Nonequilibrium dynamics of high density matter under a strong magnetic field

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Quark Confinement and the Hadrom Spectrum

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Motivation

1. Time dependence is a common feature of strong magnetic fields affecting QCD properties in the early universe, magnetars, and heavy-ion collisions
2. The quark condensate is a prominent QCD property affected by a strong magnetic field
3. Time scales of the condensate dynamics are important for hadron production in heavy-ion collisions
4. What is the effect of a magnetic field on the quark condensate dynamics?
5. Here: formalism to tackle time dependence based on a nonequilibrium QFT calculation

Work with Carlisson Miller + Arthur Frazon

Phase change - time dependence

Typical situation:

- A system is forced to change from a thermodynamic equilibrium phase to another, out-of-equilibrium phase

- Evolution to new equilibrium through spatial fluctuations that take the system (initially homogeneous) through a sequence of highly (not in equilibrium) inhomogeneous states

Theory: coarse-graining

Rational:

- It is difficult/impossible to describe the system with microscopic d.o.f.
- Focus on a small number of semi-macroscopic variables; order parameters φ
- Dynamics of φ is slower than that of the microscopic degrees of freedom; described by Ginzburg-Landau-Langevin type of equations

From A. Zee book

Dynamical equations

Landau: system's state characterized by a macroscopic free energy $F[\varphi]$

Example: $F[\varphi] = \int d^3x \left[\kappa(\nabla\varphi)^2 + V(\varphi)\right], \qquad V(\varphi) = \frac{1}{2}\, r\, \varphi^2 + \frac{1}{4}\, u\, \varphi^4$

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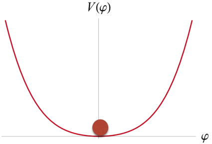
Equilibrium $(r > 0)$:



$$\frac{\delta F}{\delta \varphi} = 0$$

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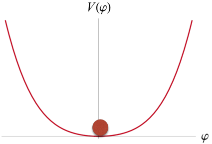

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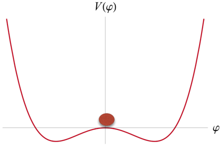
Out-of-equilibrium ($r = -|r| < 0$):



$$\varphi(x) \to \varphi(x,t): \qquad \eta\, \frac{\partial \varphi}{\partial t} = -\frac{\delta F}{\delta \varphi}$$

Near equilibrium: $\tilde{\varphi}(k,t) \approx e^{(|r| - k^2)\, t/\eta}$

Dynamical equations

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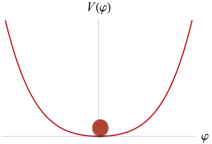

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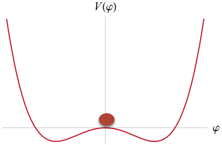
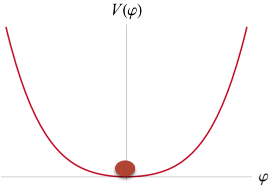


$$\varphi(x) \to \varphi(x,t): \qquad \eta\, \frac{\partial \varphi}{\partial t} = -\frac{\delta F}{\delta \varphi}$$

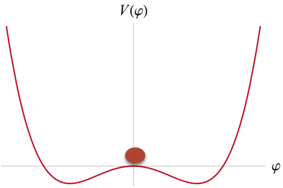
Near equilibrium: $\tilde{\varphi}(k,t) \approx e^{(|r|-k^2)\, t/\eta}$

Purely diffusive dynamics - no (thermal) fluctuations

Phase change, fluctuations



$$T > T_c$$



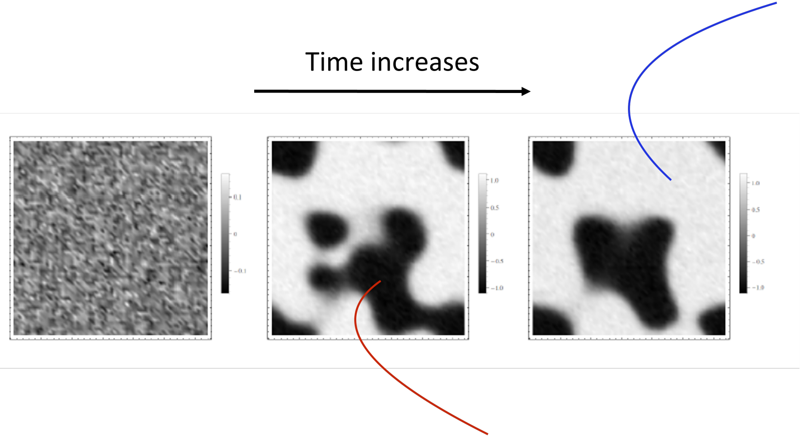
$$T < T_c$$

$$\eta \frac{\partial \varphi}{\partial t} = -\frac{\delta F}{\delta \varphi} + \xi(x,t)$$ ← Ginzburg-Landau-Langevin (GLL) equation

$$\langle \xi(x,t)\xi(x',t')\rangle = \sqrt{2\eta T}\,\delta(x-x')\delta(t-t')$$

Fluctuation-dissipation theorem

Phase change



Dynamical universality classes

Dynamical phase changes/transitions
can be classified in universality classes according
to the nature and couplings of the order parameters*

— Model A: nonconserved order parameter

— Model B: conserved order parameter

— Model C: nonconserved and conserved order parameters

— ... ...

P. C. Hohenberg & M.I Halperin, Rev. Mod. Phys. 49, 435 (1977).

Derivation of GLLeq - Linear σ model*

$$\mathcal{L} = \bar{q}[i\partial\!\!\!/ - g(\sigma + i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi})]q + \frac{1}{2}\left[\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi}\right] - U(\sigma, \boldsymbol{\pi})$$

$$U(\sigma, \boldsymbol{\pi}) = \frac{\lambda}{4}(\sigma^2 + \boldsymbol{\pi}^2 - v^2)^2 - h_q \sigma - U_0$$

Parameters: $v^2 = f_\pi^2 - \frac{m_\pi^2}{\lambda^2},\ m_\sigma^2 = 2\lambda^2 f_\pi^2 + m_\pi^2,\ h_q = f_\pi m_\pi^2,\ m_q = g\langle\sigma\rangle,\ U_0 :\ U(0,0) = 0$

Crossover at $T \simeq 150$ MeV: $g = 3.3$

Quark condensate: $m\langle\bar{q}q\rangle_{\mathrm{QCD}} = -h_q \langle\sigma\rangle$

Coupling magnetic field: $\partial_\mu \to D_\mu = \partial_\mu + iqA_\mu$ (for the charged fields)

M. Nahrgang et al. Phys. Rev. C 84, 024912 (2011).

The scenario, approximations

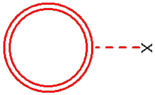
- QGP scales for temperature (T) and magnetic field (B)
- Perturbation around local equilibrium (medium at local T and B)
- No expansion & energy transfer field and medium
- Source of dissipation η: $\sigma \to \bar{q}q$ (no pions)
- Look at qualitative changes due to strong B
- Analytical understanding

Closed time path formalism

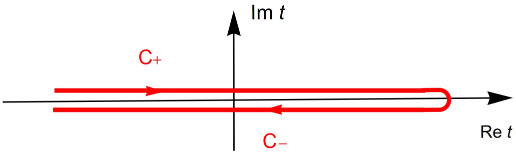
Semiclassical effective action:

$$\Gamma[\sigma, S] = \Gamma_{\rm cl}[\sigma] + i\,\mathrm{Tr}\ln S - i\,\mathrm{Tr}\left(i\not{D} - m_0\right) S + \Gamma_2[\sigma, S]$$

$$\Gamma_2[\sigma, S] = g\int_{\mathcal{C}} d^4x\,\mathrm{tr}\left[S^{++}(x,x)\sigma^+(x) + S^{--}(x,x)\sigma^-(x)\right]$$



- σ and S defined on the Schwinger-Keldysh contour (CTP contour), $\sigma^\pm$, $S^\pm$
- $\sigma^\pm$, $S^\pm$ are not independent, they are equal at some large time (CTP b.c.)



Integrate out the quarks

$$\frac{\delta\Gamma[\sigma, S]}{\delta S^{ab}(x,y)} = 0$$

$$\Downarrow$$

$$\left(i\not{D} - g\sigma_0(x)\right) S^{ab}(x,y) - \int_{\mathcal{C}} d^4z \, \frac{\delta\Gamma_2[\sigma, S]}{\delta S^{ac}(x,z)} \, S^{cb}(z,y) = i\delta^{ab}\delta^{(4)}(x-y)$$

Very difficult to solve (even numerically): expand around local equilibrium

$$\sigma^a(x) = \sigma_0^a(x) + \delta\sigma^a(x)$$

$$S^{ab}(x,y) = S^{ab}_{\text{thm}}(x,y) + \delta S^{ab}(x,y) + \delta^2 S^{ab}(x,y) + \cdots$$

where $\frac{\delta\Gamma_{\text{cl}}}{\delta\sigma_0^a(x)} = -g\text{Tr}S^{aa}(x,x)$ and $\left[i\not{D} - m_0 - g\,\sigma_0(x)\right] S^{ab}_{\text{thm}}(x,y) = -i\delta^{ab}\delta^{(4)}(x-y)$

GLL equation

- Dissipation ($\sigma \to \bar{q}q$): imaginary part of $\Gamma[\sigma, S]$
- Variation of $\Gamma[\sigma, S]$ w.r.t. σ to obtain e.o.m. not possible
- Solution: use Feynman-Vernon trick, identify the imaginary part with a noise source coupling linearly to the field
- Obtain real action, variation w.r.t. σ leads to GLL equation

$$\partial_\mu \partial^\mu \sigma(x) + \frac{\delta U[\sigma]}{\delta \sigma(x)} + g\rho_s(\sigma_0) - D_\sigma(x) = \xi_\sigma(x)$$

GLL equation

$$\partial_\mu \partial^\mu \sigma(x) + \frac{\delta U[\sigma]}{\delta \sigma(x)} + g\rho_s(\sigma_0) - D_\sigma(x) = \xi_\sigma(x)$$

Scalar Density: $\rho_s(\sigma_0) = \mathrm{tr}\, S^{++}_{thm}(x,x)$

Dissipation kernel: $D_\sigma(x) = ig^2 \int d^4y\, \theta(x^0 - y^0)\, M(x,y)\, \delta\sigma(y)$ ← Memory

$$M(x,y) = \mathrm{tr}\left[S^{+-}_{thm}(x,y) S^{-+}_{thm}(y,x) - S^{-+}_{thm}(x,y) S^{+-}_{thm}(y,x)\right]$$

Colored noise: $\langle \xi_\sigma(x) \rangle_\xi = 0$ and $\langle \xi_\sigma(x) \xi_\sigma(y) \rangle_\xi = N(x,y)$

$$N(x,y) = -\frac{1}{2} g^2 \mathrm{tr}\left[S^{+-}_{thm}(x,y) S^{-+}_{thm}(y,x) + S^{-+}_{thm}(x,y) S^{+-}_{thm}(y,x)\right]$$

$\langle \cdots \rangle_\xi$: functional average with prob. distr. $P[\xi] = \exp\left[-\frac{1}{2}\int d^4x d^4y\, \xi(x) N^{-1}(x,y)\, \xi(y)\right]$

Quark propagator $S^{ab}_{\rm thm}(x, y)$

Structure of $\rho_s(x)$, $M(x, y)$ and $N(x, y)$

— Schwinger phase drops out, use Fourier transform

In the lowest Landau level (LLL) approximation:

$$S^{++}_{thm}(p) = e^{-\boldsymbol{p}_\perp^2/|q_f B|}\, A(p) \left[\frac{i}{p_\parallel^2 - m_q^2 + i\epsilon} - 2\pi n_F(p_0)\delta(p_\parallel^2 - m_q^2) \right]$$

$$S^{+-}_{thm}(p) = e^{-\boldsymbol{p}_\perp^2/|q_f B|}\, A(p)\, 2\pi\delta(p_\parallel^2 - m_q^2)\, [\theta(-p_0) - n_F(p_0)]\,,$$

$$S^{-+}_{thm}(p) = e^{-\boldsymbol{p}_\perp^2/|q_f B|}\, A(p)\, 2\pi\delta(p_\parallel^2 - m_q^2)\, [\theta(p_0) - n_F(p_0)]$$

$$S^{--}_{thm}(p) = e^{-\boldsymbol{p}_\perp^2/|q_f B|}\, A(p) \left[\frac{-i}{p_\parallel^2 - m_q^2 - i\epsilon} - 2\pi n_F(p_0)\delta(p_\parallel^2 - m_q^2) \right]$$

where $A(p) = (\not{p}_\parallel + m_q)\left[1 + i\gamma^1\gamma^2 \mathrm{sign}(qB)\right]$ and $n_F(p_0) = \dfrac{1}{e^{|p_0|}/T + 1}$

Momentum space GLL equation

$$\frac{\partial^2 \sigma(t,\boldsymbol{p})}{\partial t^2} + \boldsymbol{p}^2\,\sigma(t,\boldsymbol{p}) + \eta(\boldsymbol{p})\,\frac{\partial \sigma(t,\boldsymbol{p})}{\partial t} + F_\sigma(t,\boldsymbol{p}) = \xi_\sigma(t,\boldsymbol{p})$$

$$\eta(\boldsymbol{p}) = g^2 \frac{1}{2E_\sigma(\boldsymbol{p})} M(\boldsymbol{p}) \quad \leftarrow M(\boldsymbol{p}) = M(E_\sigma, \boldsymbol{p}),\; E_\sigma = \sqrt{\boldsymbol{p}^2 + m_\sigma^2}$$

$$F_\sigma(t,\boldsymbol{p}) = \int d^3x\, e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} \left[\frac{\delta U[\sigma]}{\delta\sigma(t,\boldsymbol{x})} + g\,\rho_s(\sigma_0)\right]$$

$$\langle \xi_\sigma(t,\boldsymbol{p})\xi_\sigma(t',\boldsymbol{p})\rangle_\xi = (2\pi)^3\delta(\boldsymbol{p}+\boldsymbol{p}')N(t-t',\boldsymbol{p})$$

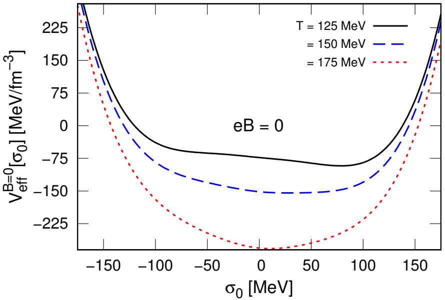
$\eta = 0$: "classical" equation of motion

$\eta \neq 0$: slows the dynamics

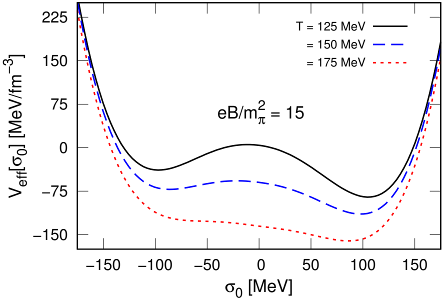
GK and C. Miller, Symmetry 13, 551 (2021).

Equilibrium*

$B = 0$



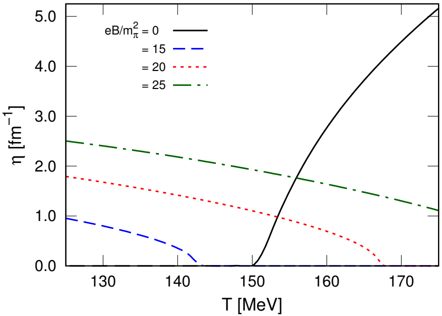
$B \neq 0$



E.S. Fraga & A.J. Mizher, Phys. Rev. D 78, 025016 (2008).

Zero-mode η

Recall, source of dissipation is $\sigma \to \bar{q}q$



$$\underline{B=0} \quad \eta_0 = g^2 \frac{2N_c}{\pi} \left[1 - 2n_F(m_\sigma/2)\right] \frac{1}{m_\sigma^2} \left(m_\sigma^2 - 4m_q^2\right)^{3/2}$$

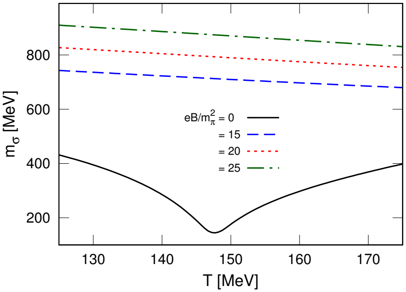
$$\underline{B \neq 0} \quad \eta_B = g^2 \frac{N_c}{4\pi} \left[1 - 2n_F(m_\sigma/2)\right] (eB) \frac{1}{m_\sigma^2} \sqrt{m_\sigma^2 - 4m_q^2} \quad (LLL)$$

σ and quark masses

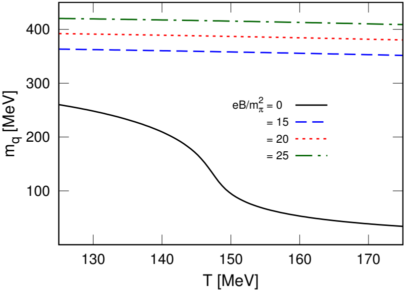
$\eta \neq 0$ when $m_\sigma > 2m_q$

B modifies minimum of V_{eff} $(\leftarrow m_q)$ and its curvature $(\leftarrow m_\sigma)$

σ mass



Quark mass



m_σ increases faster than m_q as the temperature decreases:

- η_B increases at low temperatures
- increase in η_B delays evolution of σ

Short-time dynamics

Linearized GLL equation:

$$\eta(\boldsymbol{p}_\perp)\,\frac{\partial\overline{\sigma}(t,\boldsymbol{p}_\perp)}{\partial t} - \left(\mu^2 - \boldsymbol{p}_\perp^2\right)\overline{\sigma}(t,\boldsymbol{p}_\perp) + g\rho_s(\sigma_0) - f_\pi m_\pi^2 = \overline{\xi}_\sigma(t,\boldsymbol{p}_\perp)$$

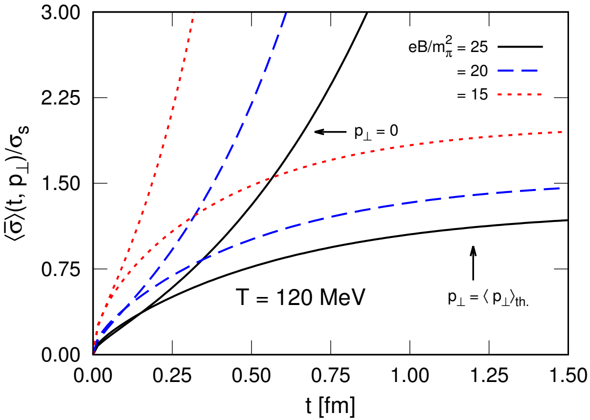
$$\mu^2 = \lambda\left(f_\pi^2 - \frac{m_\pi^2}{\lambda}\right), \qquad \overline{\sigma} = \sigma/L^3, \qquad \overline{\xi} = \xi/L^3$$

Quench from high to low T:

$$\langle\,\overline{\sigma}^2(t,\boldsymbol{p}_\perp^2)\,\rangle_\xi = \frac{\left[g\rho_s(\sigma_0) - f_\pi m_\pi^2\right]^2}{(\mu^2 - \boldsymbol{p}_\perp)^2}\left(e^{\,\lambda(\boldsymbol{p}_\perp)\,t/\tau_s} - 1\right)^2 + \frac{E(\boldsymbol{p}_\perp)\coth(E(\boldsymbol{p}_\perp))}{L^3(\mu^2 - \boldsymbol{p}_\perp^2)}\left(e^{\,2\,\lambda(\boldsymbol{p}_\perp)\,t/\tau_s} - 1\right)$$

$$\tau_s = \frac{\eta_B}{\mu^2} \qquad \text{and} \qquad \lambda(\boldsymbol{p}_\perp) = \frac{1 - \boldsymbol{p}_\perp^2/\mu^2}{\eta(\boldsymbol{p}_\perp)/\eta_B},$$

Short-time dynamics



$$\sigma_s = (g\rho_s - f_\pi m_\pi^2)/\mu^2$$

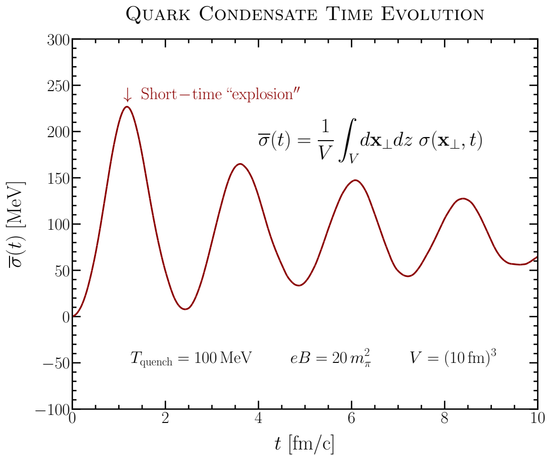
$$\langle \boldsymbol{p}_\perp^2 \rangle_{\text{th.}} = \frac{\int d^2p_\perp \, p_\perp^2 \, n_B(\boldsymbol{p}_\perp)}{\int d^2p_\perp \, n_B(p_\perp)}$$

$$n_B(p) = \frac{1}{e^{E_\sigma(p)/T} - 1}$$

Large B slows considerably the short-time dynamics

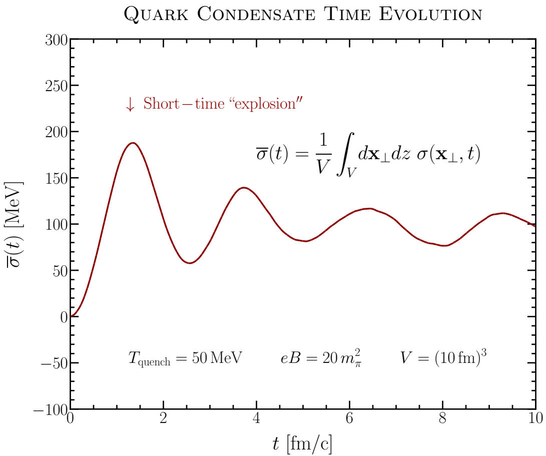
Present estimate: delays of $\simeq 1$ fm/c $\leftarrow 1/10$ of QGP lifetime

Long-time dynamics



Condensate not thermalized within QGP lifetime

Long-time dynamics



Condensate thermilized within QGP lifetime

Cold matter - nucleon superposition

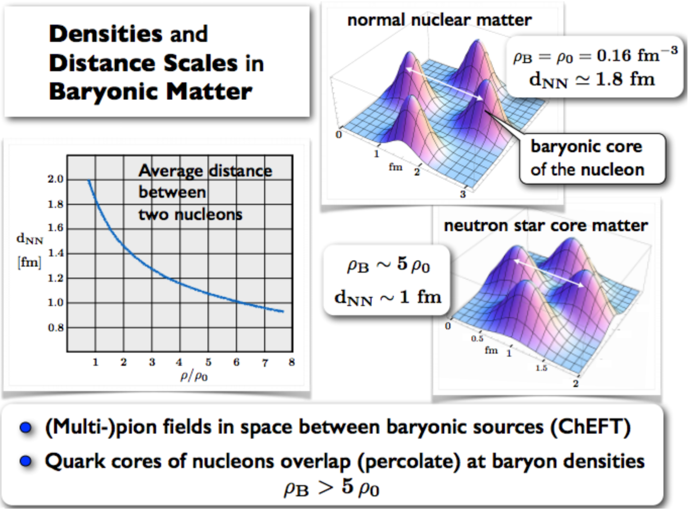


Figure from W. Weise

Nucleon compositeness: quark model

Nucleon creation operator:

$$B_n^\dagger = \frac{1}{\sqrt{3!}} \Psi_n^{\mu_1\mu_2\mu_3} \hat{q}_{\mu_1}^\dagger \hat{q}_{\mu_2}^\dagger \hat{q}_{\mu_3}^\dagger$$

$$\{\hat{B}_n, \hat{B}_{n'}^\dagger\} = \delta_{nn'} - \hat{\Delta}_{nn'}, \qquad \{\hat{B}_n, \hat{B}_{n'}\} = 0$$

$$\hat{\Delta}_{nn'} = 3\Psi_n^{*\mu_1\mu_2\mu_3} \Psi_{n'}^{\mu_1\mu_2\nu_3} \hat{q}_{\nu_3}^\dagger \hat{q}_{\mu_3} - \frac{3}{2} \Psi_n^{*\mu_1\mu_2\mu_3} \Psi_{n'}^{\mu_1\nu_2\nu_3} \hat{q}_{\nu_3}^\dagger \hat{q}_{\nu_2}^\dagger \hat{q}_{\mu_2} \hat{q}_{\mu_3}$$

Two-nucleon state:

$$|nn'\rangle = B_n^\dagger B_{n'}^\dagger |0\rangle$$

$$\langle nn'|nn'\rangle = 1 - \delta_{nn'} - \Delta N_{nn'}$$

$$\Delta N_{nn'} = 3\Psi_n^{*\mu_1\mu_2\mu_3} \Psi_{n'}^{*\nu_1\nu_2\nu_3} \left(\Psi_n^{\mu_1\mu_2\nu_3} \Psi_{n'}^{\nu_1\nu_2\mu_3} - \Psi_n^{\mu_1\nu_2\nu_3} \Psi_{n'}^{\nu_1\mu_2\mu_3}\right)$$

Field theory: "deformed" field algebra

Nucleon field operator:

$$\psi(\boldsymbol{x},t) = \sum_{\boldsymbol{k},s} \left[u_s(\boldsymbol{k})\, e^{-ik\cdot x}\, b_s^\dagger(\boldsymbol{k}) + v_s(\boldsymbol{k})\, e^{ik\cdot x}\, d_s^\dagger(\boldsymbol{k}) \right]$$

$$b_s(\boldsymbol{k}) b_{s'}^\dagger(\boldsymbol{k}') + \lambda\, b_{s'}^\dagger(\boldsymbol{k}') b_s(\boldsymbol{k}) = \delta_{ss'}\delta_{\boldsymbol{k}\boldsymbol{k}'}, \quad \lambda\, d_s(\boldsymbol{k}) d_{s'}^\dagger(\boldsymbol{k}') + d_{s'}^\dagger(\boldsymbol{k}') d_s(\boldsymbol{k}) = \delta_{ss'}\delta_{\boldsymbol{k}\boldsymbol{k}'}$$

$$\lambda \equiv 1 - x \rightarrow \{b_s(\boldsymbol{k}), b_{s'}^\dagger(\boldsymbol{k}')\} = \delta_{ss'}\delta_{\boldsymbol{k}\boldsymbol{k}'} \underline{- x\, b_s^\dagger(\boldsymbol{k}) b_s(\boldsymbol{k})} \leftarrow x:$$ in general density dependent

$$\psi(\boldsymbol{x},t)\psi^\dagger(\boldsymbol{x}',t) + \lambda\, \psi^\dagger(\boldsymbol{x}',t)\psi(\boldsymbol{x},t) = \delta(\boldsymbol{x} - \boldsymbol{x}')$$

Example: equilibrium nucleon scalar density (noninteracting)

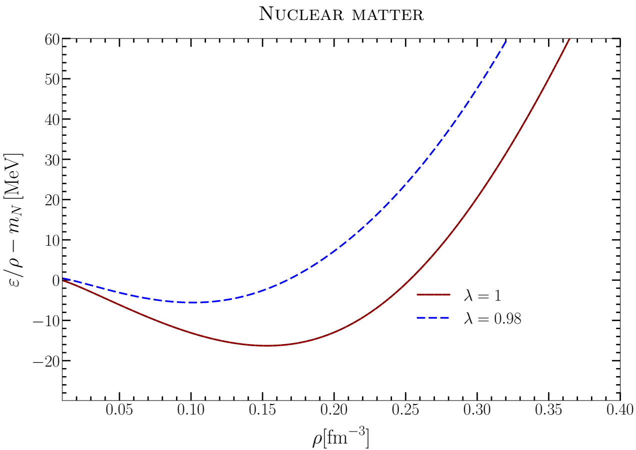
$$\rho_s = \int d^3x\, \langle \bar{\psi}(x)\psi(x) \rangle = 4 \sum_{\boldsymbol{k}} \left(\frac{1}{e^{\beta E(\boldsymbol{k}) - \mu_\lambda} + \lambda} + \frac{1/\lambda}{\lambda\, e^{\beta E(\boldsymbol{k}) + \mu_\lambda} + 1} \right)$$

$E(\boldsymbol{k}) = (\boldsymbol{k}^2 + m_N^2)^{1/2}, \quad \beta = 1/T, \quad \mu_\lambda = \mu + T \ln \lambda, \quad 0 < \lambda \leqslant 1$ (positive number density)

Nonrelativistic: S.S. Avancini and GK, J. Phys A 28, 685 (1995)
Relativistic free gas: X-Y. Hou et al., J. Stat. Mech. (2020) 113402

Walecka model: $T = 0$

λ constant



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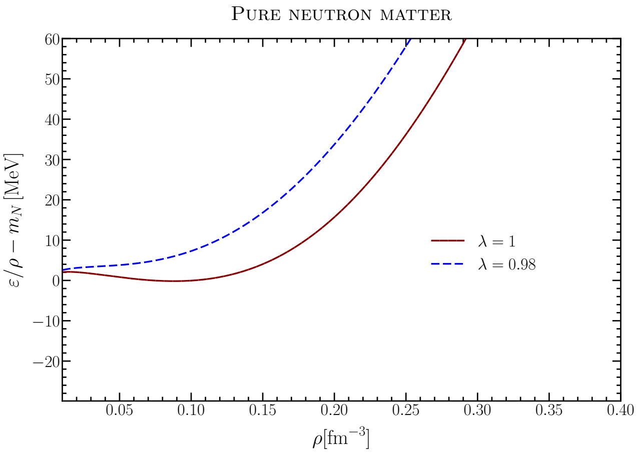

Conclusions & Perspectives

1. Presented a nonequilibrium QFT setup to tackle magnetic field effects on chiral dynamics, a first incursion into a complex many-body problem
2. Made umerical estimates, short- and long-time dynamics
3. Applications to HIC: beyond LLL approximation (weak fields), include pion dynamics, couple to hydrodynamics
4. Dense matter: presented a way to into account effectively nucleon compositeness and superposition

Thank you

Funding