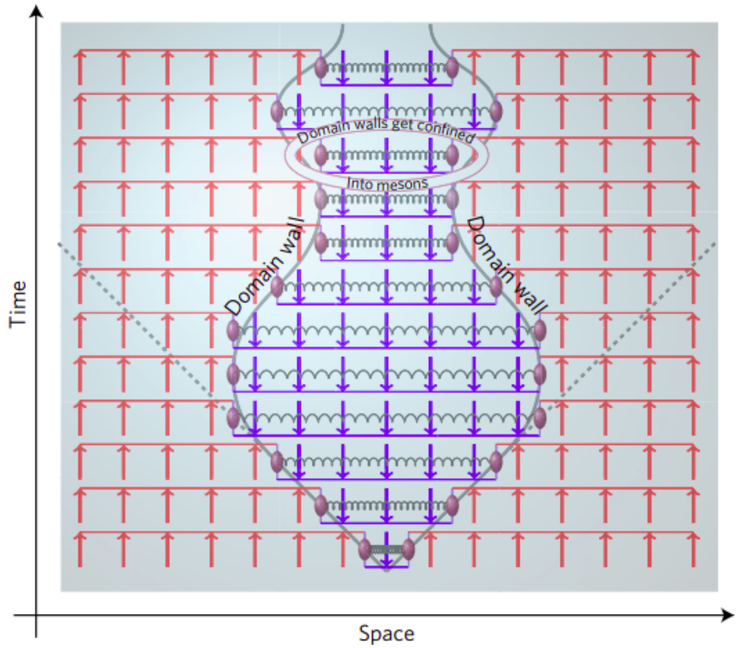
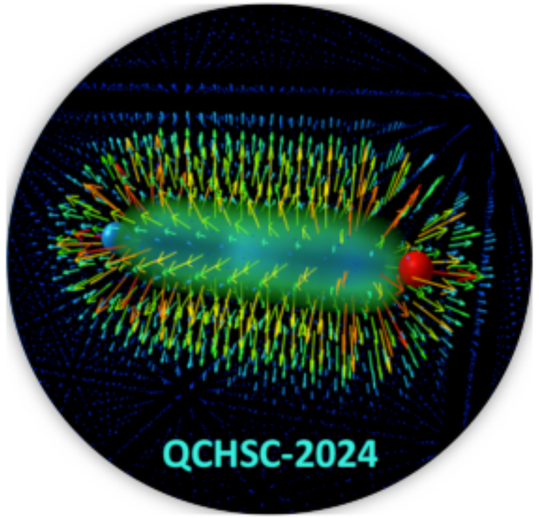
Confinement and False Vacuum Decay in Quantum Spin Chains





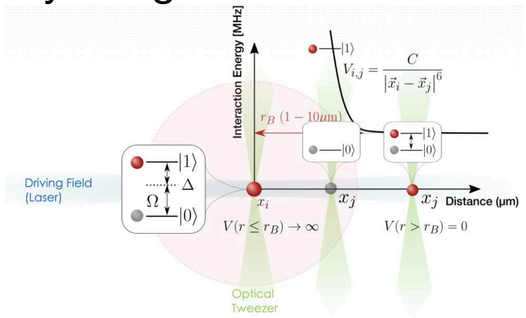
Jožef Stefan Institute, Ljubljana, Slovenia

Gianluca Lagnese

Quantum Spin Chains

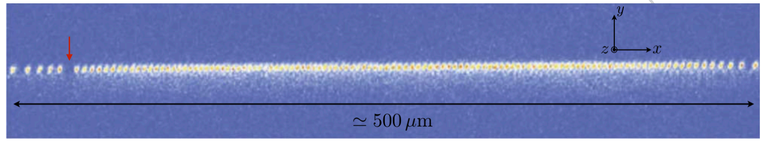
Experiments/Quantum simulators

Rydberg Neutral Atoms



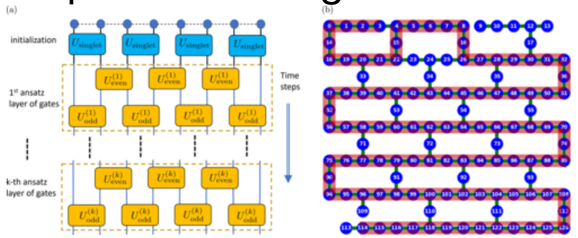
J. Wurtz et al., arXiv:2306.11727

Cryogenic Trapped Ions



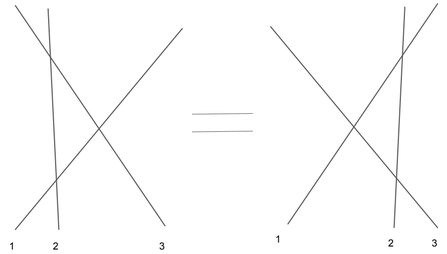
G. Pagano et al., Quantum Sci. Technol. 2019

Superconducting Qubits



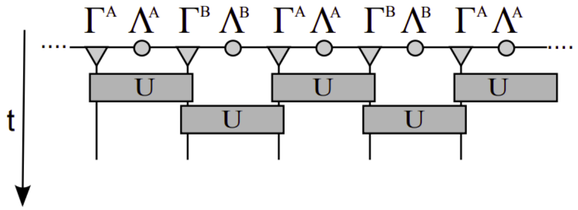
H. Yu, Y. Zhao, Tzu-Chieh Wei, Phys. Rev. Research 2023

Analytical (Integrability, CFT…)



Franchini, Springerlink LNP 940

Numerical methods (Tensor Networks, TEBD, DMRG)



J. Hauschild, F. Pollmann, SciPost Phys. 2018

Key concepts

1) Real-time out-of-equilibrium dynamics of many-body systems

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2) Isolated systems $\leftrightarrow$ Pure unitary time evolution

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“Introduction to 'Quantum Integrability in Out of Equilibrium Systems’”

Calabrese, Essler, Mussardo J. Stat. Mech. 2016

Overview

1) The Model: Transverse Field Quantum Ising Chain

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2) Confinement:

“Real time confinement following a quantum quench to a non-integrable model”

Kormos, Collura, Takaks, Calabrese, Nature Physics 2017

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4) Another Model

The Model

The model: Transverse Field Quantum Ising Chain

Classical Ising: $E = -J \sum_{\langle ij \rangle} s_i s_j \qquad s_i \in \{-1, 1\}$

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$s_i \to \sigma_i^z$

1+1 D Quantum Ising: $H = -J \sum_i \left(\sigma_i^z \sigma_{i+1}^z - h_x \sigma_i^x \right)$

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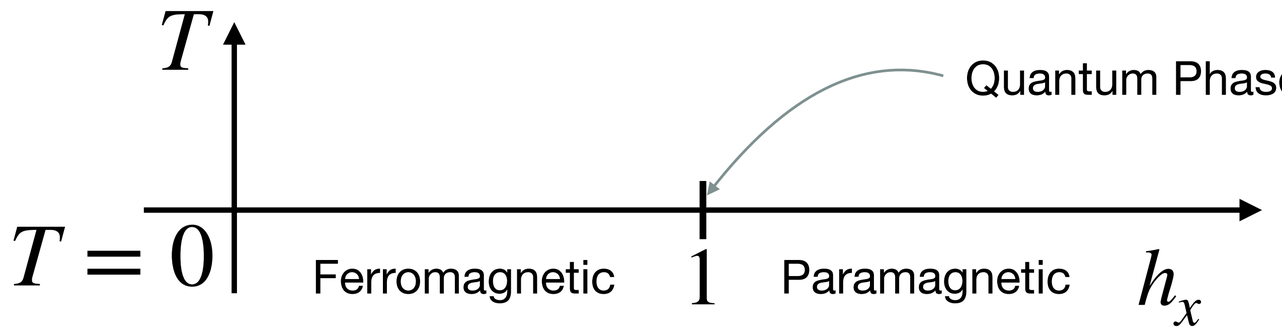
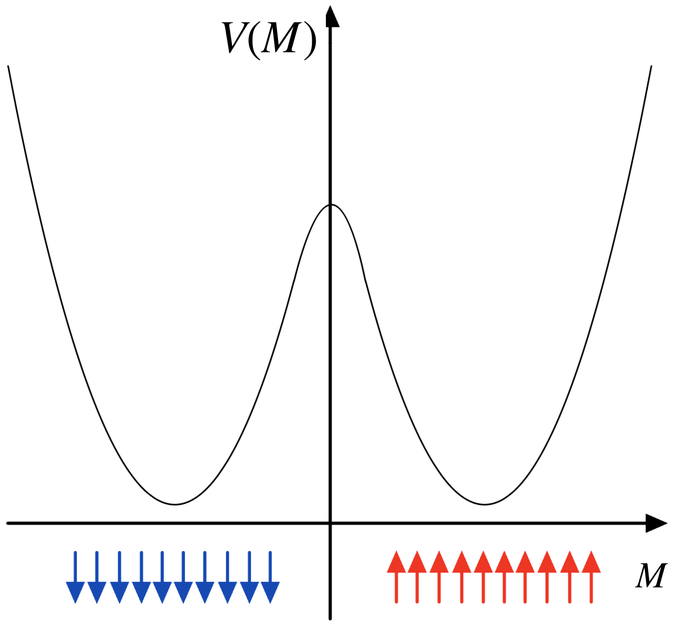


Quantum Phase Transition ~ 2D Classical Thermal

The model: Transverse Field Quantum Ising Chain

$$H = -\sum_i \sigma_i^z \sigma_{i+1}^z - h_x \sigma_i^x$$

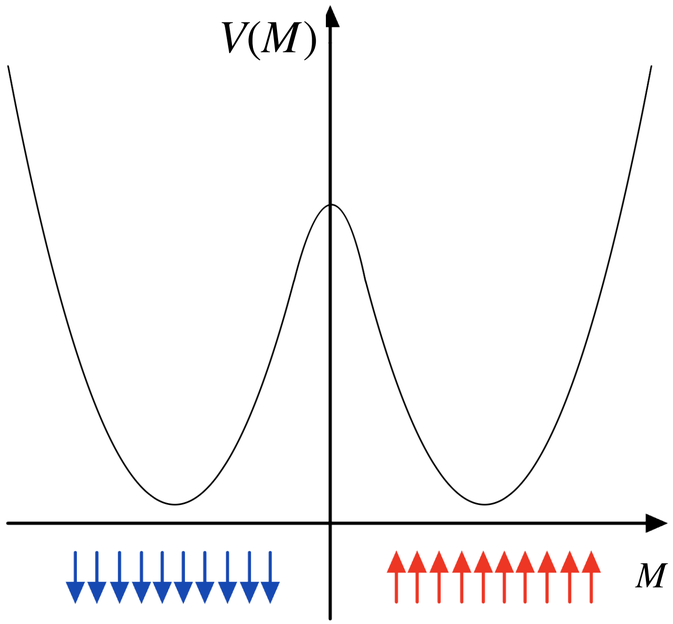
$|h_x| < 1 \Rightarrow$ Two **Degenerate** Ferromagnetic ground states



The model: Transverse Field Quantum Ising Chain

$$H = -\sum_i \sigma_i^z \sigma_{i+1}^z - h_x \sigma_i^x$$

$$|h_x| \ll 1 \quad \Rightarrow \quad H \approx -\sum_j 2\,|j\rangle\langle j| - h_x \left(|j\rangle\langle j+1| + h.c. \right)$$



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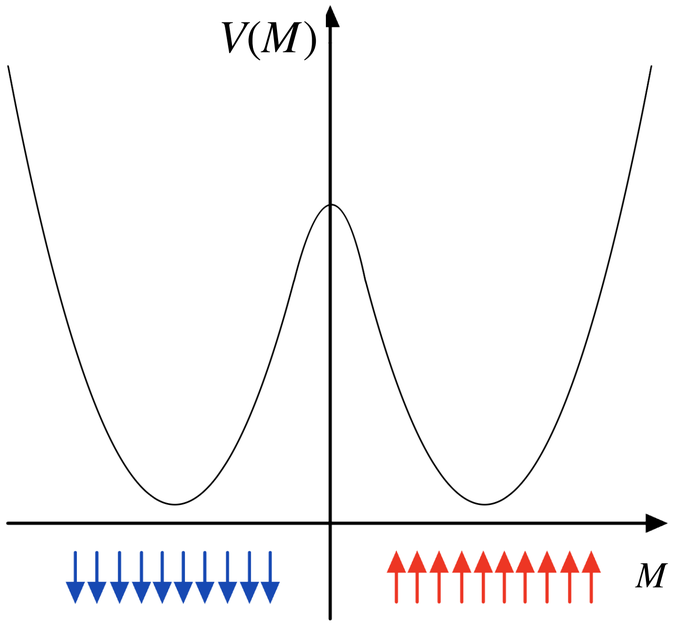
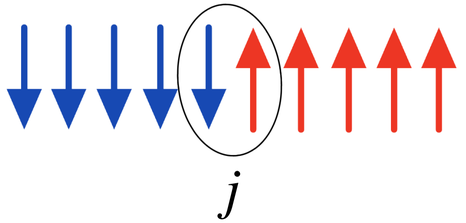


Domain-wall excitations:

$|j\rangle =$



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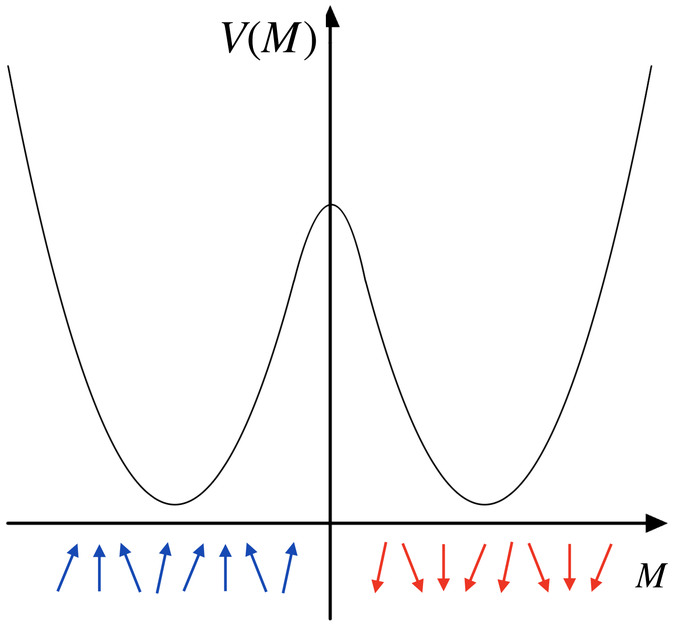
$$|h_x| < 1 \quad \Rightarrow \quad H = \sum_k \varepsilon_k \eta_k^\dagger \eta_k + const$$

Free Fermion mapping (Jordan-Wigner) + Bogoliubov + Fourier



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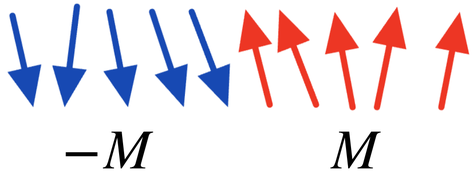
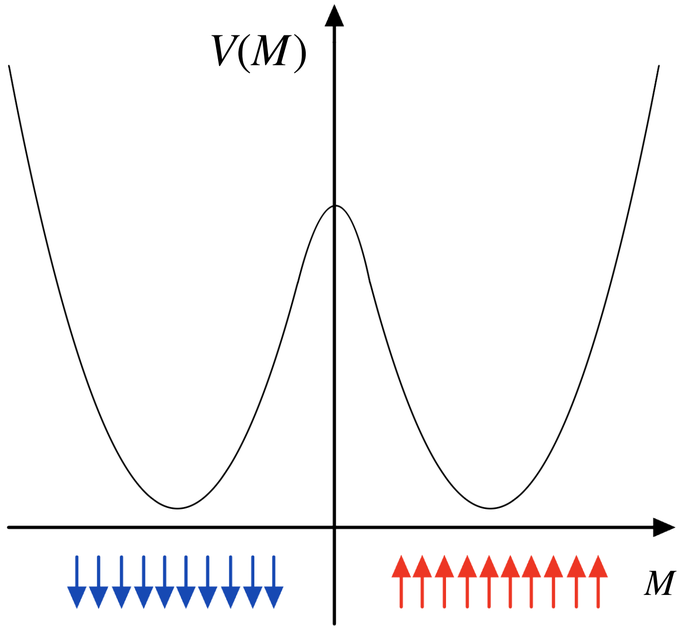


$$M = (1 - h_x)^{1/8}$$

Energy cost / Mass gap: $\Delta = 2(1 - h_x)$

The model: Transverse Field Quantum Ising Chain

$$H = -\sum_i \sigma_i^z \sigma_{i+1}^z - h_x \sigma_i^x \quad -h_z \sigma_i^z$$



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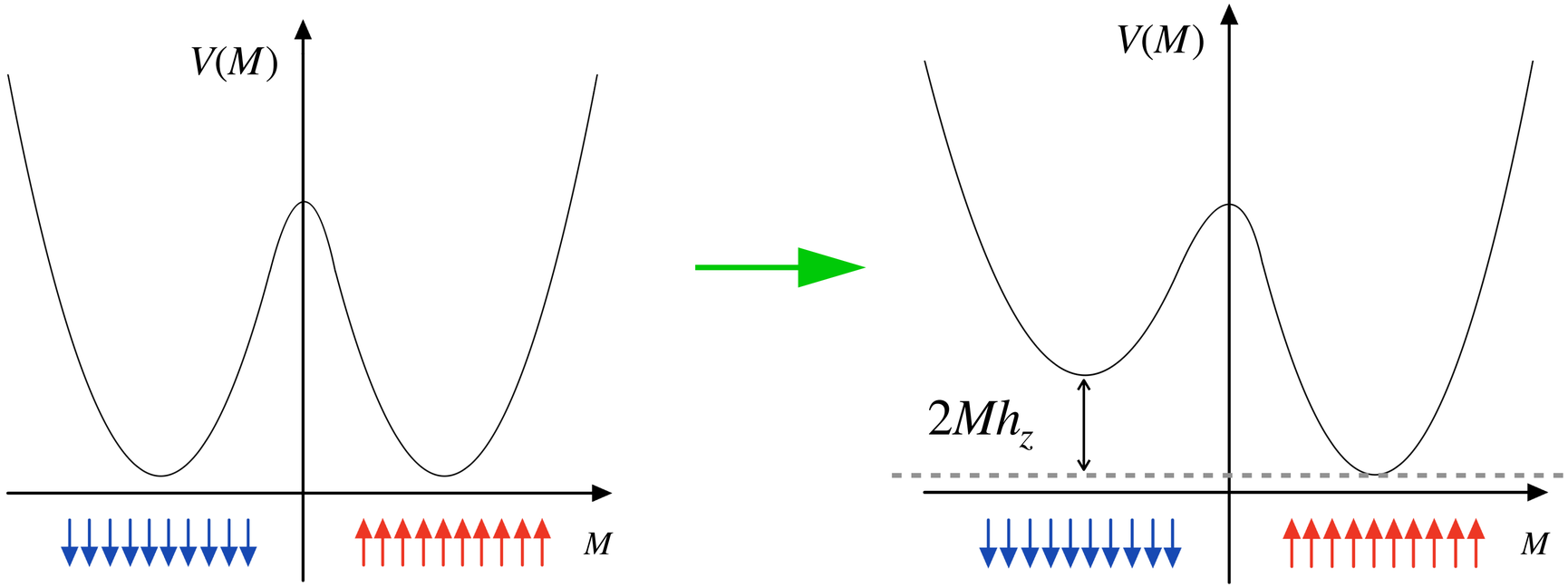
The longitudinal field h_z lifts the degeneracy



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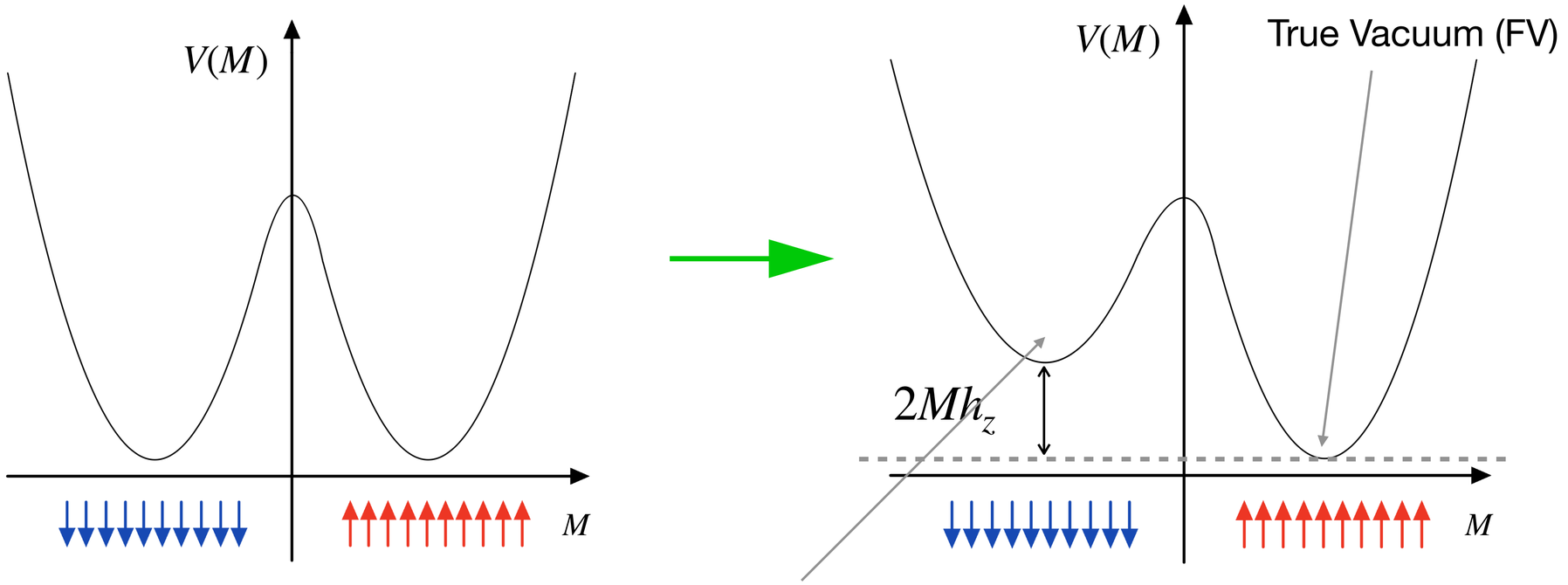

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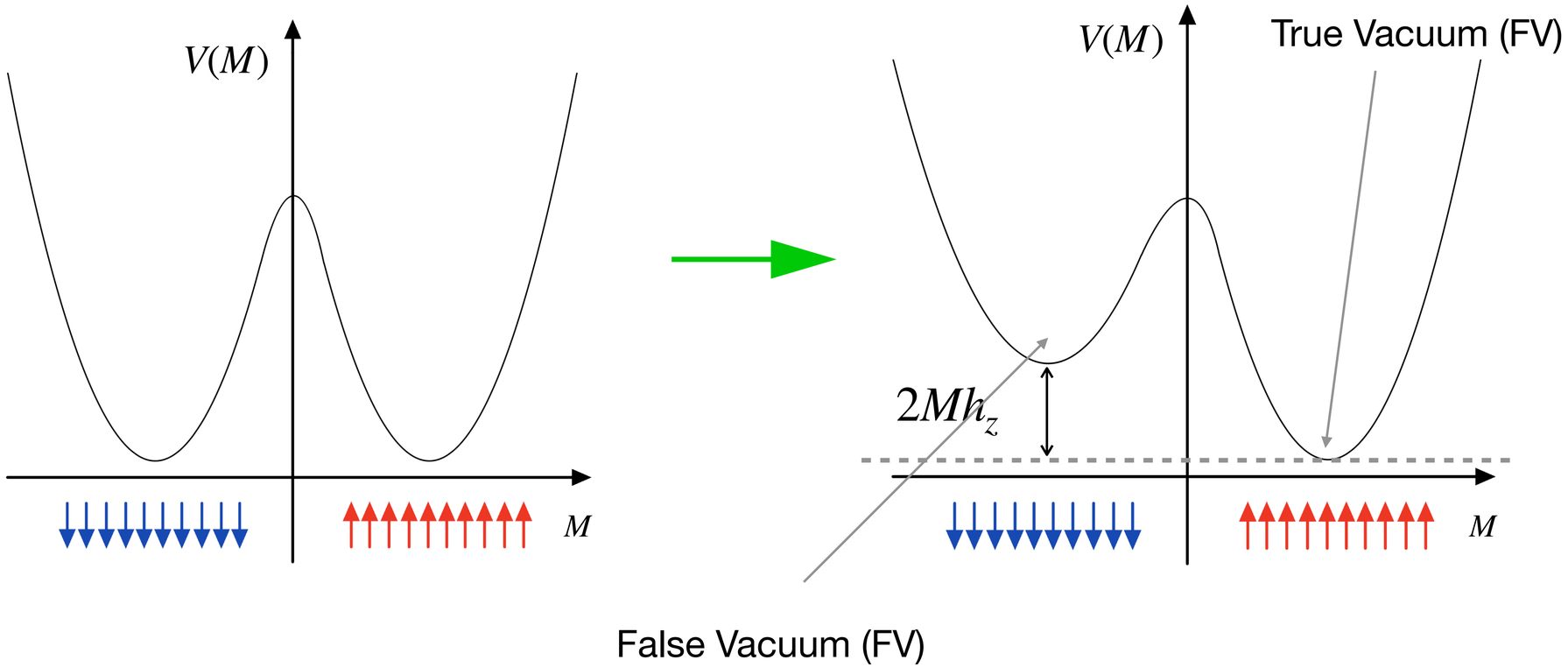
The longitudinal field h_z lifts the degeneracy → Distinctive effects in the dynamics



Confinement

The Quench Protocol

1 - Prepare ground state $|\psi_0\rangle$ of $H_0 = H(g_0)$.

$g = (h_x, h_z)$

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The Quench Protocol

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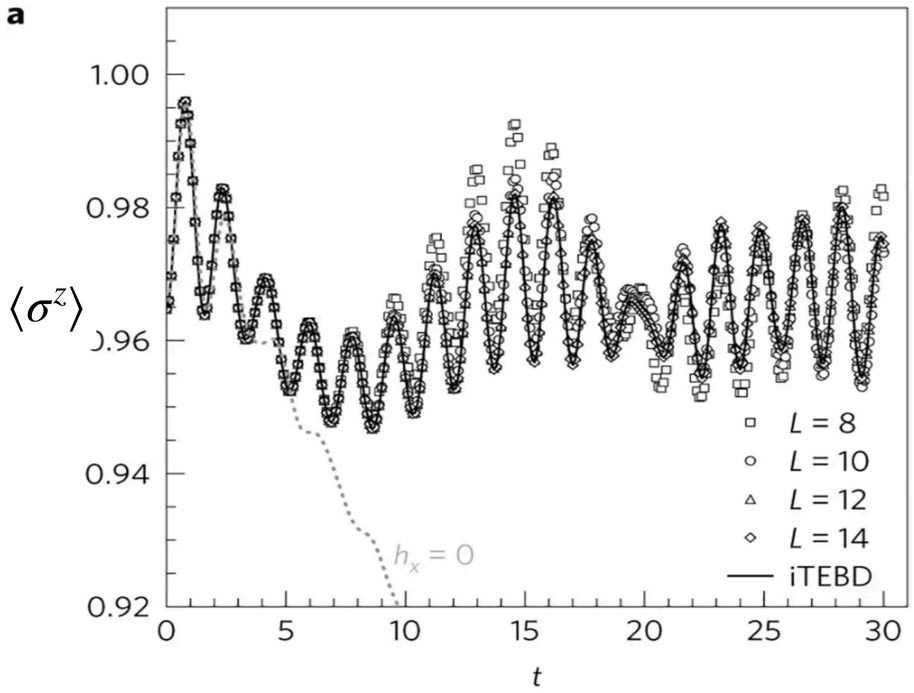
2 - At $t = 0$ **suddenly** change parameters $g_0 \to g$. $\quad H_1 = H(g)$

3 - At $t > 0$ evolve with the new Hamiltonian $|\psi(t)\rangle = \exp(-iH_1 t)\,|\psi_0\rangle$

Quench Spectroscopy

$$\langle \sigma^z \rangle(t) = \sum_i \langle \psi(t) | \sigma_i^z | \psi(t) \rangle / L$$

$$h_x = 0.5, h_z = 0 \rightarrow h_x = 0.25, h_z = 0.1$$



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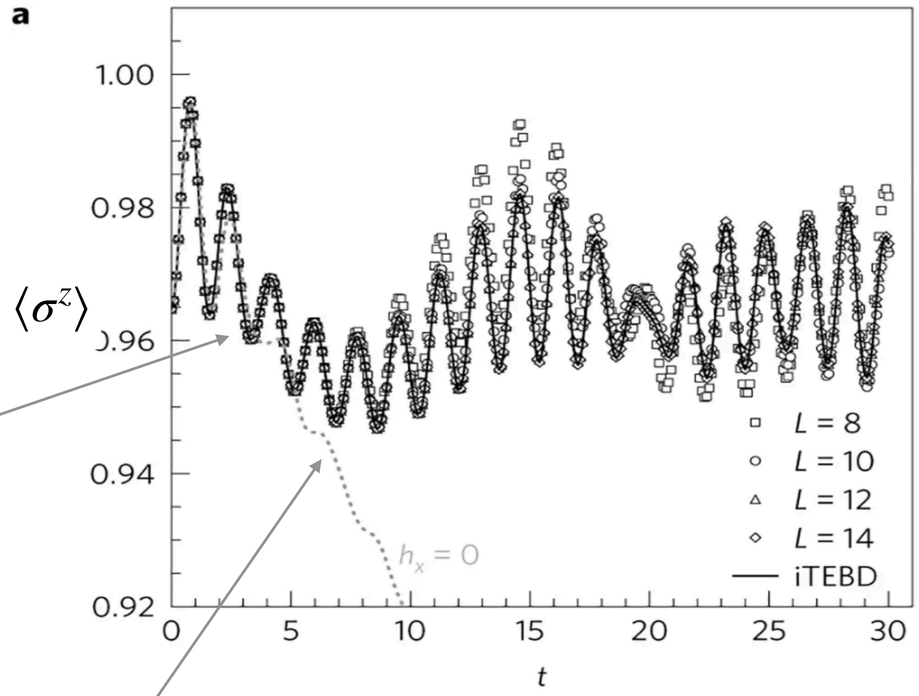

Oscillations + Slow relaxation

$$\langle \sigma^x \rangle(t) \approx e^{-kt}$$

Calabrese, Essler, Fagotti PRL 2011

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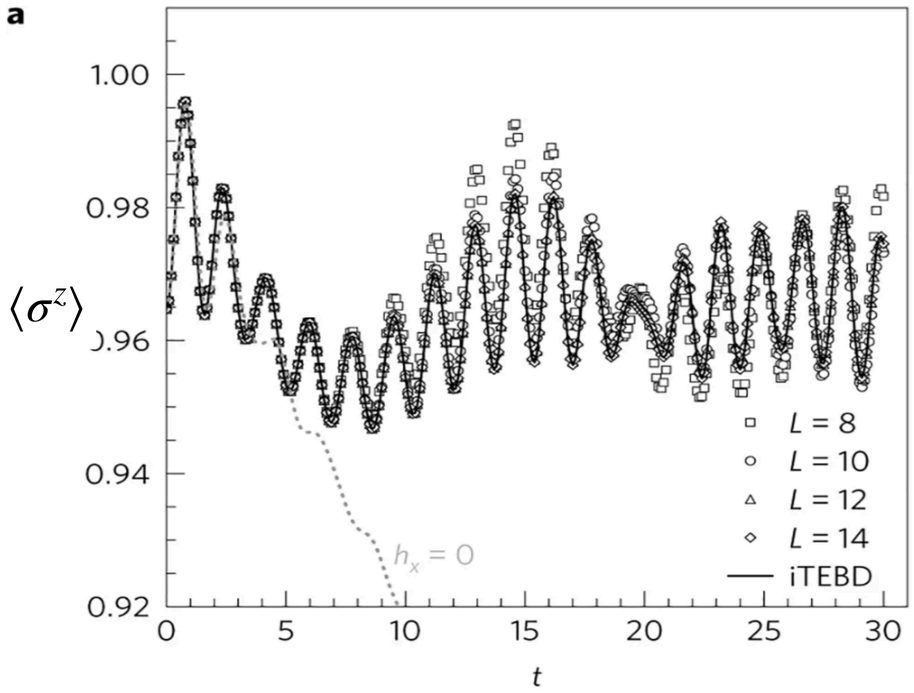
$|\Psi_0\rangle$ is not an eigenstate of H_1



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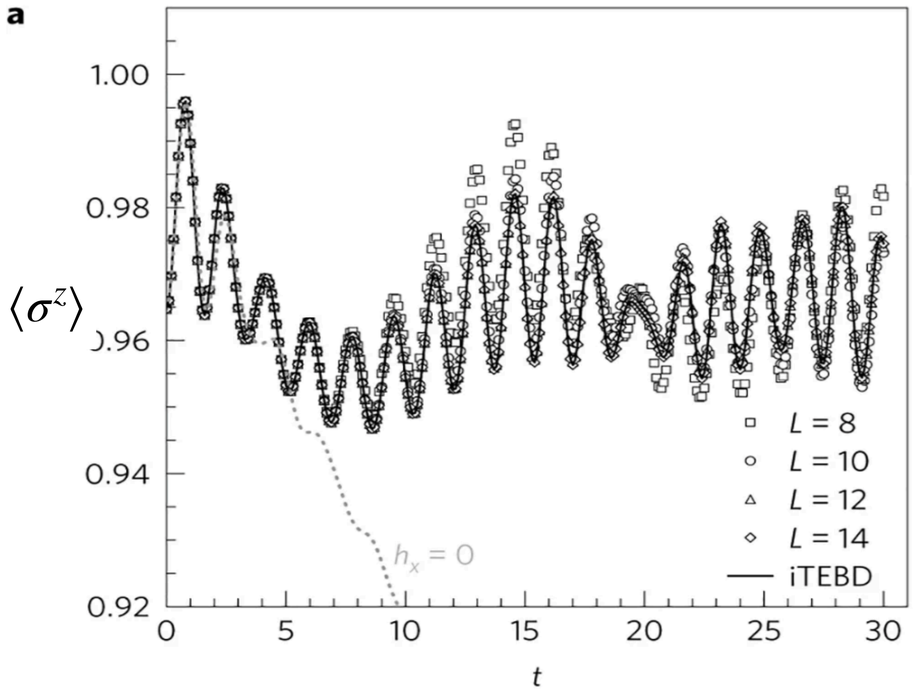

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$$\Rightarrow |\psi(t)\rangle = \exp(-iHt)|\psi_0\rangle = \sum_i a_i e^{-iE_i t} |e_i\rangle \qquad a_i = \langle e_i | \psi \rangle \qquad H_1 |e_i\rangle = E_i |e_i\rangle$$

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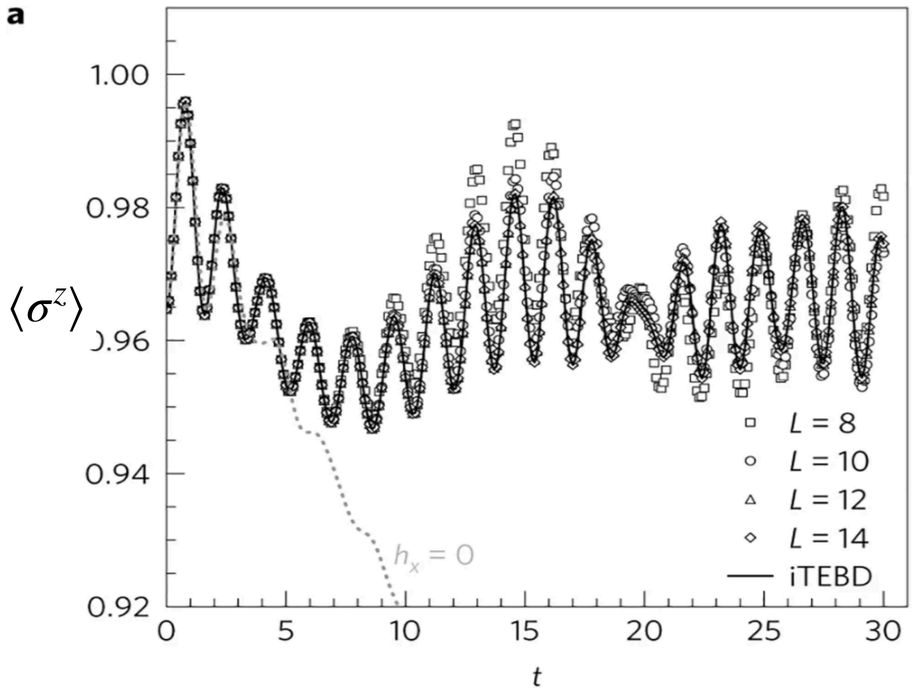

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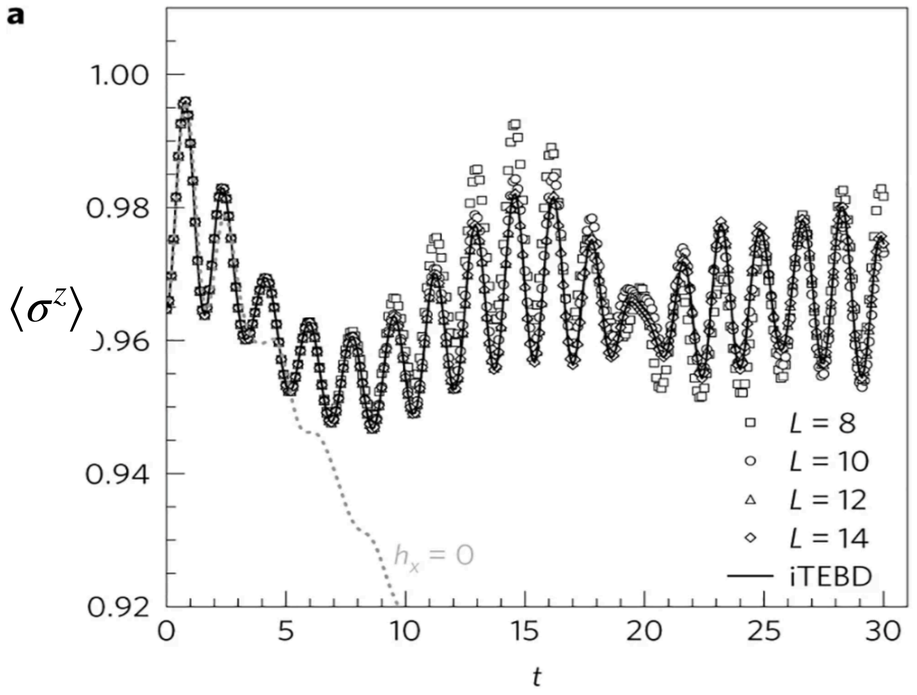
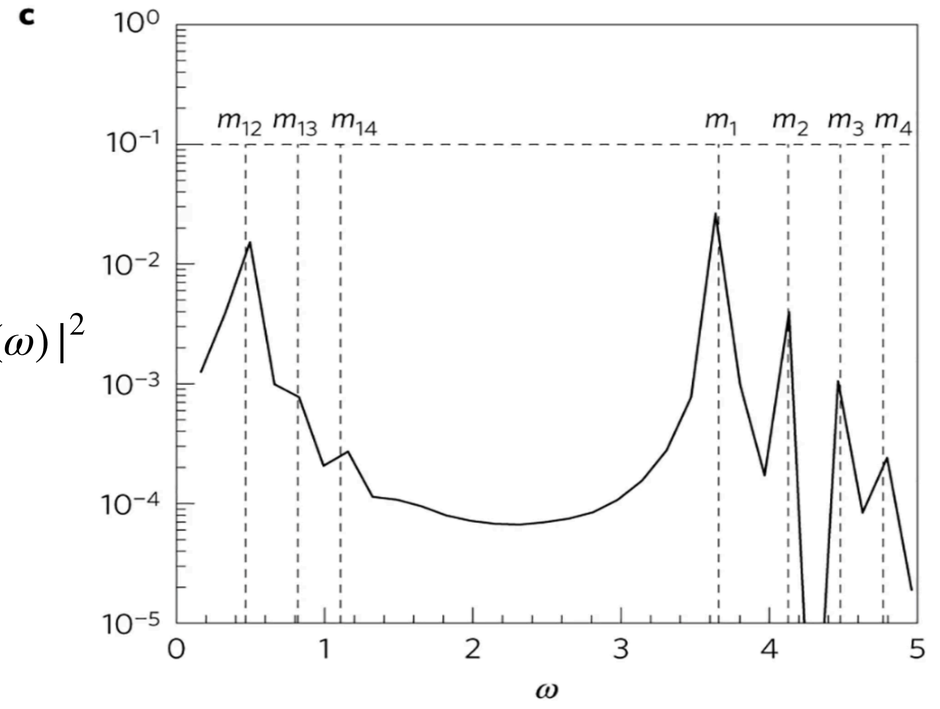
$$\Rightarrow \langle \sigma^z \rangle(t) = \sum_{ij} a_i a_j^* e^{-i(E_i - E_j)} \langle e_j | \sigma^z | e_i \rangle$$

Quench Spectroscopy

$$\langle \sigma^z \rangle(t) = \sum_i \langle \psi(t) | \sigma_i^z | \psi(t) \rangle / L$$

$$| \langle \sigma^z \rangle(\omega) |^2 = \left| \int dt e^{i\omega t} \langle \sigma^z \rangle(t) \right|^2$$





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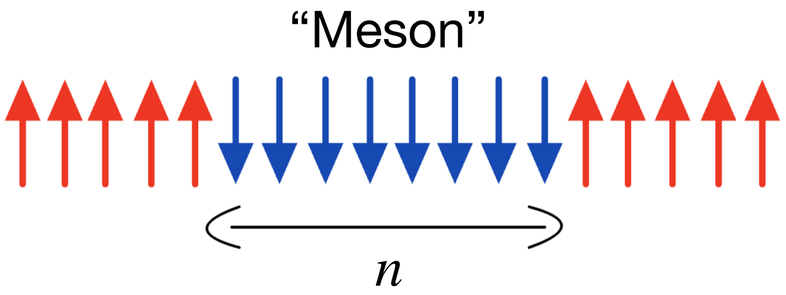
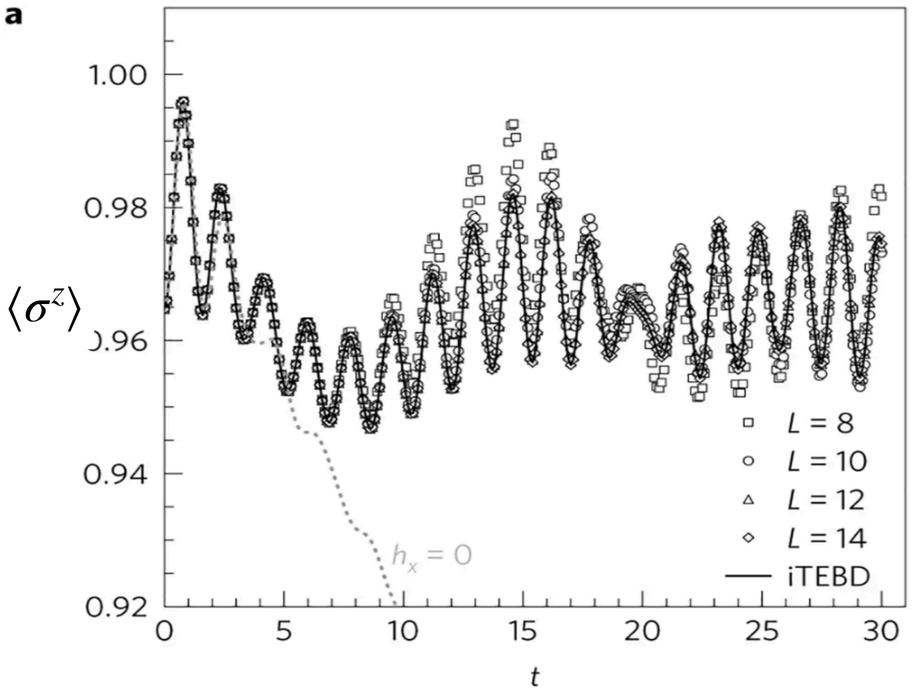
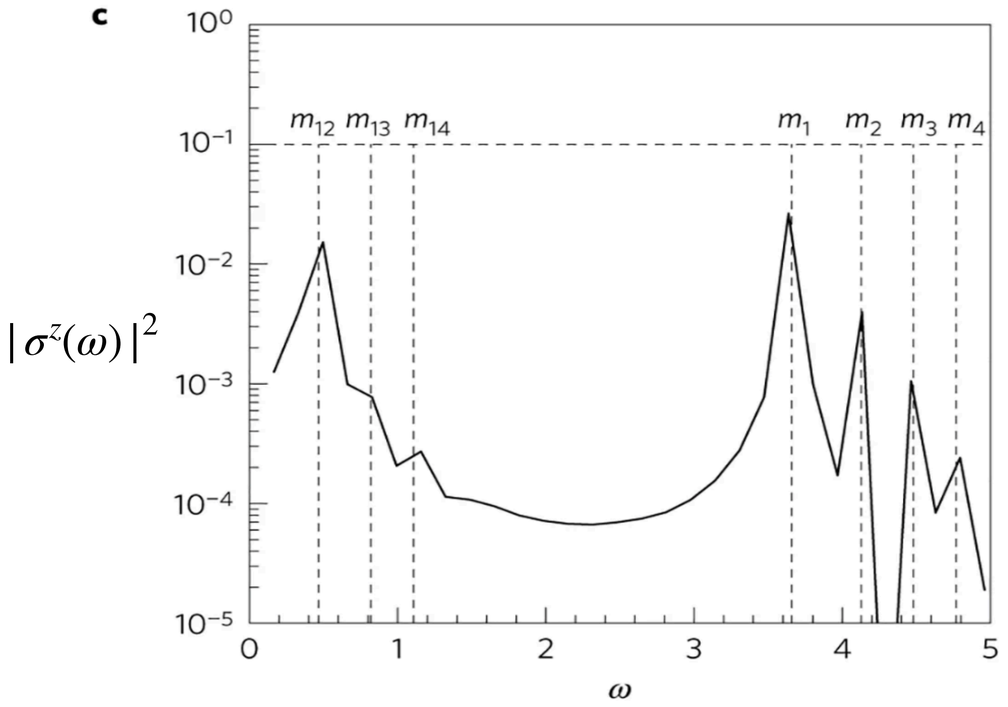


$$E_n = 2\Delta + 2Mh_z\, n$$





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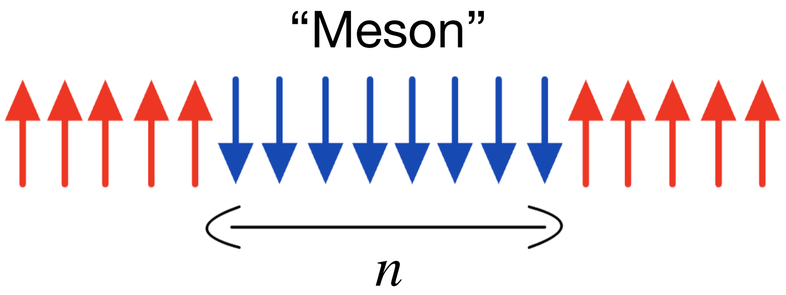
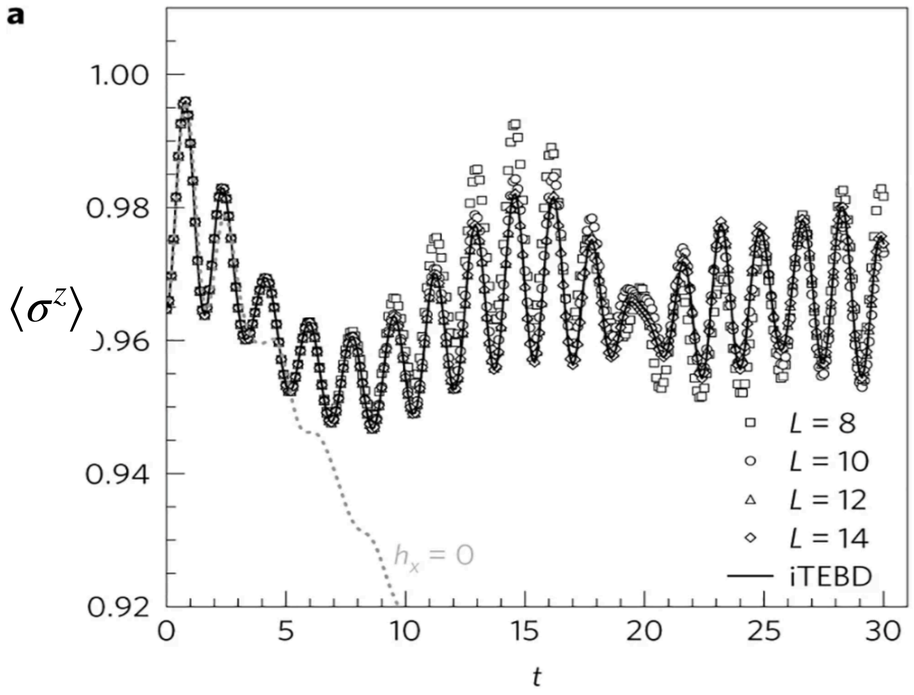
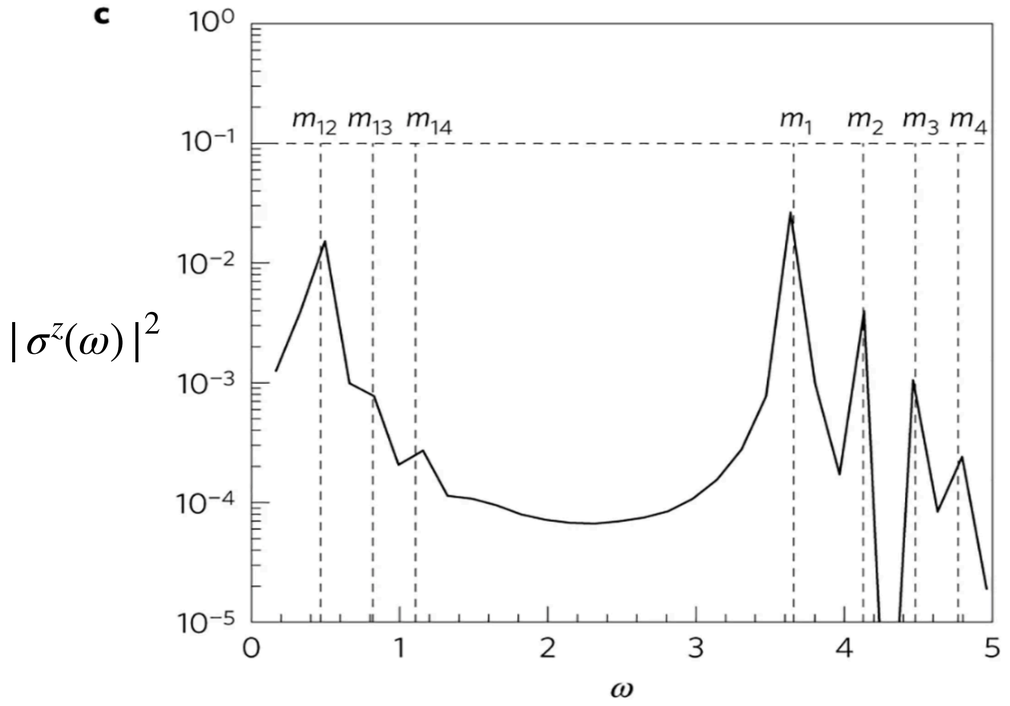


$$E_n = 2\Delta + 2Mh_z\, n$$

Kink's mass

Energy density





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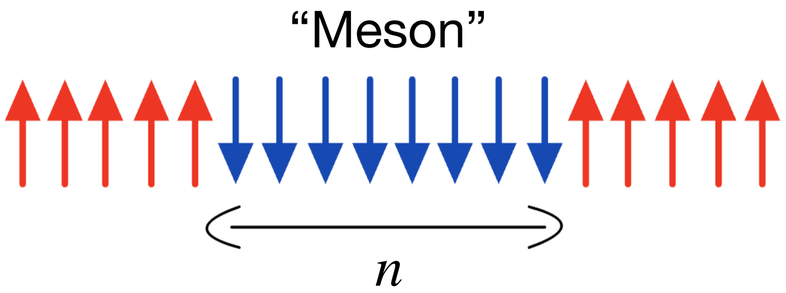
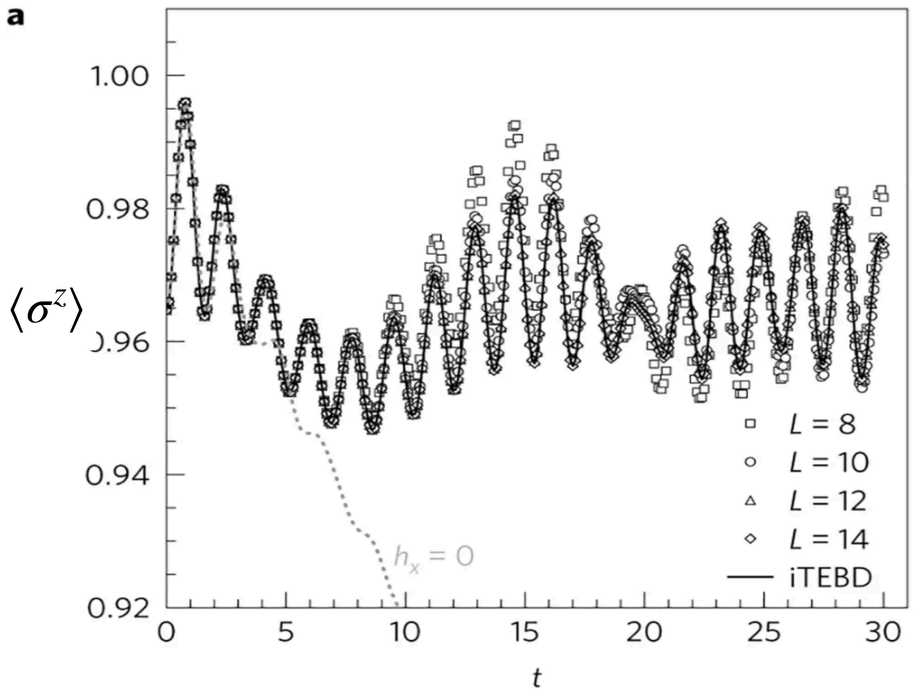
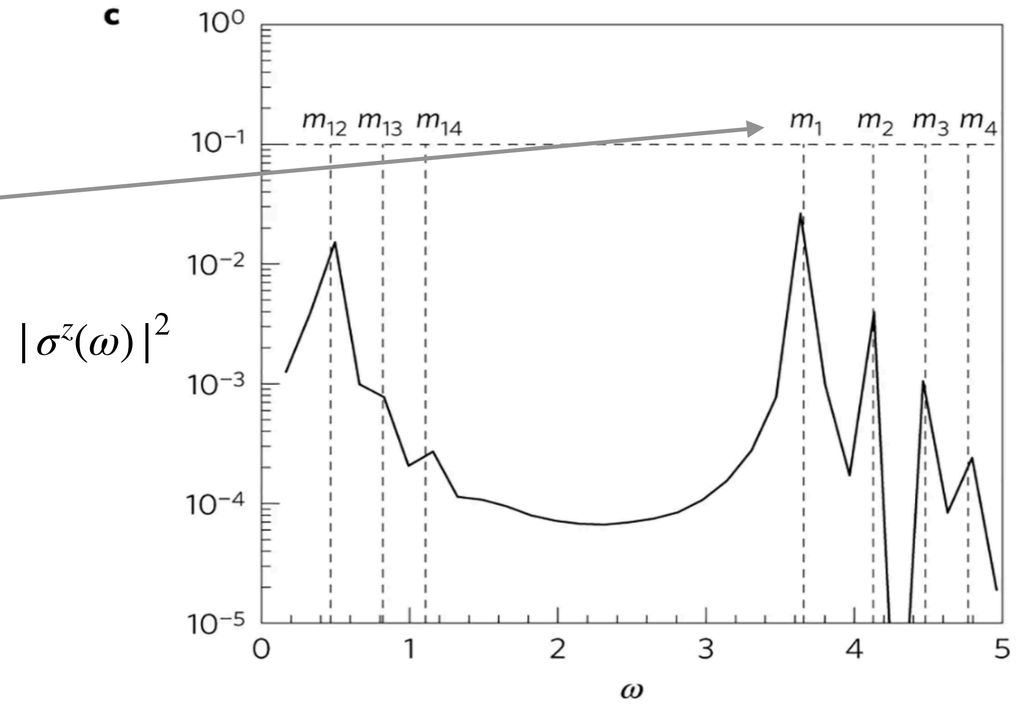


$$E_n = 2\Delta + 2Mh_z\, n$$

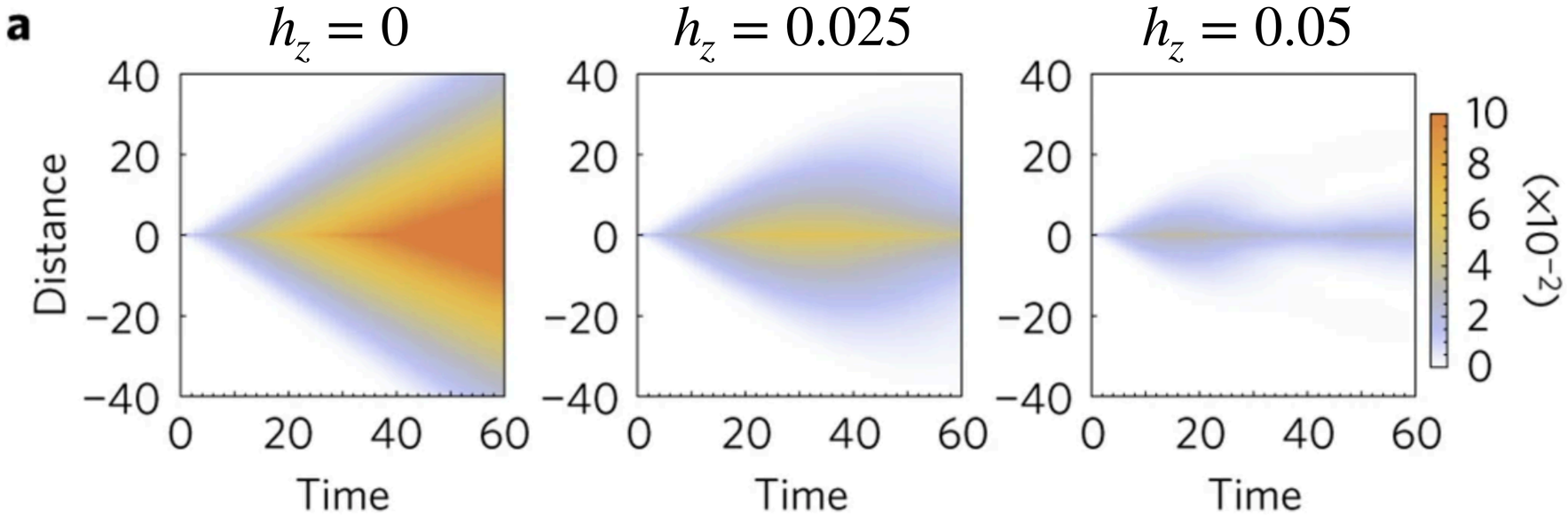
Effective two-body model:

$$H_{eff} = \varepsilon_{k_1} + \varepsilon_{k_2} + 2Mh_z\, n$$



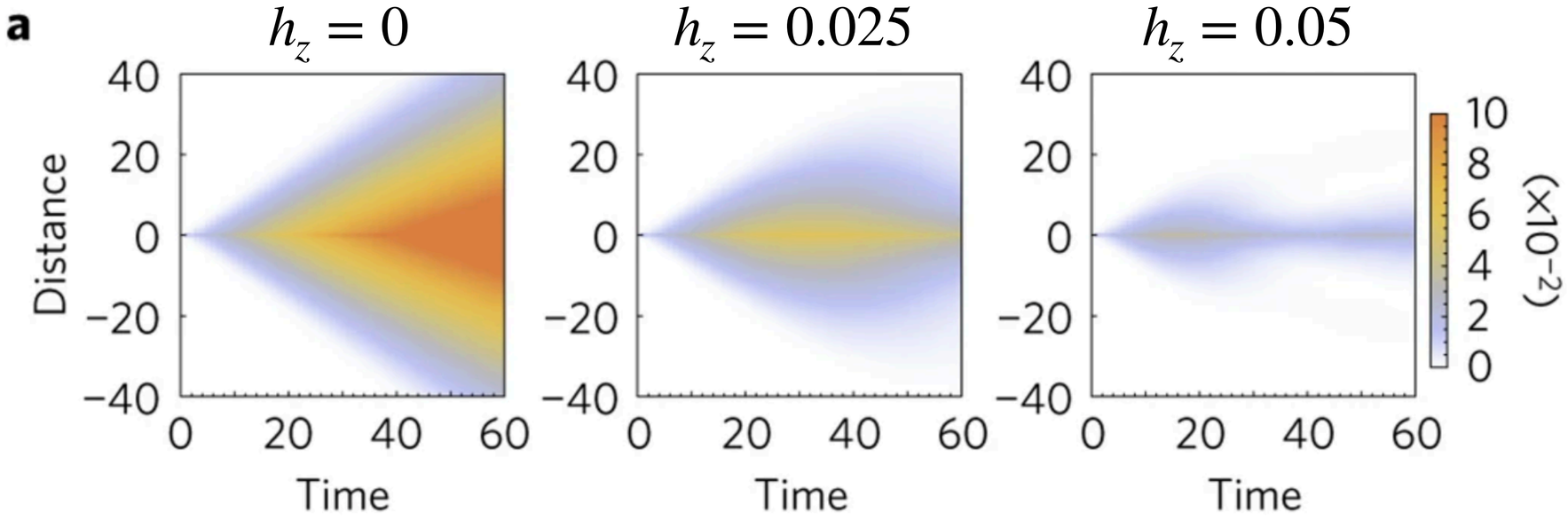


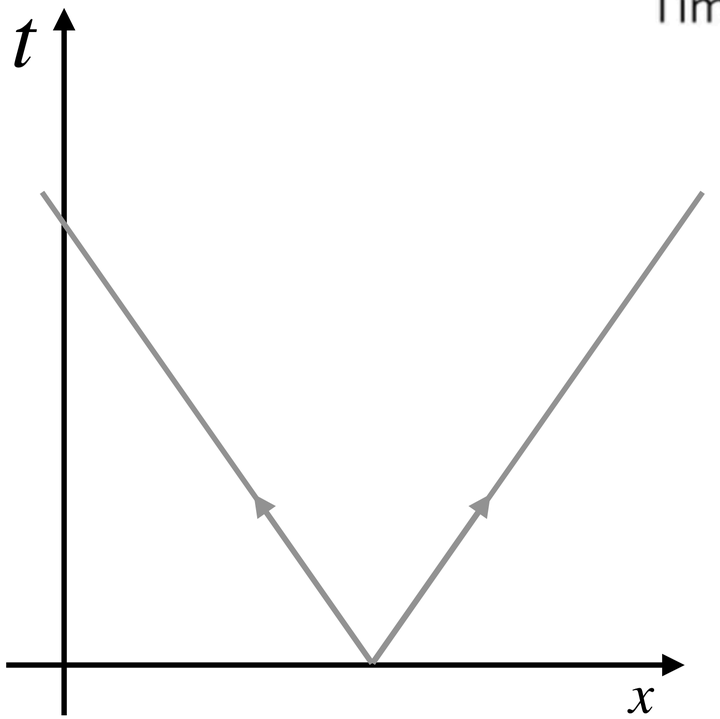
Connected Correlations



$$\langle \sigma_1^z \sigma_{1+d}^z \rangle_c = \langle \sigma_1^z \sigma_{1+d}^z \rangle - \langle \sigma_1^z \rangle \langle \sigma_{1+d}^z \rangle$$

Quasi-particle Picture





$|\Psi_0\rangle$ acts as a source of excitations

Calabrese, Cardy PRL 2006

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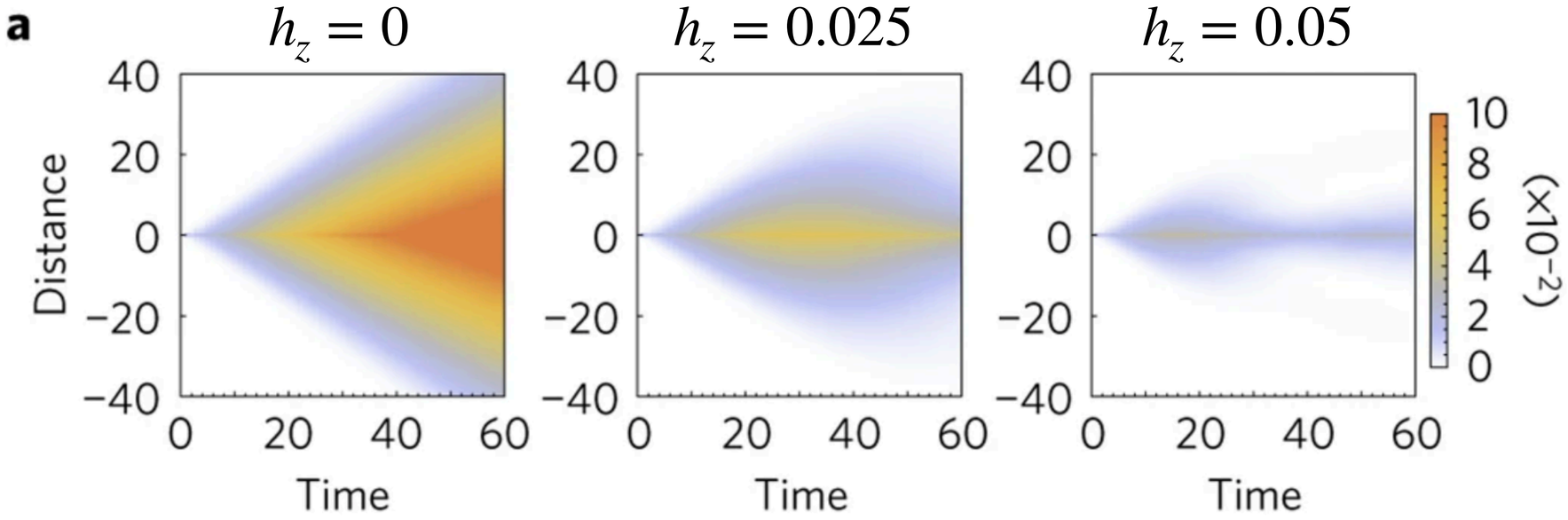

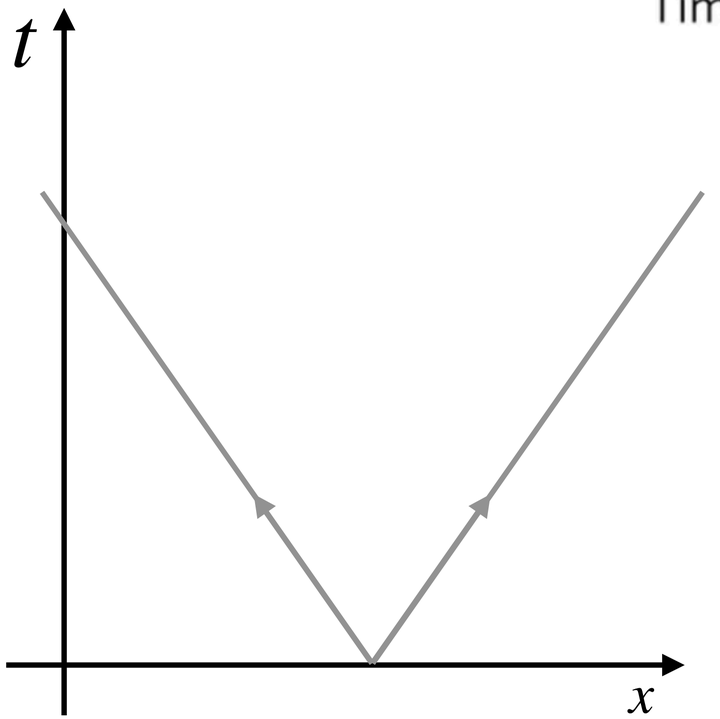

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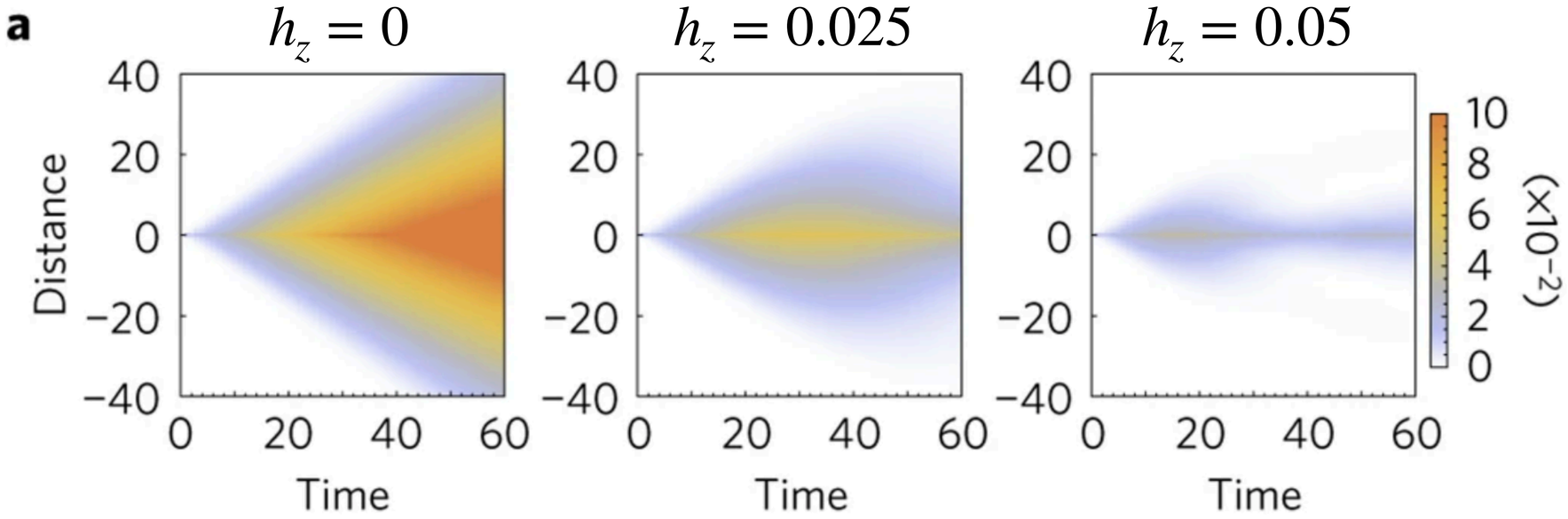
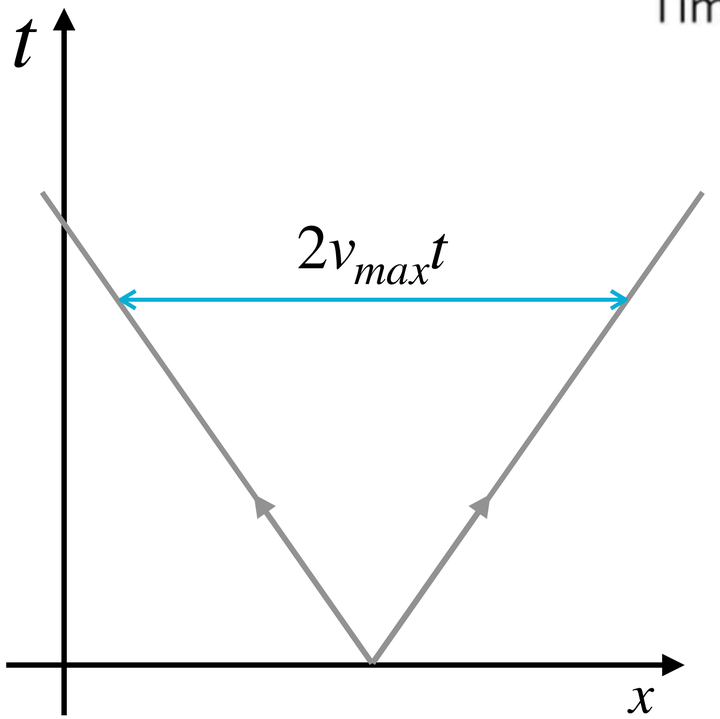
Lieb-Robinson bound

Calabrese, Cardy PRL 2006

Quasi-particle Picture





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And

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Lieb-Robinson bound

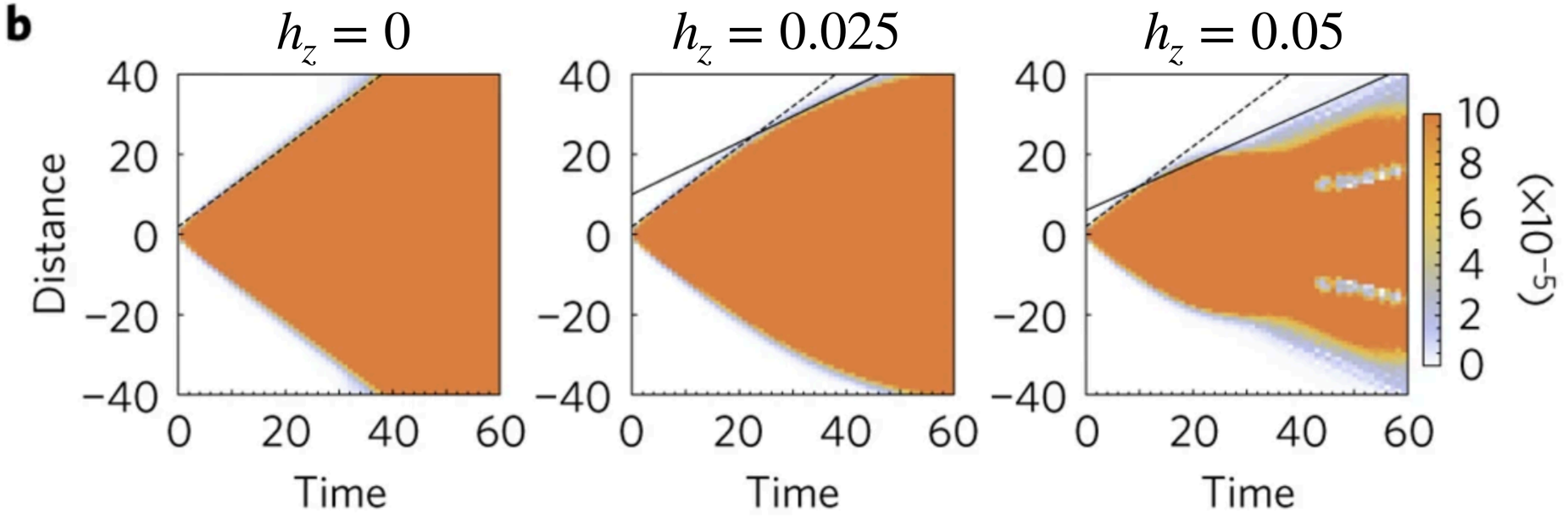
$\Rightarrow$

All correlators vanishing for

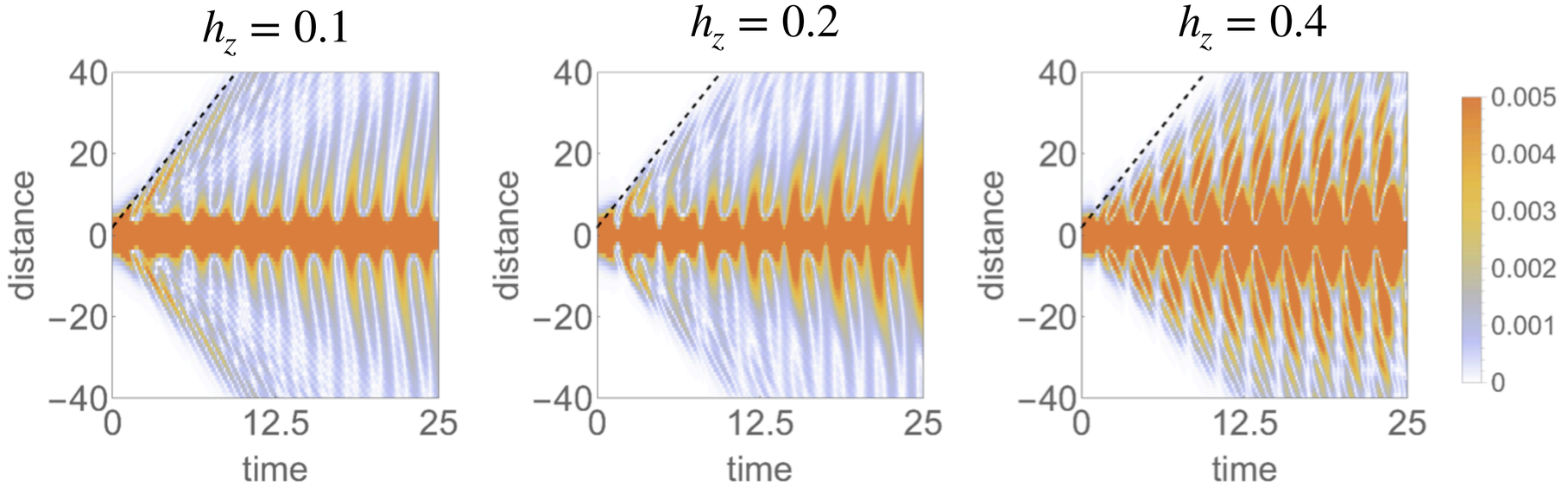
$$d > 2v_{max}t$$

Calabrese, Cardy PRL 2006

Secondary light cone

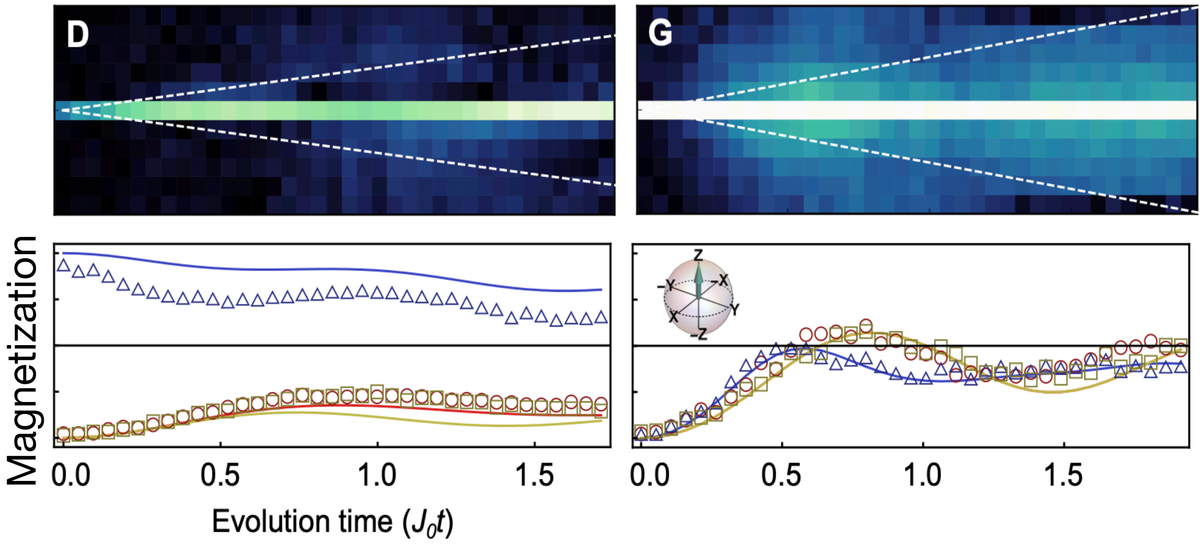


Absence for $h_x > 1$



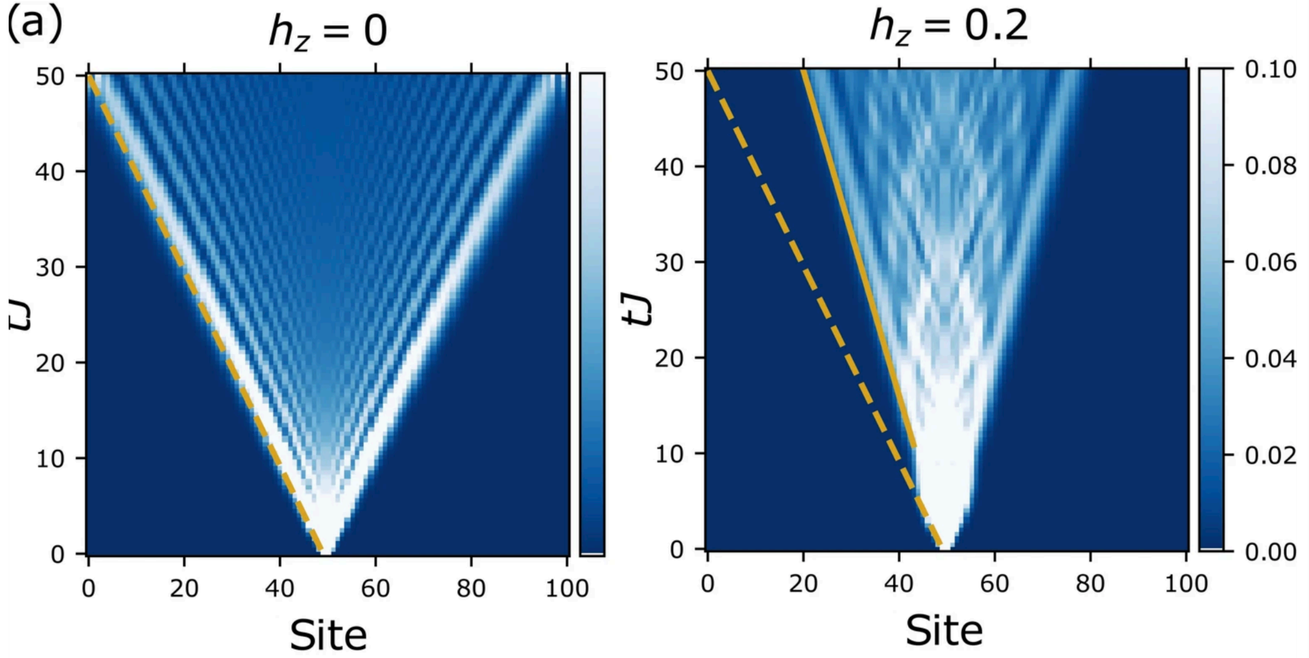
Experiments

Trapped Ions



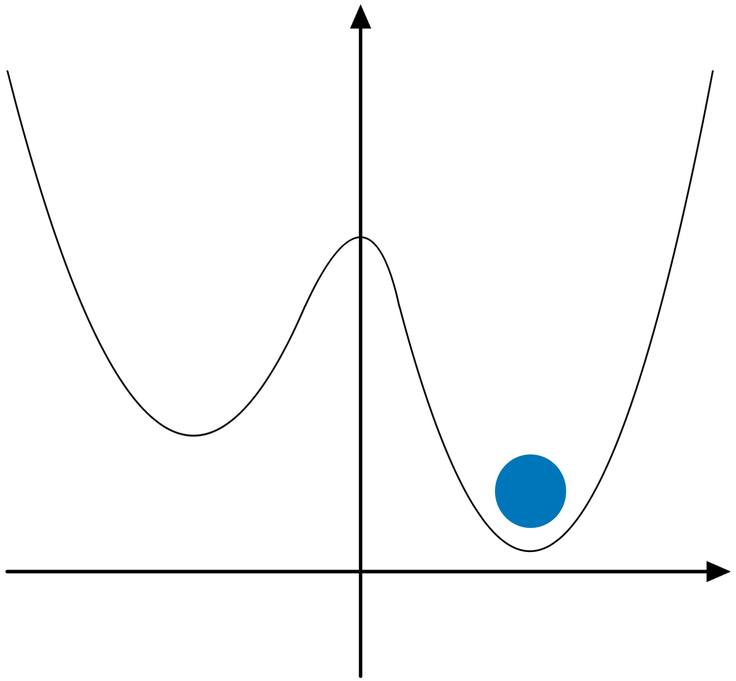
W.L. Tan et al Nature Physics 2021

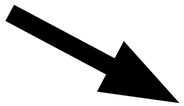
IBM Quantum computer

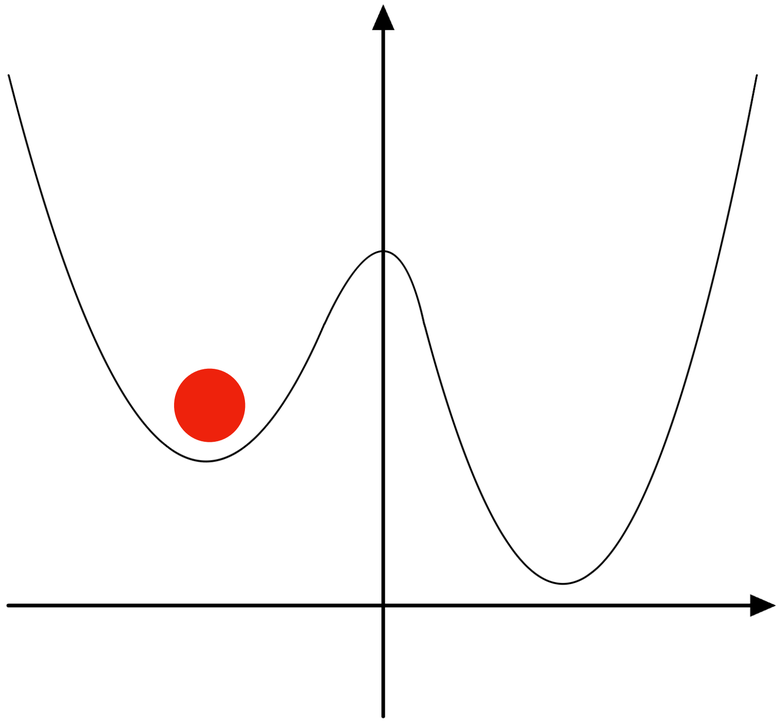


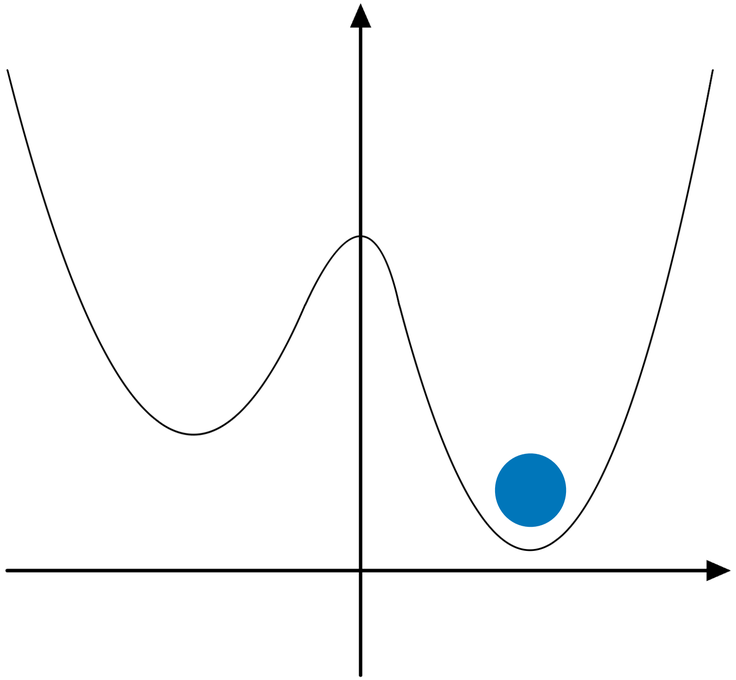
W.L. Tan et al Nature Physics 2021

False Vacuum Decay

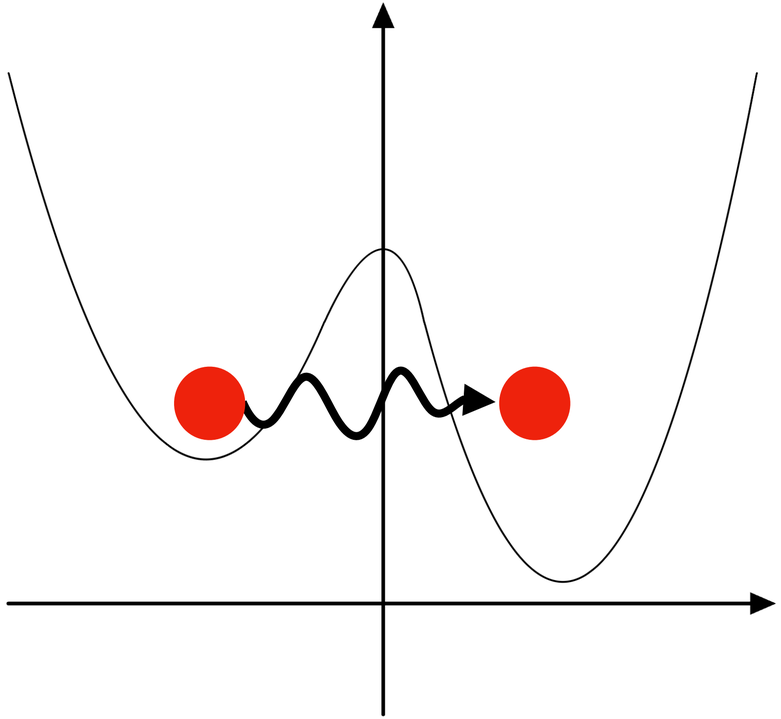


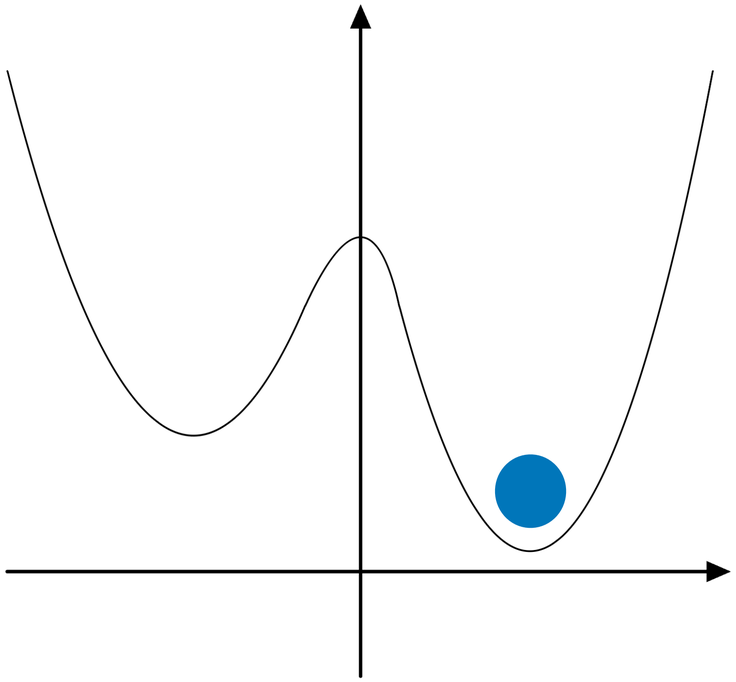






Decay by quantum fluctuations





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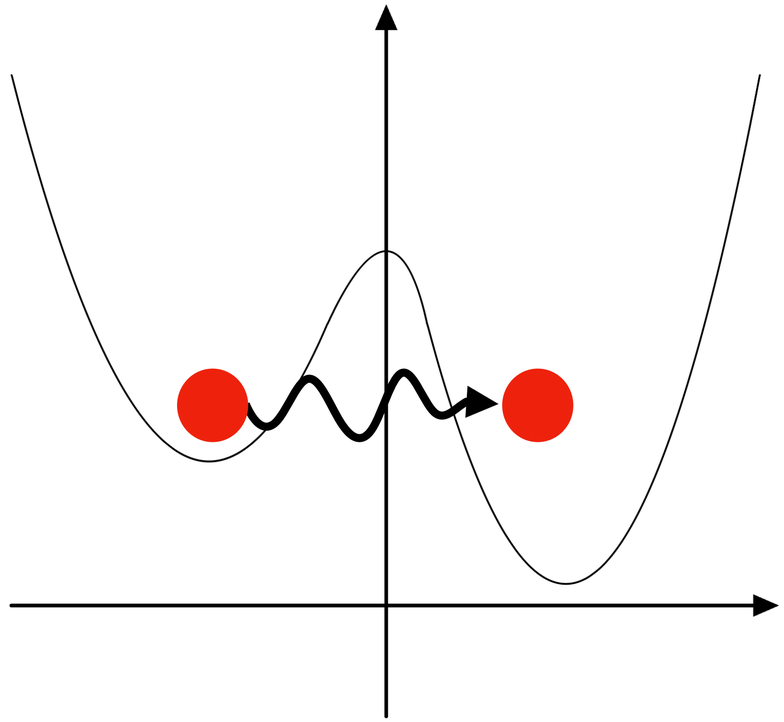
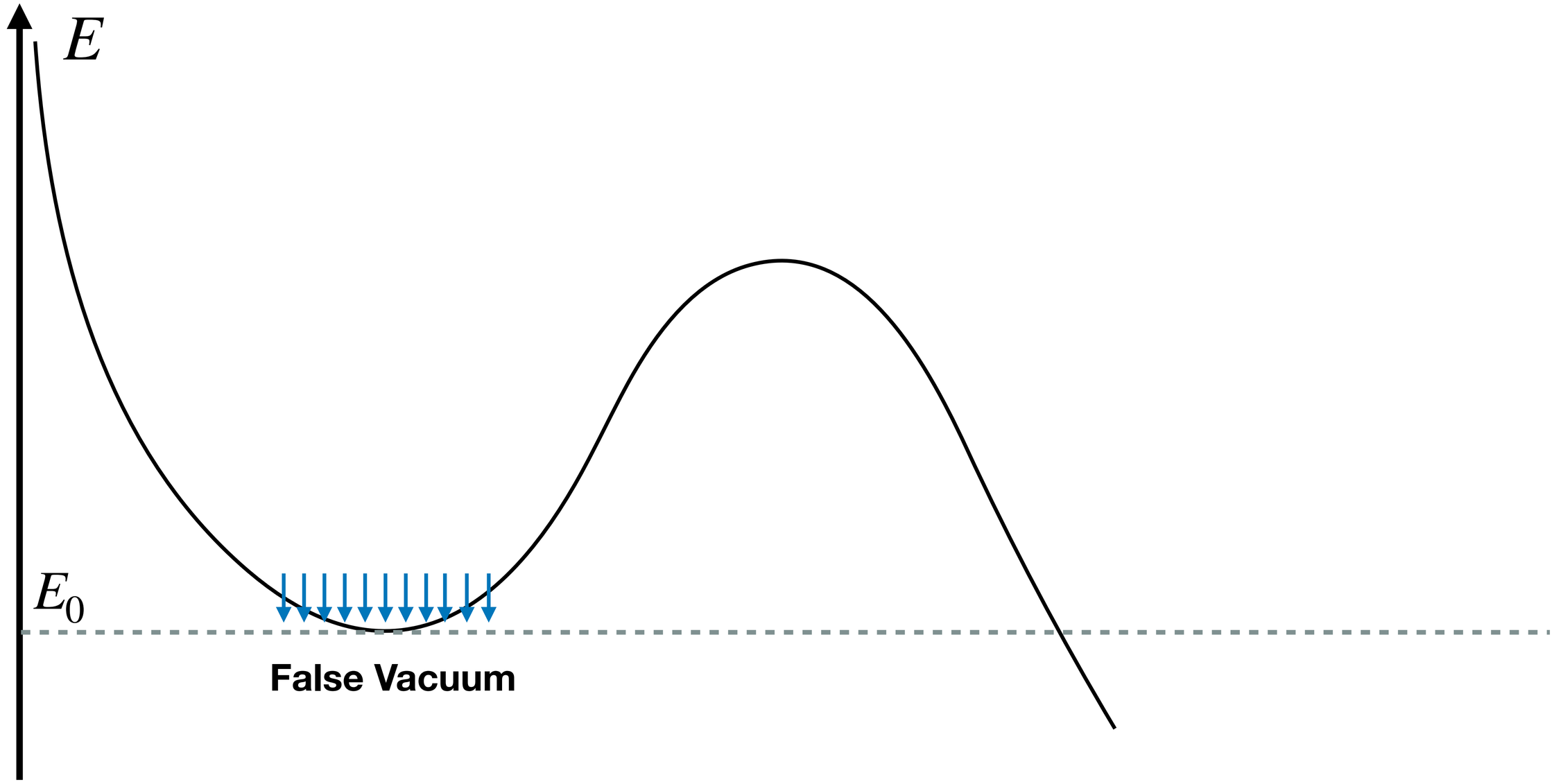
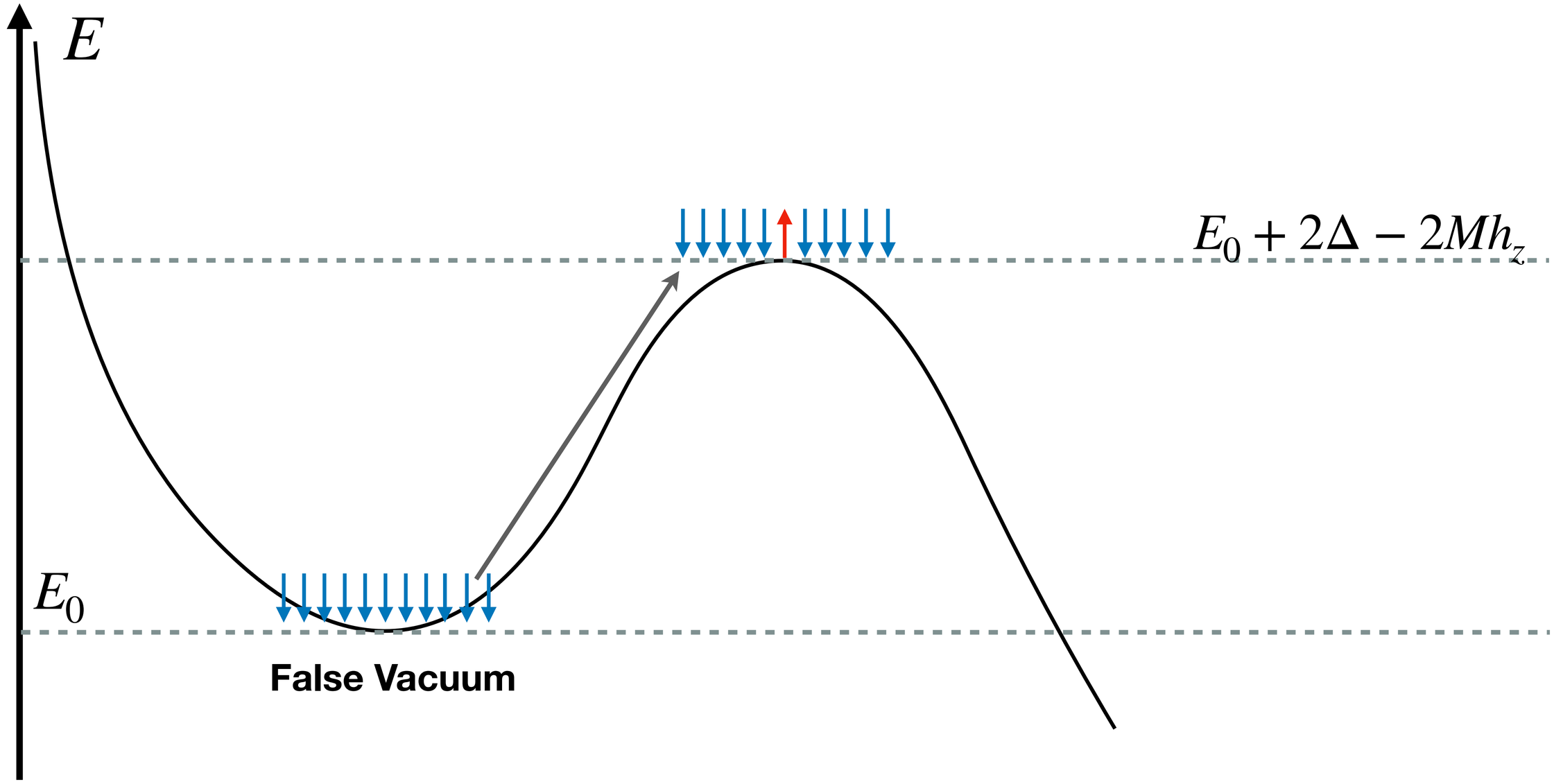
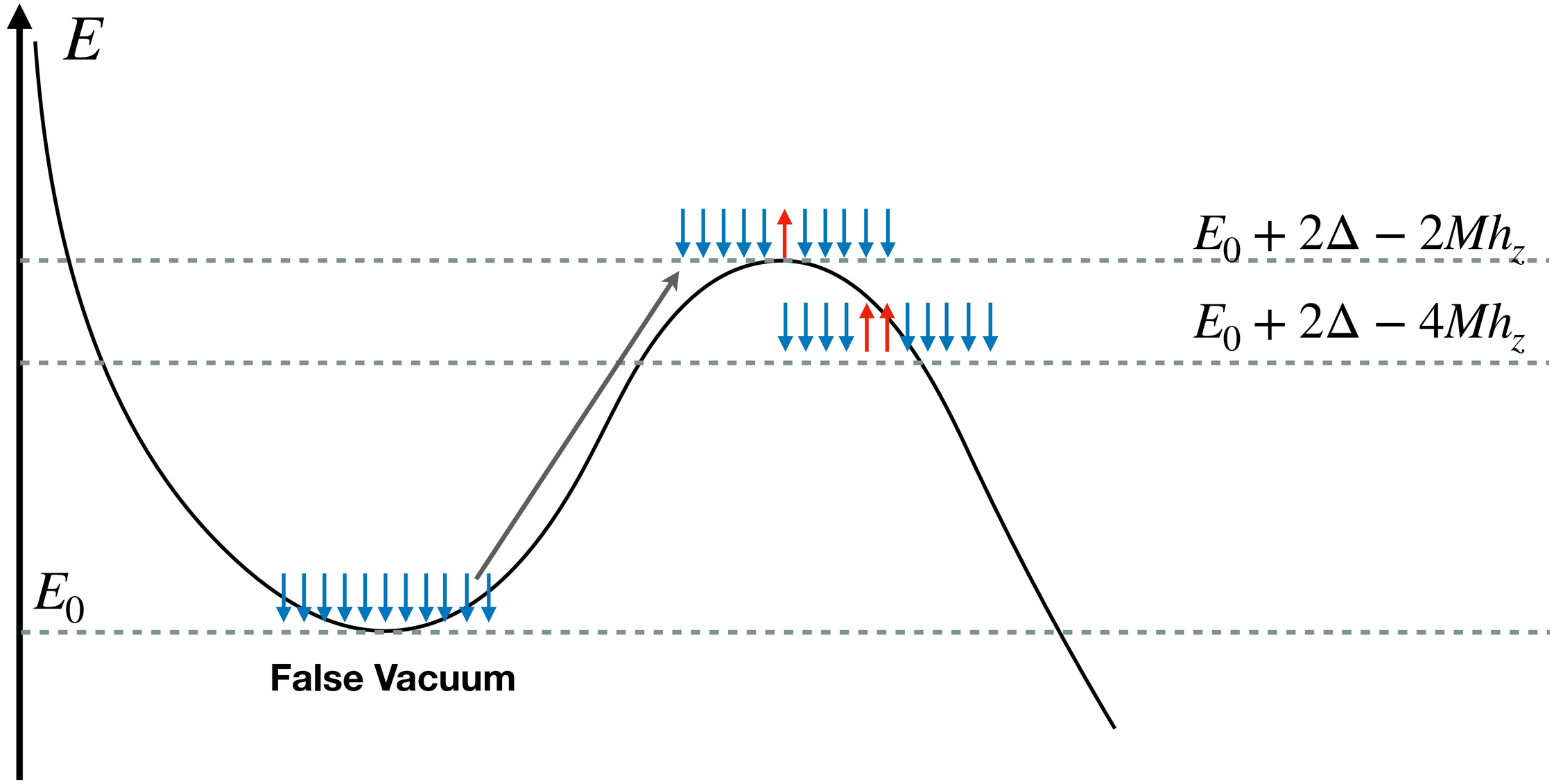


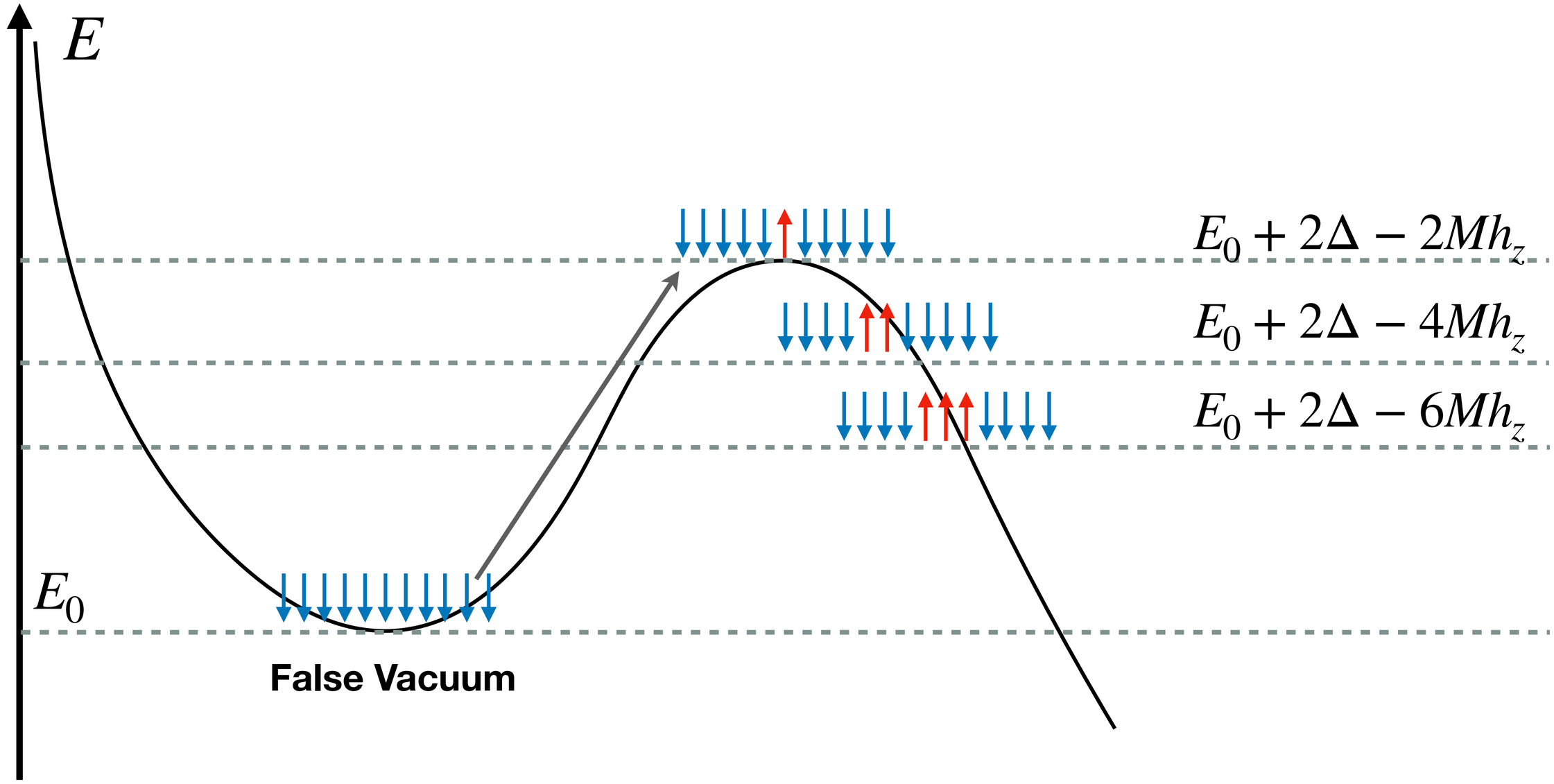
$$|\langle FV|\, U(t)\, |FV\rangle\,|^2 \approx \exp\left(-L\,\gamma\,t\right)$$

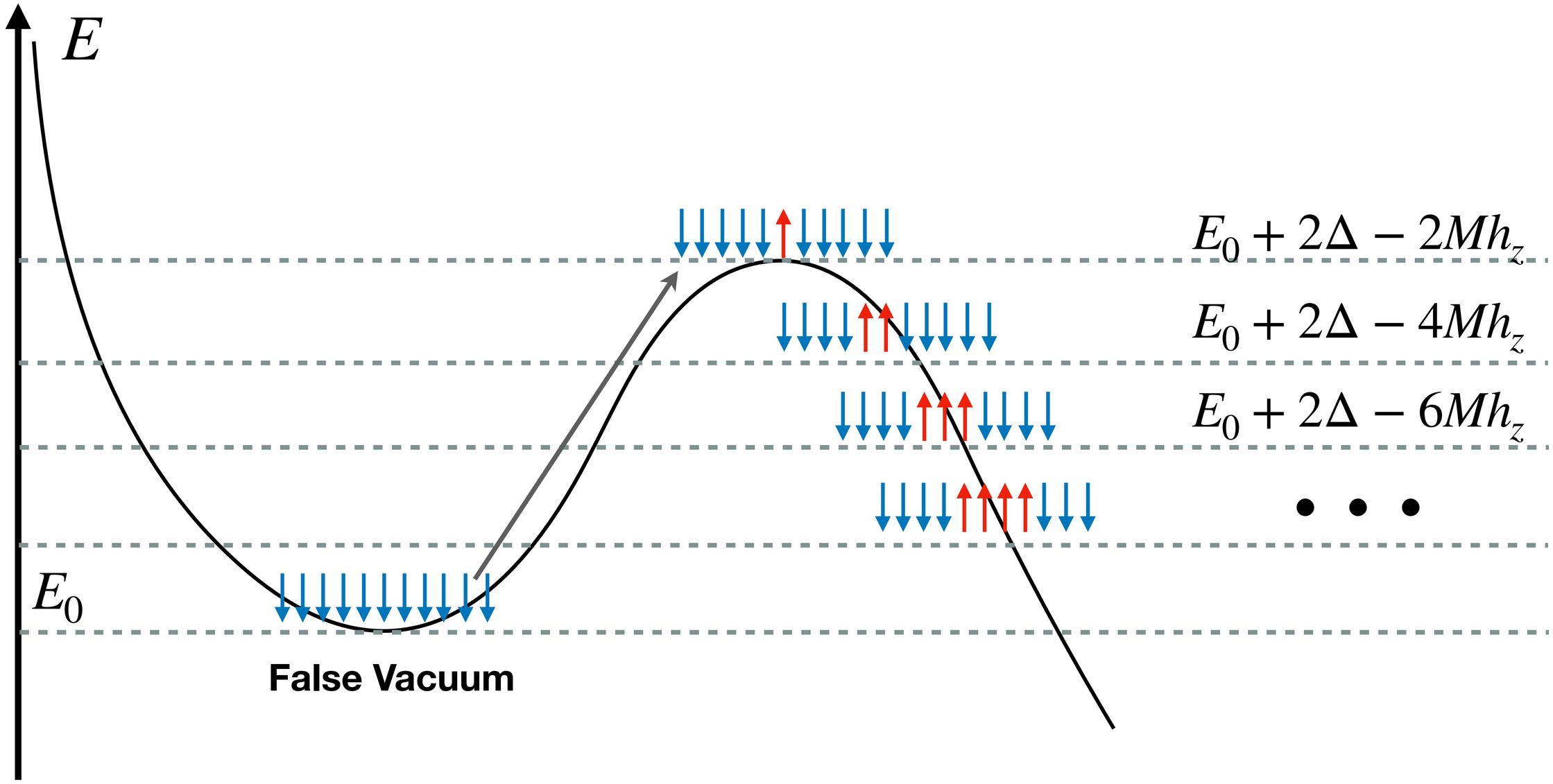
Unitary time evolution


E
E_0
False Vacuum


E
$E_0 + 2\Delta - 2Mh_z$
E_0
False Vacuum


E
$E_0 + 2\Delta - 2Mh_z$
$E_0 + 2\Delta - 4Mh_z$
E_0
False Vacuum


E
$E_0 + 2\Delta - 2Mh_z$
$E_0 + 2\Delta - 4Mh_z$
$E_0 + 2\Delta - 6Mh_z$
E_0
False Vacuum


E
$E_0 + 2\Delta - 2Mh_z$
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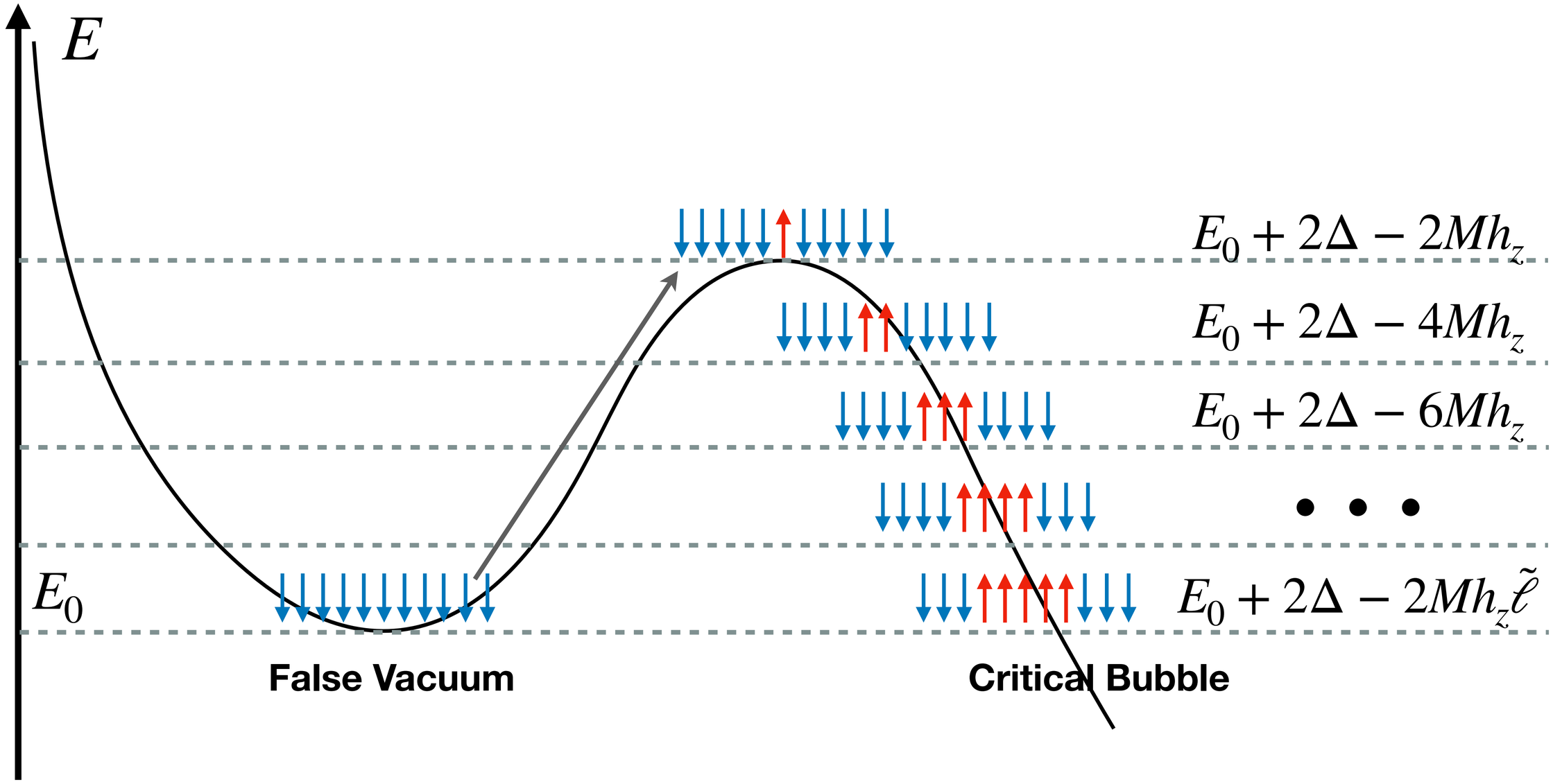
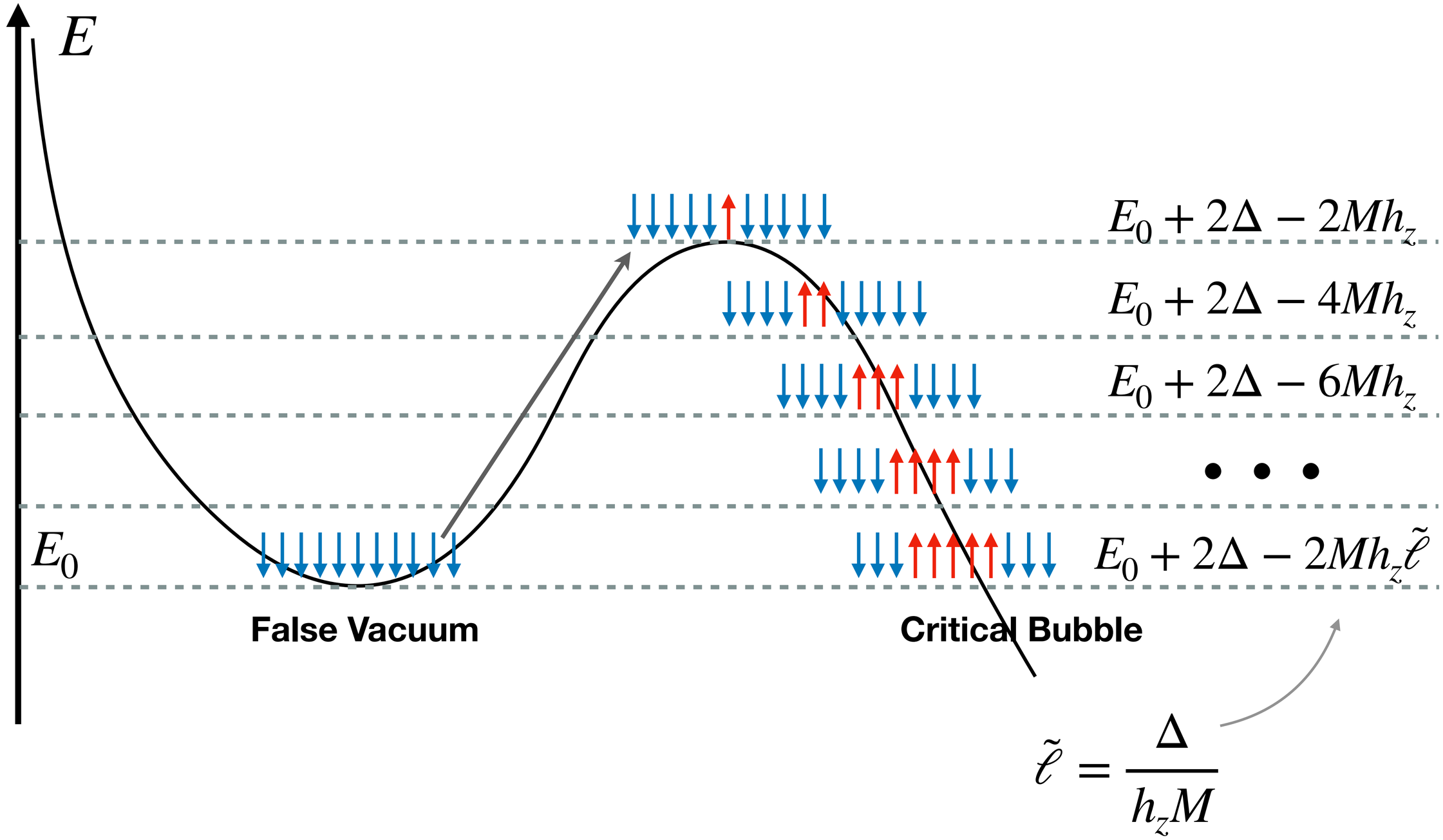
• • •
E_0
False Vacuum


E
$E_0 + 2\Delta - 2Mh_z$
$E_0 + 2\Delta - 4Mh_z$
$E_0 + 2\Delta - 6Mh_z$
• • •
E_0
$E_0 + 2\Delta - 2Mh_z\tilde{\ell}$
False Vacuum
Critical Bubble


E
$E_0 + 2\Delta - 2Mh_z$
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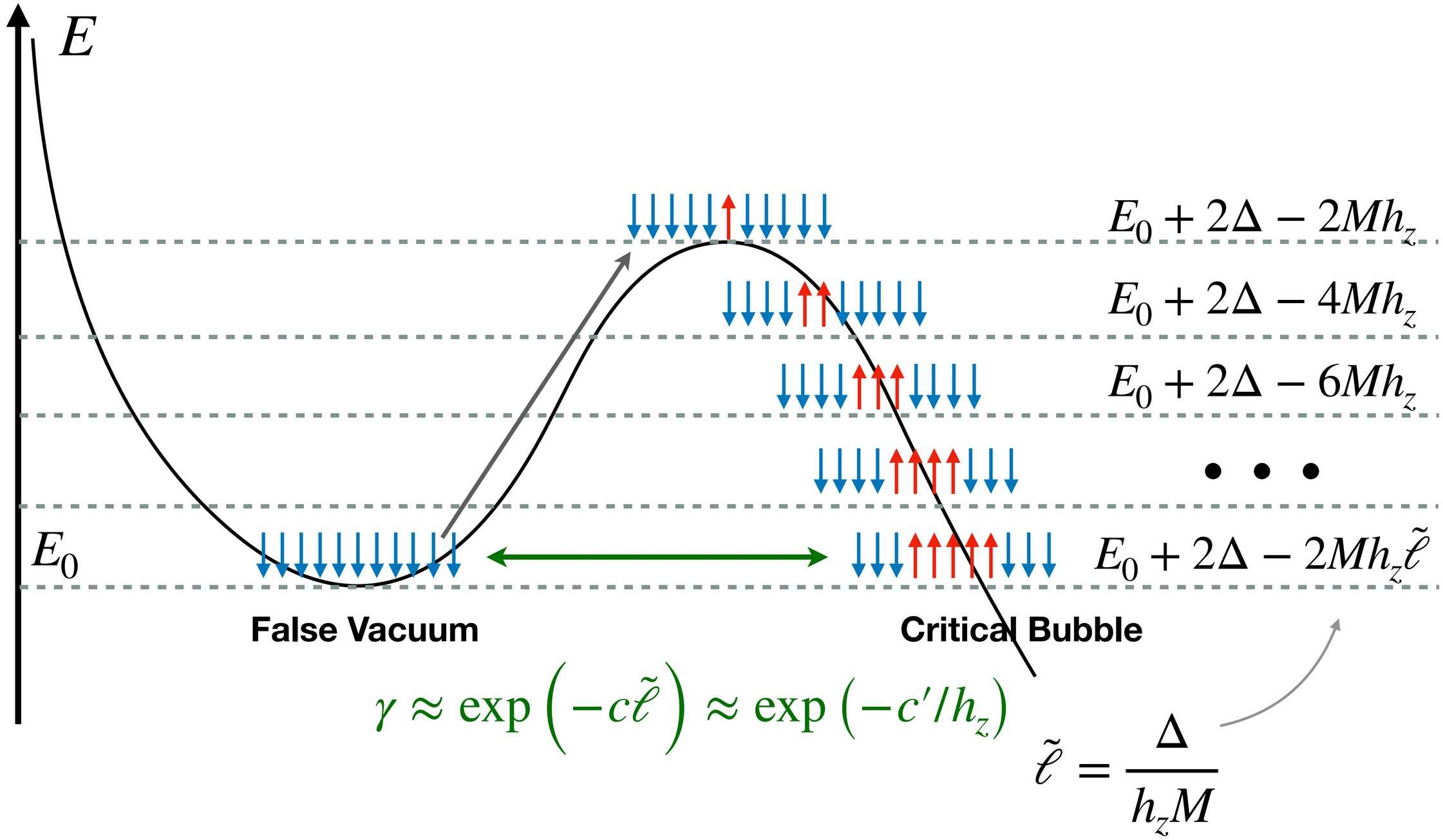
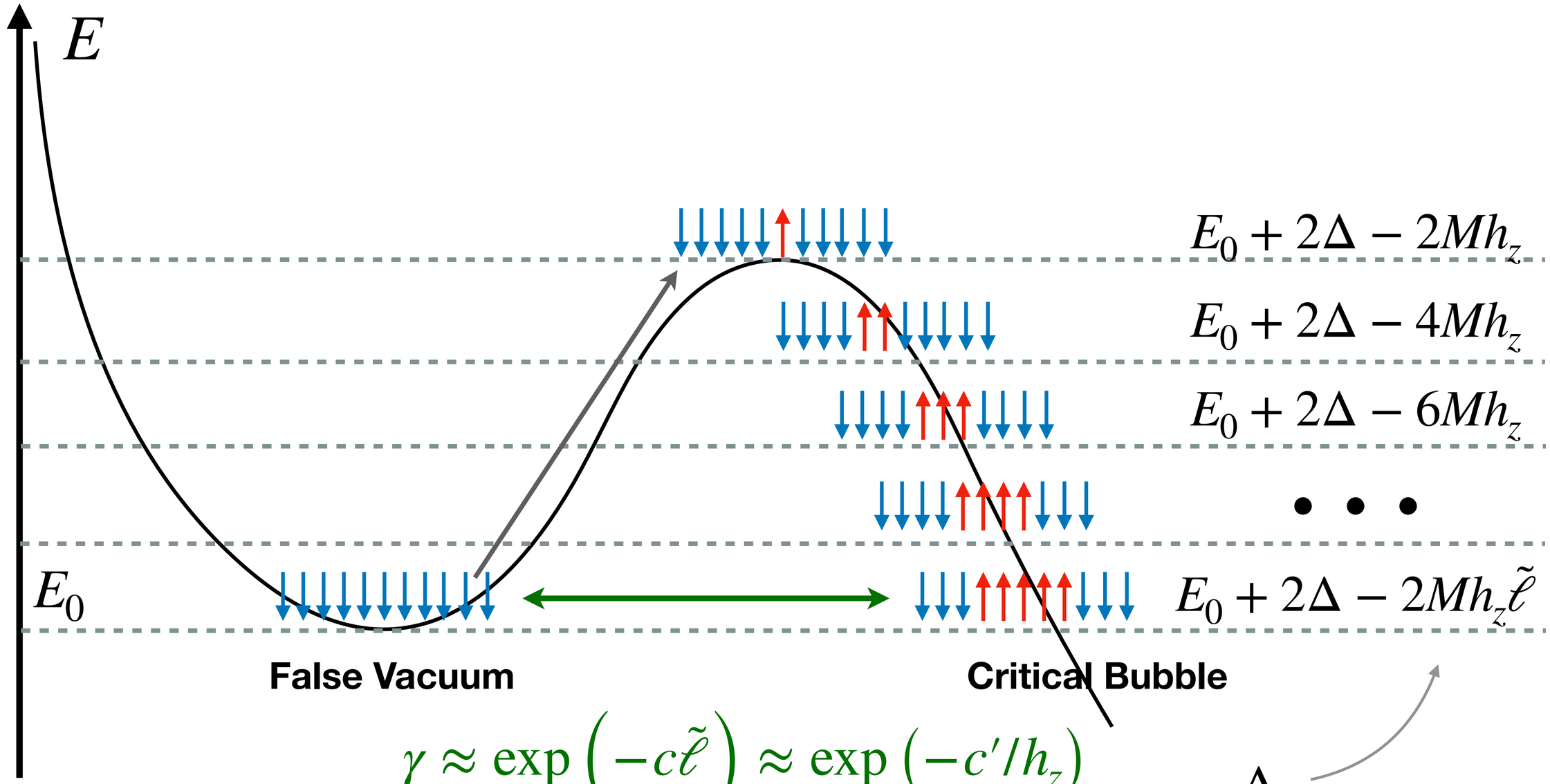
$E_0 + 2\Delta - 6Mh_z$
• • •
E_0
$E_0 + 2\Delta - 2Mh_z\tilde{\ell}$
False Vacuum
Critical Bubble
$\tilde{\ell} = \frac{\Delta}{h_z M}$


E
$E_0 + 2\Delta - 2Mh_z$
$E_0 + 2\Delta - 4Mh_z$
$E_0 + 2\Delta - 6Mh_z$
• • •
E_0
$E_0 + 2\Delta - 2Mh_z\tilde{\ell}$
False Vacuum
Critical Bubble
$\gamma \approx \exp\left(-c\tilde{\ell}\right) \approx \exp\left(-c'/h_z\right)$
$\tilde{\ell} = \dfrac{\Delta}{h_z M}$



$$\gamma \approx \exp\left(-c\tilde{\ell}\right) \approx \exp\left(-c'/h_z\right)$$

$$\tilde{\ell} = \frac{\Delta}{h_z M}$$

$h_z M \ll \Delta$ Rutkevich 1999 PRB

The Quench Protocol

1 - Prepare $|\psi_{\downarrow}\rangle = \otimes_i |\downarrow\rangle_i$: Ground state of $H(h_x = 0, h_z = 0)$

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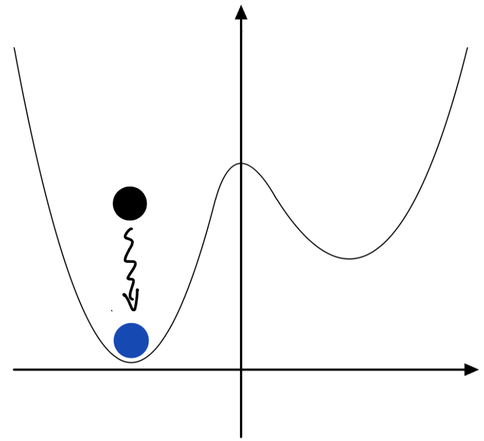

2 - Approach ground state of $H(0 < h_x < 1, -h_z)$:

$$|0\rangle = exp\left[-\tau H(h_x, -h_z)\right] |\psi_{\downarrow}\rangle$$

The Quench Protocol

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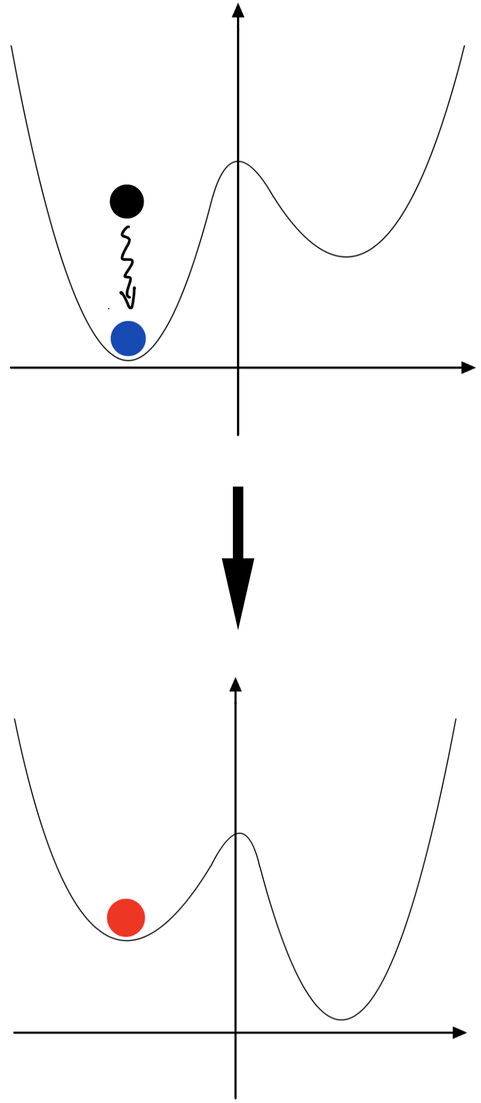
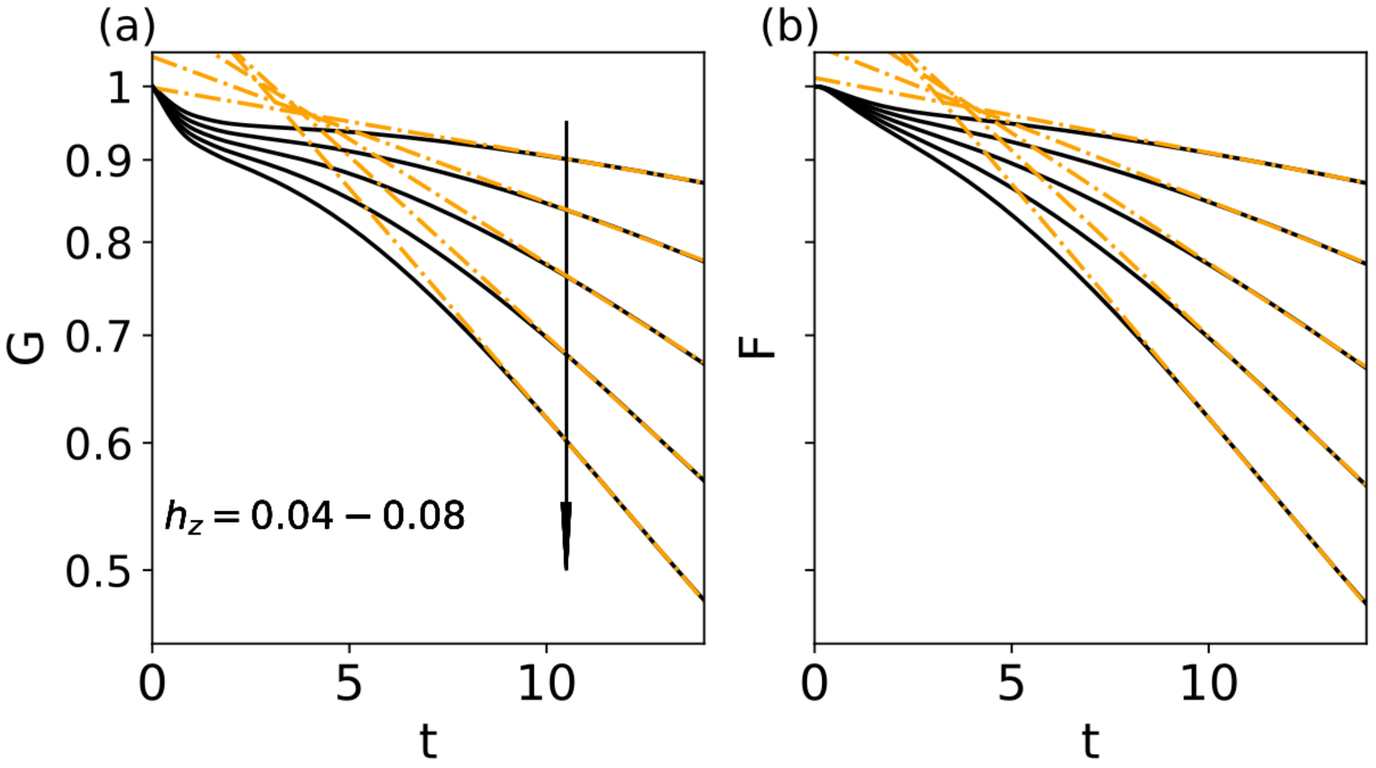
3 - Switch $-h_z$ **to** $+h_z$ and evolve with $exp\left[-i\, H(h_x, +h_z)\, t\right]$:



Rate extraction

Two-sites observable

$$G(t) = 1 - ||\rho(t) - \rho(0)||_1$$



Rescaled Magnetisation

$$F(t) = \frac{\langle \sigma^z(t) \rangle + \langle \sigma^z(0) \rangle}{2\langle \sigma^z(0) \rangle}$$

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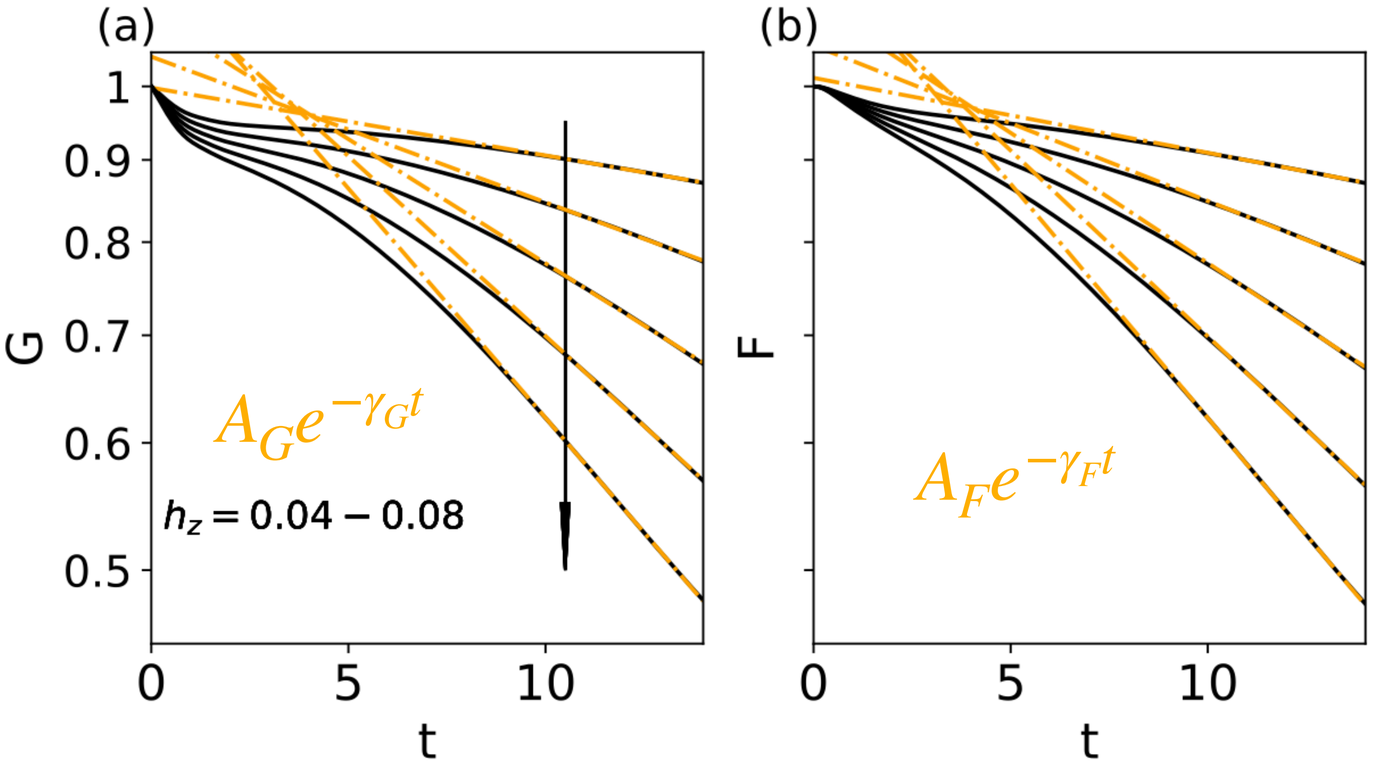

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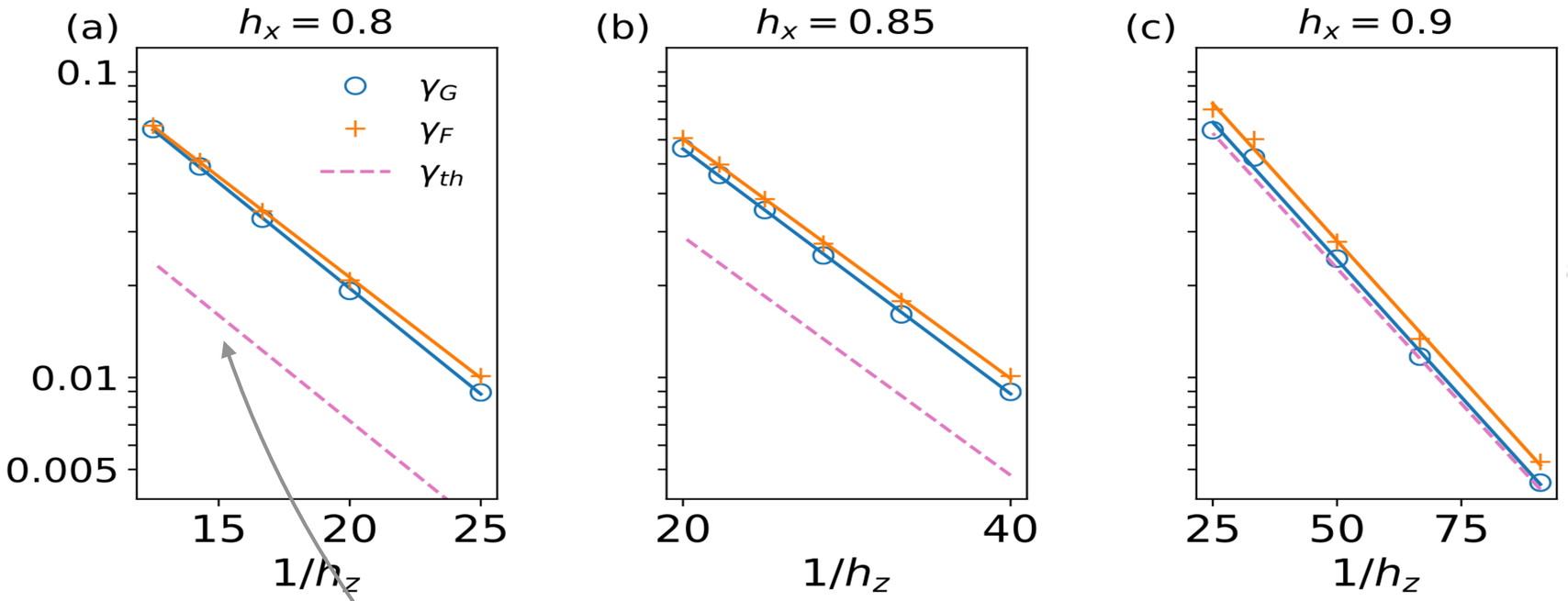
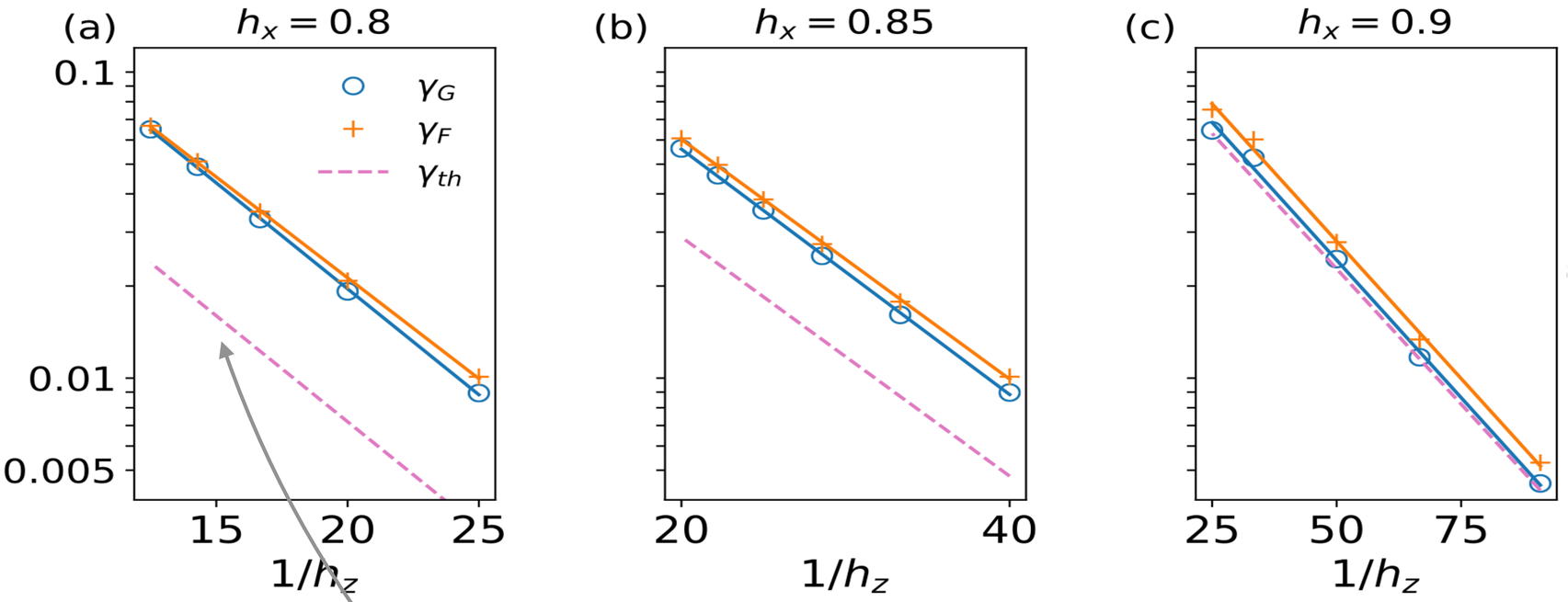
Comparison with theoretical prediction



$$\gamma_{th} = c(h_x)\exp\left(-\frac{q_h}{h_z}\right)$$

Rutkevich 1999 PRB

Comparison with theoretical prediction

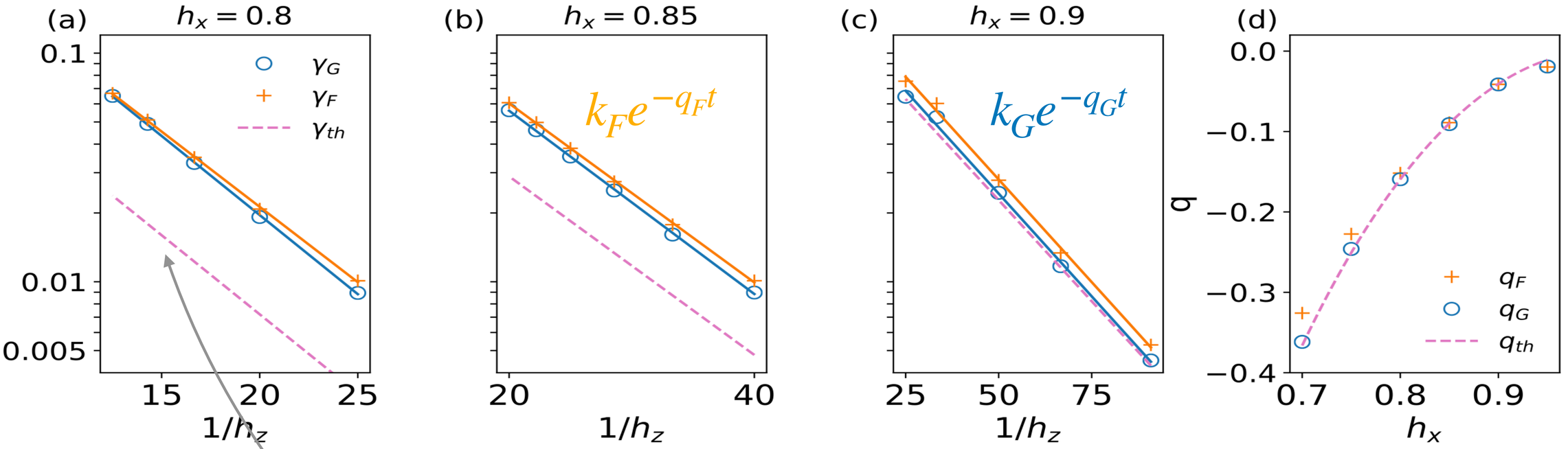


$$\gamma_{th} = c(h_x)\exp\left(-\frac{q_h}{h_z}\right)$$

Rutkevich 1999 PRB

Prefactor is off !

Comparison with theoretical prediction



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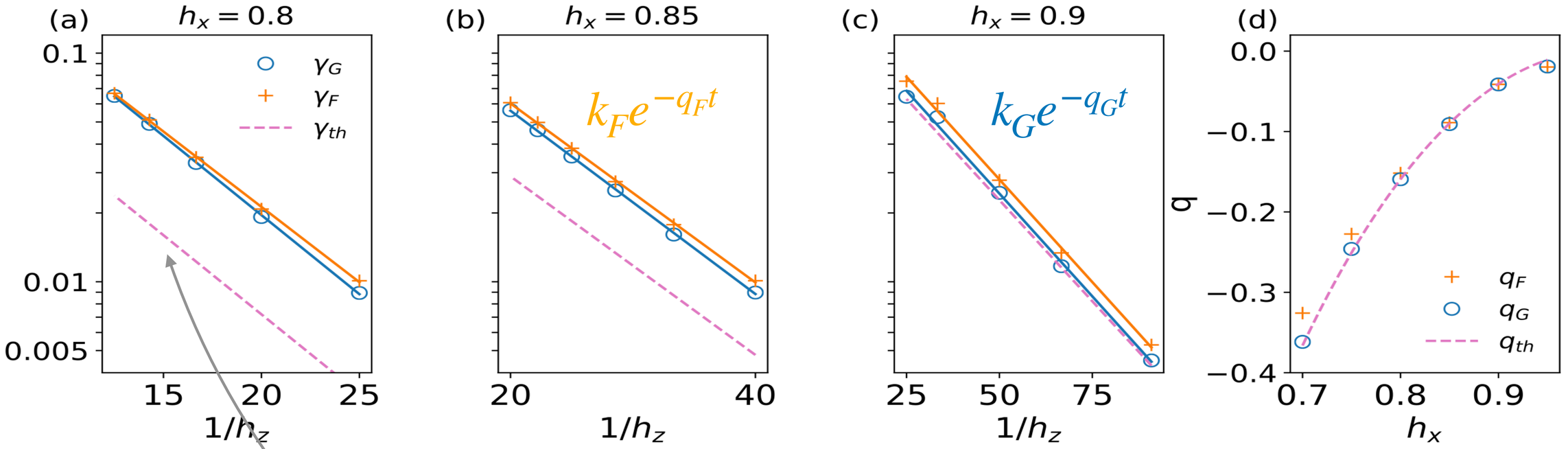

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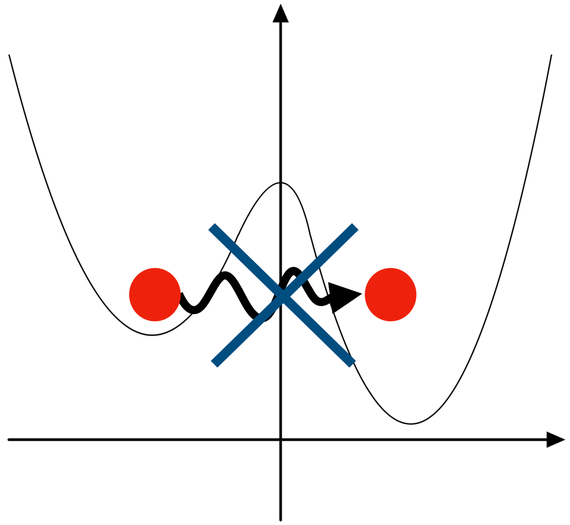
Rutkevich 1999 PRB

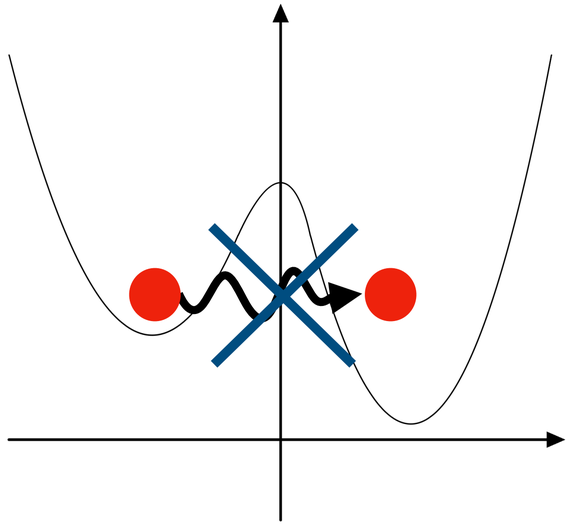
Exponential coefficient is good!

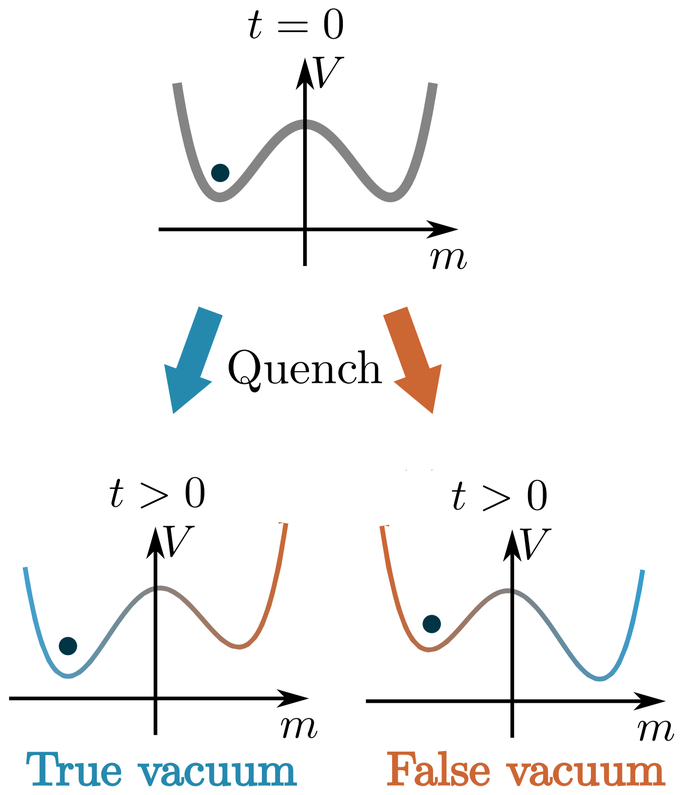
More on the oscillations



Lagnese, Surace, Morampudi, Wilczek, arXiv:2308.08340

More on the oscillations





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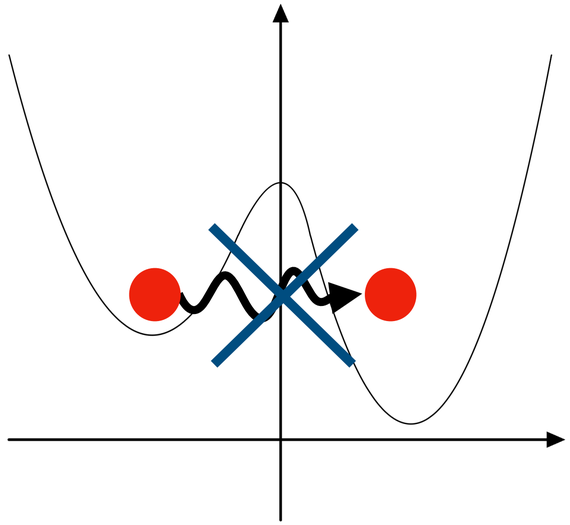

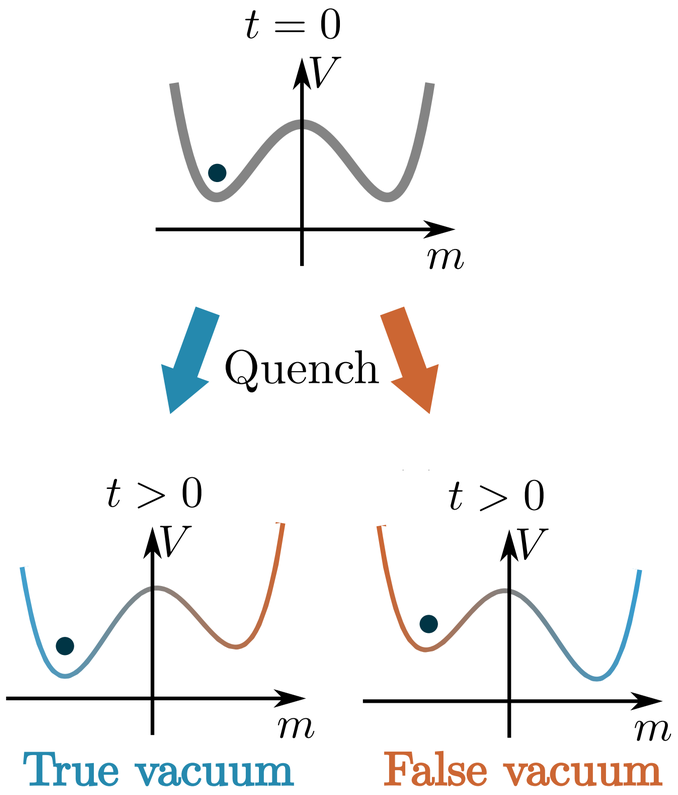

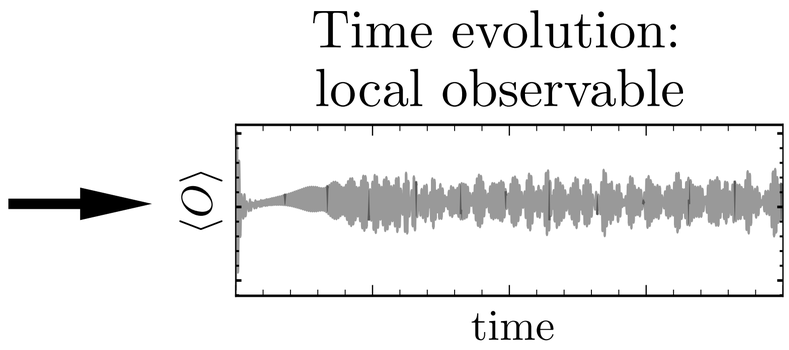

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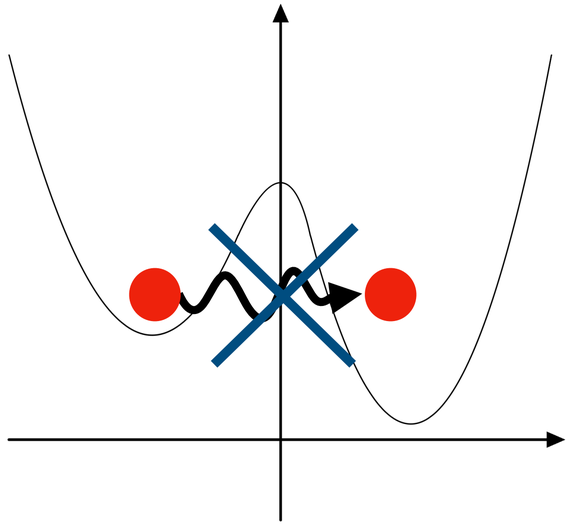

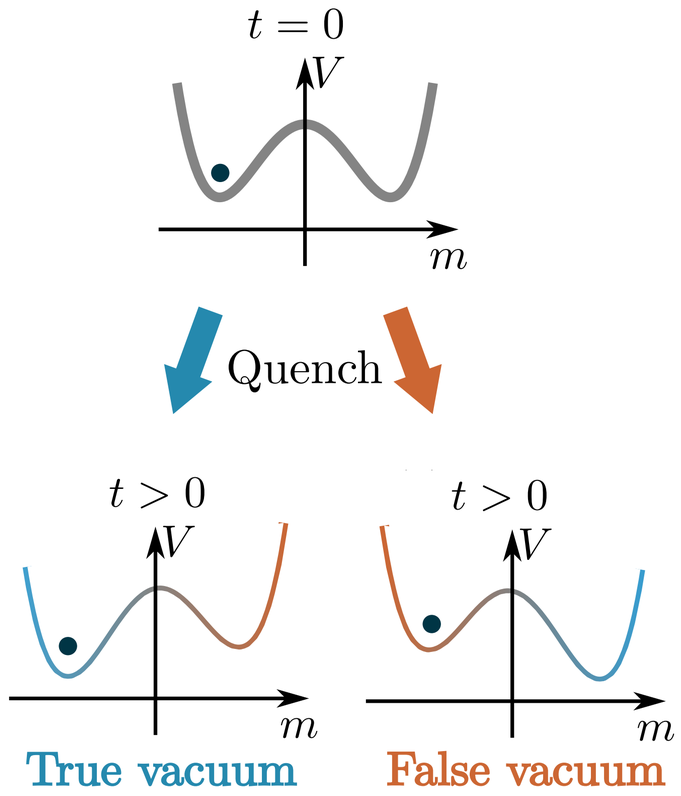

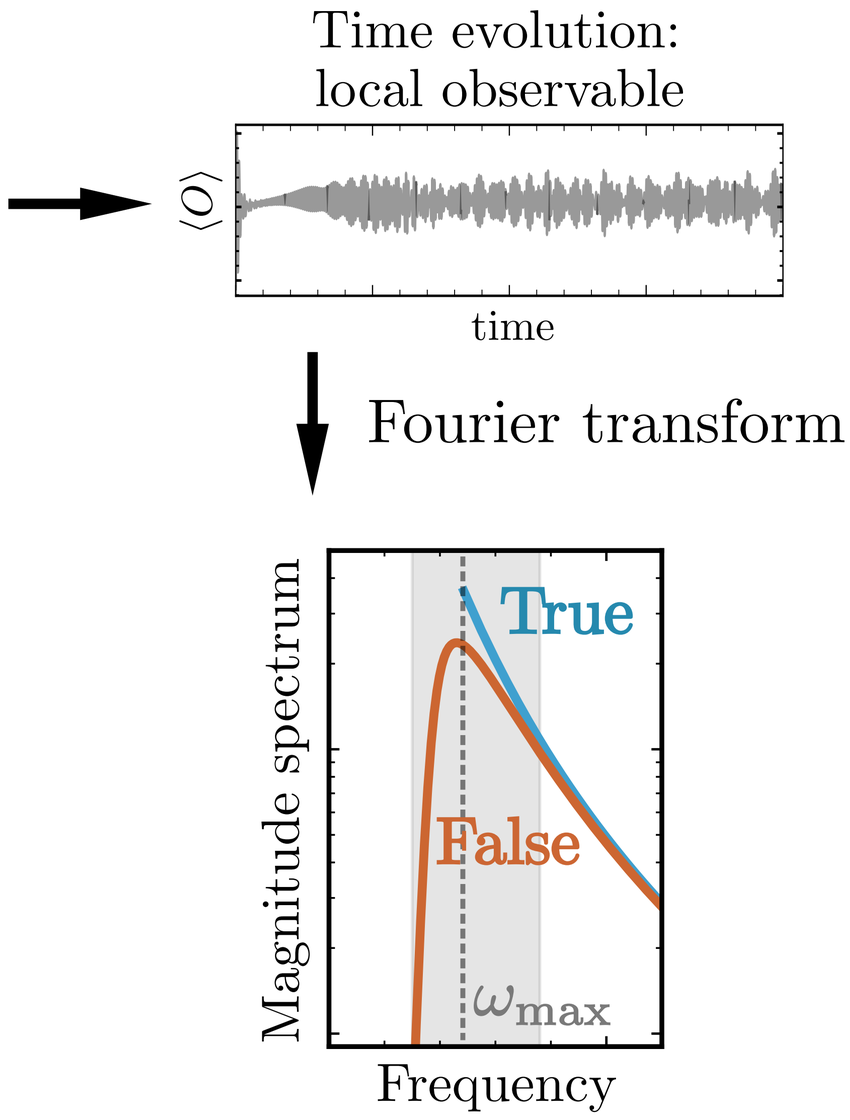

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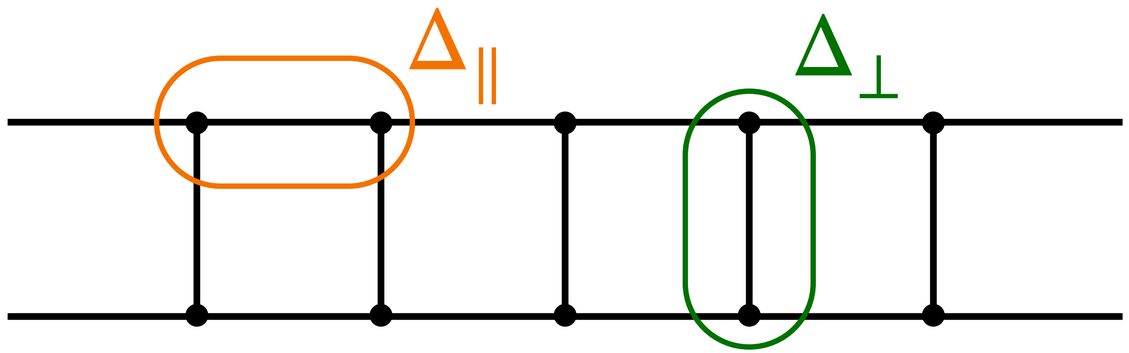
Another Model

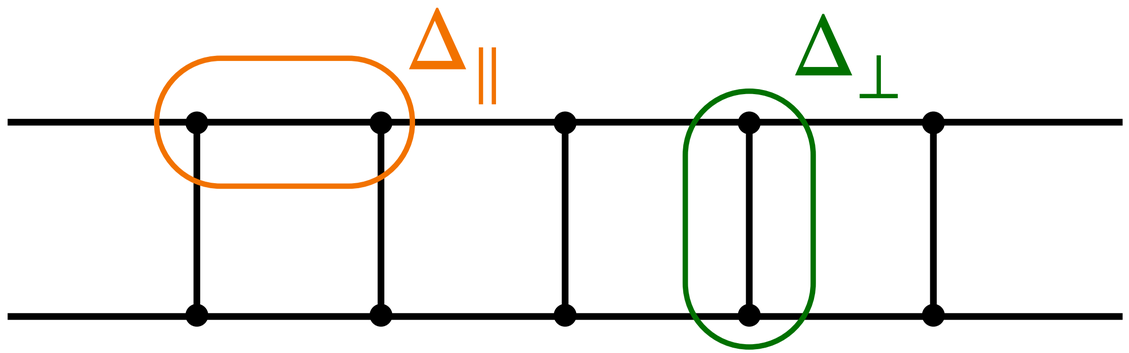
XXZ Ladder

$$H = \frac{1}{2} \sum_{j} \sum_{\alpha=1,2} \left(\sigma^x_{j,\alpha} \sigma^x_{j+1,\alpha} + \sigma^y_{j,\alpha} \sigma^y_{j+1,\alpha} + \Delta_{\parallel} \sigma^z_{j,\alpha} \sigma^z_{j+1,\alpha} \right) + \Delta_{\perp} \sigma^z_{j,1} \sigma^z_{j+1,2}$$



XXZ Ladder

$$H = \frac{1}{2}\sum_{j}\sum_{\alpha=1,2}\left(\sigma^x_{j,\alpha}\sigma^x_{j+1,\alpha} + \sigma^y_{j,\alpha}\sigma^y_{j+1,\alpha} + \Delta_{\parallel}\sigma^z_{j,\alpha}\sigma^z_{j+1,\alpha}\right) + \Delta_{\perp}\sigma^z_{j,1}\sigma^z_{j+1,2}$$

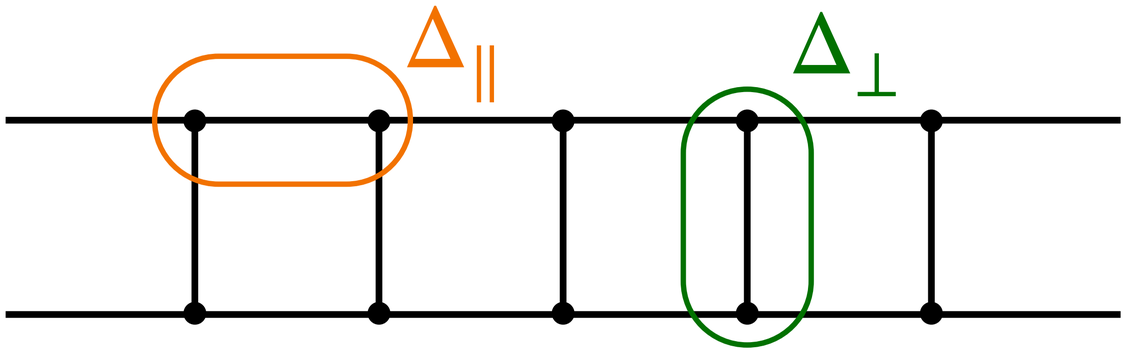


$$\Delta_{\perp} \leftrightarrow h_z$$

Interacting ↔ No Free Fermion Mapping

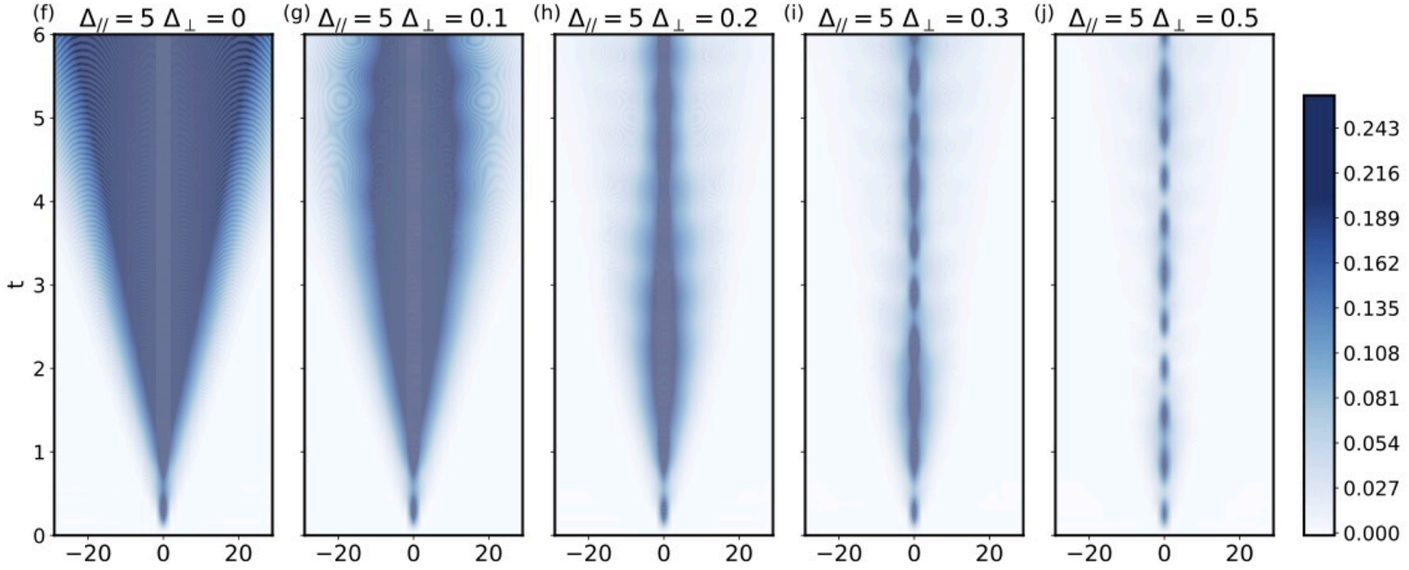
XXZ Ladder

$$H = \frac{1}{2}\sum_{j}\sum_{\alpha=1,2}\left(\sigma^x_{j,\alpha}\sigma^x_{j+1,\alpha} + \sigma^y_{j,\alpha}\sigma^y_{j+1,\alpha} + \Delta_{\parallel}\sigma^z_{j,\alpha}\sigma^z_{j+1,\alpha}\right) + \Delta_{\perp}\sigma^z_{j,1}\sigma^z_{j+1,2}$$



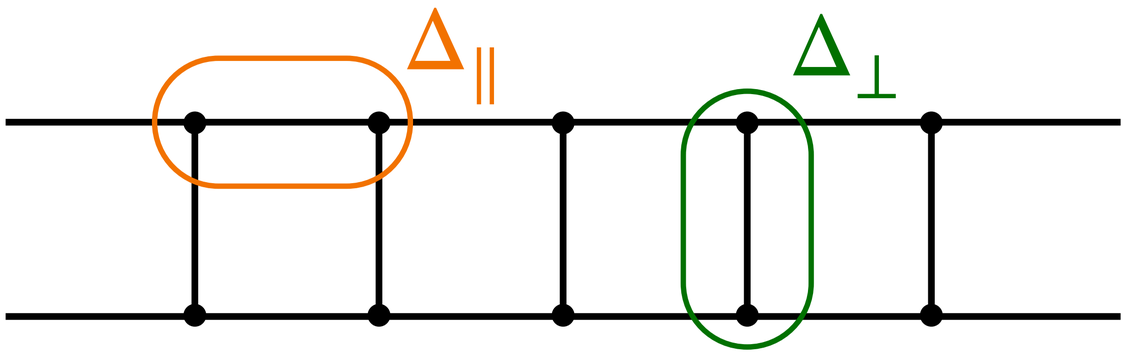
Lagnese, Surace, Kormos, Calabrese J. Phys A 2022

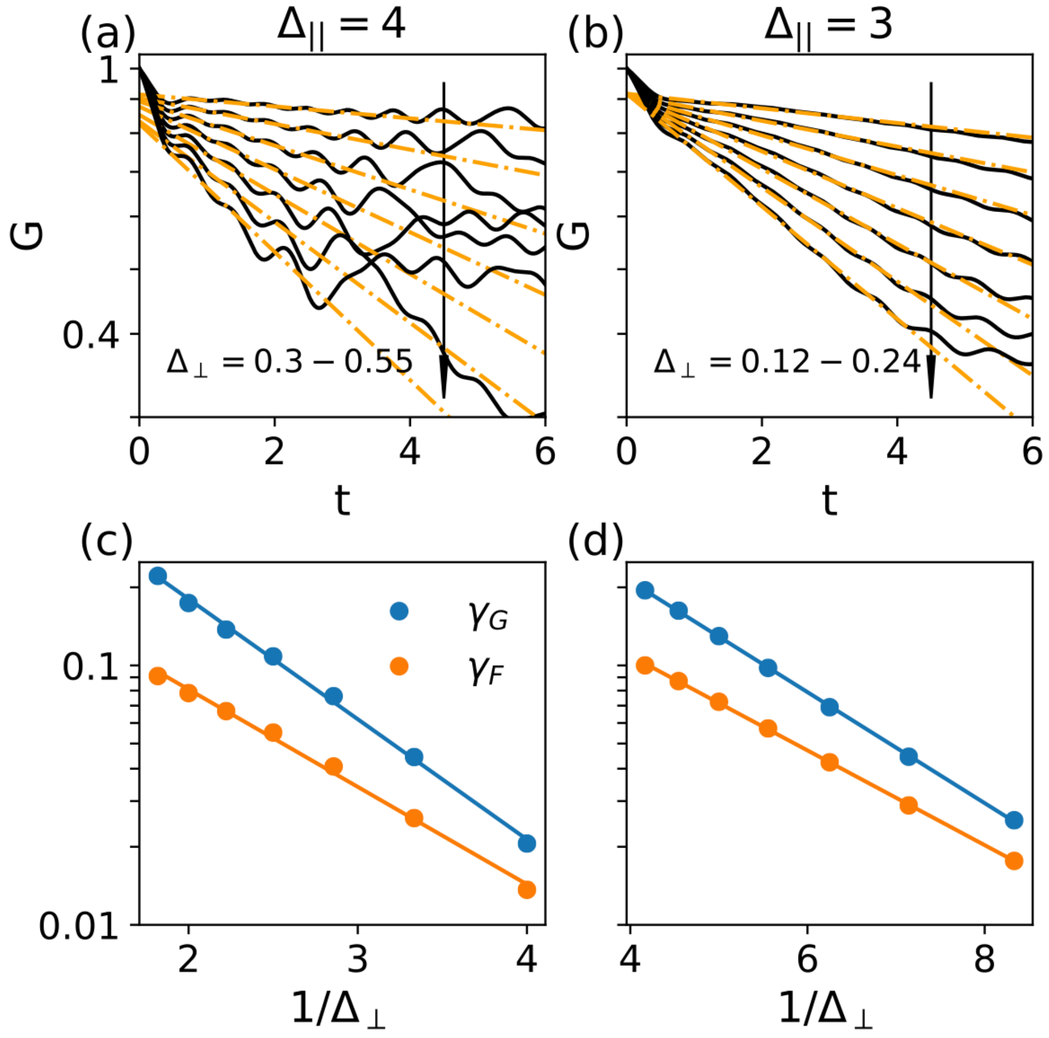
Lagnese, Surace, Kormos, Calabrese J. Stat. Mech. 2022



XXZ Ladder

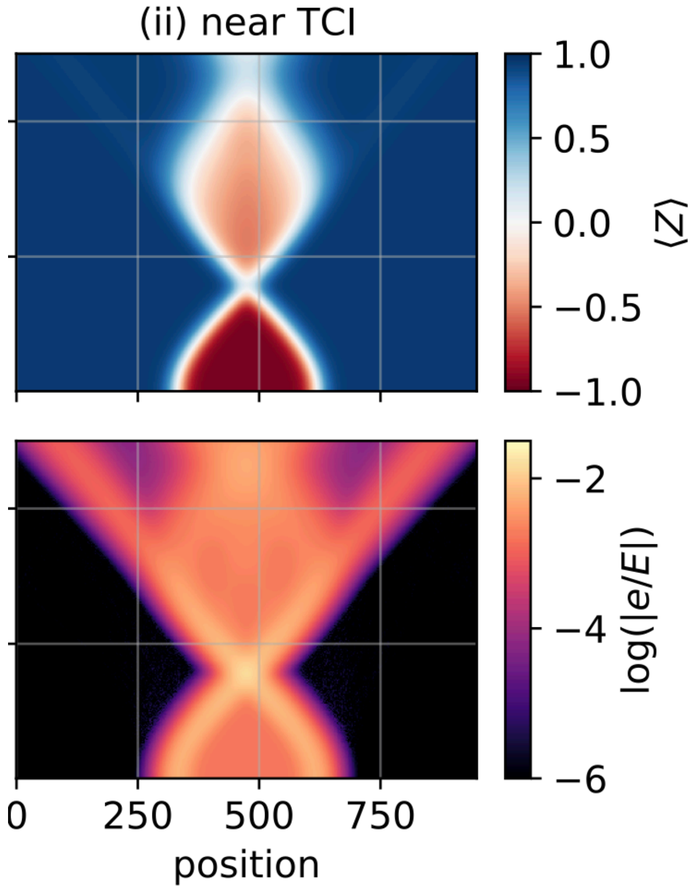
$$H = \frac{1}{2}\sum_{j}\sum_{\alpha=1,2}\left(\sigma^x_{j,\alpha}\sigma^x_{j+1,\alpha} + \sigma^y_{j,\alpha}\sigma^y_{j+1,\alpha} + \Delta_{\parallel}\sigma^z_{j,\alpha}\sigma^z_{j+1,\alpha}\right) + \Delta_{\perp}\sigma^z_{j,1}\sigma^z_{j+1,2}$$





Outlook

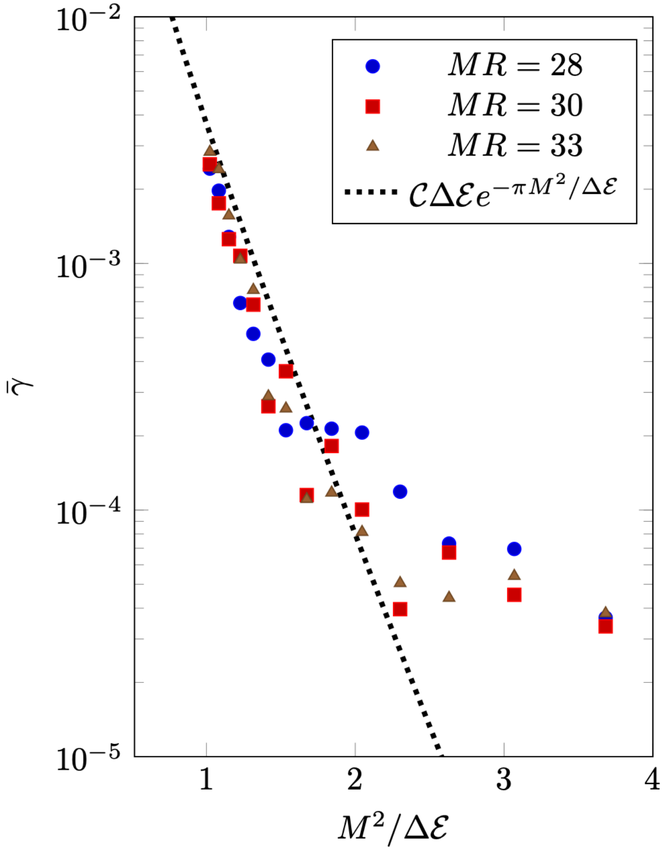
More on Bubble Dynamics



Milsted, Liu, Preskill, Vidal

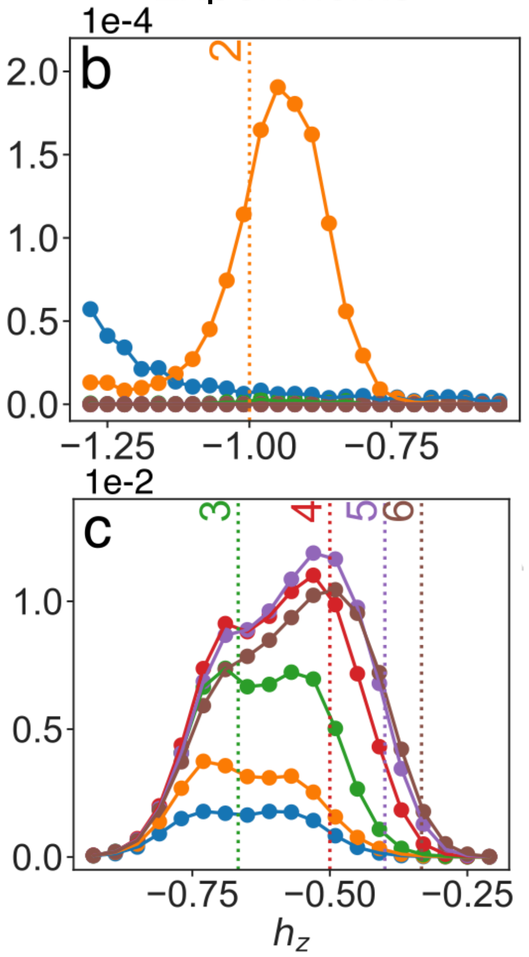
PRX Quantum 2022

Prefactor in the decay rate



Lencses, Mussardo, Tacaks PRD 2022

Experiments

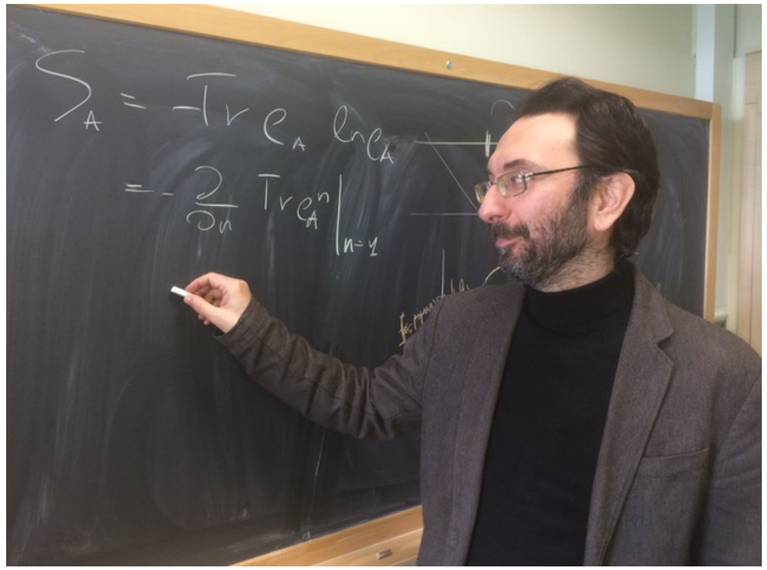


Jaka Vodeb et al. arXiv:2406.14718

Thanks for the attention !

Federica M. Surace

Pasquale Calabrese



Marton Kormos