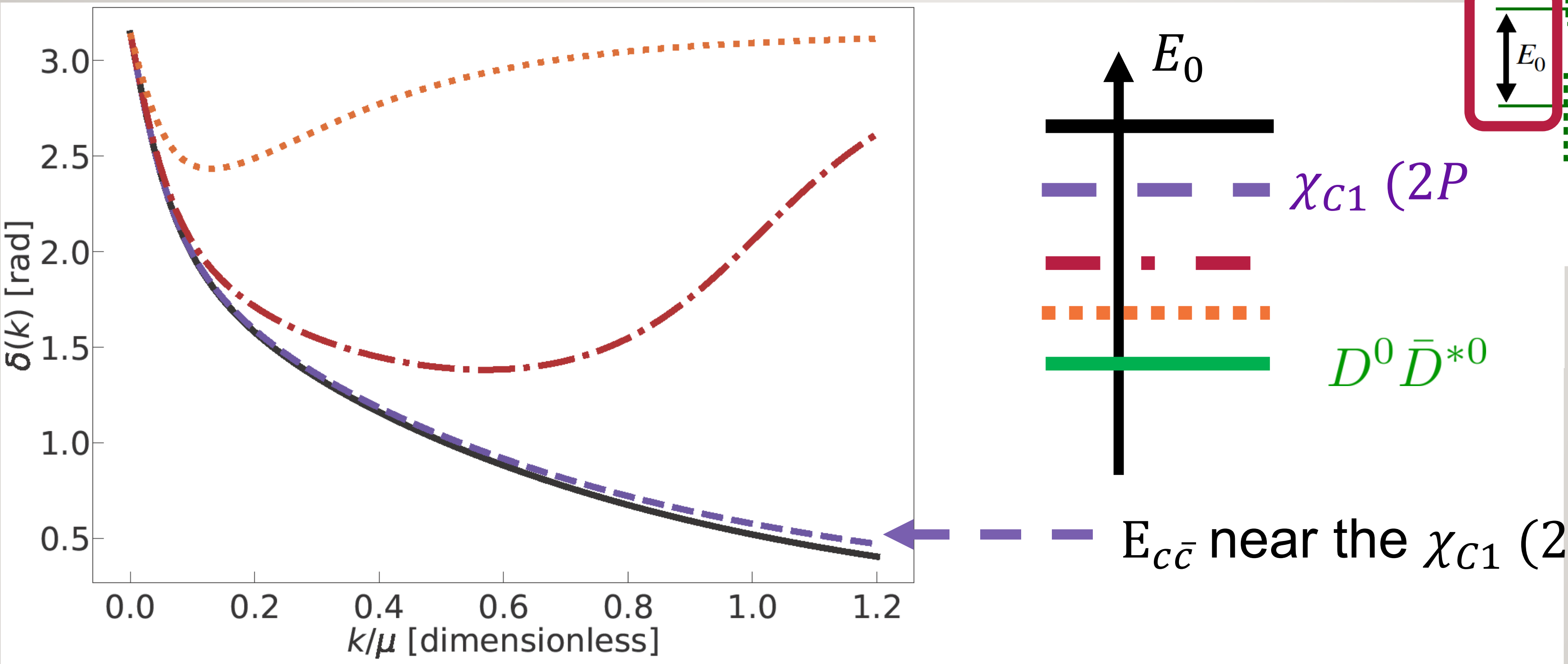
$$\langle \boldsymbol{r}'_h \mid V^h_{\rm eff}(E) \mid \boldsymbol{r}_h\rangle = \langle \boldsymbol{r}'_h \mid V^h \mid \boldsymbol{r}_h\rangle + \sum_n \frac{\langle \boldsymbol{r}'_h \mid V^t \mid \phi_n\rangle \langle \phi_n \mid V^t \mid \boldsymbol{r}_h\rangle}{E - E_n}$$

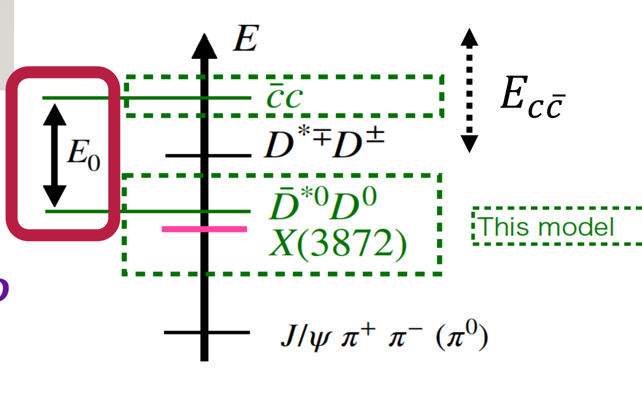
➢ Quark channel contribution. Sum of discrete eigenstates E_n

◆ Energy dependent potential (denominator depends on E)
◆ Non-local potential (numerator depends on $\boldsymbol{r}, \boldsymbol{r}'$ independently)

Result: E_0 dependence of $V^h_{\rm eff}(r, r', E)$

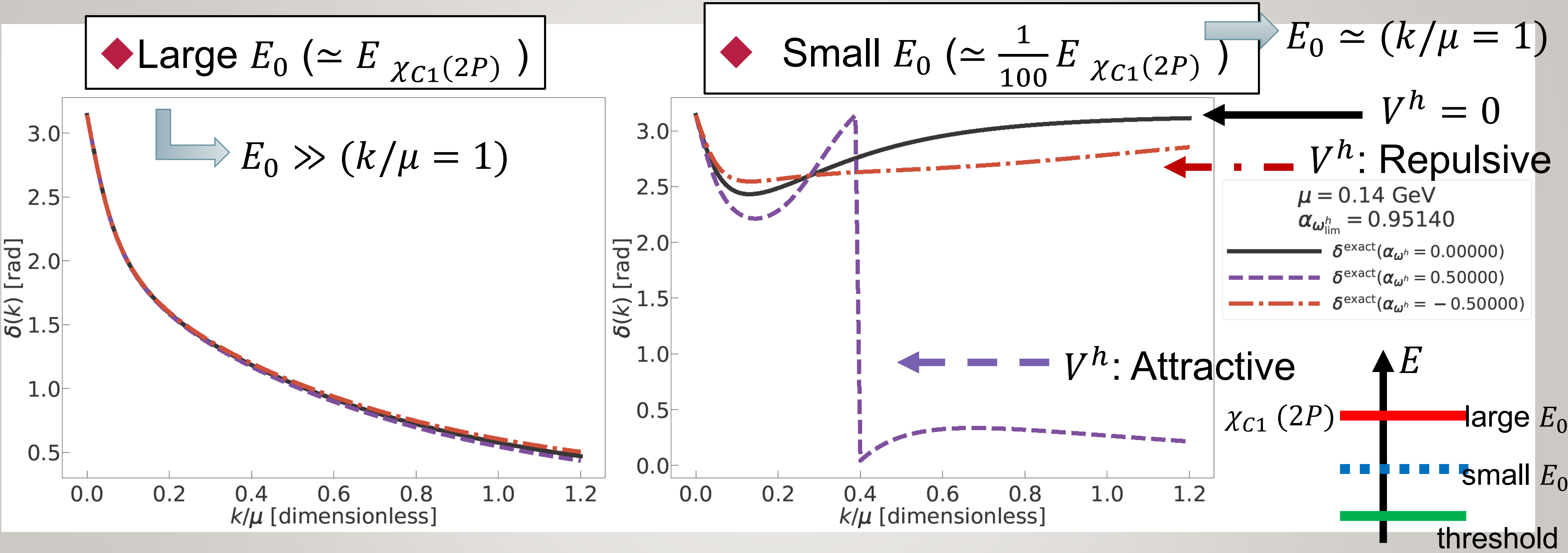
- Compare exact phase shift $\delta(k)$ with different E_0





- E_0 dependence of exact $\delta(k)$ is large for small E_0
  - Binding energy is fixed so that $\delta(k)$ does not change in small k reagion

Result: V^h dependence of exact $\delta(k)$



- V^h dependence of exact $\delta(k)$ is large for small E_0
  - ✓ Quark potential strength $\omega^q = \frac{g_0^2}{E - E_0} \approx -\frac{g_0^2}{E_0} - \frac{E g_0^2}{E_0^2}$ is suppressed when E_0 is large

Result : Compositeness

- **Compositeness** is also calculatable analytically by considering Lippmann–Schwinger equation or the bound state wave function
  - Compositeness corresponds to elementary for 0, molecule for 1
- When quark-ch. energy is close to the threshold energy of meson creation, effect of the hadron-ch. is great

Quark channel energy	Binding energy [KeV]	Hadron channel potential	Compositeness [dimensionless]	Scattering length [fm]
$\chi_{C1}(2P)$	40	None	**0.991**	24.5
$\chi_{C1}(2P)$ / 100	40	Attractive	**0.719**	20.78
$\chi_{C1}(2P)$ / 100	40	None	**0.549**	17.87
$\chi_{C1}(2P)$ / 100	40	Repulsive	**0.444**	15.77