Triangle Singularity in the $J/\psi \to \phi\pi^+ a_0^-(\pi^-\eta), \phi\pi^- a_0^+(\pi^+\eta)$ Decays

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Based on: C. W. Xiao, J. M. Dias, L. R. Dai, WHL, E. Oset, PRD109 (2024) 074033.

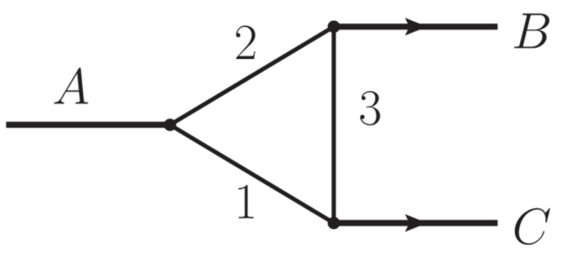
Outline

- Introduction and motivation
- Formalism
- Results and discussions
- Comparison with BESIII data
- Summary

◆ Introduction and motivation

• Triangle singularity (TS) in a reaction

$$A \to B + C$$



[Landau, Nucl. Phys. 13(1959)181]

[Coleman, Norton, Nuovo Cim. 38 (1965)438]

[Bayar, Aceti, Guo, Oset, PRD 94 (2016)074039]

[F.K. Guo, X.H. Liu, S. Sakai, Prog. Part. Nucl. Phys. 112, (2020) 103757]

When all the intermediate particles are placed on-shell and collinear in the rest frame of *A*, a singularity in the decay amplitude *T* develops.

If the internal particles have zero width, $|T| \to \infty$;

If the internal particles have non-zero width, $|T|$ **turns into a finite peak.**

Introduction and motivation

TS $\Rightarrow$
- Simulating a resonance;
- Providing a mechanism for the production of particular modes in reactions;
- …

Examples: **[F. K. Guo, X. H. Liu, and S. Sakai, Threshold cusps and triangle singularities in hadronic reactions, Prog. Part. Nucl. Phys. 112 (2020) 103757]**

✓ The $a_1(1420)$ resonance, claimed by COMPASS, would not be a real state but the effect of TS in $a_1(1260) \to \pi f_0(980)$;

[COMPASS, PRL115(2015)082001]; [X.H. Liu, M. Oka, Q. Zhao, PLB753(2016)297];

✓ $f_1(1420)$ corresponds to TS in $f_1(1285) \to \pi f_0(980)$;

[Debastiani, Aceti, WHL, Oset, PRD95(2017)034015];

✓ $f_2(1810)$ peak comes from TS involving $K^* \bar{K}^*$ production;

[Xie, Geng, Oset, PRD95(2017)034004];

…

◆ Introduction and motivation

- **Motivation** Study of the decay $J/\psi \to \phi\pi^0\eta$

[Jing, Sakai, F. K.Guo, B.S.Zou, PRD100 (2019) 114010]:

Triangle singularities in $J/\psi \to \eta\pi^0\phi$ and $\pi^0\pi^0\phi$

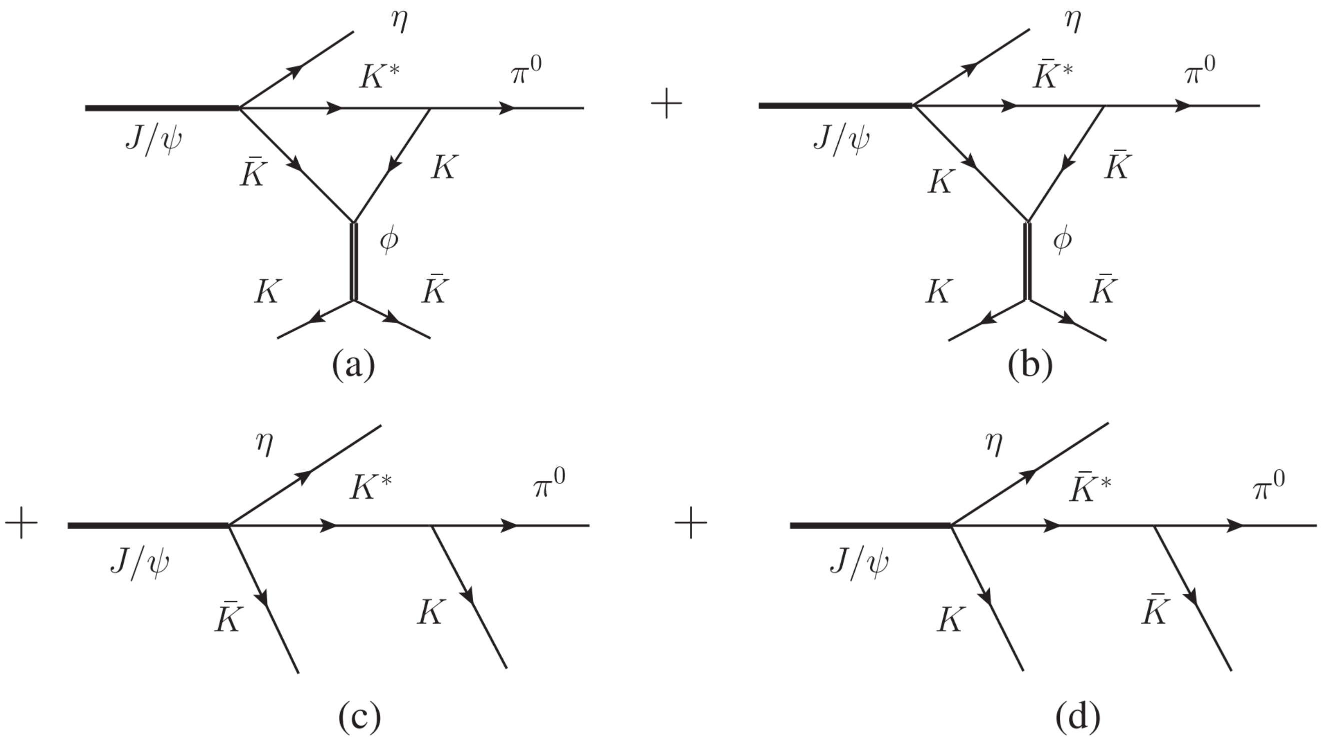
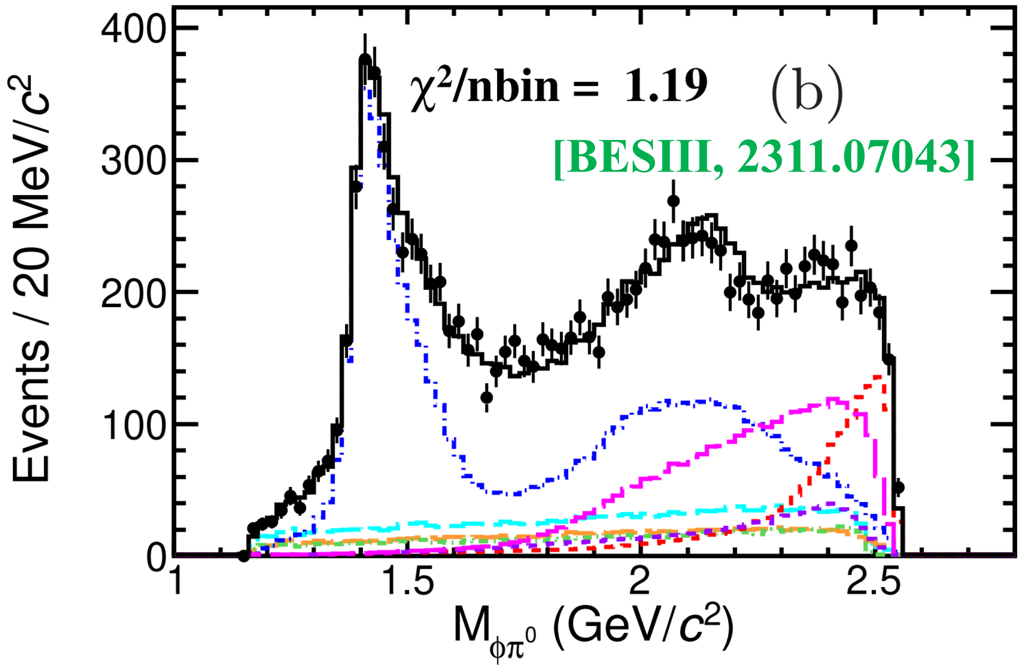


FIG. 1. Mechanisms in $J/\psi \to \eta\pi^0\phi(\phi \to K\bar{K})$: (a),(b) TS; (c),(d) the tree level.

The triangle diagram develops a TS around 1.4GeV.



A clear signal shown around 1.4GeV.

Schmid theorem: TS can be reabsorbed into tree level with a change in the phase.

⇓

The effect of TS is diluted in the distribution.

◆ Introduction and motivation

- **Our work:** $J/\psi \to \phi\pi^+ a_0(980)^- \to \phi\pi^+\pi^-\eta, \quad J/\psi \to \phi\pi^- a_0(980)^+ \to \phi\pi^-\pi^+\eta$

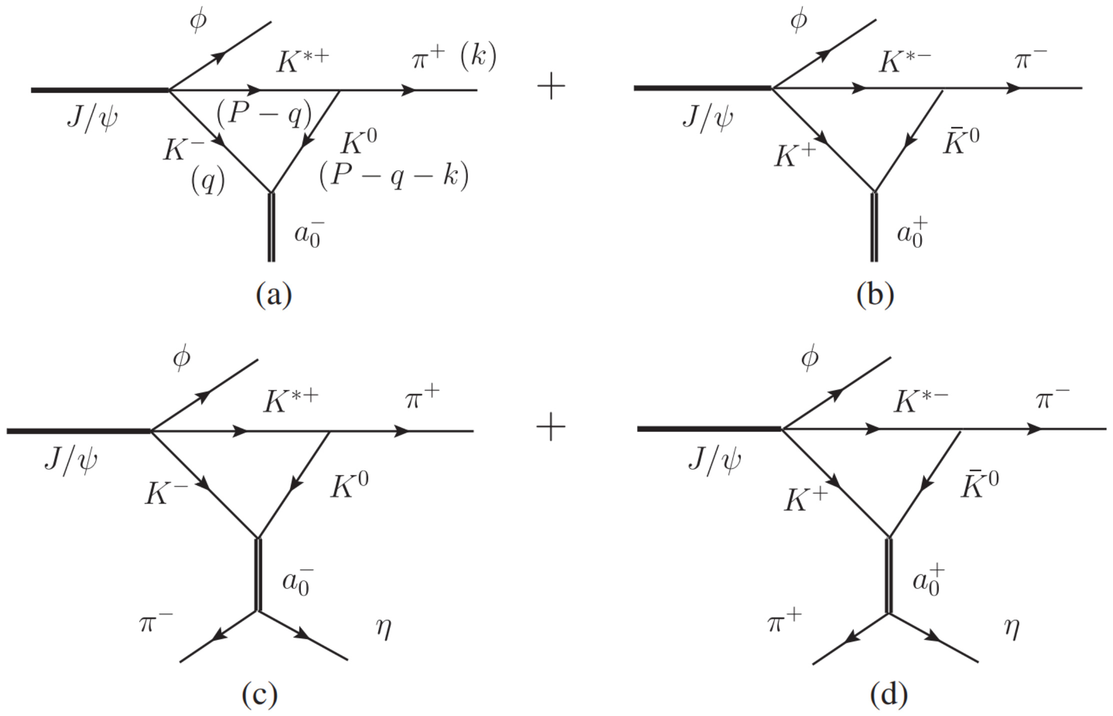


FIG. 2. Triangle diagrams for $J/\psi \to \phi\pi^+ a_0^-$ decay (a) and $J/\psi \to \phi\pi^- a_0^+$ decay (b). (c) and (d) illustrate the processes of (a) and (b) respectively, with a clear depiction of the decay channel of a_0^- and a_0^+. In (a), the momenta of the particles are shown, where $P = p_{J/\psi} - p_\phi$.

✓ A TS appears at $M_{\rm inv}(\pi a_0) \simeq 1416$ MeV;

✓ without tree level diagrams interfering with the triangle diagrams.

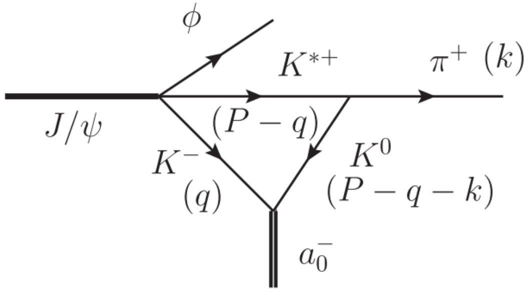
The purpose: to make a realistic prediction of the shape and size of the $\pi a_0(980)$ mass distribution in the reaction.

Formalism

We need: 1) information on $J/\psi \to \phi K^* \bar{K}$; (taken from the experiment)

2) the dynamics of $K^* \to K\pi$ and $K\bar{K} \to a_0 \to \pi\eta$; (well known)

3) the $K\bar{K} \to a_0 \to \pi\eta$ amplitudes. (using the chiral unitary approach)

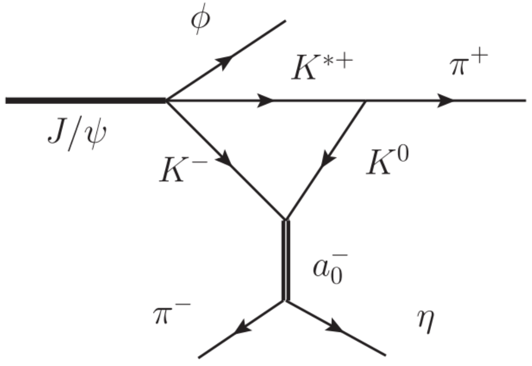


(a)

A. The $J/\psi \to \phi K^* \bar{K}$ reaction

From PDG: $\mathrm{Br}(J/\psi \to \phi K^*(892)\bar{K} + \mathrm{c.c.}) = (2.18 \pm 0.23) \times 10^{-3}$.

But we need that for $J/\psi \to \phi K^{*+} K^-$.



(c)

The $K^* \bar{K}$ must have C-parity positive, then

$$J/\psi \to \phi(K^{*+}K^- + K^{*0}\bar{K}^0 - K^{*-}K^+ - \bar{K}^{*0}K^0), \quad (3)$$

$$\mathrm{Br}(J/\psi \to \phi K^{*+}K^-) = (0.55 \pm 0.06) \times 10^{-3}. \quad (4)$$

Formalism

The structure of the $J/\psi \to \phi K^{*+} K^-$ amplitude in S-wave is given by

$$t_{J/\psi,\phi K^{*+} K^-} = C \vec{\epsilon}_{J/\psi} \cdot (\vec{\epsilon}_\phi \times \vec{\epsilon}_{K^*}),$$ (with C being a constant.)

We can determine C from the rate of Eq. (4) using

$$\frac{d\Gamma_{J/\psi \to \phi K^{*+} K^-}}{dM_{\text{inv}}(K^{*+} K^-)} = \frac{1}{(2\pi)^3} \frac{1}{4M_{J/\psi}^2} p_\phi \tilde{p}_{K^-} \bar{\sum} \sum |t|^2,$$

$$\Longrightarrow \quad \frac{C^2}{\Gamma_{J/\psi}} = \frac{\text{Br}(J/\psi \to \phi K^{*+} K^-)}{\int \frac{2}{(2\pi)^3} \frac{1}{4M_{J/\psi}^2} p_\phi \tilde{p}_{K^-} dM_{\text{inv}}(K^{*+} K^-)} = 1.381 \times 10^{-2} \ (\text{MeV}^{-1}), \tag{10}$$

(be used to evaluate the strength of the triangle mechanism)

Formalism

B. The $a_0^- \to K^- K^0$ coupling and $K^* \to K\pi$ vertex

✓ $K^{*+} \to K^0\pi^+$ vertex :

$$\mathcal{L} = -ig\langle[P, \partial_\mu P]V^\mu\rangle, \quad g = \frac{M_V}{2f},\ M_V = 800\ \text{MeV},\ f = 93\ \text{MeV}$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} \end{pmatrix} \qquad V = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}$$

$$-it = -ig\epsilon_j(K^*)(2k+q)^j,$$ (evaluated in the frame with $\vec{P} = \vec{p}_{J/\psi} - \vec{p}_\phi = 0.$)

◆ Formalism

✓ $a_0^- \to K^- K^0$ coupling, g_{a_0,K^-K^0} : (a_0 (980) is a cusp, no clear couplings.)

Assuming that, close to the peak of the $a_0(980)$,

$$t_{K^-K^0,K^-K^0}(M_{\rm inv}) = \frac{g^2_{a_0,K^-K^0}}{M^2_{\rm inv} - m^2_{a_0} + iM_{\rm inv}\Gamma_{a_0}},$$

Cauchy's integration ⟹

$$g^2_{a_0,K^-K^0} = -\frac{1}{\pi}\int {\rm d}M^2_{\rm inv}{\rm Im}\,t_{K^-K^0,K^-K^0}(M_{\rm inv}), \quad (17)$$

2-body scattering amplitude, obtained from the chiral unitary approach.

Coupled channels: $K\bar{K}$, $\pi\eta$, $\pi\pi$, and $\eta\eta$.

$t_{K^-K^0,K^-K^0}$ amplitude is extracted by solving Bether-Salpeter equation in coupled-channels,

$$T = [1 - VG]^{-1}V,$$

A diagonal matrix with its elements G_l being the loop function of l-channel.

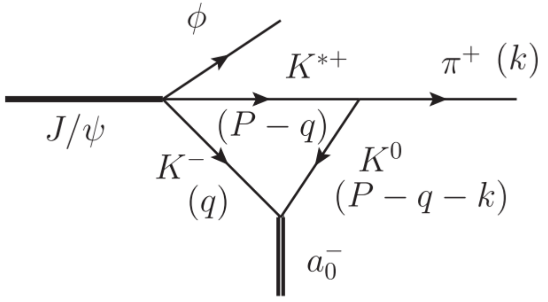
the kernel encoding the V_{ij} potential from i- to j-channel.

Formalism

$$G_l = \int_{|\vec{q}|<q_{\max}} \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{\omega_1+\omega_2}{2\omega_1\omega_2} \frac{1}{s-(\omega_1+\omega_2)^2+i\varepsilon}.$$

($q_{\max} = 600$ MeV/c)

C. The triangle amplitude and the differential decay width



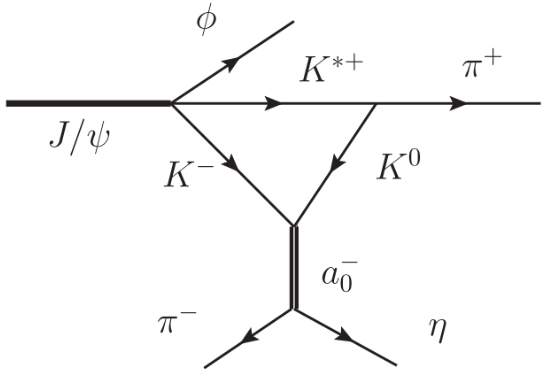
(a)

K^- propagator :

$$\frac{1}{q^2-m_K^2+i\varepsilon} = \frac{1}{2\omega(\vec{q})}\left(\frac{1}{q^0-\omega_K(\vec{q})+i\varepsilon} - \frac{1}{q^0+\omega_K(\vec{q})-i\varepsilon}\right),$$

$$\omega_K(\vec{q}) = \sqrt{\vec{q}^2+m_K^2},$$

With q^0 positive, only this term can go on shell.



(c)

Formalism

The triangle amplitude reads, removing the $g_{a_0,\,K^-K^0}$ coupling,

$$-i\tilde{t}_{TS} = -iC\int \frac{d^4q}{(2\pi)^4}\varepsilon_{ijl}\epsilon_i(J/\psi)\epsilon_j(\phi)\epsilon_l(K^*)(-i)g\epsilon_m(K^*)$$
$$\times (2k+q)_m(-i)\frac{1}{2\omega_{K^-}(\vec{q})}\frac{1}{2\omega_{K^0}(\vec{q}+\vec{k})}$$
$$\times \frac{1}{2\omega_{K^{*+}}(\vec{q})}\frac{i}{q^0-\omega_{K^-}(\vec{q})+i\varepsilon}$$
$$\times \frac{i}{P^0-q^0-\omega_{K^{*+}}(\vec{q})+i\frac{\Gamma_{K^*}}{2}}$$
$$\times \frac{i}{P^0-q^0-k^0-\omega_{K^0}(\vec{q}+\vec{k})+i\varepsilon}, \qquad (24)$$

Integrating over q^0

$$\tilde{t}_{TS} = gC\varepsilon_{ijl}\epsilon_i(J/\psi)\epsilon_j(\phi)\int\frac{d^3q}{(2\pi)^3}(2k+q)_l$$
$$\times \frac{1}{2\omega_{K^-}(\vec{q})}\frac{1}{2\omega_{K^{*+}}(\vec{q})}\frac{1}{2\omega_{K^0}(\vec{q}+\vec{k})}$$
$$\times \frac{i}{P^0-\omega_{K^-}(\vec{q})-\omega_{K^{*+}}(\vec{q})+i\frac{\Gamma_{K^*}}{2}}$$
$$\times \frac{i}{P^0-k^0-\omega_{K^-}(\vec{q})-\omega_{K^0}(\vec{q}+\vec{k})+i\varepsilon}.$$

$$P^0 = M_{\rm inv}(\pi^+a_0^-), \qquad k^0 = \frac{P^{02}+m_{\pi^+}^2-M_{\rm inv}^2(\pi^-\eta)}{2P^0}.$$

$$\sum_{\rm pol}\epsilon_l(K^*)\epsilon_m(K^*) = \delta_{lm}$$

$$\tilde{t}_{TS} = gC\epsilon_{ijl}\epsilon_i(J/\psi)\epsilon_j(\phi)k_l\tilde{t}'_{TS}, \qquad (28)$$

◆ Formalism

$$\tilde{t}'_{\rm TS} = \int \frac{{\rm d}^3 q}{(2\pi)^3}\, \theta(q_{max} - |\vec{q}^{\,*}|)\, \left(2 + \frac{\vec{q}\cdot\vec{k}}{\vec{k}^{\,2}}\right) \frac{1}{2\,\omega_{K^-}(\vec{q}\,)} \frac{1}{2\,\omega_{K^{*+}}(\vec{q}\,)} \frac{1}{2\,\omega_{K^0}(\vec{q}+\vec{k}\,)}$$
$$\times \frac{i}{P^0 - \omega_{K^-}(\vec{q}\,) - \omega_{K^{*+}}(\vec{q}\,) + i\frac{\Gamma_{K^*}}{2}} \frac{i}{P^0 - k^0 - \omega_{K^-}(\vec{q}\,) - \omega_{K^0}(\vec{q}+\vec{k}\,) + i\varepsilon}, \tag{29}$$

$\vec{q}^{\,*}$ is the K^- momentum in the $\pi^-\eta$ rest frame given by

$$\vec{q}^{*} = \left[\left(\frac{E_{a_0}}{M_{\rm inv}(\pi^-\eta)} - 1\right)\frac{\vec{q}\cdot\vec{k}}{\vec{k}^2} + \frac{q^0}{M_{\rm inv}(\pi^-\eta)}\right]\vec{k} + \vec{q},$$

$$E_{a_0} = \sqrt{M_{\rm inv}^2 + \vec{k}^2}, \quad \text{and} \quad q^0 = \sqrt{m_K^2 + \vec{q}^2}.$$

◆ Formalism

The differential decay width:

$$\frac{d^2\Gamma_{J/\psi,\to\phi\pi^+a_0^-(\pi^-\eta)}}{dM_{\rm inv}(\pi^-\eta)\, dM_{\rm inv}(\pi^+a_0^-)} = \underline{-\frac{1}{\pi}\, 2M_{\rm inv}(\pi^-\eta)\, {\rm Im}\, t_{K^-K^0,\,K^-K^0}(M_{\rm inv}(\pi^-\eta))} \times \frac{1}{(2\pi)^3}\, \frac{1}{4\, M_{J/\psi}^2}\, p_\phi\, \tilde{p}'_{\pi^+} \overline{\sum}|\tilde{t}_{\rm TS}|^2, \quad (22)$$

A

$$\bar{\sum} |\tilde{t}_{\rm TS}|^2 = \frac{2}{3}\vec{k}^2 g^2 C^2 |\tilde{t}'_{\rm TS}|^2. \quad (30)$$

◆ Results and discussions

$$\frac{d^2\Gamma_{J/\psi,\to\phi\pi^+a_0^-(\pi^-\eta)}}{dM_{\rm inv}(\pi^-\eta)\, dM_{\rm inv}(\pi^+a_0^-)} = \underline{-\frac{1}{\pi}\, 2M_{\rm inv}(\pi^-\eta)\, {\rm Im}\, t_{K^-K^0,\, K^-K^0}(M_{\rm inv}(\pi^-\eta))} \times \frac{1}{(2\pi)^3}\frac{1}{4M_{J/\psi}^2}\, p_\phi\, \tilde{p}'_{\pi^+} \overline{\sum}|\tilde{t}_{\rm TS}|^2, \quad (22)$$

→ A

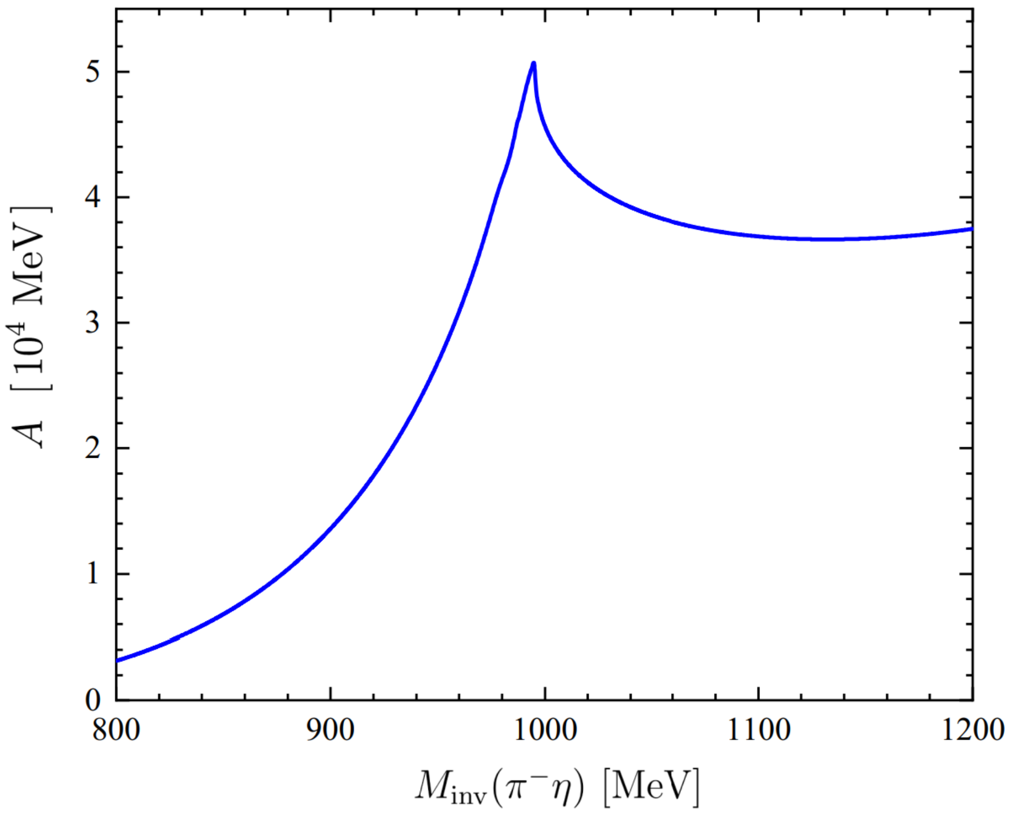


FIG. 3. Factor $A \equiv [-\frac{2}{\pi}M_{\rm inv}(\pi^-\eta){\rm Im}t_{K^-K^0,K^-K^0}(M_{\rm inv}(\pi^-\eta))]$ as a function of $M_{\rm inv}(\pi^-\eta)$.

- Factor A as a function of $M_{\rm inv}(\pi^-\eta)$

✓ A cusp-like structure around $M_{\rm inv}(\pi^-\eta) = m_{a_0} = 980$ MeV, reflecting the spectral function of the $a_0(980)$.

✓ The shape of Fig.3 does not reflect $|t_{K\bar{K},\pi\eta}|^2$, because, through the optical theorem, ${\rm Im}t_{K^-K^0,K^-K^0}$ contains a part from $K^-K^- \to K\bar{K}$, and also $K^-K^0 \to K\bar{K}$.

This is the reason for the flattening of factor A when going away from the KKbar threshold.

$$\tilde{t}'_{\rm TS} = \int \frac{{\rm d}^3 q}{(2\pi)^3}\, \theta(q_{max} - |\vec{q}^{\,*}|)\, \left(2 + \frac{\vec{q}\cdot\vec{k}}{\vec{k}^{\,2}}\right) \frac{1}{2\,\omega_{K^-}(\vec{q})}\, \frac{1}{2\,\omega_{K^{*+}}(\vec{q})}\, \frac{1}{2\,\omega_{K^0}(\vec{q}+\vec{k})}$$
$$\times \frac{i}{P^0 - \omega_{K^-}(\vec{q}) - \omega_{K^{*+}}(\vec{q}) + i\frac{\Gamma_{K^*}}{2}}\, \frac{i}{P^0 - k^0 - \omega_{K^-}(\vec{q}) - \omega_{K^0}(\vec{q}+\vec{k}) + i\varepsilon}, \quad (29)$$

$$\frac{{\rm d}^2\Gamma_{J/\psi,\to \phi\pi^+ a_0^-(\pi^-\eta)}}{{\rm d}M_{\rm inv}(\pi^-\eta)\, {\rm d}M_{\rm inv}(\pi^+ a_0^-)} = -\frac{1}{\pi}\, 2M_{\rm inv}(\pi^-\eta)\, {\rm Im}\, t_{K^-K^0,\, K^-K^0}(M_{\rm inv}(\pi^-\eta))$$
$$\times \frac{1}{(2\pi)^3}\, \frac{1}{4\, M^2_{J/\psi}}\, p_\phi\, \tilde{p}'_{\pi^+} \overline{\sum} |\tilde{t}_{\rm TS}|^2, \quad (22)$$

The triangle amplitude $\tilde{t}'_{\rm TS}$ and the differential decay width $\frac{1}{\Gamma_{J/\psi}} \frac{{\rm d}^2\Gamma_{J/\psi\to\phi\pi^+ a_0(980)^-}}{{\rm d}M_{\rm inv}(\pi^-\eta){\rm d}M_{\rm inv}(\pi^+ a_0^-)}$ are functions of both $M_{\rm inv}(\pi^-\eta)$ and $M_{\rm inv}(\pi^+ a_0^-)$.

The results will be presented in three cases:

1) fixing $M_{\rm inv}(\pi^-\eta) = m_{a_0} = 980$ MeV,

2) fixing $M_{\rm inv}(\pi^+ a_0^-) = 1416$ MeV, where the TS occurs

3) integrating $\frac{1}{\Gamma_{J/\psi}} \frac{{\rm d}^2\Gamma_{J/\psi\to\phi\pi^+ a_0(980)^-}}{{\rm d}M_{\rm inv}(\pi^-\eta){\rm d}M_{\rm inv}(\pi^+ a_0^-)}$ over $M_{\rm inv}(\pi^-\eta)$.

Results and discussions

Case 1): **Fixing $M_{\text{inv}}(\pi^- \eta) = m_{a_0} = 980$ MeV**

The picture of the triangle amplitude:

✓ The structure of the triangle amplitude exhibits features typical of TS observed in other cases.

✓ $|\tilde{t}'_{\text{TS}}|$ has a clear peak, looking like the structure of a resonance.

✓ The origin of this structure comes from the triangle diagram, tied to the kinematical singularity, not from the interaction of quarks or hadrons.

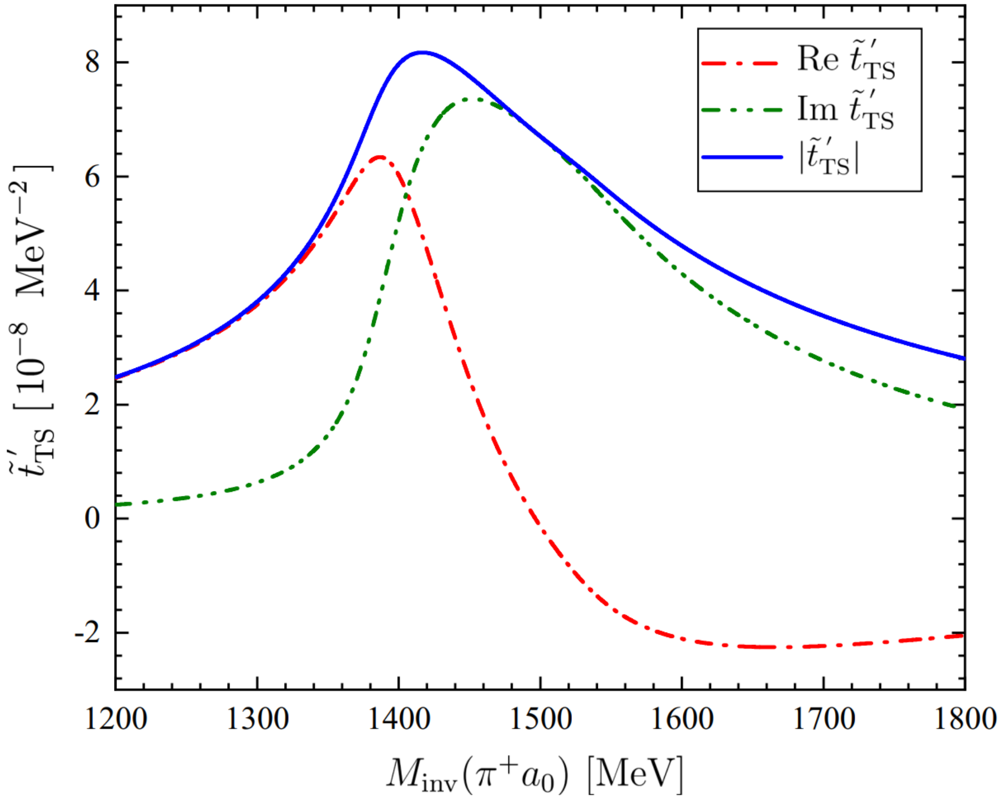


FIG. 4. $\tilde{t}'_{\text{TS}}$ given by Eq. (29) as a function of $M_{\text{inv}}(\pi^+ a_0^-)$ when fixing $M_{\text{inv}}(\pi^- \eta) = m_{a_0}$.

Results and discussions

The picture of $\frac{1}{\Gamma_{J/\psi}}\frac{\mathrm{d}^2\Gamma_{J/\psi\to\phi\pi^+a_0(980)^-}}{\mathrm{d}M_{\mathrm{inv}}(\pi^-\eta)\mathrm{d}M_{\mathrm{inv}}(\pi^+a_0^-)}$, fixing $M_{\mathrm{inv}}(\pi^-\eta)=m_{a_0}=980$ MeV:

- ✓ A clear peak is seen around $M_{\mathrm{inv}}(\pi^+a_0^-)=1440$ MeV, coming from $|\tilde{t}'_{\mathrm{TS}}|$.

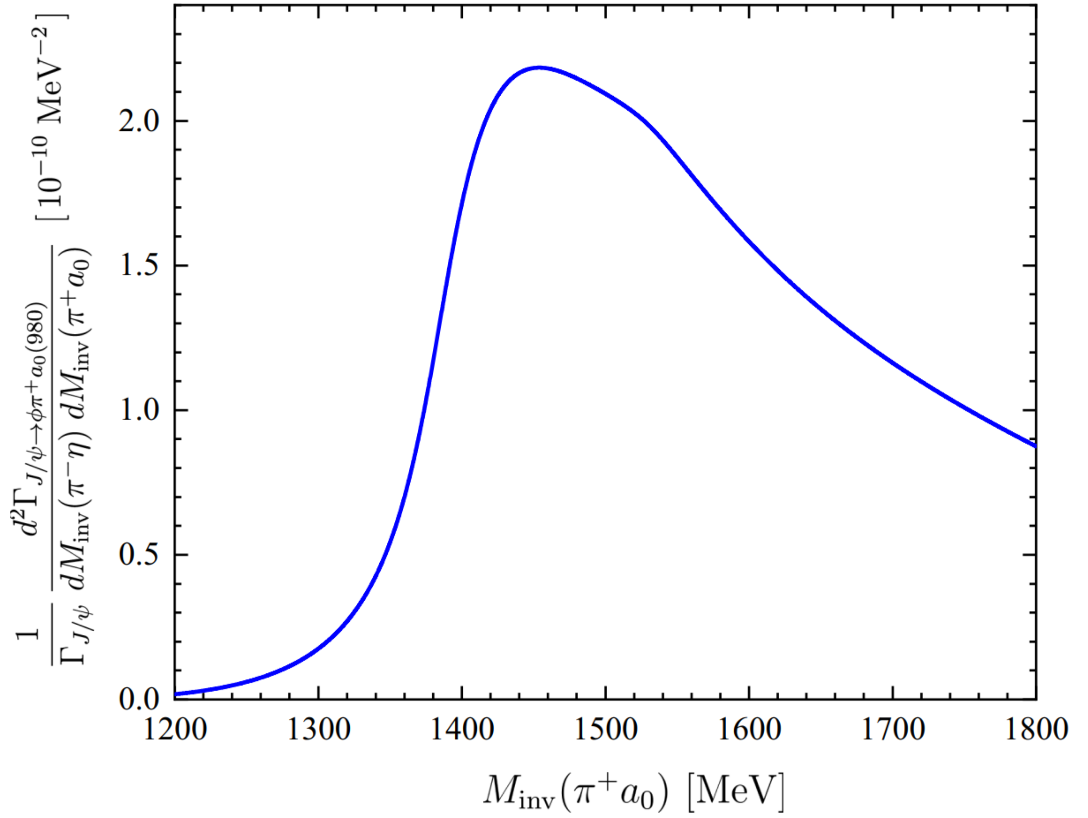


FIG. 5. $\frac{1}{\Gamma_{J/\psi}}\frac{\mathrm{d}^2\Gamma_{J/\psi\to\phi\pi^+a_0(980)^-}}{\mathrm{d}M_{\mathrm{inv}}(\pi^-\eta)\mathrm{d}M_{\mathrm{inv}}(\pi^+a_0^-)}$ as a function of $M_{\mathrm{inv}}(\pi^+a_0^-)$ when fixing $M_{\mathrm{inv}}(\pi^-\eta)=m_{a_0}$.

Results and discussions

Case 2): **Fixing $M_{inv}(\pi^+ a_0^-) = 1416$ MeV**

The picture of the triangle amplitude:

✓ Again, the imaginary part and the modulus delineate the shape of the $a_0(980)$ resonance.

✓ The real part changes sign at the peak of the $a_0(980)$, reflecting a typical resonance behavior.

✓ Even if the $a_0(980)$ appears as cusp, corresponding to a nearly missed bound state, or virtual state, it still exhibits the typical shape of a resonance amplitude.

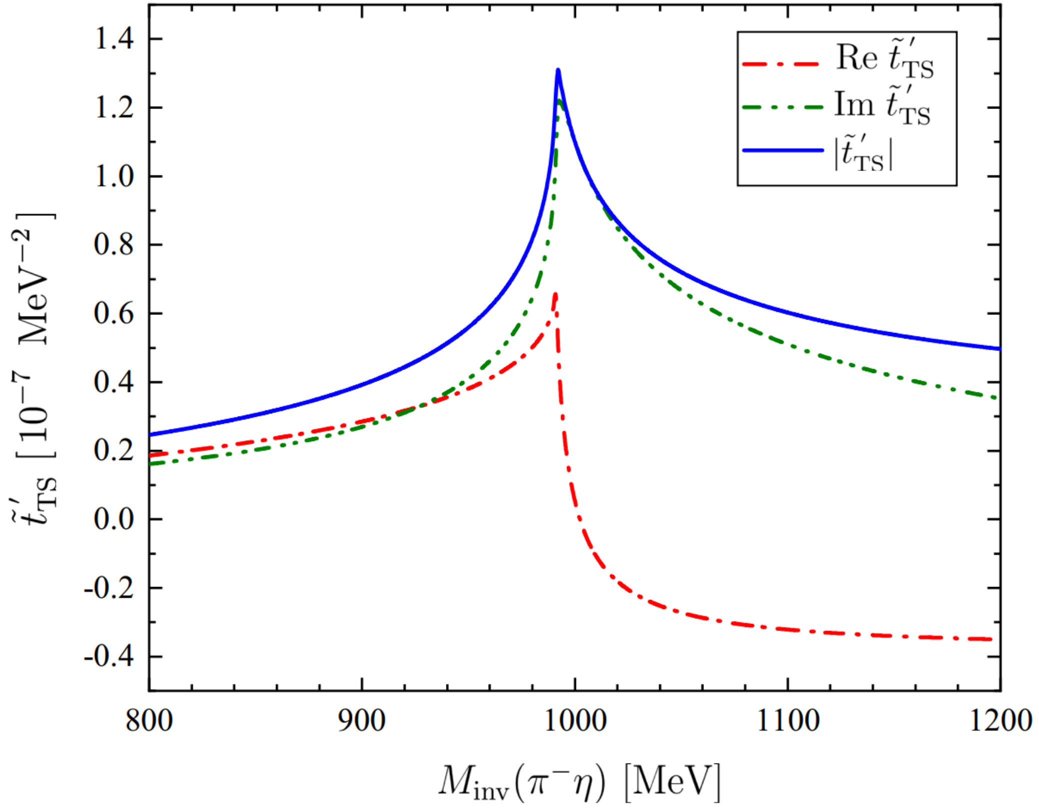


FIG. 6. $\tilde{t}'_{TS}$ given by Eq. (29) as a function of $M_{inv}(\pi^-\eta)$ when fixing $M_{inv}(\pi^+ a_0^-) = 1416$ MeV.

Results and discussions

The picture of $\frac{1}{\Gamma_{J/\psi}} \frac{d^2\Gamma_{J/\psi\to\phi\pi^+ a_0(980)^-}}{dM_{\rm inv}(\pi^-\eta) dM_{\rm inv}(\pi^+ a_0^-)}$, fixing $M_{\rm inv}(\pi^+ a_0^-) = 1416$ MeV:

✓ The shape of the $a_0(980)$ resonance shows up as a clear cusp structure.

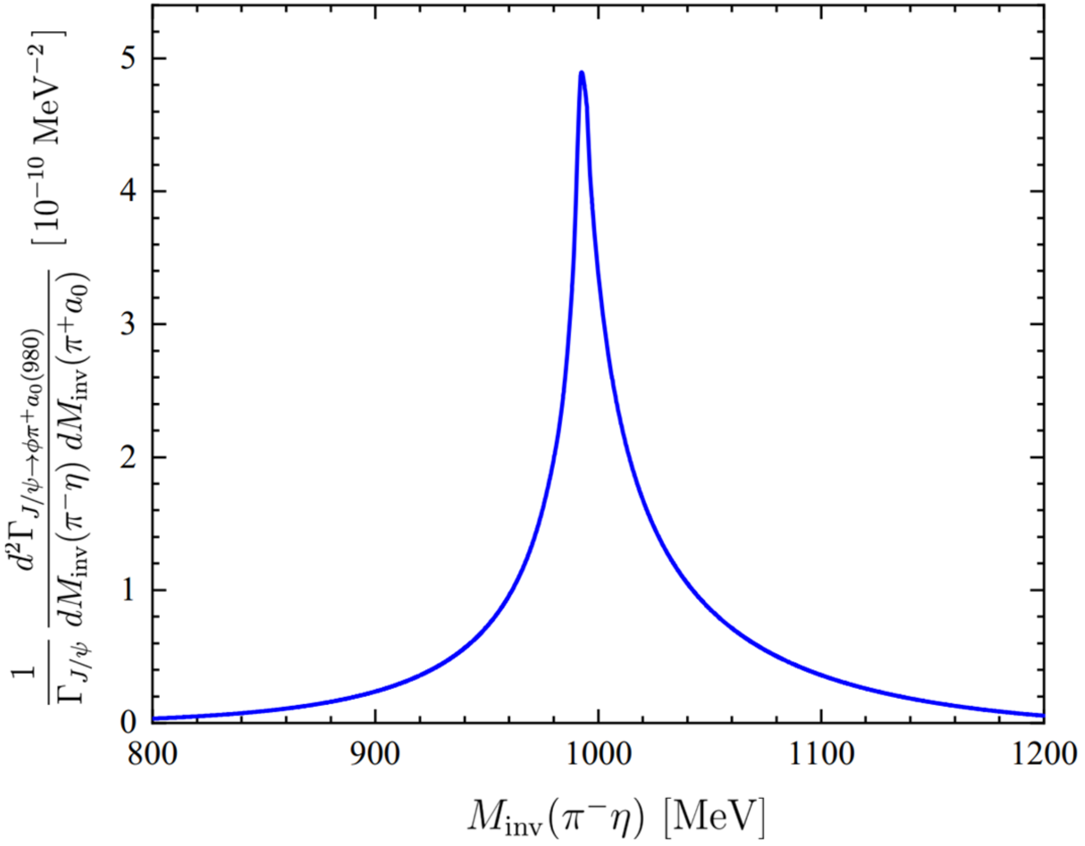


FIG. 7. $\frac{1}{\Gamma_{J/\psi}} \frac{d^2\Gamma_{J/\psi\to\phi\pi^+ a_0(980)^-}}{dM_{\rm inv}(\pi^-\eta) dM_{\rm inv}(\pi^+ a_0^-)}$ as a function of $M_{\rm inv}(\pi^-\eta)$ when fixing $M_{\rm inv}(\pi^+ a_0^-) = 1416$ MeV.

Results and discussions

Case 3): Integrating $\frac{1}{\Gamma_{J/\psi}} \frac{d^2\Gamma_{J/\psi\to\phi\pi^+ a_0(980)^-}}{dM_{\rm inv}(\pi^-\eta)dM_{\rm inv}(\pi^+ a_0^-)}$ over $M_{\rm inv}(\pi^-\eta)$

✓ The shape of the TS is clearly seen, and should be observed in the experiments.

✓ Integrating the double mass distribution over $M_{\rm inv}(\pi^-\eta)$ within the range $m_{a_0} \pm 100$ MeV accounts for the whole strength of the $a_0(980)$ resonance.

✓ For the case where $M_{\rm inv}(\pi^-\eta) \in [m_{a_0} - 100, m_{a_0} + 100]$ MeV, integrating over $M_{\rm inv}(\pi^+ a_0^-)$ in the range $[m_{\pi^+} + m_{a_0}, M_{J/\psi} - m_\phi]$ gives the branching ratio

$$\mathrm{Br}(J/\psi \to \phi\pi^+ a_0^-) = 1.07 \times 10^{-5}, \qquad (35)$$

which is easily reachable in BESIII, where branching ratios of 10^{-7} can be reached.

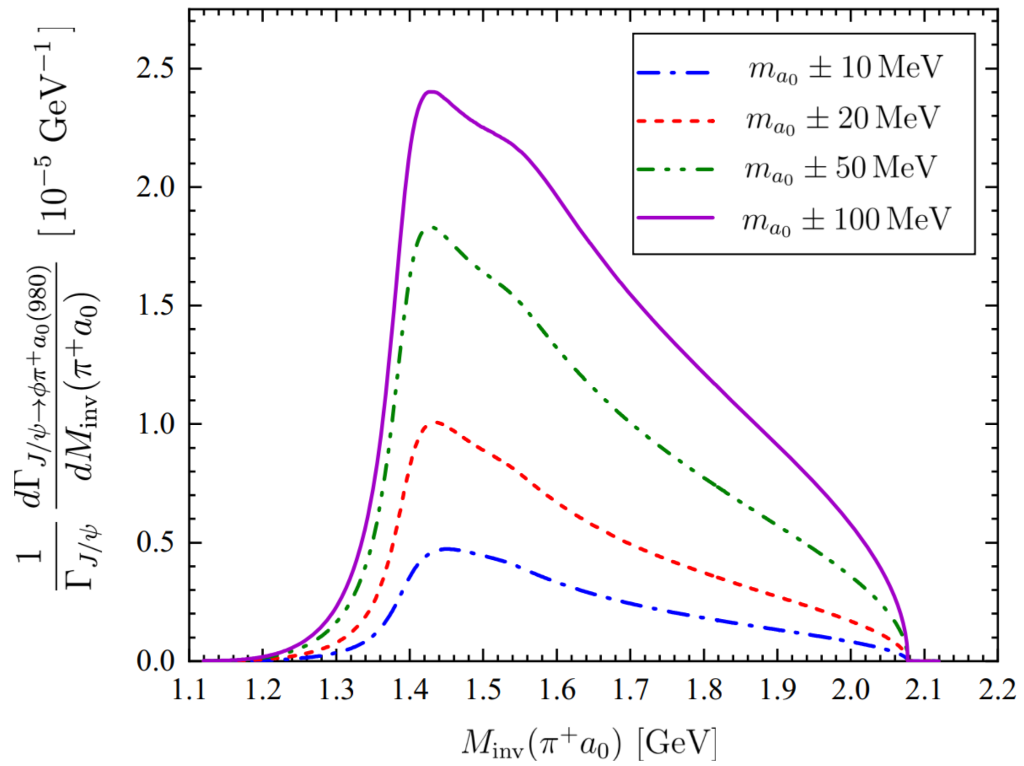


FIG. 8. $\frac{1}{\Gamma_{J/\psi}} \frac{d^2\Gamma_{J/\psi\to\phi\pi^+ a_0(980)^-}}{dM_{\rm inv}(\pi^-\eta)dM_{\rm inv}(\pi^+ a_0^-)}$ as a function of $M_{\rm inv}(\pi^+ a_0^-)$ when integrating over $M_{\rm inv}(\pi^-\eta)$ in the ranges $m_{a_0} \pm 10$ MeV, $m_{a_0} \pm 20$ MeV, $m_{a_0} \pm 50$ MeV and $m_{a_0} \pm 100$ MeV.

✓ Twice our rate of Eq. (35), to account also for $\phi\pi^{-}a_0^{+}$ decay, with 30% uncertainty, gives

$$\mathrm{Br}(J/\psi \to \phi\pi a_0) = (2.14 \pm 0.64) \times 10^{-5}.$$

Comes from the uncertainties when calculating the constant C^2 and the experimental error in $\mathrm{Br}(J/\psi \to \phi K^*(892)\bar{K} + c.c.)$, summing in quadrature.

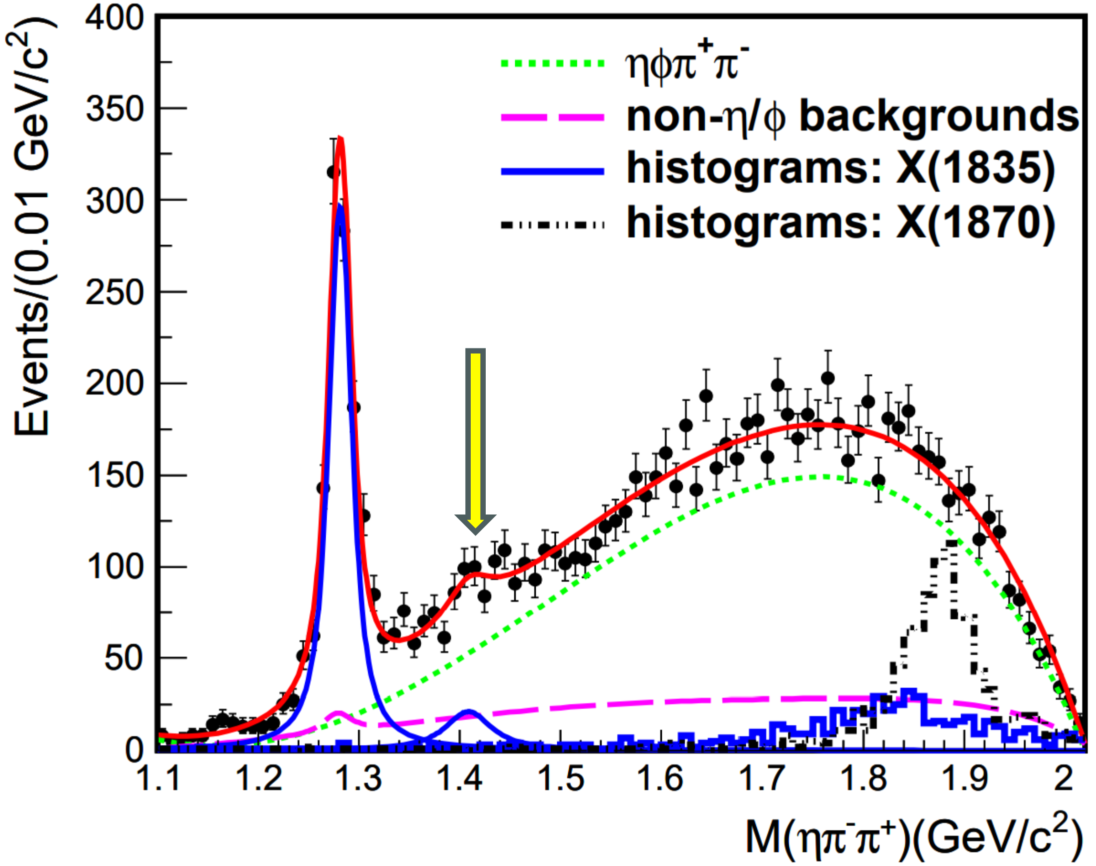
◆ Comparison with BESIII data

✓ There is a clear bump in the $\eta\pi^+\pi^-$ mass distribution stretching from 1.4 ~ 1.53 GeV.

✓ In BESIII paper, this bump is associated to the excitation of $\eta(1405)$, which has the $\eta\pi^+\pi^-$ as one of the decay modes.

✓ In BESIII paper, the branching ratio of the bump was estimated to $(2.01 \pm 0.58 \pm 0.82) \times 10^{-5}$, compatible with our estimate for the TS.

✓ This coincidence and the position of the peak compared to our Fig. 8 give us strong arguments to encourage the reanalysis of this decay mode from the perspective given in this work.



BESIII Collaboration, Study of $J/\psi \to \eta\phi\pi^+\pi^-$ at BESIII, Phys. Rev. D 91, 052017 (2015).

TABLE III. Measurements of the branching fractions for the decay modes. Upper limits are given at the 90% C.L.

Decay mode	Branching fraction $\mathcal{B}$
$J/\psi \to \eta Y(2175),\ Y(2175) \to \phi f_0(980),\ f_0(980) \to \pi^+\pi^-$	$(1.20 \pm 0.14 \pm 0.37) \times 10^{-4}$
$J/\psi \to \phi f_1(1285),\ f_1(1285) \to \eta\pi^+\pi^-$	$(1.20 \pm 0.06 \pm 0.14) \times 10^{-4}$
$J/\psi \to \phi\eta(1405),\ \eta(1405) \to \eta\pi^+\pi^-$	$(2.01 \pm 0.58 \pm 0.82)(< 4.45) \times 10^{-5}$
$J/\psi \to \phi X(1835),\ X(1835) \to \eta\pi^+\pi^-$	$< 2.80 \times 10^{-4}$
$J/\psi \to \phi X(1870),\ X(1870) \to \eta\pi^+\pi^-$	$< 6.13 \times 10^{-5}$

(From BESIII paper.)

Summary

- We propose the $J/\psi \to \phi\pi^+ a_0(980)^-$ $(a_0^- \to \pi^-\eta)$ decay, showing that it develops a TS at $M_{\rm inv}(\pi a_0) \simeq 1.42$ GeV. There is no tree level competing mechanism, and then the TS appearing can be clearly interpreted.

- We evaluate the mass distributions in terms of $M_{\rm inv}(\pi^-\eta)$ and $M_{\rm inv}(\pi^+ a_0^-)$. A clear cusp structure shows up in the $\pi^-\eta$ mass distribution, and the TS peak appears in the $\pi^+ a_0^-$ mass distribution at $M_{\rm inv}(\pi^+ a_0^-)$ ~1420 MeV.

- The results obtained are consistent with a peak seen in a recent BESIII experiment.

- We predict a branching ratio for the reaction of the order of 10^{-5}, within present measurable range. and encourage the experimental teams to look into the $\phi\pi^+ a_0^-$ and $\phi\pi^- a_0^+$ decay channels of J/ψ to further clarify experimentally the $\eta(1405)$ excitation mode from the TS mechanism suggested here.

Thank you for your attention!

[BESIII, 2311.07043]

Study of the decay $J/\psi \to \phi\pi^0\eta$

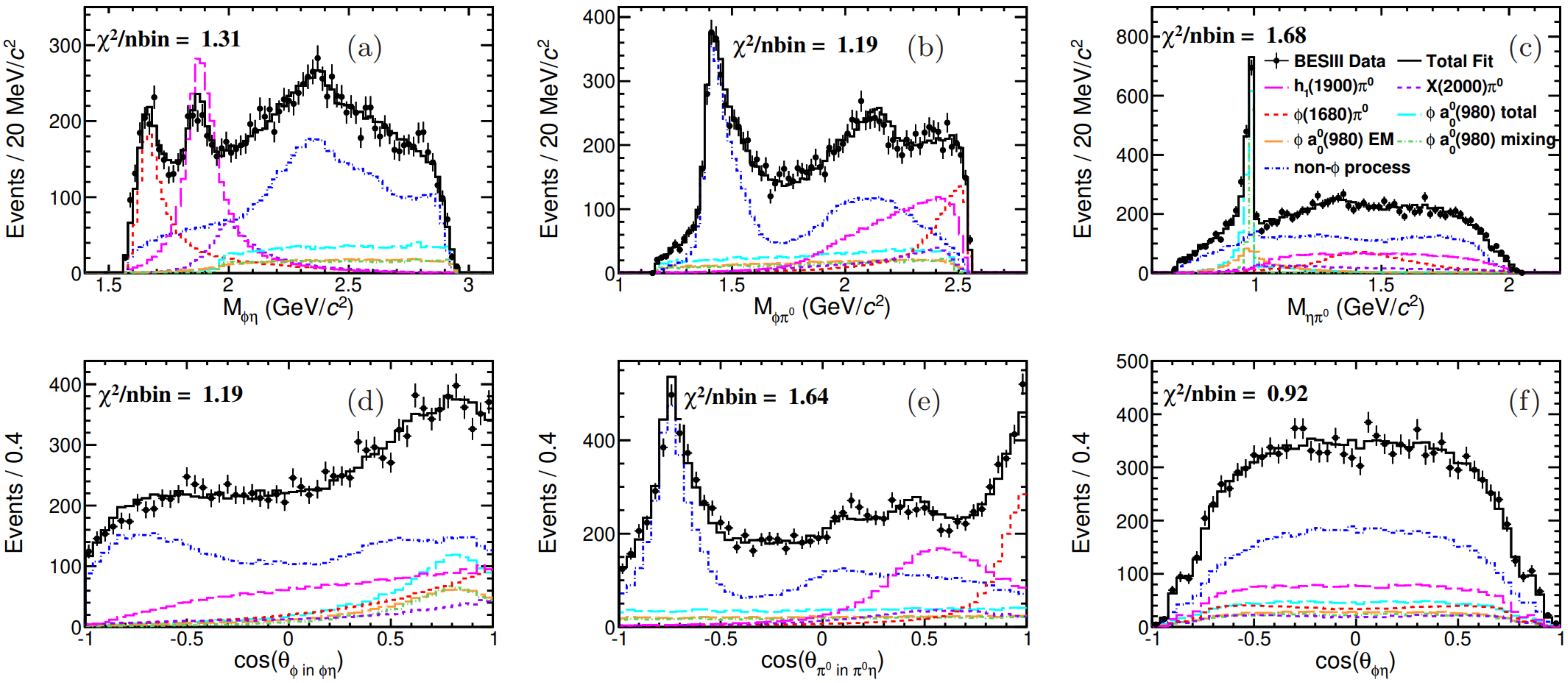


FIG. 2. Invariant mass distributions of (a) $\phi\eta$, (b) $\phi\pi^0$, and (c) $\eta\pi^0$, and angular distributions of (d) $\cos\theta_\phi$ in the $\phi\eta$ helicity frame, (e) $\cos\theta_{\pi^0}$ in the $\pi^0\eta$ helicity frame, (f) $\cos\theta_{\phi\eta}$ in the center-of-mass rest frame. The black dots are the background-subtracted data, the black continuous lines are the PWA fit projections, the colored non-continuous lines show the components of the fit model.