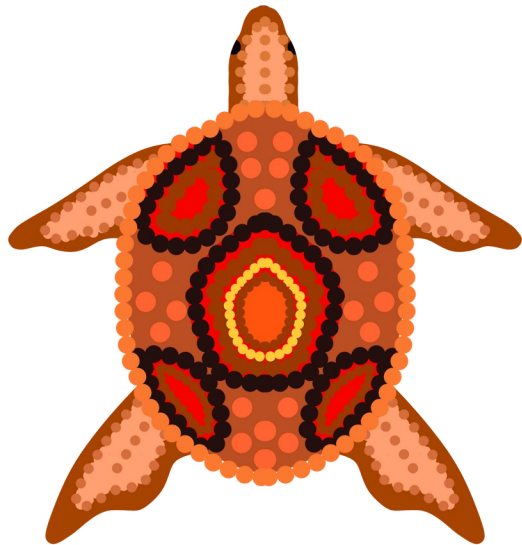


# Gravitational Waves from the innermost parts of Core-Collapse Supernovae

Dr. Pia Jakobus  
University of Hamburg



By Maria Pereira ✨



XVIth Quark  
Confinement and the  
Hadron Spectrum  
Conference 2024



Equation of State

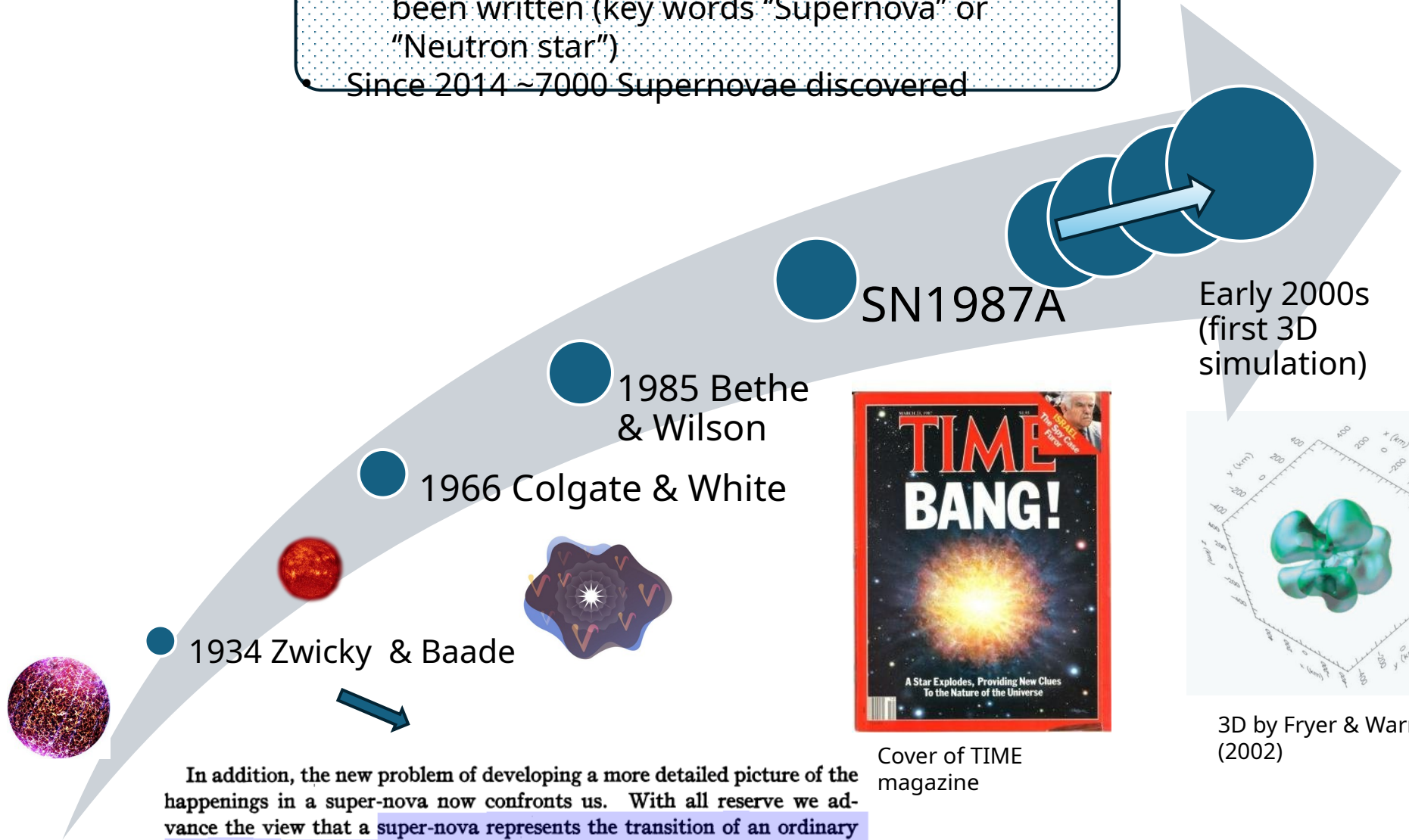
Neutrinos

Stellar evolution

Hydrodynamics

Modelling

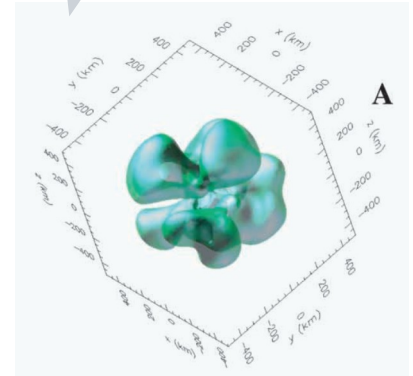
- Since 1932, more than 160.000 papers have been written (key words "Supernova" or "Neutron star")
- Since 2014 ~7000 Supernovae discovered



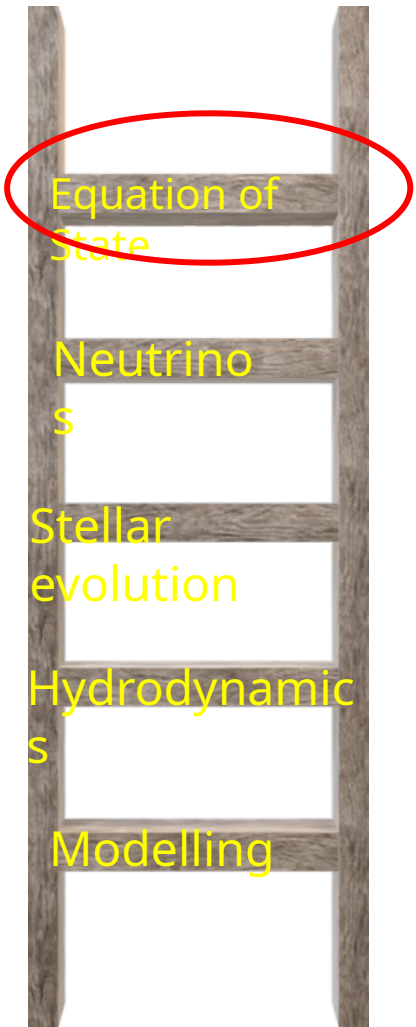
In addition, the new problem of developing a more detailed picture of the happenings in a super-nova now confronts us. With all reserve we advance the view that a super-nova represents the transition of an ordinary star into a *neutron star*, consisting mainly of neutrons. Such a star may possess a very small radius and an extremely high density. As neutrons can be packed much more closely than ordinary nuclei and electrons, the



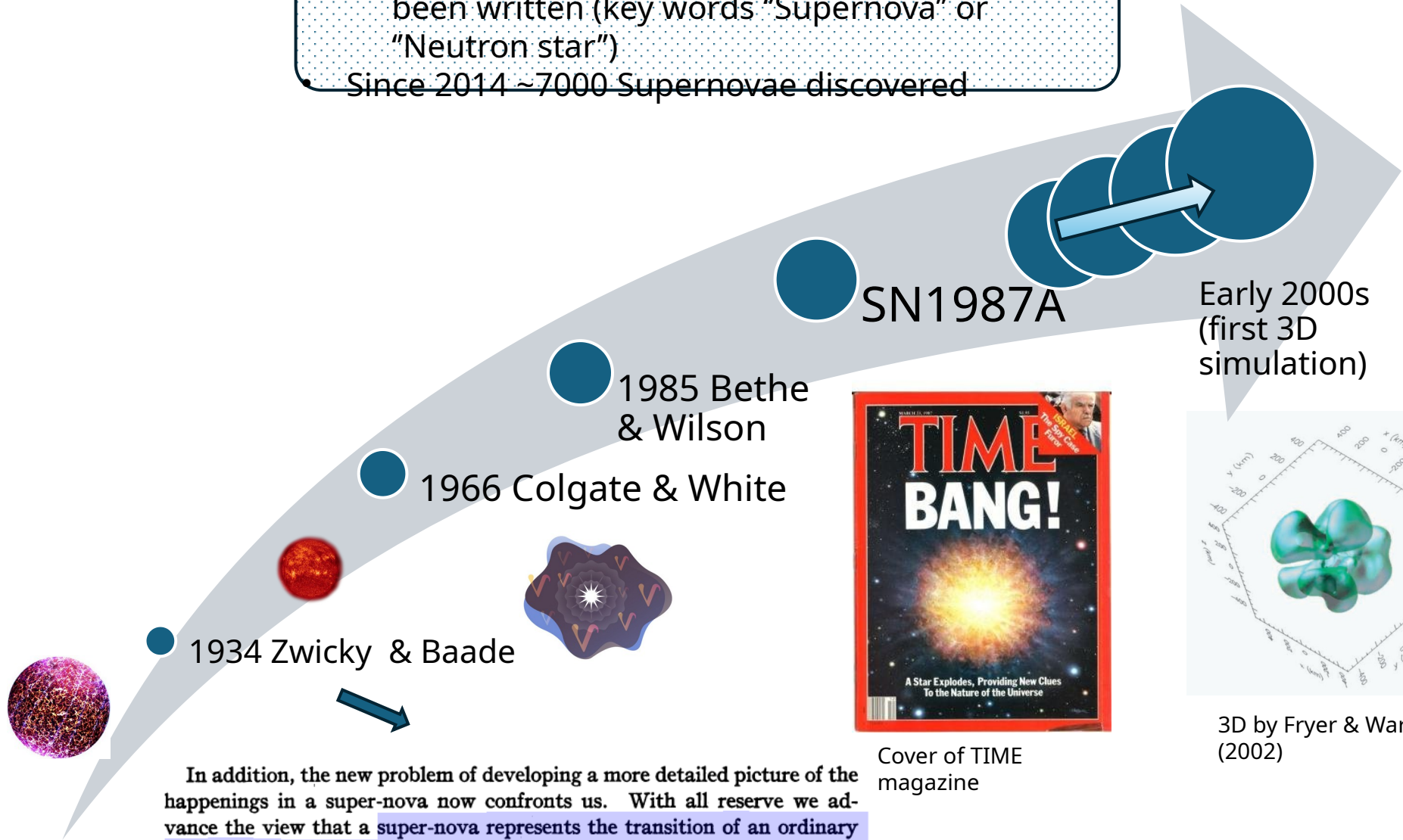
Cover of TIME magazine



3D by Fryer & Warren (2002)



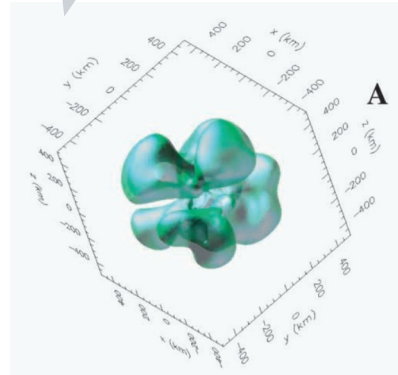
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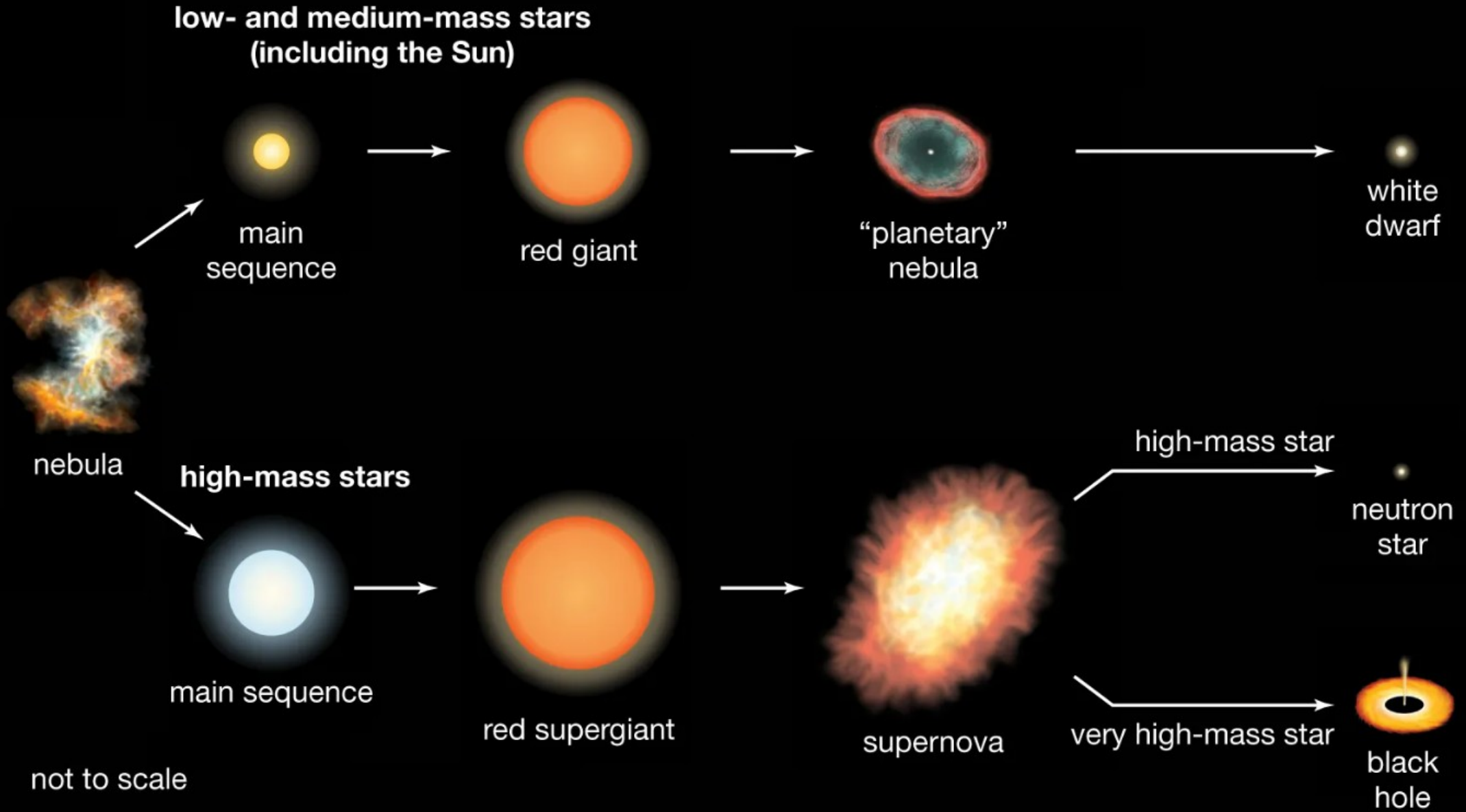


Cover of TIME magazine



3D by Fryer & Warren (2002)

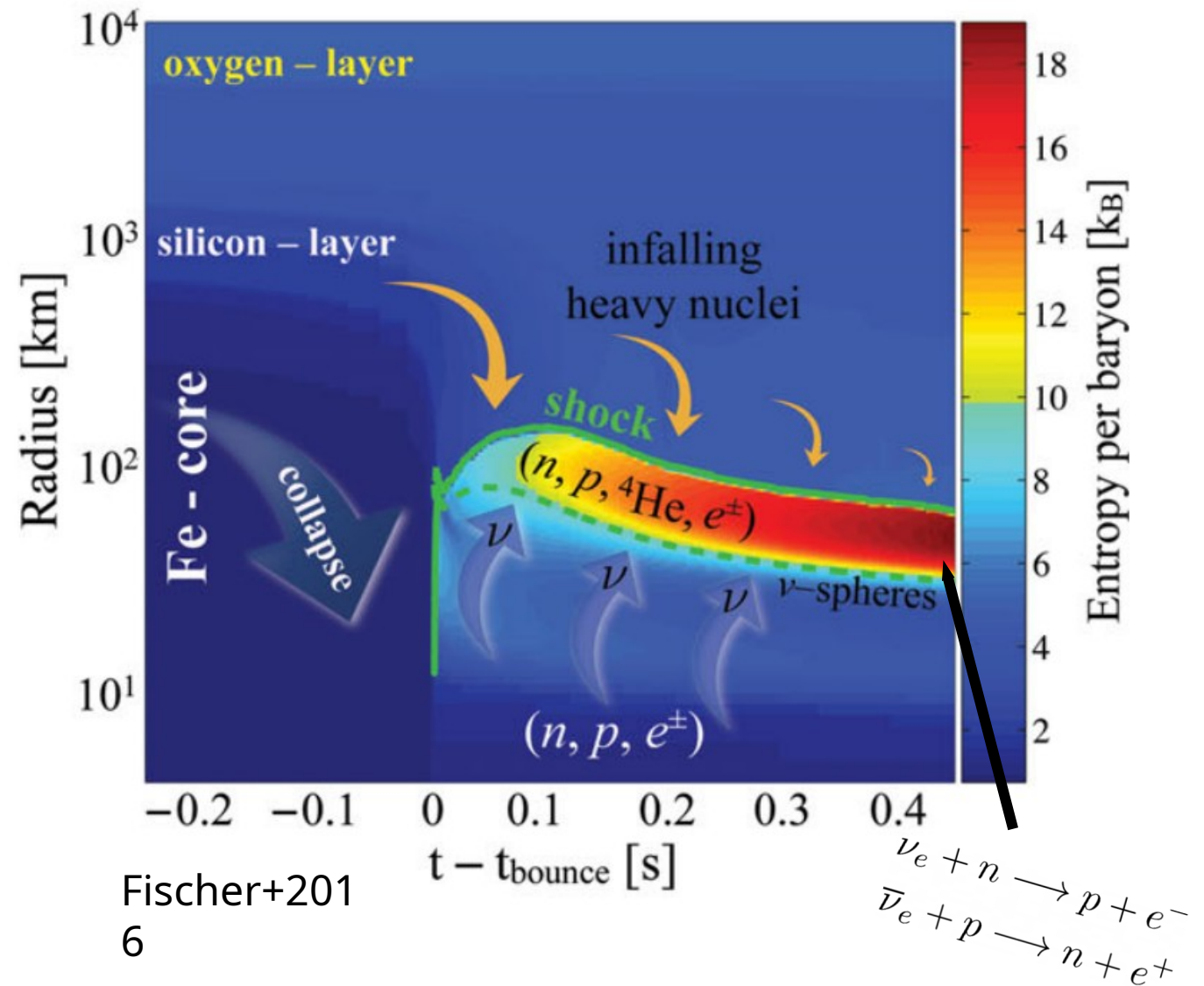
# Stellar evolution



# I: Collapse phase

- Iron core: No exotherm reaction possible
  - Iron core accreting material from Silicon shell burning
  - Exceeds Chandrasekhar mass limit
  - “Time zero” of what (can) be called a CCSN
  - Core collapses

Collapse further accelerated by electron captures on Iron peak nuclei



## II: Core-bounce

Around nuclear saturation density:

Neutrinos become trapped

à Iron core can either form a BH or a NS

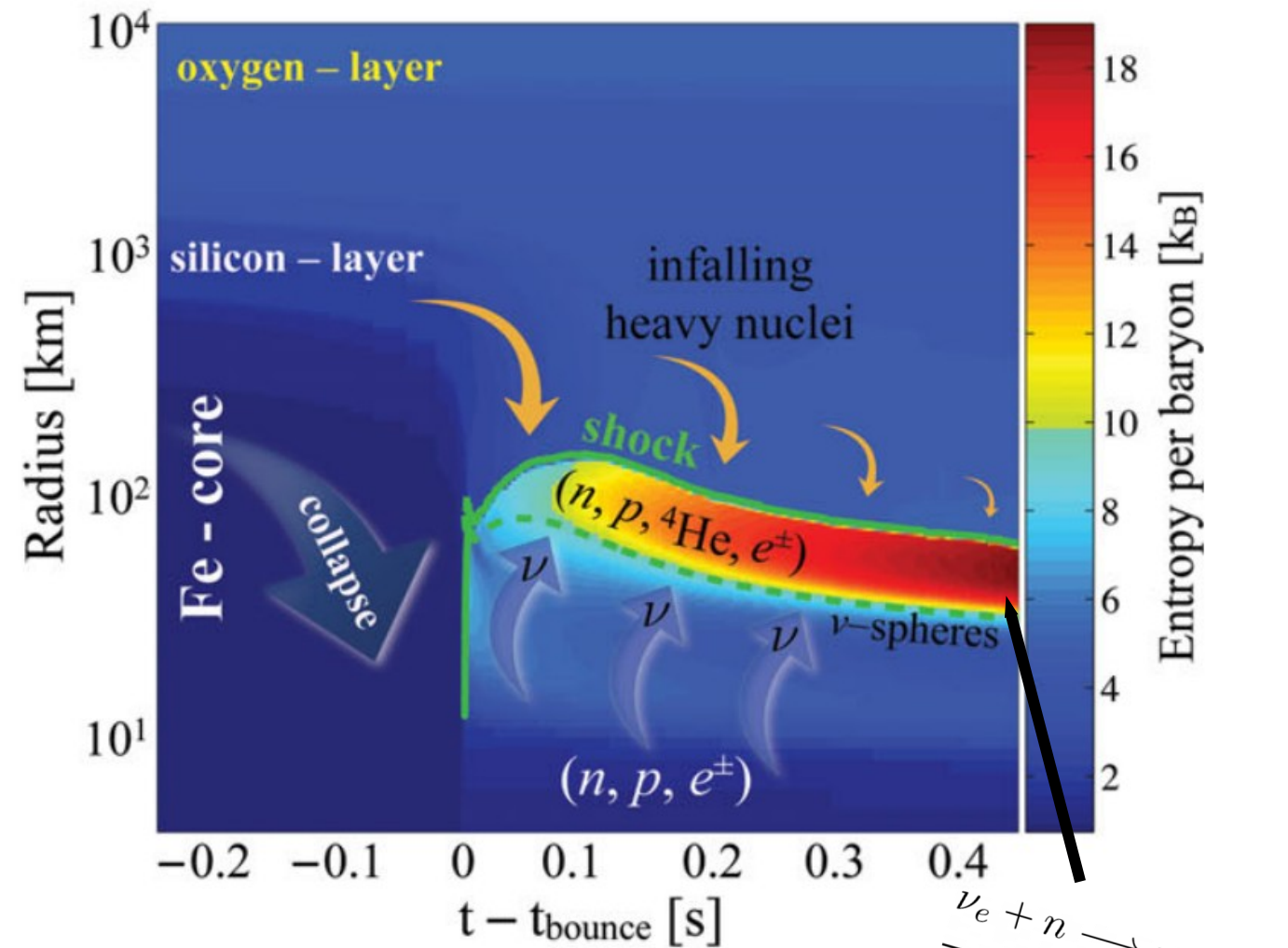
à Core decelerates and bounces

à Shock wave forms  $\sim 1e51$  erg

## III: Explosion phase

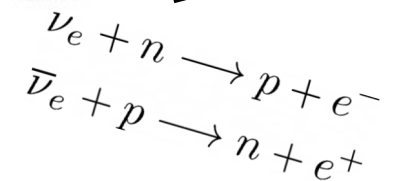
Shock wave stalls...

- “real explosion” sets in 100-200ms post-bounce
- Driven by neutrino heating behind the shock
- Explosion can outshine entire galaxy!
- Asymmetric effects pivotal



Fischer+201

6



CCSN & NSM

?  $T_{crit}$   
?

$10^{12}$  K

Quark-gluon plasma

Hadronic matter

Chiral restoration / Phase transition

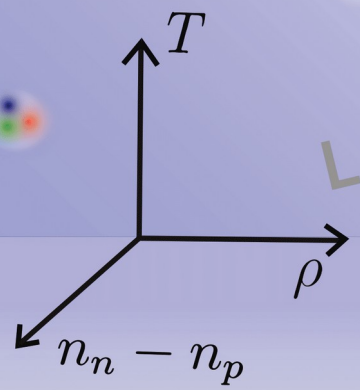
Liquid gas

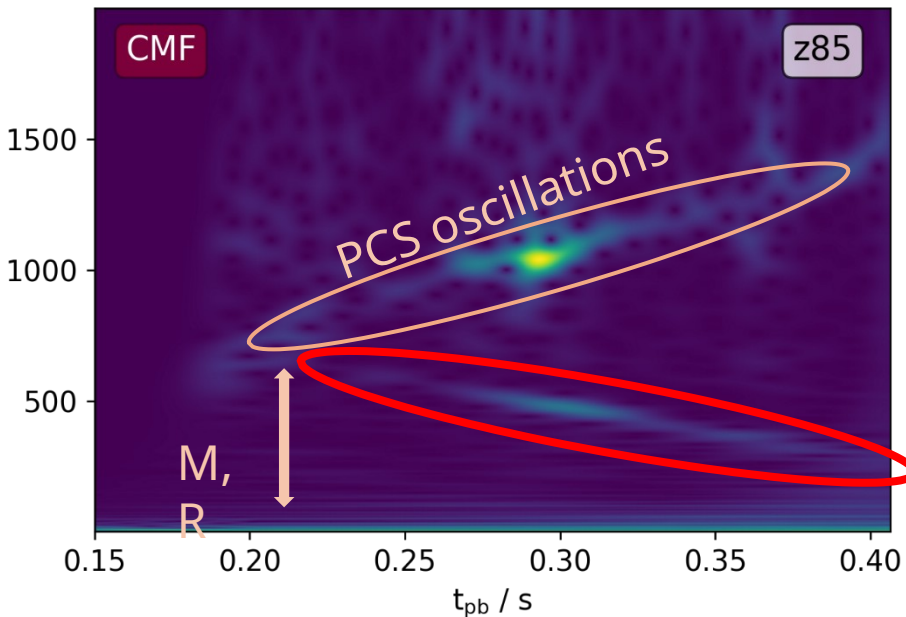
Nuclear matter

$2.3 \cdot 10^{14} \text{gcm}^{-3}$

Neutron stars

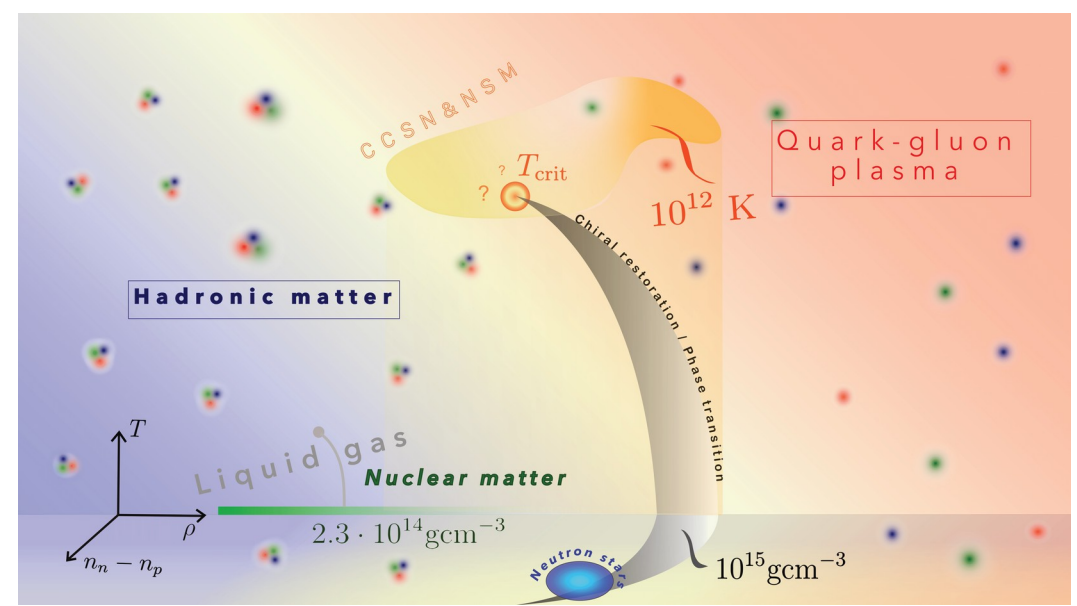
$10^{15} \text{gcm}^{-3}$



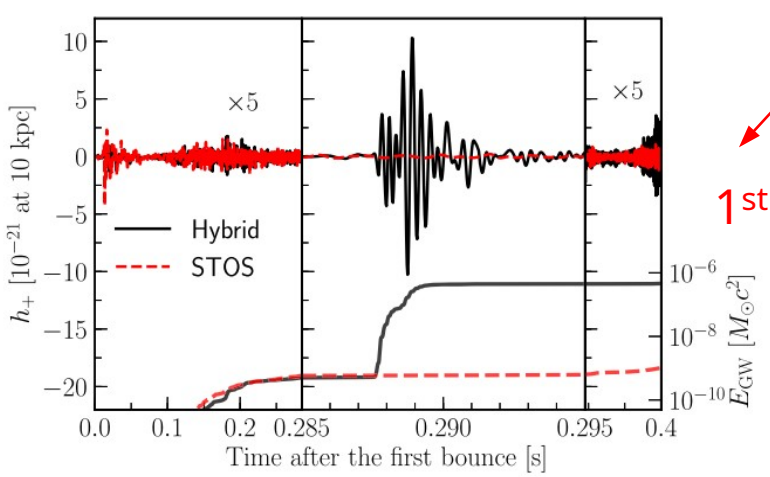


PCS evolution ←

→ Constrain EoS



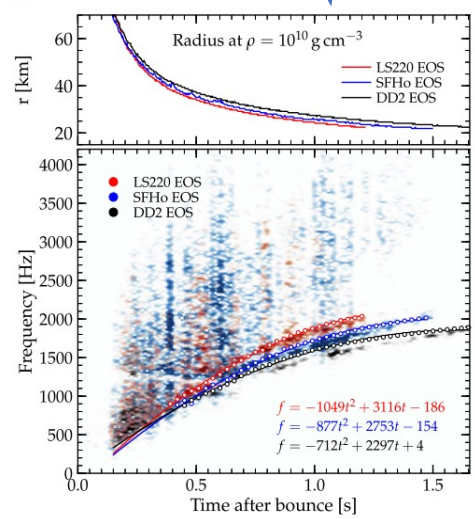
$$f_{\text{peak}} \approx \frac{1}{2\pi} \frac{GM}{R^2} \sqrt{1.1 \frac{m_n}{\langle E_\nu \rangle} \left(1 - \frac{GM}{Rc^2}\right)^2}$$



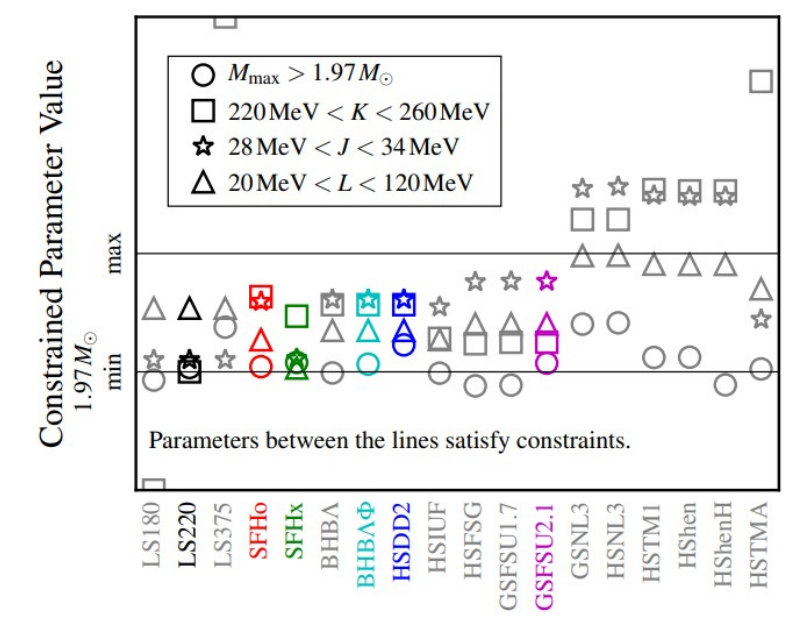
(Zha et al., 2020)

Free quarks?

1<sup>st</sup> order PT



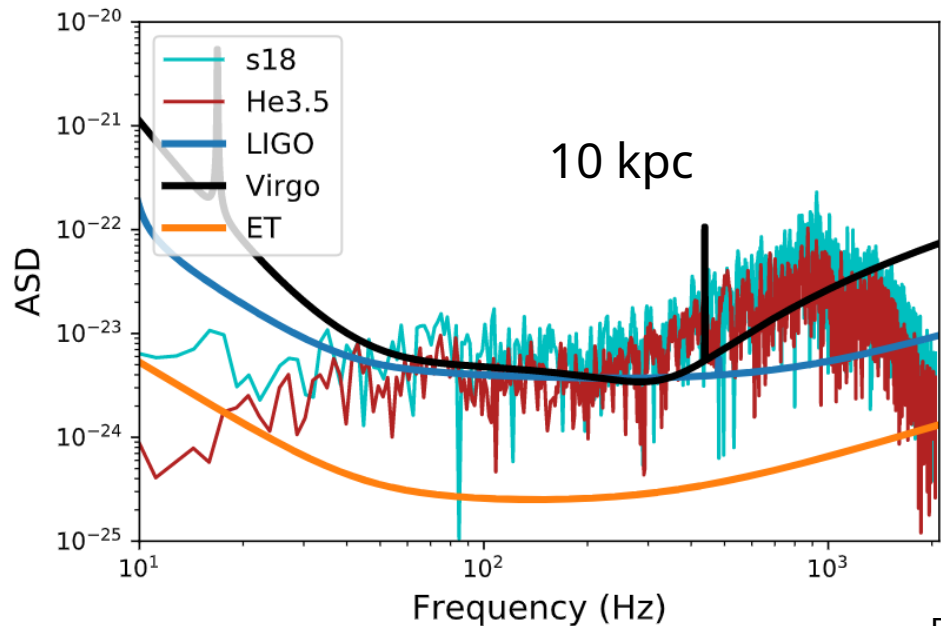
(Morozova et al., 2018)



(Richers et al., 2017)



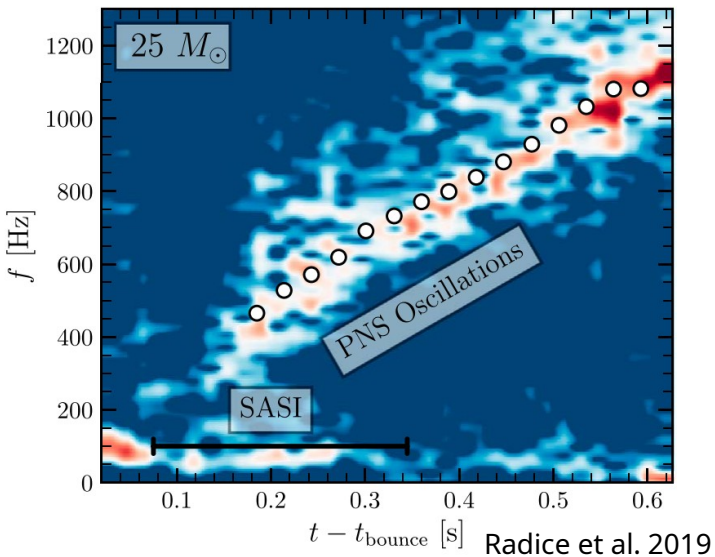
# Gravitational Waves from Core-Collapse Supernovae



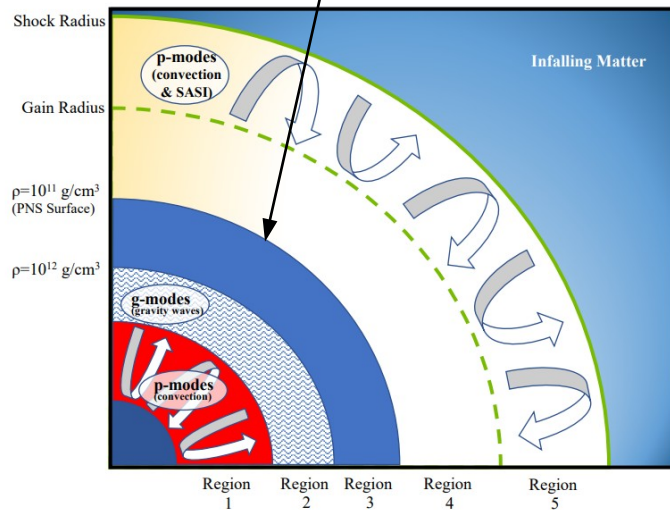
- CCSNe ~3 event per century in our Galaxy
- Observable distance ~10-20 kpc
- Time changing mass quadrupole moment
- GWs provide access to compact inner core

Powell & Mueller (2018)

Radius of Proto-Compact Stars ~40km to ~10km  
 Mass accretion on PCS: ~1.4-2M<sub>⊙</sub>



Radice et al. 2019



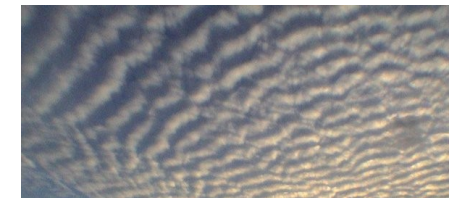
Mezzacappa et al. 2020

- Different regions susceptible to different mode oscillations (p-,g- f-modes)
- Positive entropy gradient: g-mode oscillations
- Characteristic frequency of g-modes:

*Bruno Väisälä frequency*

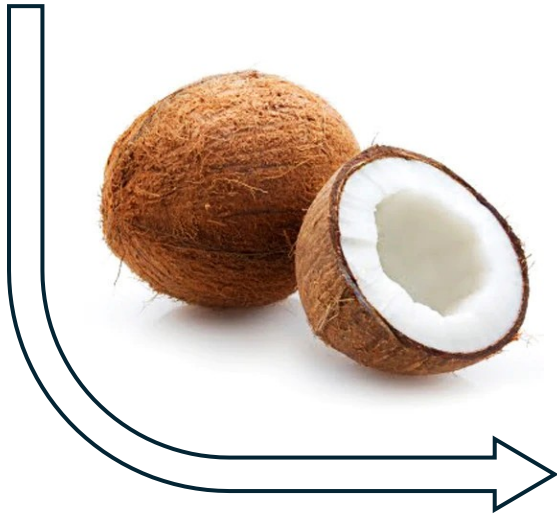
$$\omega_{BV}^2 = \frac{d\alpha}{dr} \frac{\alpha}{\rho h \Phi^4} \cdot \left( \frac{1}{c_s^2} \frac{dP}{dr} - \frac{d\rho}{dr} \right)$$

Stripy clouds →



# Setup

CoCoNut-FMT

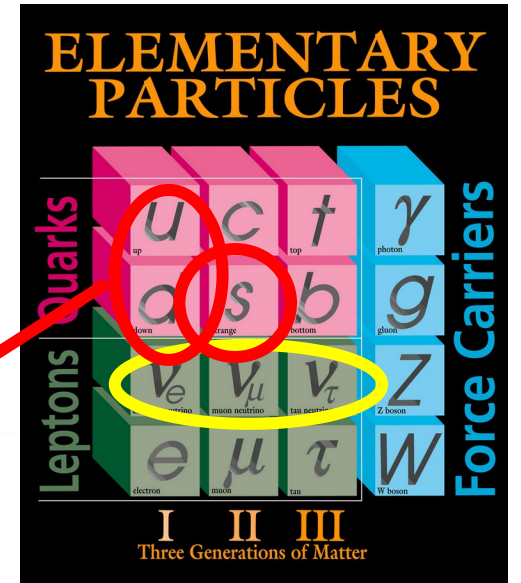


Kepler



Feed-in (1D)

Incorporated  
hadron-quark  
EoS into  
CoCoNuT-FMT



neutrinos.fnal.gov

- Ø **Godunov-based** finite-volume solver (Dimmelmeier et al. 2002; Müller et al. 2013)
- Ø 3+1 splitting formalism; **CFC approximation**
- Ø Three flavour **Fast Multigroup Transport (FMT)**

Up to 50 million core hours 😊

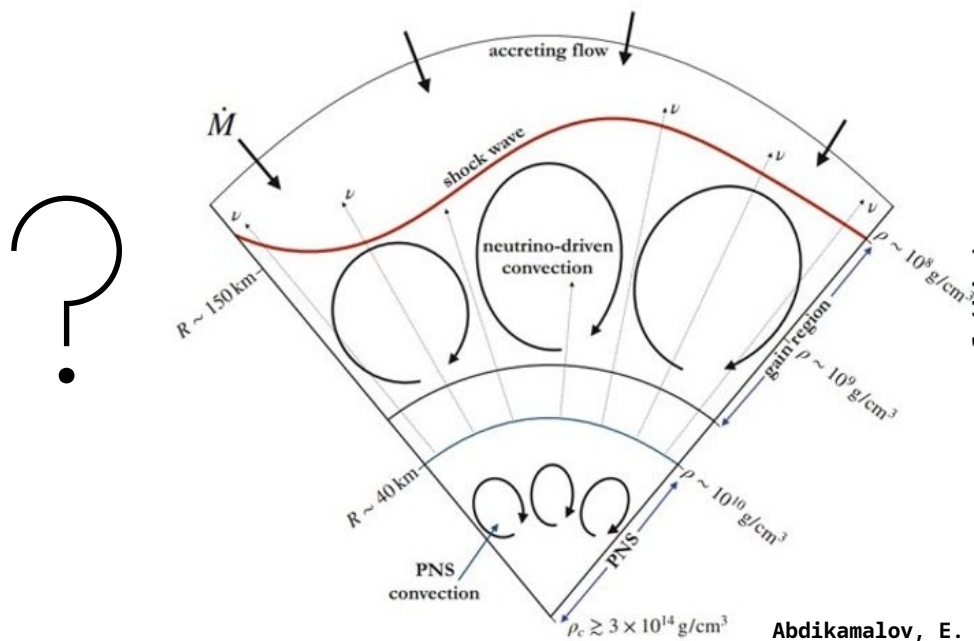


CoCoNuT-FMT: Müller et al. (2010), Müller & Janka (2015)  
Kepler: Weaver et al. (1978), Heger & Woosley (2010)

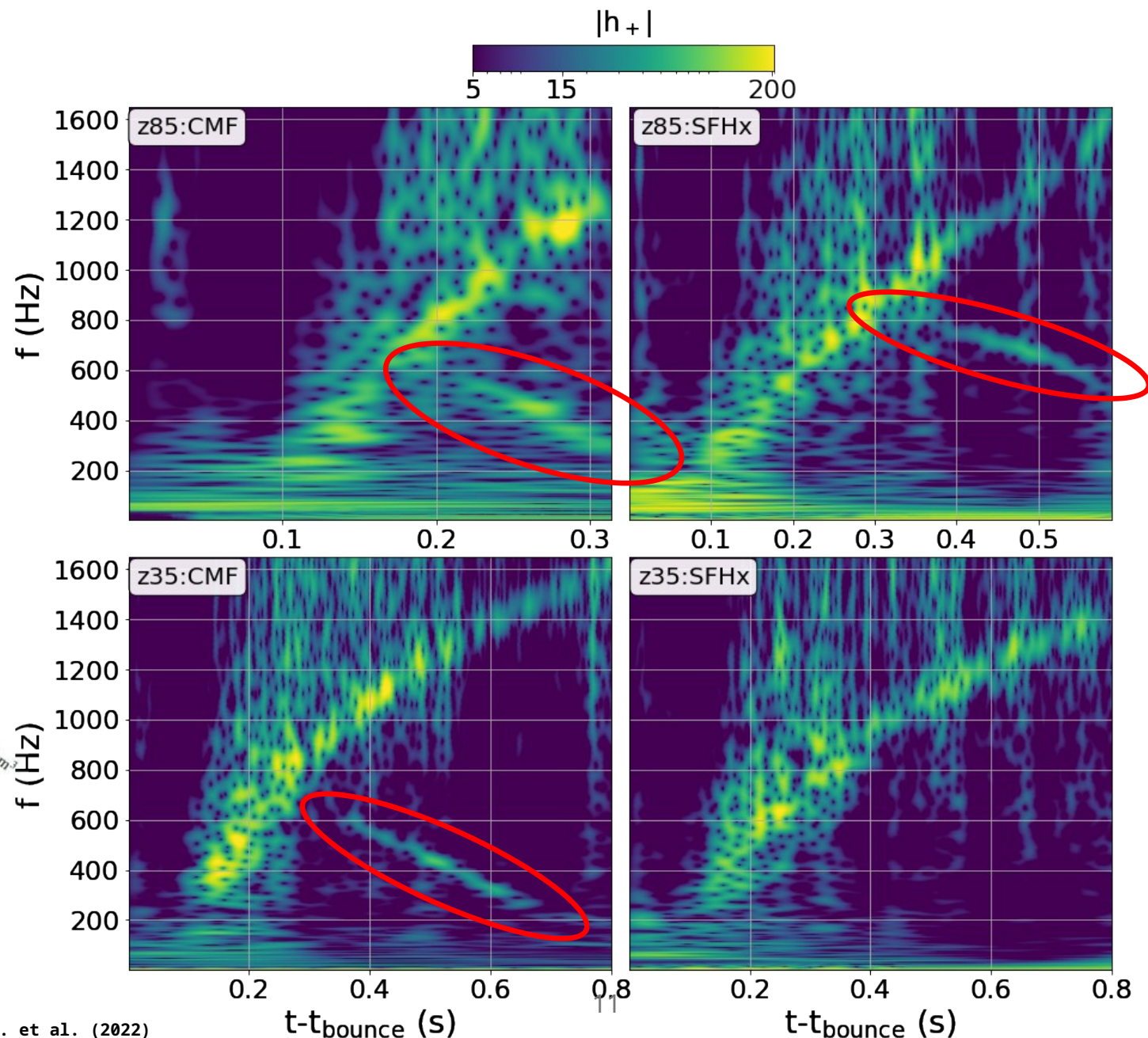
- Gravitational wave signals show lower frequency g-modes
- Frequency depends on Equation of State
  - Where is signal originating from?
  - No signal in z35:SFhx?
  - Lower frequencies in CMF?

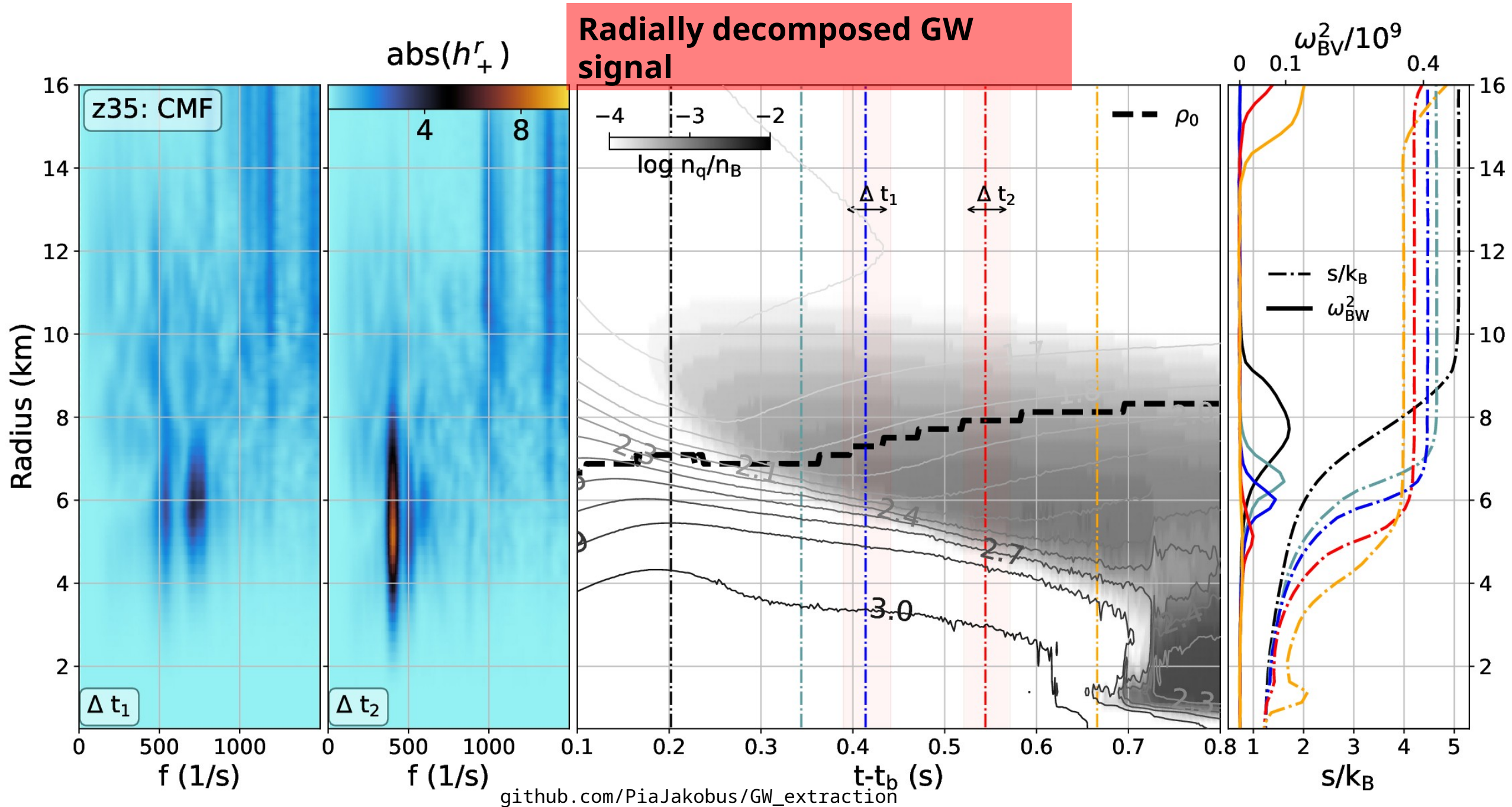


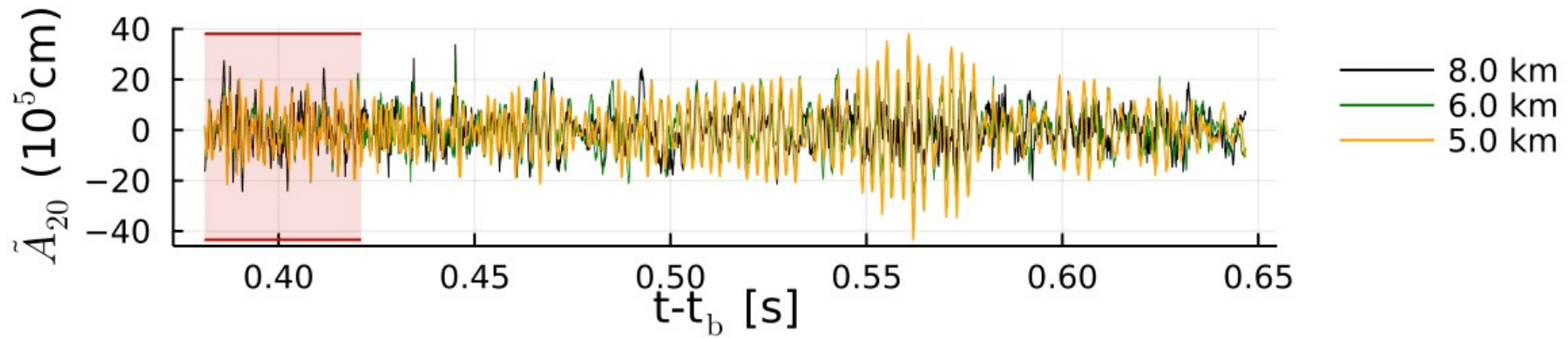
1) Where is the signal originating from?



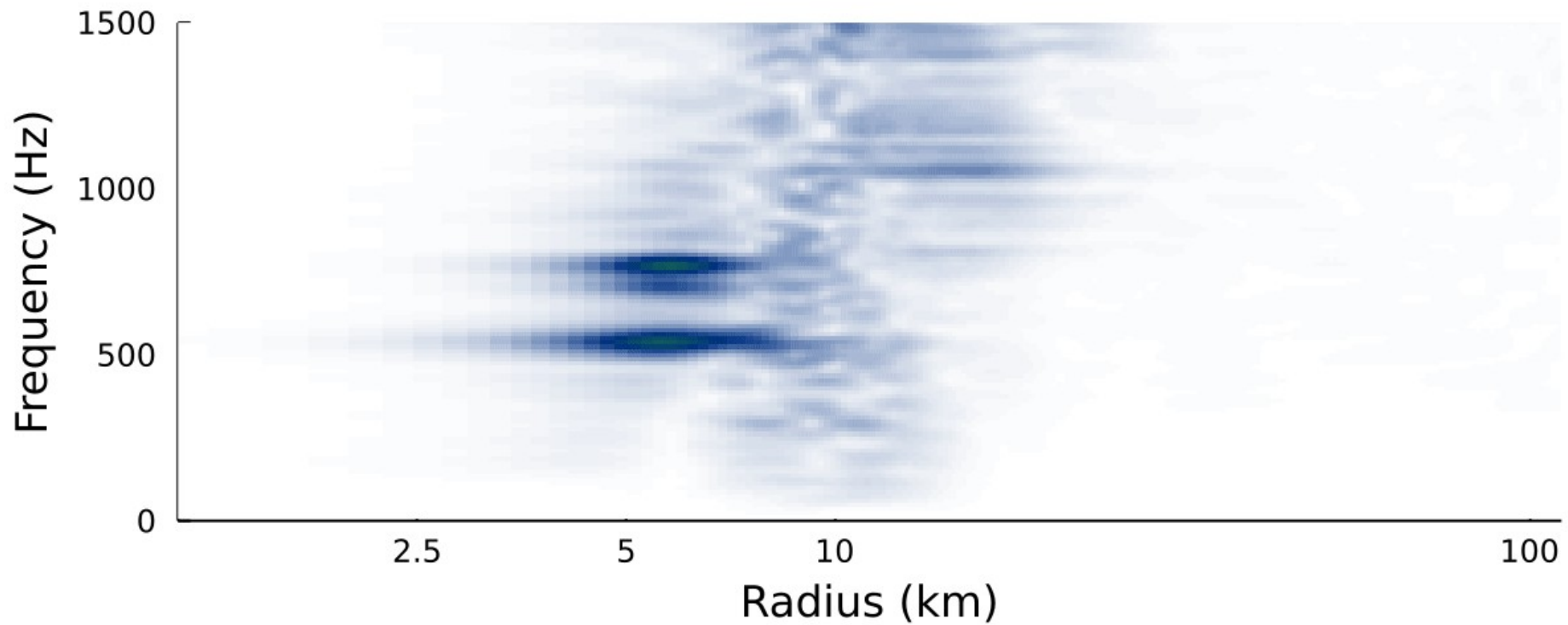
Abdikamalov, E. et al. (2022)



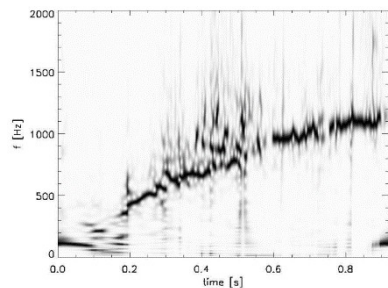
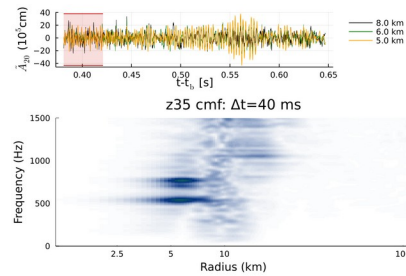
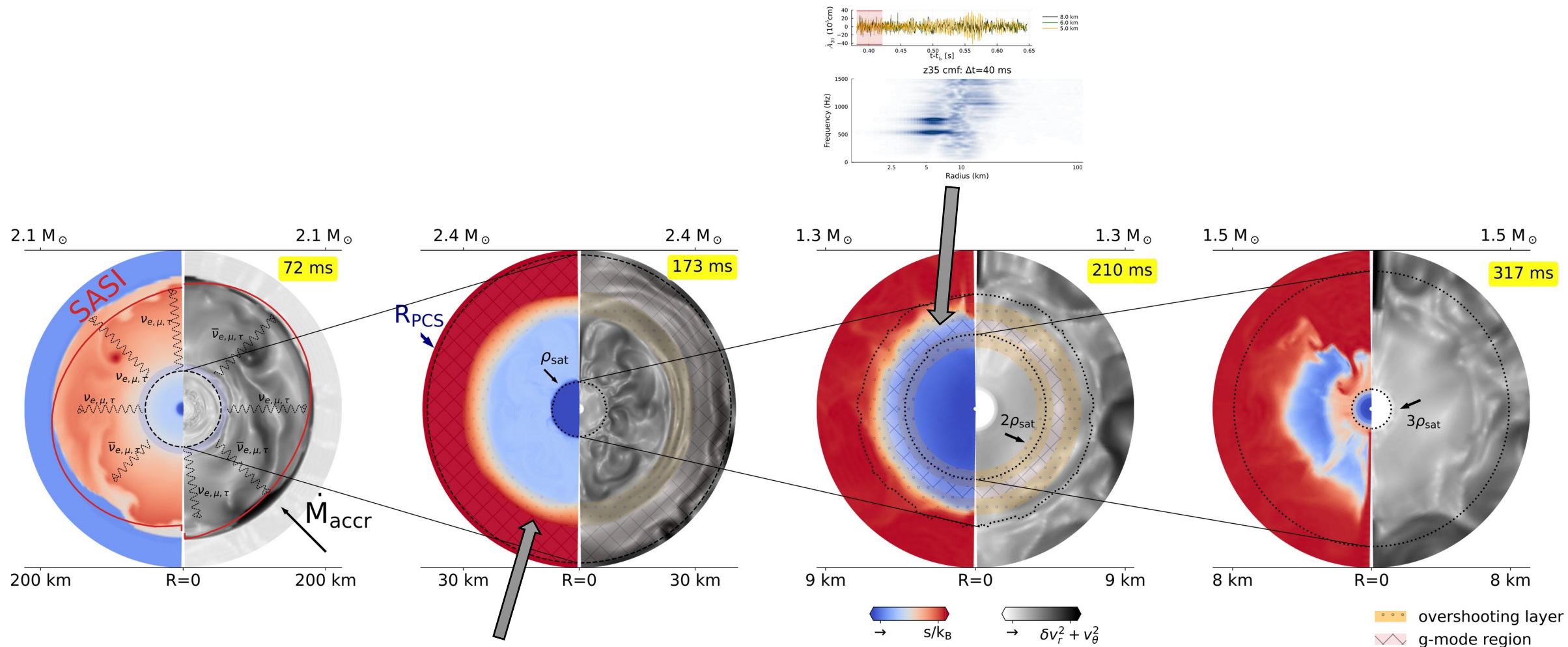




z35 cmf:  $\Delta t=40$  ms

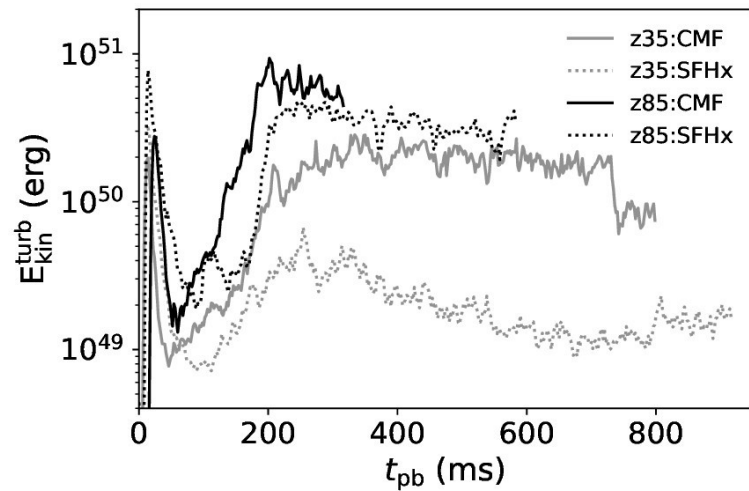




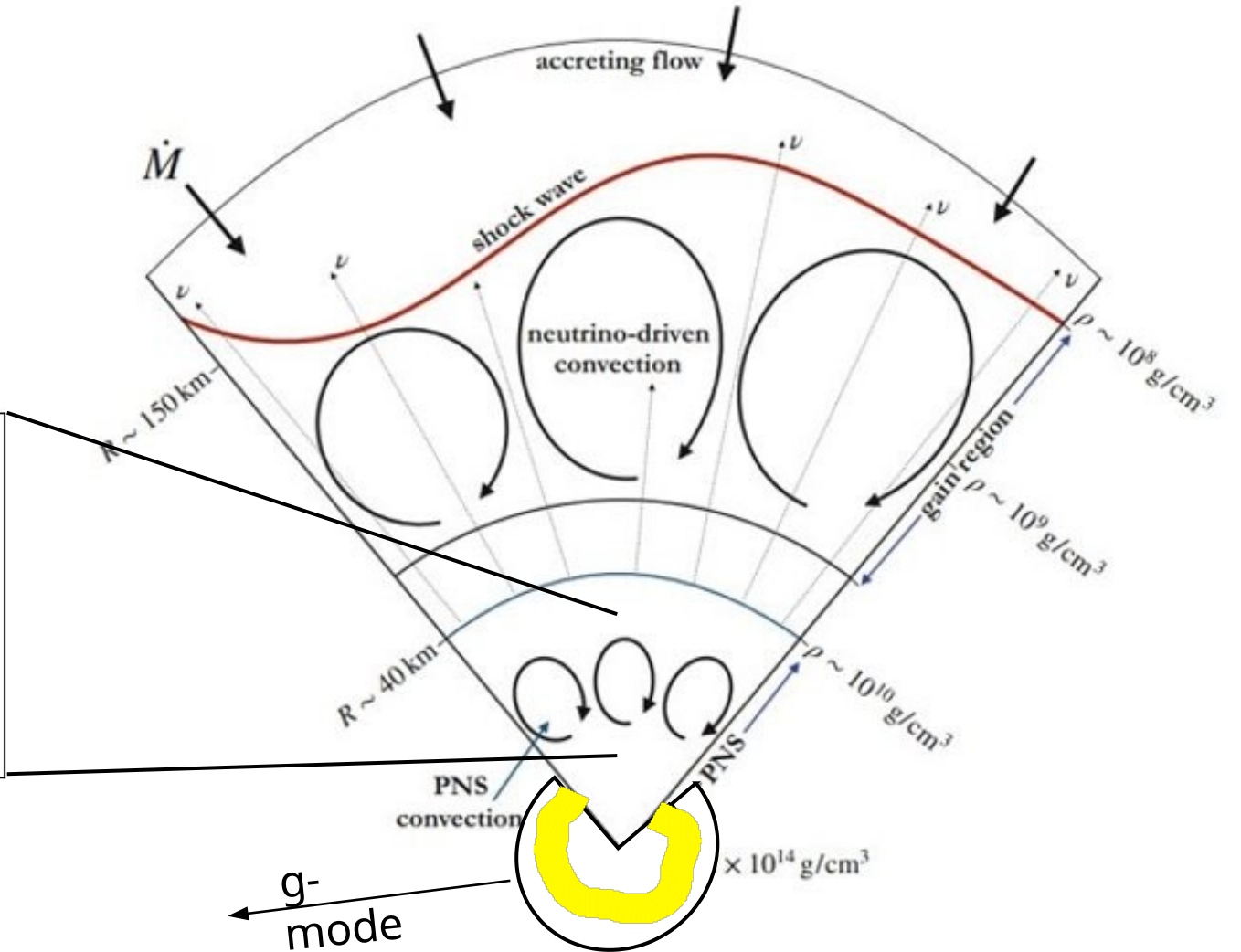


## Where is signal originating from?

1. **g-mode "lives" in convectively stable PCS region beneath the PCS convection zone**
2. Higher turbulent convective energies seen in CMF, particularly z35



$$E_{\text{kin}}^{\text{turb}} = \frac{1}{2} \int_{\text{PCS}} \delta v^2 \rho r^2 dV$$



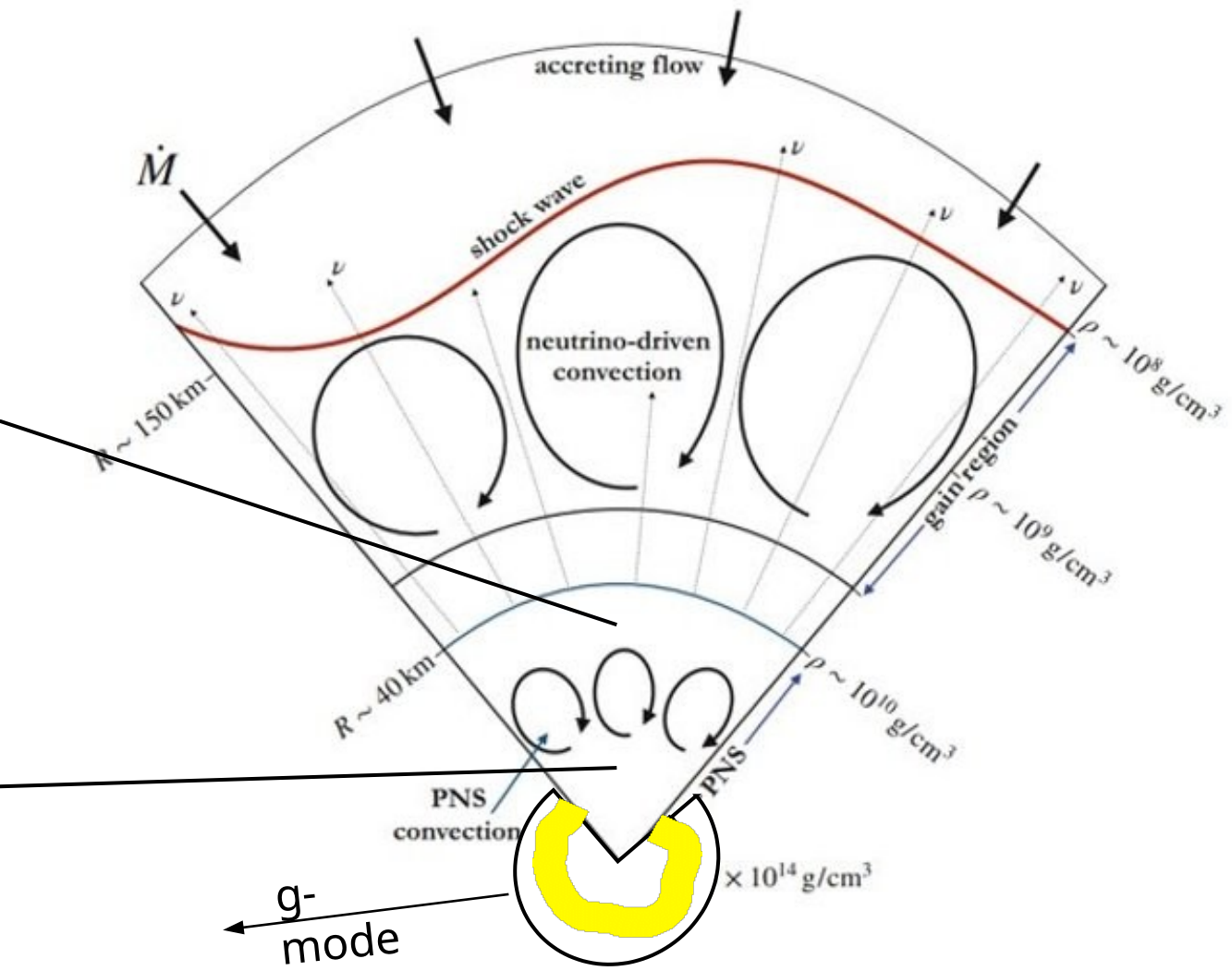
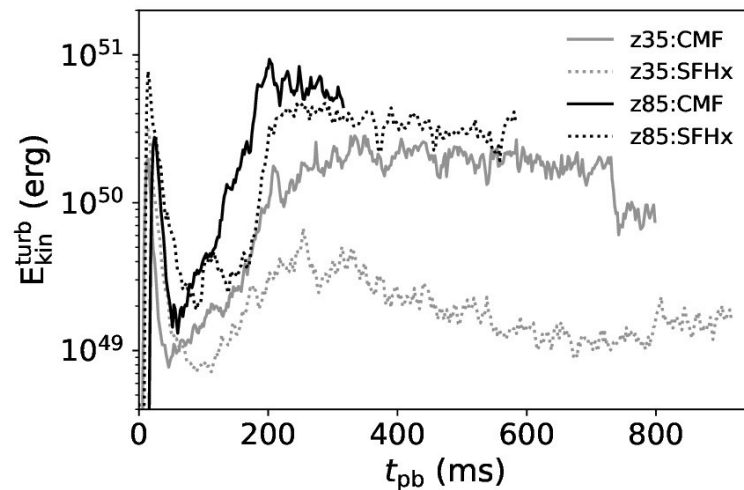
(Abdikamalov et al. 2022)



## No signal in z35:SFhx?

1. g-mode "lives" in convectively stable PCS region beneath the PCS convection zone

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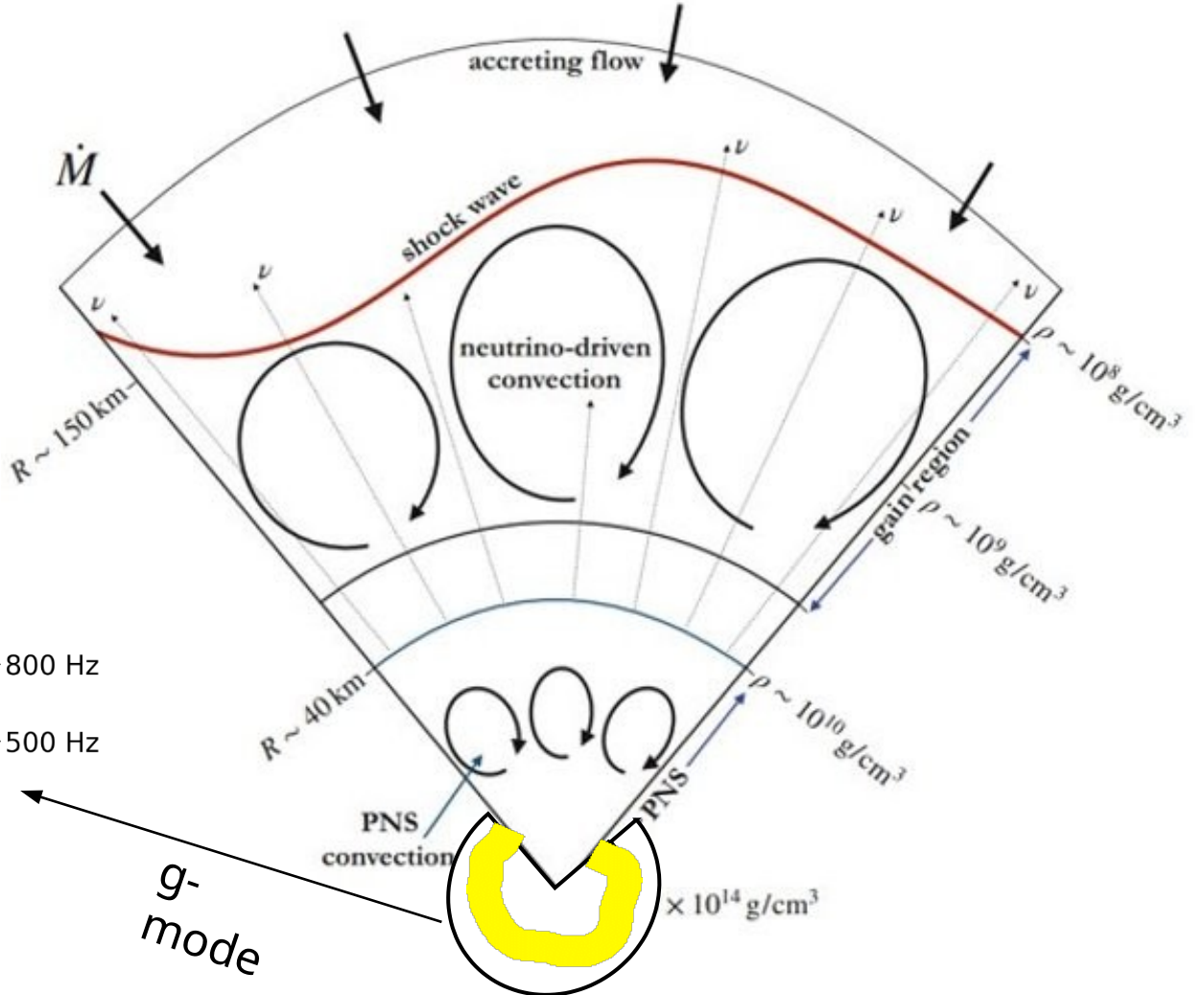
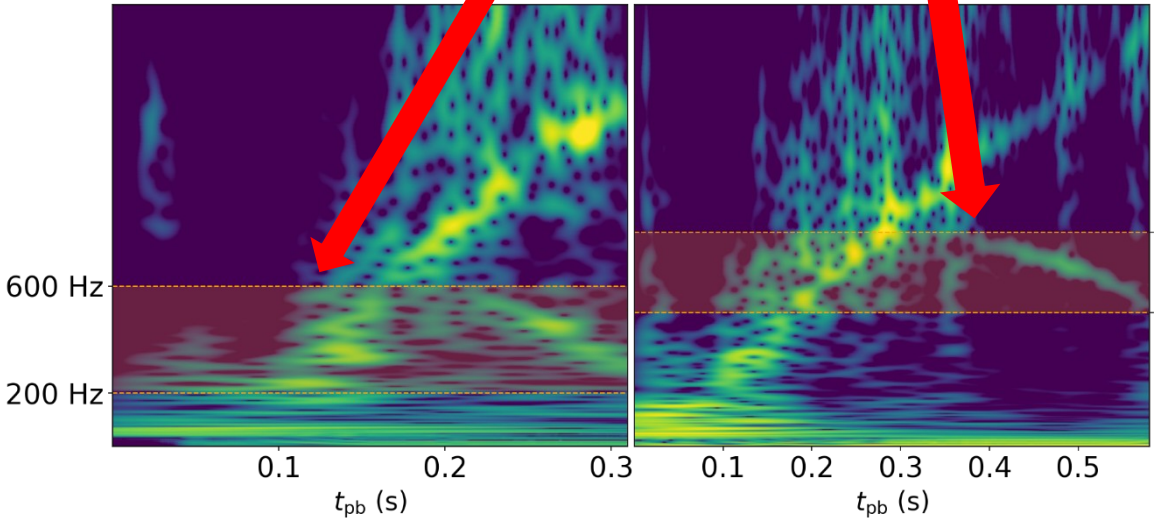


$$E_{\text{kin}}^{\text{turb}} = \frac{1}{2} \int_{\text{PCS}} \delta v^2 \rho r^2 dV$$

(Abdikamalov et al. 2022)

3. What EoS properties cause ~20% lower g-mode frequencies in CMF?

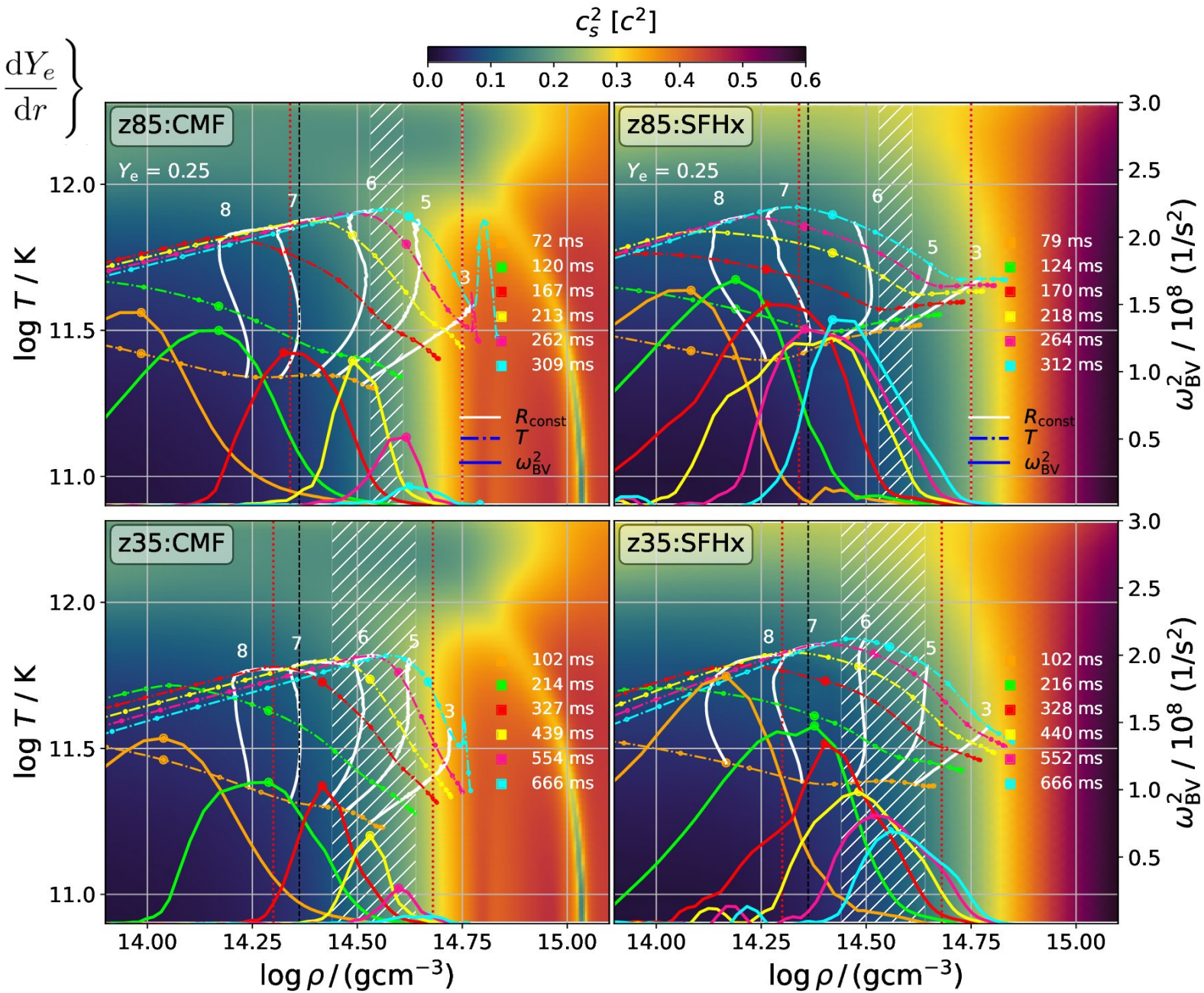
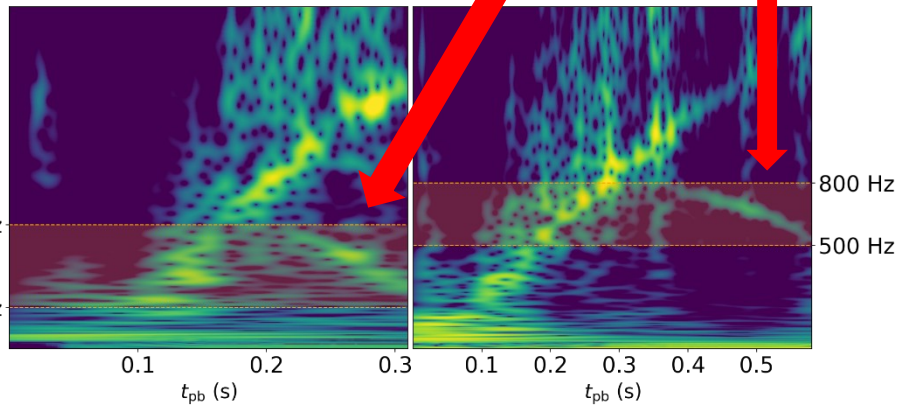
?

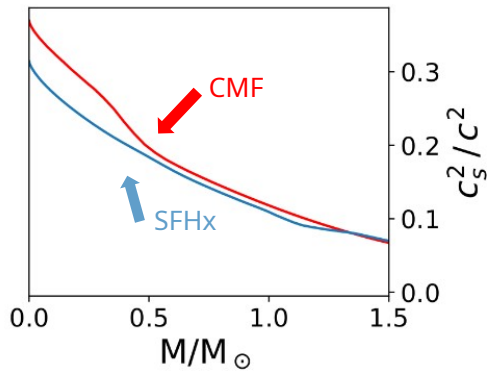


(Abdikamalov et al. 2022)

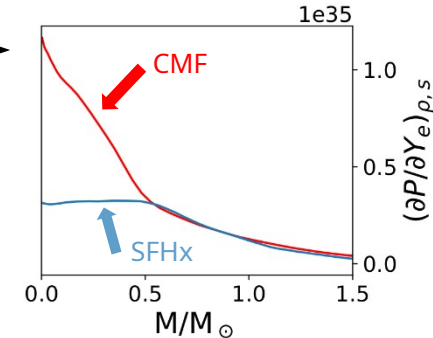
$$\omega_{\text{BV}}^2 = \frac{d\alpha}{dr} \frac{\alpha}{\rho h \Phi^4} \cdot \frac{1}{c_s^2} \left\{ \left( \frac{\partial P}{\partial s} \right)_{\tilde{\rho}, Y_e} \frac{ds}{dr} + \left( \frac{\partial P}{\partial Y_e} \right)_{\tilde{\rho}, s} \frac{dY_e}{dr} \right\}$$

What EoS properties cause a ~20% lower g-mode frequency in CMF?

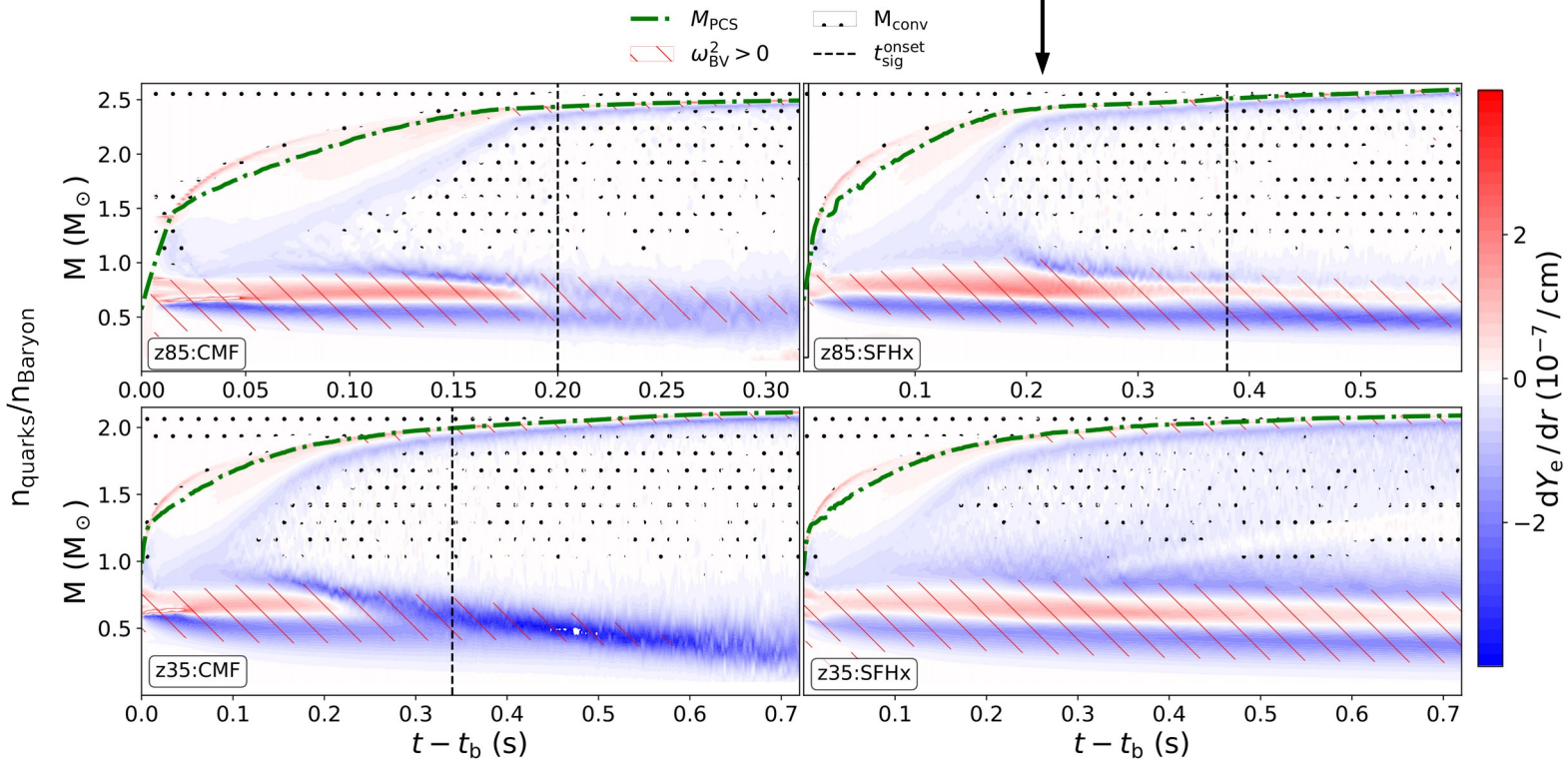
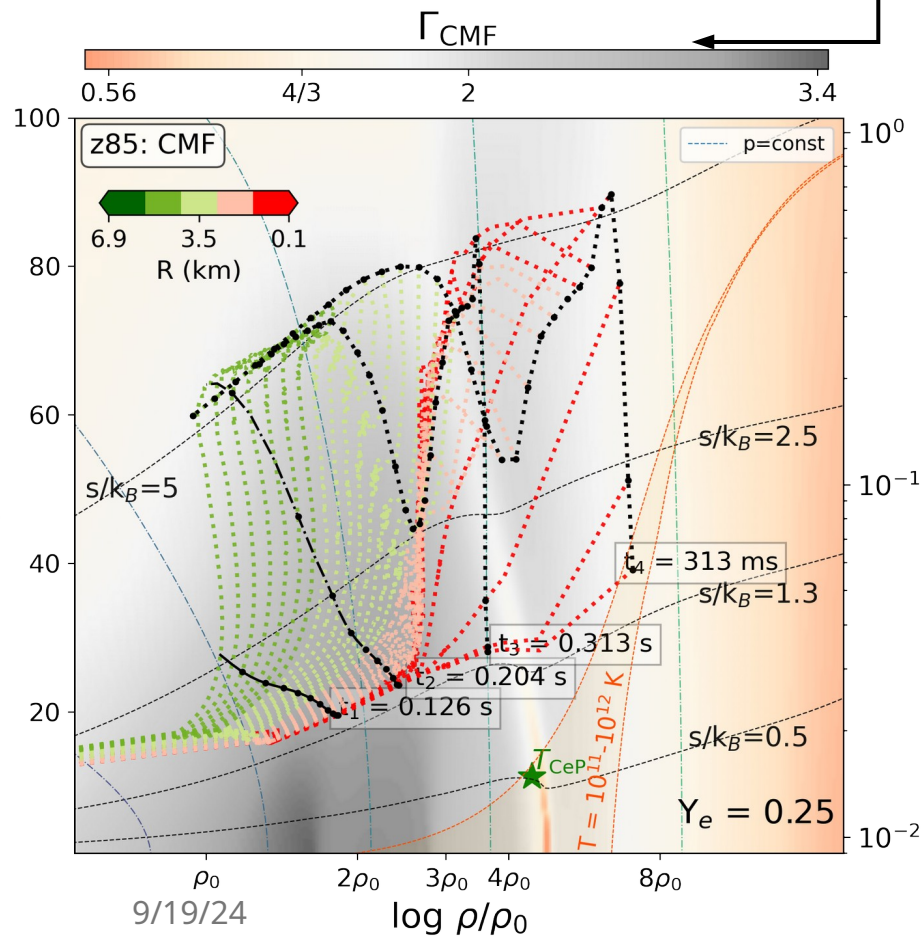


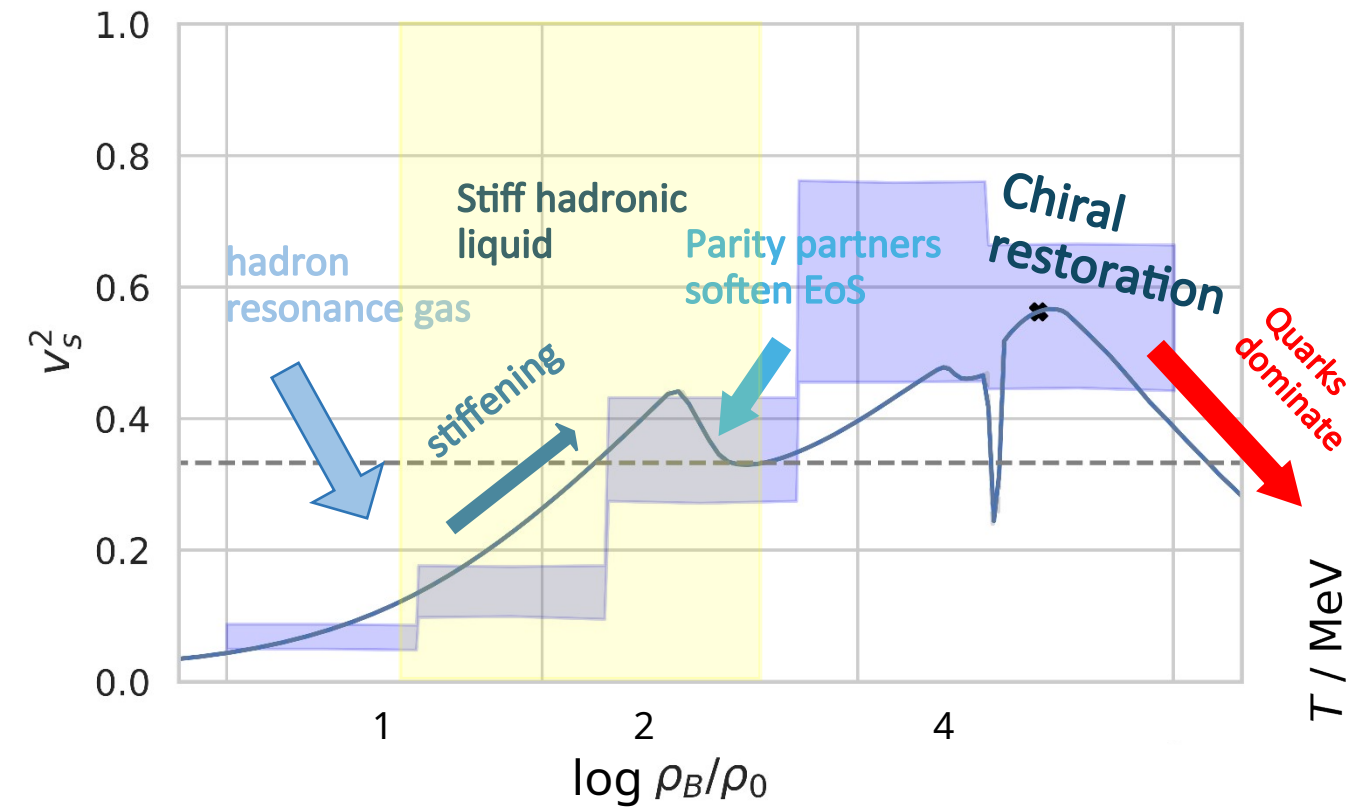


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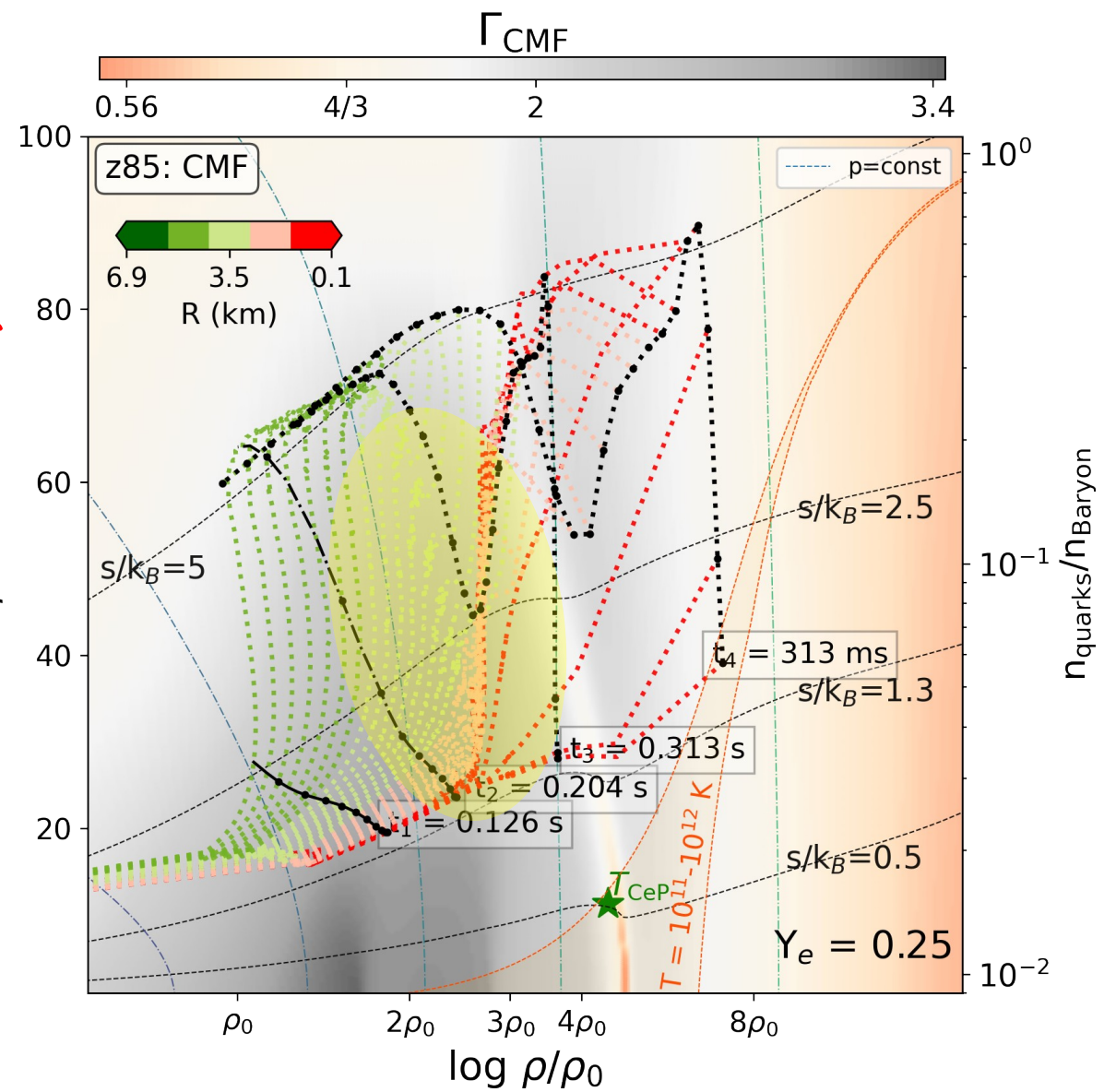


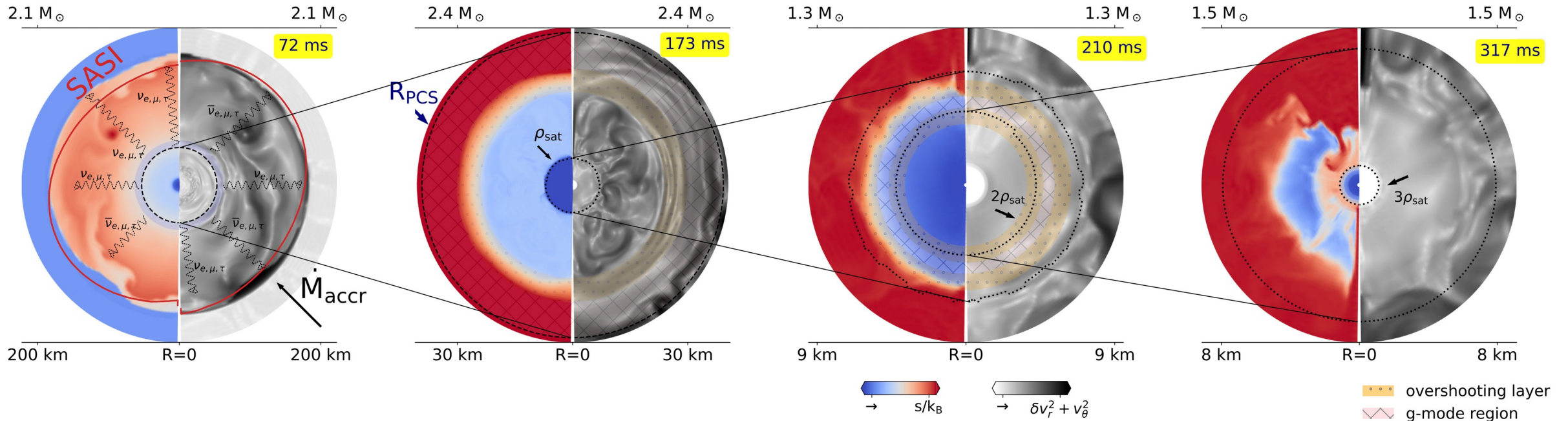
What EoS properties cause ~20% lower g-mode frequencies in CMF?





- Quarks are abundant in very low numbers
- Matter dominated by Nucleons / Quarks
- Interplay of electron gradient in combination with a large  $(\partial \mu_e / \partial Y_e)_{\rho, s}$  drives down B.V.f.



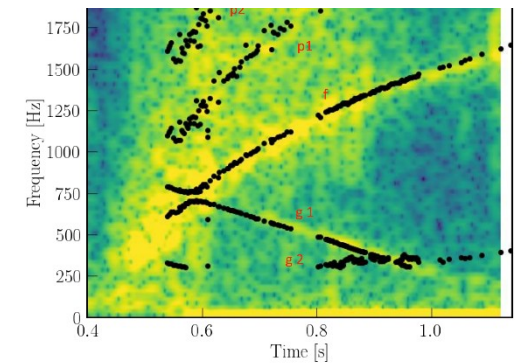


# Summary

✓ Signal corresponds to g-mode oscillations in the PCS

→ ~5-10 km (200-500 ms post-bounce)

→ l-mode [Zha et al., paper in prep]



✓ Mode frequency very sensitive to speed of sound at around twice saturation density

✓ Low frequency feature of mode lies within the sensitivity range of current GW detectors

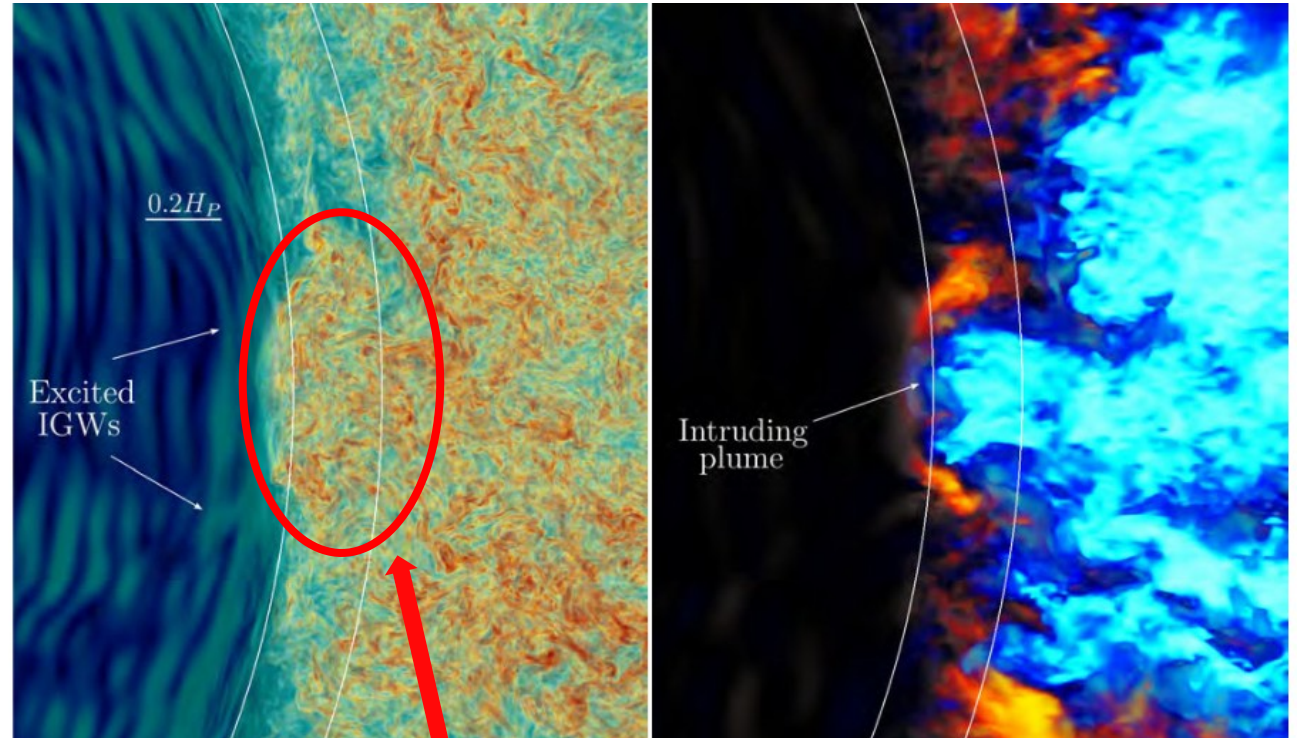
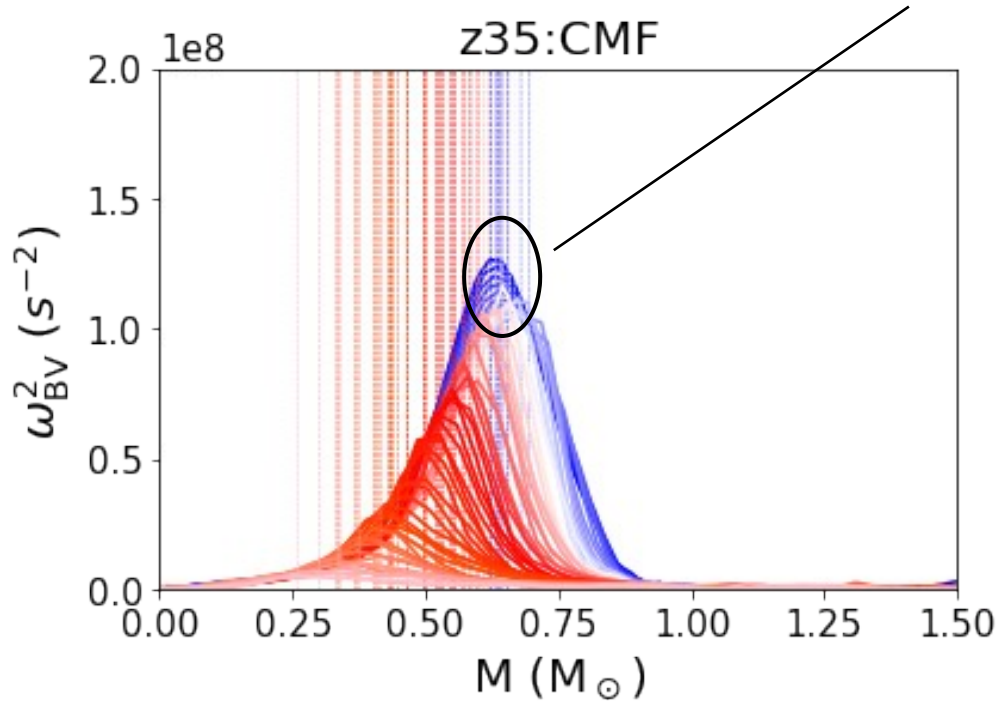
**2D versus 3D: Jakobus et al. 2024**

# If a fluid parcel is perturbed- will it settle back into its equilibrium position?



Oscillation of a displaced fluid element  $\longrightarrow$

Higher frequencies give rise to larger 2<sup>nd</sup> order time-derivatives of the mass quadrupole moment



**Blouin+23**

Based on the Ledoux criterion:

$$\omega_{\text{BV}}^2 = \frac{d\alpha}{dr} \frac{\alpha}{\rho h \Phi^4} \cdot \frac{1}{c_s^2} \left\{ \left( \frac{\partial P}{\partial s} \right)_{\tilde{\rho}, Y_e} \frac{ds}{dr} + \left( \frac{\partial P}{\partial Y_e} \right)_{\tilde{\rho}, s} \frac{dY_e}{dr} \right\}$$

Overshoot into convectively stable region  
Buoyancy force back toward convective region

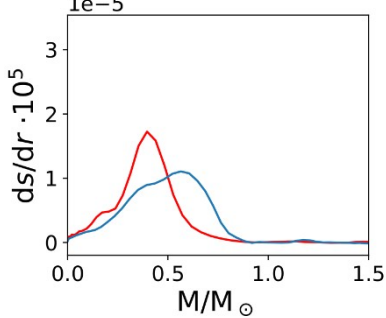
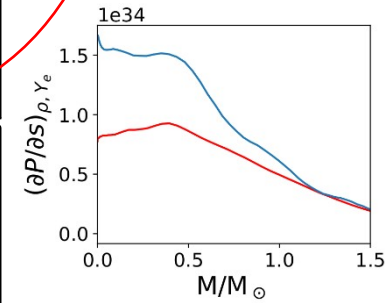
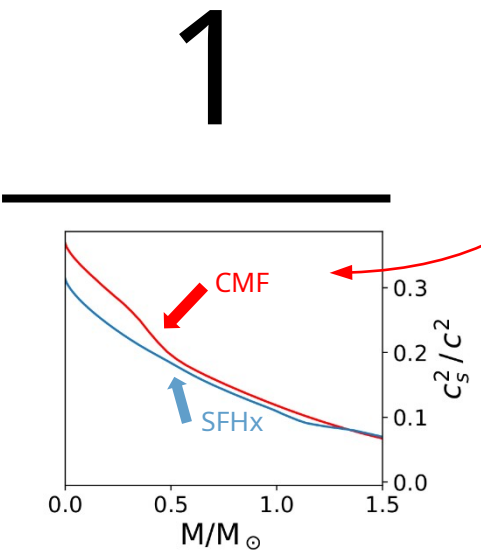
**z35  
progenitor  
550 ms**

$$\omega_{\text{BV}}^2 = \frac{d\alpha}{dr} \frac{\alpha}{\rho h \Phi^4} \cdot \frac{1}{c_s^2} \left\{ \left( \frac{\partial P}{\partial s} \right)_{\tilde{\rho}, Y_e} \frac{ds}{dr} + \left( \frac{\partial P}{\partial Y_e} \right)_{\tilde{\rho}, s} \frac{dY_e}{dr} \right\}$$

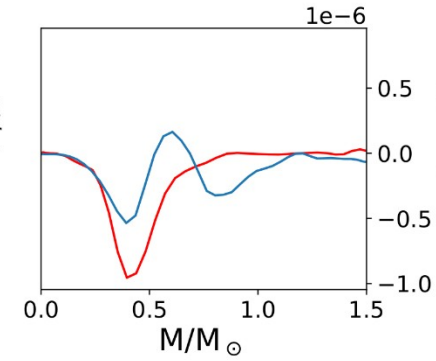
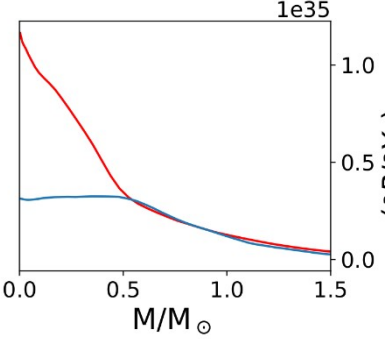
Speed of sound  
larger  
(CMF)

Increase  
s  
over  
time  
(CMF)

Changes sign  
Absolute becomes  
larger (CMF)

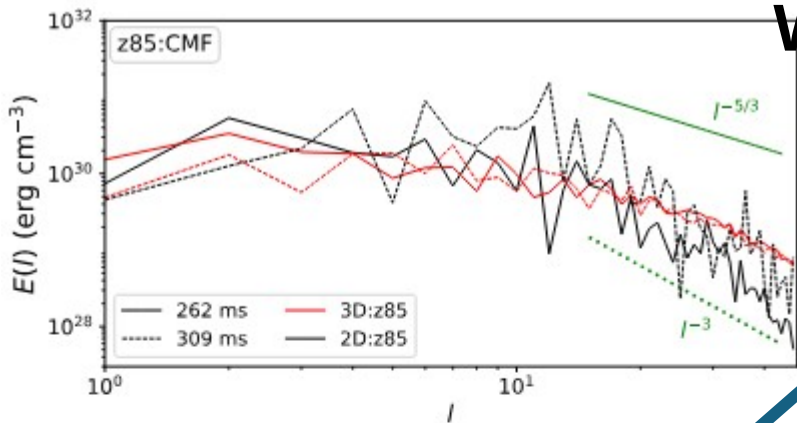


+



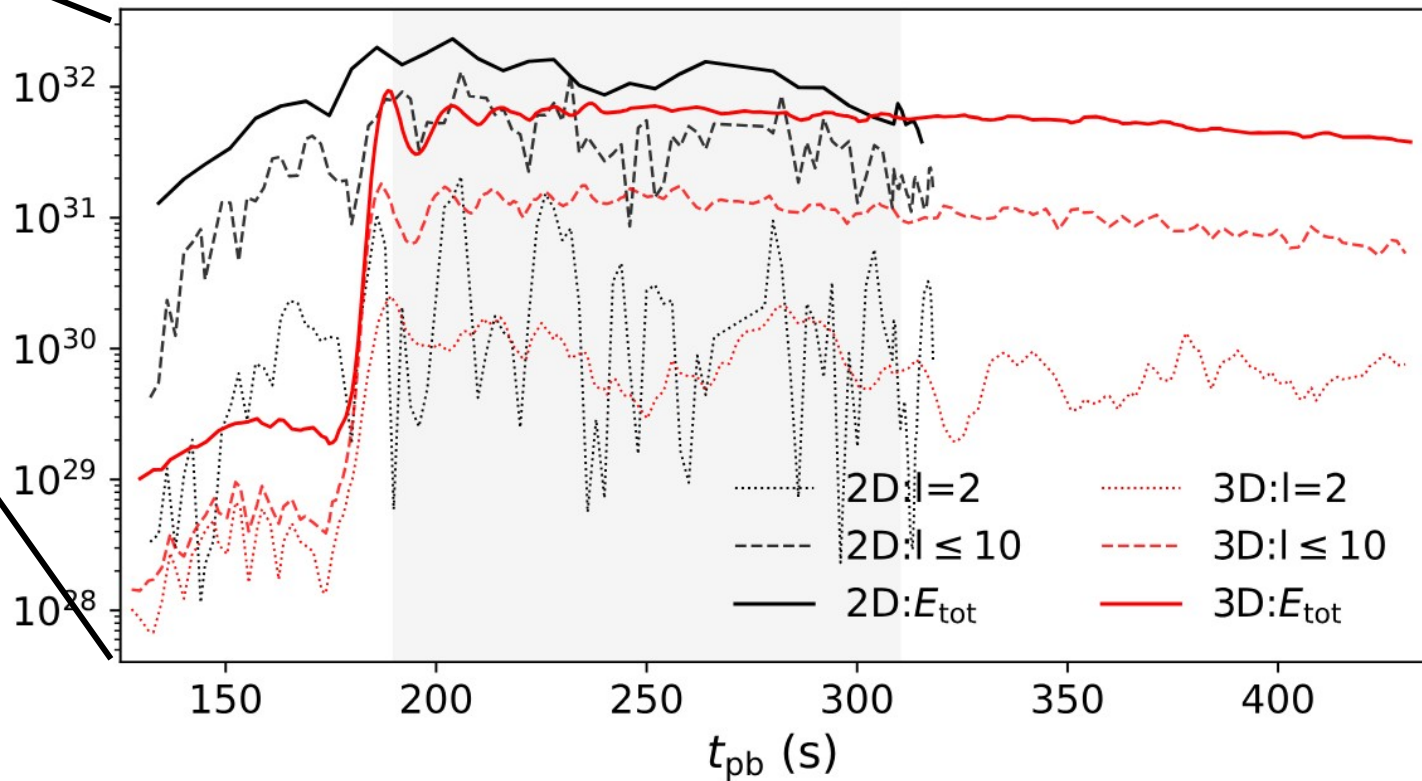
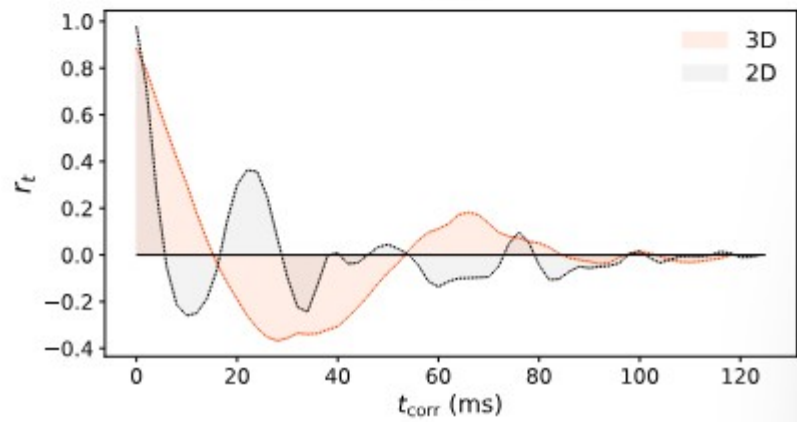
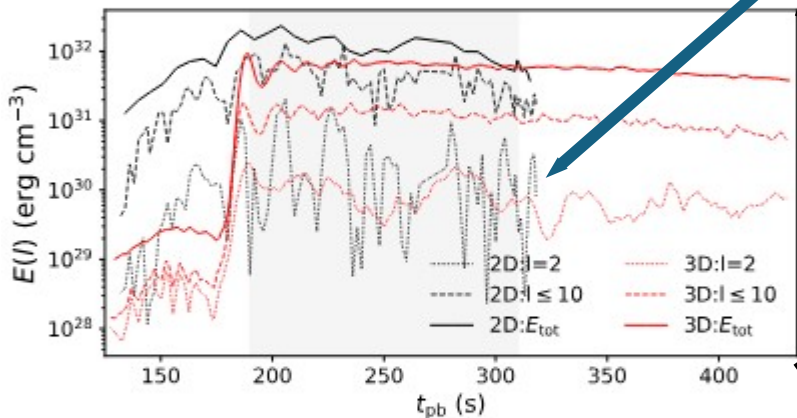


# Why is the GW signal absent in 3D?

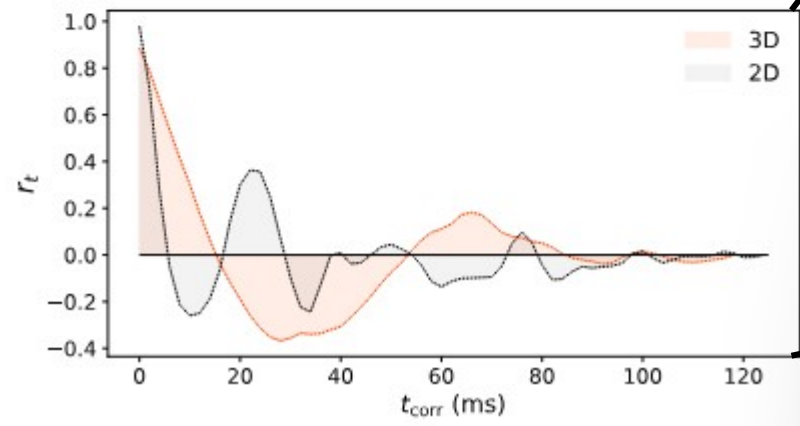
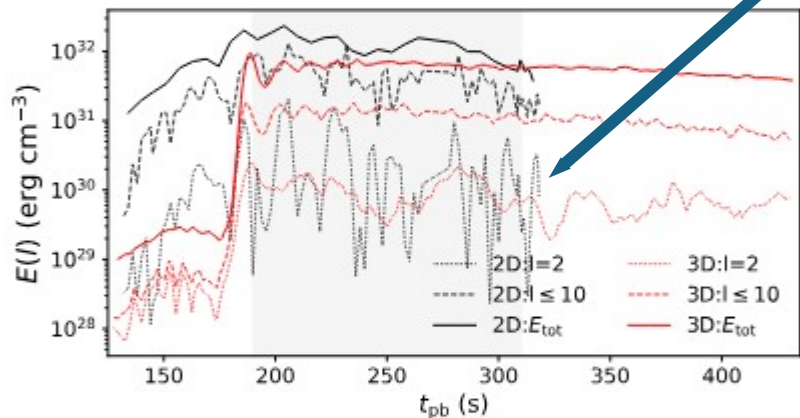
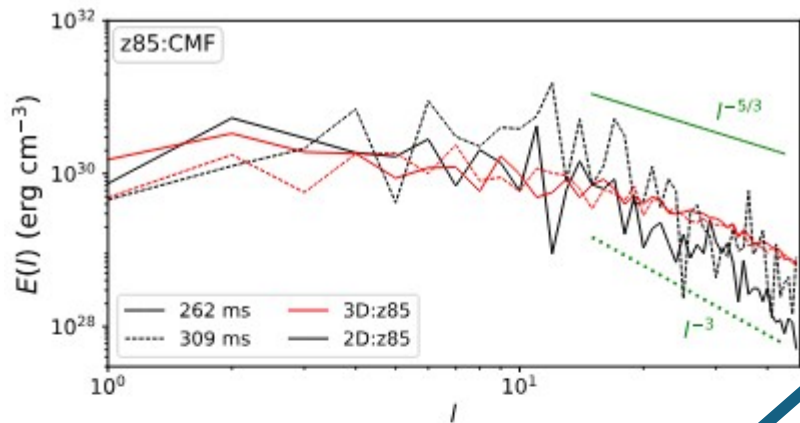


- 1<sup>st</sup> guess: Inverse cascade leads to more quadrupolar turbulent motion in 2D and more resonant excitation?

Absolute strength very similar...



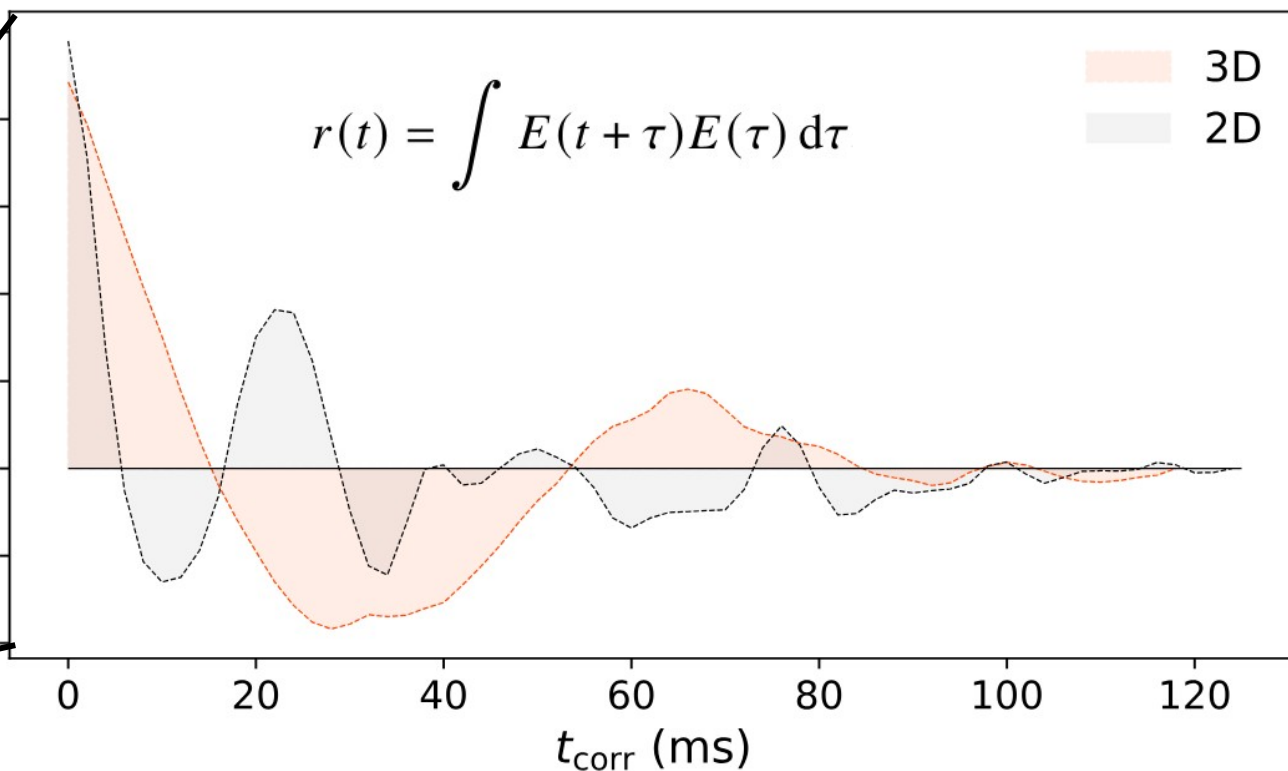
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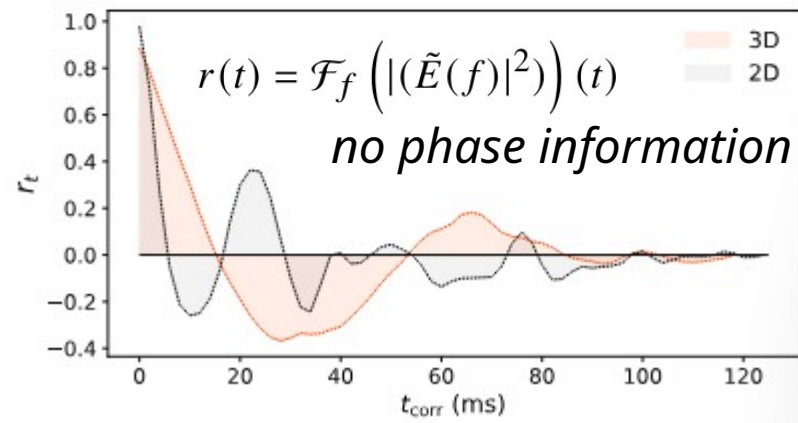
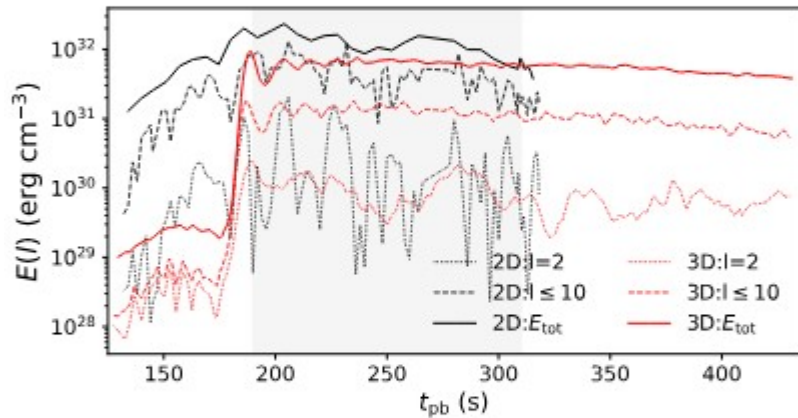
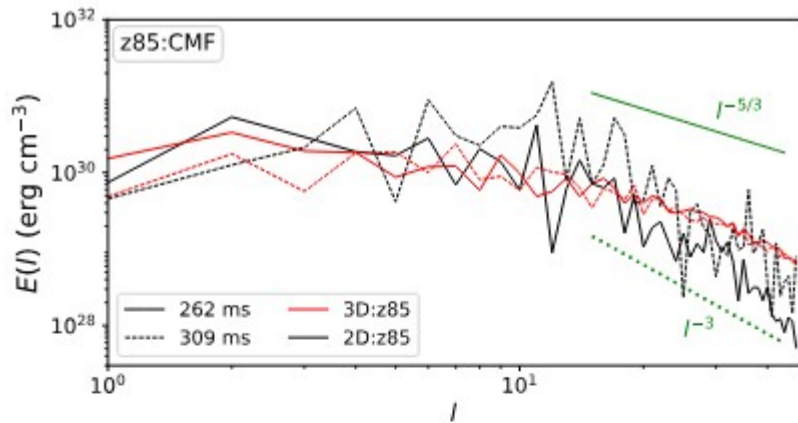


2<sup>nd</sup> guess : Larger temporal variability:

- Reflected in shorter auto-correlation time of  $E(2)$
- Increased power at higher frequencies in the Fourier spectra

Physical meaning: faster decay rates of eddies in 2D  
greater velocity dispersion

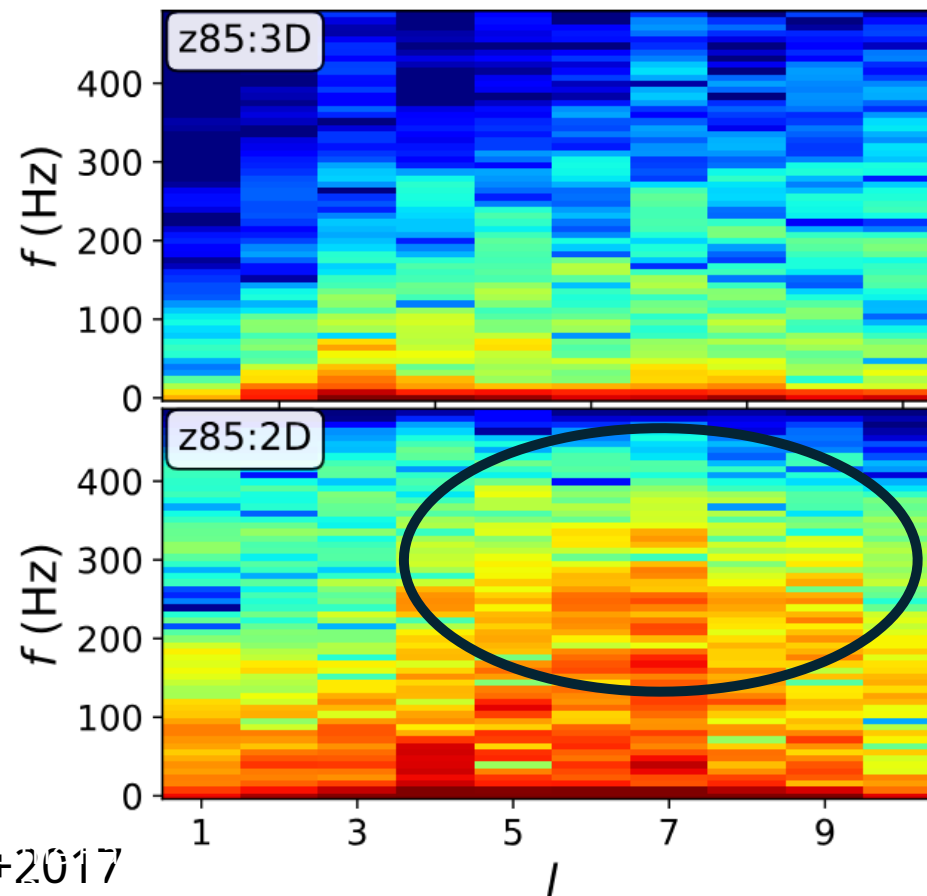




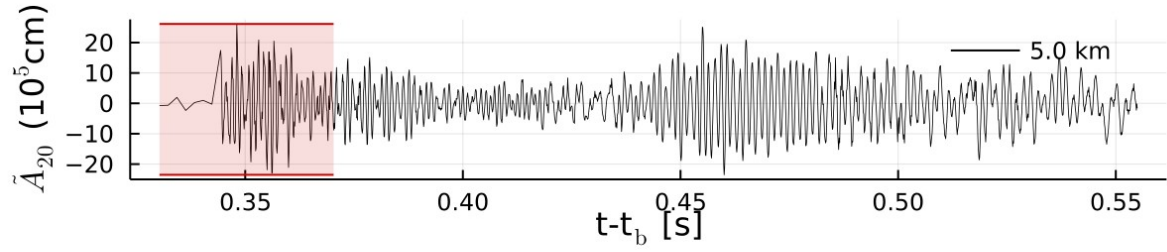
## Why is the GW signal absent in 3D?

Autocorrelation function and Fourier transform look very different!

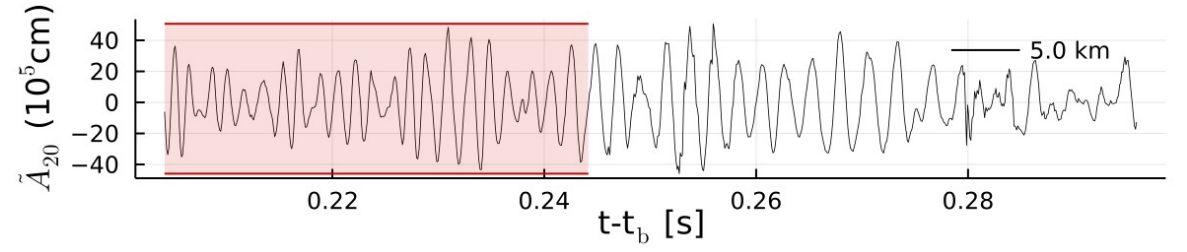
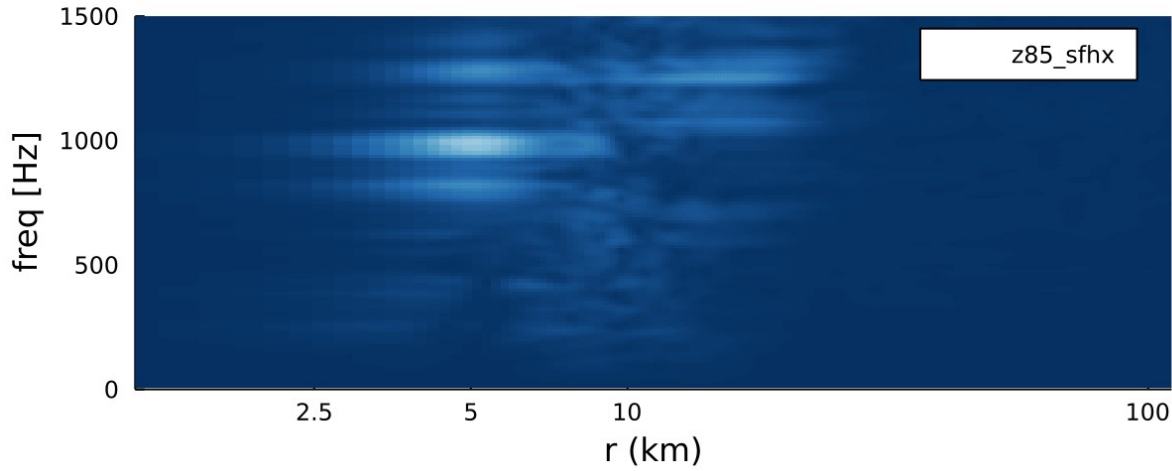
$$\tilde{E}(f) = 1/\sqrt{2\pi} \int E(t) e^{-2\pi i f t} dt \quad \log \tilde{E}_f \text{ (erg s cm}^{-3}\text{)}$$



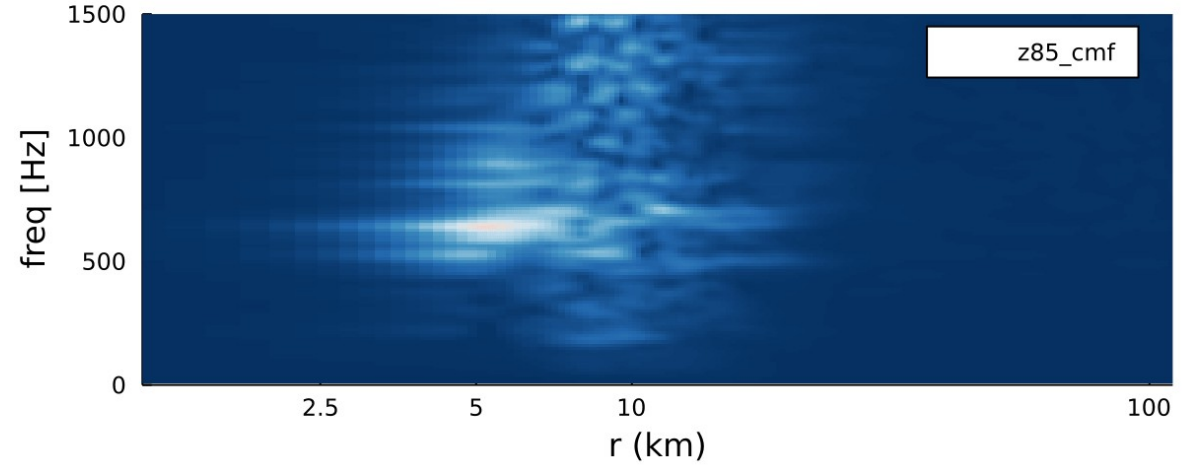
See also Andresen+2017



$\Delta t=40$  ms



$\Delta t=40$  ms



[github.com/PiaJakobus/GW\\_extraction](https://github.com/PiaJakobus/GW_extraction)