Estimate for the neutrino magnetic moment from pulsar kick velocities induced at the birth of strange quark matter neutron stars

Alejandro Ayala,
Universe 2024, 10, 301

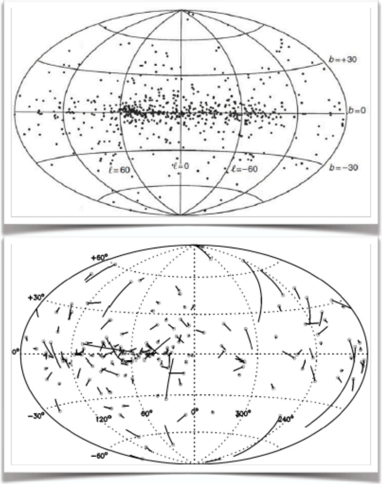
Cairns, August 2024

Outline

- Pulsar kicks.
- Strange quark matter stars.
- Anisotropic neutrino emission in a magnetic field.
- Chirality flip
- Calculation of the kick velocity.
- Total reaction rate for chirality flip.
- Estimate of the neutrino magnetic moment.
- Summary and Conclusions.

Galactic pulsar kicks

- A large portion of young pulsar kicks are located in the galactic plane.
- 1974: Pulsars move with velocities larger than the those of the star from which they were born.
- **Kick velocities** are in the range 12 – 1645 km/s. These velocities follow a Maxwell distribution with an average of 400 km/s.
- The escape velocity in the Milky Way is 400 km/s.

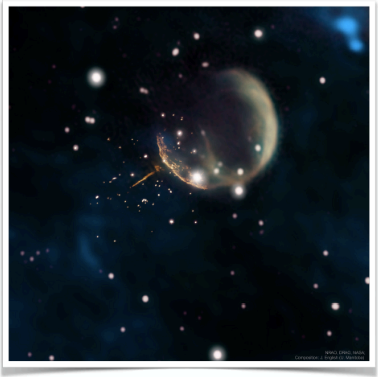


A. Lyne, F. Graham - Smith

G. Hobbs, D. E. Lorimer, A. Lyne, M. Kramer

Existing models

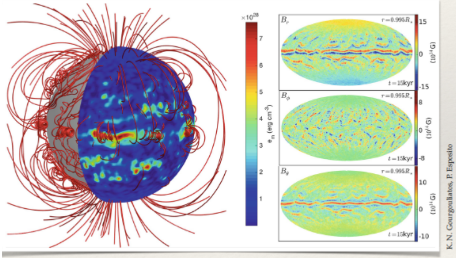
- Hydrodynamical perturbations (SN1987A?)
- Breaking of a binary system (Crab pulsar?)
- Electromagnetic rocket: radiation from a spinning magnetic dipole (Crab pulsar and Vela?)
- **Anisotropic neutrino emission**



PSR J0002 + 6216

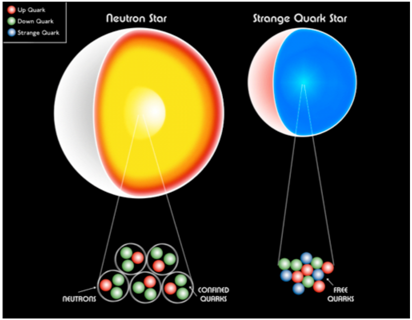
Magnetic field in NS

- Neutron Stars are compact objects with a large magnetic field ($B > 10^{12}$ G).
- At a macroscopic (global) level, the magnetic field is of a dipole or toroidal shape.
- At the microscopic (local) level, the magnetic field can be considered constant and uniform.



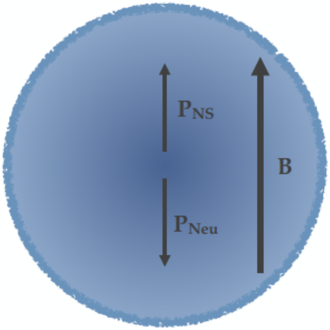
Strange QM stars

- We consider a hybrid NS with the core as a plasma made out of magnetized strange quark matter (SQM), a gas composed of quarks u, d, and s and e in a magnetic field.
- Core temperature about 50 MeV.
- Neutrino mean free path much smaller than NS radius.
- Fast and slow neutrino production processes at work.



Neutrino anisotropic emission

- Different neutrino production mechanisms in the Neutron Star: DURCA, MURCA, bremsstrahlung, pair anihilation, etc.
- Weak interaction violates Parity: preferred direction for neutrino emission opposite to the magnetic field. Neutrino emission is anisotropic for weak decay processes (DURCA).
- Number of emitted neutrinos in one and the other direction (with respect to the magnetic field) is obtained from the spin asymmetry.



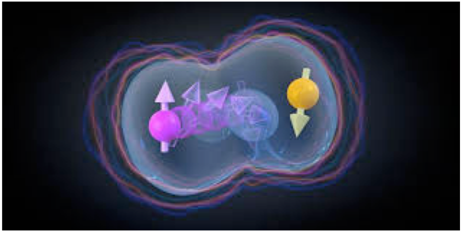
- **Kick velocity naturally aligned with direction of magnetic field.**

Chirality flip

- If neutrinos possess a **magnetic dipole moment** μ_ν, they can flip their chirality due to interactions within the NS core mediated by photons.
- The interaction can be modelled by a magnetic dipole interaction term.

$$\mu_\nu \sigma_{\alpha\mu} K^\mu$$

where K^μ is the photon four-momentum and $\sigma_{\alpha\mu}$ the neutrino spin operator.



- **A fraction of the emitted left-handed neutrinos can escape from the NS as right-handed ones producing the kick.**

Kick velocity

The resulting kick velocity v for the NS can be calculated from

$$dv = \frac{\chi}{M_{NS}}\frac{4}{3}\pi R^3 \epsilon dt$$

Kick velocity

The resulting kick velocity for NS can be calculated from

$$dv = \frac{\chi}{M_{NS}} \frac{4}{3} \pi R^3 \epsilon dt$$

χ : **electron spin polarization (a function of T, n and B)**

Kick velocity

The resulting kick velocity for NS can be calculated from

$$dv = \frac{\chi}{M_{NS}} \frac{4}{3} \pi R^3 \epsilon dt$$

M_{NS} : **Neutron Star Mass**

Kick velocity

The resulting kick velocity for NS can be calculated from

$$dv = \frac{\chi}{M_{NS}} \frac{4}{3} \pi R^3 \epsilon dt$$

R : **Neutron Star Radius**

Kick velocity

The resulting kick velocity for NS can be calculated from

$$dv = \frac{\chi}{M_{NS}} \frac{4}{3} \pi R^3 \epsilon dt$$

ϵ : **Right-handed neutrino emissivity**

Computation of kick velocity elements: Electron spin polarization

- The fraction of neutrinos asymmetrically emitted is equal to the number of spin polarized electrons

$$\chi \equiv \frac{n_- - n_+}{n_- + n_+}$$

Where $n_\pm$ are the the number densities for electrons whose spin is oriented parallel and antiparallel to the magnetic field

$$n_+ = \frac{m^3}{2\pi^2} b \sum_{l=1}^{\infty} \int_0^{\infty} dx_3 \, \frac{1}{\exp\left\{ \frac{m}{T} \left(\sqrt{x_3^2 + 1 + 2lb} - x \right) \right\} + 1}$$

$$n_- = \frac{m^3}{2\pi^2} b \sum_{l=0}^{\infty} \int_0^{\infty} dx_3 \, \frac{1}{\exp\left\{ \frac{m}{T} \left(\sqrt{x_3^2 + 1 + 2lb} - x \right) \right\} + 1}$$

Computation of kick velocity elements: Electron spin polarization

- Where use is made of the expression for the dispersion relation of a charged fermion in a magnetic field

$$E_l = (p_3^2 + 2|e|Bl + m^2)^{1/2}$$

m is the electron mass, $x_3 = p_3/T$ and $x = \mu/m$

$$\chi = \left\{ 1 + \frac{2\sum_{\nu=1}^{\infty}\int_0^{\infty} dx_3 \frac{1}{e^{\frac{m}{T}\left(\sqrt{x_3^2+1+2\nu b}-x\right)}+1}}{\int_0^{\infty} dx_3 \frac{1}{e^{\frac{m}{T}\left(\sqrt{x_3^2+1}-x\right)}+1}} \right\}^{-1}$$

- Where $b = B/B_c$, $B_c = m/|e| = 4.41 \times 10^{13}$ G is the *critical field*.

Computation of kick velocity elements: Right-handed neutrino emissivity

- The right-handed neutrino emissivity ϵ can be expressed as

$$\epsilon = g\tau\Gamma\varepsilon$$

Computation of kick velocity elements: Right-handed neutrino emissivity

- The right-handed neutrino emissivity ϵ can be expressed as

$$\epsilon = g\tau\Gamma\varepsilon$$

Γ : Total reaction rate for the chirality flip

Computation of kick velocity elements: Right-handed neutrino emissivity

- The right-handed neutrino emissivity ϵ can be expressed as

$$\epsilon = g\tau\Gamma\varepsilon$$

τ : **Emission time**

Computation of kick velocity elements: Right-handed neutrino emissivity

- The right-handed neutrino emissivity ϵ can be expressed as

$$\epsilon = g\tau\Gamma\varepsilon$$

ε : **Left-handed neutrino emissivity**

Computation of kick velocity elements: Right-handed neutrino emissivity

- The right-handed neutrino emissivity ϵ can be expressed as

$$\epsilon = g\tau\Gamma\varepsilon$$

g : fraction of energy carried away by right-handed neutrinos

Left-handed neutrino emisivity

- When the left-handed neutrino emissivity changes with temperature, the cooling equation can be used

$$\varepsilon = -\frac{dU}{dt} = -\frac{dU}{dT}\frac{dT}{dt} = -C_V\frac{dT}{dt}$$

- where U is the internal energy and C_V is the heat capacity given by

$$\begin{aligned}
C_V &= \sum_f C_{Vf}; \quad f \text{ represents the fermion spieces} \\
C_{Vf} &= \frac{d_f m^2}{4\pi^2 T^2} b_f \int_0^\infty dp_3 \sum_{l=0}^{\infty} (2-\delta_{l0}) \frac{(E_{lf}-\mu_f)^2}{[1+\cosh\frac{E_{lf}-\mu_f}{T}]} \\
E_{lf} &= (p_3^2 + 2|e_f|Bl + m_f^2)^{1/2}
\end{aligned}$$

d_f is a degeneration factor; for quarks $d_f = 3$ and for leptons $d_f = 1$.

Stellar equillibrium conditions

- β equillibrium

$$\begin{array}{rcl} d & \to & u + e + \overline{\nu}_e \\ s & \to & u + e + \overline{\nu}_e \\ u + e & \to & d + \nu_e \\ u + d & \to & u + s \end{array} \Rightarrow \left\{ \begin{array}{rcl} \mu_d - \mu_u - \mu_e + \mu_{\nu_e} & = & 0 \\ \mu_d - \mu_s & = & 0 \\ \mu_{\nu_e} - \mu_{\overline{\nu}_e} & = & 0 \end{array} \right.$$

- Charge neutrality

$$2n_u - n_d - n_s - 3n_e = 0$$

- Baryon number conservation

$$n_u + n_d + n_s - 3n_B = 0$$

- Lepton fraction

$$Y_L = \frac{n_e + n_{\nu_e}}{n_B} = 0.4$$

Total reaction rate for chirality flip

- The production rate of a right-handed neutrino, from a left-handed one with a four-momentum P is

$$\Gamma(p_0) = \frac{\tilde{f}(p_0)}{2p_0} \mathrm{Tr}\left[\not{P} R (\mathrm{Im}\Sigma) \right]$$

Total reaction rate for chirality flip

- The production rate of a right-handed neutrino, from a left-handed one with a four-momentum P is

$$\Gamma(p_0) = \frac{\tilde{f}(p_0)}{2p_0} \mathrm{Tr}\left[\not{P} R(\mathrm{Im}\Sigma) \right]$$

$\tilde{f}$: **Fermi-Dirac distribution**

Total reaction rate for chirality flip

- The production rate of a right-handed neutrino, from a left-handed one with a four-momentum P is

$$\Gamma(p_0) = \frac{\tilde{f}(p_0)}{2p_0}\mathrm{Tr}\left[\not{P}\,R(\mathrm{Im}\Sigma)\right]$$

$$R, L = \frac{1 \pm \gamma^5}{2}$$: **Right- and Left-chirality projectors**

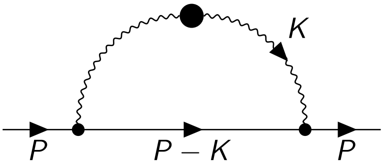
Total reaction rate for chirality flip

- The production rate of a right-handed neutrino, from a left-handed one with a four-momentum P is

$$\Gamma(p_0) = \frac{\tilde{f}(p_0)}{2p_0} \mathrm{Tr}\left[\not{P} R(\mathrm{Im}\Sigma)\right]$$

Σ : neutrino self-energy in the medium

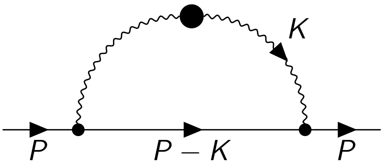
Total reaction rate for chirality flip



$$\Sigma(P) = T\sum_n \int \frac{d^3k}{(2\pi)^3} V^\rho(K)\, S_F(\not{P} - \not{K})\, L\, V^\lambda(K)\, D_{\rho\lambda}(K),$$

$V^\rho = \mu_\nu \sigma^{\rho\alpha} K_\alpha$: **magnetic dipole interaction**

Total reaction rate for chirality flip



$$\Sigma(P) = T\sum_n \int \frac{d^3k}{(2\pi)^3} V^\rho(K)\, S_F(\not{P} - \not{K})\, L\, V^\lambda(K)\, D_{\rho\lambda}(K),$$

$D_{\rho\lambda}$: **in-medium photon propagator**

In-medium photon propagator

$$D_{\rho\lambda}(K) = \Delta_L(K) P^L_{\rho\lambda} + \Delta_T(K) P^T_{\rho\lambda}$$

- Sums over Matsubara frequencies

$$S_{L,T} = T \sum_n \Delta_{L,T}(i\omega_n) \tilde{\Delta}_F(i(\omega - \omega_n))$$

- Imaginary part of the sum over Matsubara frequencies

$$\mathrm{Im}\left[S_{L,T}\right] = \pi \left(e^{\beta(p_0-\mu)} + 1\right) \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \int_{-\infty}^{\infty} \frac{dp_0'}{2\pi} f(k_0) \tilde{f}(p_0' - \mu) \times \delta(p_0 - k_0 - p_0') \rho_{L,T}(k_0, k) \rho_F(p_0')$$

Spectral densities

$$\rho_F(p_0') = 2\pi\delta({p_0'}^2 - E_p^2)\,\mathrm{sign}(p_0')$$

$$\rho_L(k_0,k) = \frac{x}{1-x^2}\frac{2\pi m_\gamma^2\theta(k^2-k_0^2)}{\left[k^2+2m_\gamma^2\left(1-\frac{x}{2}\ln|(1+x)/(1-x)|\right)\right]^2+\left[\pi m_\gamma^2 x\right]^2}$$

$$\begin{aligned}\rho_T(k_0,k) &= \pi m_\gamma^2 x(1-x^2)\theta(k^2-k_0^2)\\ &\times\left(\left[k^2(1-x^2)+m_\gamma^2\left(x^2+\frac{x}{2}(1-x^2)\ln|(1+x)/(1-x)|\right)\right]^2\right.\\ &+\left.\left[(\pi/2)m_\gamma^2 x(1-x^2)\right]^2\right)^{-1}\end{aligned}$$

- Thermal photon mass

$$m_\gamma^2 = \frac{e^2}{2\pi^2}\left(\mu_e^2+\frac{\pi^2 T^2}{3}\right)$$

Reaction rate

- The term $\tilde{f}(E_p - \mu)$ corresponds to a left-handed neutrino in the initial state. After a bit of algebra

$$\begin{aligned}\Gamma(p_0) &= \frac{\mu_\nu^2}{32\pi^2 p_0^2}\int_0^\infty dk\, k^3 \int_{-k}^{k} dk_0\; \theta(2p_0 + k_0 - k) \\ &\times [1 + f(k_0)]\, \tilde{f}(p_0 + k_0 - \mu)\,(2p_0 + k_0)^2 \left(1 - \frac{k_0^2}{k^2}\right)^2 \\ &\times \left[\rho_L(k_0) + \left(1 - \frac{k^2}{(2p_0 + k_0)^2}\right)\rho_T(k_0)\right]\end{aligned}$$

- The total reaction rate obtained integrating over phase space

$$\Gamma = V \int \frac{d^3 p}{(2\pi)^3}\Gamma(p_0) = \frac{V}{2\pi^2}\int_0^{p_0^{max}} dp_0\; p_0^2\; \Gamma(p_0)$$

V : volume where flip takes place

Pulsar kick velocity

- The kick velocity is given by

$$v = -\frac{g}{M_{NS}}\frac{4}{3}\pi R^3 \tau \int_{T_i}^{T_f} \chi \, \Gamma \, C_v dT$$

- and can be written in the form

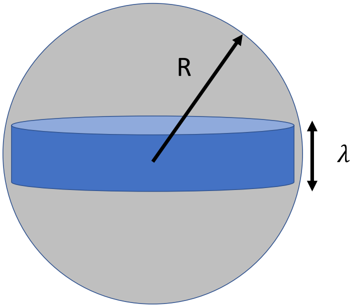
$$v = -g \; 804 \; \frac{\text{km}}{\text{s}} \left(\frac{1.4 M_\odot}{M_{NS}}\right) \left(\frac{R}{10 \text{ km}}\right)^3 \left(\frac{\tau I}{\text{MeV fm}^{-3}}\right)$$

- with

$$I = \int_{T_i}^{T_f} \Gamma \, \chi \, C_v dT$$

Estimate for the neutrino magnetic moment

- Use typical values of NS kick velocities, average values of NS mass, radii, magnetic field intensities, temperature, and chemical potentials at birth.
- Reaction volume **cylinder** with height given by mean free path $\lambda = 1$ cm
- $\tau = \lambda/c = 3 \times 10^{-11}$ s.
- g from fraction of product of volume times the emission time where the flip takes place to the total NS core volume times the total neutrino emission time $g \sim 7.5 \times 10^{-19}$



- Energy released in right-handed neutrinos $E \sim 10^{34}$ erg, which is a small fraction of the 10^{53} erg in a typical SN explosion.

Estimate for the neutrino magnetic moment

- Using these estimates, we find that the NS kick velocity is

$$v \sim 3.1 \times 10^{37} \left(\frac{\mu_\nu}{\mu_B}\right)^2 \text{ km s}^{-1}$$

- The average observed birth velocity for pulsars whose characteristic age is less than three million years is $v \approx 400$ km s^{-1}. Using this value, we estimate that the neutrino magnetic moment is of the order

$$\mu_\nu \sim 3.6 \times 10^{-18} \mu_B$$

- For comparison, in the minimal extension of the SM

$$\mu_\nu \geq (4 \times 10^{-20})\mu_B$$

Summary and conclusions

- We studied the neutrino chirality flip during the birth of an SN, produced by the possible existence of a neutrino magnetic moment.
- The flip is caused by the interaction of the neutrino with photons of the medium in the core of a magnetized strange quark matter NS.
- The calculation is performed using average values of NS mass, radii, magnetic field intensities, temperature, and chemical potentials at the birth of NS.
- Resorting to average values of the known kick velocities, we estimate the value $\mu_\nu \sim 3.6 \times 10^{-18} \mu_B$ for the neutrino magnetic moment.
- This estimate is more stringent than the SM bound obtained from bounds on the neutrino mass from solar, atmospheric, and reactor neutrinos.

Collaborators

Daryel Manreza

Santiago Bernal

THANKS!

BACKUP

Summation over Landau levels

- For small field strengths (below the critical value), the sum over Landau levels appearing in the particle density, heat capacity, and electron spin polarization expressions can be approximated by resorting to the Euler–McLaurin formula

$$
\begin{aligned}
h\left[\frac{f_0}{2}+f_1+\ldots+\frac{f_N}{2}\right] &= \int_0^{Nh} dx\, f(x) \\
&+ \sum_k \frac{B_{k+1}}{(k+1)!} h^{k+1}\left(f_N^{(k)}-f_0^{(k)}\right)
\end{aligned}
$$

- B_k are the Bernoulli numbers. All the calculations where performed keeping terms up to order h^2.