Gribov copies in the quark propagator

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Introduction

In QCD, gauge-fixing is not unique. This leads to the appearance of Gribov copies, which are multiple gauge field configurations that satisfy the same gauge-fixing condition, but still belong to the same gauge orbit. There have been a number of lattice studies investigating the impact of Gribov copies on gluon and ghost propagator such as [1]. In contrast, to our knowledge, no studies on the effect of Gribov copies on the quark propagator had previously been carried out. This poster contains results from a study on Gribov copies in the quark propagator [2, 3].

Computational Setup

β	κ	a (fm)	m_π/m_ρ	m_q (MeV)	N_s	N_τ	V (fm^3)	T (MeV)
1.9	0.1680	0.178(6)	0.805(9)	56	24	12	78	94
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Table: Simulation parameters: gauge coupling β, hopping parameter κ, lattice spacing a, pseudoscalar-to-vector mass ratio m_π/m_ρ, subtracted bare quark mass m_q, spatial and temporal extent N_s, N_τ, lattice volume V and temperature T.

- Simulated QCD with gauge group SU(2) (QC$_2$D) using a Wilson gauge action and $N_f = 2$ Wilson fermions [4, 5].
- Three different lattice volumes have been used to assess temperature and volume effects.
- The lattice configurations have been fixed to Landau gauge by maximising the functional $F[U;g] = \sum_{x,\mu} U^g_\mu(x)$ using a standard overrelaxation algorithm (with convergence precision 10^{-12}).
- Different Gribov copies have been obtained by repeating this procedure 100 times after a random gauge transformation, this has been done for 5 different gauge configurations for each of the three lattice volumes.

Quark Propagator

- The tree-level lattice fermion propagator with the Wilson action is given by

$$S(p) = \frac{1}{i\not{K}(p) + m_0 + \frac{a}{2}Q^2(p)}, \tag{1}$$

where we have introduced the lattice momentum variables

$$K_\mu(p) = \frac{1}{a}\sin(ap_\mu), \qquad Q_\mu(p) = \frac{2}{a}\sin\left(\frac{ap_\mu}{2}\right), \tag{2}$$

- The non-perturbative propagator is given by

$$S(p) = \frac{Z(p^2)}{i\not{K}(p) + M(p^2)}, \tag{3}$$

where $Z(p)$ is the wave function and $M(p)$ is the tree-level corrected mass function, defined as

$$M(p) = \frac{i\,\mathrm{Tr}\left[\not{K}(p)S(p)\right]}{K^2(p)\,\mathrm{Tr}\left(S(p)\right)}\frac{m}{m + \frac{a}{2}Q^2(p)}, \tag{4}$$

where m is the subtracted bare quark mass.

Form Factor Results

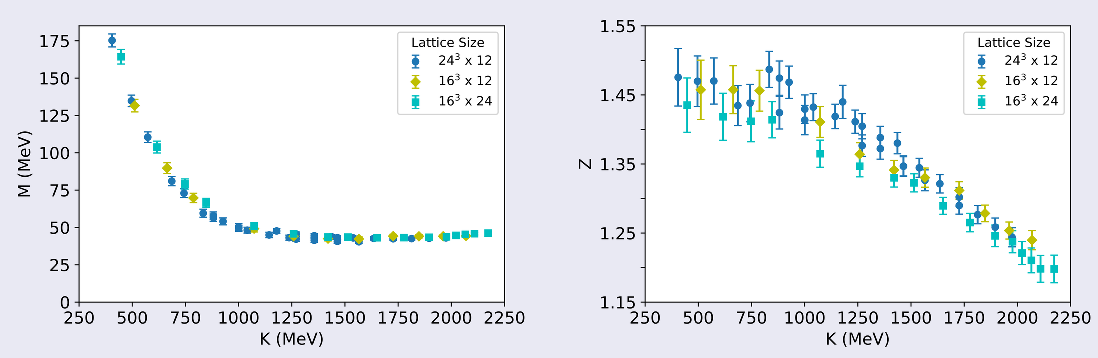

Figure: The tree-level corrected mass function $M(p)$ (left) and wave function $Z(p)$ (right) versus four-momentum $K(p)$ for all three lattice volumes, each averaged over 50 configurations. The data have been cylinder cut to reduce lattice artefacts.

- The mass function exhibits a clear infrared enhancement signalling dynamical chiral symmetry breaking
- The wave function $Z(p)$ is enhanced in the infrared, unlike what has been found in SU(3) with various fermion formulations
- We do not find any strong temperature or finite-volume effects in either quantity

Gribov Copy Results

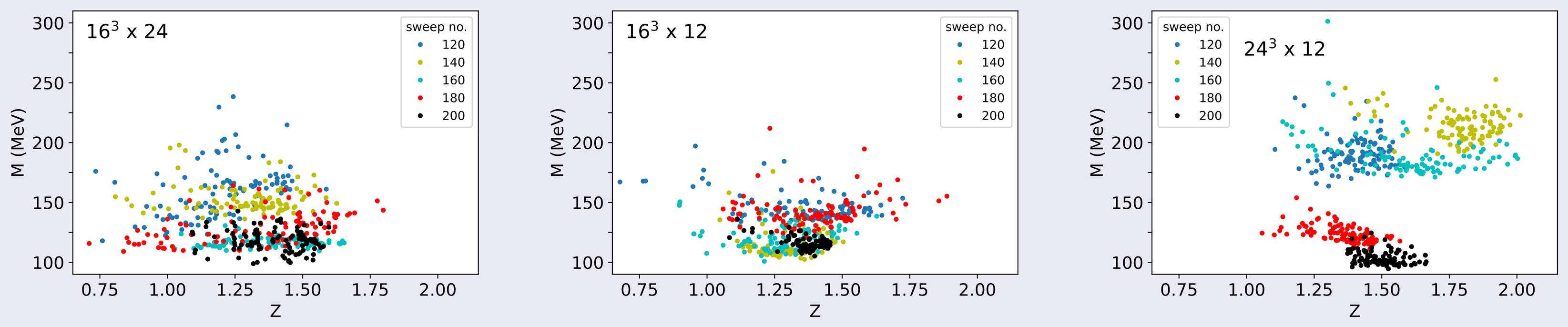

Figure: The quark mass function $M(p)$ and wave function $Z(p)$ at four-momentum $K(p) \approx 500$ MeV, for the three lattice volumes. Different colours represent the 5 different gauge configurations; each point represents a Gribov copy, of which there are 100.

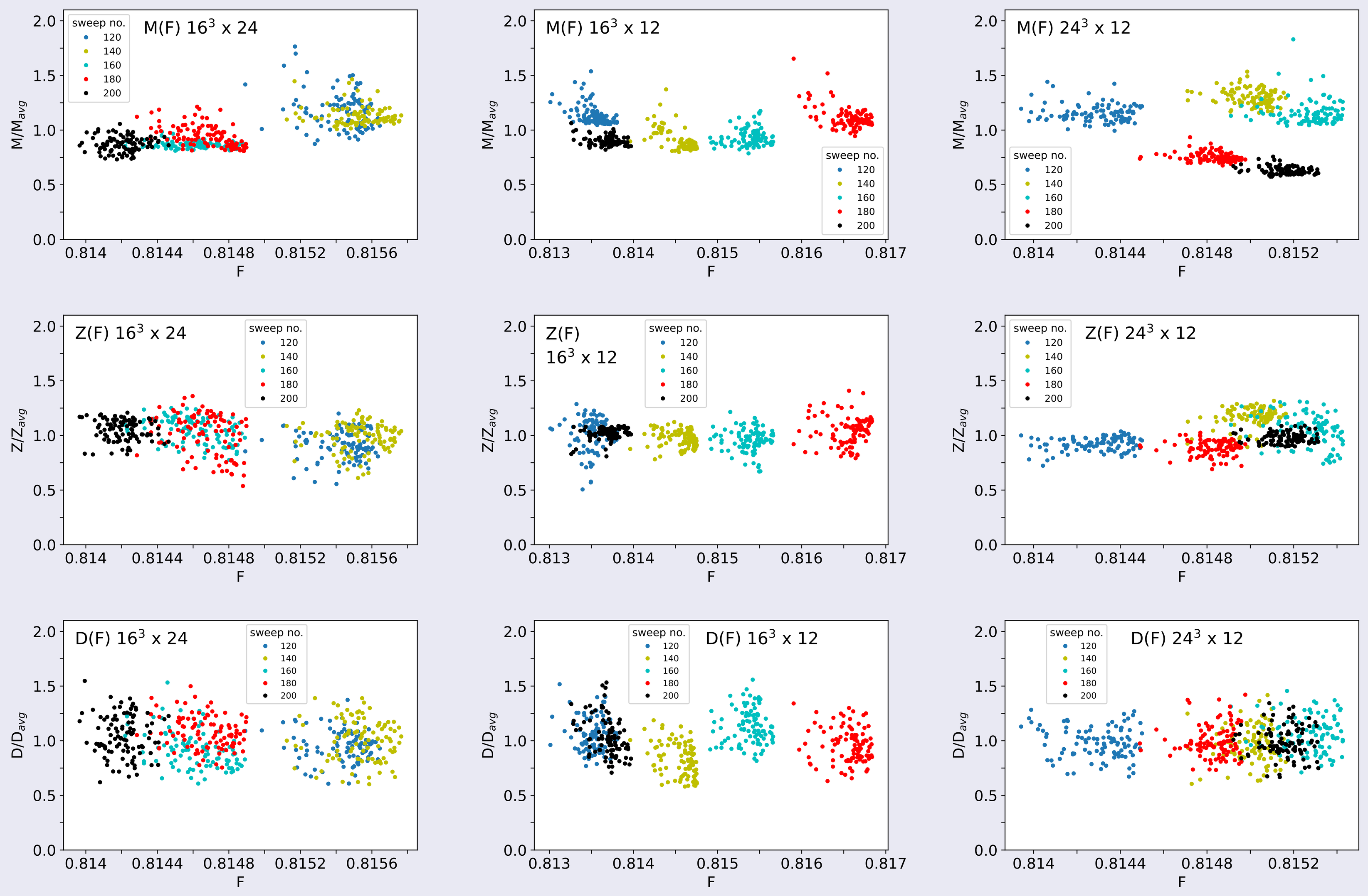

Figure: The quark mass function $M(p)$ (top), wave function $Z(p)$ (middle) and gluon propagator $D(p)$ (bottom) at four-momentum $K(p) \approx 500$ MeV, for each of the three lattice volumes, versus the gauge fixing functional F. Different colours represent different gauge configurations, while each dot represents a Gribov copy.

Conclusions

- The Gribov noise is somewhat smaller for the quark propagator than for the gluon propagator, for all 3 lattice volumes
- In the quark propagator;
  - In the 16^3 spatial volumes, the Gribov noise is comparable to the gauge noise
  - In the 24^3 volume, it is significantly smaller, with the Gribov noise more significant for $Z(p)$ than for $M(p)$
- We see no clear evidence of any temperature dependence, as the results for the $16^3 \times 24$ and $16^3 \times 12$ are very similar
- No correlation was found between the form factors
- We find no evidence of any correlation between the values of the quark or gluon propagator and the gauge fixing functional

References

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