General behavior of near-threshold hadron scattering for exotic hadrons

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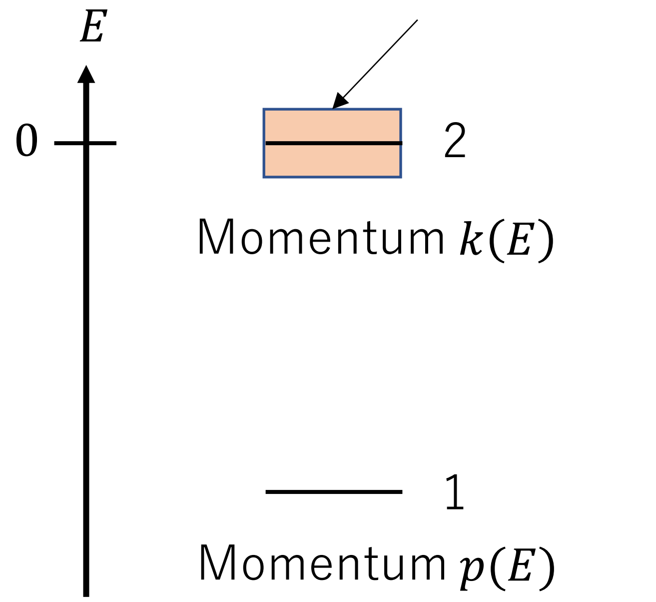
Background

Exotic hadrons ⇨ $X(3872), f_0(980)$

Internal structure ⇔ Scattering length a

Recent analysis of near-threshold hadron scattering

⇨ **Flatté amplitude** has been used[1].

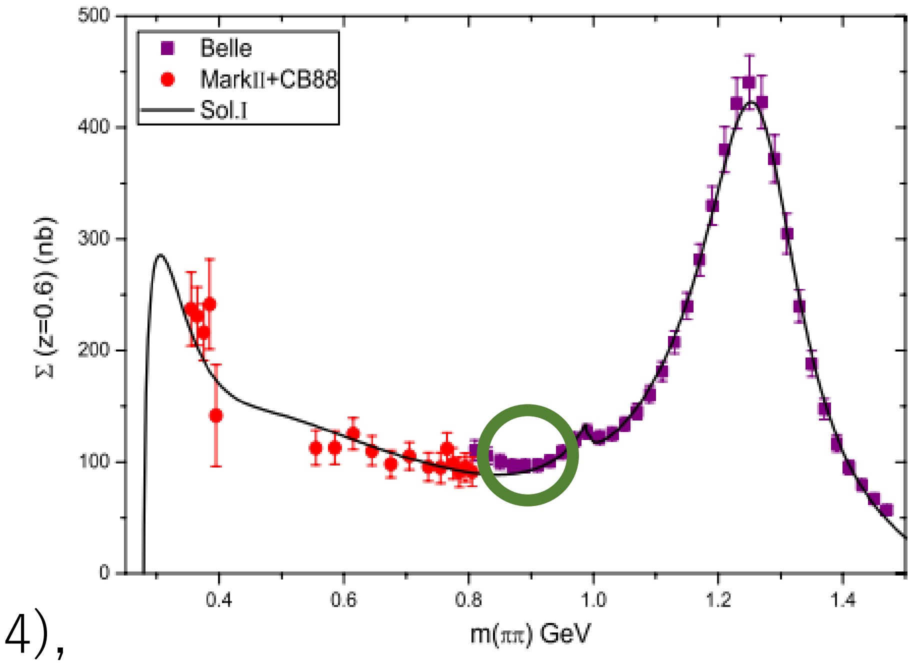


Dip structure emerges near the threshold[2]

⇨ some explanations

This work: We discuss the behavior of cross section

⇨ focus on a **zero point**



[1] R.Aaij et al. [LHCb], Phys. Rev. D 102, no.9, 092005 (2020)
[2] L.-Y. Dai and M. R. Pennington, Phys. Rev. D 90, 036004 (2014),

Dip structure

A dip emerges near the threshold similarly a peak

Dip is caused by **the interference**

⇨ left-hand cuts, backgrounds, zero points…

this work

・**Zero point of an amplitude**

Scattering amplitudes may have zero points

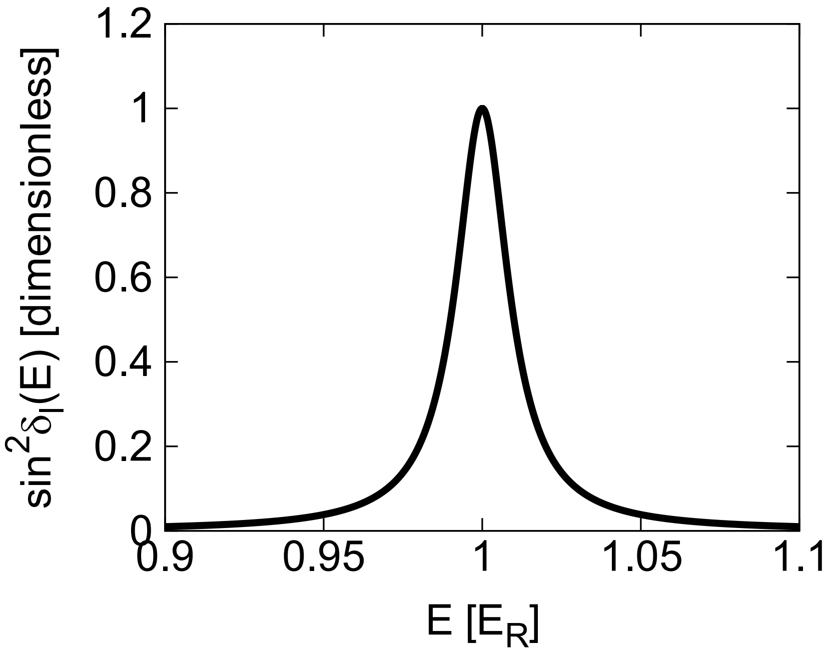
EX.)

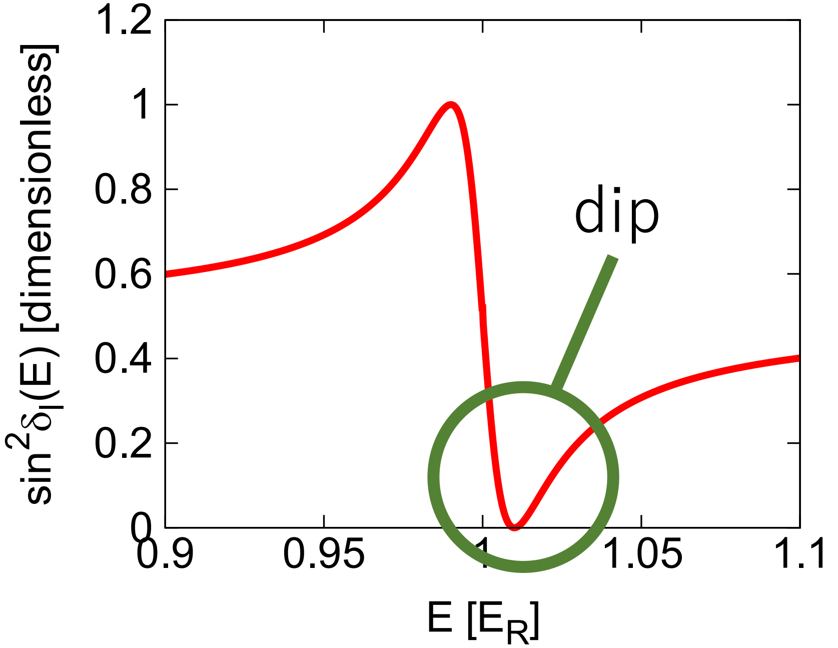
$$f(E) = f^{BW}(E) + f^{BG}(E)$$

⇨ Interference between $f^{BW}(E)$ and $f^{BG}(E)$

・At the zero point: $Im f(E) = \mathrm{Re} f(E) = 0$

⇨ **Dip structure**





General form : Contact amplitude

The general form of the scattering amplitude is derived from the optical theorem.

One of the general form derived from EFT.

Contact amplitude[3] up to first order of k.

$$f^C = \left\{\frac{1}{a_{12}^2} - \left(\frac{1}{a_{22}} + ik\right)\left(\frac{1}{a_{11}} + ip_0\right)\right\}^{-1} \begin{pmatrix} \left(\frac{1}{a_{22}} + ik\right) & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & \left(\frac{1}{a_{11}} + ip_0\right) \end{pmatrix}$$

The Contact amplitude has three parameters a_{11}, a_{12}, a_{22} near the threshold.

11 component of the Contact amplitude has **a zero point** of the amplitude

$$k^C_{zero} = i/a_{22}$$

[3]T.D.Cohen et al. , Phys. Lett. B 588 (2004) 57-66

Flatté amplitude

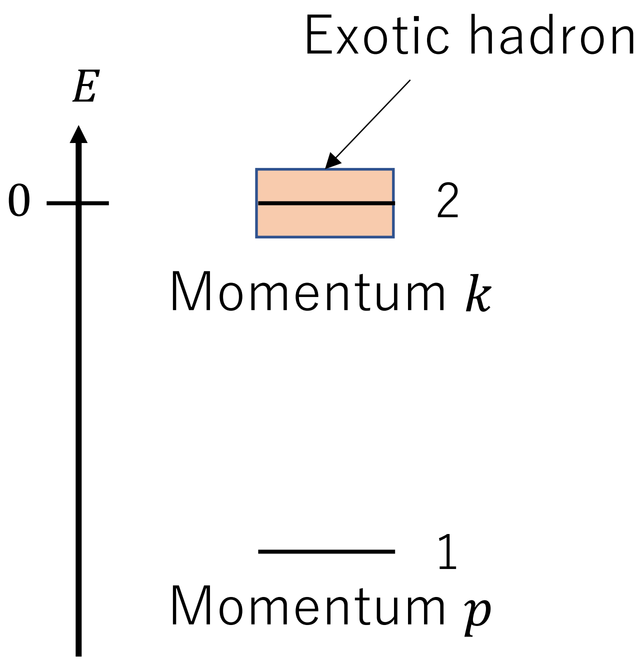
The Flatté amplitude

$$f^F = h(E)\begin{pmatrix} g_1^2 & g_1 g_2 \\ g_1 g_2 & g_2^2 \end{pmatrix}$$

The Flatté parameters

g_1, g_2 : Real coupling constants

E_{BW} : Bare energy



The Flatté amplitude has the threshold effect.

$$h(E) = -\frac{1}{2}\frac{1}{E - E_{BW} + i\, g_1^2 p(E)/2 + i g_2^2\, k(E)/2}$$

Flatté amplitude **does not have any zero point**

f_{11}^F, f_{22}^F can be written as the effective range expansion in k.

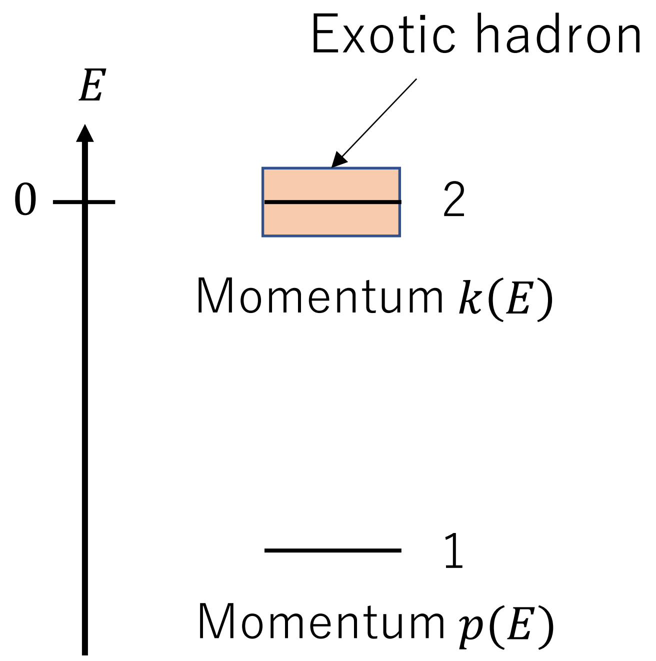
$$f_{11}^F, f_{22}^F \propto \left(-\frac{1}{a_F} + \frac{1}{2} r_F k^2 - ik + O(k^4)\right)^{-1}$$

a_F : Scattering length

r_F : Effective range

Problem of Flatté amplitude

$1/f_{11}^F$ up to order k^1 can be written only by two parameters R, α[4].



$$f_{11}^F = \frac{g_1^2}{2E_{BW} - ig_1^2 p_0 - ig_2^2 k} = \frac{1/R}{\alpha p_0/R - ip_0/R - ik}$$

$$\alpha = \frac{2E_{BW}}{g_1^2 p_0} \qquad R = \frac{g_2^2}{g_1^2}$$

We find $1/f_{22}^F$ up to k^1 can also be written only by two parameters R, α.

$$f_{22}^F = \frac{g_2^2}{2E_{BW} - ig_1^2 p_0 - ig_2^2 k} = \frac{1}{\alpha p_0/R - ip_0/R - ik}$$

p_0 : channel 1 momentum at $E = 0$

$f^F(g_1^2, g_2^2, E_{BW})$ three parameters ⇨ $f^F(R, \alpha)$ two parameters(near the threshold)

⇨ The number of parameters is less than that of the general form(Contact)

[4] V. Baru et al. Eur. Phys. J. A, 23, 523-533 (2005)

Comparison

What is the difference between the Contact and Flatté ?

Contact

$$(f^C)^{-1} = \begin{pmatrix} -\frac{1}{a_{11}} - ip_0 & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & -\frac{1}{a_{22}} - ik \end{pmatrix}$$

・has a zero point

Flatté

$$f^F = h(E)\begin{pmatrix} g_1^2 & g_1 g_2 \\ g_1 g_2 & g_2^2 \end{pmatrix}$$

⇨ $(f^F)^{-1} =$ does not exist

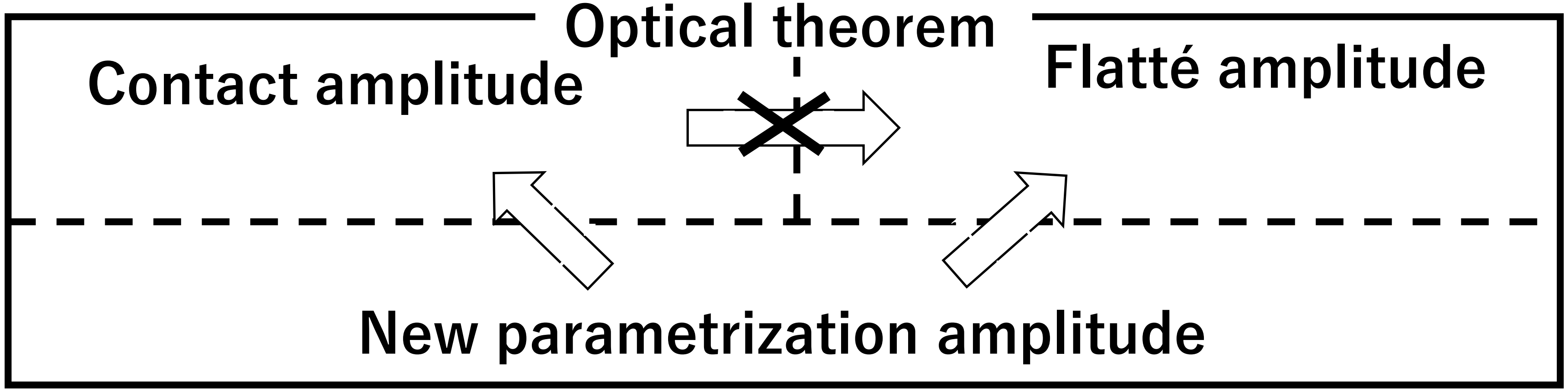
・no zero point

unitarity relation

$\boldsymbol{f^C(a_{11}, a_{12}, a_{22})}$		$\boldsymbol{f^F(\alpha, R)}$
・Inverse matrix exists	⇏	**・Inverse matrix does not exist**

Contact amplitude does not reduce to Flatté amplitude directly

New parametrization amplitude



We construct the new representation including Contact and Flatté.

⇨ **General amplitude $f^G(A_{22}, \gamma, \epsilon)$**

$$(f^C)^{-1} = \begin{pmatrix} -\frac{1}{a_{11}} - ip_0 & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & -\frac{1}{a_{22}} - ik \end{pmatrix} \quad \Rightarrow \quad (f^G)^{-1} = \begin{pmatrix} -\frac{1}{A_{22}}\frac{1}{\gamma} - ip_0 & \frac{1}{A_{22}}\frac{\sqrt{\epsilon-\gamma}}{\gamma} \\ \frac{1}{A_{22}}\frac{\sqrt{\epsilon-\gamma}}{\gamma} & -\frac{1}{A_{22}}\frac{\epsilon}{\gamma} - ik \end{pmatrix}$$

A_{22} : scattering length of channel two in the absence of channel couplings

Property

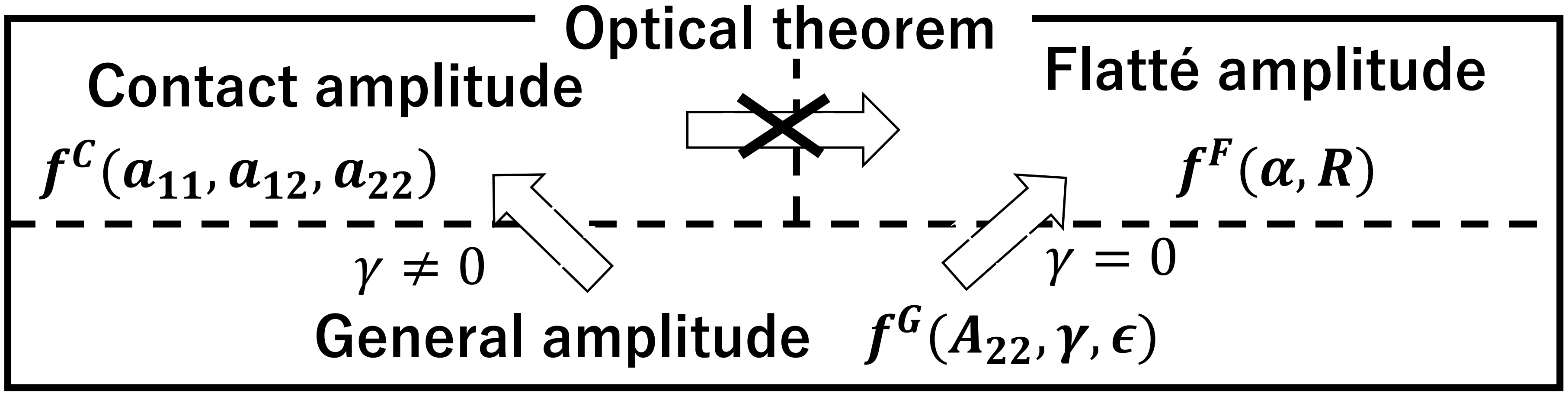
$\gamma \neq 0$: $f^G(A_{22}, \gamma, \epsilon) \Longrightarrow$ Contact amplitude $f^C(a_{11}, a_{12}, a_{22})$

$\gamma = 0$: $f^G(A_{22}, \gamma, \epsilon) \Longrightarrow$ Flatté form

$$f^G(A_{22}, 0, \epsilon) = \frac{1}{-\frac{1}{A_{22}} - i\epsilon p_0 - ik} \begin{pmatrix} \epsilon & \sqrt{\epsilon} \\ \sqrt{\epsilon} & 1 \end{pmatrix}$$

$$(f^G)^{-1}(A_{22}, \gamma, \epsilon) = \begin{pmatrix} -\frac{1}{A_{22}}\frac{1}{\gamma} - ip_0 & \frac{1}{A_{22}}\frac{\sqrt{\epsilon - \gamma}}{\gamma} \\ \frac{1}{A_{22}}\frac{\sqrt{\epsilon - \gamma}}{\gamma} & -\frac{1}{A_{22}}\frac{\epsilon}{\gamma} - ik \end{pmatrix}$$

$\gamma = 0$: $\Longrightarrow$ $(f^G)^{-1}$ = does not exist



Zero point

General amplitude has a zero point in f_{11}^G

$$f_{11}^G = \frac{-\frac{1}{A_{22}}\frac{\epsilon}{\gamma} - ik}{\left(-\frac{1}{A_{22}}\frac{1}{\gamma} - ip_0\right)\left(-\frac{1}{A_{22}}\frac{\epsilon}{\gamma} - ik\right) - \frac{\epsilon-\gamma}{A_{22}^2\gamma^2}} \Rightarrow k_{zero}^G = \frac{i}{A_{22}}\frac{\epsilon}{\gamma}$$

⇓ $\gamma \to 0$

$$f_{11}^F = \frac{\epsilon}{-\frac{1}{A_{22}} - i\epsilon p_0 - ik} \Rightarrow |k_{zero}^F| \to \infty$$

The zero point of Flatté amplitude goes to infinity

⇒ Consistent with the Flatté property

Comparison of the cross section

We study the behavior of the scattering cross section near the threshold when the scattering length is stable.

$$\sigma_{ij} = \frac{p_j}{p_i}\int \left|f_{ij}\right|^2 d\Omega = 4\pi \frac{p_j}{p_i}\left|f_{ij}\right|^2$$

We focus on σ_{11} and σ_{21}

The Flatté amplitude up to first order of k.

$$f_{21}^F = \frac{1/\sqrt{R}}{\alpha p_0/R - ip_0/R - ik} \propto \frac{1}{-\frac{1}{a_F} - ik} \qquad f_{11}^F = \frac{1/R}{\alpha p_0/R - ip_0/R - ik} \propto \frac{1}{-\frac{1}{a_F} - ik}$$

$$\Rightarrow \quad \sigma_{21}^F, \sigma_{11}^F \propto \left|\frac{1}{-\frac{1}{a_F} - ik}\right|^2$$

The Flatté cross sections near the threshold $\boldsymbol{\sigma_{21}^F, \sigma_{11}^F}$ are determined only by $\boldsymbol{a_F}$.

Comparison of the cross section

The General amplitude up to first order of k.

$$f_{21}^G = \frac{C_{21}^G}{-\frac{1}{A_{22}}\left(\frac{\frac{1}{A_{22}}+i\epsilon p_0}{\frac{1}{A_{22}}+i\gamma p_0}\right)-ik} \propto \frac{1}{-\frac{1}{a_G}-ik}$$

$$f_{11}^G \cong \frac{1}{-\frac{1}{a_G-b(a_G,\gamma)}-ik}$$

$b(Re(a_G), Im(a_G), \gamma)$: Real constant

⇨ $$\sigma_{21}^G \propto \left|\frac{1}{-\frac{1}{a_G}-ik}\right|^2$$

⇨ $$\sigma_{11}^G \propto \left|\frac{1}{-\frac{1}{a_G-b(a_G,\gamma)}-ik}\right|^2$$

$\sigma_{21}^G(A_{22},\epsilon,\gamma)$ $\sigma_{11}^G(A_{22},\epsilon,\gamma)$ ⇨ $\sigma_{21}^G(Re(a_G), Im(a_G))$ $\sigma_{11}^G(Re(a_G), Im(a_G), \gamma)$

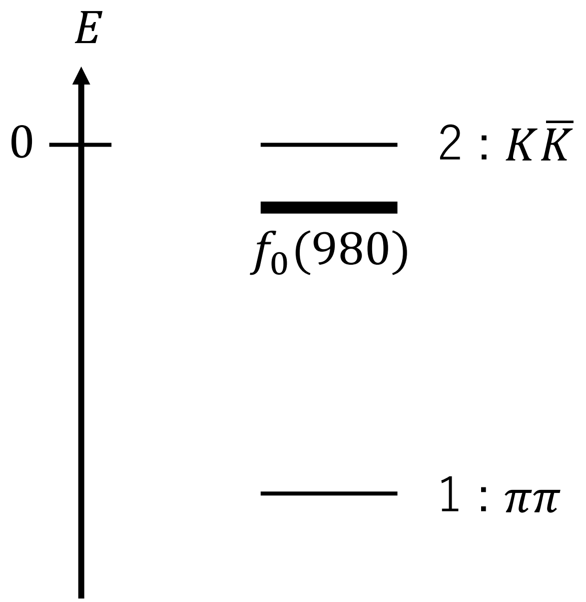
⇨ **When a_G is fixed, σ_{21}^G is stable, but σ_{11}^G changes for variation of γ.**

Application

We apply the General amplitude to $\pi\pi$-$K\bar{K}$ scattering

⇨ $f_0(980)$ locates below the $K\bar{K}$ threshold(ch. 2)

$f_0(980)$ corresponds to **quasibound state**



・Pole position

The scattering amplitude up to first order of k

$$\textbf{pole}: \; k_p^G \cong i/a_G$$

The pole position can be written only by the scattering length a_G

fixed a_G ⇔ fixed k_p^G

Cross section

$\pi\pi$-$K\bar{K}$ system with $f_0(980)$ **as an example**

$$a_G = a_F = +1.0 - i0.8 \text{ [fm]}$$

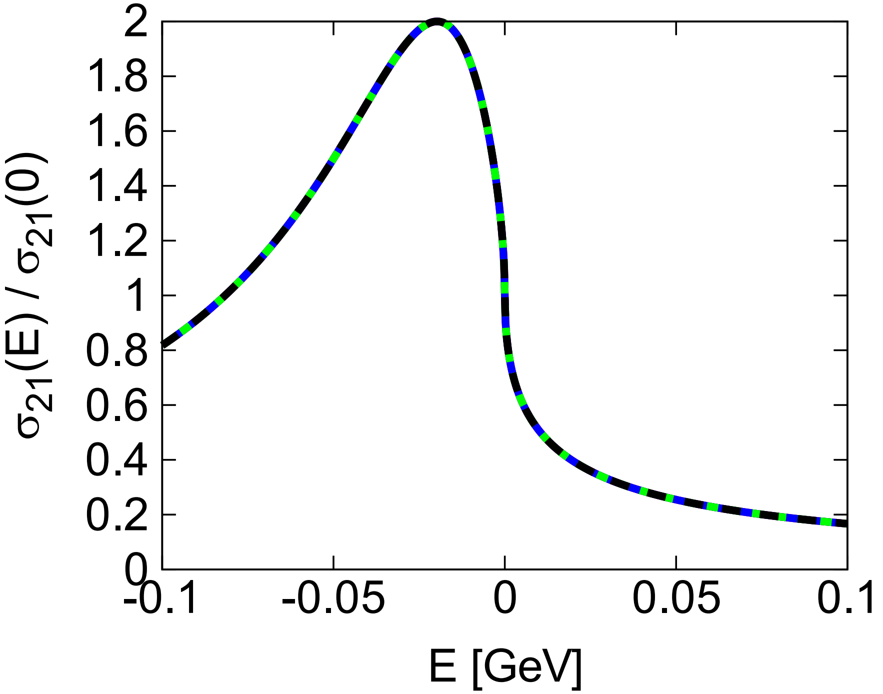
a_G makes the sharp peak in σ_{12}^G

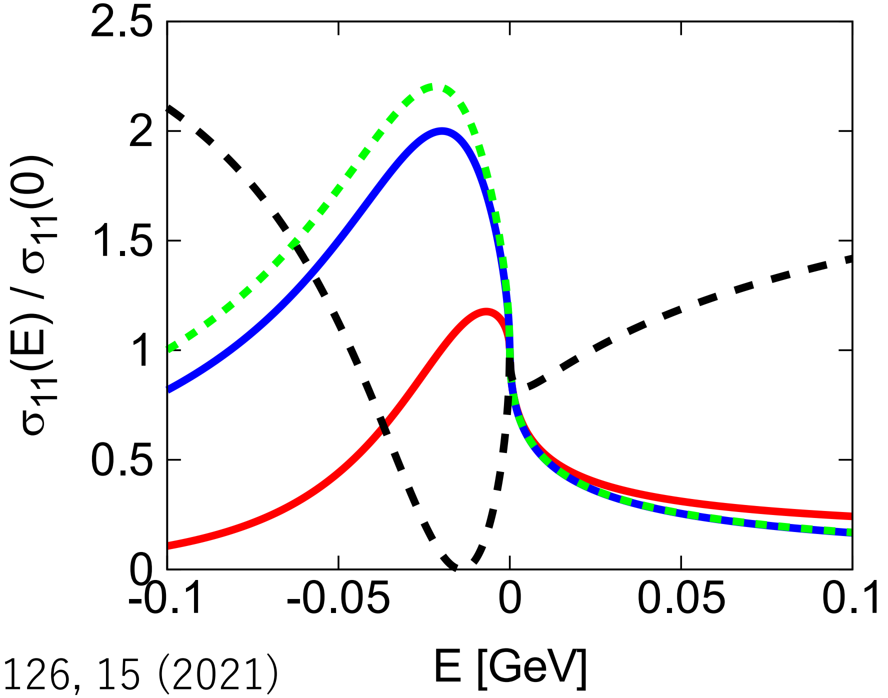
(1) $A_{22} = 3.4\text{[fm]}, \epsilon = 0.3, \gamma = 0.05$

(2) $\gamma = 0.0$ (Flatté)

(3) $A_{22} = 1.9\text{[fm]}, \epsilon = 0.2, \gamma = -0.01$

(4) $A_{22} = 0.27\text{[fm]}, \epsilon = -1.1, \gamma = -10.0$





However, σ^G changes significantly for same a_G. In particular, when $\epsilon < 0$, the dip emerge below the threshold[5].

[5]Dong, Xiang-Kun and Guo, Feng-Kun and Zou, Bing-Song, Phys. Rev. Lett.126, 15 (2021)

Summary

- We focus on the number of parameters of the scattering amplitude near the threshold.

⇨ The Flatté amplitude is written by only **two parameters**.
The Contact amplitude is written by **three parameters**.

- We propose a new parametrization of the Contact amplitude .

⇨ The general amplitude $f^G(A_{22}, \gamma, \epsilon)$ reduces to the Flatté amplitude when $\gamma = 0$.

- We study the behavior of the scattering cross section near the threshold.

⇨ The cross section σ_{11} have a **dip**. In this case Flatté cross section **does not work**

The scattering length

The scattering length a is obtained from the effective range expansion

$$f_{22}(k) = \frac{1}{-\frac{1}{a} + \frac{r}{2}k^2 + O(k^4) - ik}$$

a : the scattering length

r : the effective range

・Flatté scattering length a_F

$$a_F = \frac{1}{\frac{1}{A_{22}} + i\epsilon p_0}$$

・General scattering length a_G

$$a_G = A_{22}\left(\frac{\frac{1}{A_{22}} + i\gamma p_0}{\frac{1}{A_{22}} + i\epsilon p_0}\right)$$

When a pole is near the threshold, the pole position is related to a

Pole position $k \sim i/a$

Pole term

f_{11}^{G} can be divided into two parts, the pole term and the constant background term.

$$f_{11}^{G}(k) = \frac{-\frac{1}{A_{22}}\frac{\epsilon}{\gamma} - ik}{\left(-\frac{1}{A_{22}}\frac{1}{\gamma} - ip_0\right)\left(-\frac{1}{A_{22}}\frac{\epsilon}{\gamma} - ik\right) - \frac{\epsilon-\gamma}{A_{22}^2\gamma^2}} = \underbrace{\frac{i(\gamma-\epsilon)}{(A_{22}\gamma p_0 - i)^2}\frac{1}{k-k_p}}_{\text{Pole term}} - \underbrace{\frac{iA_{22}\gamma}{A_{22}\gamma p_0 - i}}_{\text{BG term}} \qquad k_p = \frac{i}{a_G}$$

$\Downarrow$ $\gamma = 0$

$$f_{11}^{F} = \frac{\epsilon}{-\frac{1}{A_{22}} - i\epsilon p_0 - ik} = \underbrace{\frac{i\epsilon}{k-k_p}}_{\text{Pole term}} \qquad k_p = \frac{i}{a_F}$$

When $\gamma = 0$, the BG term vanishes.

⇨ The parameter γ determines the magnitude of the BG term.

Comparison of Flatté with EFT

The relation between A_{22}, γ, ϵ and a_{11}, a_{12}, a_{22}.

$$a_{11} = A_{22}\gamma \qquad a_{12} = \frac{A_{22}\gamma}{\sqrt{\epsilon - \gamma}} \qquad a_{22} = \frac{A_{22}\gamma}{\epsilon}$$

⇩

When $\gamma = 0$: $a_{11} \to 0$ $a_{12} \to 0$ $a_{22} \to 0$

The relation between EFT amplitude and Flatté amplitude.

$$\lim_{a_{11}, a_{12}, a_{22} \to 0} f^{EFT}(k: a_{11}, a_{12}, a_{22}) = f^{F}(k: R, \alpha)$$

Flatté amplitude cannot be written by EFT parameters a_{11}, a_{12}, a_{22}.

⇨ EFT amplitude does not reduce to Flatté amplitude directly.

Cross section

We calculate σ_G varying γ for same value of scattering length:

$$a_G = a_F = -1.0 - i1.0 \text{ [fm]}$$

This a_G makes the shaper cusp at $E = 0$ for σ_{21}.

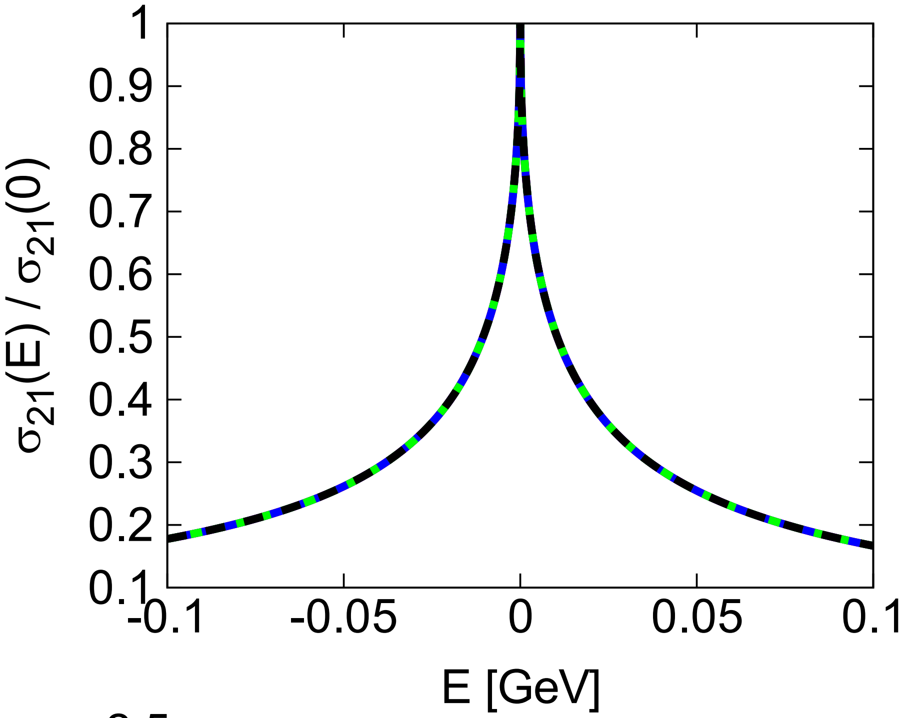
(1) $A_{22} = -3.4[\text{fm}], \epsilon = 0.3, \gamma = 0.05$

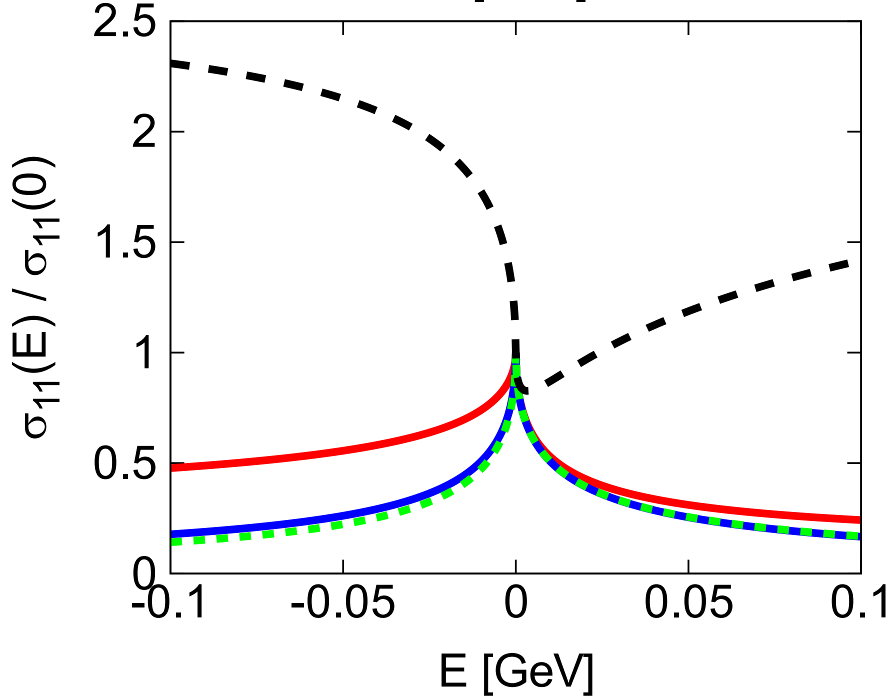
(2) $\gamma = 0.0$ (Flatté)

(3) $A_{22} = -1.9 \text{ [fm]}, \epsilon = 0.2, \gamma = -0.01$

(4) $A_{22} = -0.27[\text{fm}], \epsilon = -1.1, \gamma = -10.0$

However, σ^G changes significantly for same a_G. In particular, when $\epsilon < 0$, the dip emerge near the threshold.





The phase shift δ

The S-matrix for two channel :

$$S = \begin{pmatrix} \eta e^{2i\delta_1} & i(1-\eta^2)^{1/2} e^{i(\delta_1+\delta_2)} \\ i(1-\eta^2)^{1/2} e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix}$$

η : inelasticity

δ_1, δ_2 : phase shift

The relation between S_{11} and f_{11}^G :

$$S_{11} = \eta e^{2i\delta_1} = 1 + 2ip_0 f_{11}^G$$

$$\eta e^{2i\delta_1} = \frac{-\frac{1}{A_{22}} + i\epsilon p_0 - ik - A_{22}\gamma p_0}{-\frac{1}{A_{22}} - i\epsilon p_0 - ik + A_{22}\gamma p_0 k}$$

$$S_{11} = \frac{d(-p_0, k)}{d(p_0, k)}$$

S_{11} can be written by the pole k_p^G.

$$\eta e^{2i\delta_1} = -\frac{A_{22}\gamma p_0 + i}{A_{22}\gamma p_0 - i} \times \frac{k + \left(k_p^G\right)^*}{k - k_p^G}$$

$$k_p = \frac{i}{a_G}$$

The background phase shift $\boldsymbol{\delta_{BG}}$

We represent the phase shift δ_1 by A_{22}, γ, ϵ.

$$\eta e^{2i\delta_1} = -\frac{A_{22}\gamma p_0 + i}{A_{22}\gamma p_0 - i} \times \frac{k + \left(k_p^G\right)^*}{k - k_p^G}$$

⇩

$$2\delta_1 = \underline{arg\left(-\frac{A_{22}\gamma p_0 + i}{A_{22}\gamma p_0 - i}\right)} + \underline{arg\left(\frac{k + \left(k_p^G\right)^*}{k - k_p^G}\right)}$$

BG phase shift $= 2\delta_{BG}$

pole phase shift $= 2\delta_P$

⇨ $\delta_1 = \delta_P + \delta_{BG}$

When $\gamma = 0$, δ_{BG} becomes 0 : $2\delta_{BG} = arg\left(-\frac{i}{-i}\right) = 0$ (Flatté amplitude)

The effect of $\boldsymbol{\delta_{BG}}$

We focus on the energy region **below the threshold** $(\boldsymbol{E < 0})$

⇨ $$S_{11} = e^{2i\delta_1}$$

The cross section represented by δ_1 :

$$\sigma_{11}^{G} = 4\pi\left|f_{11}^{G}\right|^2 = 4\pi\left|\frac{S_{11}-1}{2ip_0}\right|^2 \qquad (S_{11} = e^{2i\delta_1} = 1 + 2ip_0 f_{11}^{G})$$

$$\sigma_{11}^{G} \propto sin^2(\delta_P + \delta_{BG})$$

If $\delta_{BG} > \pi/2$ and δ_P is larger than $\pi/2$,

$$\sigma_{11}^{G} \propto sin^2(\pi) = 0$$

The cross section $\boldsymbol{\sigma_{11}^{G}}$ has a dip.

⇨ **ex. $\boldsymbol{\pi\pi}$-$\boldsymbol{\pi\pi}$ scattering : the peak of $\boldsymbol{f_0(980)}$ is affected by the pole of $\boldsymbol{\sigma}$**

Backup

scattering length : $a_G = \alpha + i\beta$ ($\beta < 0$)

⇩

$$A_{22}\left(\frac{\frac{1}{A_{22}}+i\gamma p_0}{\frac{1}{A_{22}}+i\epsilon p_0}\right) = \alpha + i\beta \ (\beta < 0)$$

⇩

$$A_{22}(\gamma) = \frac{-\frac{\alpha}{\beta} \pm \sqrt{\left(\frac{\alpha}{\beta}\right)^2 + 4\gamma p_0\left(\beta + \frac{\alpha^2}{\beta}\right)}}{2\gamma p_0}$$

$$\epsilon = \frac{1}{\beta p_0}\left(\frac{\alpha}{A_{22}(\gamma)} - 1\right)$$

⇩

$$a_G = \alpha + i\beta \begin{cases} A_{22}^{+}, \epsilon^{+}, \gamma \\ A_{22}^{-}, \epsilon^{-}, \gamma \end{cases}$$

: There are two sets of parameters for a_G

Backup

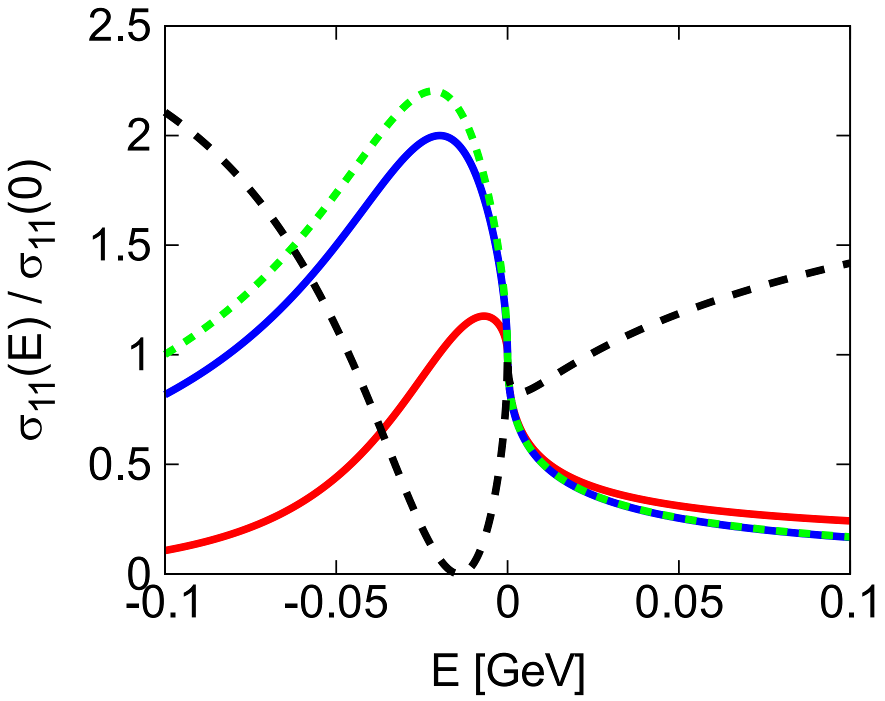
scattering length : $a_G = 1.0 - i1.0$

(1) $\sigma^G_{11}(E\!:A^-_{22}, \epsilon^-, \gamma)$

(2)

(3) In the page 17

(4)



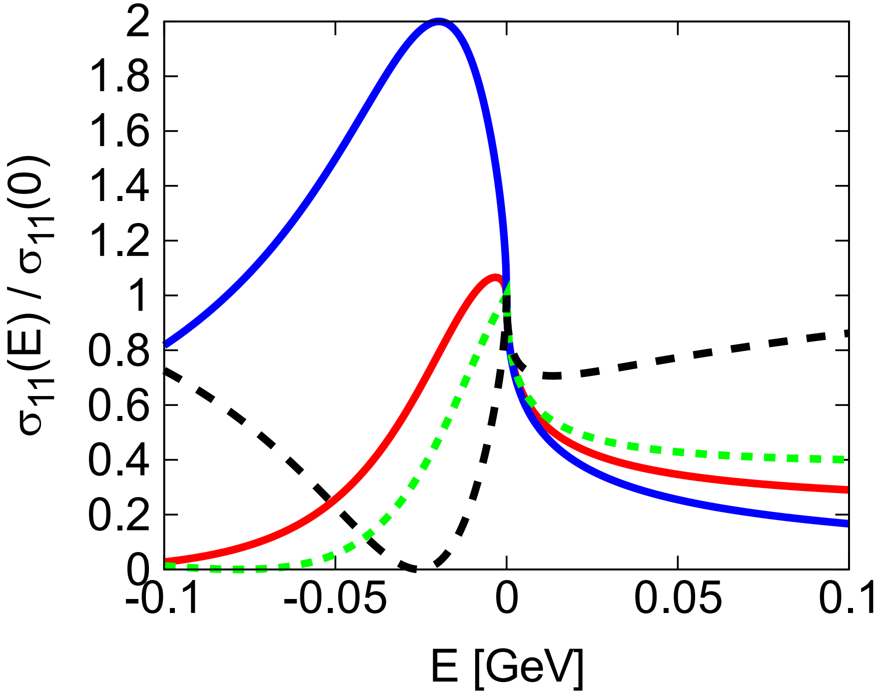
$\sigma^G_{11}(E\!:A^+_{22}, \epsilon^+, \gamma)$

(1) $A_{22} = 4.9[\text{fm}],\ \epsilon = 0.33,\ \gamma = 0.05$

(2) $\gamma = 0.0$ (Flatté) $(\sigma^G_{11}(E\!:A^-_{22}, \epsilon^-, \gamma))$

(3) $A_{22} = -43.3[\text{fm}],\ \epsilon = 0.42,\ \gamma = -0.01$

(4) $A_{22} = -0.31[\text{fm}],\ \epsilon = 1.75,\ \gamma = -10.0$



Backup

scattering length : $\boldsymbol{a_G = -1.0 - i1.0}$

(1) ▬ (red)

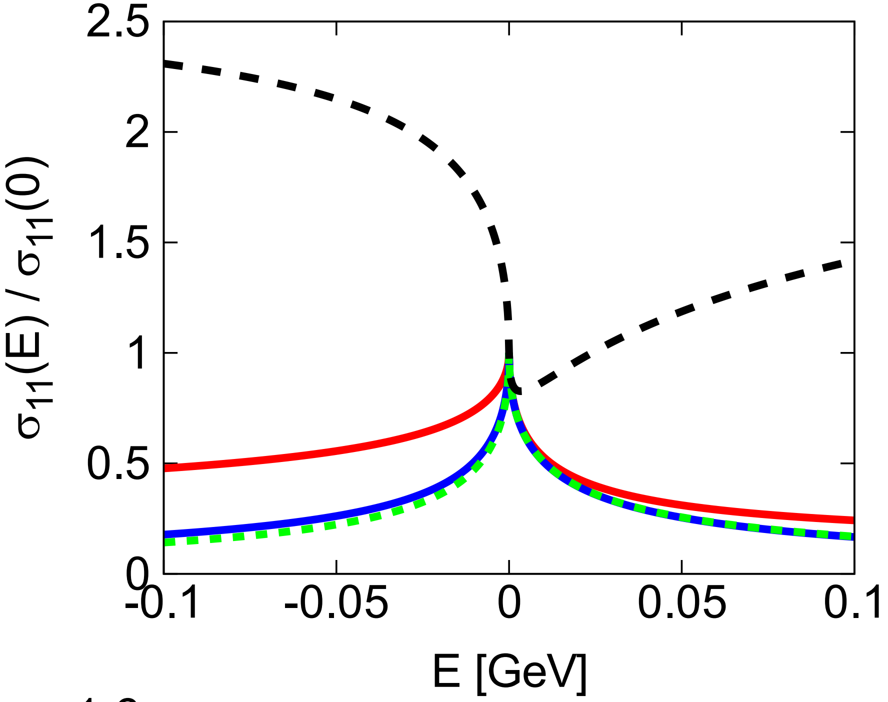
$\sigma_{11}^{G}(E\!:A_{22}^{+},\epsilon^{+},\gamma)$

(2) ▬ (blue)

⇨ In the page 18

(3) ▪▪▪ (green dotted)

(4) ▬ ▬ (black dashed)

⇨ 

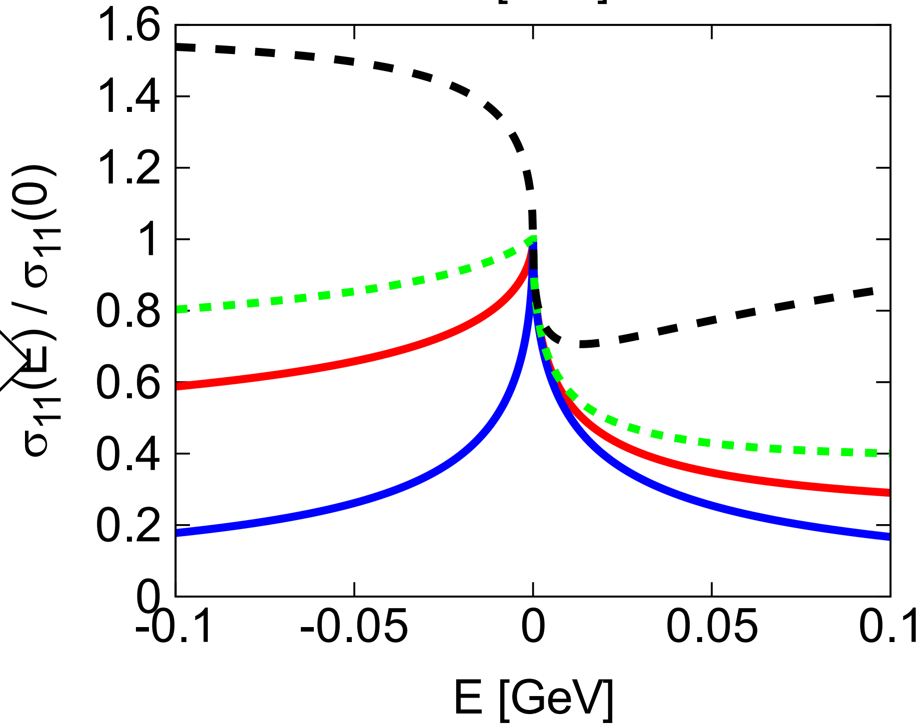
$\sigma_{11}^{G}(E\!:A_{22}^{-},\epsilon^{-},\gamma)$

(1) $A_{22} = -4.9[\mathrm{fm}],\ \epsilon = 0.33,\ \gamma = 0.05$

(2) $\gamma = 0.0$ (Flatté) $(\sigma_{11}^{G}(E\!:A_{22}^{+},\epsilon^{+},\gamma))$

(3) $A_{22} = 43.3[\mathrm{fm}],\ \epsilon = 0.42,\ \gamma = -0.01$

(4) $A_{22} = 0.31[\mathrm{fm}],\ \epsilon = 1.75,\ \gamma = -10.0$

⇨ 

EFT parameters near-threshold

$1/f_{11}^{EFT}$ up to order k^1

$$f_{11}^{EFT}=\frac{a_{12}^2/a_{22}^2}{\frac{1}{a_{22}}-\frac{a_{12}^2}{a_{11}a_{22}^2}-i\frac{a_{12}^2}{a_{22}^2}p_0-ik}$$

$1/f_{22}^{EFT}$ up to order k^1

$$f_{22}^{EFT}=\frac{1}{\frac{1}{a_{12}^2\left(\frac{1}{a_{11}}+ip_0\right)}-\frac{1}{a_{22}}-ik}$$

$f^{EFT}(a_{11},a_{12},a_{22})$ three parameters（near the threshold）

Are there any relations between the EFT amplitude and the Flatté amplitude?

Determination of a_F

We consider the region near the threshold 2(region II and III).

2→2 scattering does not occur in region II.

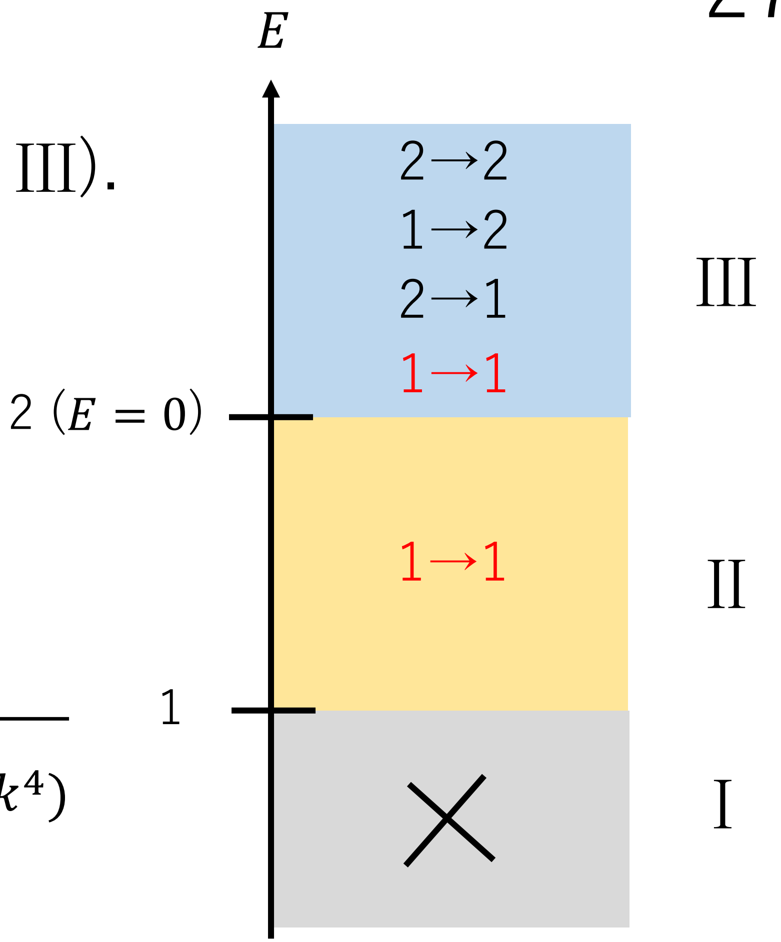
1→1 scattering occurs in both region II and III.

⇨ a_F is determined from f_{11}^F
(ex. [1][4] for $X(3872)$).

$$f_{11}^F = \frac{\frac{g_1^2}{g_2^2}}{\left(\frac{2E_{BW}}{g_2^2} - i\frac{g_1^2}{g_2^2}p_0\right) - \left(\frac{2}{m_k g_2^2} + i\frac{g_1^2}{2p_0 g_2^2}\right)k^2 - ik + O(k^4)}$$

$$a_F = -\frac{g_2^2}{2E_{BW} - i g_1^2 p_0}$$

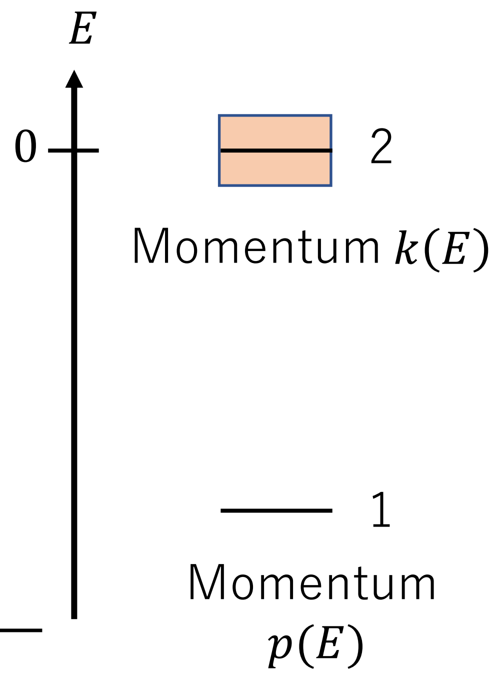
Scattering length



[1] R. Aaij et al. [LHCb], Phys. Rev. D102, no.9, 092005 (2020)

[4] A. Esposito et al., Phys. Rev. D 105 (2022) 3, L031503

f_{22} component



Effective range expansion for f_{22}^G

$$f_{22}^G = \frac{-\frac{1}{A_{22}}\frac{1}{\gamma} - ip_0}{\left(-\frac{1}{A_{22}}\frac{1}{\gamma} - ip_0\right)\left(-\frac{1}{A_{22}}\frac{\epsilon}{\gamma} - ik\right) - \frac{\epsilon-\gamma}{A_{22}^2\gamma^2}}$$

$$= \frac{1}{-\frac{1}{A_{22}}\left(\frac{\frac{1}{A_{22}} + i\epsilon p_0}{\frac{1}{A_{22}} + i\gamma p_0}\right) - \frac{i(\epsilon-\gamma)}{2(1+iA_{22}\gamma p_0)^2 p_0}k^2 - ik + O(k^4)}$$

$$a_G = A_{22}\left(\frac{\frac{1}{A_{22}} + i\gamma p_0}{\frac{1}{A_{22}} + i\epsilon p_0}\right)$$

: scattering length

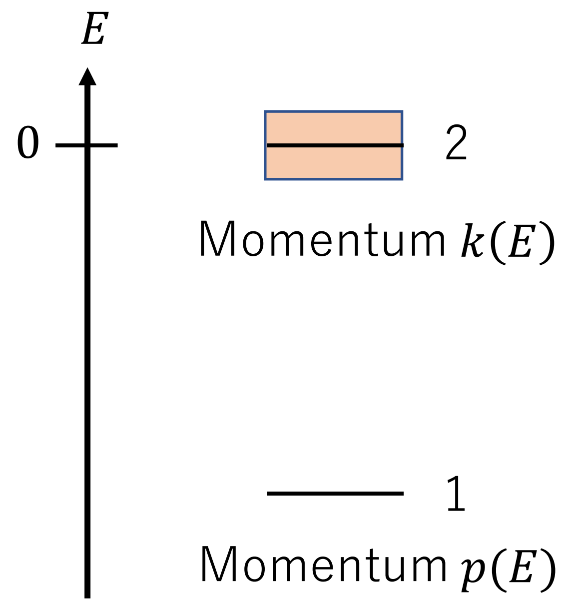
f_{22}^G can be written as the effective range expansion in k.

f_{11} component

Effective range expansion for f_{11}^G

$$f_{11}^G = \frac{-\frac{1}{A_{22}}\frac{\epsilon}{\gamma} - ik}{\left(-\frac{1}{A_{22}}\frac{1}{\gamma} - ip_0\right)\left(-\frac{1}{A_{22}}\frac{\epsilon}{\gamma} - ik\right) - \frac{\epsilon-\gamma}{A_{22}^2\gamma^2}}$$

$$= \frac{\frac{\epsilon^2}{\epsilon-\gamma}}{-\frac{1}{A_{22}}\frac{\epsilon}{\epsilon-\gamma} - i\frac{\epsilon^2}{\epsilon-\gamma}p_0 - \left(A_{22}\frac{\gamma}{\epsilon} + i\frac{\epsilon^2}{2(\epsilon-\gamma)p_0}\right)k^2 - ik + O(k^3)}$$



f_{11}^G cannot be written as the effective range expansion in k.

⇨ a_G should not be defined in f_{11}^G.

The correct scattering length must be defined by f_{22}.

Scattering length

The constant term of the denominator of f_{22}^{G}

⇨ general scattering length a_G

$$a_G = A_{22}\left(\frac{\frac{1}{A_{22}} + i\gamma p_0}{\frac{1}{A_{22}} + i\epsilon p_0}\right)$$

$\gamma = 0$ ⇨

Flatté scattering length a_F

$$a_F = \frac{1}{\frac{1}{A_{22}} + i\epsilon p_0}$$

The constant term of the denominator of f_{11}^{G}

$$\frac{1}{\frac{1}{A_{22}}\frac{\epsilon}{\epsilon - \gamma} + i\frac{\epsilon^2}{\epsilon - \gamma}p_0}$$

$\gamma = 0$ ⇨

$$a_F = \frac{1}{\frac{1}{A_{22}} + i\epsilon p_0}$$

Except for the case with gamma being zero we should not use the Flatté amplitude

Application

We study the effect of γ on the scattering length a_G.

Analysis of the $\pi\pi$-$K\bar{K}$ system with $f_0(980)$ [5].

⇨ We determine the constant term of the denominator of $f_{11}^G(f_{\pi\pi})$

$$-\frac{1}{A_{22}}\frac{\epsilon}{\epsilon-\gamma}-i\frac{\epsilon^2}{\epsilon-\gamma}p_0=-1.0-1.0\mathrm{i}\ [\mathrm{GeV}]$$

⇩

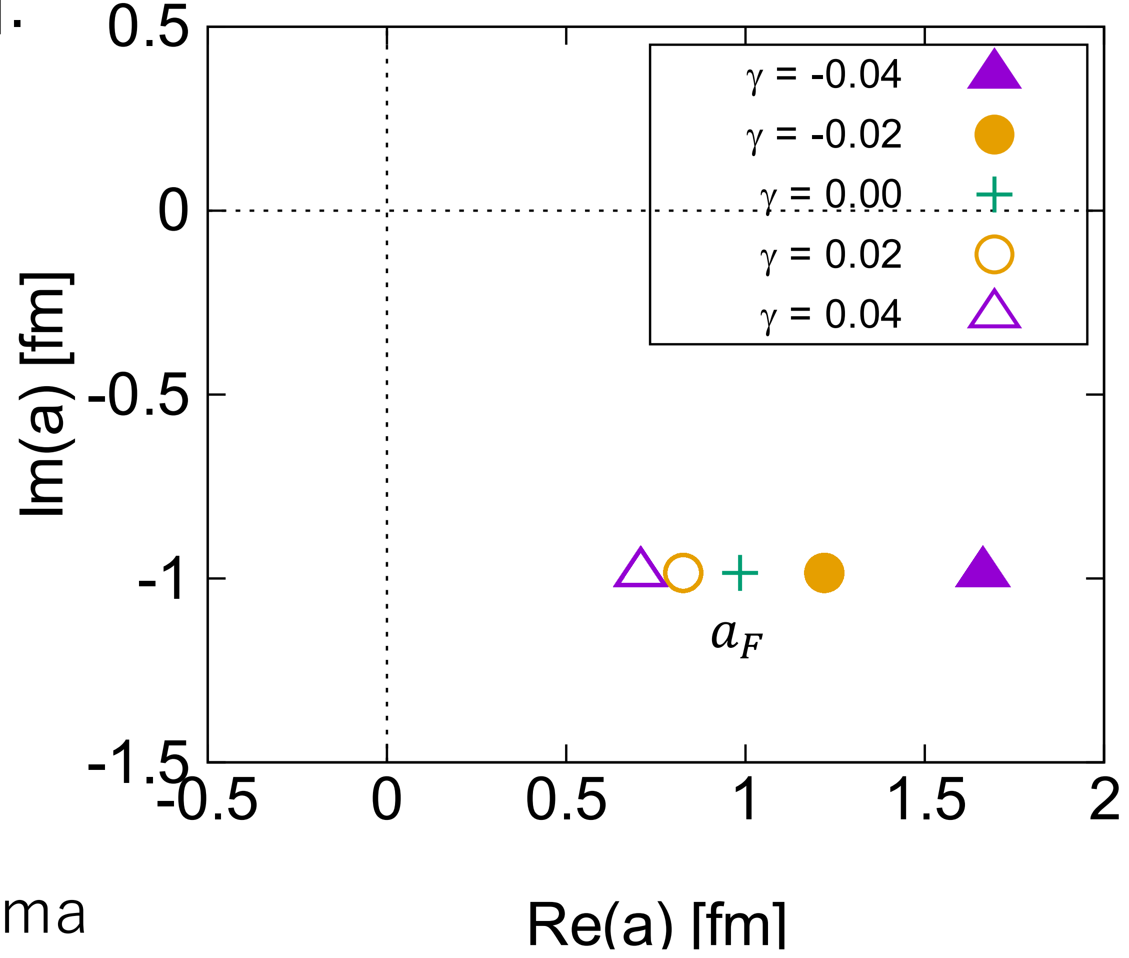
Two conditions

$A_{22}(\gamma), \epsilon(\gamma)$

⇩

We can determine $a_G(\gamma)$as a function of gamma

The imaginary part is invariant under the variation of γ.



[5] R.R. Akhametshin et al., Phys. Lett B 462, 380 (1999)