Quantum real-time evolution of entanglement and hadronization in jet production: lessons from the massive Schwinger model

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in collaboration with

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why quantum simulation?

"first principle" microscopic theory: quantum field theory $L \leftrightarrow H$

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perturbative calculation: scattering process, thermodynamics, transport

non-perturbative calculation(lattice QFT): thermodynamics, transport

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non-perturbative calculation(lattice QFT): thermodynamics, transport

real time dynamics of non-perturbative theory?

time evolution of a quantum (field) state:

$$\partial_t | \psi(t) \rangle = - i \hat{H} | \psi(t) \rangle$$

$$\partial_t \hat{\rho} = - i [\hat{H}, \hat{\rho}]$$

Ideally, ***quantum simulation*** for ***full QCD in 3+1 D***, but …

Hamiltonian

1+1D Schwinger model

$$H = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1 \partial_x - g\gamma^1 A - m)\psi \right) \mathrm{d}x \,.$$

$$L(t) = \int \left(-\frac{F^{\mu\nu}F_{\mu\nu}}{4} + \bar{\psi}(i\gamma^\mu \partial_\mu - g\gamma^\mu A_\mu - m)\psi \right) \mathrm{d}x \,.$$

E: electric field

A: electric potential

$\psi, \bar{\psi}$: fermion field

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1+1D Schwinger model with time-dependent external source

$$H(t) = \int \left(\frac{E^2}{2} - \bar{\psi}(i\gamma^1\partial_x - g\gamma^1 A - m)\psi - j^1_{\text{ext}}(t)A \right) \mathrm{d}x\,.$$

$$j^1_{\text{ext}}(x,t) = g\,[\delta(x-t) + \delta(x+t)]\,\theta(t)$$

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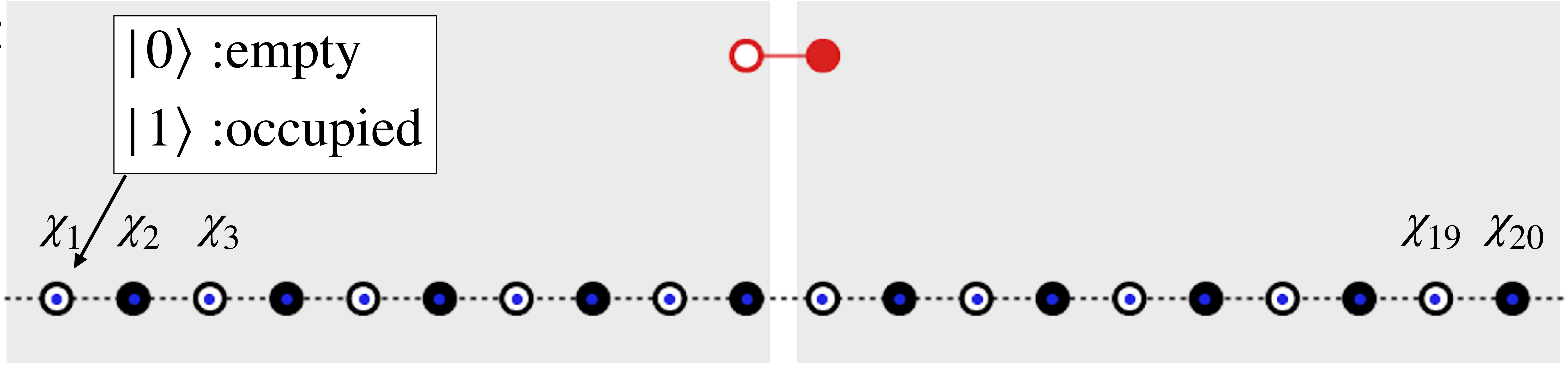
discretize and matrix(gate) representation:

staggered fermion that satisfied anti-commutation: $\{\psi_a(x), \psi_b^\dagger(y)\} = \delta_{a,b}\delta(x-y)$

Hamiltonian

1+1D Sc



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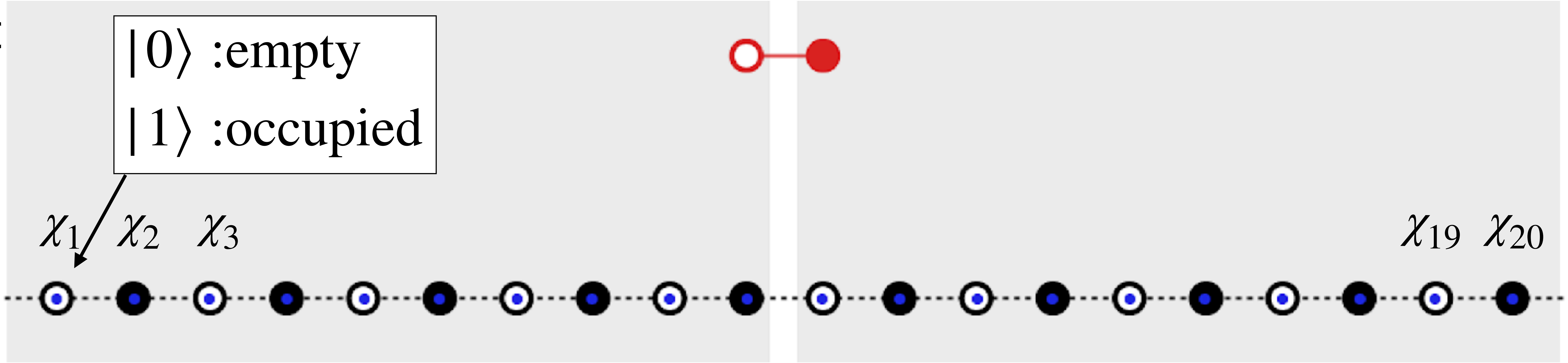
$$\psi(x = a\ n) \quad \leftrightarrow \quad \frac{1}{\sqrt{a}}\begin{pmatrix}\chi_{2n} \\ \chi_{2n-1}\end{pmatrix} \qquad \{\chi_n^\dagger, \chi_m\} = \delta_{nm}, \quad \{\chi_n^\dagger, \chi_m^\dagger\} = \{\chi_n, \chi_m\} = 0\,.$$

Kogut-Susskind

Hamiltonian

1+1D Sc



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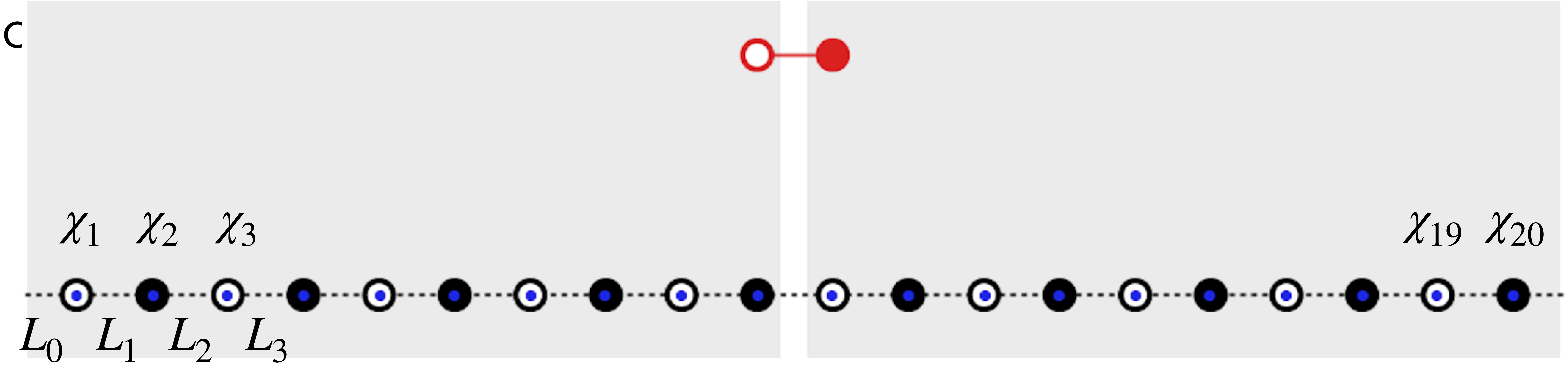
Jordan-Wigner

$$\chi_n = \frac{X_n - iY_n}{2}\prod_{m=1}^{n-1}(-iZ_m)$$

$$X_n \equiv I \otimes \cdots \otimes I \otimes \underset{n^{\text{th}}}{\sigma_x} \otimes I \otimes \cdots \otimes I$$

Hamiltonian

1+1D Sc



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gauge field fixed by Gauss' law: $\partial_1 E - g\,\bar{\psi}\gamma^0\psi = j^0_{\mathrm{ext}}$

$E(x = an) \quad \leftrightarrow \quad L_n$

$$L_n - L_{n-1} - \frac{Z_n + (-1)^n}{2} = \frac{1}{g}\int_{(n-1/2)a}^{(n+1/2)a} \mathrm{d}x\, j^0_{\mathrm{ext}}(x,t)\ ,$$

Hamiltonian

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$$H(t) = \frac{1}{4a}\sum_{n=1}^{N-1}(X_n X_{n+1} + Y_n Y_{n+1}) + \frac{m}{2}\sum_{n=1}^{N}(-1)^n Z_n + \frac{a\,g^2}{2}\sum_{n=1}^{N-1} L_n^2(t) \,.$$

initial state: vacuum $H_{\text{Schwinger}} | \psi(t=0) \rangle = E_0 | \psi(t=0) \rangle$

$$\frac{\partial}{\partial t} | \psi(t) \rangle = -i \left(H_{\text{Schwinger}} + H_{\text{source}}(t) \right) | \psi(t) \rangle$$

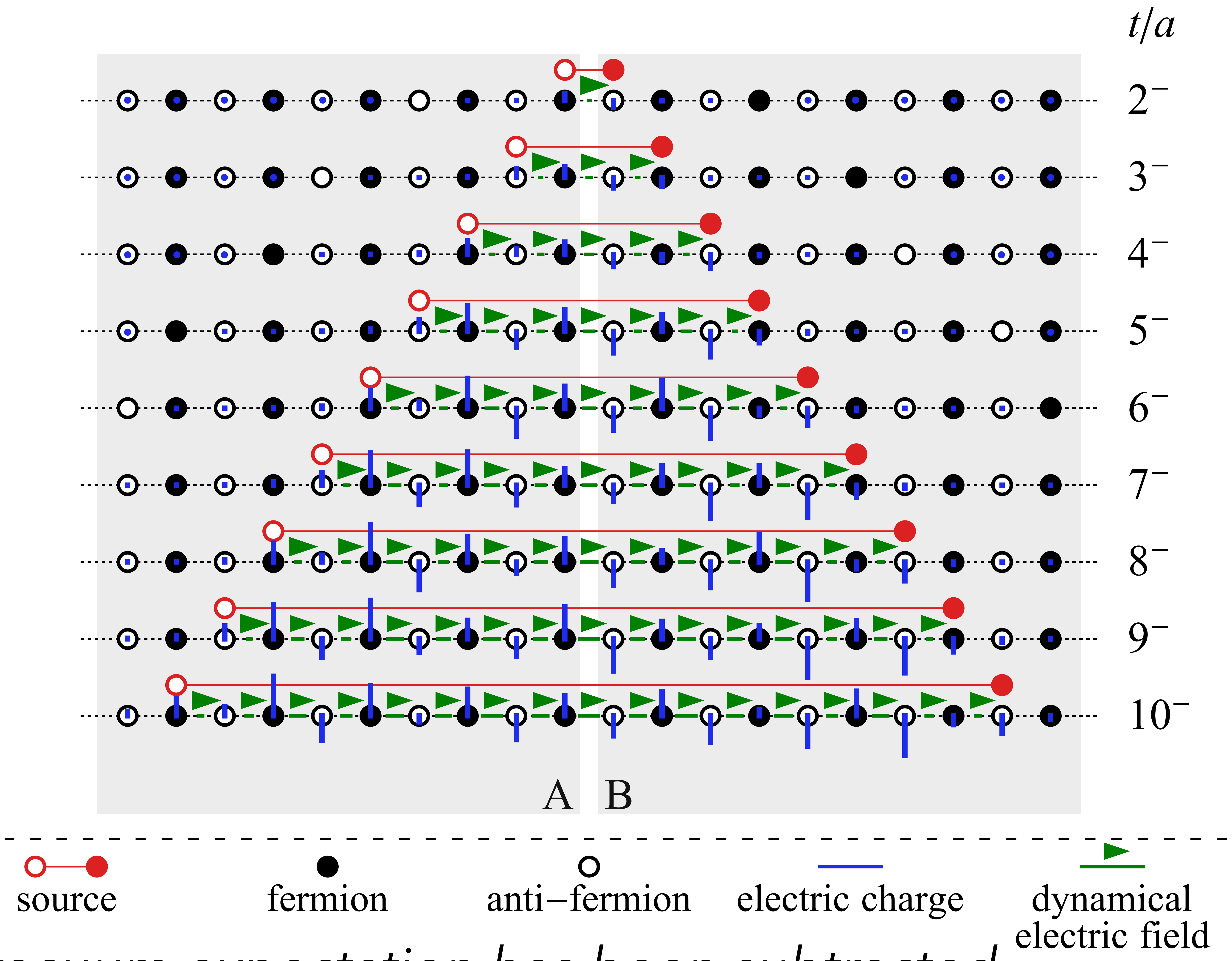
charge density: $q_{n,t} \equiv \langle \psi^\dagger(a\,n) \psi(a\,n) \rangle_t = \dfrac{\langle Z_n \rangle_t + (-1)^n}{2a},$

scalar condensate density: $\nu_{n,t} \equiv \langle \bar{\psi}(a\,n) \psi(a\,n) \rangle_t = \dfrac{(-1)^n \langle Z_n \rangle_t}{2a},$

electric field: $\Pi_{n,t} \equiv \langle E(a\,n) \rangle_t = g \, \langle L_n \rangle_t,$

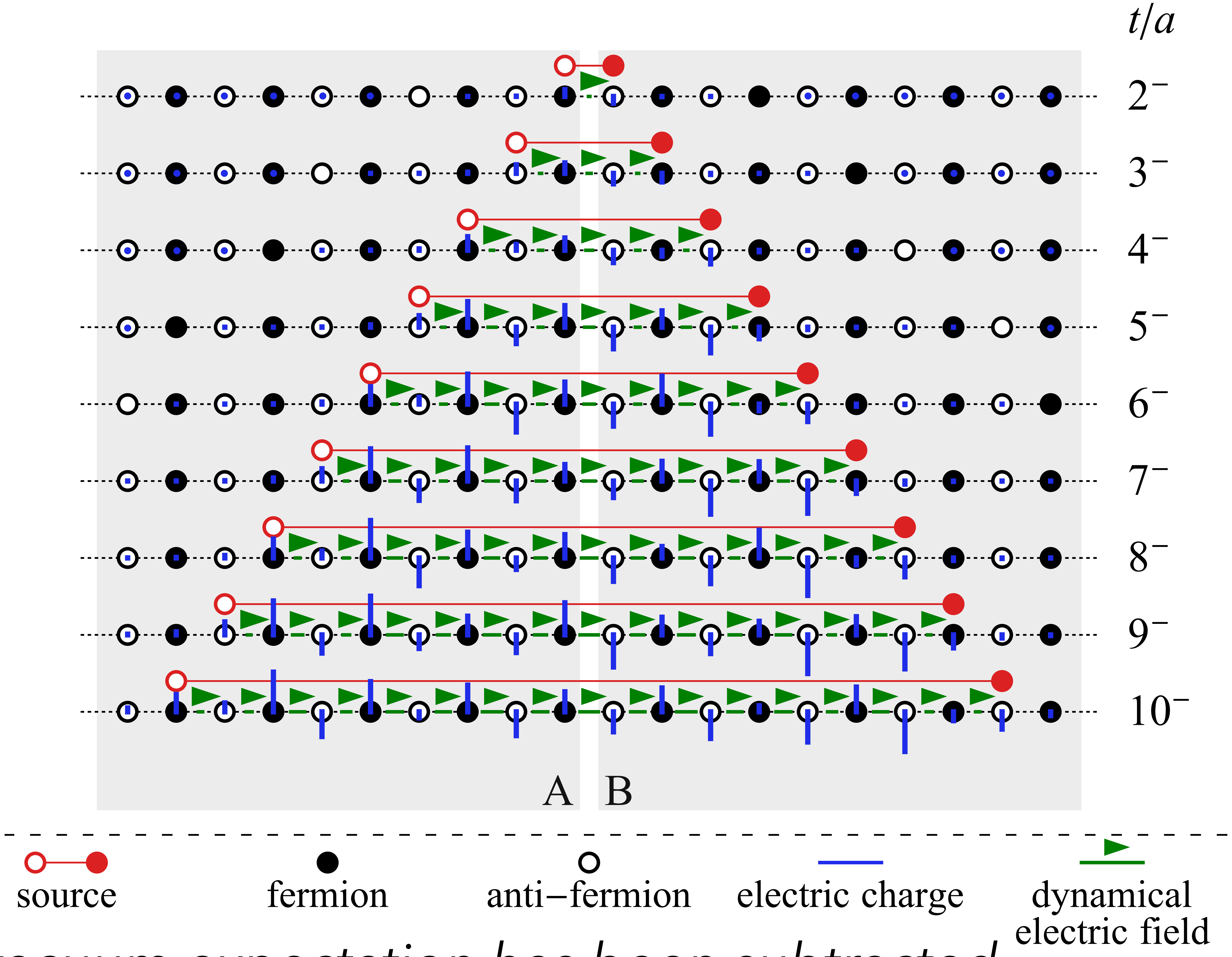
$\vdots$

vacuum modification



vacuum expectation has been subtracted

vacuum modification



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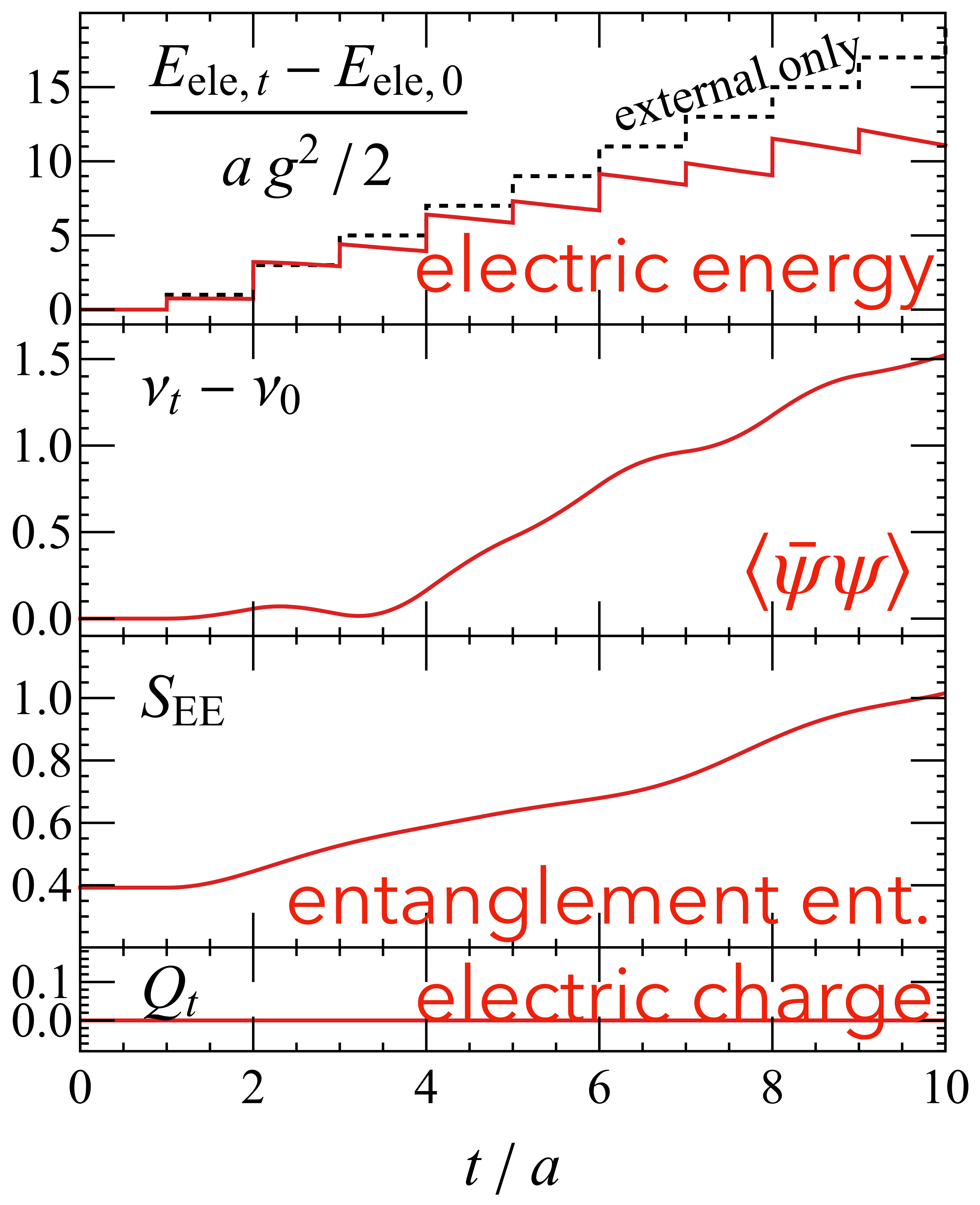
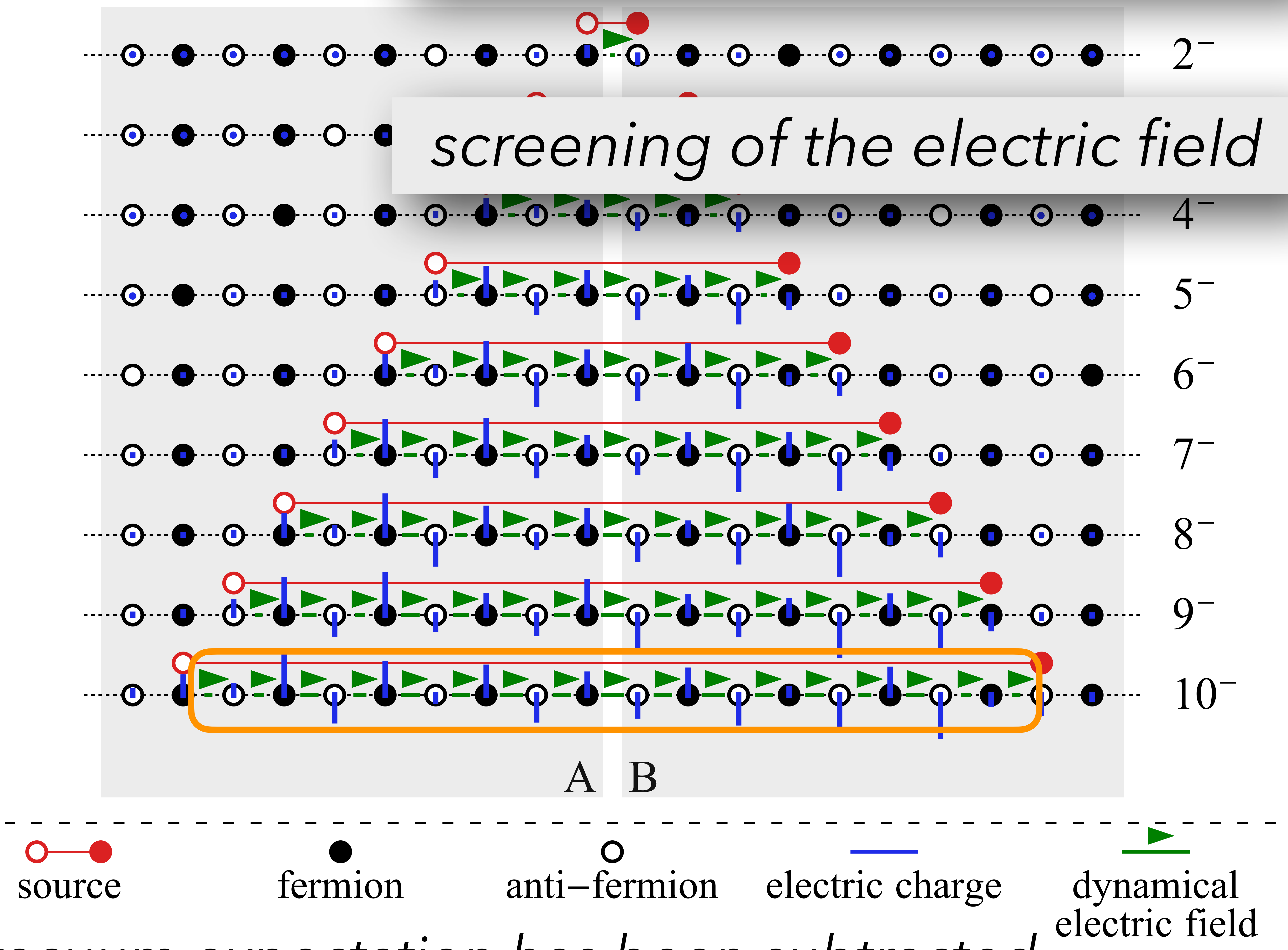


vacuum modification

effects of pair production:

screening of the electric field



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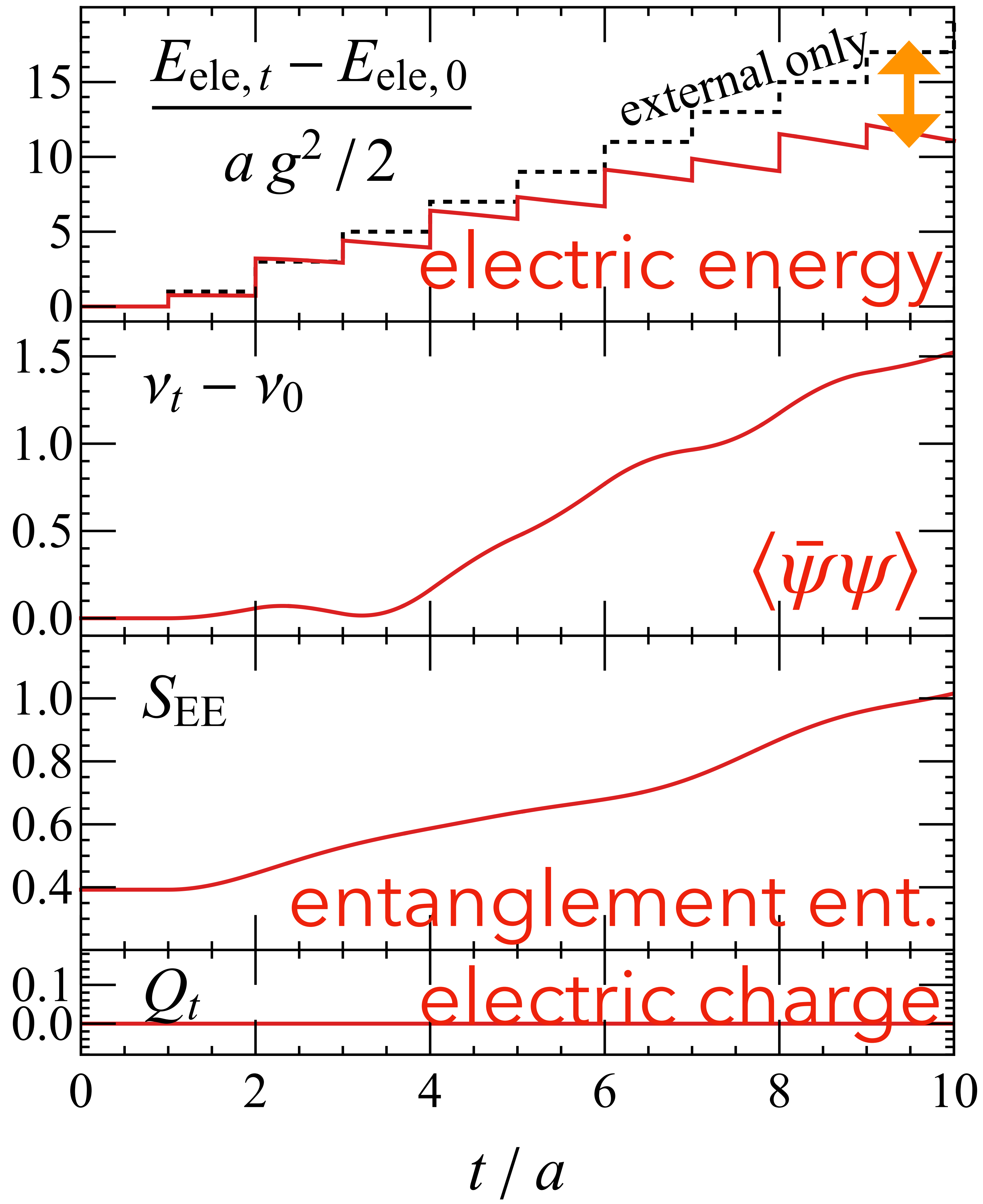

vacuum modification

effects of pair production:

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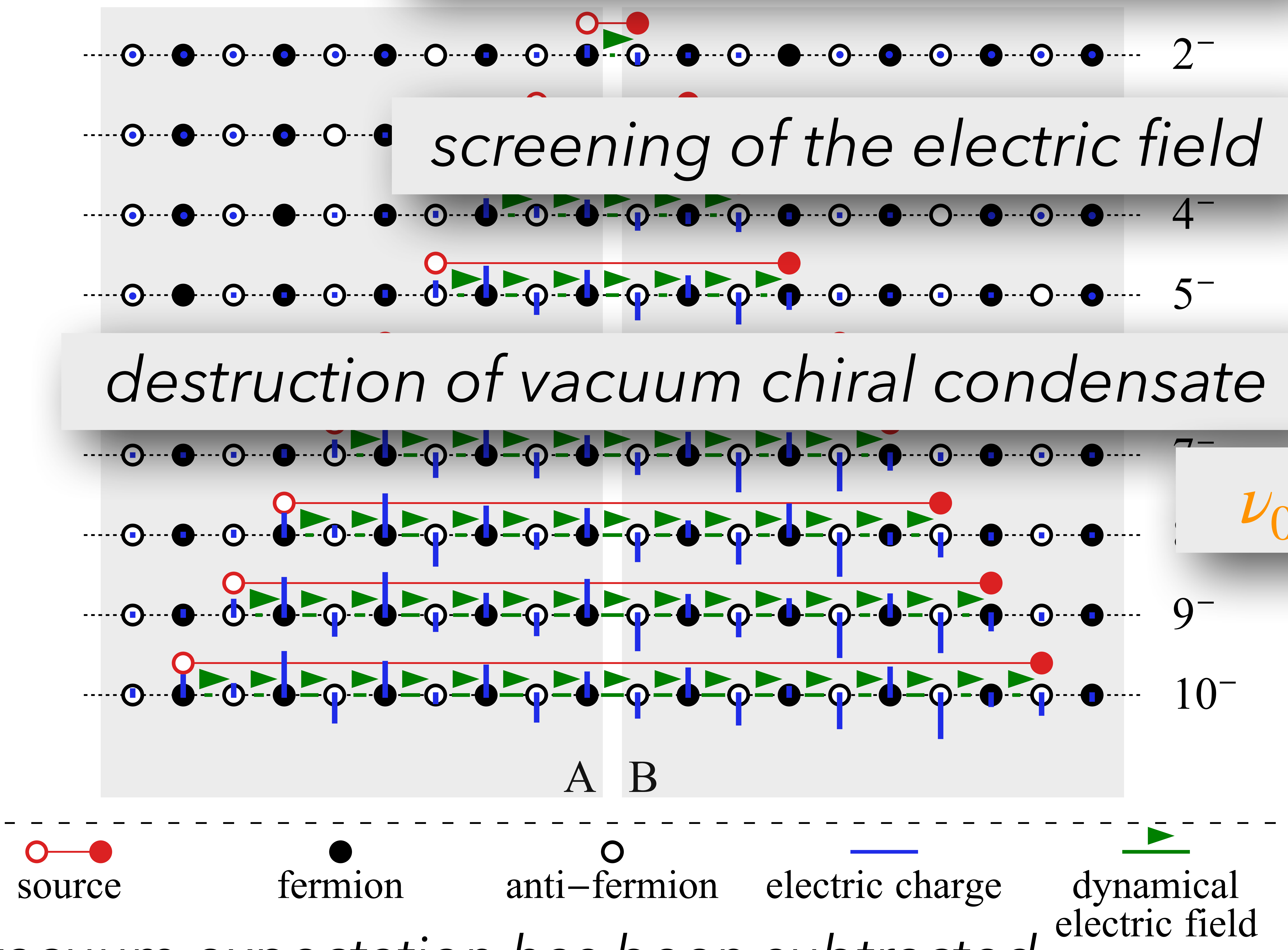
destruction of vacuum chiral condensate

$\nu_0 < 0$



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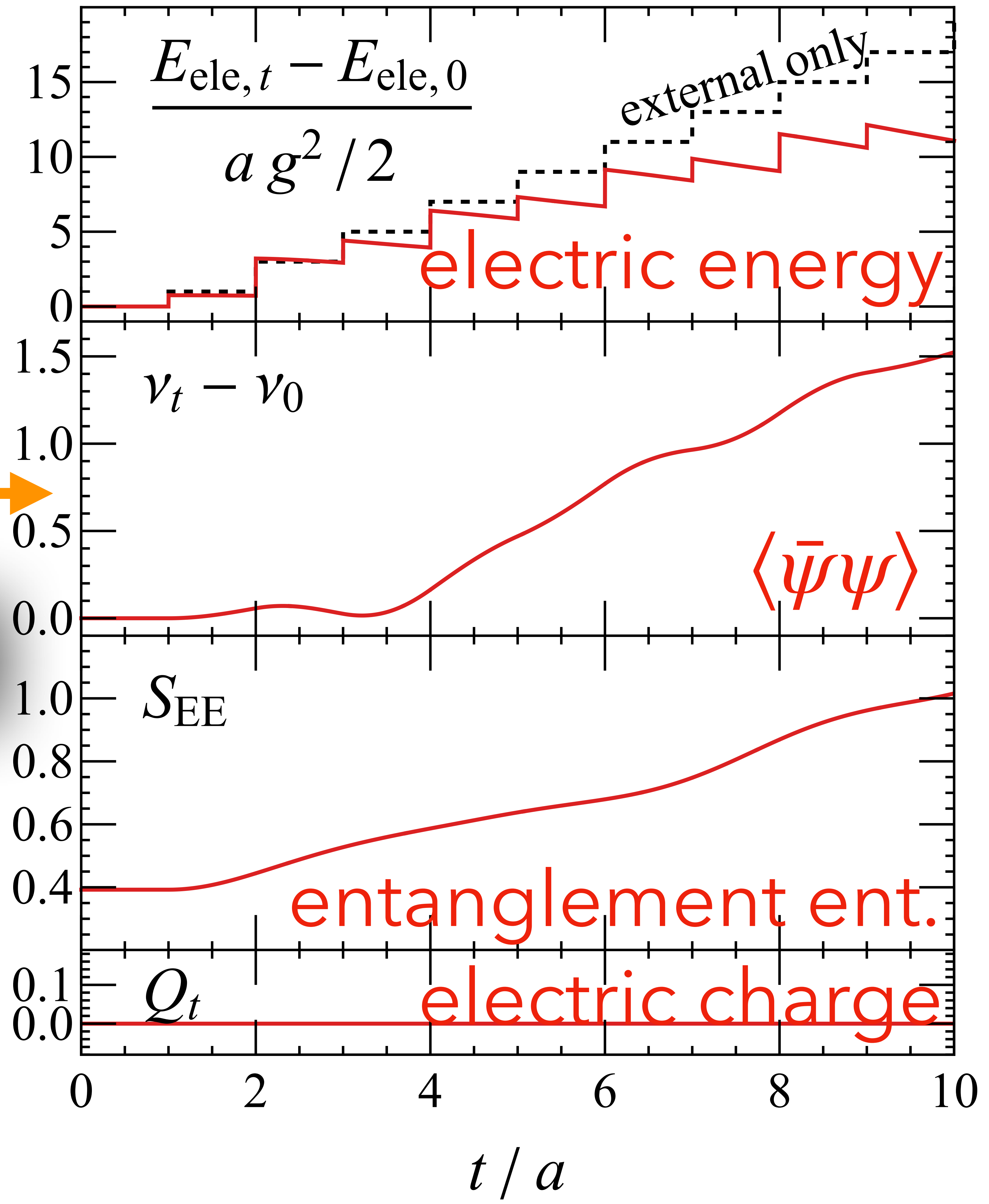

vacuum modification

effects of pair production:

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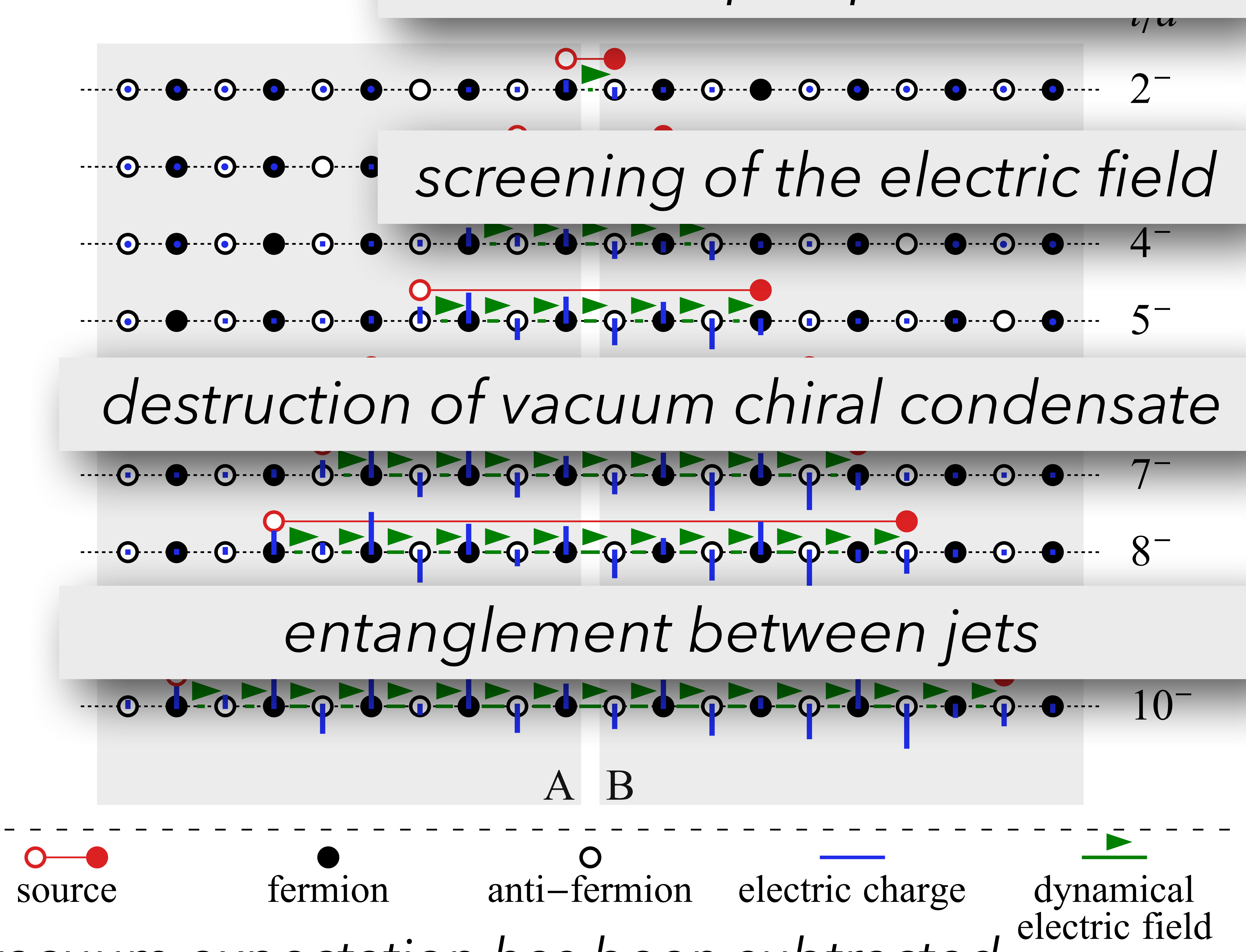
entanglement between jets



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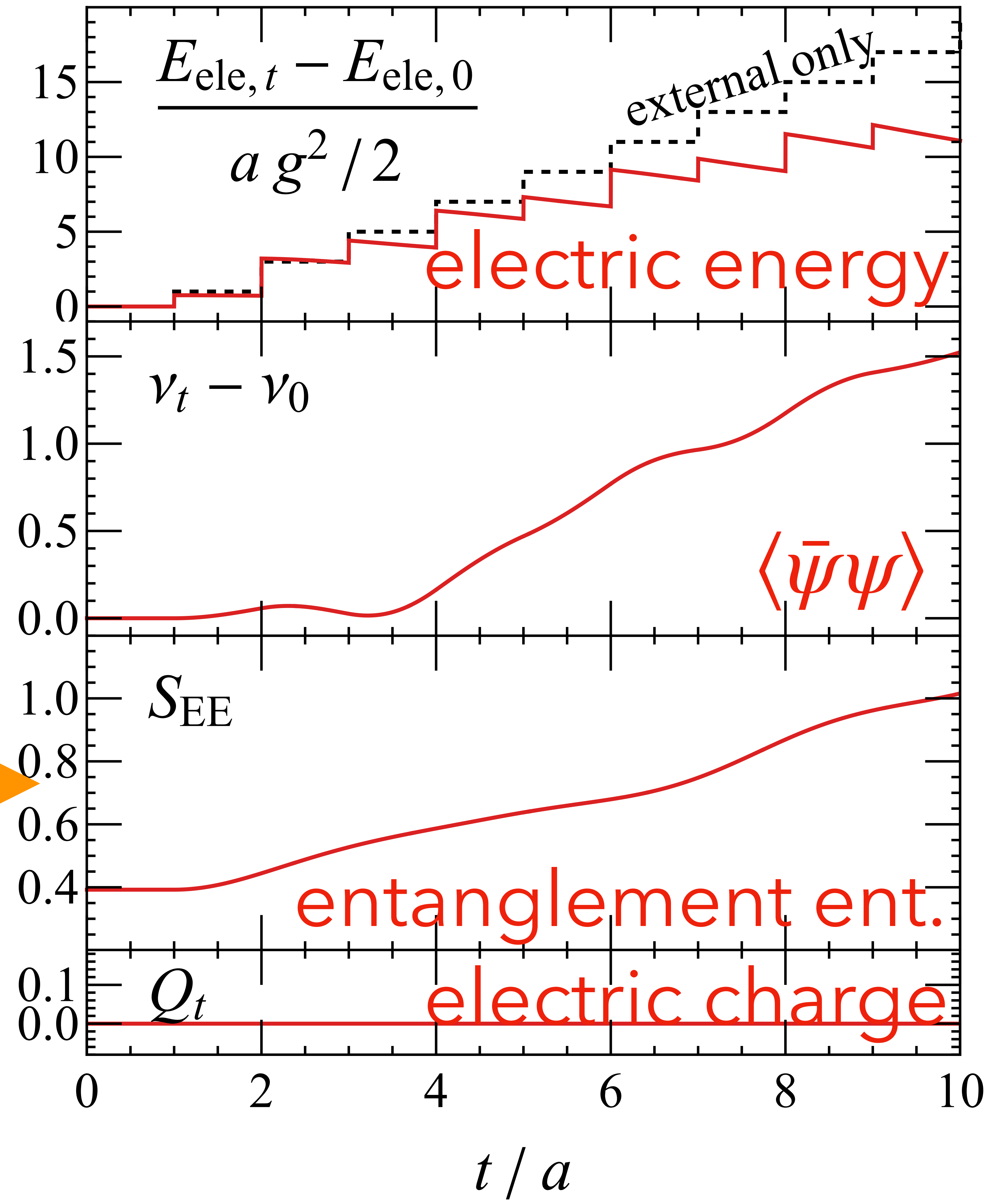
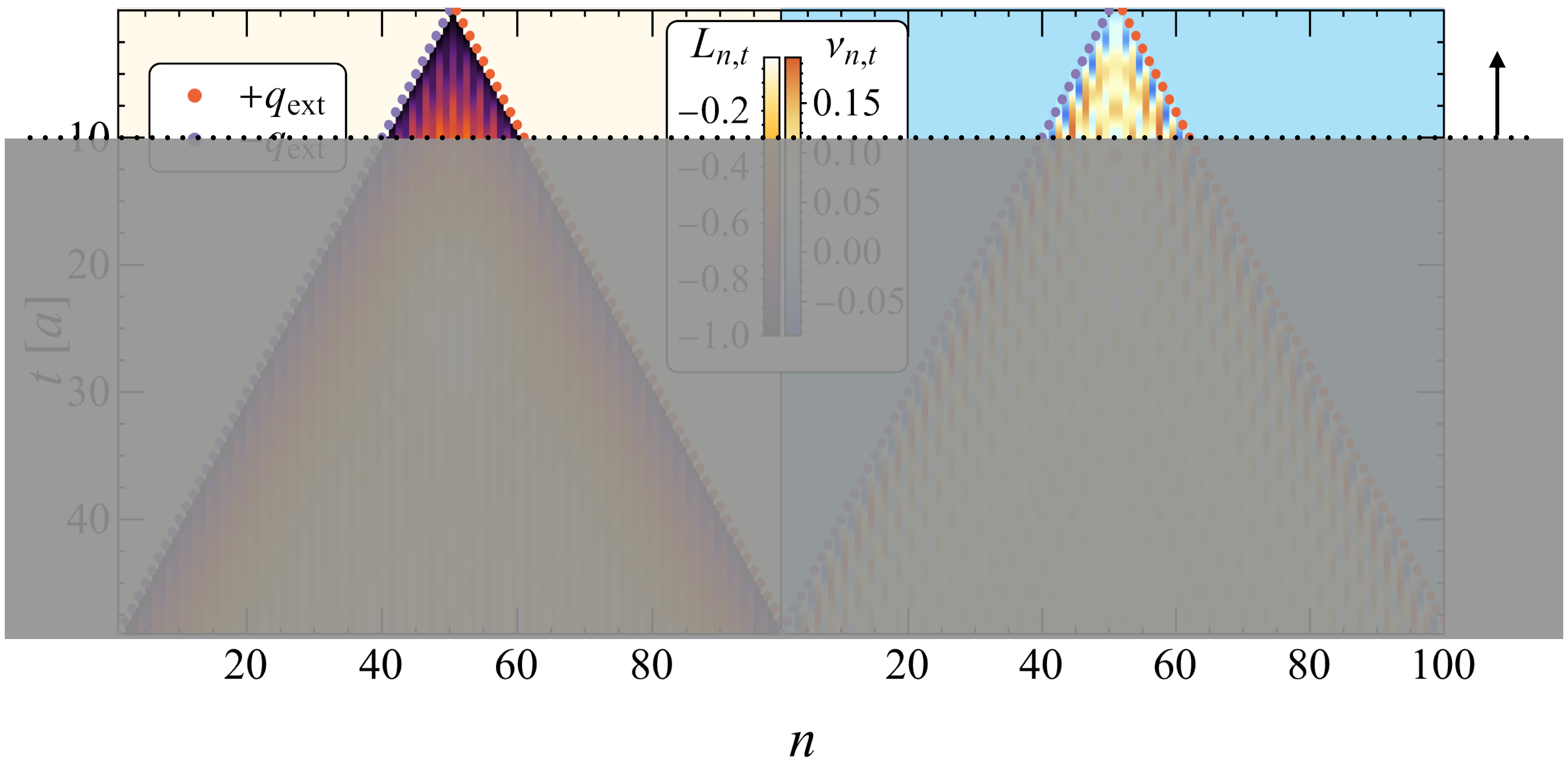


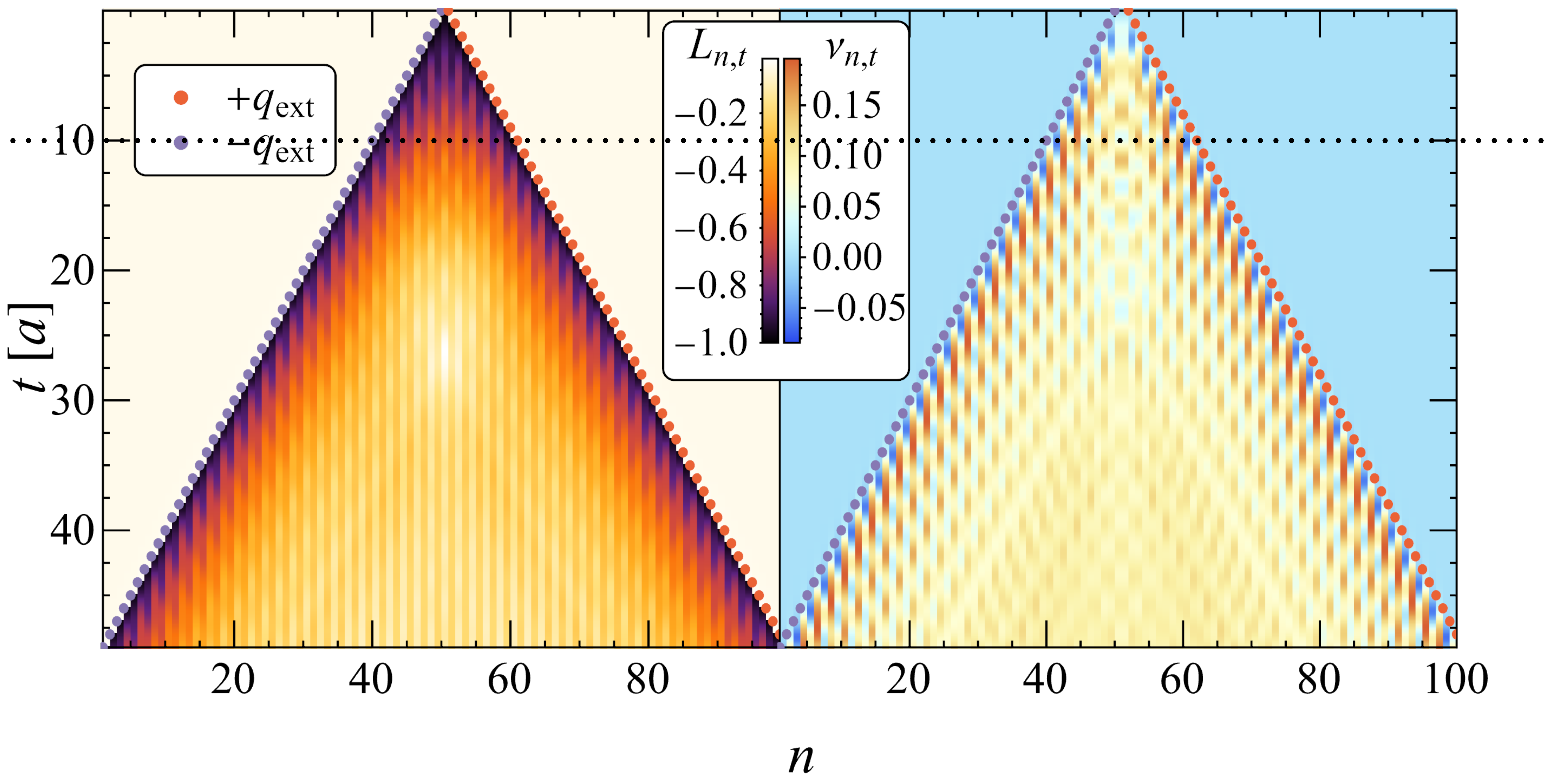
approach to the continuum limit



what have been learned from exact diagonalization (N=20, t<10a)

calculation using Tensor Network (keeping only the most essential states)

approach to the continuum limit



calculation using Tensor Network
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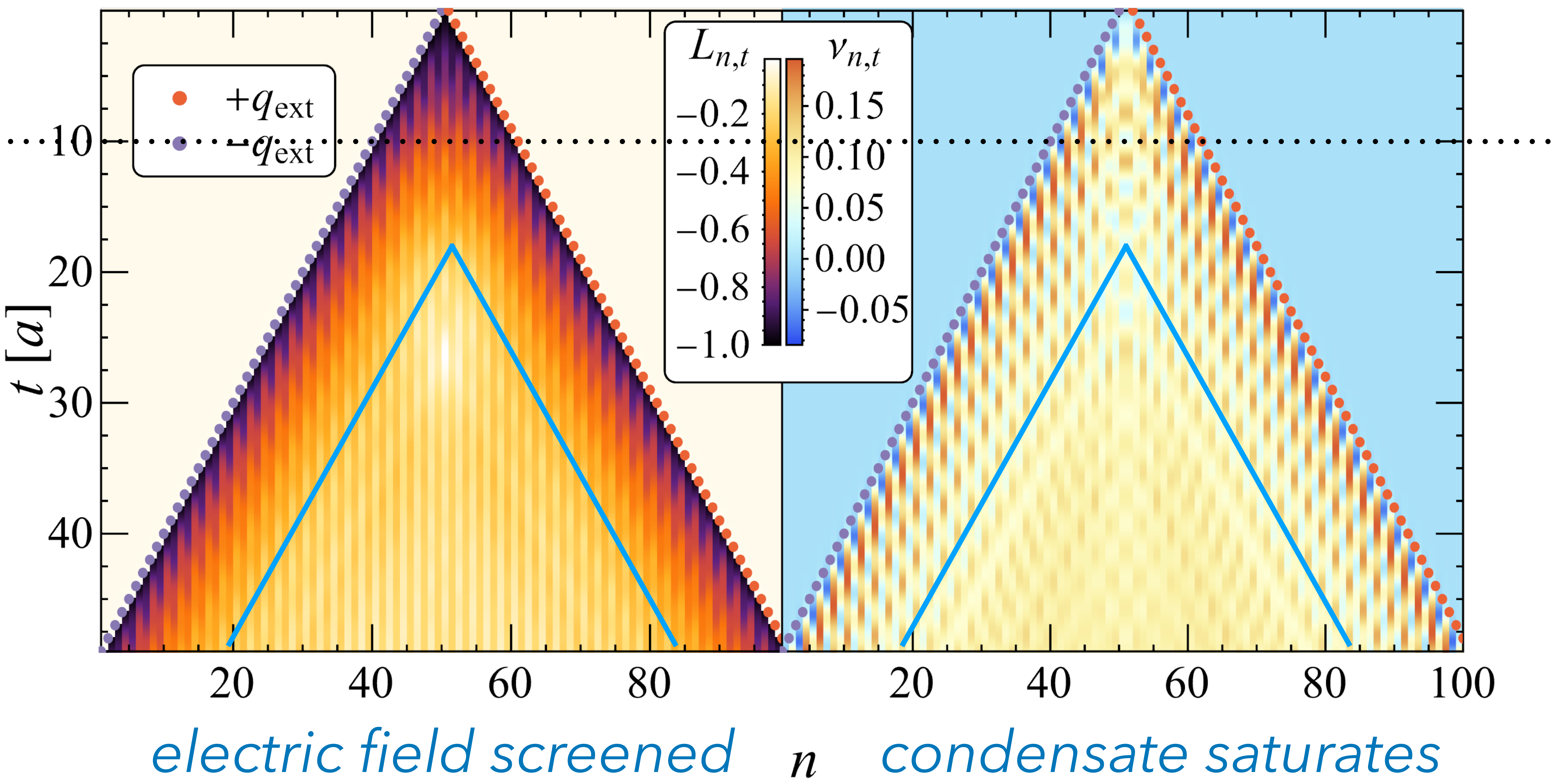
approach to the continuum limit



electric field screened

condensate saturates

calculation using Tensor Network
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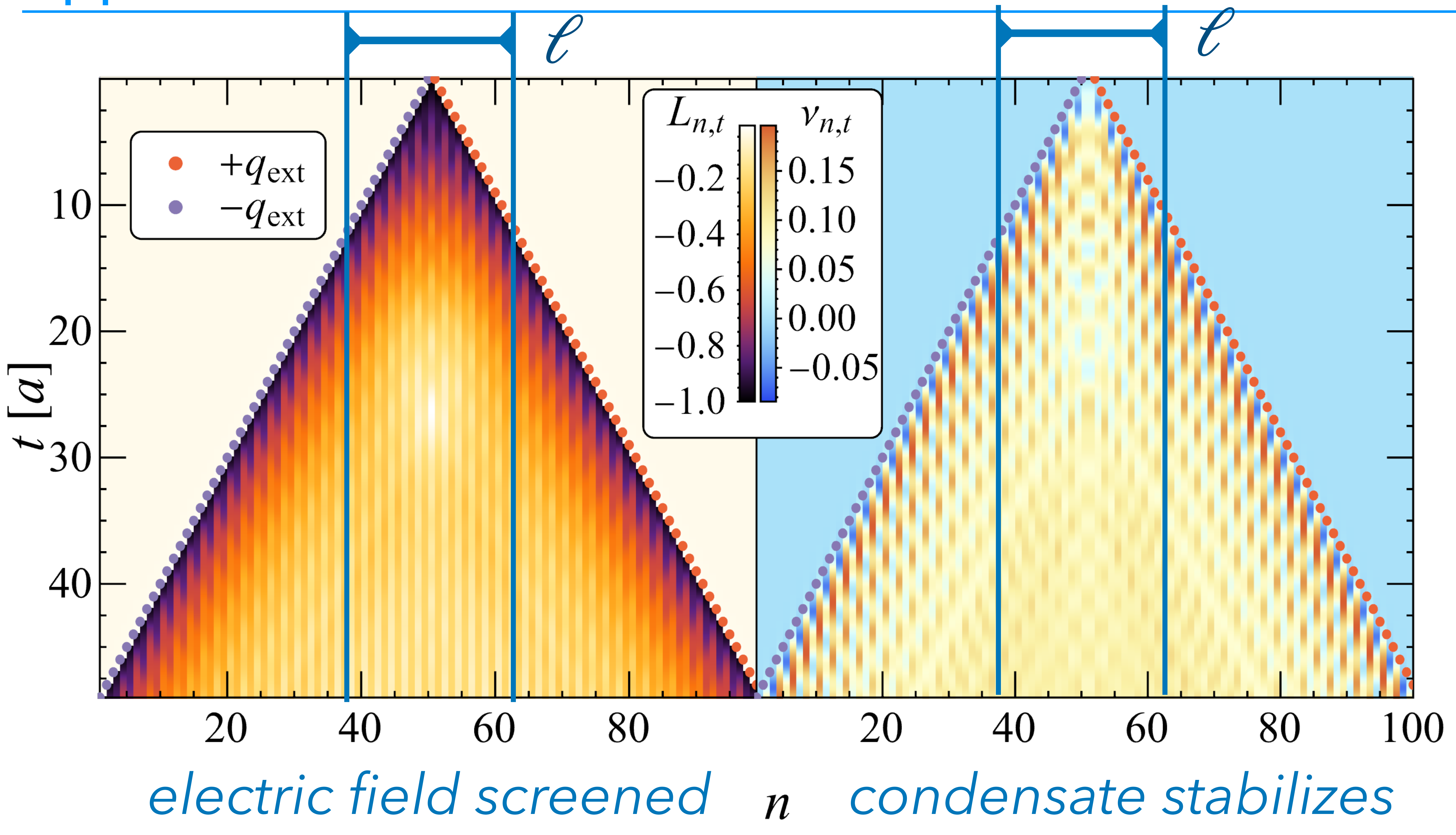
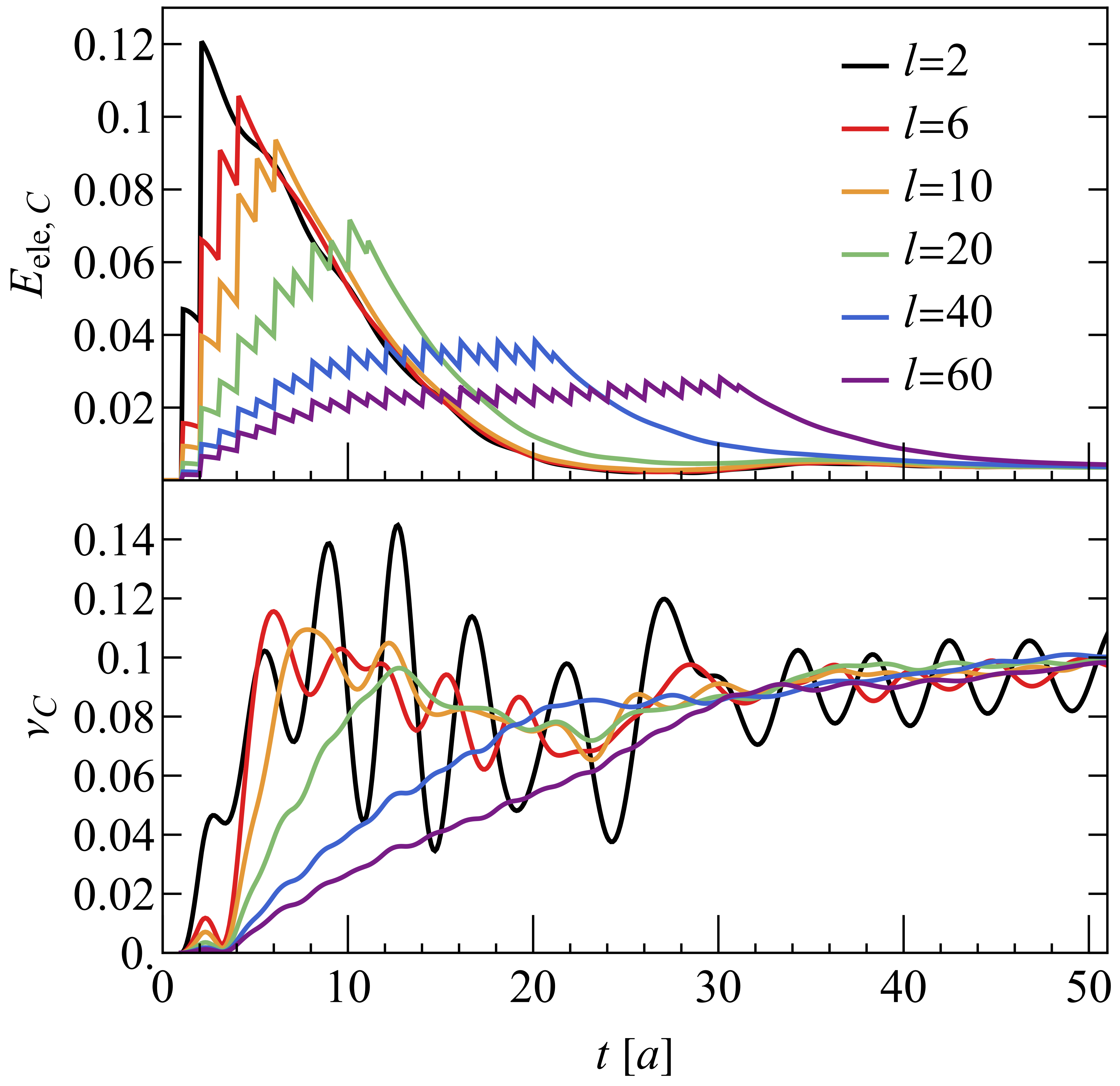
approach to the continuum limit



electric field screened

condensate stabilizes

observables at the center



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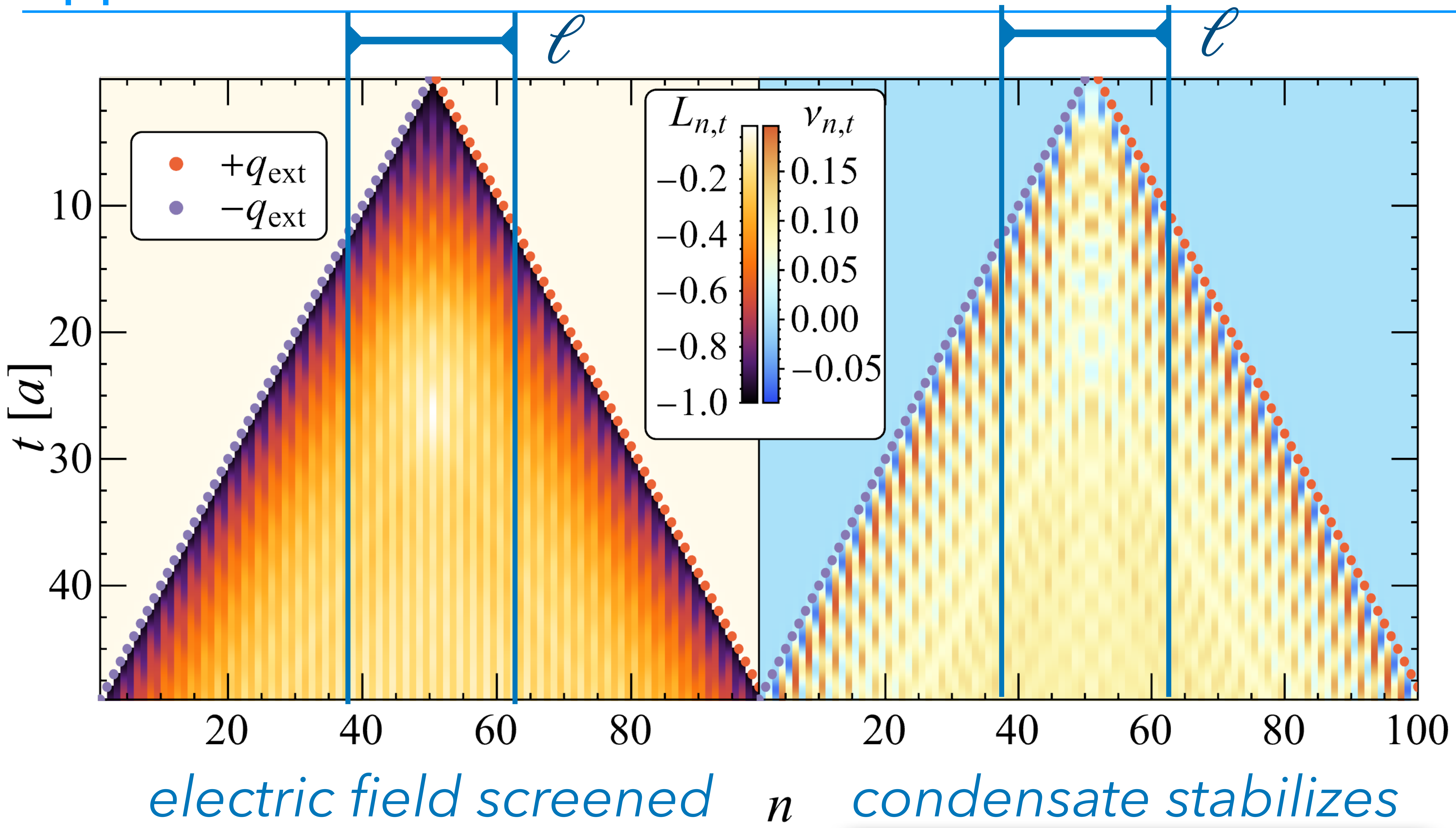

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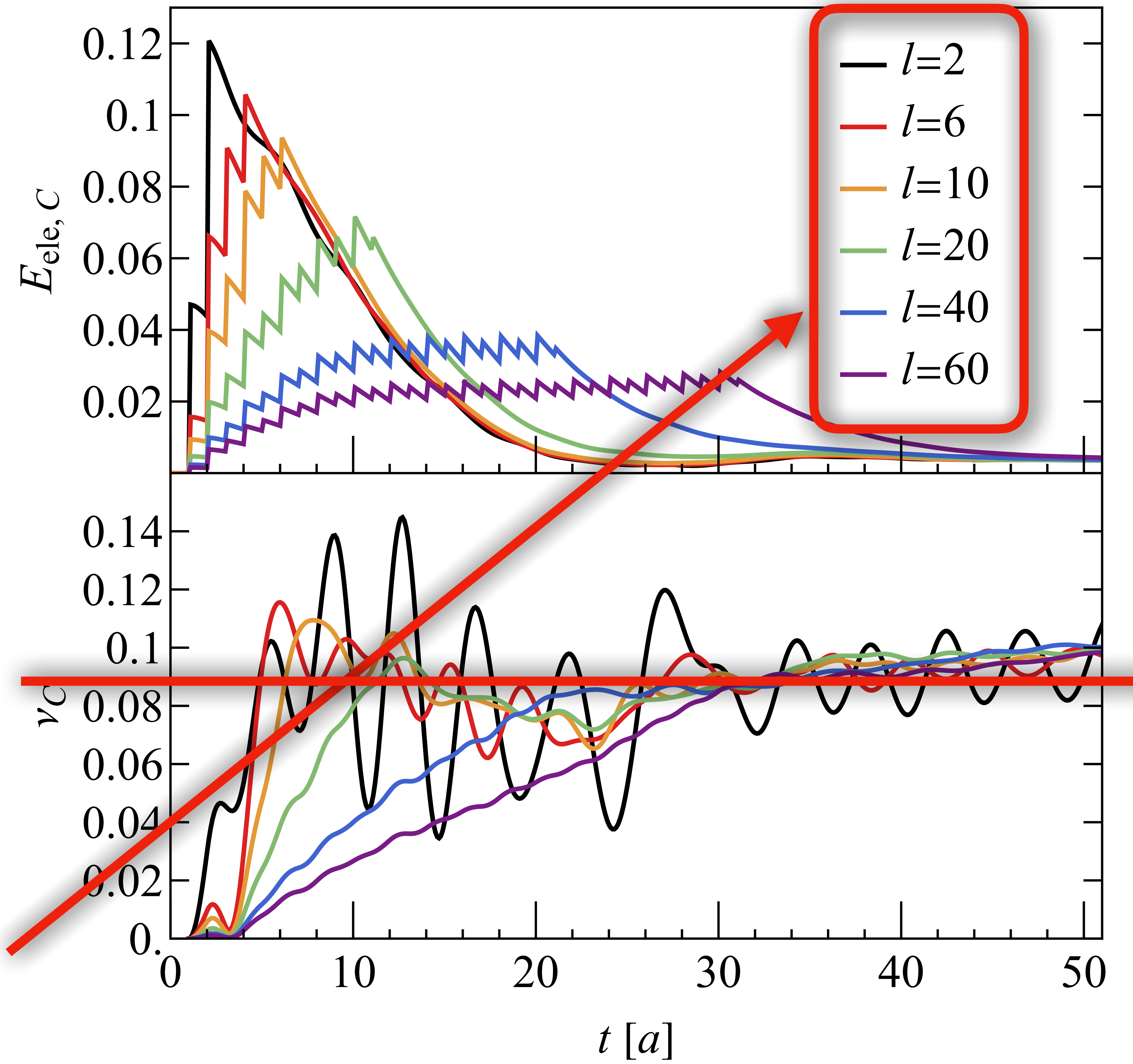


value independent of subsystem length

calculation using Tensor Network
(keeping only the most essential states)

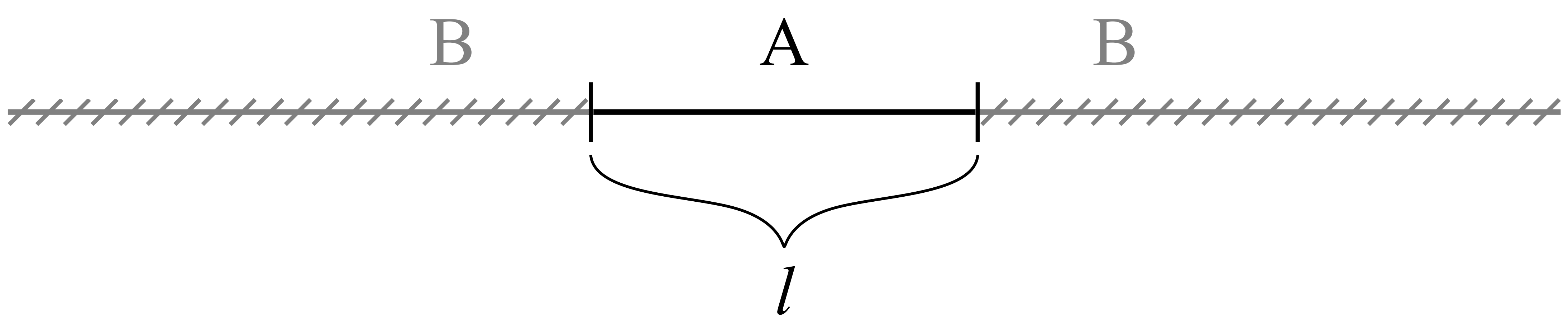
entanglement entropy

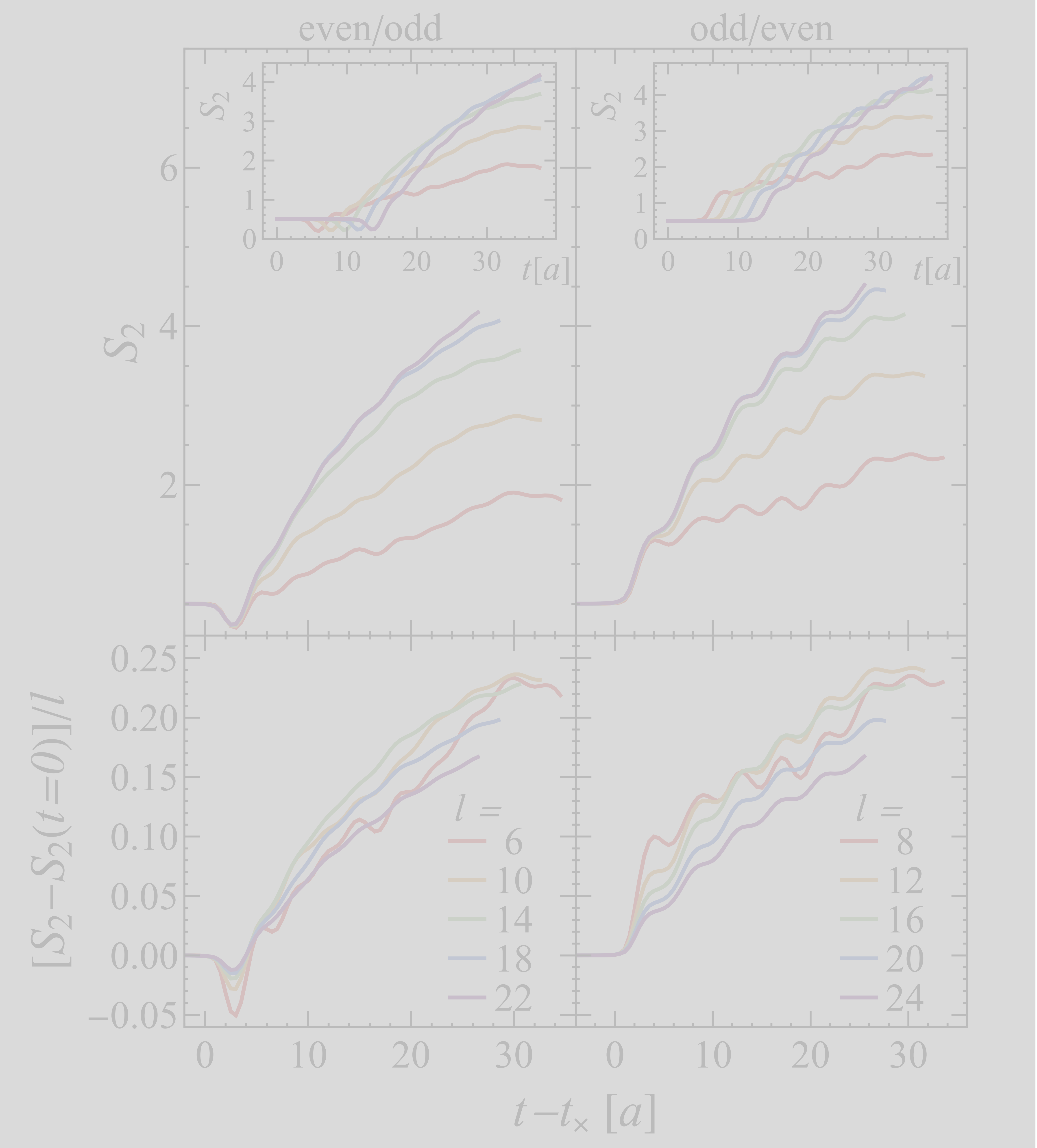
reduced density matrix: $\rho_A = \mathrm{tr}_B\, \rho$

diagonalization: $\rho_A = \sum_i \lambda_i^2 |\Psi_i\rangle\langle\Psi_i|$

entropies: $S_{\mathrm{vN}} = -\sum_i \lambda_i^2 \ln \lambda_i^2$

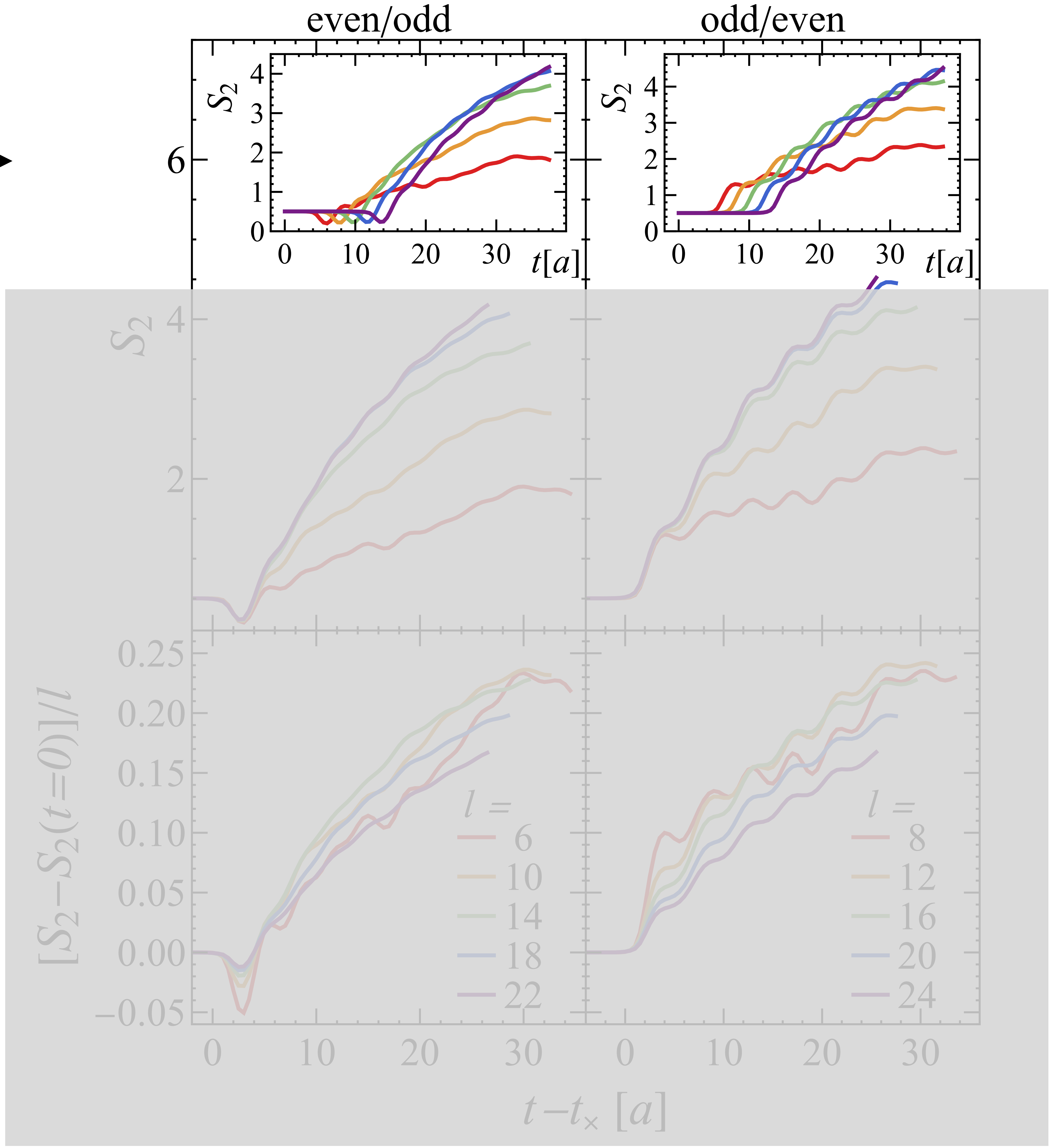
$$S_\alpha = -\frac{\ln \sum_i \ln \lambda_i^{2\alpha}}{1-\alpha}$$



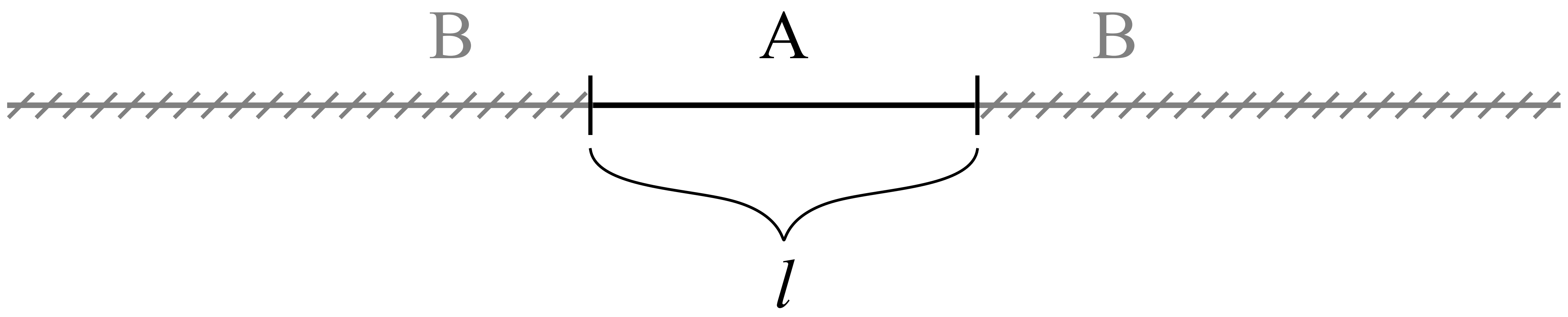


entanglement entropy

"raw" result



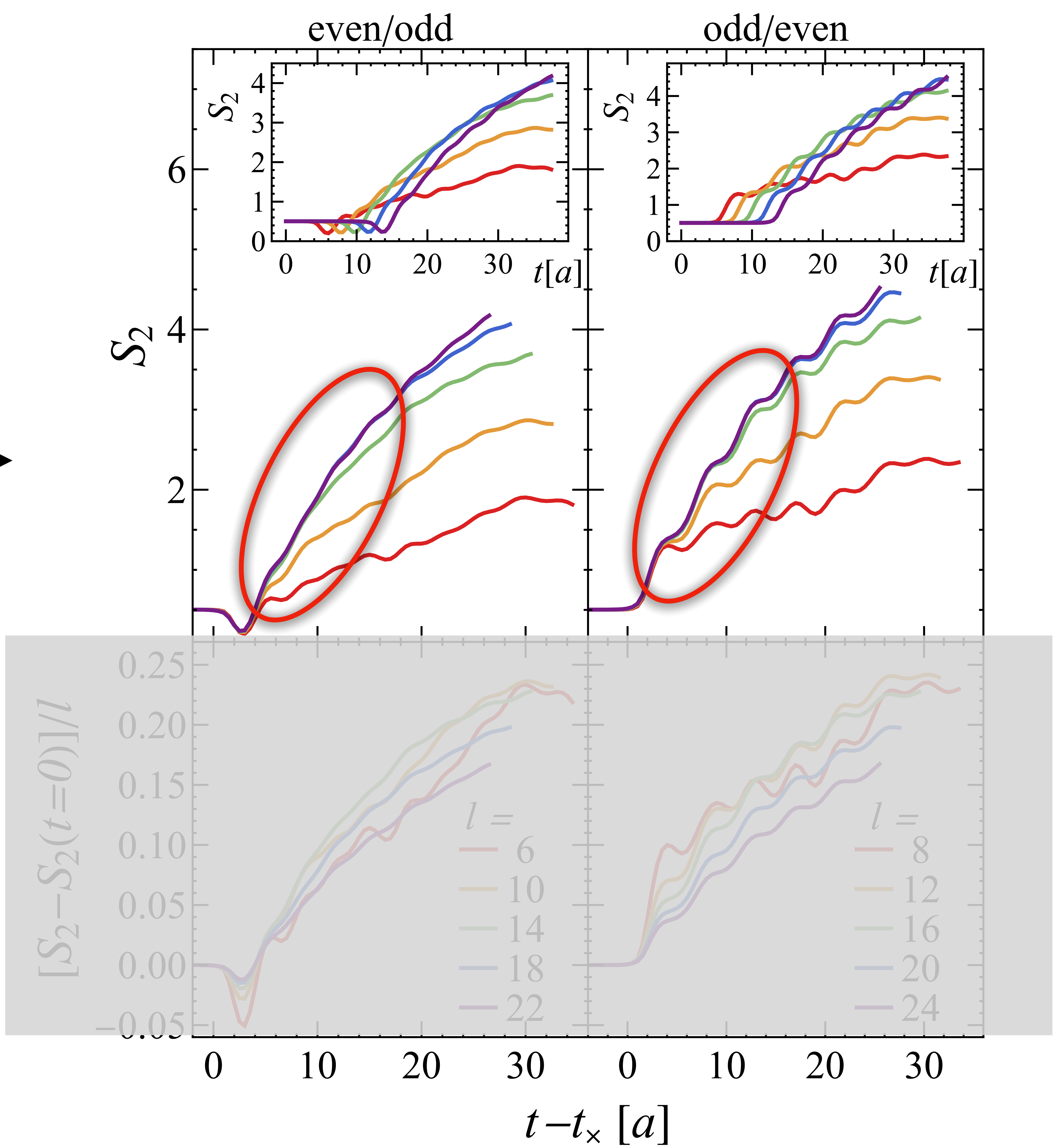
increasing of (entanglement) entropy

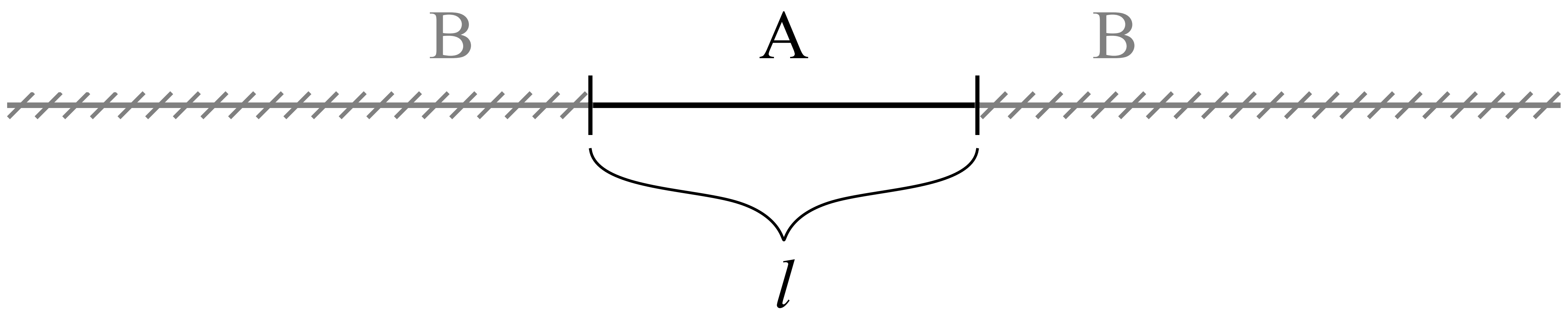


entanglement entropy

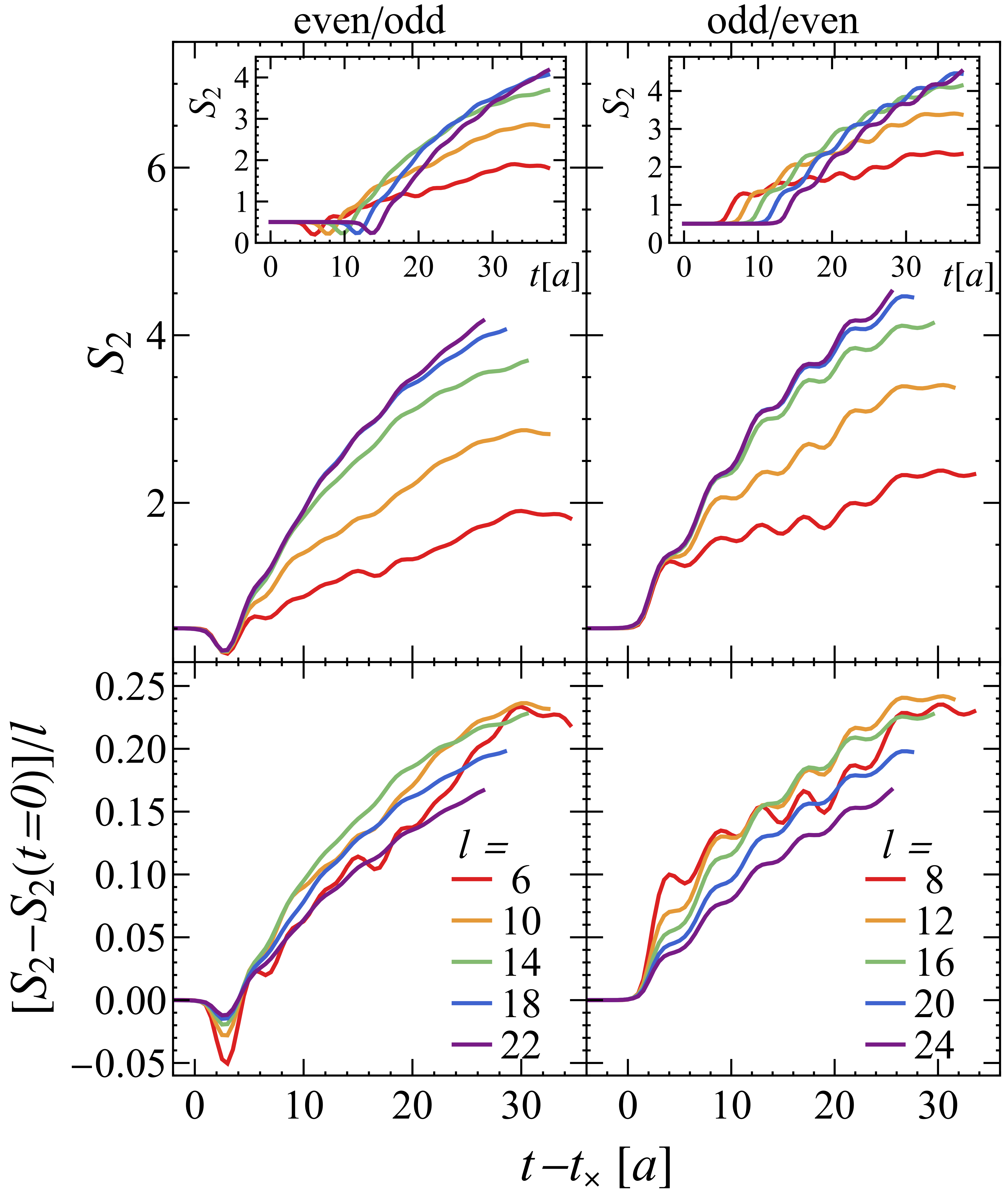
shift horizontally by the time that the jets pass the boundary

the same early-time rising



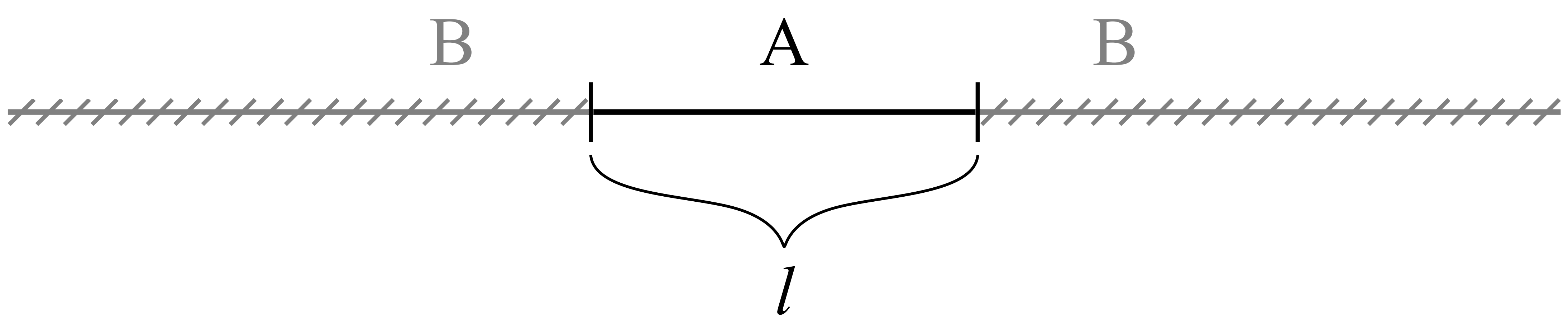


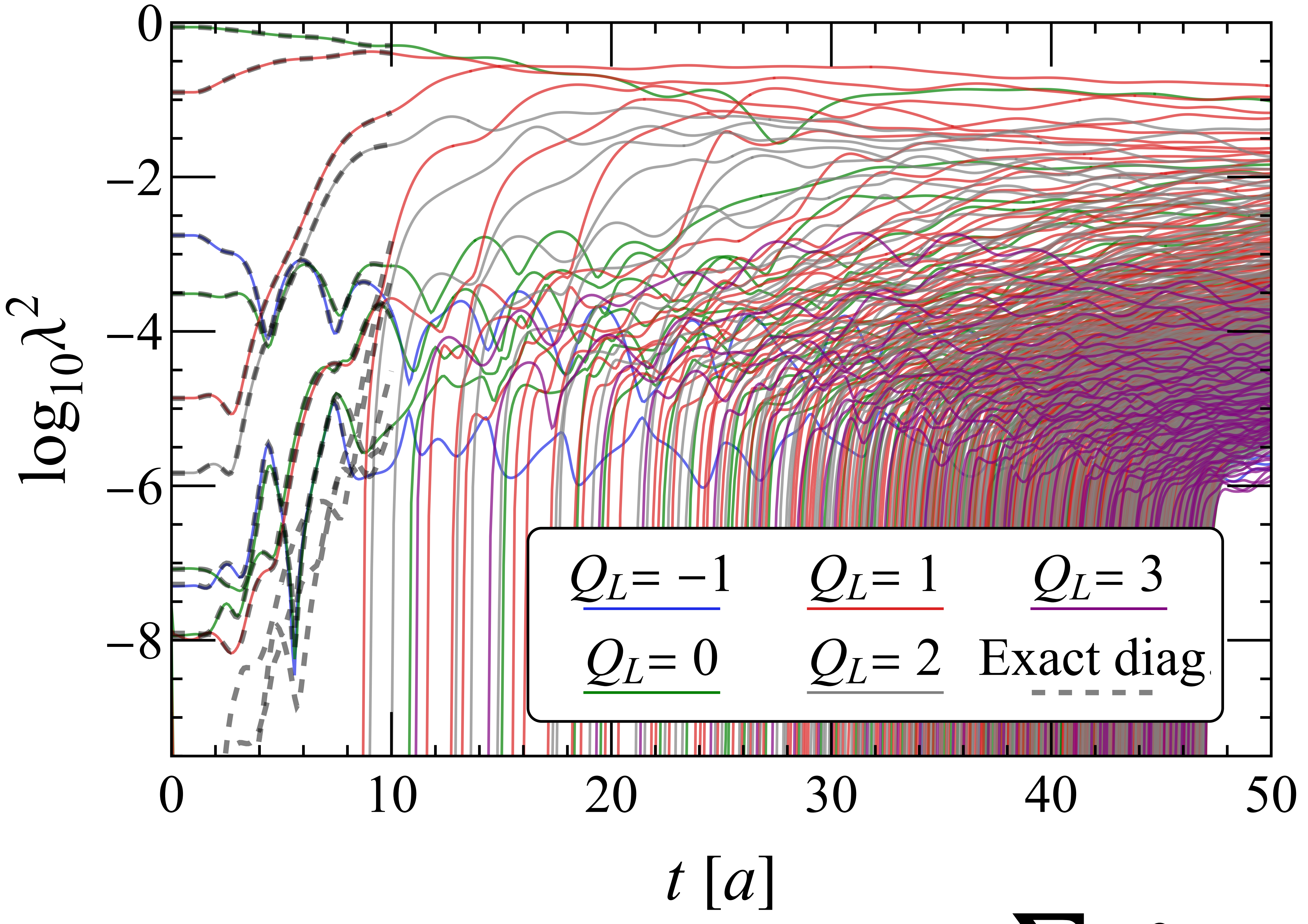
entanglement entropy

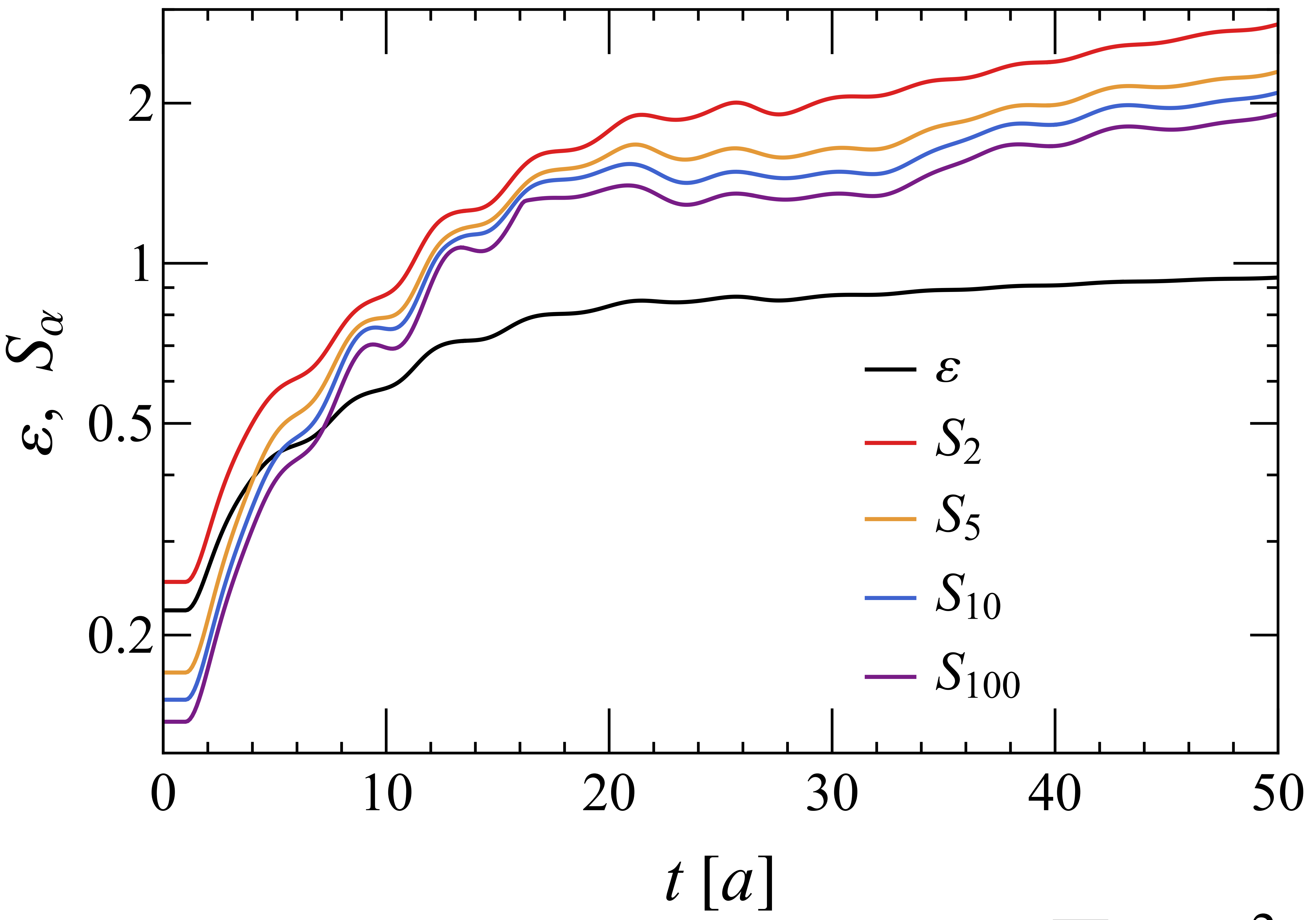


entropy proportional to volume!

entropy scaled by subsystem size →







$$\rho_A = \sum_i \lambda_i^2 |\Psi_i\rangle\langle\Psi_i|$$

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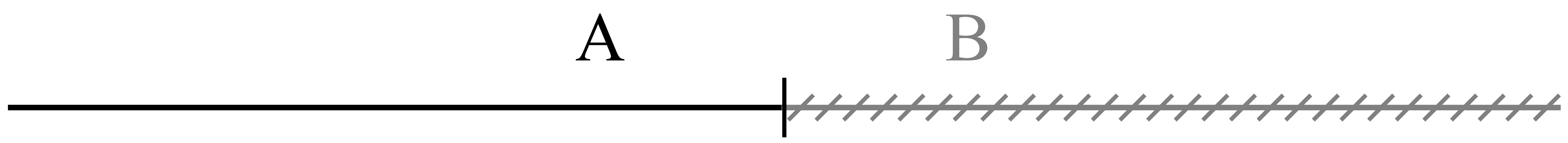

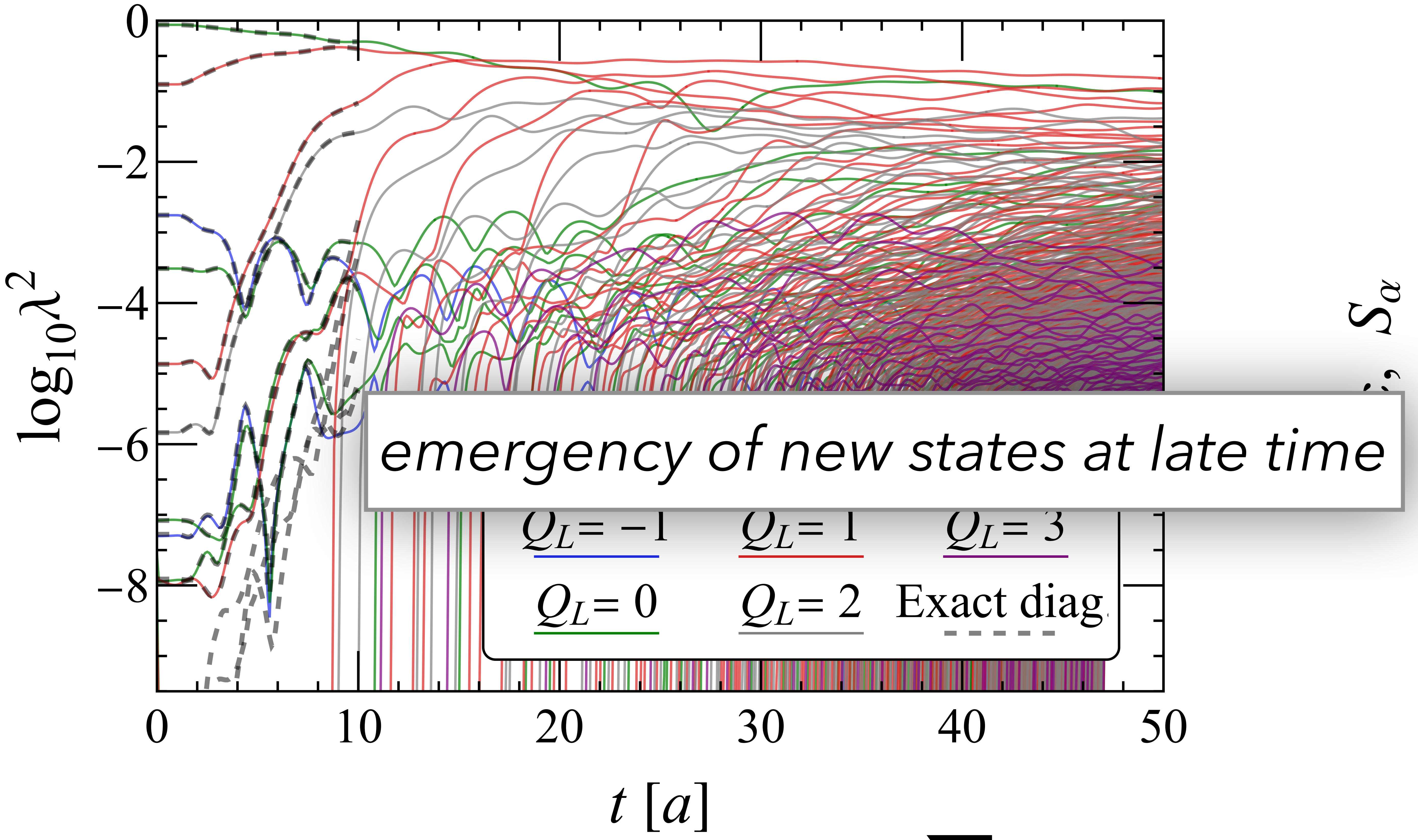

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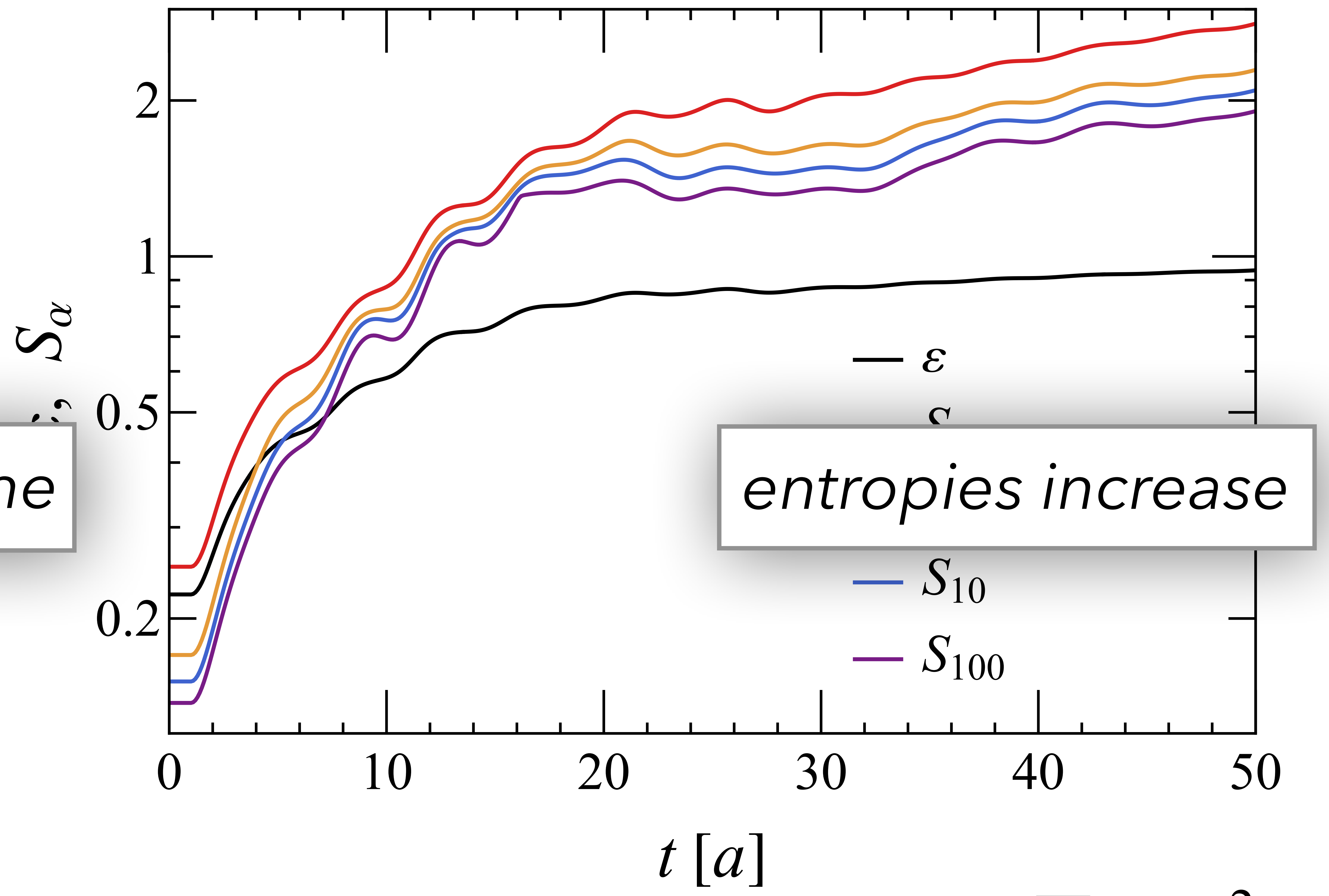

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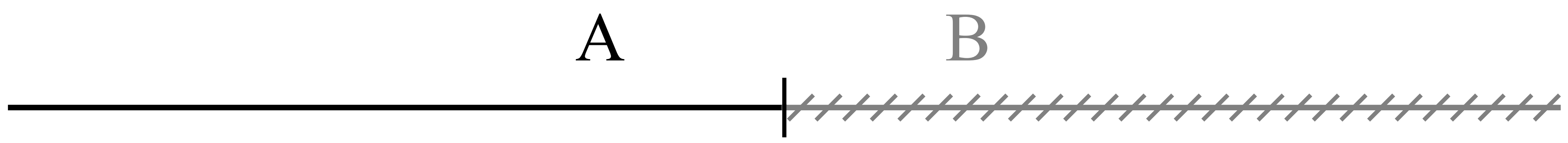
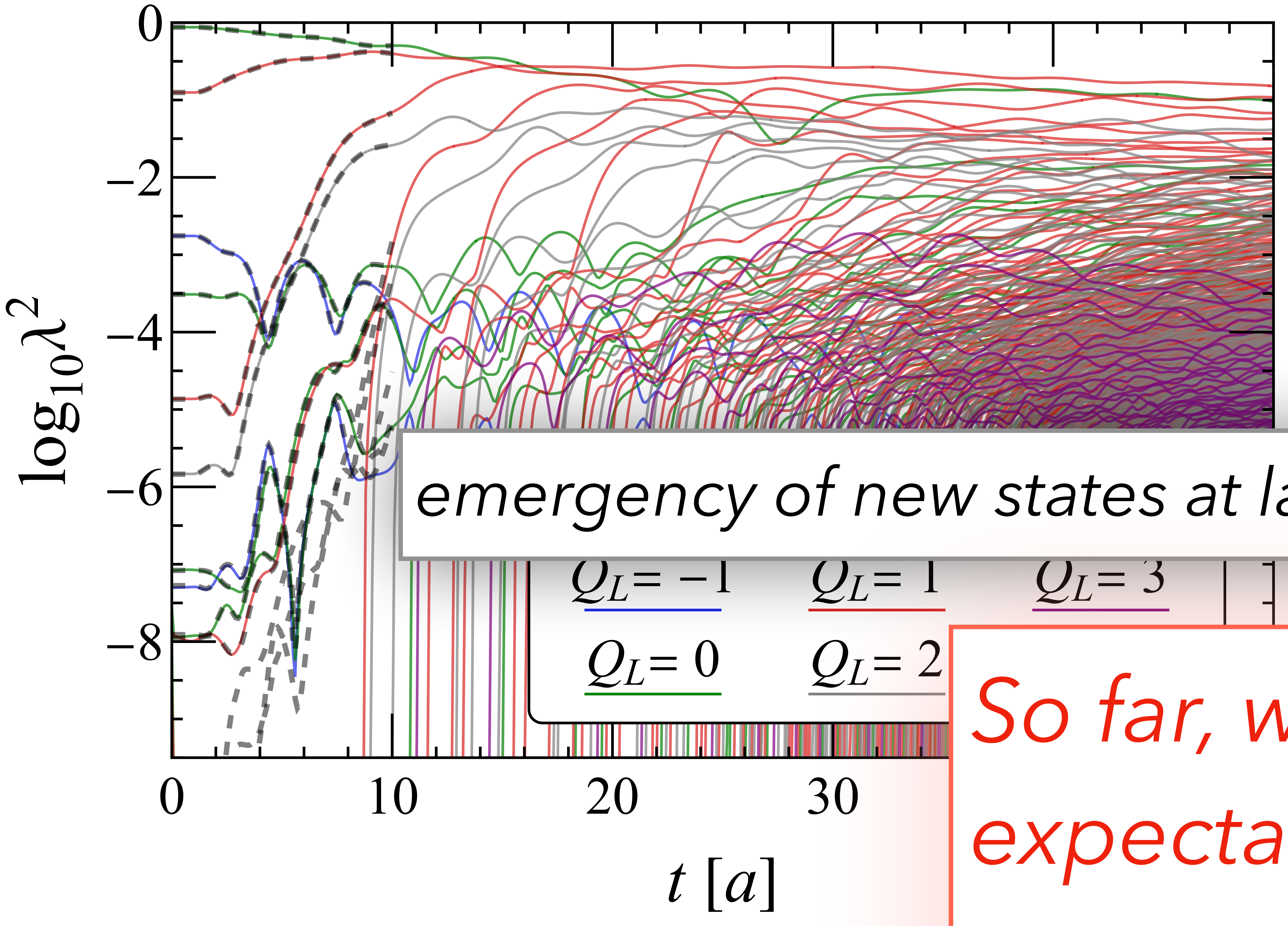
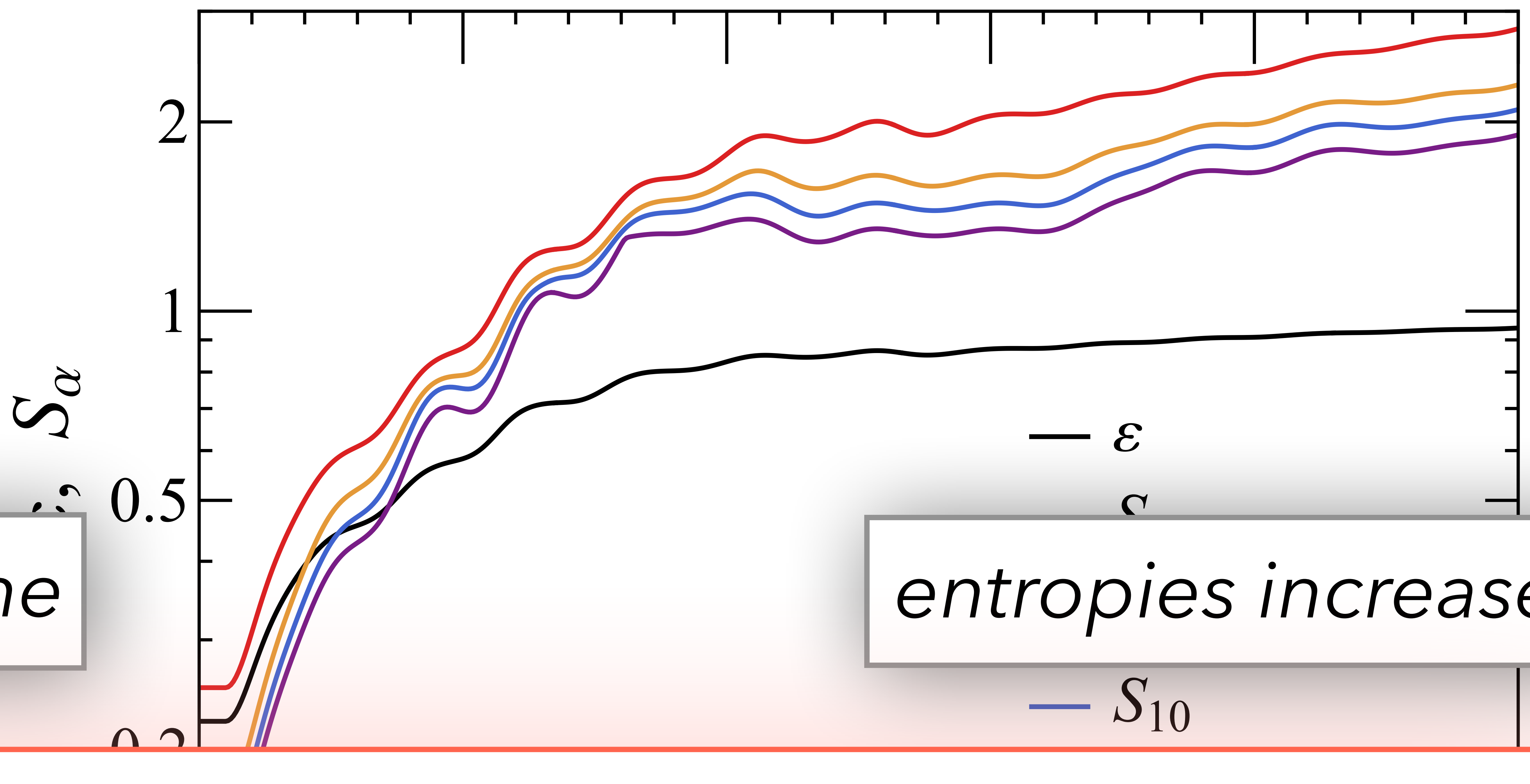


entanglement entropy

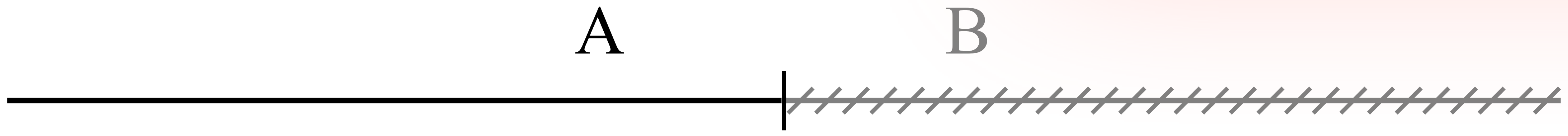


emergency of new states at late time



entropies increase

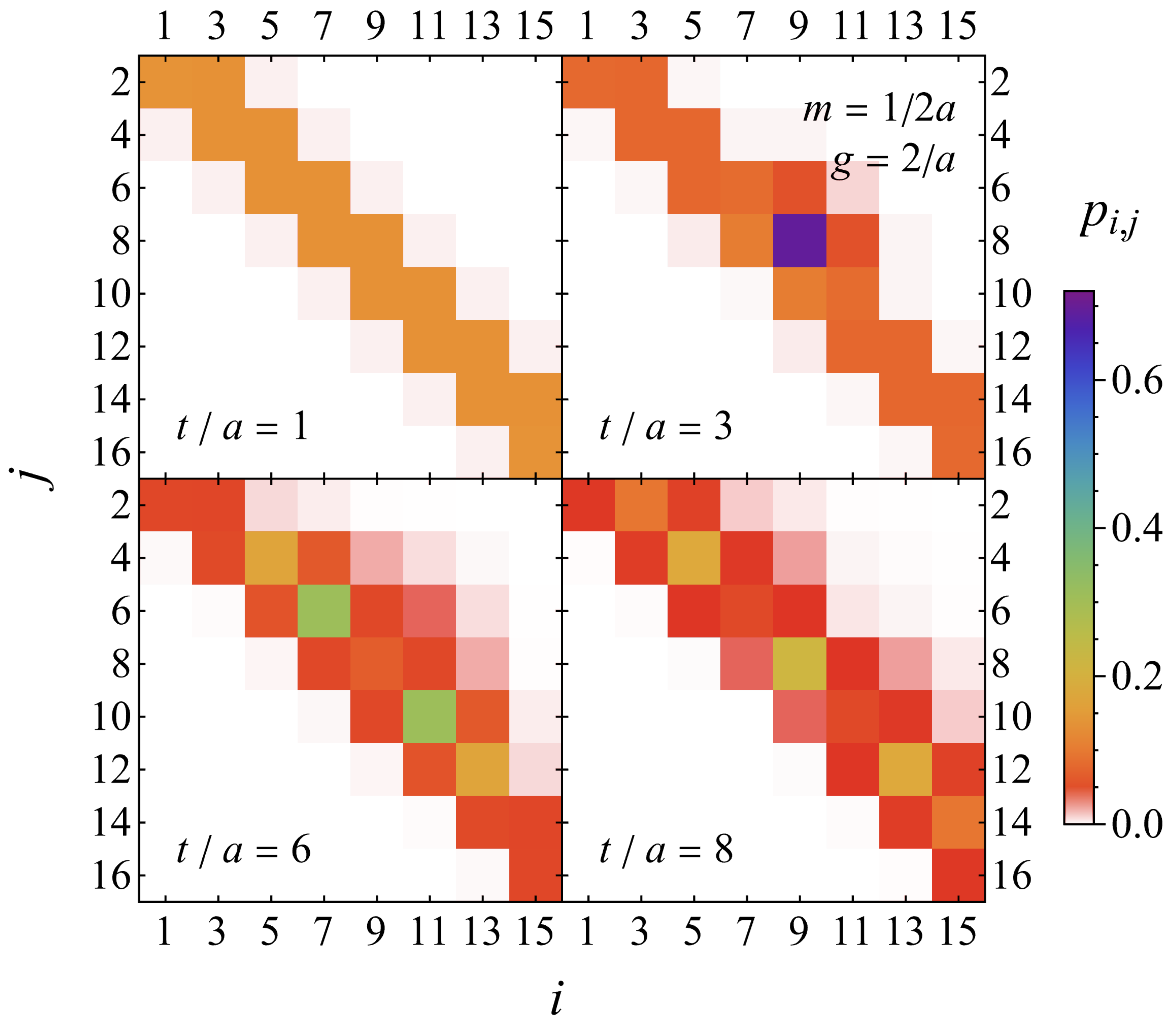
ρ_A =



So far, what have been observed fit the expectation of thermalization.

What are there in the "thermalized" state?

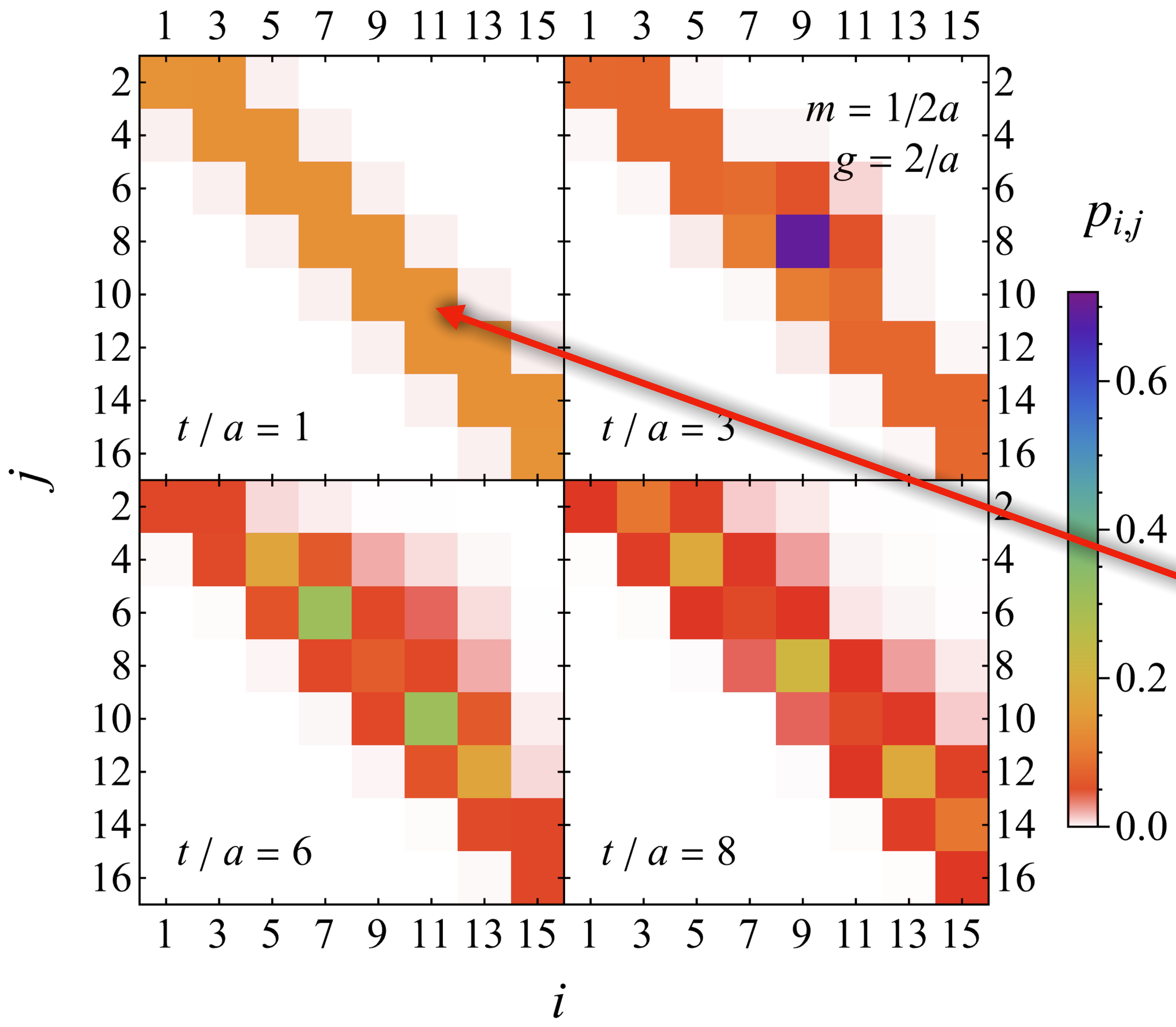
overlap with one-pair state



$$p_{i,j} = |\langle \Psi_t | \chi_i \chi_j^\dagger | \text{Neel} \rangle|^2$$

probability of exciting the i^{th} antiquark and j^{th} quark

overlap with one-pair state

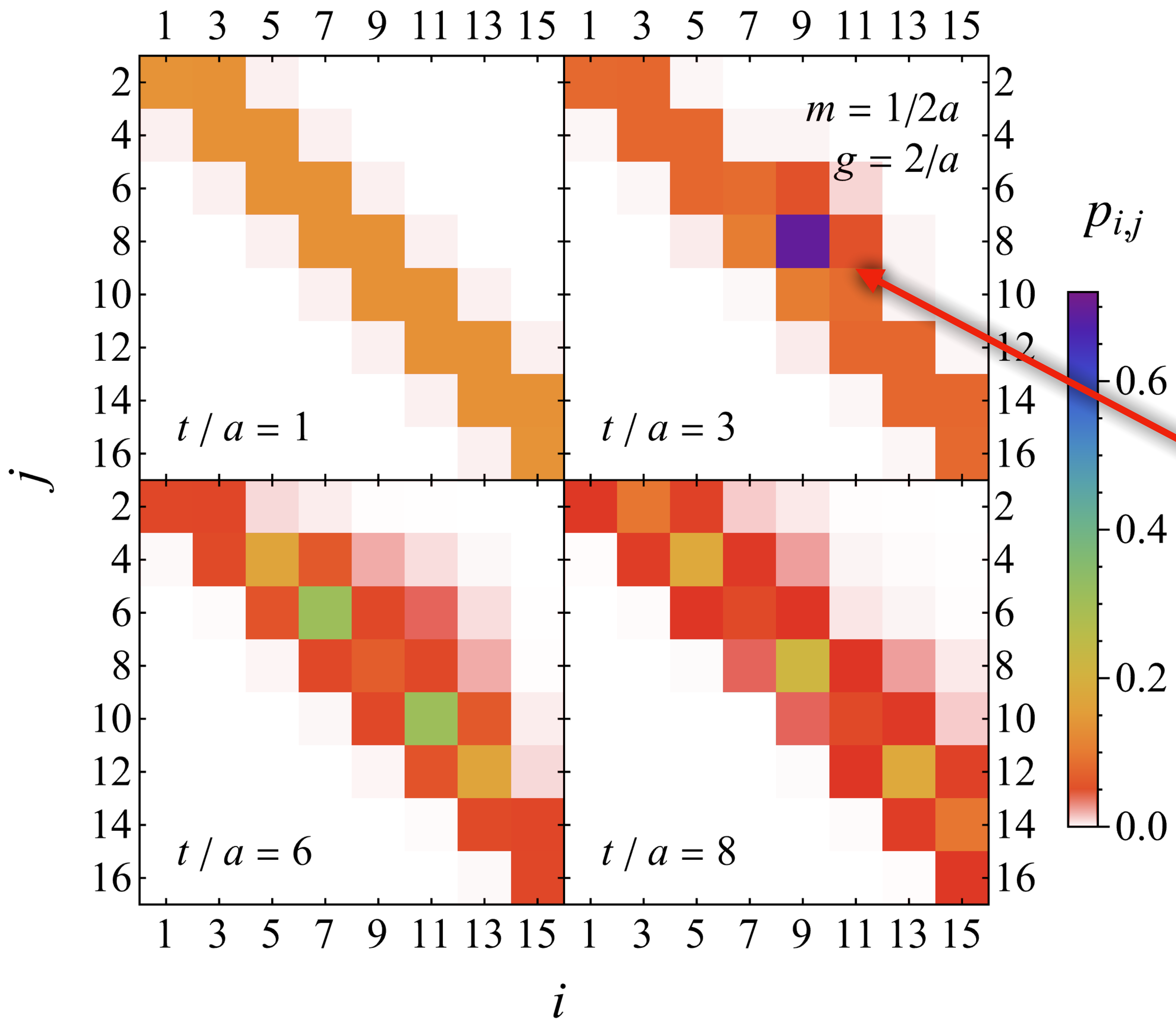


$$p_{i,j} = |\langle \Psi_t | \chi_i \chi_j^\dagger | \mathrm{Neel} \rangle|^2$$

probability of exciting the i^{th} antiquark and j^{th} quark

homogeneous vacuum structure

overlap with one-pair state

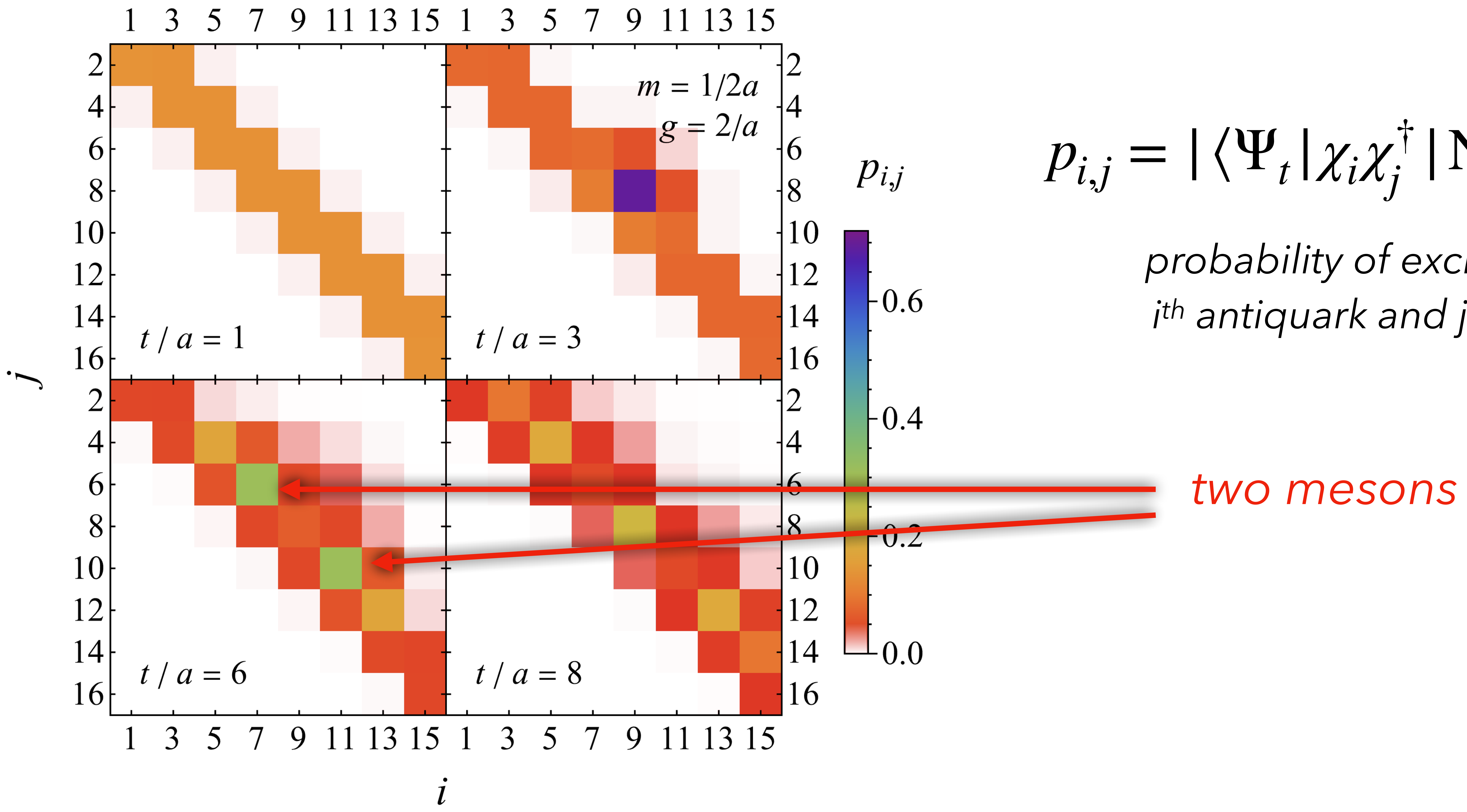


$$p_{i,j} = |\langle \Psi_t | \chi_i \chi_j^\dagger | \mathrm{Neel} \rangle|^2$$

probability of exciting the i^{th} antiquark and j^{th} quark

creation of meson

overlap with one-pair state



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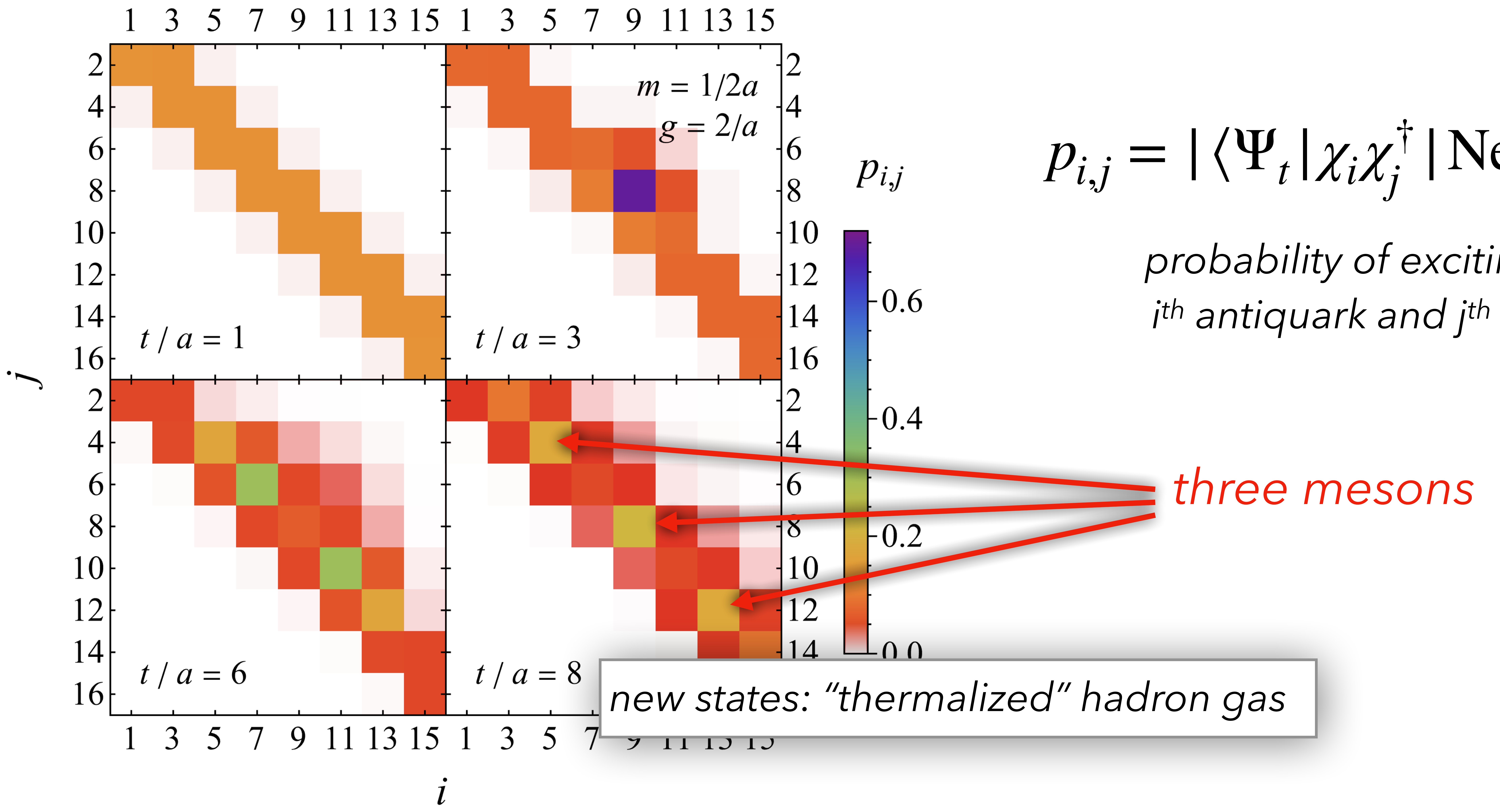


$$p_{i,j} = |\langle \Psi_t | \chi_i \chi_j^\dagger | \mathrm{Neel} \rangle|^2$$

probability of exciting the i^{th} antiquark and j^{th} quark

- real-time dynamics of jet production:
  - spread out of light-cone
  - creation of fermion-antifermion pairs

- thermalized hadron gas in the final state:
  - screening of electric field
  - saturation of destructed chiral condensate
  - entropy that increase in time and proportional to volume

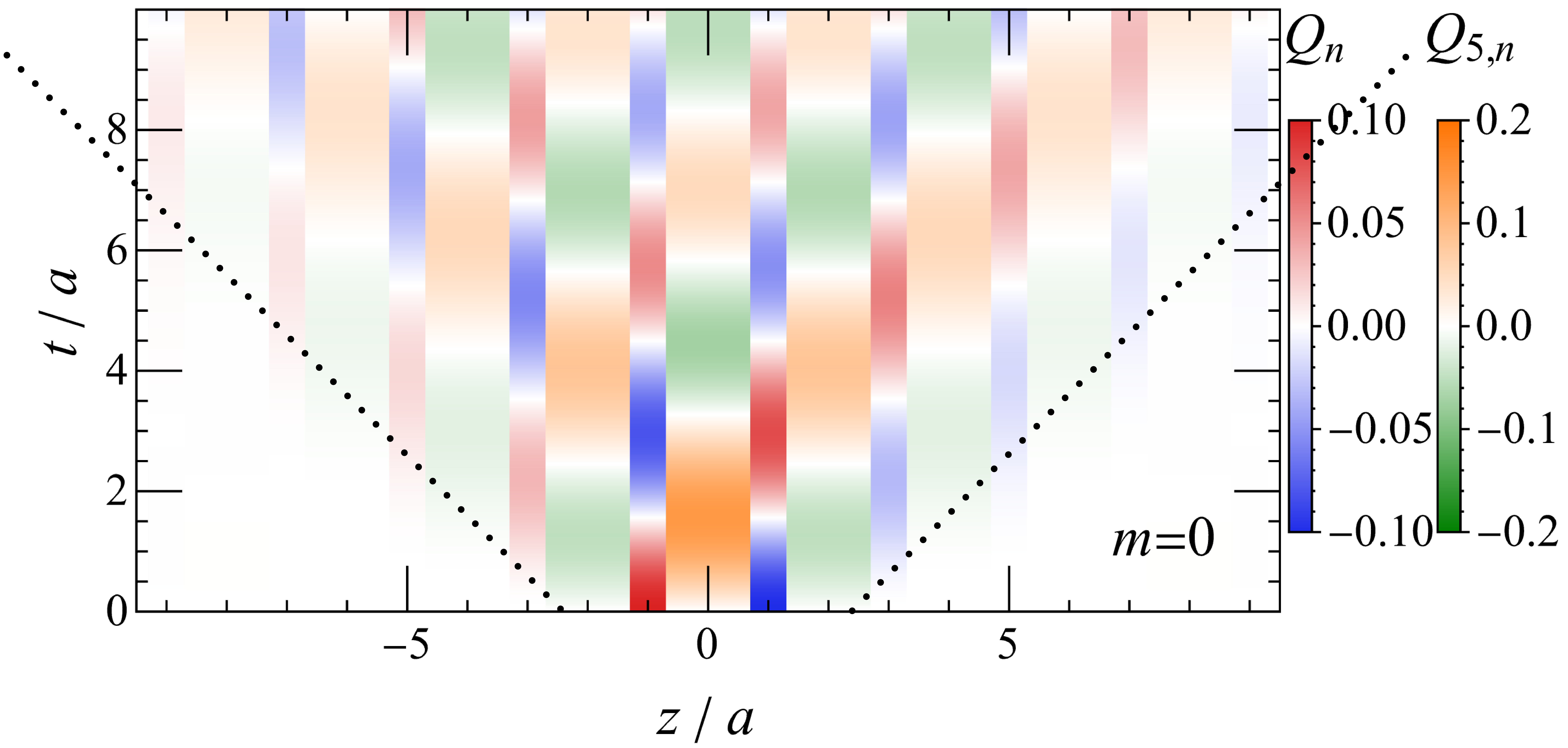
thermodynamic properties and phase structure

- finite temperature, finite chemical potential:

$$\langle O \rangle_{\rm th} \equiv {\rm Tr}(\rho_{\rm th}\, O) \qquad \rho_{\rm th} \equiv \frac{e^{-(H-\mu Q)/T}}{{\rm Tr}(e^{-(H-\mu Q)/T})}$$

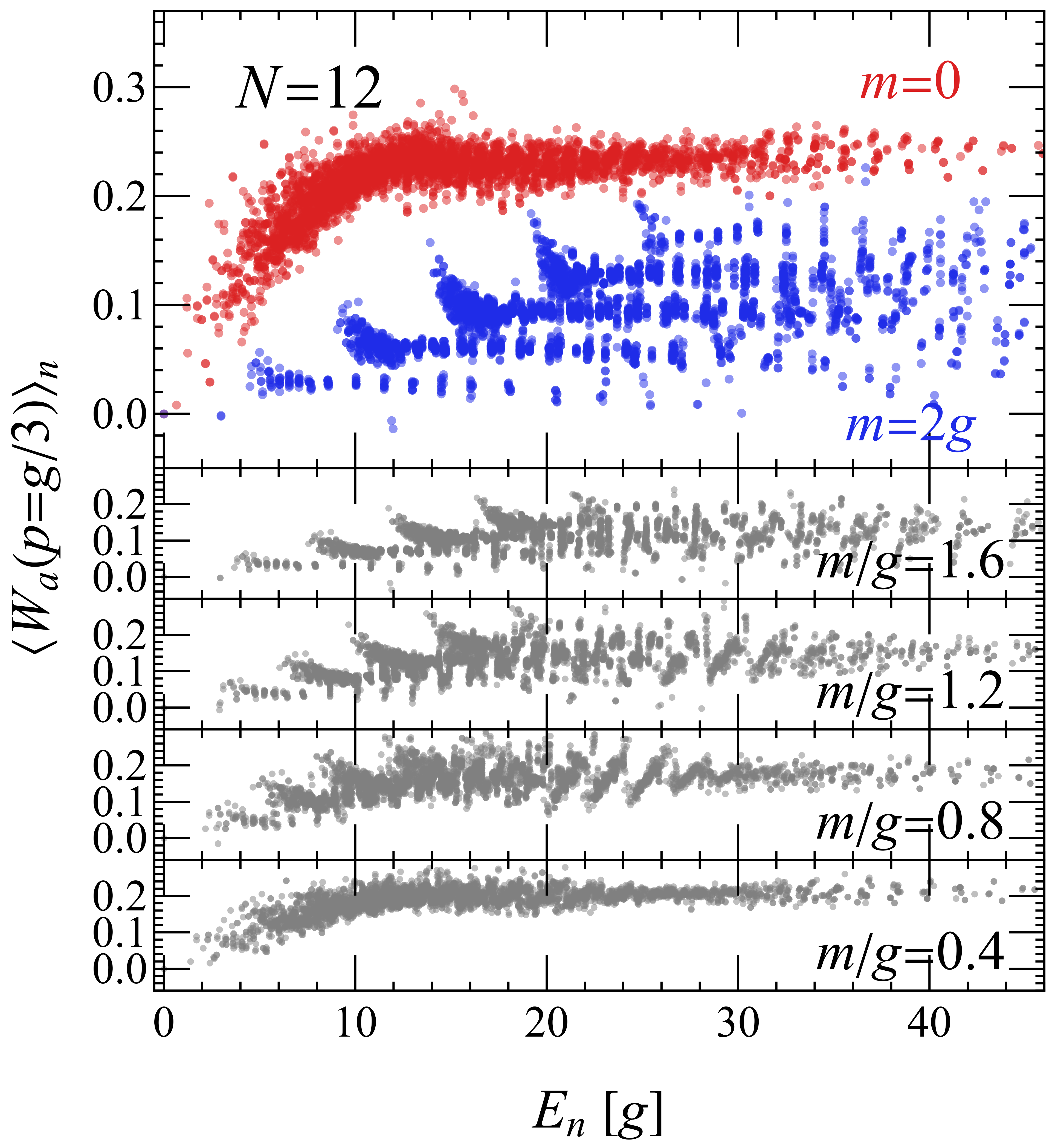
K. Ikeda, D. Kharzeev, R. Meyer, *SS*, PhysRevD.108.L091501

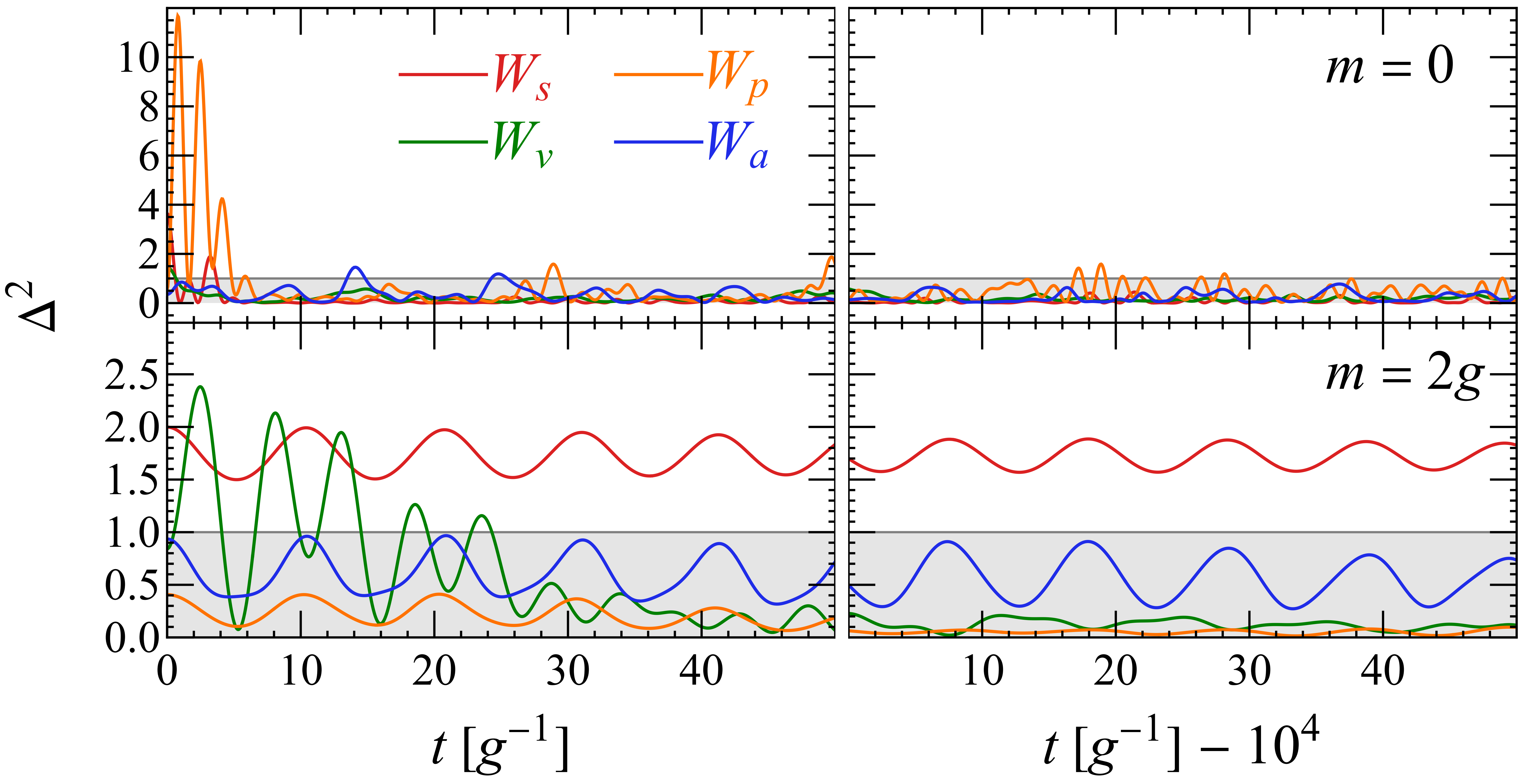
vector and axial charge propagation



K. Ikeda, D. Kharzeev, *SS*, PhysRevD.108.074001

thermalization of quantum distribution function





w/ Shile Chen and Li Yan,
in final preparation

Shile Chen's talk @ XQCD24 [link]

back up slides

thermal equilibrium property

$$\hat{\rho}_{\rm th} \equiv Z^{-1}\, e^{-(\hat{H}-\mu\hat{Q})/T}$$

$$\langle \hat{O} \rangle_{\rm th} \equiv {\rm Tr}(\hat{\rho}_{\rm th}\, \hat{O})$$

real time evolution

$$\partial_t \hat{\rho} = -\, i[\hat{H}, \hat{\rho}]$$

$$\frac{\partial}{\partial t} | \psi(t) \rangle = -\, i\, \hat{H} | \psi(t) \rangle$$

$$q_{n,t} \equiv \langle \psi^\dagger(a\,n)\psi(a\,n) \rangle_t = \frac{\langle Z_n \rangle_t + (-1)^n}{2a},$$

$$\nu_{n,t} \equiv \langle \bar{\psi}(a\,n)\psi(a\,n) \rangle_t = \frac{(-1)^n \langle Z_n \rangle_t}{2a},$$

$$\Pi_{n,t} \equiv \langle E(a\,n) \rangle_t = g\, \langle L_n \rangle_t,$$

$$\vdots$$

why quantum computer?

dimension of state vector $= 2^N$

N : number of lattice sides

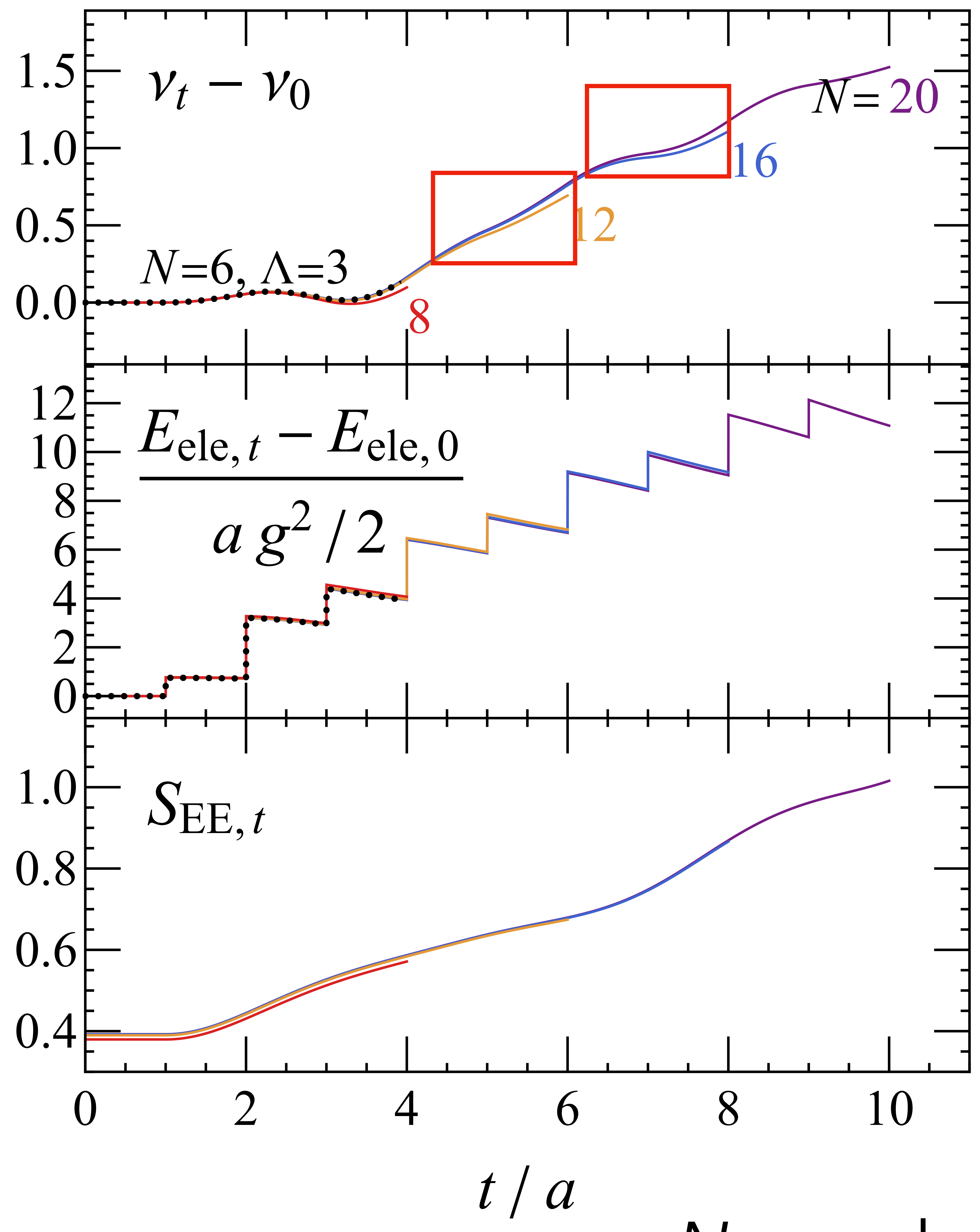
dimension of Hamiltonian $=$ ~~$2^N \times 2^N$~~ sparse $\sim 2N \times 2^N$

N	dimension	memory of Hamiltonian	# of qubit (N)
8	256	~ 131 kB	8
12	4,096	~ 3.1 MB	12
16	65,536	~ 67 MB	16
20	1,048,576	~ 1.3 GB	20
24	16,777,216	~ 26	
~~28~~	~~268,435,456~~	~~~ 481 GB~~	~~28~~

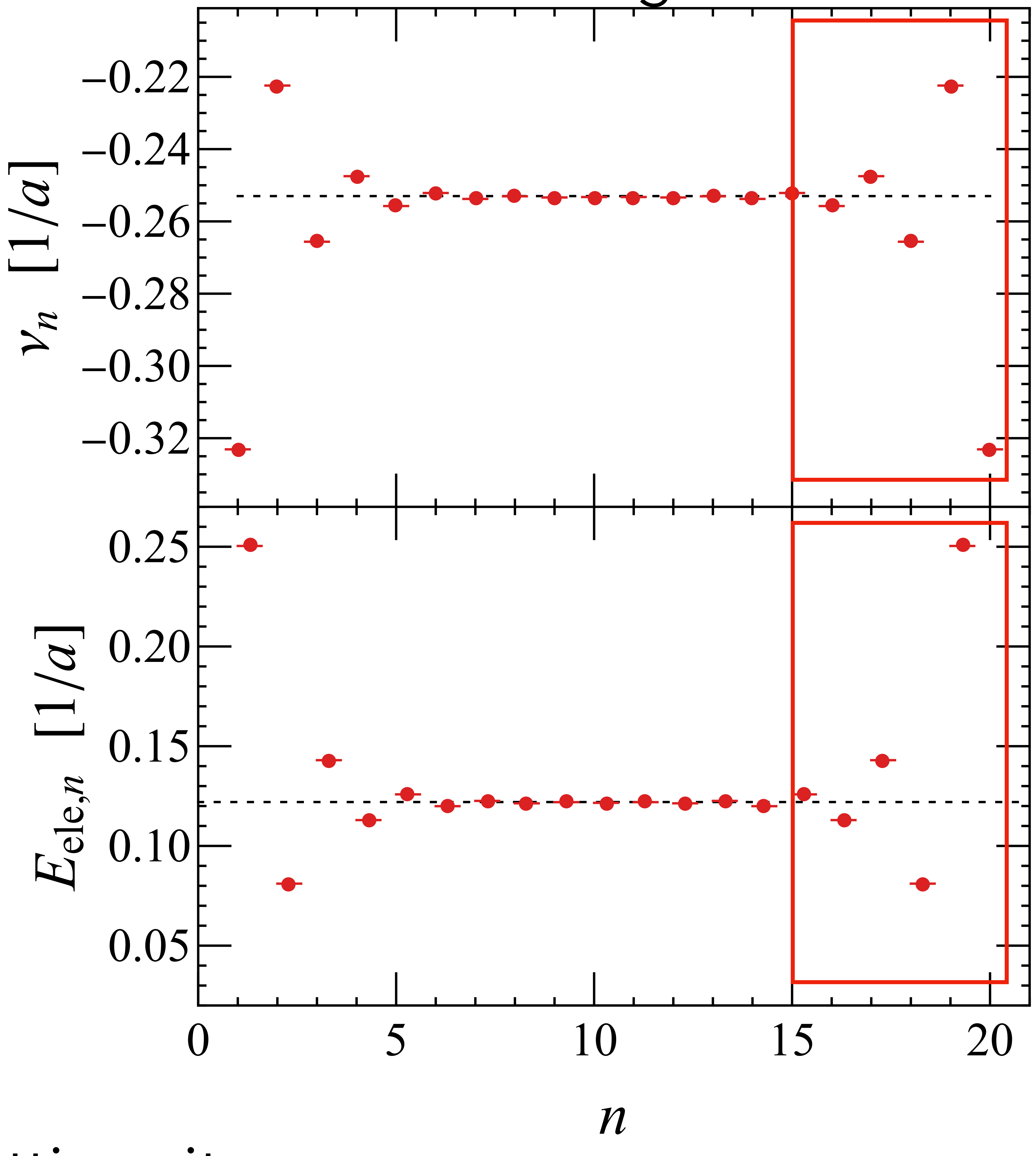
classical hardware in this work

performance not satisfying…

size dependence and boundary effect

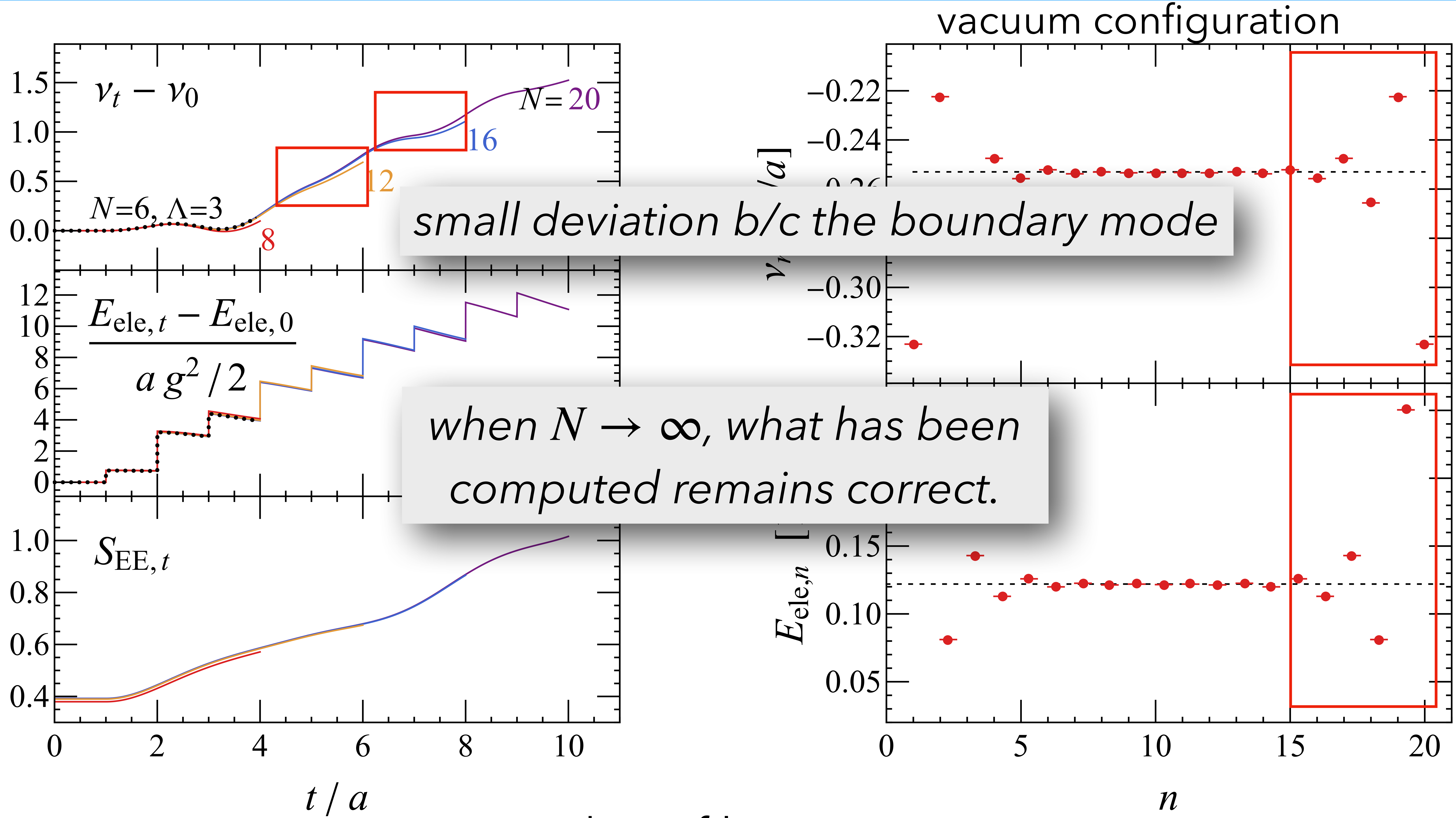


vacuum configuration



N: number of lattice sites

size dependence and boundary effect

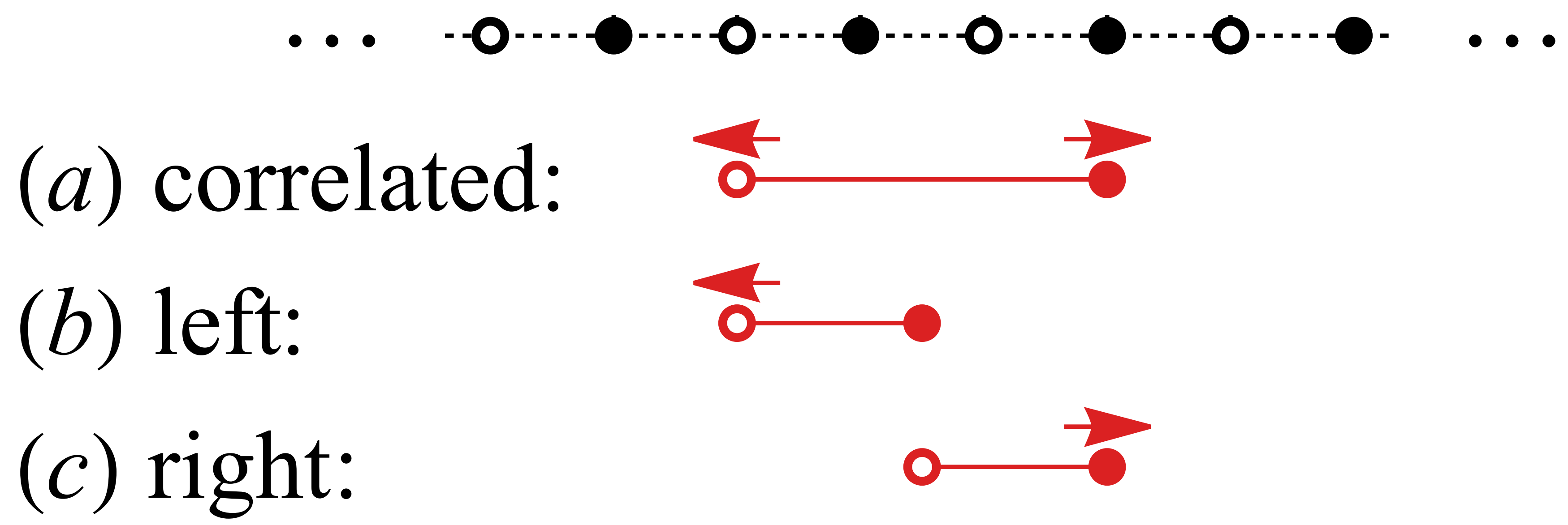


N: number of lattice sites

Background estimation: uncorrelated sources

$$|\psi_{\text{uncorr}}\rangle = \frac{1}{\sqrt{2}}|\psi_{\text{left}}\rangle + \frac{e^{i\varphi}}{\sqrt{2}}|\psi_{\text{right}}\rangle$$

$$\begin{aligned}
&\langle\langle \psi_{\text{uncorr}} | O | \psi_{\text{uncorr}} \rangle\rangle \\
&\equiv \int \langle \psi_{\text{uncorr}} | O | \psi_{\text{uncorr}} \rangle \frac{\mathrm{d}\varphi}{2\pi} \\
&= \frac{\langle \psi_{\text{left}} | O | \psi_{\text{left}} \rangle}{2} + \frac{\langle \psi_{\text{right}} | O | \psi_{\text{right}} \rangle}{2}
\end{aligned}$$



vacuum response to uncorrelated sources

vacuum response to uncorrelated sources

vacuum response to uncorrelated sources

vacuum response to uncorrelated sources

vacuum response to uncorrelated sources

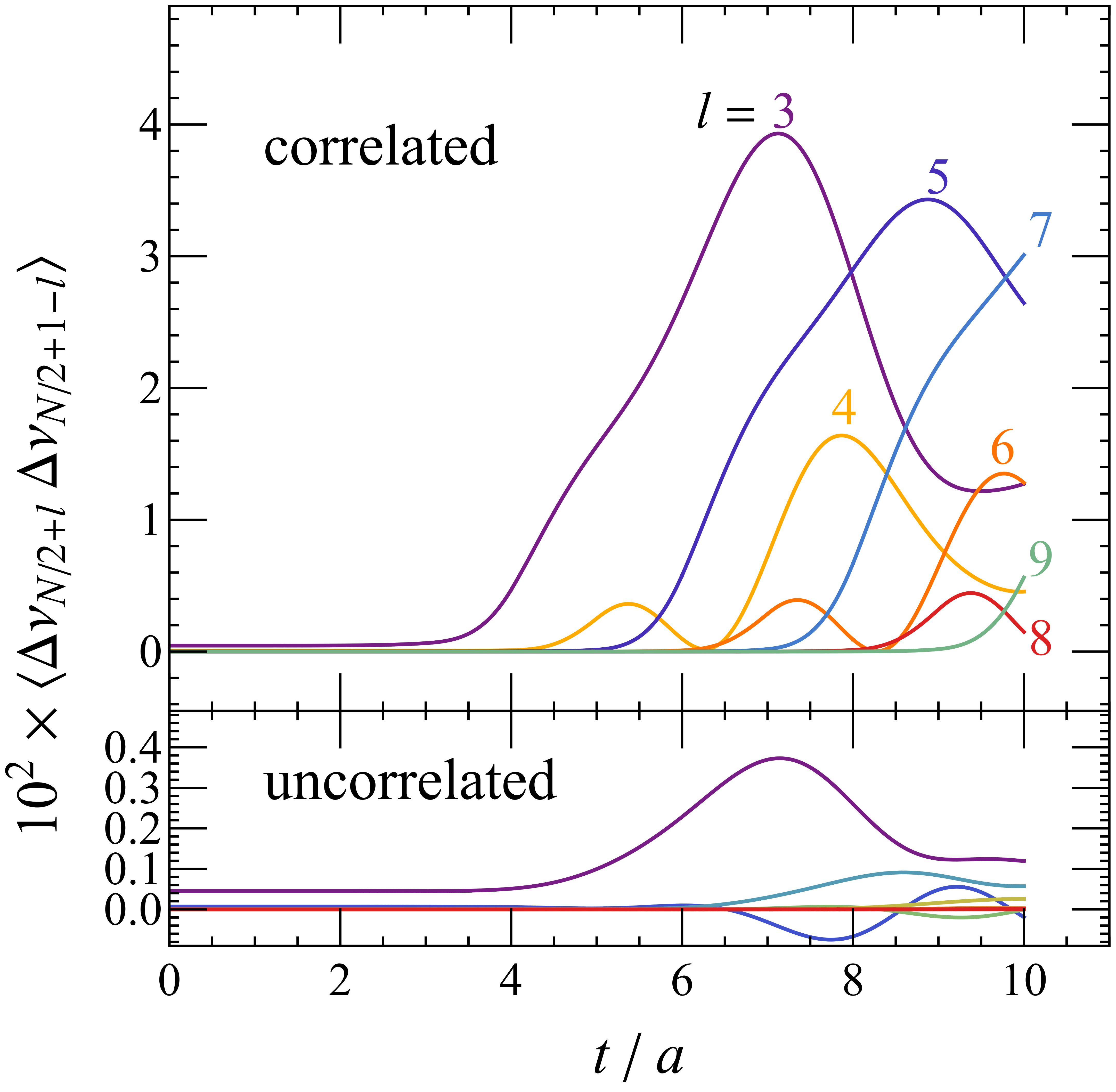
near light-front:

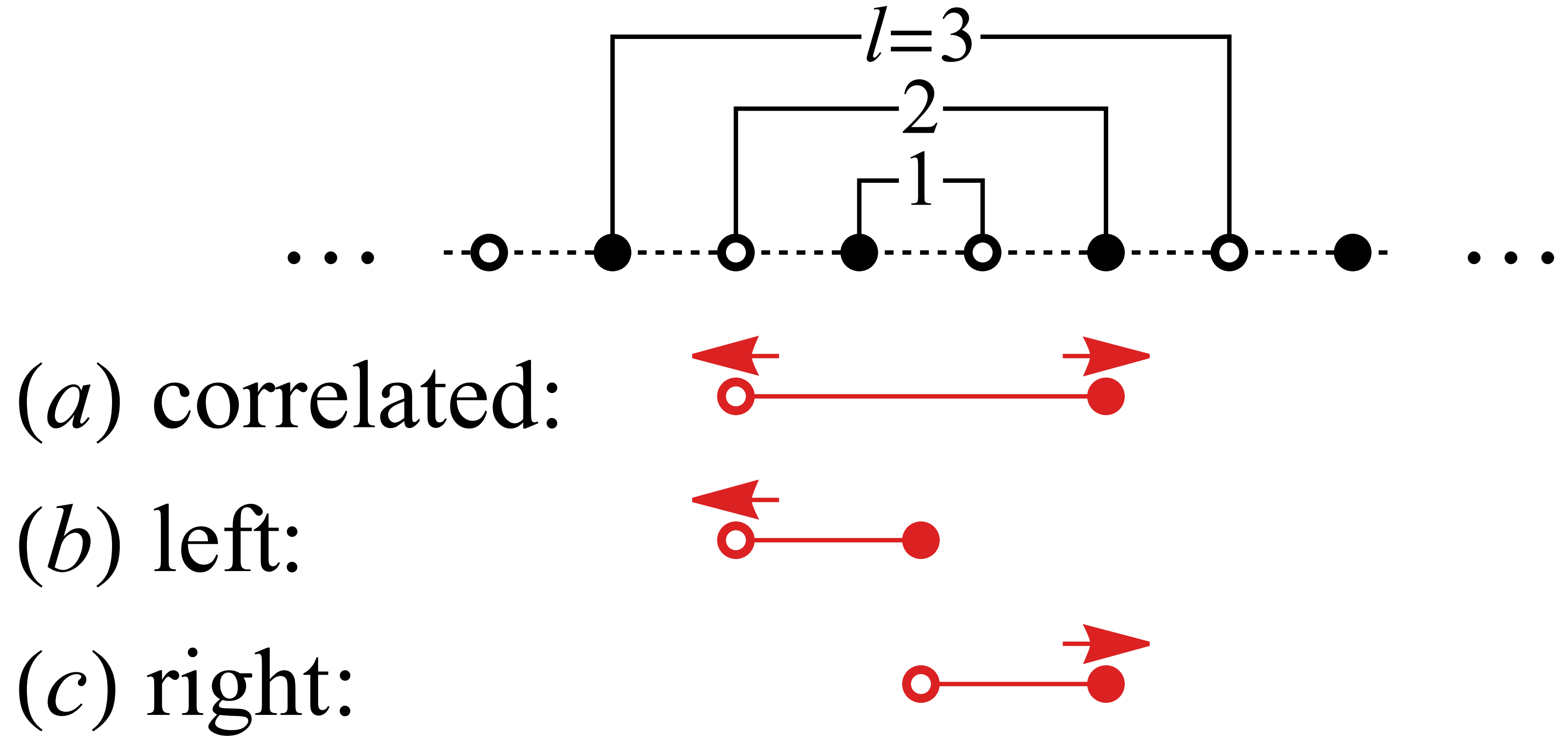
uncorrelated states reproduce one-point observables

$\Longrightarrow$ *reliable baseline for two-point correlations*

observables of entanglement

$\nu_n \equiv \bar{\psi}_n \psi_n \qquad \Delta\nu_n \equiv \nu_n - \langle \nu_n \rangle_{\text{vac}}$





experimental measurement of entanglement