NSTC 國家科學及技術委員會
National Science and Technology Council

Quarkonium Polarization Transport in Medium from Open Quantum Systems and Effective Field Theories

XVIth Quark Confinement and the Hadron Spectrum Conference
Cairns Convention Centre, Cairns, Queensland, Australia
19-24 August 2024 (inclusive)

QCHSC2024

Di-Lun Yang

Institute of Physics, Academia Sinica

(QCHSC2024, Aug. 22, 2024)

Based on D-L Y & Xiaojun Yao, arXiv:2405.20280

Quarkonium in heavy ion collisions

- Quarkonium is a useful probe for the quark gluon plasma (QGP) in heavy ion collisions. e.g., J/ψ suppression
- Heavy-quark potential : Vacuum : $V(r) = -\dfrac{A}{r} + Br$

Finite T :

static screening

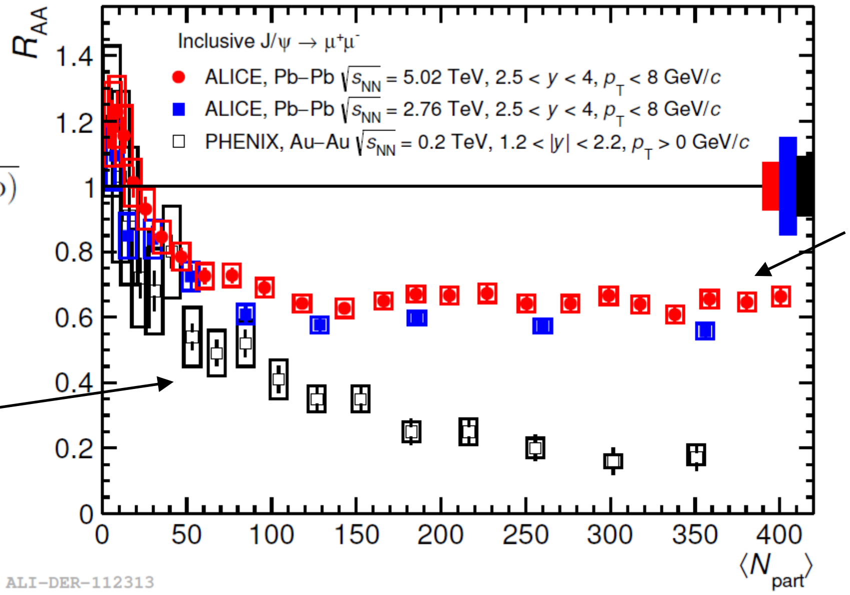
T. Matsui & H. Satz, PLB 178 (1986) 416.
F. Karsch, M. T. Mehr and H. Satz, Z. Phys. C 37 (1988) 617

$$V(r, t \to \infty) = -\frac{A}{r} e^{-m_D r} - i\phi(m_D r)$$

dynamical dissociation

M. Laine et al., JHEP 03 (2007) 054.
A. Beraudo, J. P. Blaizot, C. Ratti, Nucl. Phys. A 806 (2008) 312.

$$R_{\mathrm{AA}} = \frac{d^2N/dp_{\mathrm{T}}d\eta(\mathrm{A+A})}{N_{\mathrm{coll}} d^2N/dp_{\mathrm{T}}d\eta(\mathrm{p+p})}$$



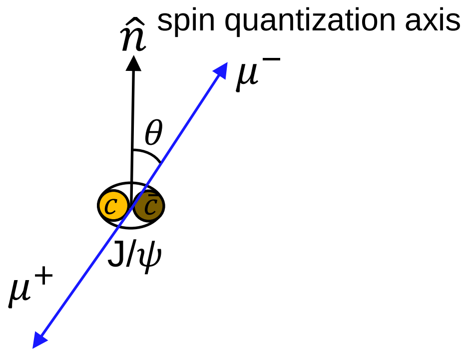
J/ψ suppression: screening +dissociation

recombination

R. L. Thews, M. Schroedter, J. Rafelski, PRC 63 (2001) 054905

Polarization of vector quarkonia

- Spin alignment of J/ψ (polarization of quarkonia) :



$$\frac{dN}{d\cos\theta} \propto \left[(1+\rho_{00}) + (1-3\rho_{00})\cos^2\theta\right]$$

(normalized) 00th component of the spin density matrix

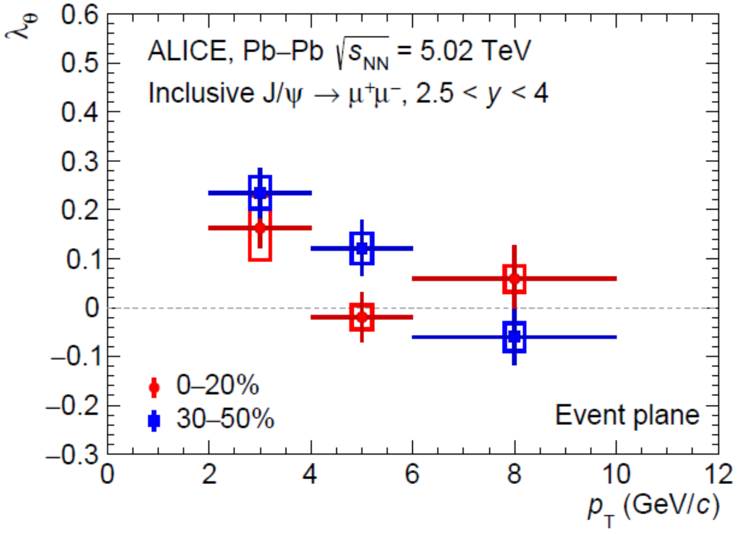
Transverse pol : $\rho_{00} < 1/3$

Longitudinal pol : $\rho_{00} > 1/3$

Unpolarized : $\rho_{00} = 1/3$ (no spin alignment)

- Spin alignment w.r.t the reaction plane :

$$\lambda_\theta = \frac{1-3\rho_{00}}{1+\rho_{00}} > 0 \Rightarrow \rho_{00} < \frac{1}{3}$$



S. Acharya et al. , PRL 131,042303 (2023)

- Unexpectedly large spin alignment cannot be explained by vorticity responsible for the spin polarization of light quarks (for Λ hyperons).

Theoretical frameworks

- How to construct the transport theory for quarkonia from the first principle?

❖ Quarkonia (or heavy quarks) are dilute : open quantum system (OQS)

Total system= subsystem + environment $H = H_S + H_E + \boxed{H_I}$ weakly coupled

X. Yao, Int. J. Mod. Phys. A 36 (2021) 2130010

Y. Akamatsu, Prog. Part. Nucl. Phys. 123 (2022) 103932

see the plenary talk by Akamatsu

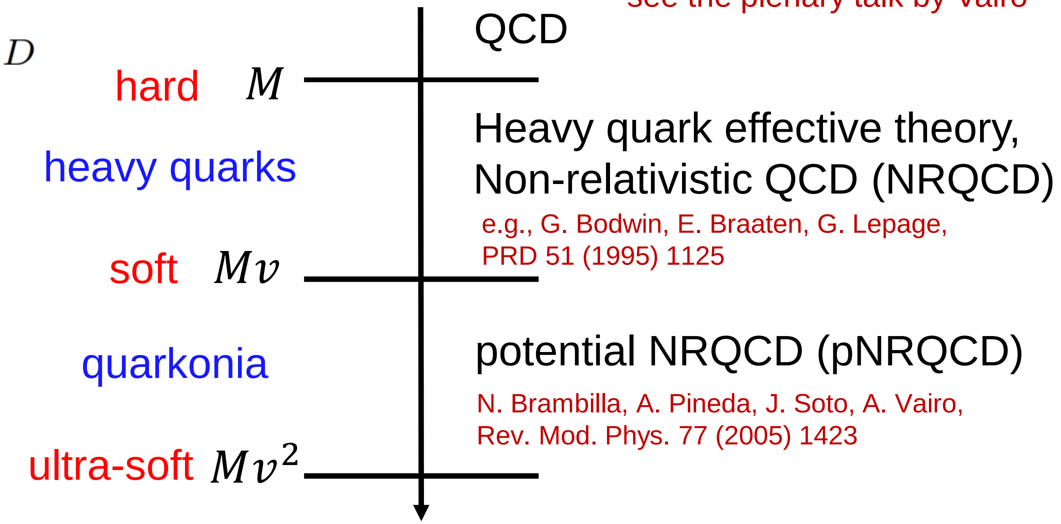
❖ Separation of scales : non-relativistic effective fields theories (NREFT)

see the plenary talk by Vairo

$$M \gg Mv \gg Mv^2, T, \Lambda_{QCD}$$

M : heavy quark mass

v : relative velocity



QCD

hard M

heavy quarks

Heavy quark effective theory,
Non-relativistic QCD (NRQCD)

e.g., G. Bodwin, E. Braaten, G. Lepage,
PRD 51 (1995) 1125

soft Mv

quarkonia

potential NRQCD (pNRQCD)

N. Brambilla, A. Pineda, J. Soto, A. Vairo,
Rev. Mod. Phys. 77 (2005) 1423

ultra-soft Mv^2

➢ OQS + pNRQCD ⟹ kinetic theory for quarkonia

Evolution of the density matrix

- Tracking the evolution of the density matrix (unitary → time reversible):

$$\frac{\mathrm{d}\rho^{(\mathrm{int})}(t)}{\mathrm{d}t} = -i[H_I^{(\mathrm{int})}(t), \rho^{(\mathrm{int})}(t)] \qquad H_I = \sum_\alpha O_\alpha^{(S)} \otimes O_\alpha^{(E)}$$

- Tracing over the environment (non-unitary → time irreversible):

$$\rho^{(\mathrm{int})}(t_i) = \rho_S^{(\mathrm{int})}(t_i) \otimes \boxed{\rho_E^{(\mathrm{int})}(t_i)}, \quad \rho_S^{(\mathrm{int})}(t) = \mathrm{Tr}_E[\rho^{(\mathrm{int})}(t)]. \longrightarrow$$

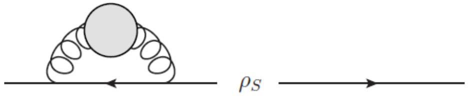
a matrix in both color & spin spaces

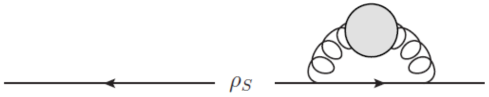
$\approx \rho_T(0)$: static thermal equilibrium medium

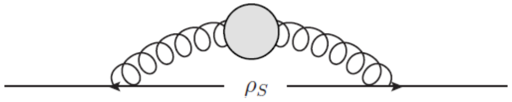
⇨ master eq :

$$\rho_S^{(\mathrm{int})}(t) = \rho_S^{(\mathrm{int})}(0) - \int_0^t \mathrm{d}t_1 \int_0^t \mathrm{d}t_2 \frac{\mathrm{sign}(t_1 - t_2)}{2} D_{\alpha\beta}(t_1, t_2) \left[O_\alpha^{(S)}(t_1) O_\beta^{(S)}(t_2), \rho_S^{(\mathrm{int})}(0) \right]$$
$$+ \int_0^t \mathrm{d}t_1 \int_0^t \mathrm{d}t_2 D_{\alpha\beta}(t_1, t_2) \left(O_\beta^{(S)}(t_2) \rho_S^{(\mathrm{int})}(0) O_\alpha^{(S)}(t_1) - \frac{1}{2} \left\{ O_\alpha^{(S)}(t_1) O_\beta^{(S)}(t_2), \rho_S^{(\mathrm{int})}(0) \right\} \right),$$

environment correlation function : $D_{\alpha\beta}(t_1, t_2) = \mathrm{Tr}_E \left(O_\alpha^{(E)}(t_1) O_\beta^{(E)}(t_2) \rho_T(0) \right).$







From OQS to kinetic equations

- Master eq. :

Quantum optical limit (low T) → Boltzmann eq.

$\tau_R \gg \tau_E \sim T^{-1}, \tau_R \gg \tau_S \sim 1/(Mv^2)$

Wigner transform

pol indep. : X. Yao, T. Mehen, PRD 99 (2019) 096028; JHEP 02 (2021) 062.

Quantum Brownian motion (high T) → Langevin/Fokker-Planck eq.

$\tau_R \gg \tau_E, \tau_S \gg \tau_E$

- Markovian approximation (coarse graining such that $\tau_R \gg t \gg \tau_E$) : the time-difference eq. becomes a differential eq.

recombination dissociation

$$\frac{\partial}{\partial t} f_\lambda(\boldsymbol{x},\boldsymbol{k},t) + \frac{\boldsymbol{k}}{2M}\cdot\nabla_{\boldsymbol{x}} f_\lambda(\boldsymbol{x},\boldsymbol{k},t) = \boxed{\mathcal{C}^+_\lambda(\boldsymbol{x},\boldsymbol{k},t)[f^{(8)}_{Q\bar{Q}\lambda}]} - \boxed{\mathcal{C}^-_\lambda(\boldsymbol{x},\boldsymbol{k},t)[f_\lambda]\,,}$$

color singlet : $f_\lambda(\boldsymbol{x},\boldsymbol{k},t) \equiv \int \frac{\mathrm{d}^3k'}{(2\pi)^3} e^{i\boldsymbol{k}'\cdot\boldsymbol{x}} \left\langle \boldsymbol{k}+\frac{\boldsymbol{k}'}{2},\lambda \middle| \rho_S(t) \middle| \boldsymbol{k}-\frac{\boldsymbol{k}'}{2},\lambda\right\rangle,$ $\lambda = \pm 1, 0.$

polarization (vector mesons)

color octet :

$$\int \frac{\mathrm{d}^3k'}{(2\pi)^3} e^{i\boldsymbol{k}'\cdot\boldsymbol{x}} \left\langle \boldsymbol{k}+\boldsymbol{q}+\frac{\boldsymbol{k}'}{2}, \boldsymbol{p}_{1\mathrm{rel}},\lambda,a \middle| \rho^{(8)}_S(t) \middle| \boldsymbol{k}+\boldsymbol{q}-\frac{\boldsymbol{k}'}{2}, \boldsymbol{p}_{2\mathrm{rel}},\lambda,a'\right\rangle$$

$$\approx (2\pi)^3\delta^3(\boldsymbol{p}_{1\mathrm{rel}}-\boldsymbol{p}_{2\mathrm{rel}})\delta^{aa'} f^{(8)}_{Q\bar{Q}\lambda}\left(\boldsymbol{x},\boldsymbol{k}+\boldsymbol{q},\boldsymbol{x}_0,\frac{\boldsymbol{p}_{1\mathrm{rel}}+\boldsymbol{p}_{2\mathrm{rel}}}{2},t\right).$$

$a = 1, 2, \ldots N_c^2 - 1$

(keeping diag. com. & gradient exp.)

Building blocks of pNRQCD

- Collision terms depend on the details of interaction : $H_I = \sum_\alpha \underbrace{O^{(S)}_\alpha \otimes O^{(E)}_\alpha}_{\text{from an effective Lagrangian}}$

- Adopted NRQCD Lagrangian : $(M^{-1}$ expansion)

$$\mathcal{L}_{\rm NRQCD} = \psi^\dagger \left(iD_0 + \frac{\boldsymbol{D}^2}{2M} + c_4 \frac{g\boldsymbol{B}\cdot\boldsymbol{\sigma}}{2M} \right) \psi + \chi^\dagger \left(iD_0 - \frac{\boldsymbol{D}^2}{2M} - c_4 \frac{g\boldsymbol{B}\cdot\boldsymbol{\sigma}}{2M} \right) \chi$$

- Introducing the composite field : $\Phi^{\overset{\text{spin}}{s_1 s_2}}_{\underset{\text{color}}{ij}}(\boldsymbol{x}_1, \boldsymbol{x}_2, t) = \psi^{s_1}_i(\boldsymbol{x}_1, t)\chi^{\dagger s_2}_j(\boldsymbol{x}_2, t)$

➢ color decomposition :

$$\Phi_{ij}(\boldsymbol{x}_1, \boldsymbol{x}_2, t) = U_{ik}(\boldsymbol{x}_1, \boldsymbol{R}) \left(\frac{\delta_{k\ell}}{\sqrt{N_c}} \overset{\text{singlet}}{S(\boldsymbol{R}, \boldsymbol{r}, t)} + \frac{T^a_{k\ell}}{\sqrt{T_F}} \overset{\text{octet}}{O^a(\boldsymbol{R}, \boldsymbol{r}, t)} \right) U_{\ell j}(\boldsymbol{R}, \boldsymbol{x}_2),$$

$$\boldsymbol{R} = (\boldsymbol{x}_1 + \boldsymbol{x}_2)/2, \quad \boldsymbol{r} = \boldsymbol{x}_1 - \boldsymbol{x}_2, \quad T_F = 1/2.$$

➢ spin decomposition :

(for pseudo scalar & vector)

e.g., η_c & J/ψ

$$S = \frac{1}{\sqrt{2}} \left(\overset{\text{singlet}}{IS_1} + \sum_\lambda \overset{\text{triplet}}{S_{\lambda i}\sigma_i} \right), \quad O^a = \frac{1}{\sqrt{2}} \left(IO^a_1 + \sum_\lambda O^a_{\lambda i}\sigma_i \right),$$

i : spatial indices

$$S_{\lambda i}, O^a_{\lambda i} \propto \varepsilon_{\lambda i}. \quad \underbrace{(\varepsilon_{0\,i} = \hat{n}_i)}_{\text{polarization vector}}$$

Effective Lagrangian in pNRQCD

- Spin-independent case (with multipole expansion, small $r \sim (Mv)^{-1}$) :

$$\mathcal{L} = \boxed{S^\dagger(\boldsymbol{R},\boldsymbol{r},t)(i\partial_0 - \mathcal{H}_s)S(\boldsymbol{R},\boldsymbol{r},t) + O^{a\dagger}(\boldsymbol{R},\boldsymbol{r},t)(iD_0 - \mathcal{H}_o)O^a(\boldsymbol{R},\boldsymbol{r},t)}$$

kinetic terms &
static potentials

$$+ V_A\sqrt{\frac{T_F}{N_c}}\boxed{\left(S^\dagger(\boldsymbol{R},\boldsymbol{r},t)\boldsymbol{r}\cdot g\boldsymbol{E}^a(\boldsymbol{R},t)O^a(\boldsymbol{R},\boldsymbol{r},t) + O^{a\dagger}(\boldsymbol{R},\boldsymbol{r},t)\boldsymbol{r}\cdot g\boldsymbol{E}^a(\boldsymbol{R},t)S(\boldsymbol{R},\boldsymbol{r},t)\right)}$$

$$+ V_B d^{abc}O^{a\dagger}(\boldsymbol{R},\boldsymbol{r},t)\boldsymbol{r}\cdot g\boldsymbol{E}^b(\boldsymbol{R},t)O^c(\boldsymbol{R},\boldsymbol{r},t) + \mathcal{O}(r^2)\,,$$

color singlet-octet transitions

e.g., $J/\psi + g \leftrightarrow c + \bar{c}$

- Spin-dependent case (the transition terms) :

(pseudo) scalar-scalar transitions vector-vector transitions

$$\mathcal{L}_t = V_A\sqrt{\frac{T_F}{N_c}}\left(S_1^\dagger(\boldsymbol{R},\boldsymbol{r},t)\boldsymbol{r}\cdot g\boldsymbol{E}^a(\boldsymbol{R},t)O_1^a(\boldsymbol{R},\boldsymbol{r},t) + \sum_\lambda S_{\lambda i}^\dagger(\boldsymbol{R},\boldsymbol{r},t)\boldsymbol{r}\cdot g\boldsymbol{E}^a(\boldsymbol{R},t)O_{\lambda i}^a(\boldsymbol{R},\boldsymbol{r},t) + \text{h.c.}\right)$$

$$+\frac{c_4}{M}V_A^s\sqrt{\frac{T_F}{N_c}}\sum_\lambda\left[S_1^\dagger(\boldsymbol{R},\boldsymbol{r},t)gB_i^a(\boldsymbol{R},t)O_{\lambda i}^a(\boldsymbol{R},\boldsymbol{r},t) + S_{\lambda i}^\dagger(\boldsymbol{R},\boldsymbol{r},t)gB_i^a(\boldsymbol{R},t)O_1^a(\boldsymbol{R},\boldsymbol{r},t) + \text{h.c.}\right]$$

pseudo scalar-vector transitions :
vector mesons polarized by B fields

Dissociation for vector quarkonia

- Dissociation term :

E of J/ψ E of $Q\bar{Q}$ E of gluons

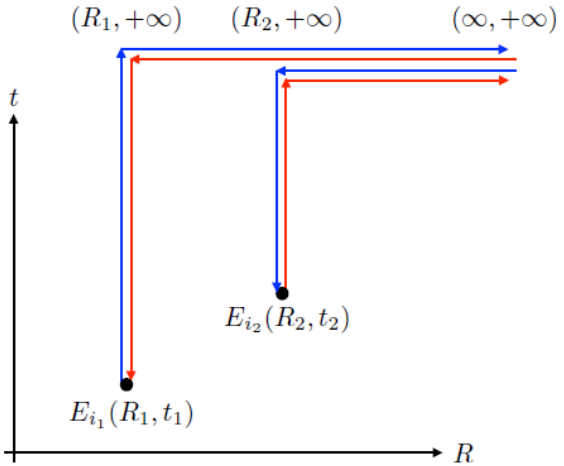
$$\mathcal{C}_\lambda^-(\boldsymbol{x},\boldsymbol{k},t)[f_\lambda]=\int\frac{\mathrm{d}^3p_{\mathrm{cm}}}{(2\pi)^3}\frac{\mathrm{d}^3p_{\mathrm{rel}}}{(2\pi)^3}\int\mathrm{d}^4q\,\delta(E_k^\lambda-E_p-q_0)\delta^3(\boldsymbol{k}-\boldsymbol{p}_{\mathrm{cm}}-\boldsymbol{q})|\mathcal{M}_d|^2f_\lambda(\boldsymbol{x},\boldsymbol{k},t)\,,$$

$$|\mathcal{M}_d|^2=\frac{V_A^2T_F}{N_c}\tilde{g}_{ij}^{E++}(q)\langle\psi^\lambda|r_i|\Psi_{\boldsymbol{p}_{\mathrm{rel}}}^\lambda\rangle\langle\Psi_{\boldsymbol{p}_{\mathrm{rel}}}^\lambda|r_j|\psi^\lambda\rangle+\frac{(c_4V_A^s)^2T_F}{M^2N_c}\tilde{g}_{ij}^{B++}(q)\varepsilon_{\lambda i}^*\varepsilon_{\lambda j}|\langle\psi^\lambda|\Psi_{\boldsymbol{p}_{\mathrm{rel}}}\rangle|^2\,,$$

field-field correlators : $\tilde{g}_{ij}^{V++}(q)=\int\mathrm{d}\delta t\mathrm{d}^3\delta R\,e^{iq_0\delta t-i\boldsymbol{q}\cdot\delta\boldsymbol{R}}g_{ij'}^{V++}(t_1,t_2,\boldsymbol{R}_1,\boldsymbol{R}_2)\,,\quad V=E,B,$

$$\delta t=t_1-t_2,\ \ \delta\boldsymbol{R}=\boldsymbol{R}_1-\boldsymbol{R}_2,$$

$$g_{ij}^{V++}(t)=\mathrm{Tr}_E\Big\{gV_i^a(\boldsymbol{R},t)W^{ac}[(\boldsymbol{R},t),(\boldsymbol{R},\infty)W^{cb}[(\boldsymbol{R},\infty),(\boldsymbol{R},0)]gV_j^b(\boldsymbol{R},0)\rho_T(0)\Big\}.$$



Recombination for vector quarkonia

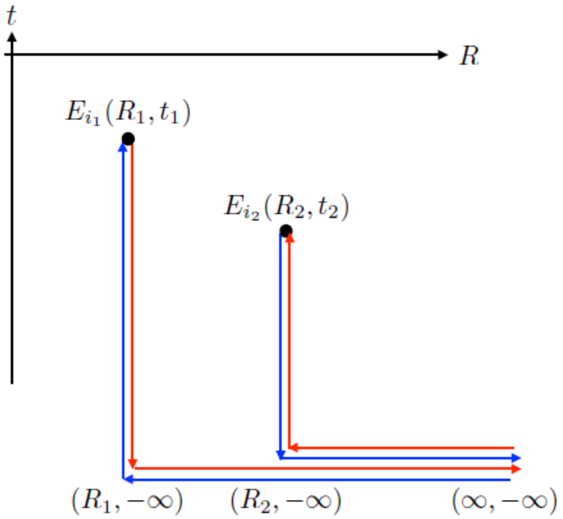
- Recombination term :

$$\mathcal{C}^{+}_{\lambda}(\boldsymbol{x},\boldsymbol{k},t)[f^{(8)}_{Q\bar{Q}}, f^{(8)}_{Q\bar{Q}\lambda}] = \int \frac{\mathrm{d}^3p_{\mathrm{cm}}}{(2\pi)^3}\frac{\mathrm{d}^3p_{\mathrm{rel}}}{(2\pi)^3}\int \mathrm{d}^4q\,\delta(E^{\lambda}_{k} - E_{p} + q_0)\delta^3(\boldsymbol{k} - \boldsymbol{p}_{\mathrm{cm}} + \boldsymbol{q})$$

$$\times\Big(|\mathcal{M}_{r,e}|^2 f^{(8)}_{Q\bar{Q}\lambda}(\boldsymbol{x},\boldsymbol{p}_{\mathrm{cm}},\boldsymbol{x}_0,\boldsymbol{p}_{\mathrm{rel}},t) + |\mathcal{M}_{r,b}|^2 f^{(8)}_{Q\bar{Q}}(\boldsymbol{x},\boldsymbol{p}_{\mathrm{cm}},\boldsymbol{x}_0,\boldsymbol{p}_{\mathrm{rel}},t)\Big)\,,$$

$$|\mathcal{M}_{r,e}|^2 = \frac{V_A^2 T_F}{N_c}\tilde{g}^{E--}_{ji}(q)\langle\psi^{\lambda}|r_j|\Psi^{\lambda}_{\boldsymbol{p}_{\mathrm{rel}}}\rangle\langle\Psi^{\lambda}_{\boldsymbol{p}_{\mathrm{rel}}}|r_i|\psi^{\lambda}\rangle\,,$$

$$|\mathcal{M}_{r,b}|^2 = \frac{(c_4 V_A^s)^2 T_F}{M^2 N_c}\tilde{g}^{B--}_{ji}(q)\varepsilon^{*}_{\lambda i}\varepsilon_{\lambda j}|\langle\psi^{\lambda}|\Psi_{\boldsymbol{p}_{\mathrm{rel}}}\rangle|^2\,.$$



field-field correlators :

$$\tilde{g}^{V--}_{ij}(q) = \int \mathrm{d}\delta t\mathrm{d}^3\delta R\, e^{-iq_0\delta t + i\boldsymbol{q}\cdot\delta\boldsymbol{R}}\big[g^{V--}_{ij}(t_2,t_1,\boldsymbol{R}_2,\boldsymbol{R}_1)\big]^{aa}\,,$$

$$\big[g^{V--}_{ji}(t_2,t_1,\boldsymbol{R}_2,\boldsymbol{R}_1)\big]^{a_2a_1} \equiv \mathrm{Tr}_E\Big\{W^{a_2b}[(\boldsymbol{R}_1,-\infty),(\boldsymbol{R}_2,-\infty)]$$

$$\times W^{bc}[(\boldsymbol{R}_2,-\infty),(\boldsymbol{R}_2,t_2)]gV^{c}_{j}(\boldsymbol{R}_2,t_2)gV^{d}_{i}(\boldsymbol{R}_1,t_1)W^{da_1}[(\boldsymbol{R}_1,t_1),(\boldsymbol{R}_1,-\infty)]\rho_T(0)\Big\}\,.$$

Phenomenological applications

- From the quarkonium distribution functions to spin alignment :

$$\rho_{\lambda\lambda}(\boldsymbol{k}) = \frac{\int \mathrm{d}\Sigma_x \cdot k\, f_\lambda(\boldsymbol{x}, \boldsymbol{k}, t)}{\int \mathrm{d}\Sigma_x \cdot k \sum_{\lambda'=\pm 1,0} f_{\lambda'}(\boldsymbol{x}, \boldsymbol{k}, t)},$$

ρ_{00}: spin alignment

$\rho_{11} - \rho_{-1-1}$: spin polarization

- Chromo-electric & magnetic correlators of QGP are needed.
  - ❖ The correlators for quarkonia and heavy quarks differ in terms of operator orderings in the Wilson lines. B. Scheihing-Hitschfeld, X. Yao, PRL 130 (2023) 052302
  - ❖ chromo-electric correlator (for quarkonia):

perturbative theory : T. Binder et al., JHEP 01 (2022) 137

AdS/CFT correspondence : G. Nijs, B. Scheihing-Hitschfeld, X. Yao, JHEP 06 (2023) 007; arXiv:2310.09325

lattice (preliminary) : V. Leino, PoS LATTICE2023 (2024) 385 [2401.06733]

- ❖ chromo-magnetic correlator :

lattice (for heavy quarks) : J. Mayer-Steudte et al., PoS LATTICE2021 (2022) 318 [2111.10340].
L. Altenkort et al., arXiv:2402.09337

- Further studies of **chromo-magnetic correlators for quarkonia** are needed.

Summary

❑ Take-home messages :

- ✓ We utilize the **OPS & pNRQCD** to derive the **polarization-dependent Boltzmann equation for vector quarkonia** in the quantum optical limit.
- ✓ Quarkonium polarization can be induced by **chromo-magnetic correlators** via the **spin singlet-triplet transition**.
- ✓ A Lindblad equation with the spin singlet-triplet transition can also be derived in the quantum Brownian motion limit (not discussed in this talk).

❑ Outlook :

- ➢ We may first calculate chromo-magnetic correlators perturbatively.
- ➢ We also need to track $f^{(8)}_{Q\bar{Q}} \sim f_Q f_{\bar{Q}}$. ⇨ solving coupled Boltzmann eqs.

  X. Yao et al., JHEP 01 (2021) 046
- ➢ There exists implicit spin dependence for the chromo-electric int. through $f^{(8)}_{Q\bar{Q}\lambda}$ in recombination. ⇨ e.g., initial polarization of heavy quarks from glasma?

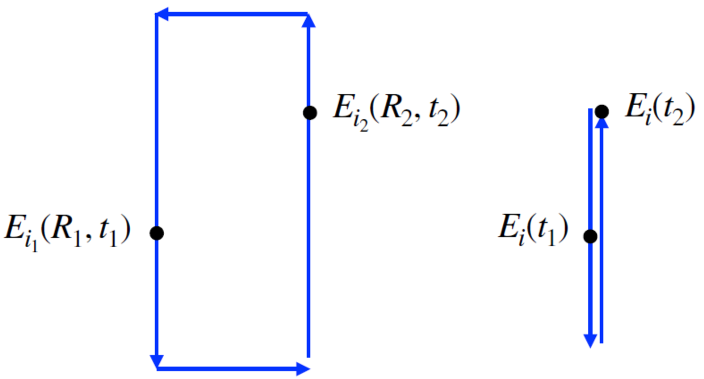
  A. Kumar, B. Müller, DY, PRD 108, 016020 (2023); 107, 076025 (2023).

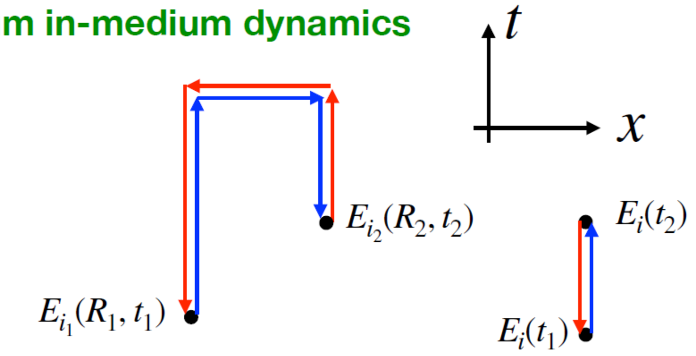
Thank you!

Heavy quark v.s. quarkonia

Backup: Two Chromoelectric Field Correlators

- Correlators for heavy quark and quarkonium in-medium dynamics





$$\langle O \rangle_T = \mathrm{Tr}(O \rho_T)$$

Single heavy quark

$$g_E^{\mathrm{Q}}(t) = g^2 \left\langle \mathrm{Tr}_c \left(U_{[-\infty,t]} E_i(t) U_{[t,0]} E_i(0) U_{[0,-\infty]} \right) \right\rangle_T$$

J.Casalderrey-Solana, D.Teaney, hep-ph/0605199

Heavy quark antiquark pair

$$g_E^{\mathrm{Q}\bar{\mathrm{Q}}}(t) = g^2 T_F \left\langle \left(E_i^a(t) \mathcal{W}^{ab}_{[t,0]} E_i^b(0) \right) \right\rangle_T$$

Thomas Mehen, XY, 2009.02408

Color interactions in
both initial **and** final states
since HQ carries color

Color interactions in
either initial **or** final state
since quarkonium colorless

credit : X. Yao, QCD theory seminar
https://ribf.riken.jp/~hidaka/QCD/20240118XiaojunYao.html