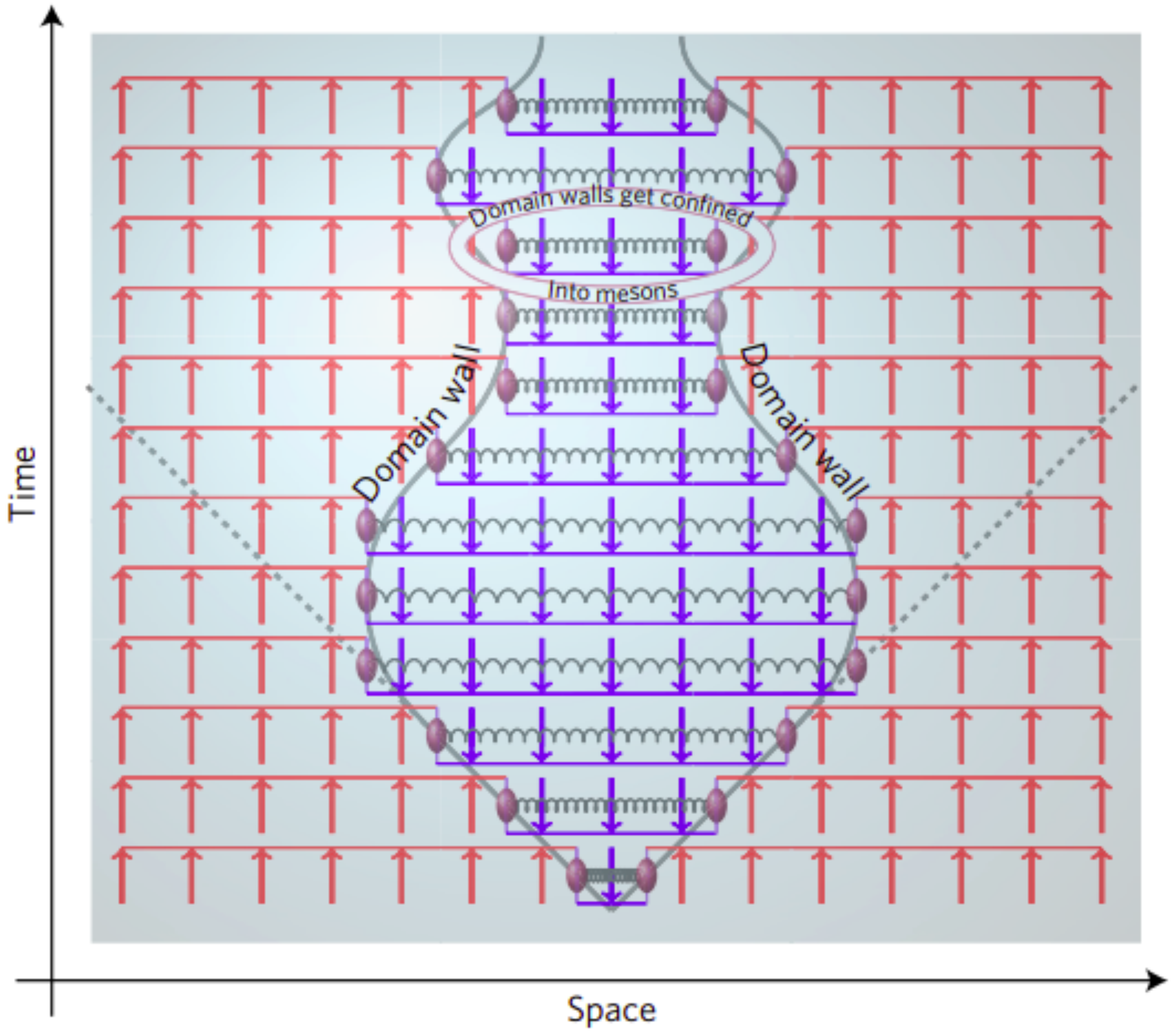
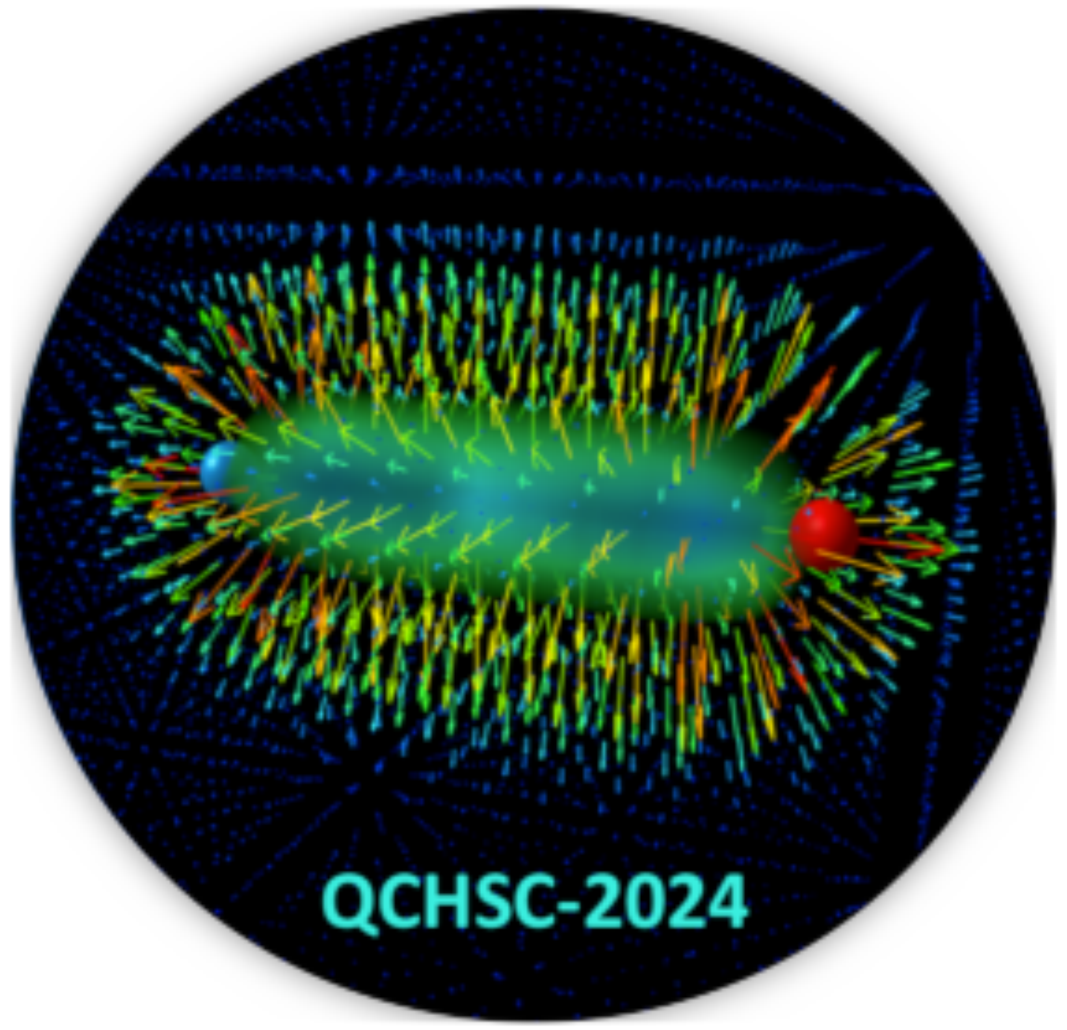
# Confinement and False Vacuum Decay in Quantum Spin Chains



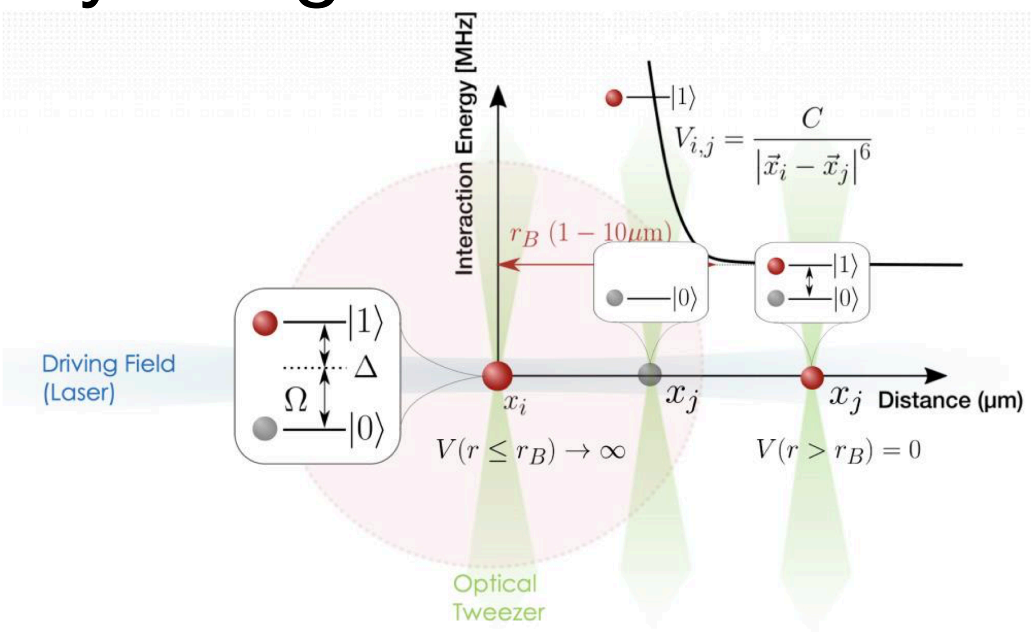




Jožef Stefan Institute, Ljubljana, Slovenia

Gianluca Lagnese

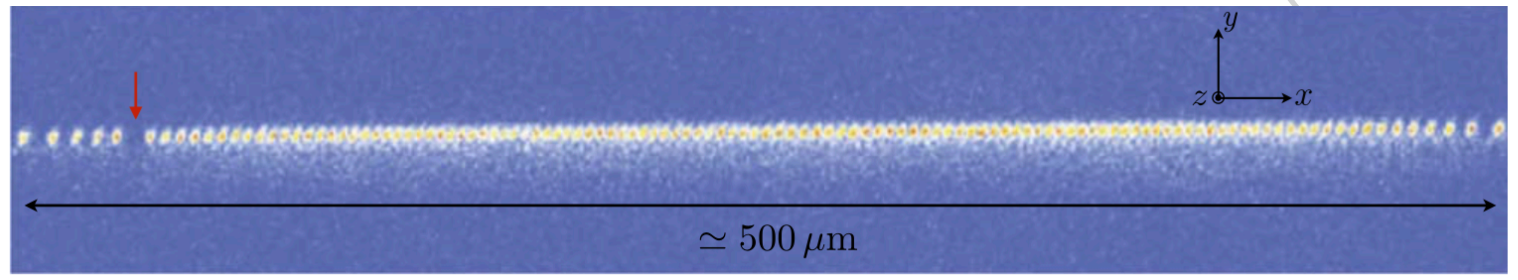
## Experiments/Quantum simulators

### Rydberg Neutral Atoms



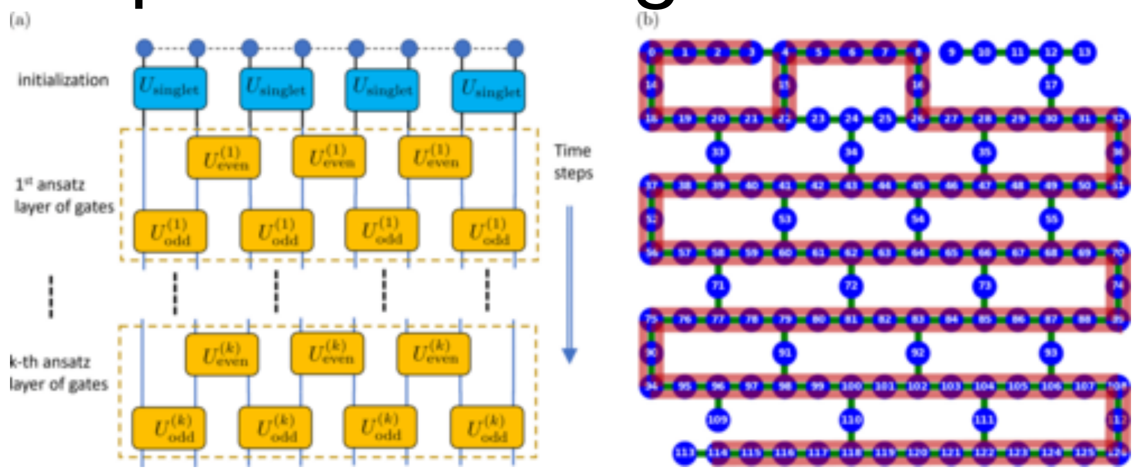

J. Wurtz et al., arXiv:2306.11727

### Cryogenic Trapped Ions



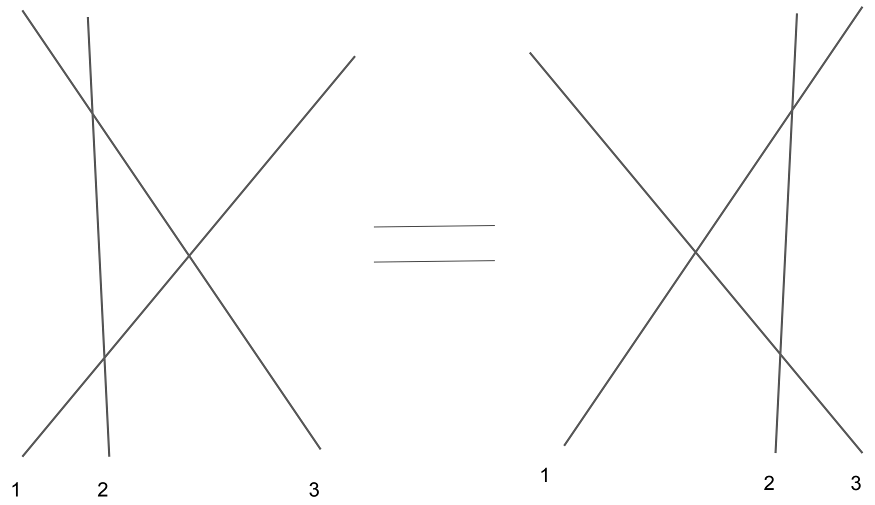

G. Pagano et al., Quantum Sci. Technol. 2019

### Superconducting Qubits



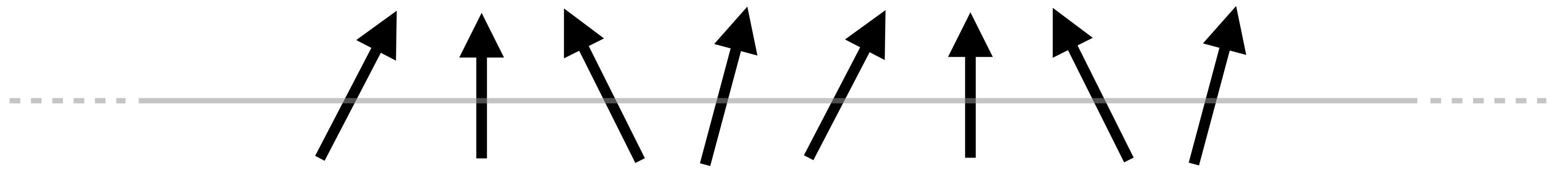

H. Yu, Y. Zhao, Tzu-Chieh Wei, Phys. Rev. Research 2023

## Analytical (Integrability, CFT…)

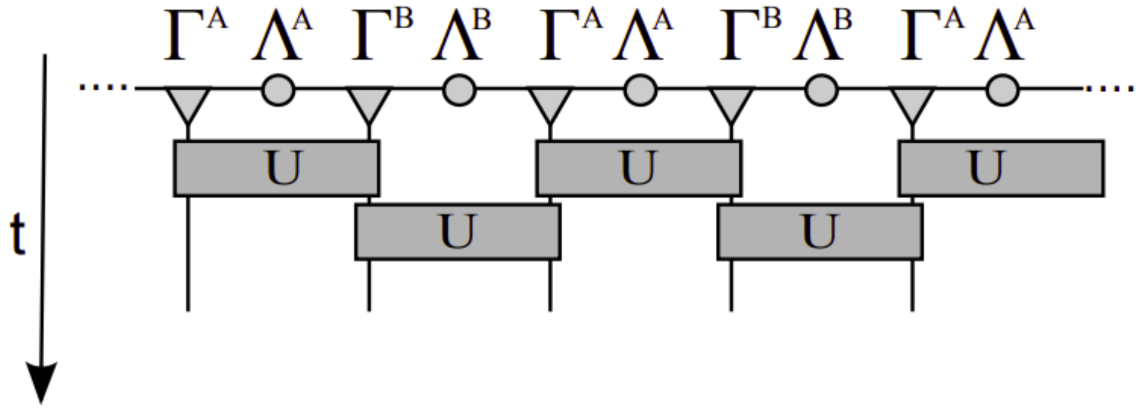


Franchini, Springerlink LNP 940

# Quantum Spin Chains



## Numerical methods (Tensor Networks, TEBD, DMRG)




J. Hauschild, F. Pollmann, SciPost Phys. 2018

# Key concepts

1) Real-time out-of-equilibrium dynamics of many-body systems

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*“Introduction to 'Quantum Integrability in Out of Equilibrium Systems’”*

*Calabrese, Essler, Mussardo J. Stat. Mech.* 2016

# Overview

1) The Model: Transverse Field Quantum Ising Chain

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2) Confinement:

*“Real time confinement following a quantum quench to a non-integrable model”*

Kormos, Collura, Takaks, Calabrese, Nature Physics 2017

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*"Real time confinement following a quantum quench to a non-integrable model"*

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*"False vacuum decay in quantum spin chains"*

Lagnese, Surace, Kormos, Calabrese PRB 2021

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Kormos, Collura, Takaks, Calabrese, Nature Physics 2017

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Lagnese, Surace, Kormos, Calabrese PRB 2021

4) Another Model

# The Model

# The model: Transverse Field Quantum Ising Chain

Classical Ising: $E = -J \sum_{\langle ij \rangle} s_i s_j \qquad s_i \in \{-1, 1\}$

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$s_i \to \sigma_i^z$

1+1 D Quantum Ising: $H = -J \sum_i \left( \sigma_i^z \sigma_{i+1}^z - h_x \sigma_i^x \right)$

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Pauli Operators

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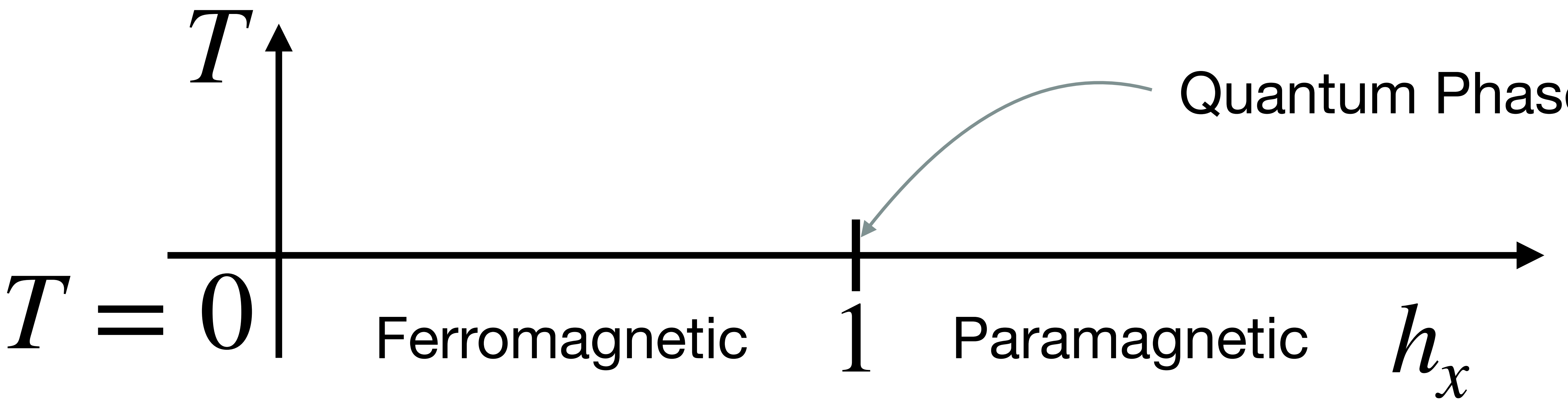
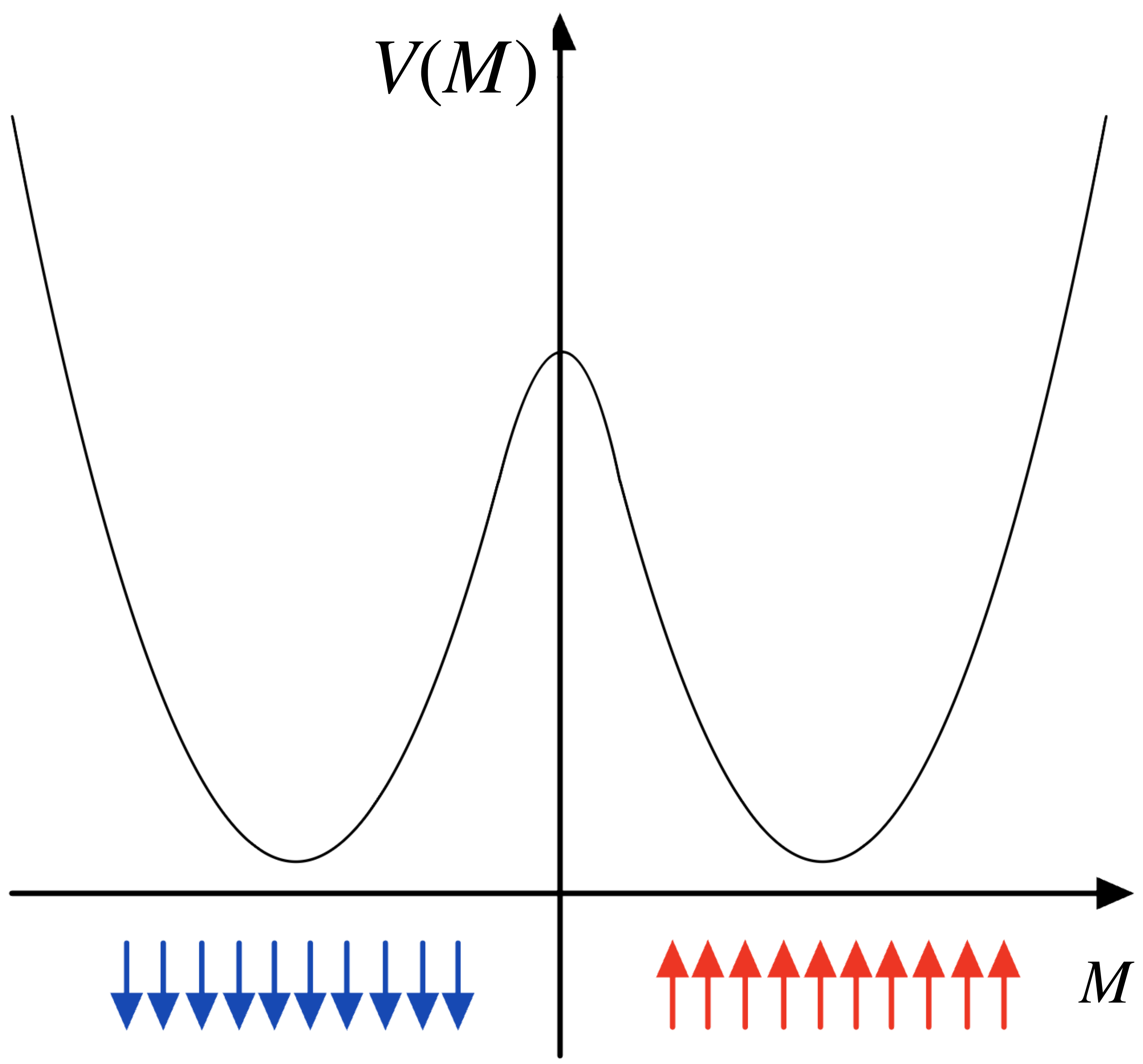
Quantum Phase Transition ∼ 2D Classical Thermal



# The model: Transverse Field Quantum Ising Chain

$$H = -\sum_i \sigma_i^z \sigma_{i+1}^z - h_x \sigma_i^x$$

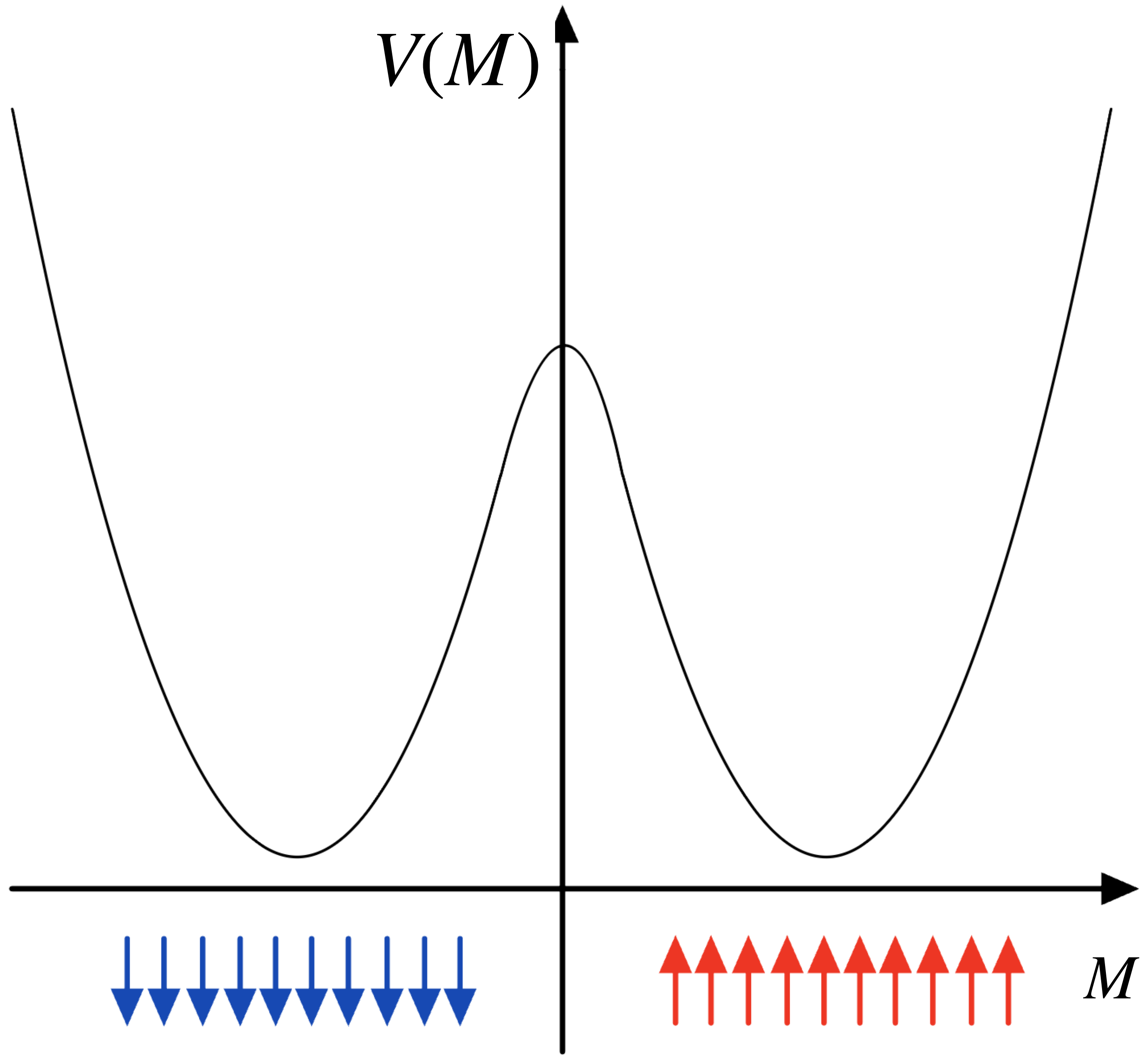
$|h_x| < 1 \Rightarrow$ Two **Degenerate** Ferromagnetic ground states



# The model: Transverse Field Quantum Ising Chain

$$H = -\sum_i \sigma_i^z \sigma_{i+1}^z - h_x \sigma_i^x$$

$$|h_x| \ll 1 \quad \Rightarrow \quad H \approx -\sum_j 2\,|j\rangle\langle j| - h_x \left( |j\rangle\langle j+1| + h.c. \right)$$



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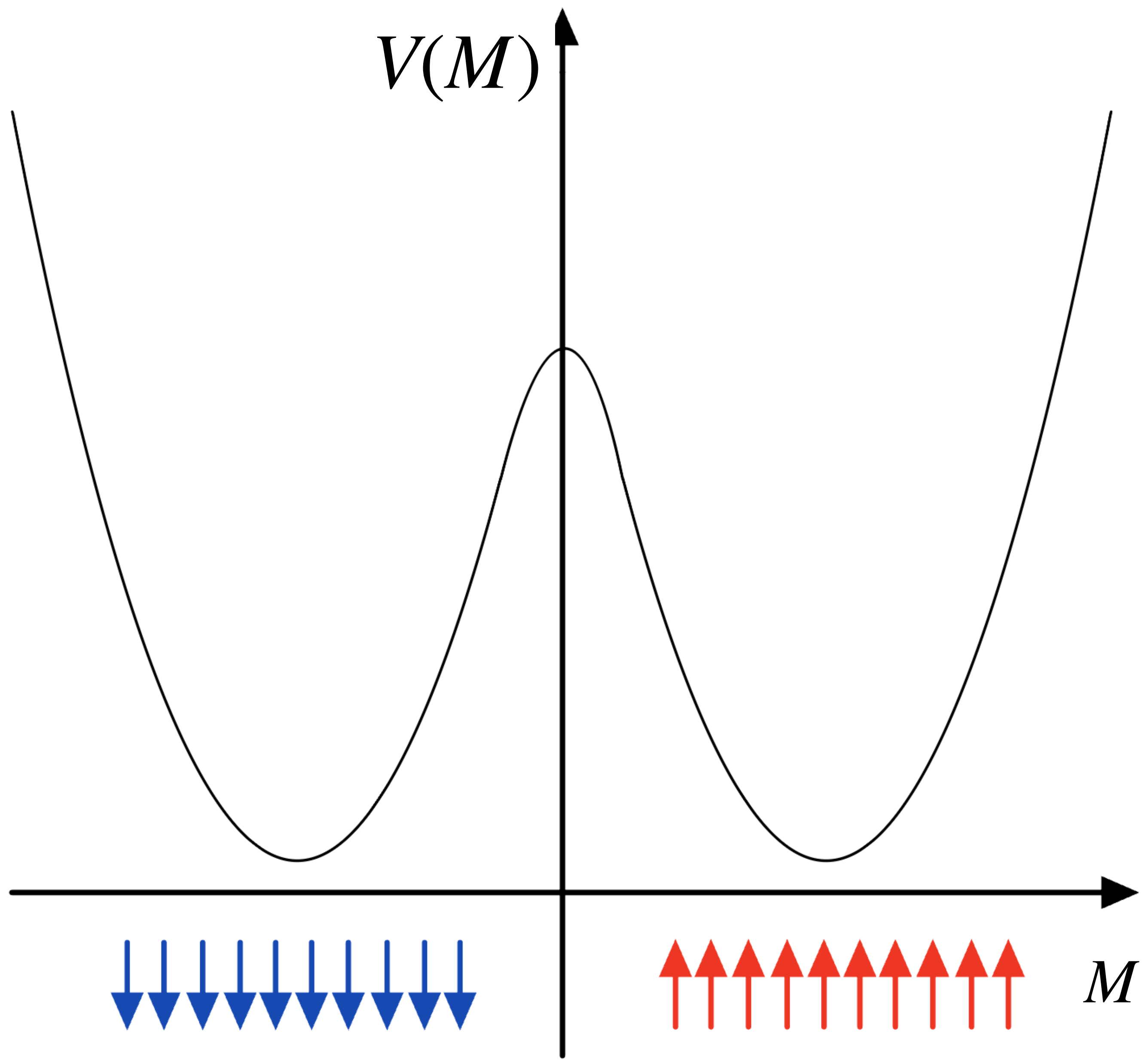
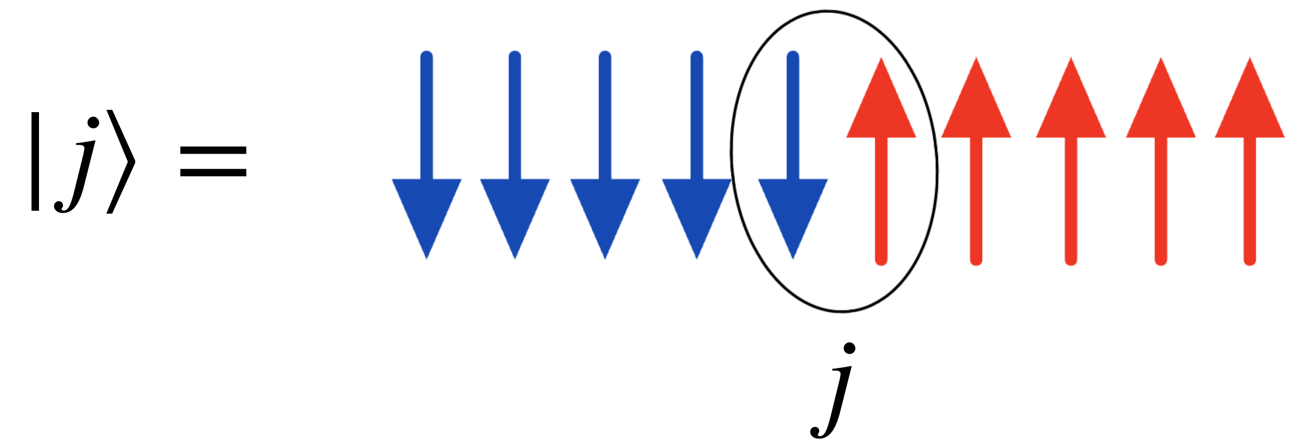



Domain-wall excitations:



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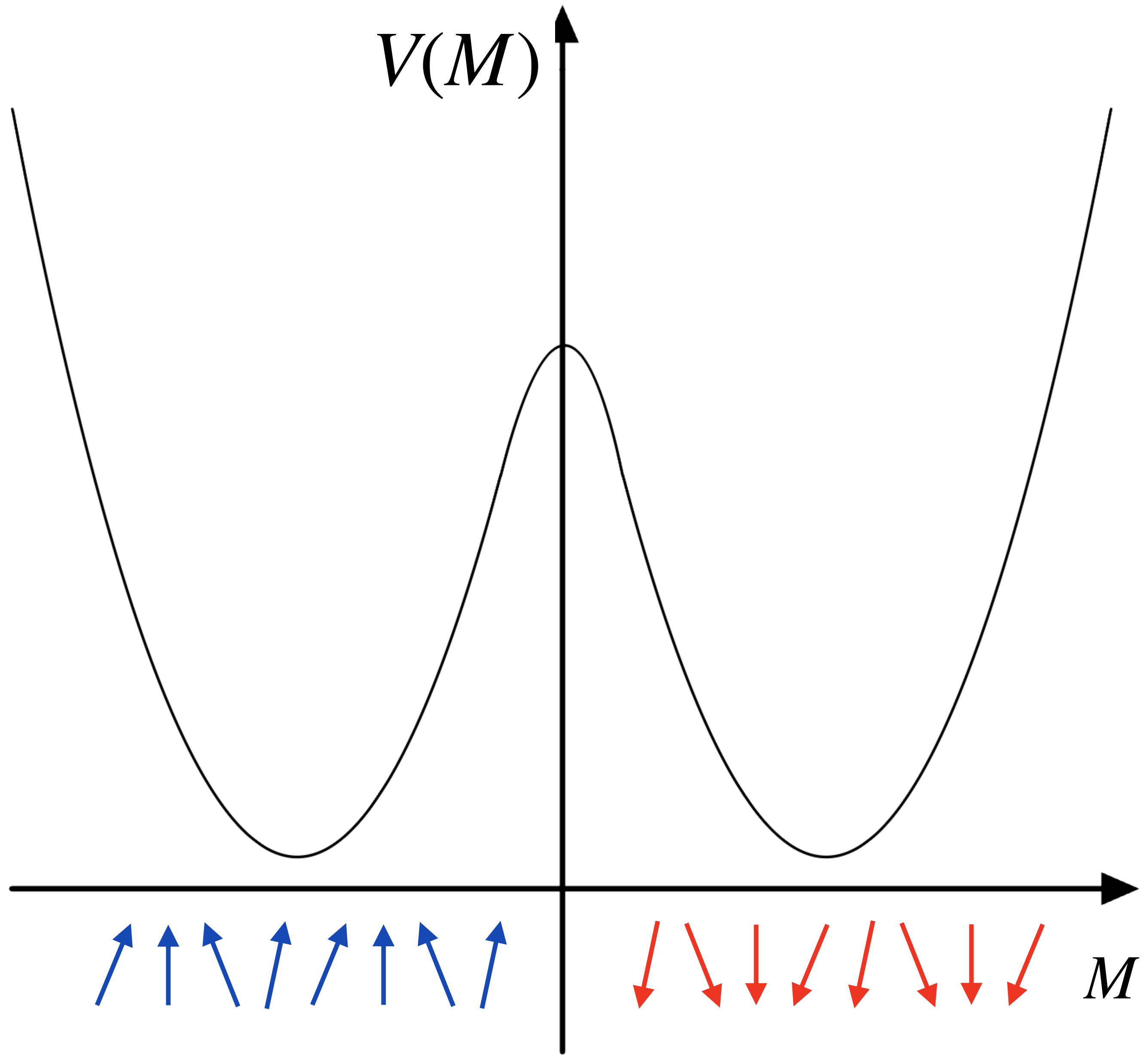
$$|h_x| < 1 \quad \Rightarrow \quad H = \sum_k \varepsilon_k \eta_k^\dagger \eta_k + const$$

Free Fermion mapping (Jordan-Wigner) + Bogoliubov + Fourier

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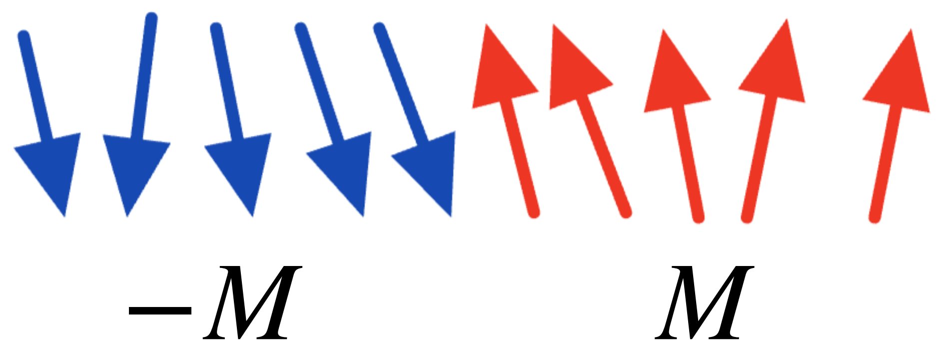
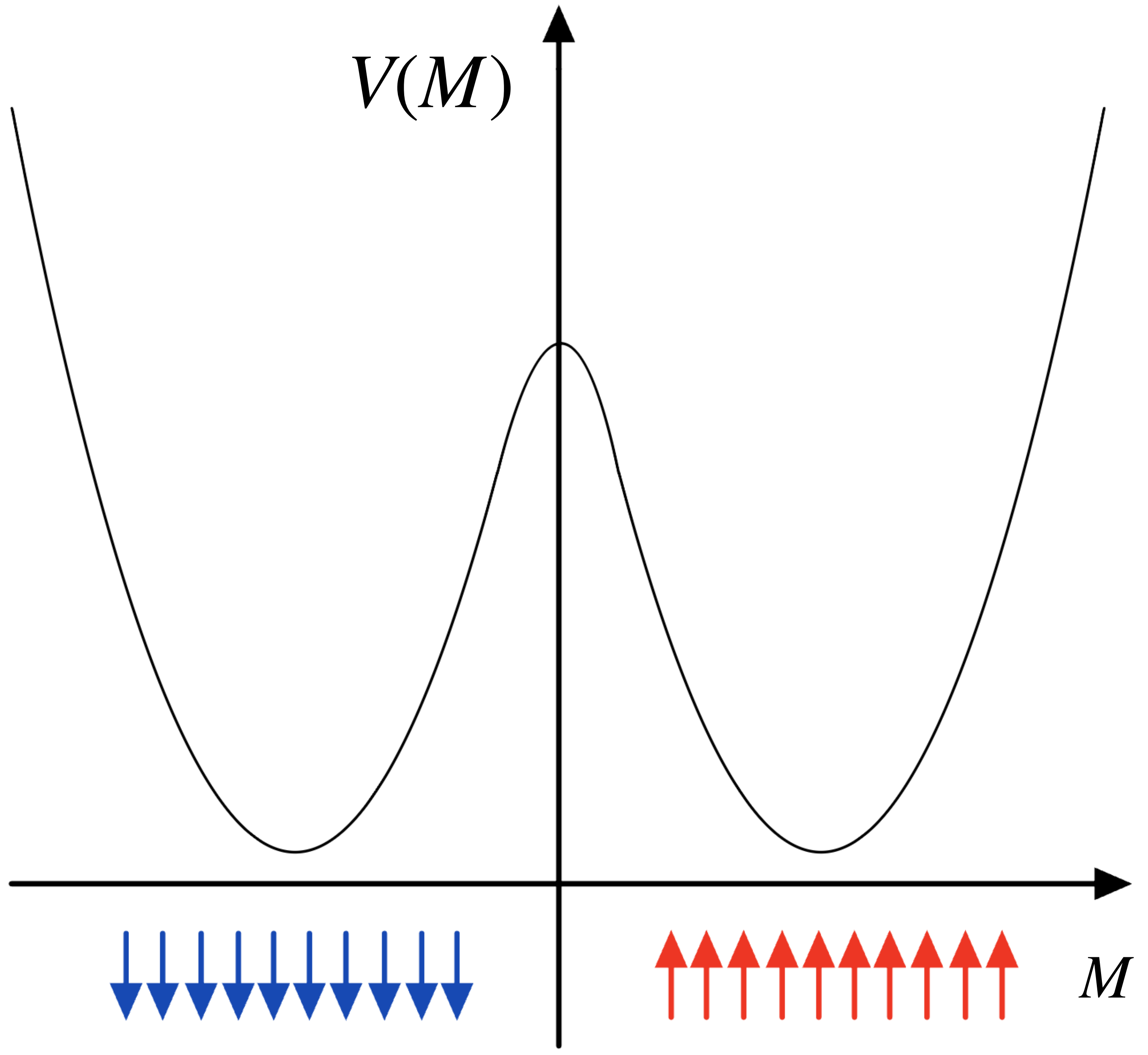



$$M = (1 - h_x)^{1/8}$$

Energy cost / Mass gap: $\Delta = 2(1 - h_x)$

# The model: Transverse Field Quantum Ising Chain

$$H = -\sum_i \sigma_i^z \sigma_{i+1}^z - h_x \sigma_i^x \quad -h_z \sigma_i^z$$



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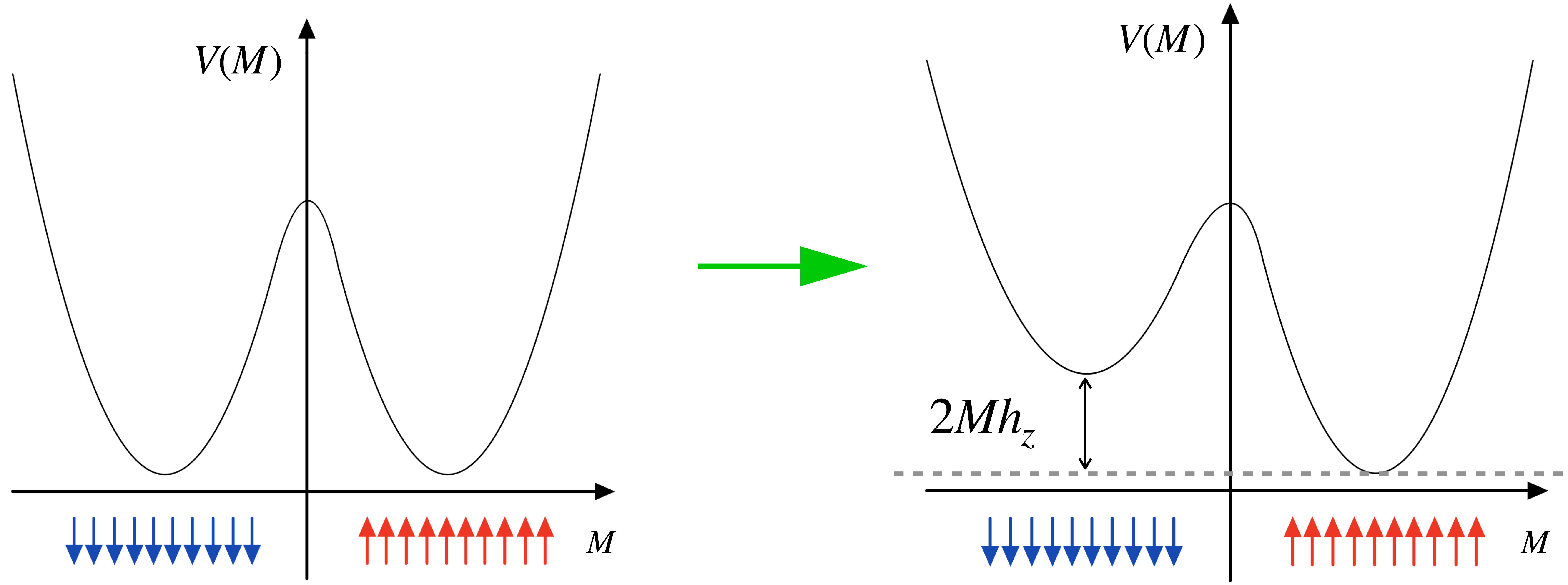
$$H = -\sum_i \sigma_i^z \sigma_{i+1}^z - h_x \sigma_i^x \quad {\color{green} - h_z \sigma_i^z}$$

The longitudinal field $h_z$ lifts the degeneracy



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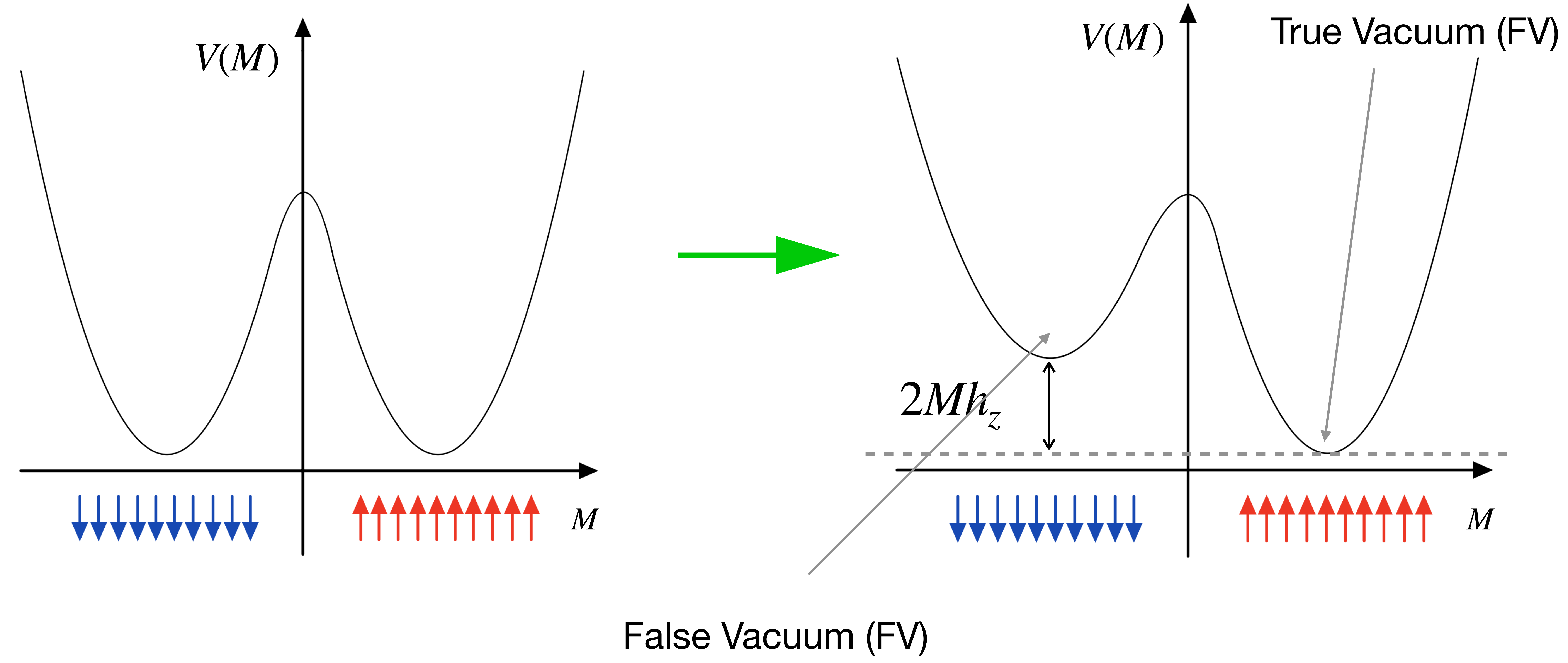

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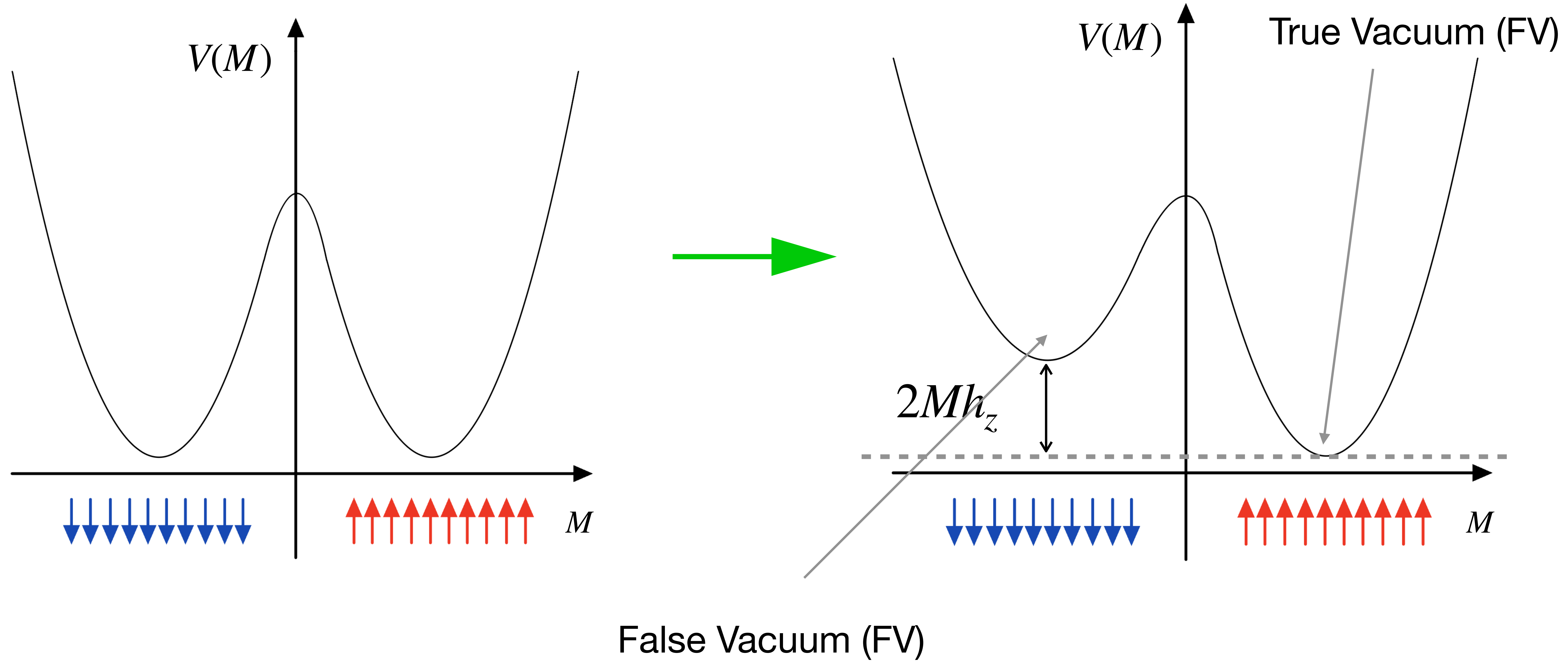
$$H = -\sum_i \sigma_i^z \sigma_{i+1}^z - h_x \sigma_i^x \quad -h_z \sigma_i^z$$

The longitudinal field $h_z$ lifts the degeneracy → Distinctive effects in the dynamics



# Confinement

# The Quench Protocol

1 - Prepare ground state $|\psi_0\rangle$ of $H_0 = H(g_0)$.

$g = (h_x, h_z)$

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# The Quench Protocol

1 - Prepare ground state $|\psi_0\rangle$ of $H_0 = H(g_0)$. $g = (h_x, h_z)$

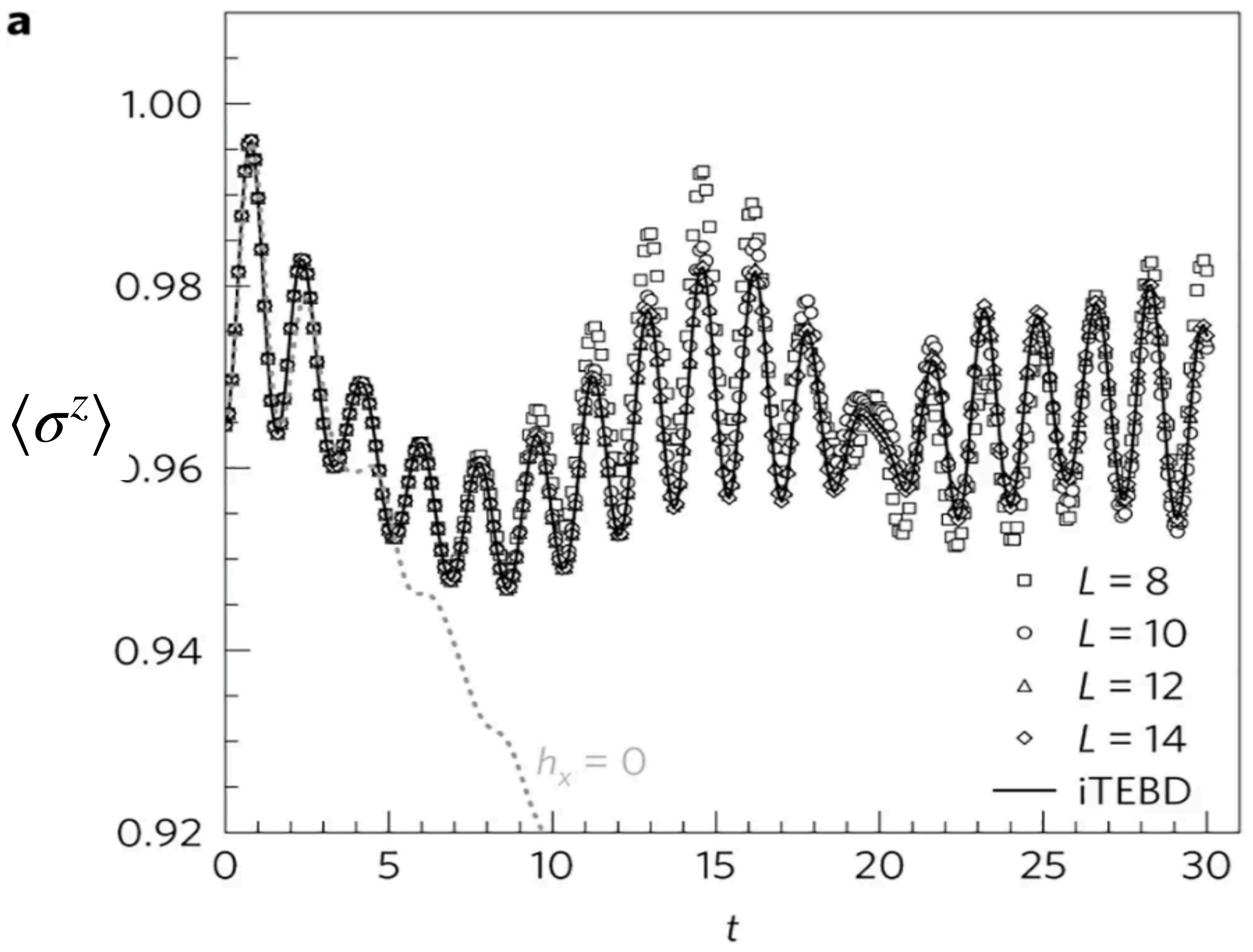
2 - At $t = 0$ **suddenly** change parameters $g_0 \to g$. $H_1 = H(g)$

3 - At $t > 0$ evolve with the new Hamiltonian $|\psi(t)\rangle = \exp(-iH_1 t)\,|\psi_0\rangle$

# Quench Spectroscopy

$$\langle \sigma^z \rangle(t) = \sum_i \langle \psi(t) | \sigma_i^z | \psi(t) \rangle / L$$

$$h_x = 0.5, h_z = 0 \rightarrow h_x = 0.25, h_z = 0.1$$



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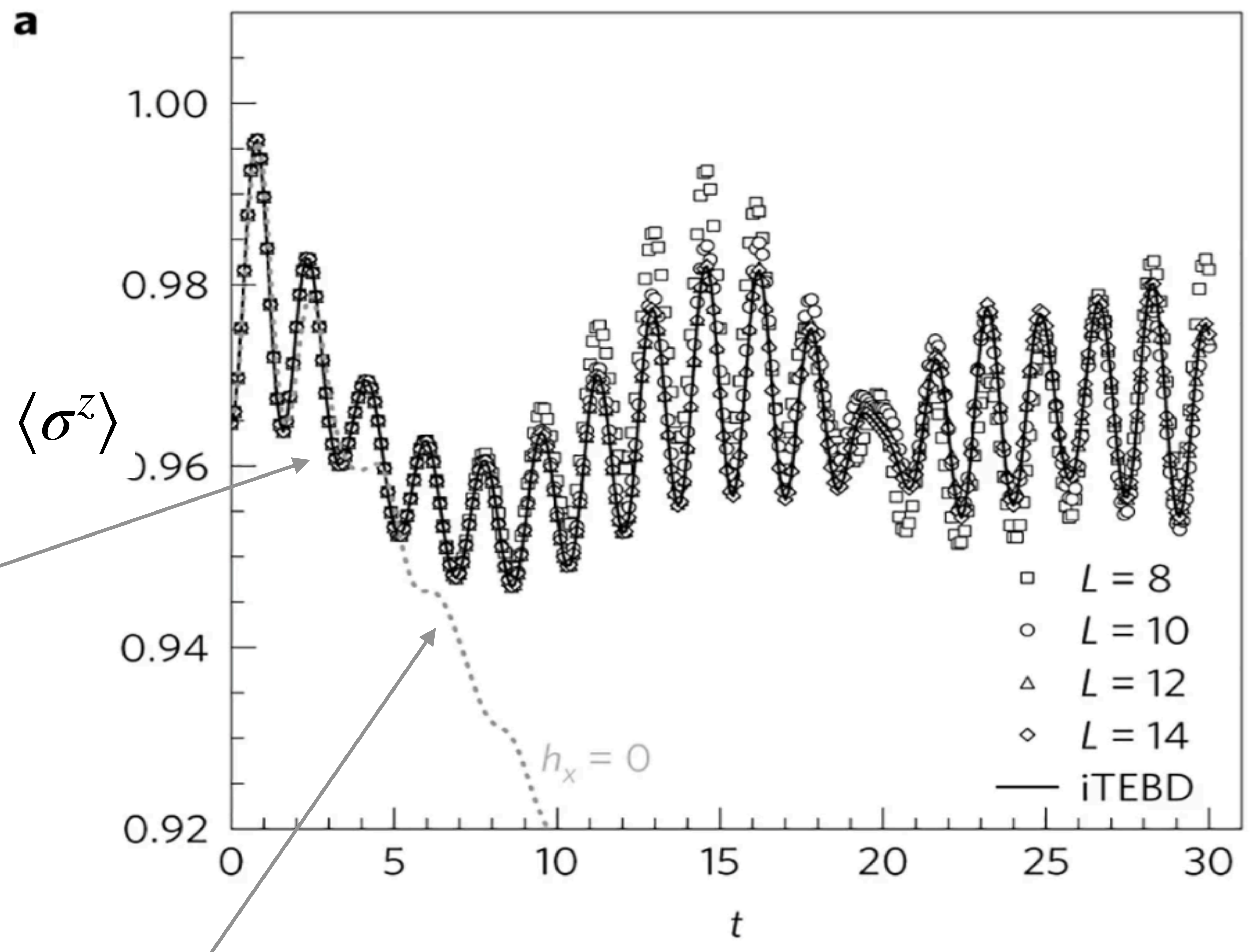


Oscillations + Slow relaxation

$$\langle \sigma^x \rangle(t) \approx e^{-kt}$$

Calabrese, Essler, Fagotti PRL 2011

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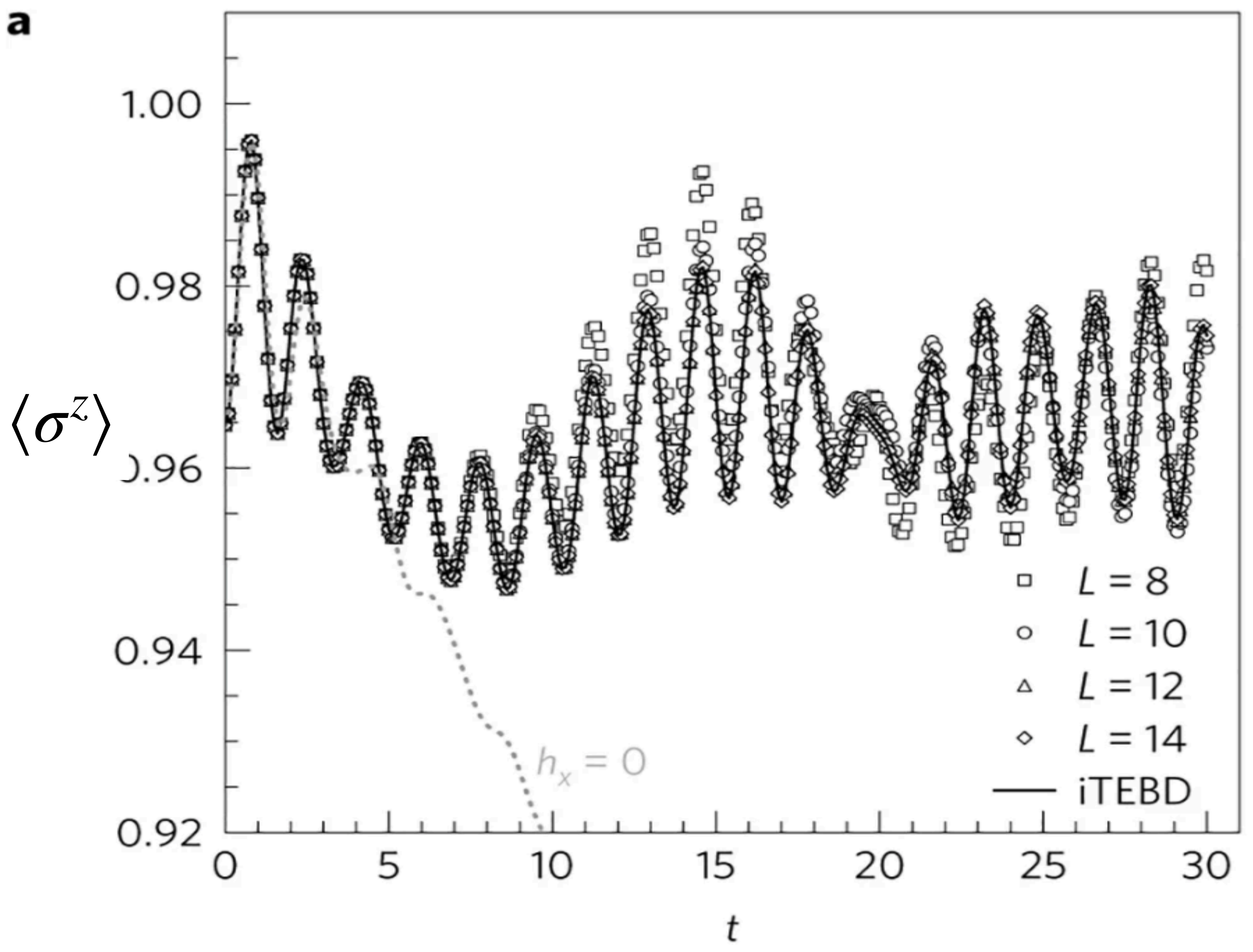
$|\Psi_0\rangle$ is not an eigenstate of $H_1$



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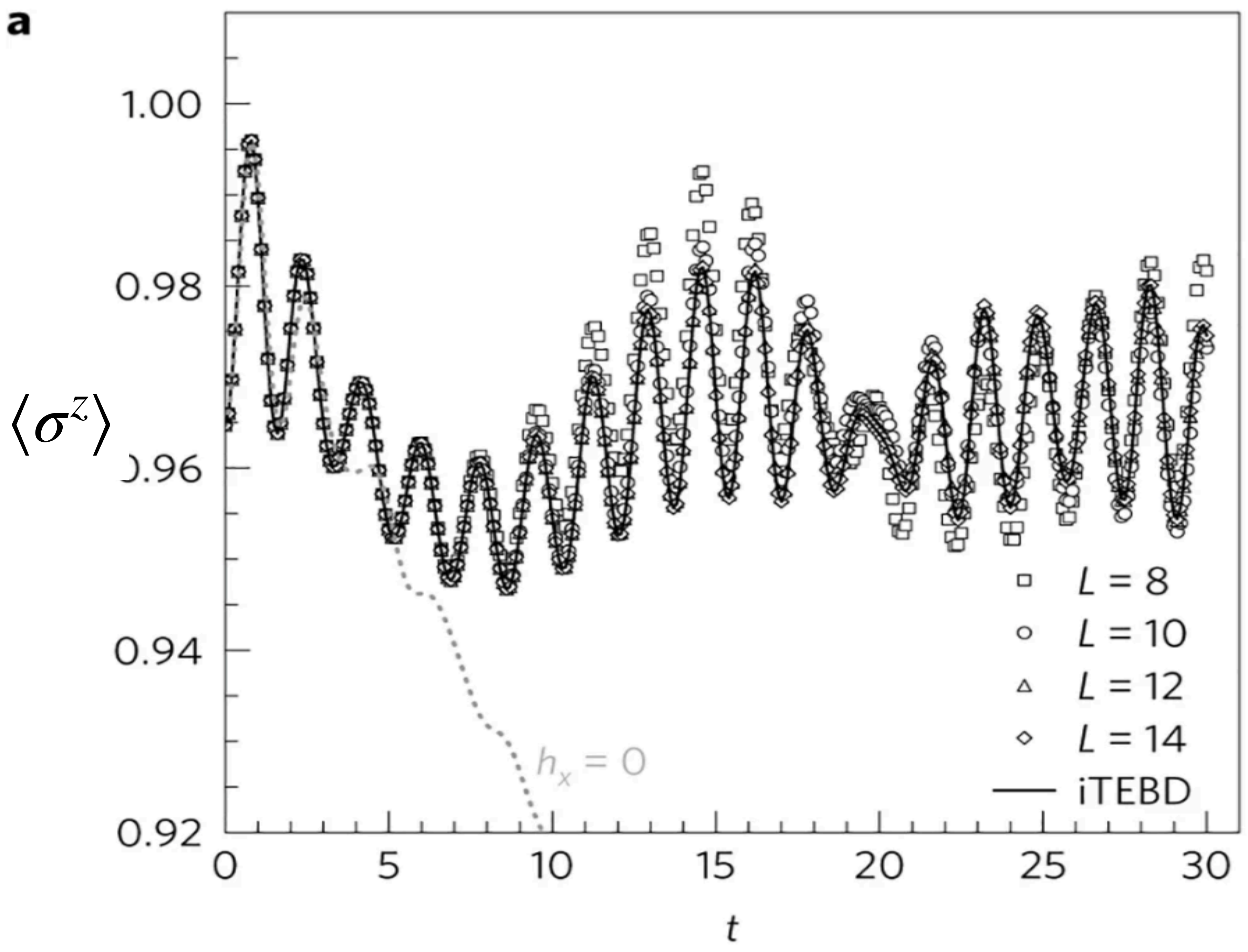


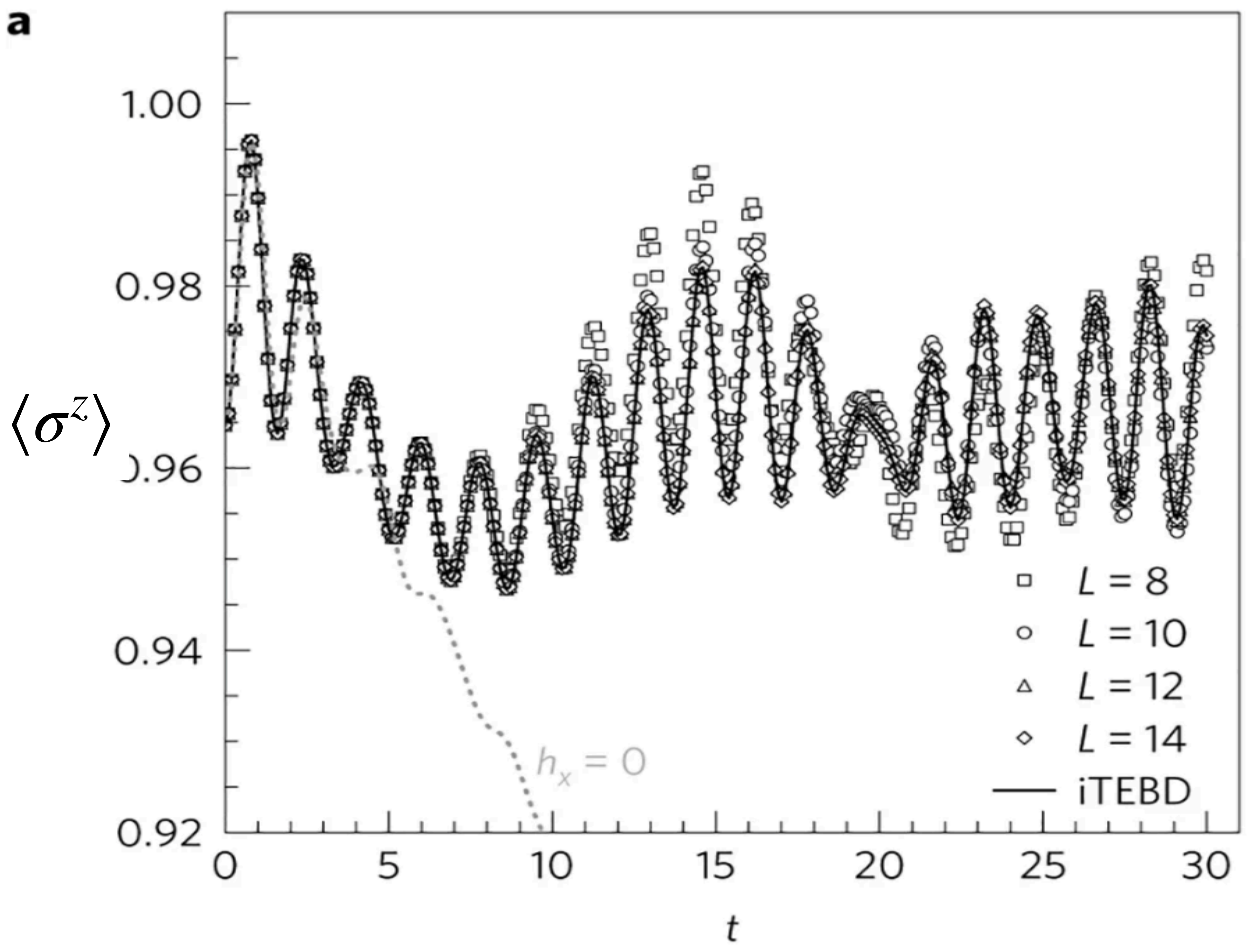

$|\Psi_0\rangle$ is not an eigenstate of $H_1$

$$\Rightarrow |\psi(t)\rangle = \exp(-iHt)|\psi_0\rangle = \sum_i a_i e^{-iE_i t}|e_i\rangle$$

$$a_i = \langle e_i | \psi \rangle$$

$$H_1 |e_i\rangle = E_i |e_i\rangle$$

# Quench Spectroscopy

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$$a_i = \langle e_i\,|\,\psi\rangle$$

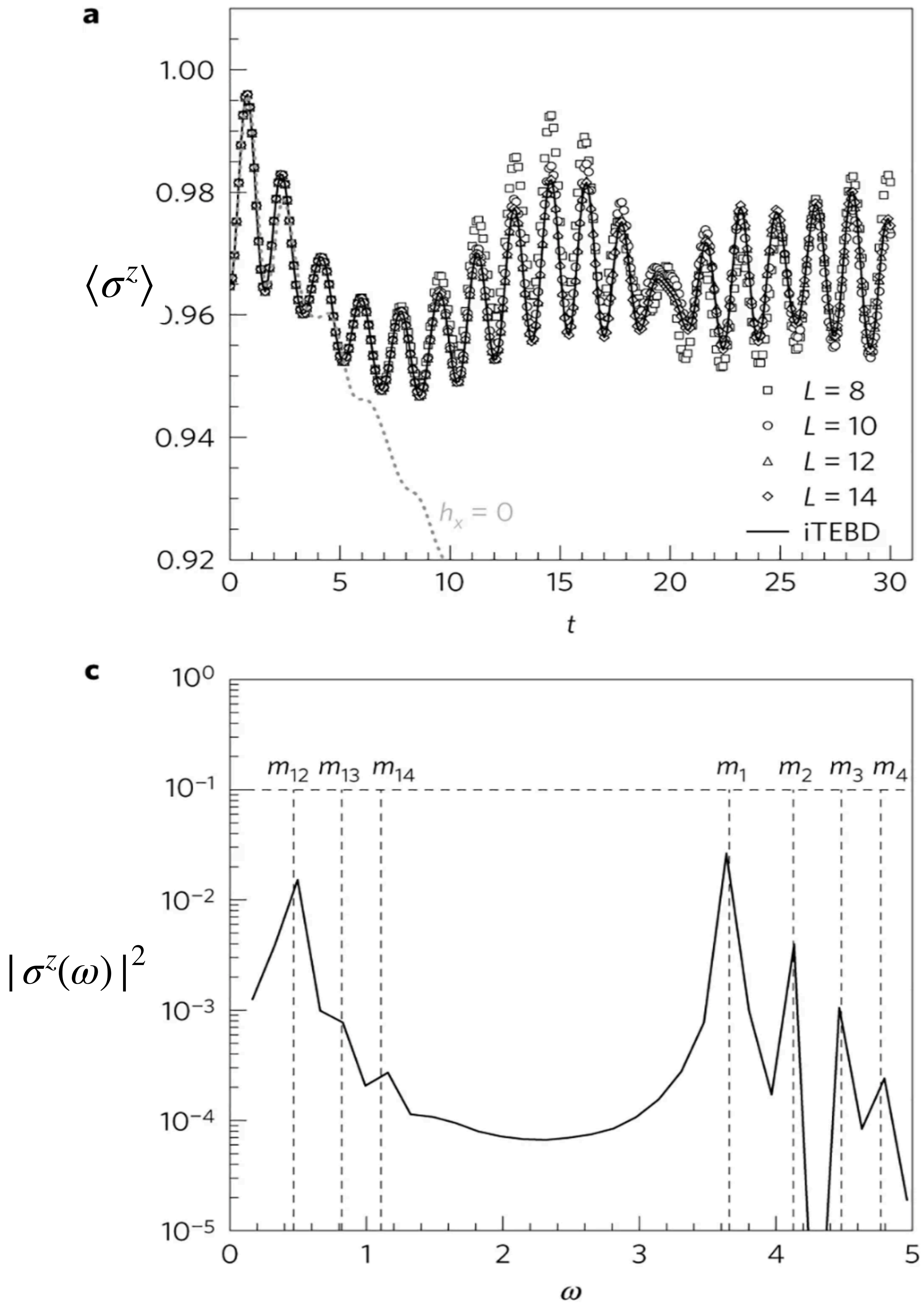
$$H_1\,|\,e_i\rangle = E_i\,|\,e_i\rangle$$

$$\Rightarrow \langle\sigma^z\rangle(t) = \sum_{ij} a_i a_j^* e^{-i(E_i - E_j)}\langle e_j\,|\,\sigma^z\,|\,e_i\rangle$$

# Quench Spectroscopy

$$\langle \sigma^z \rangle(t) = \sum_i \langle \psi(t) | \sigma_i^z | \psi(t) \rangle / L$$

$$|\langle \sigma^z \rangle(\omega)|^2 = \left| \int dt e^{i\omega t} \langle \sigma^z \rangle(t) \right|^2$$



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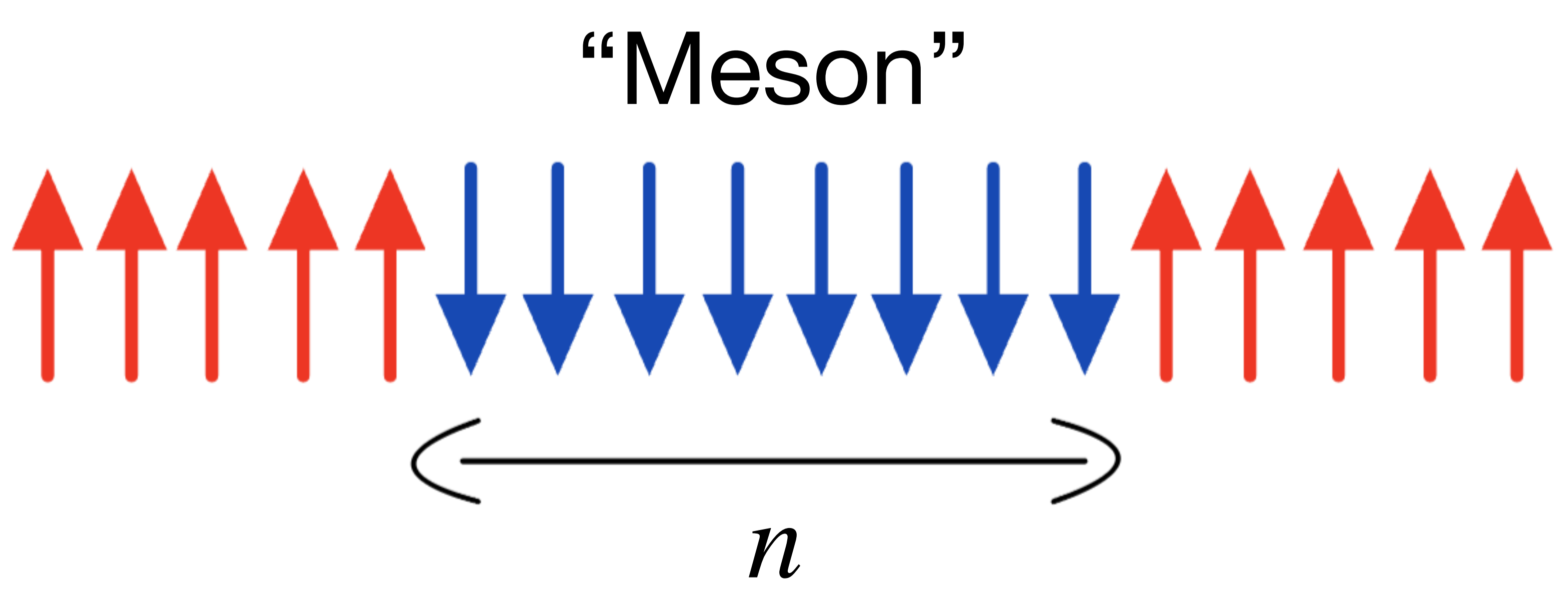
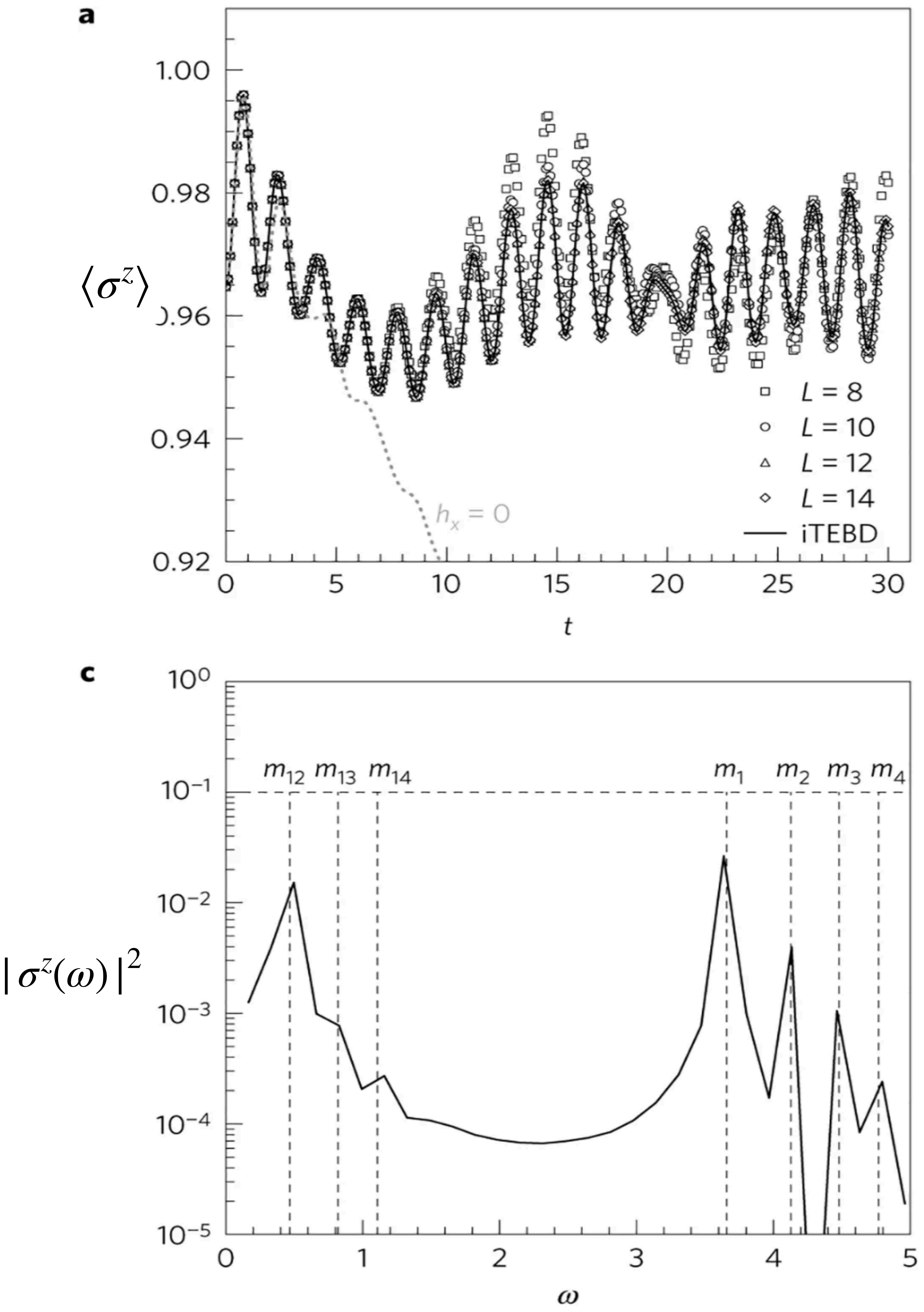



$$E_n = 2\Delta + 2Mh_z\, n$$



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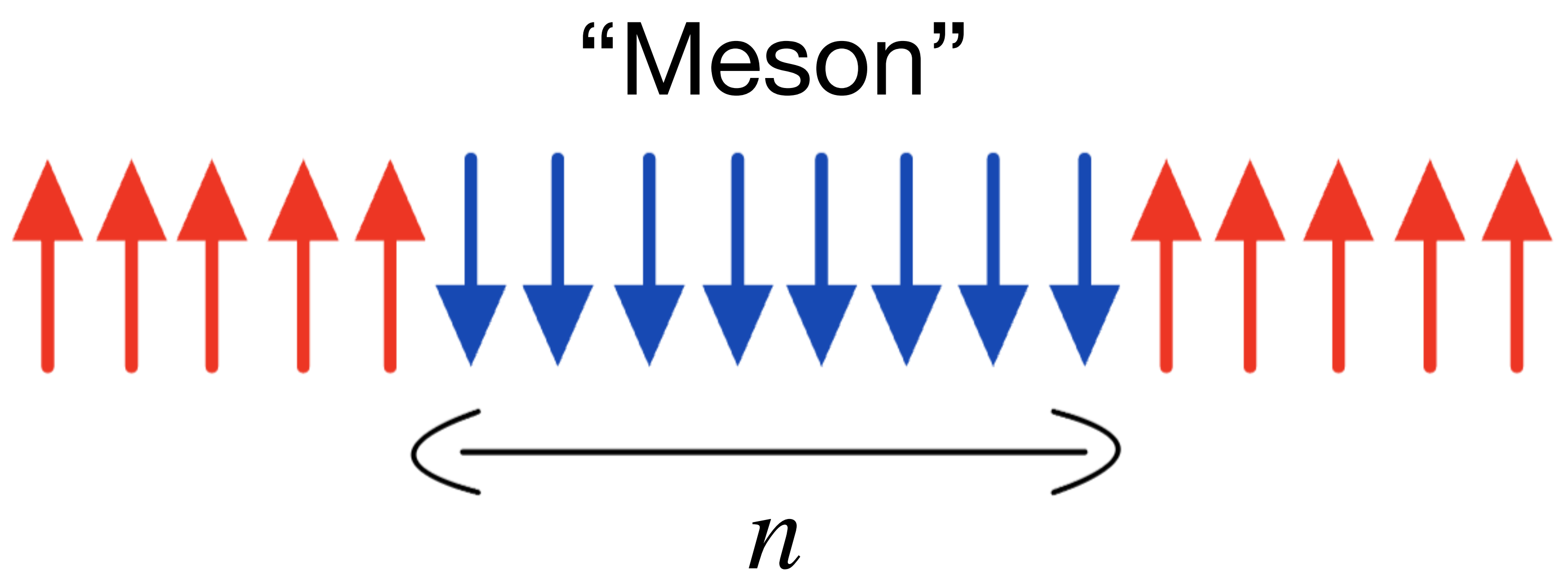


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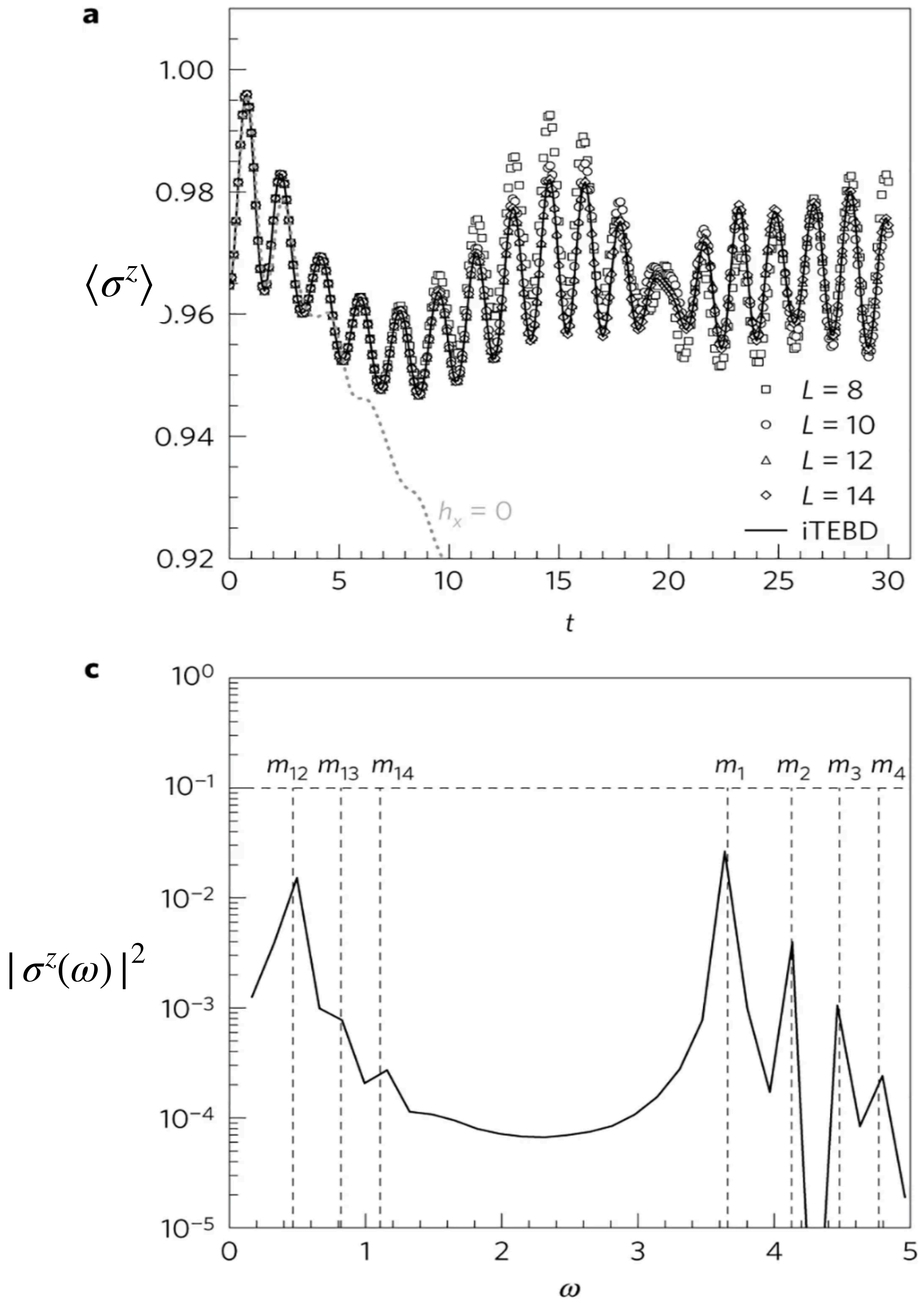
Kink’s mass

Energy density



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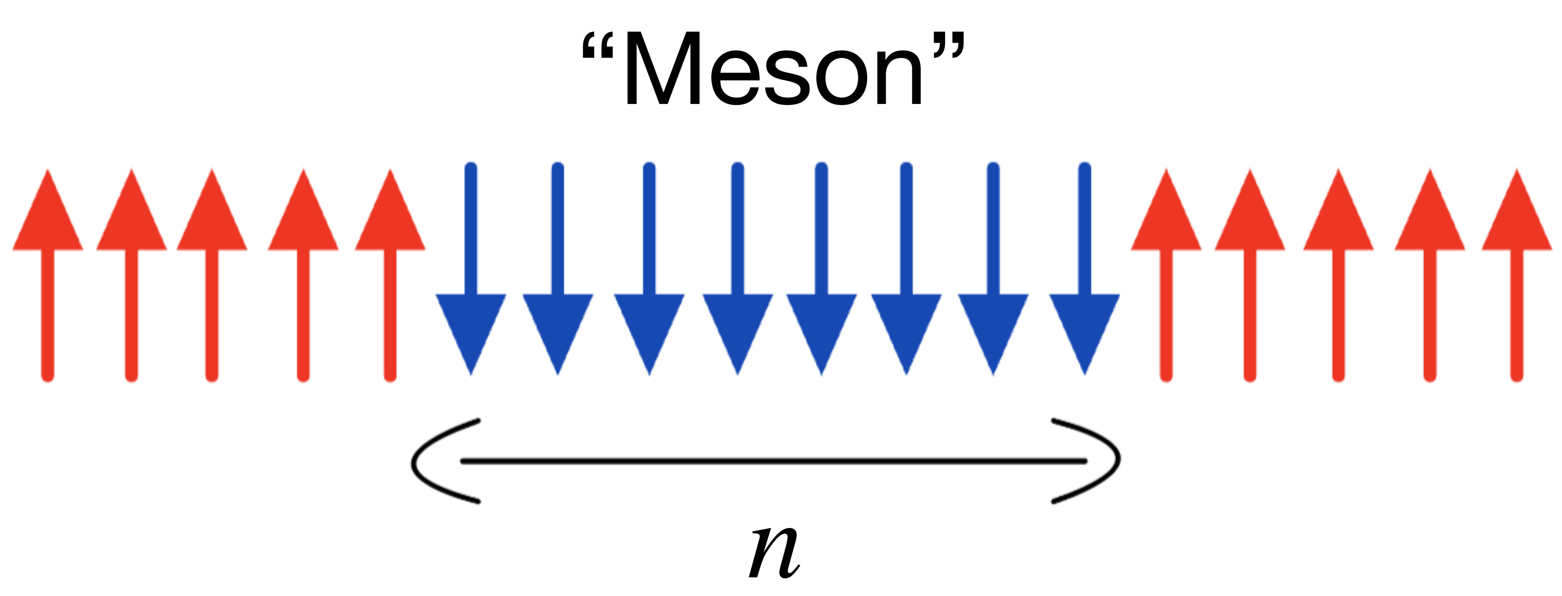
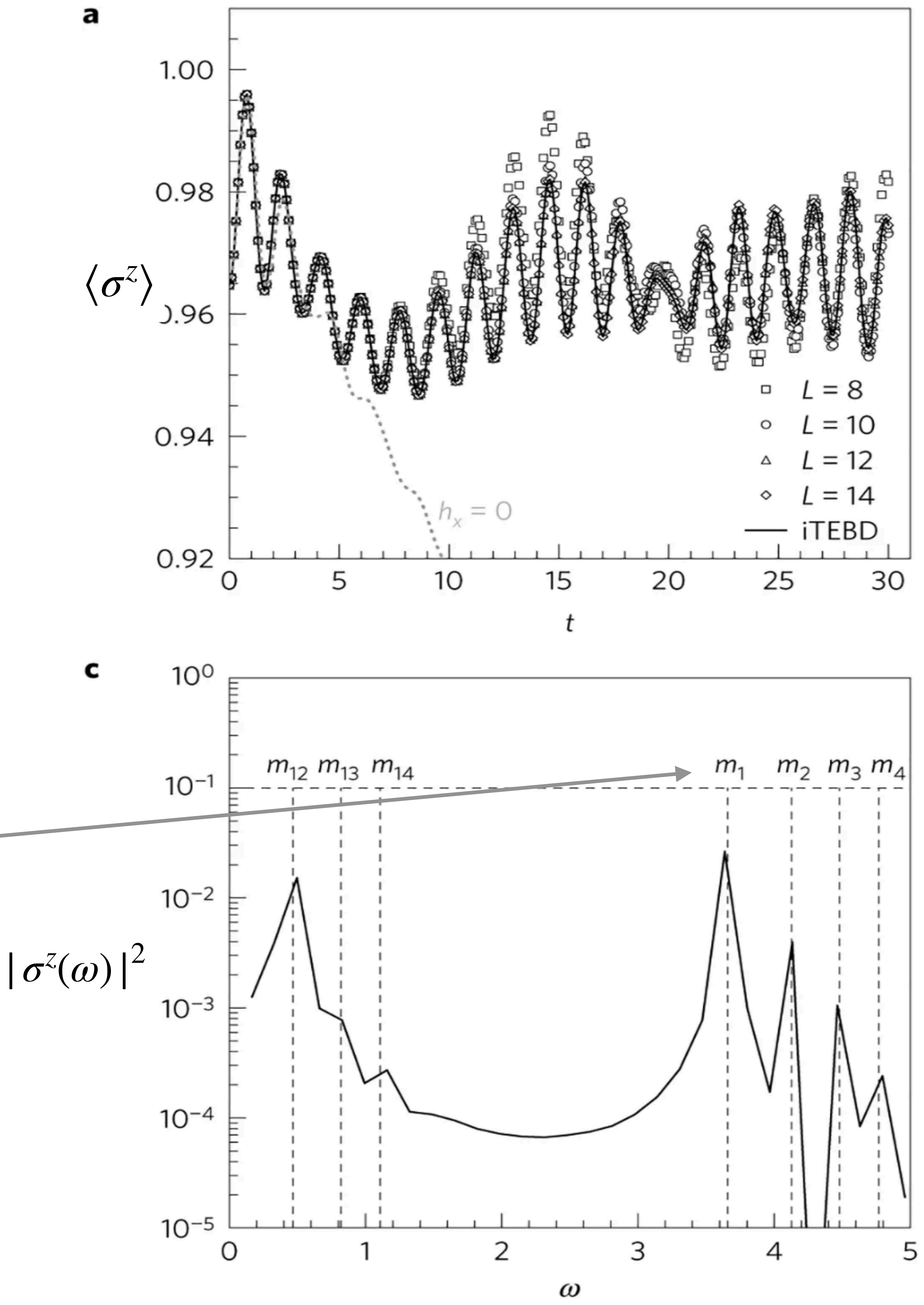


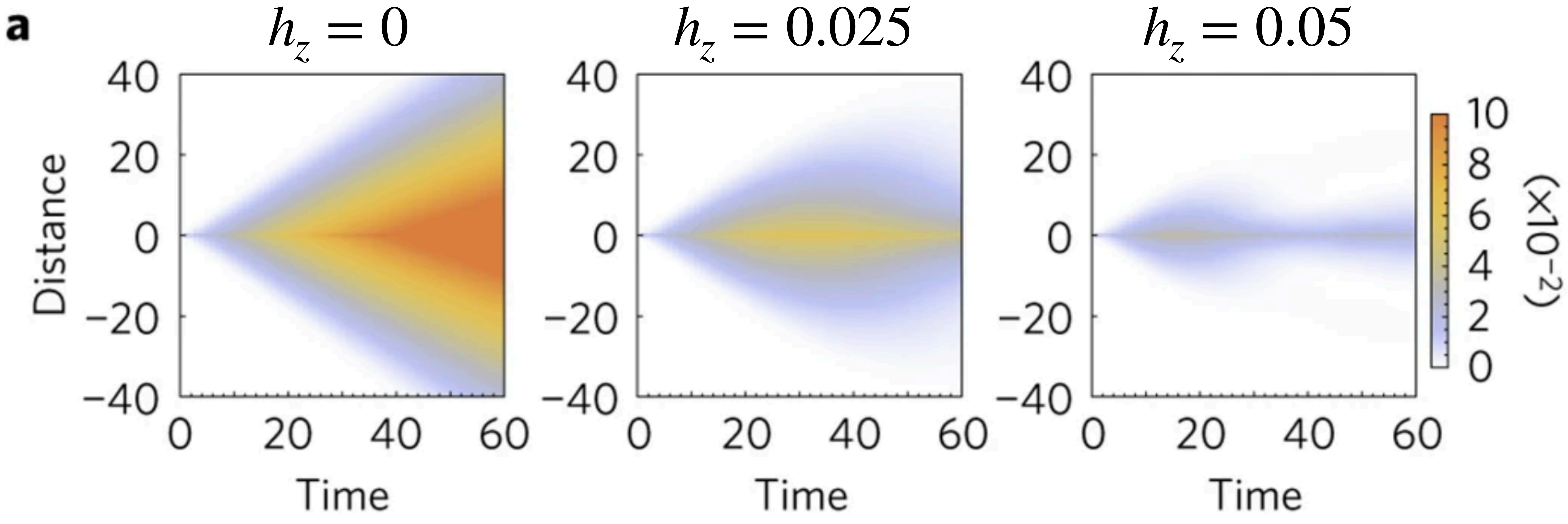

$$E_n = 2\Delta + 2Mh_z\, n$$

Effective two-body model:

$$H_{eff} = \varepsilon_{k_1} + \varepsilon_{k_2} + 2Mh_z\, n$$

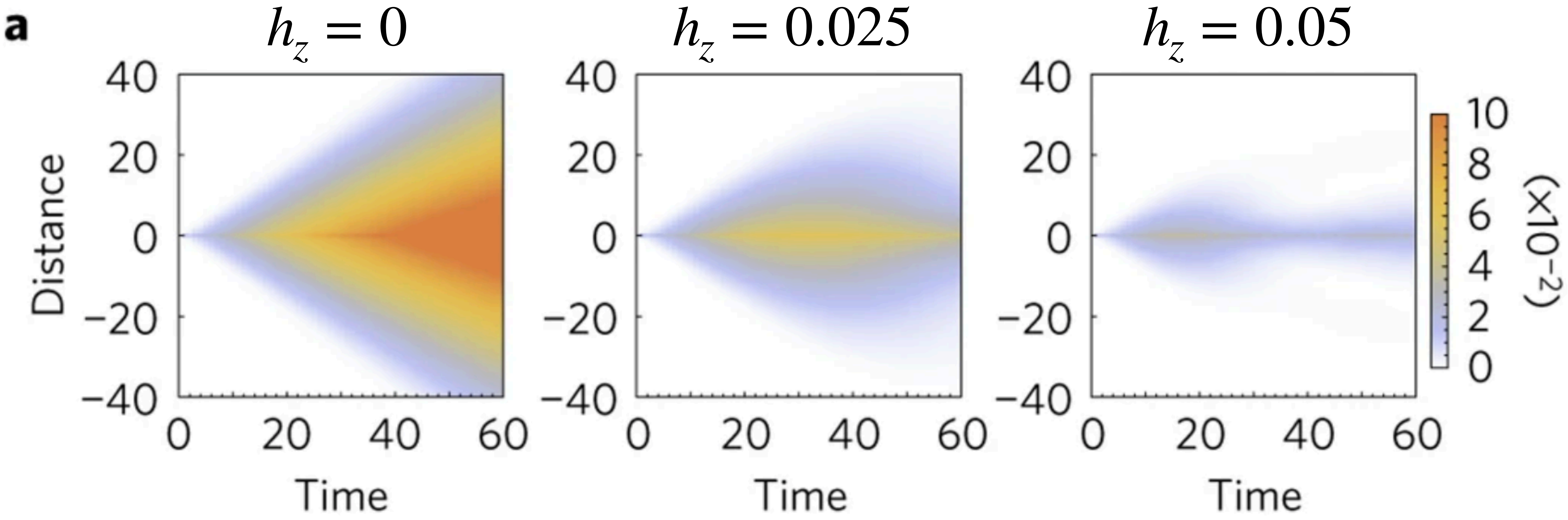


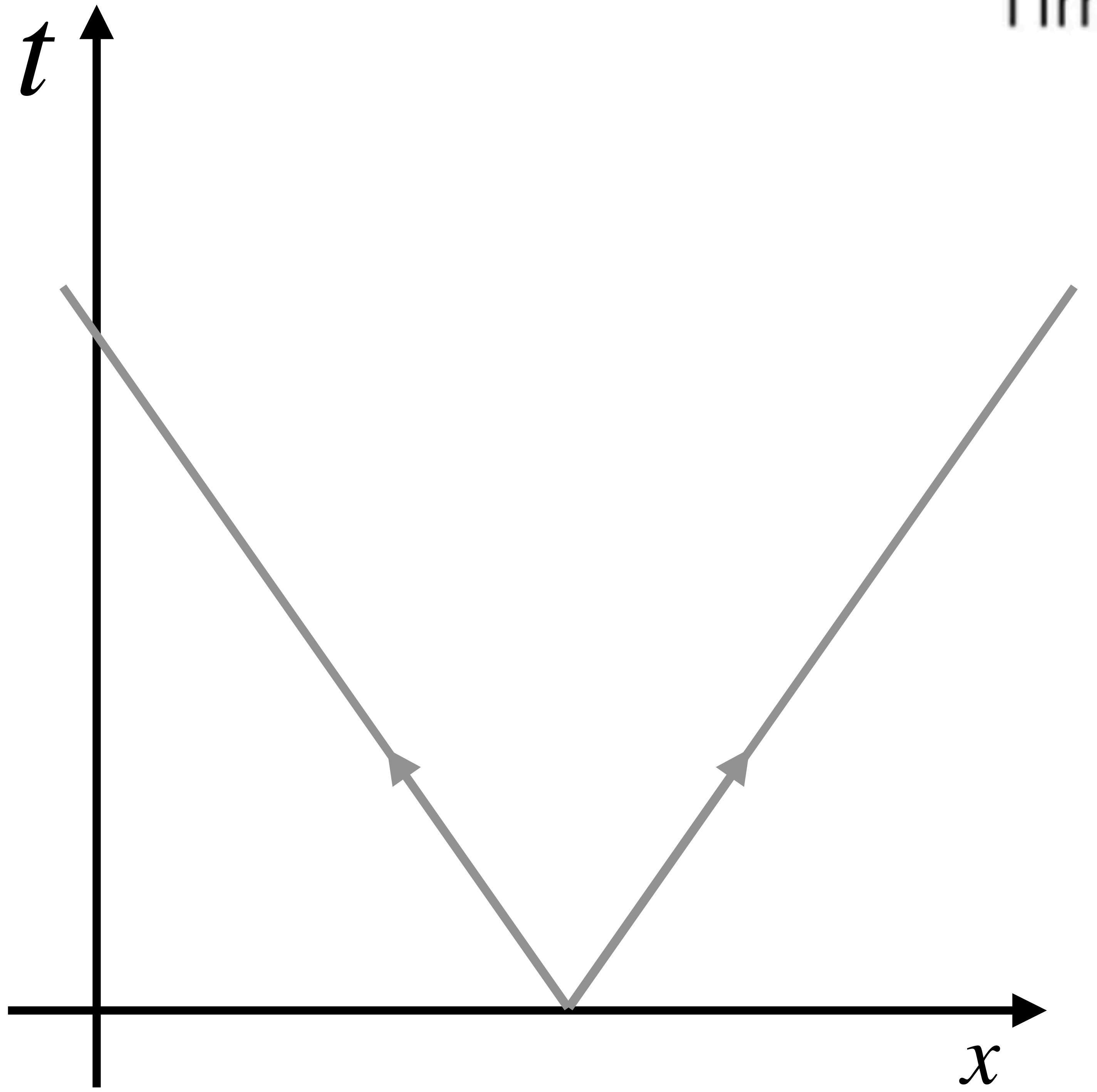
# Connected Correlations




$$\langle \sigma_1^z \sigma_{1+d}^z \rangle_c = \langle \sigma_1^z \sigma_{1+d}^z \rangle - \langle \sigma_1^z \rangle \langle \sigma_{1+d}^z \rangle$$

# Quasi-particle Picture

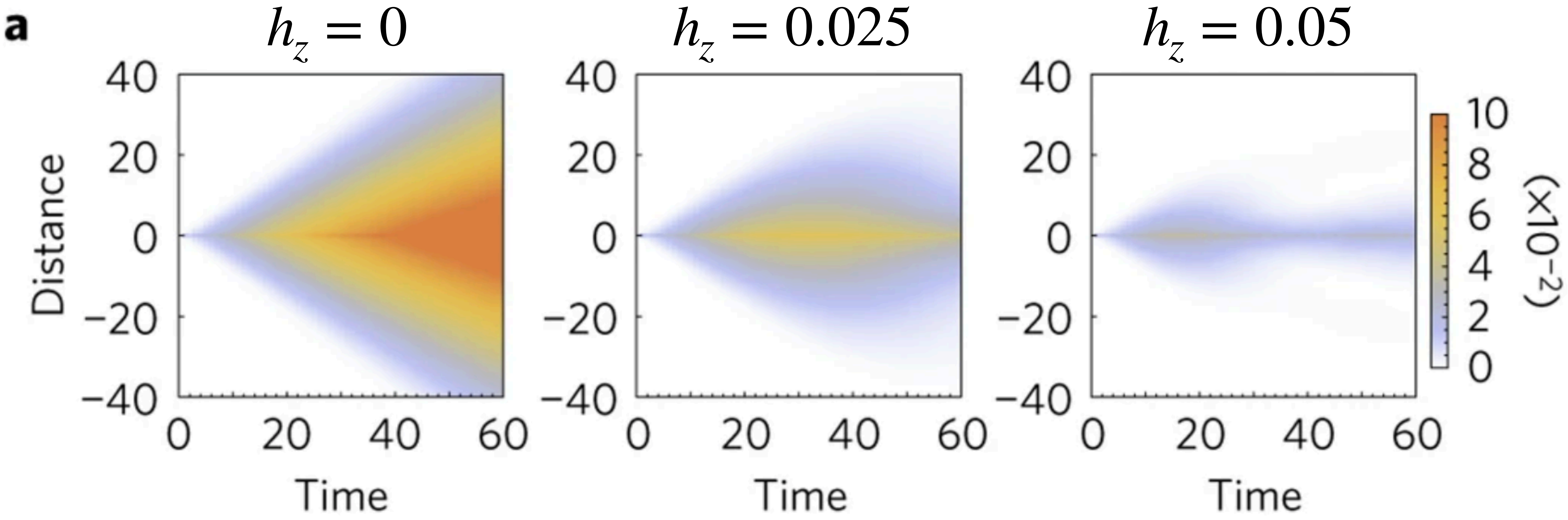


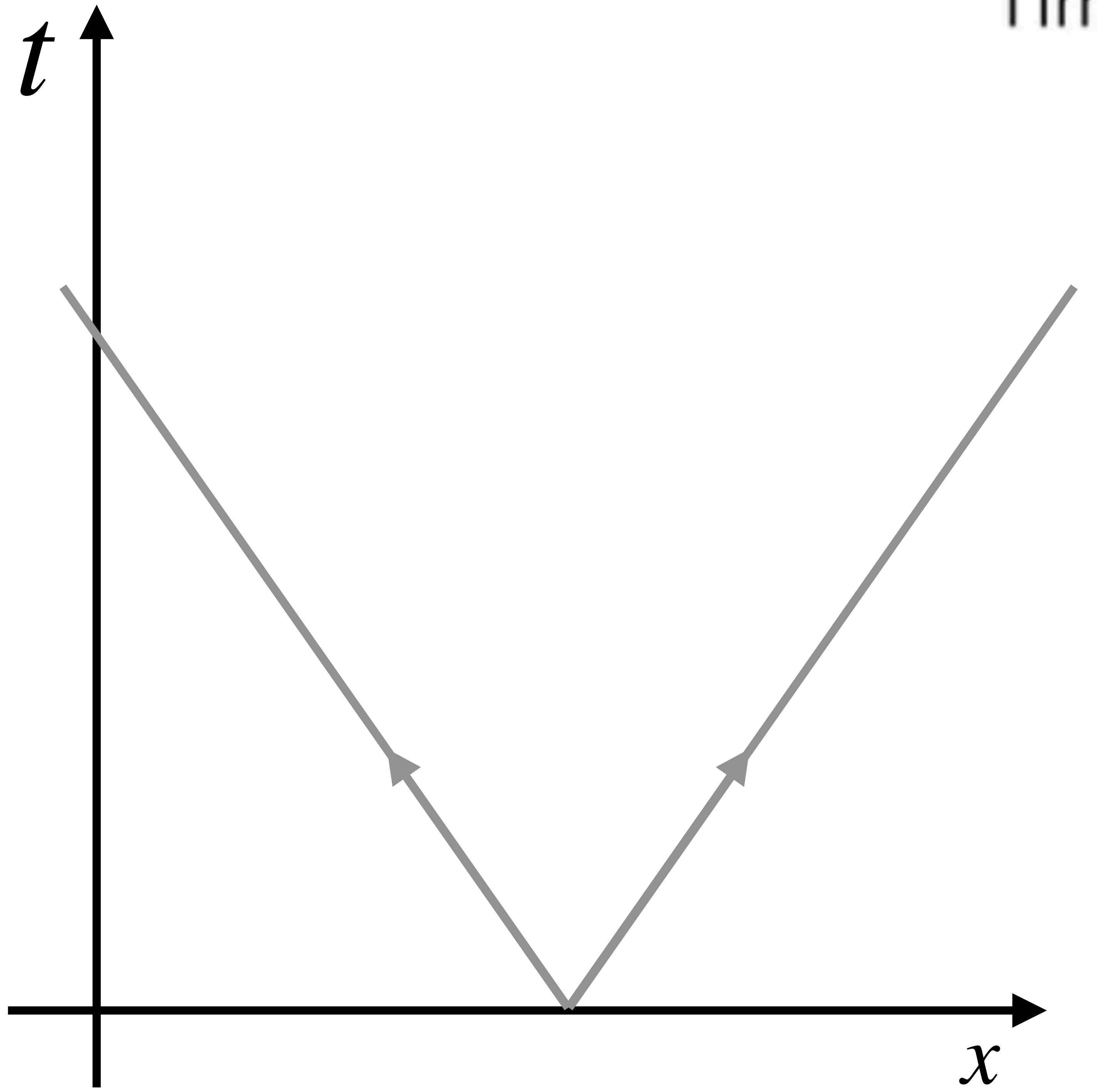




$|\Psi_0\rangle$ acts as a source of excitations

Calabrese, Cardy PRL 2006

# Quasi-particle Picture

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And

$$v_k = \frac{d\varepsilon}{dk} \qquad v_{max} \geq v_k \ \forall k$$

Lieb-Robinson bound

Calabrese, Cardy PRL 2006

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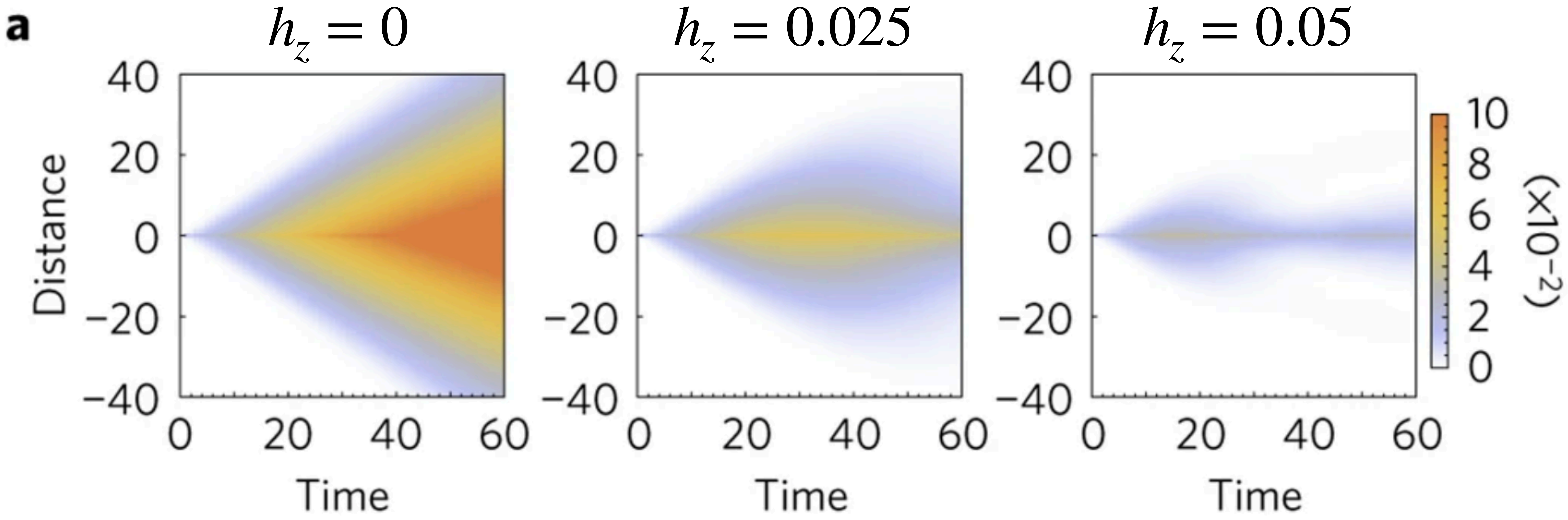


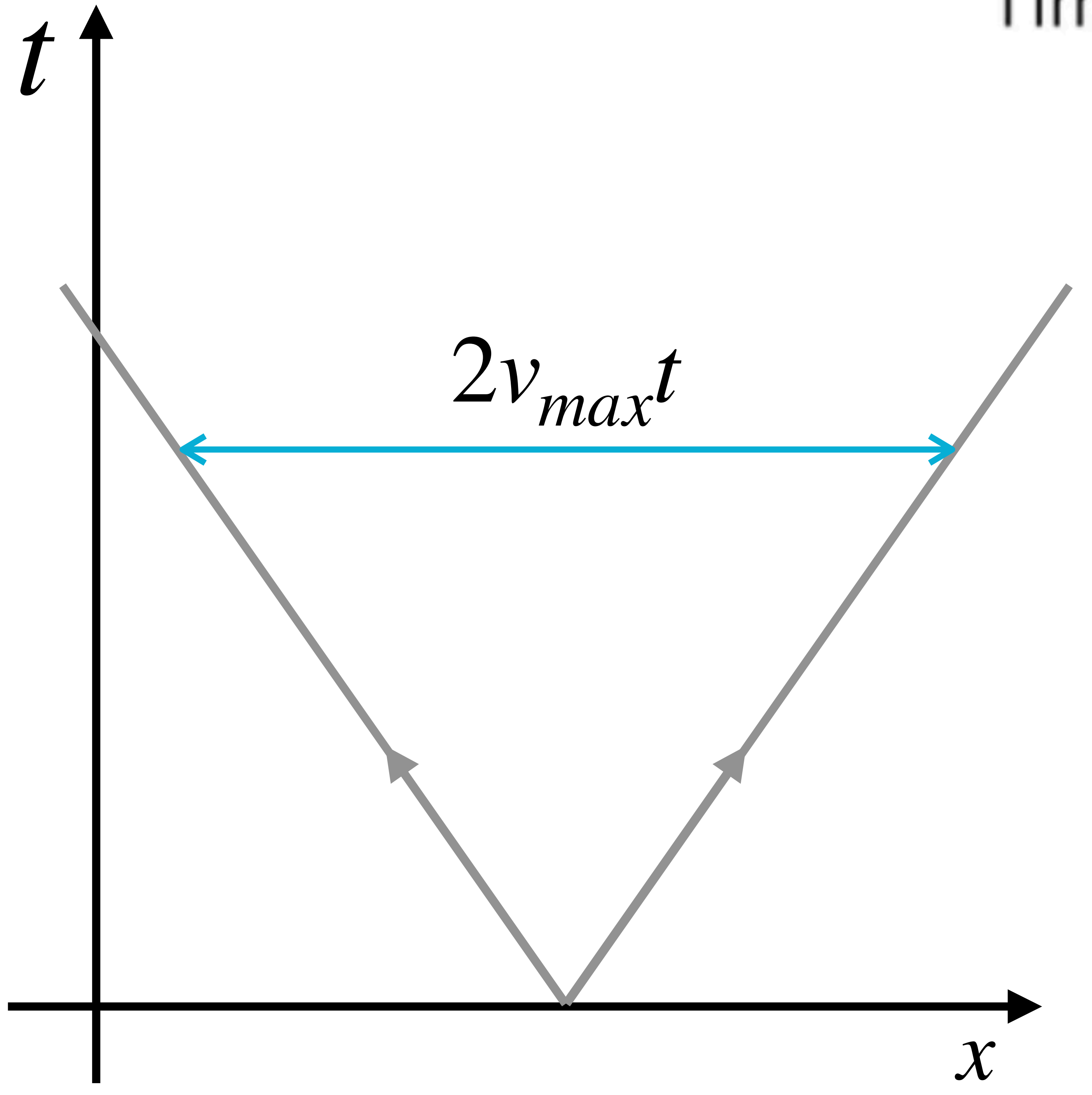


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And

$$v_k = \frac{d\varepsilon}{dk} \qquad v_{max} \geq v_k \; \forall k$$

Lieb-Robinson bound

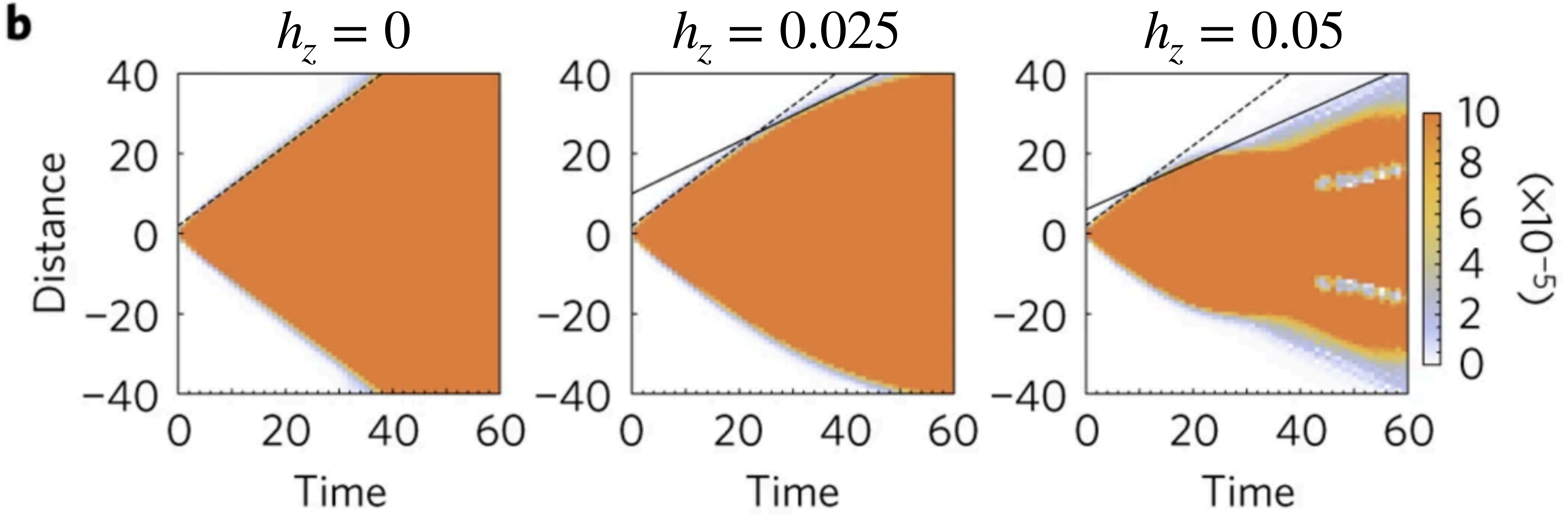
$\Rightarrow$

All correlators vanishing for

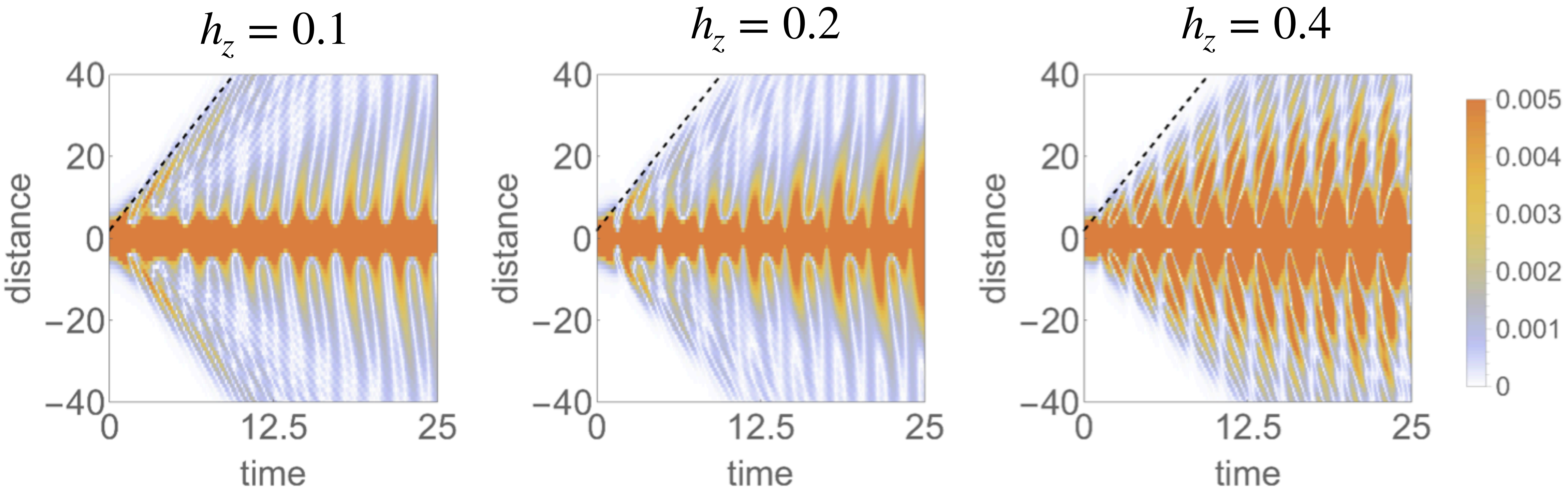
$$d > 2v_{max}t$$

Calabrese, Cardy PRL 2006

# Secondary light cone

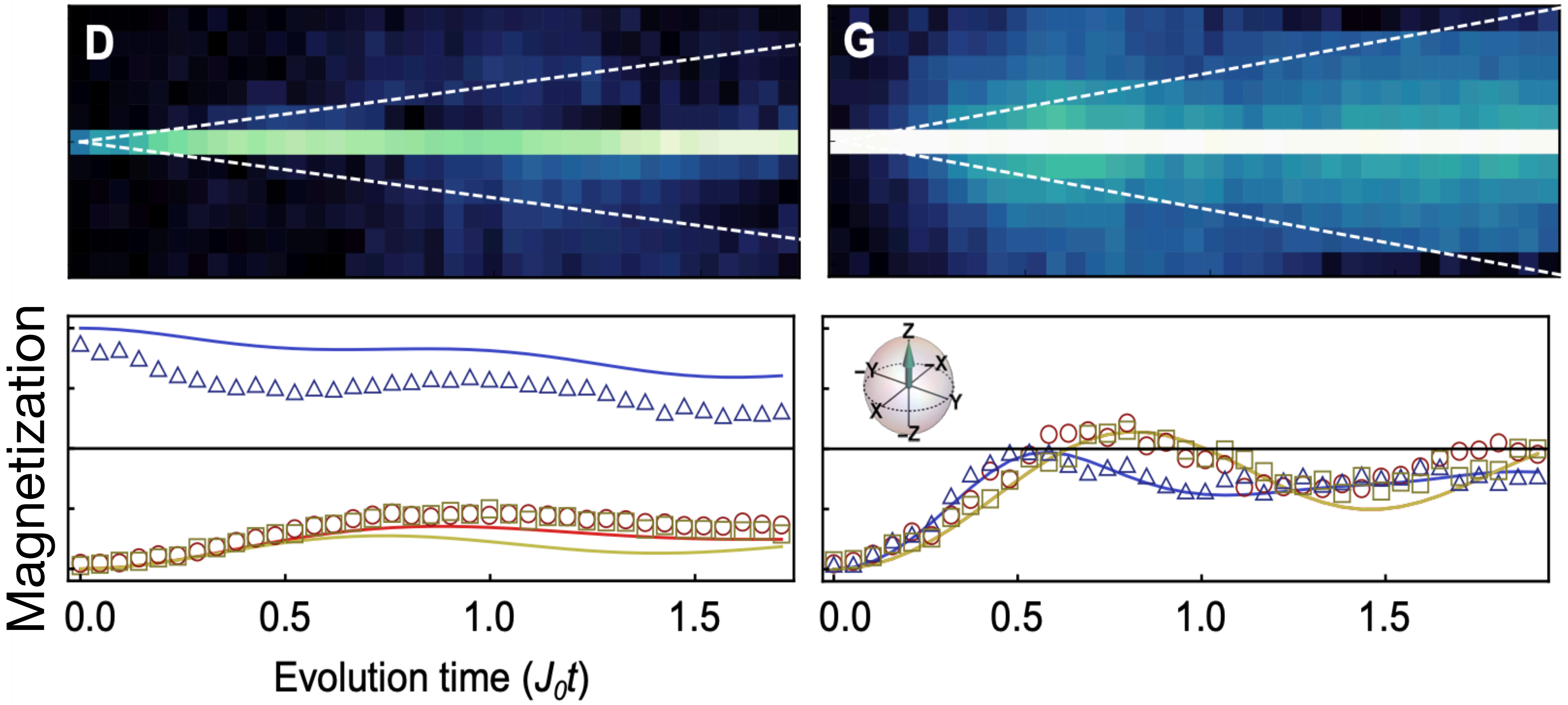



# Absence for $h_x > 1$



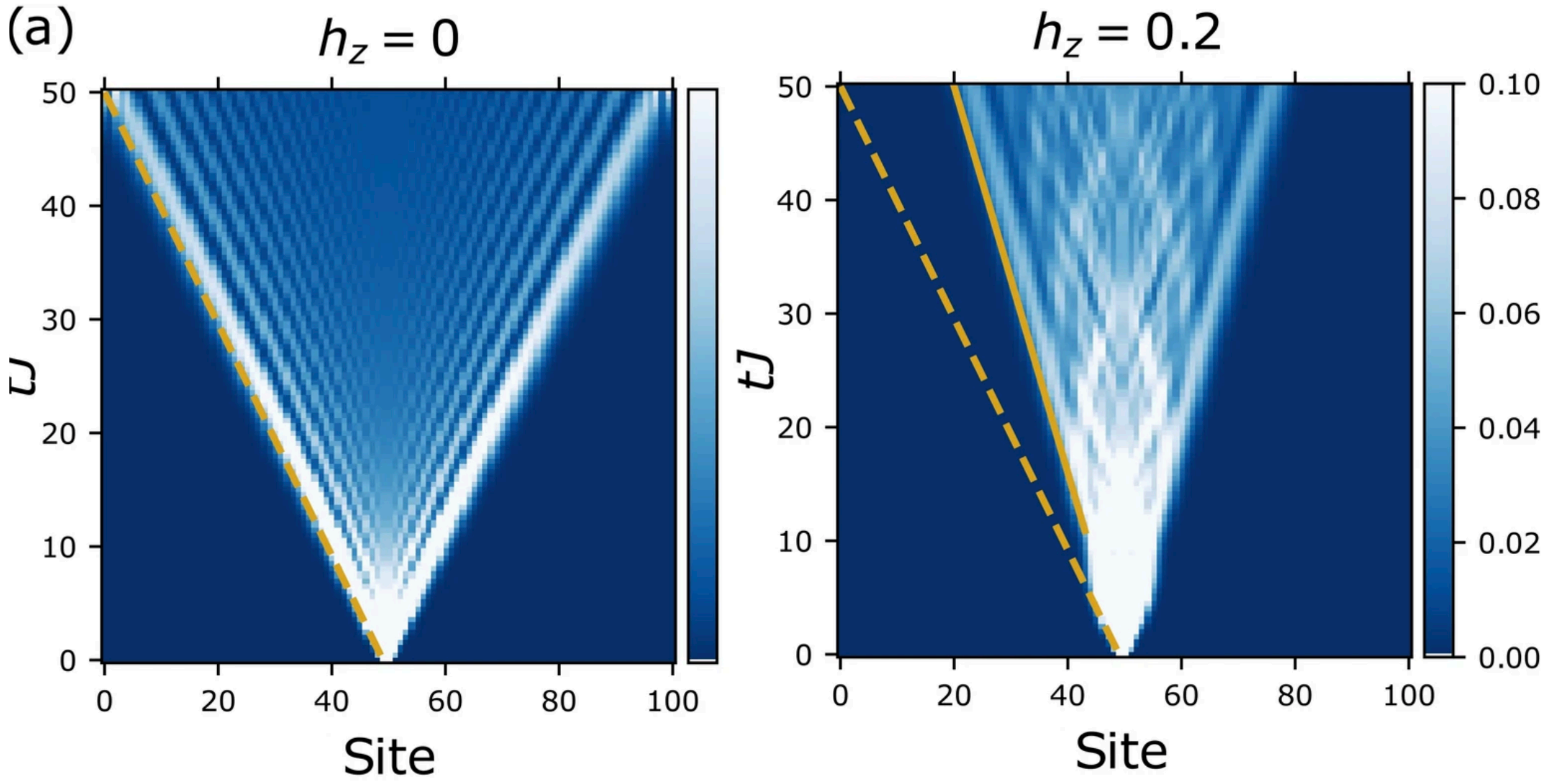
# Experiments

## Trapped Ions




W.L. Tan et al Nature Physics 2021

## IBM Quantum computer

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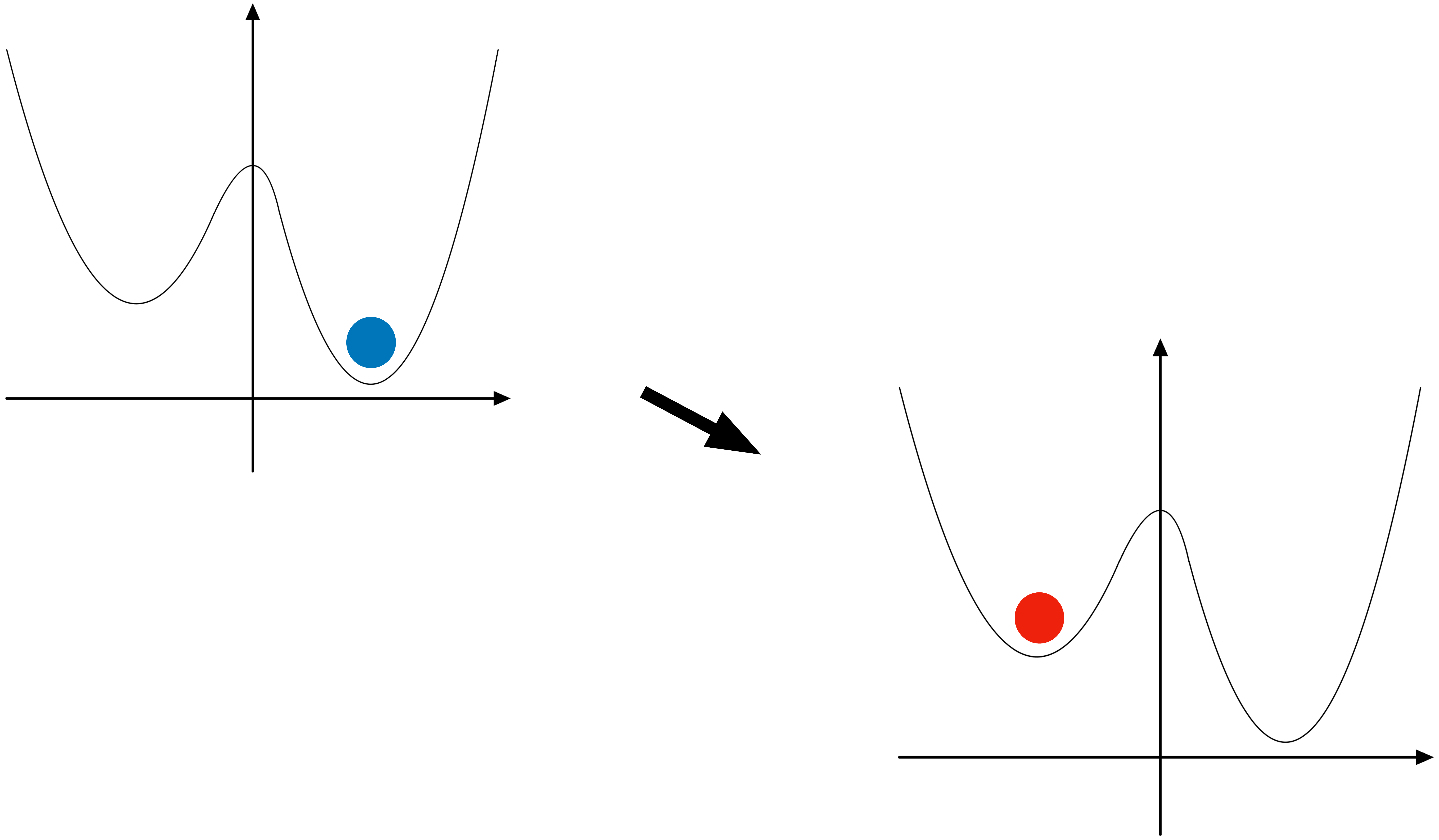
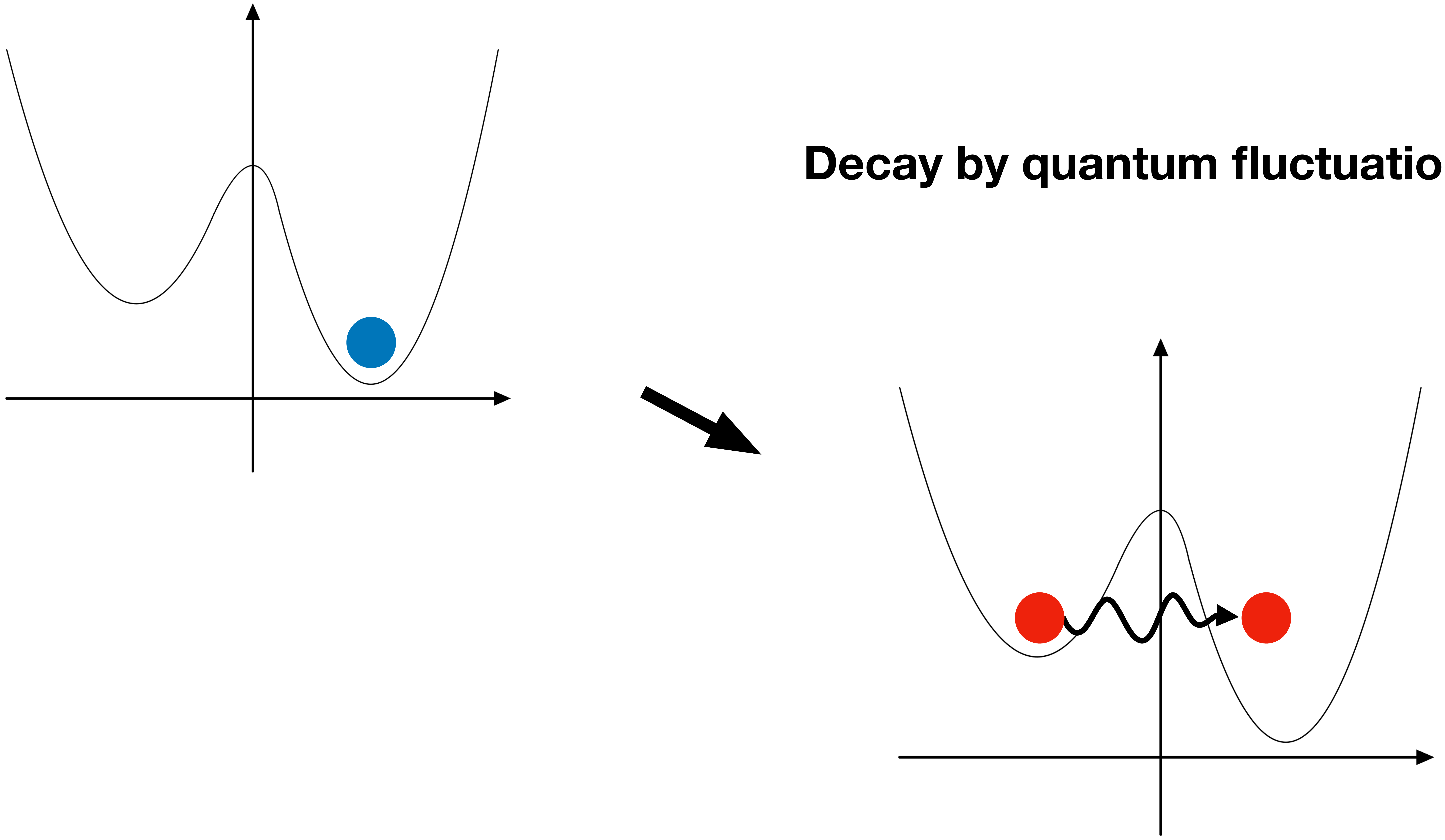
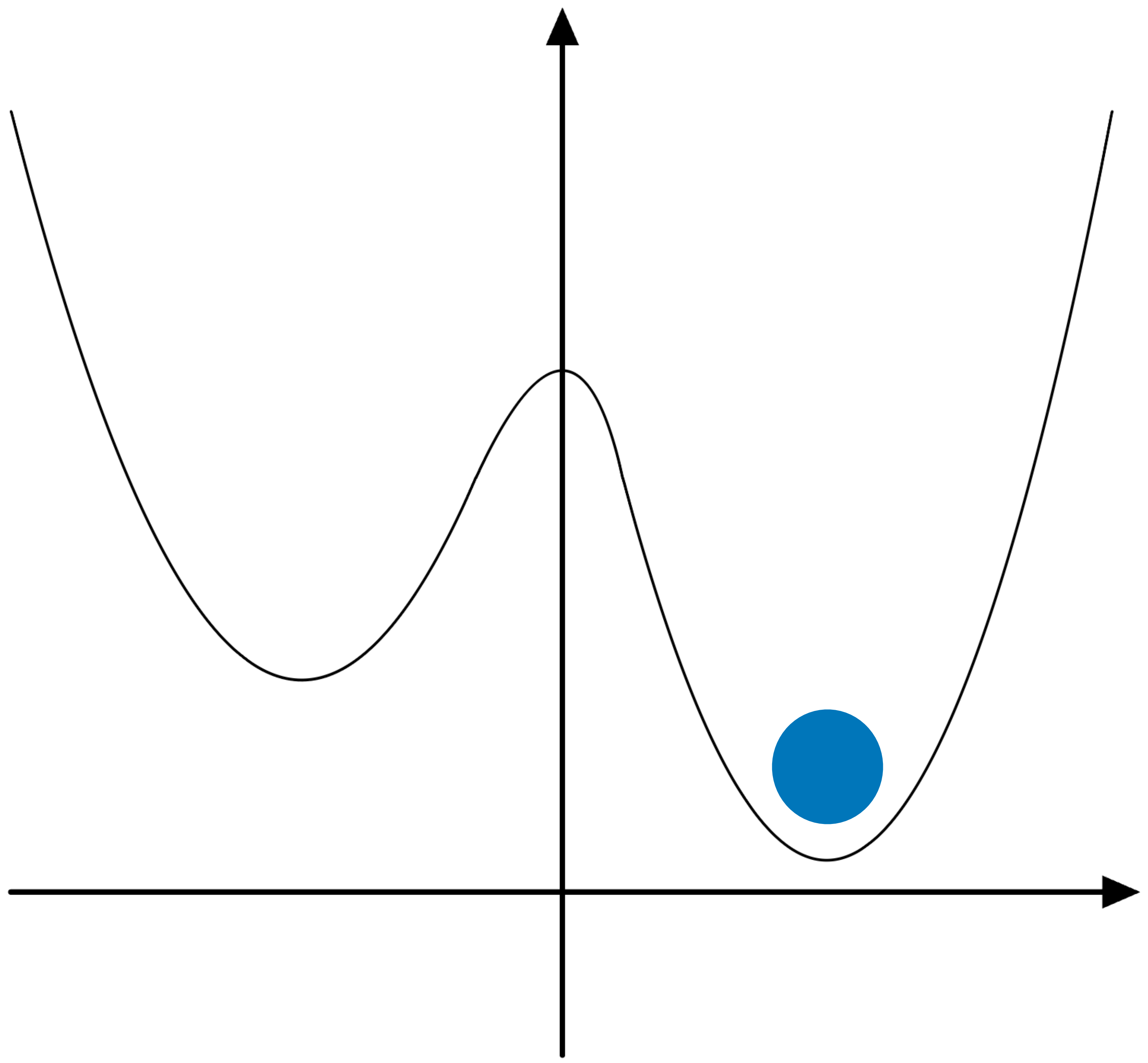
# False Vacuum Decay

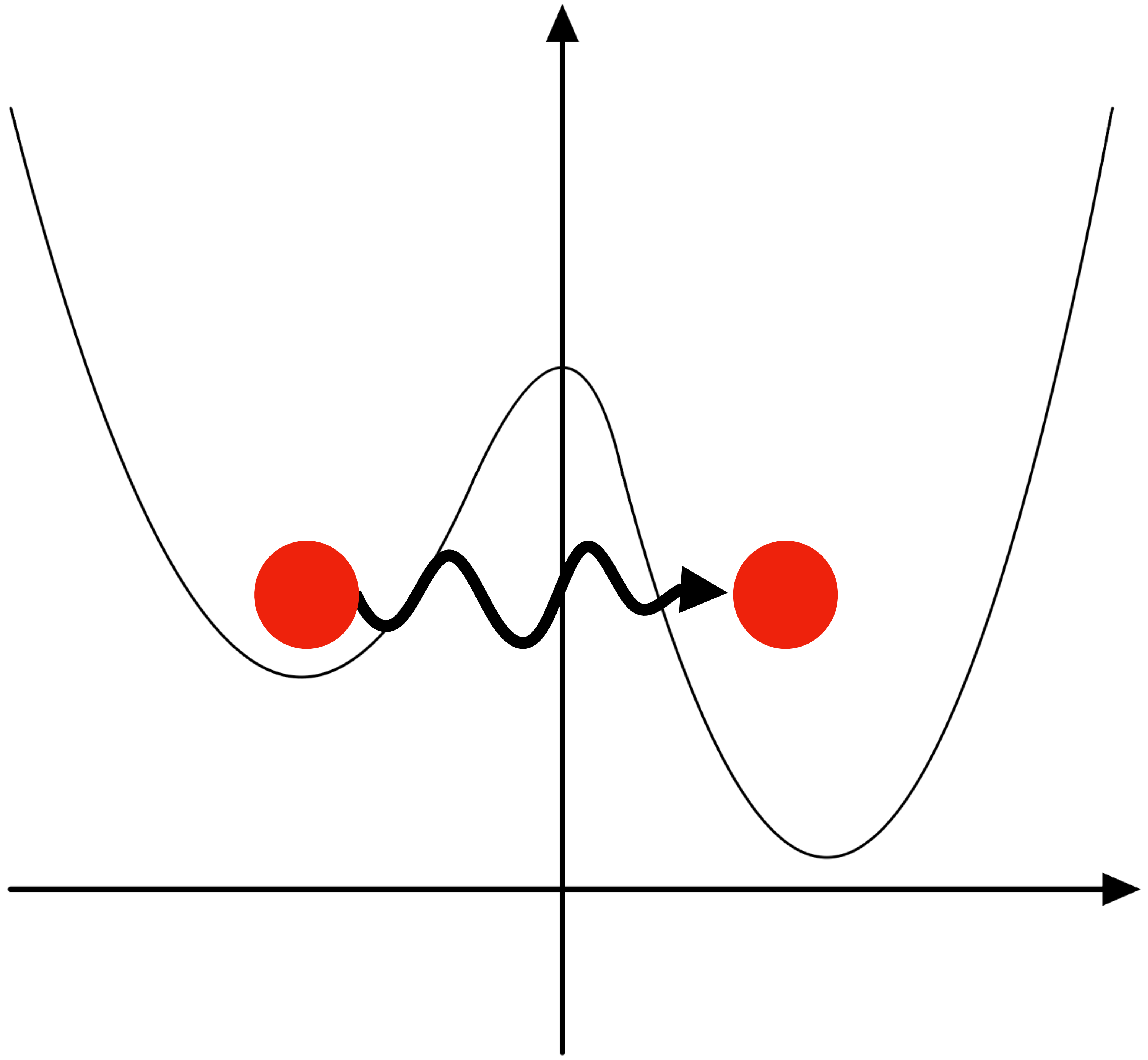


# Decay by quantum fluctuations



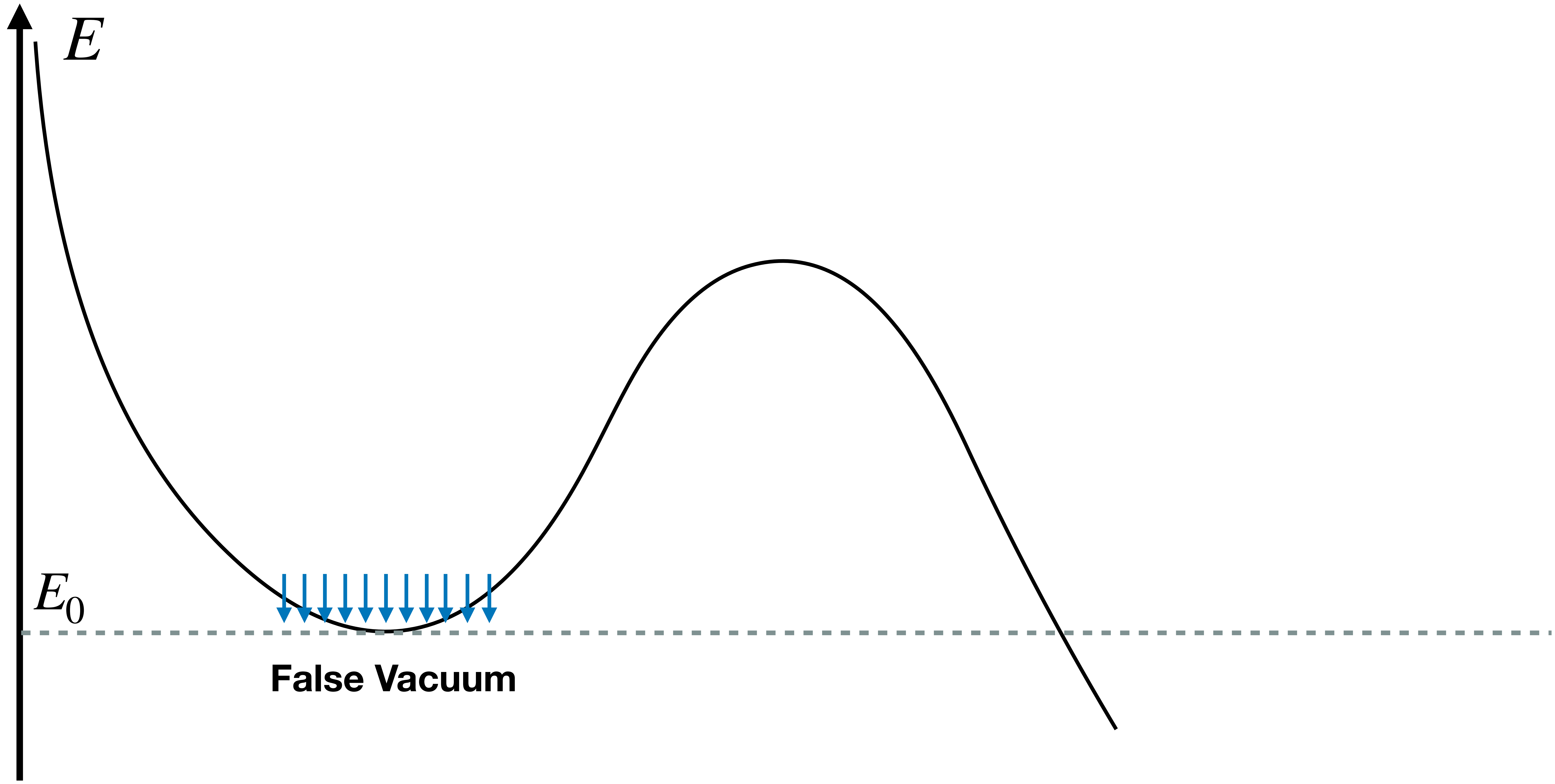


## Decay by quantum fluctuations

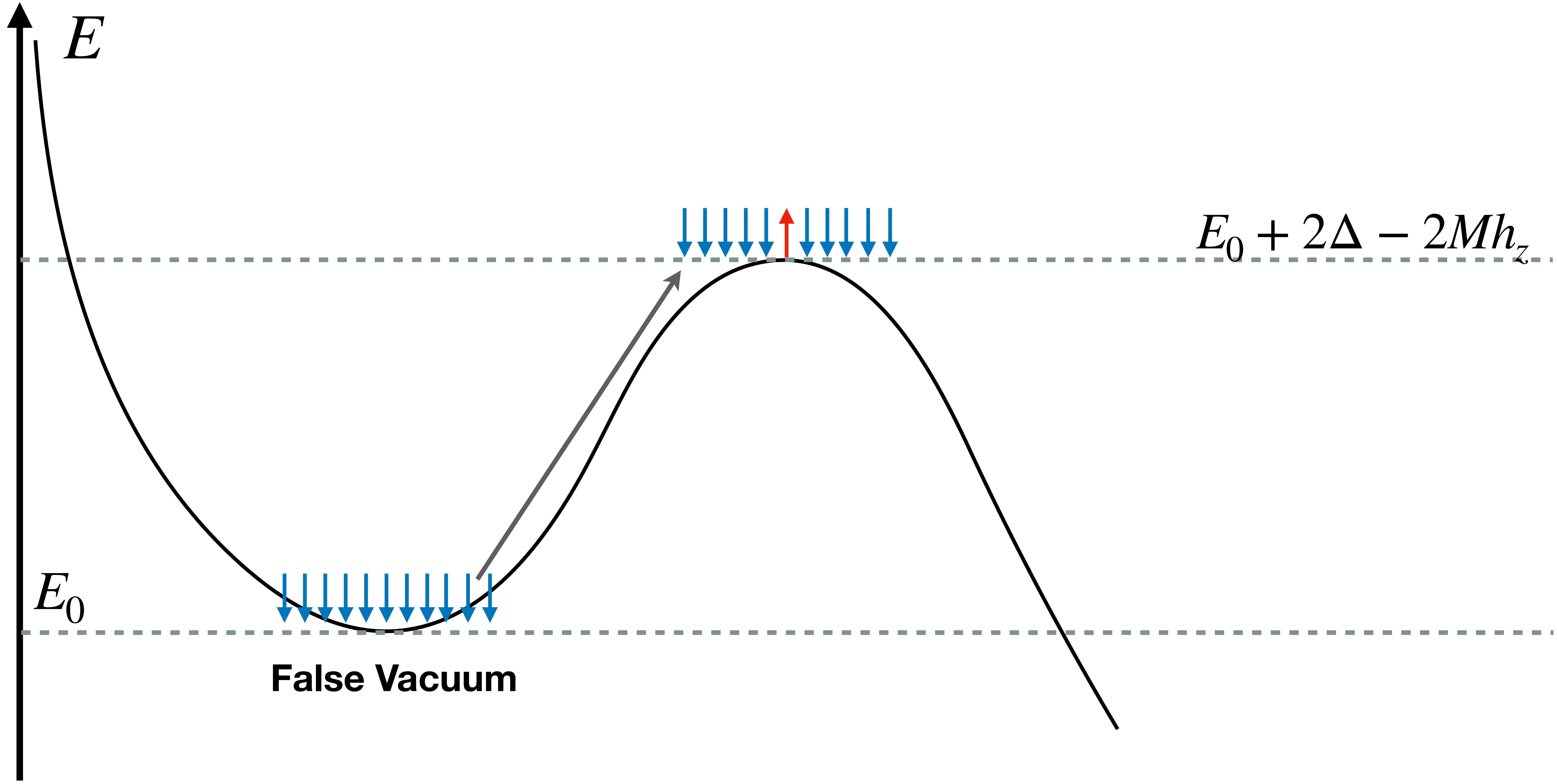


$$|\langle FV|\,U(t)\,|FV\rangle|^2 \approx \exp\left(-L\,\gamma\,t\right)$$

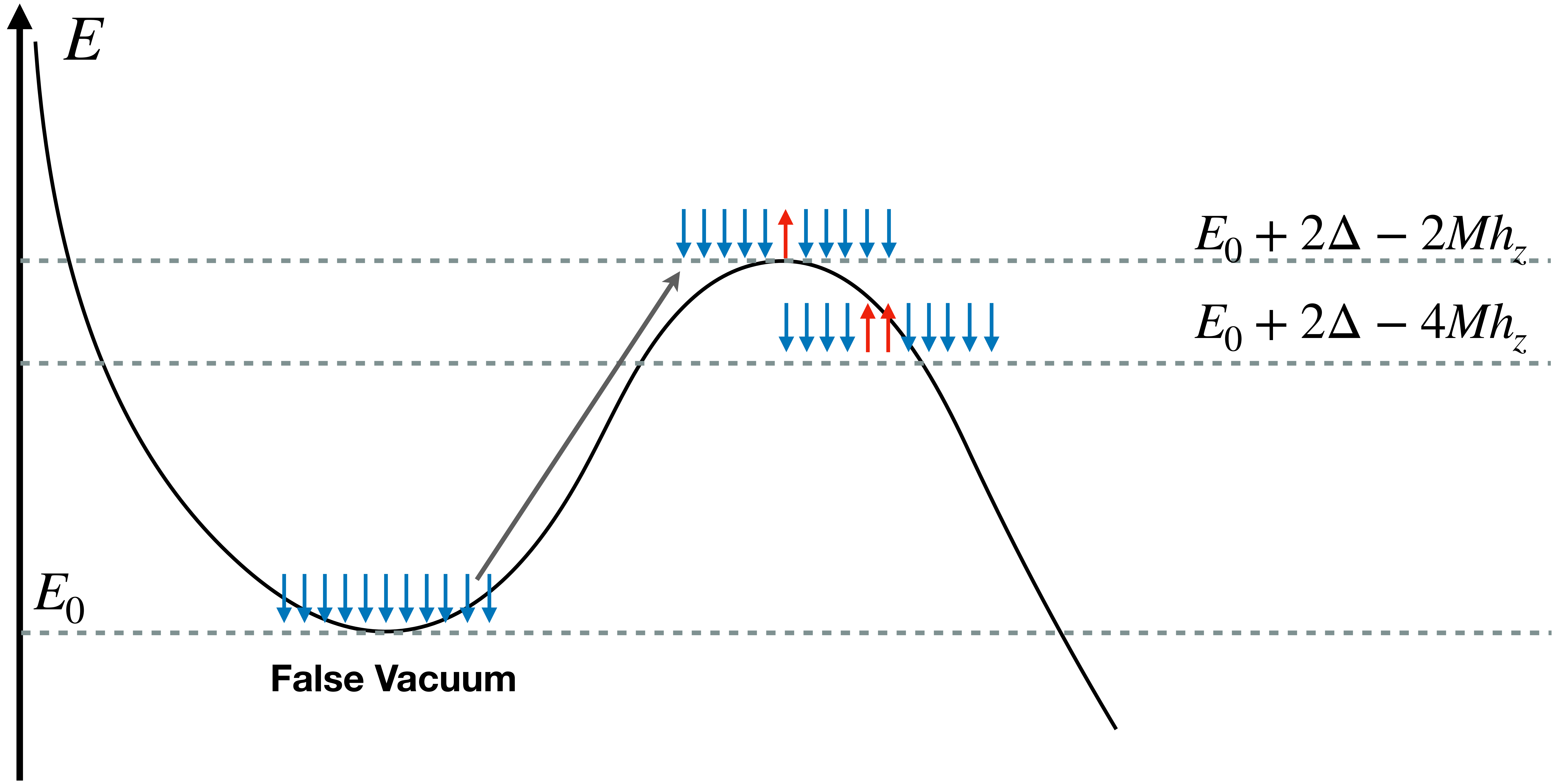
**Unitary time evolution**

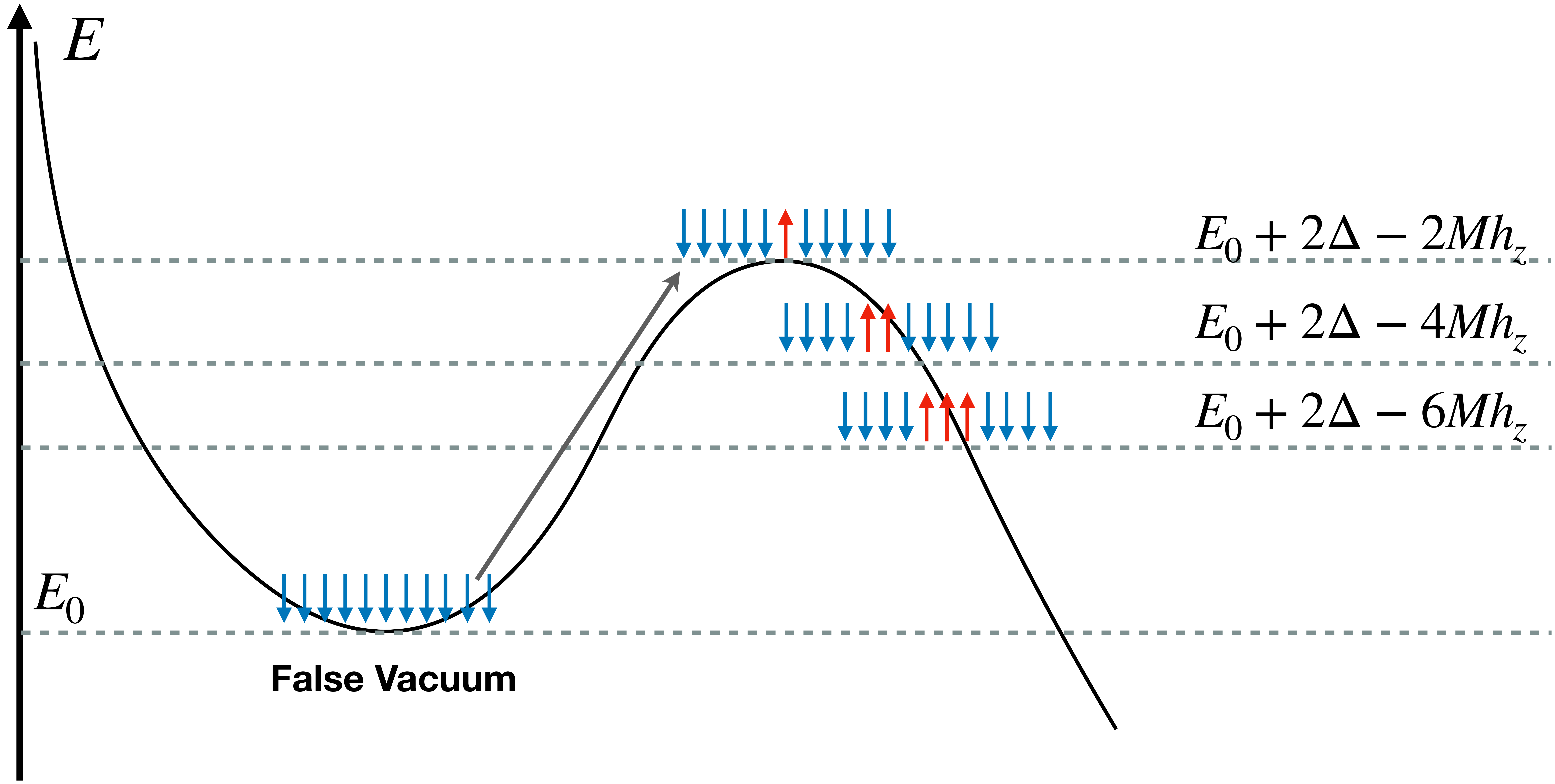


$E$
$E_0$
False Vacuum

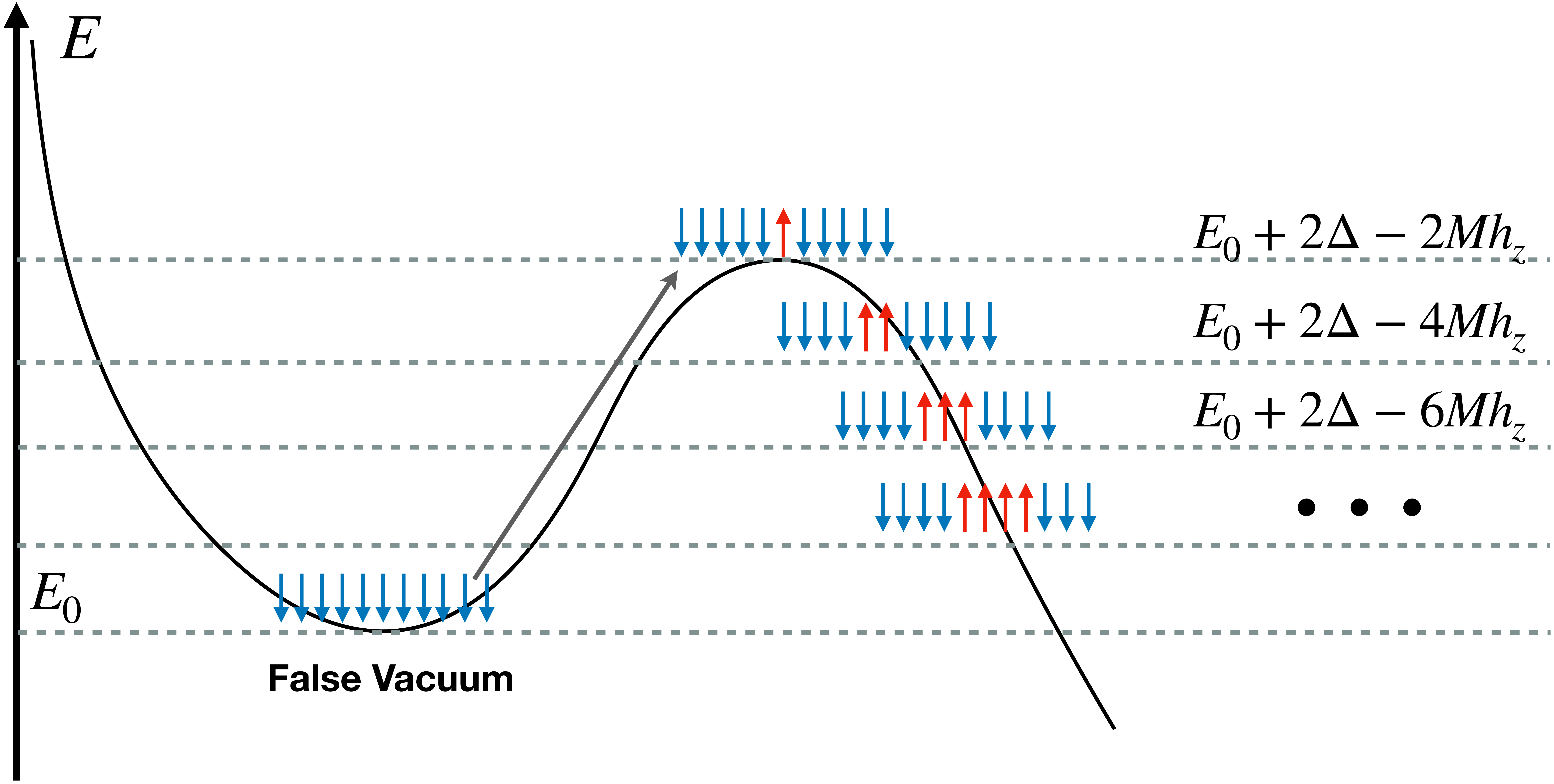


$E$
$E_0 + 2\Delta - 2Mh_z$
$E_0$
False Vacuum


$E$
$E_0 + 2\Delta - 2Mh_z$
$E_0 + 2\Delta - 4Mh_z$
$E_0$
False Vacuum



$E$
$E_0 + 2\Delta - 2Mh_z$
$E_0 + 2\Delta - 4Mh_z$
$E_0 + 2\Delta - 6Mh_z$
$E_0$
False Vacuum



$E$
$E_0 + 2\Delta - 2Mh_z$
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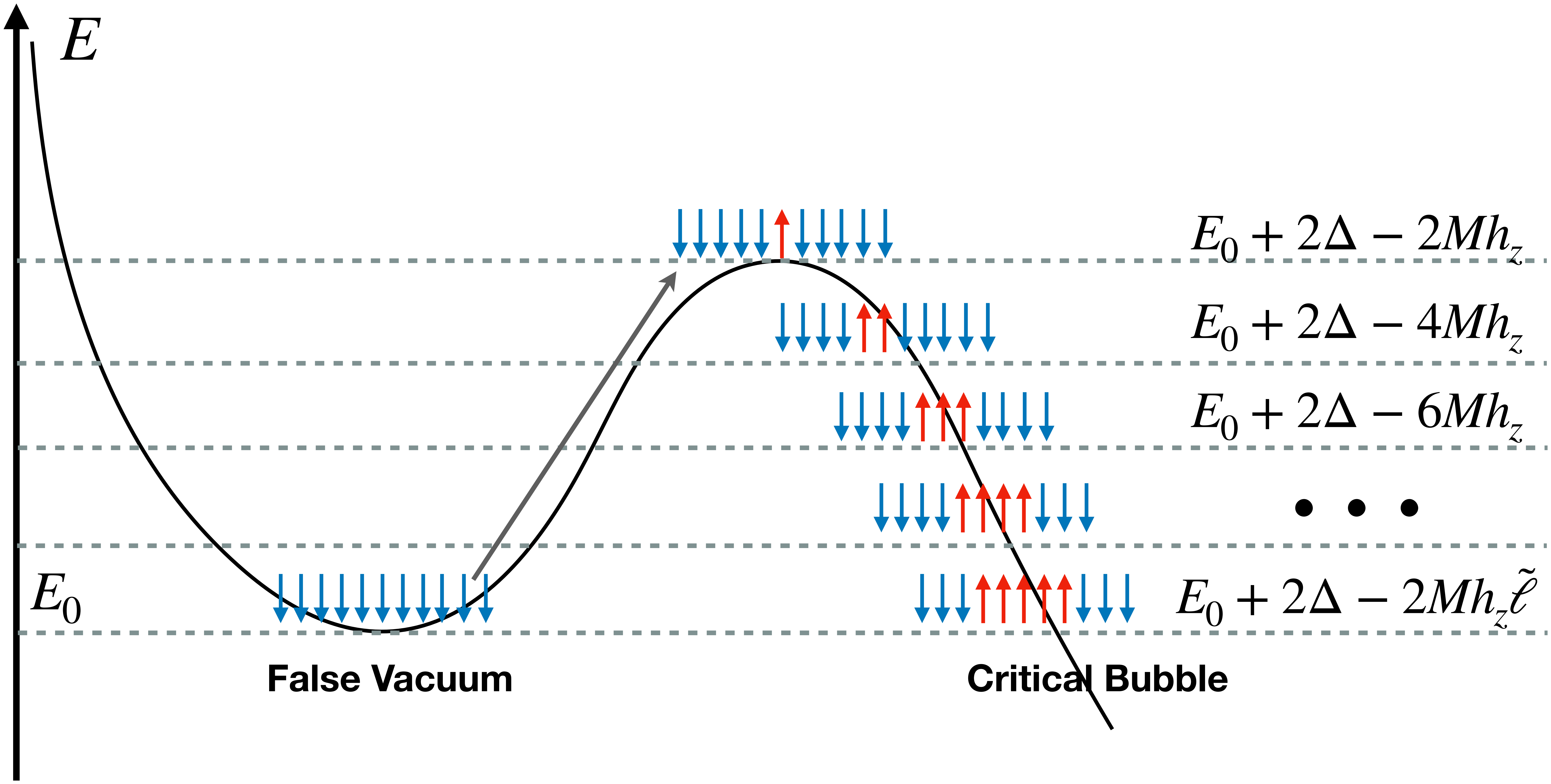
$E_0 + 2\Delta - 6Mh_z$
• • •
$E_0$
False Vacuum



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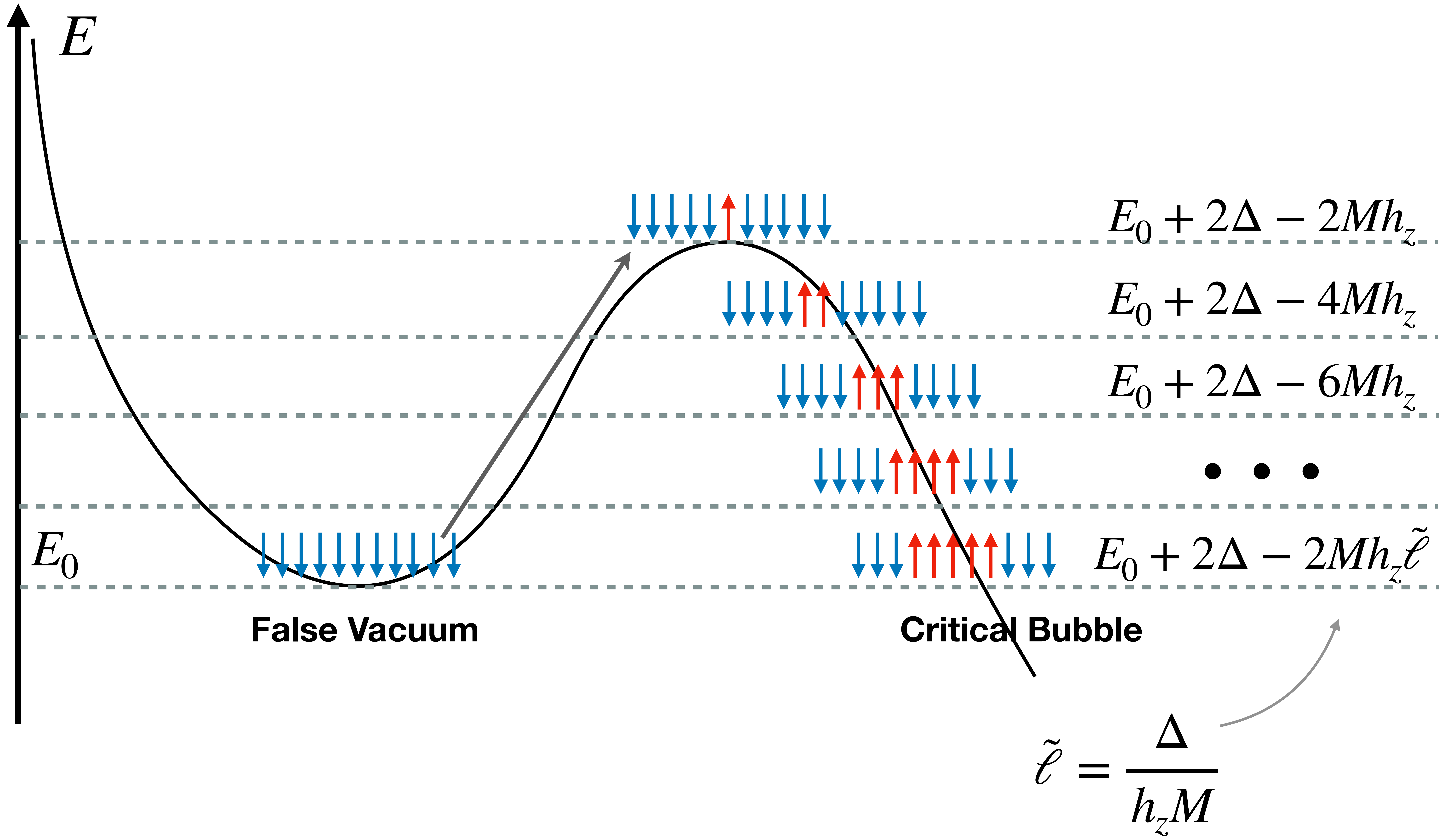
$E_0$
$E_0 + 2\Delta - 2Mh_z\tilde{\ell}$
False Vacuum
Critical Bubble



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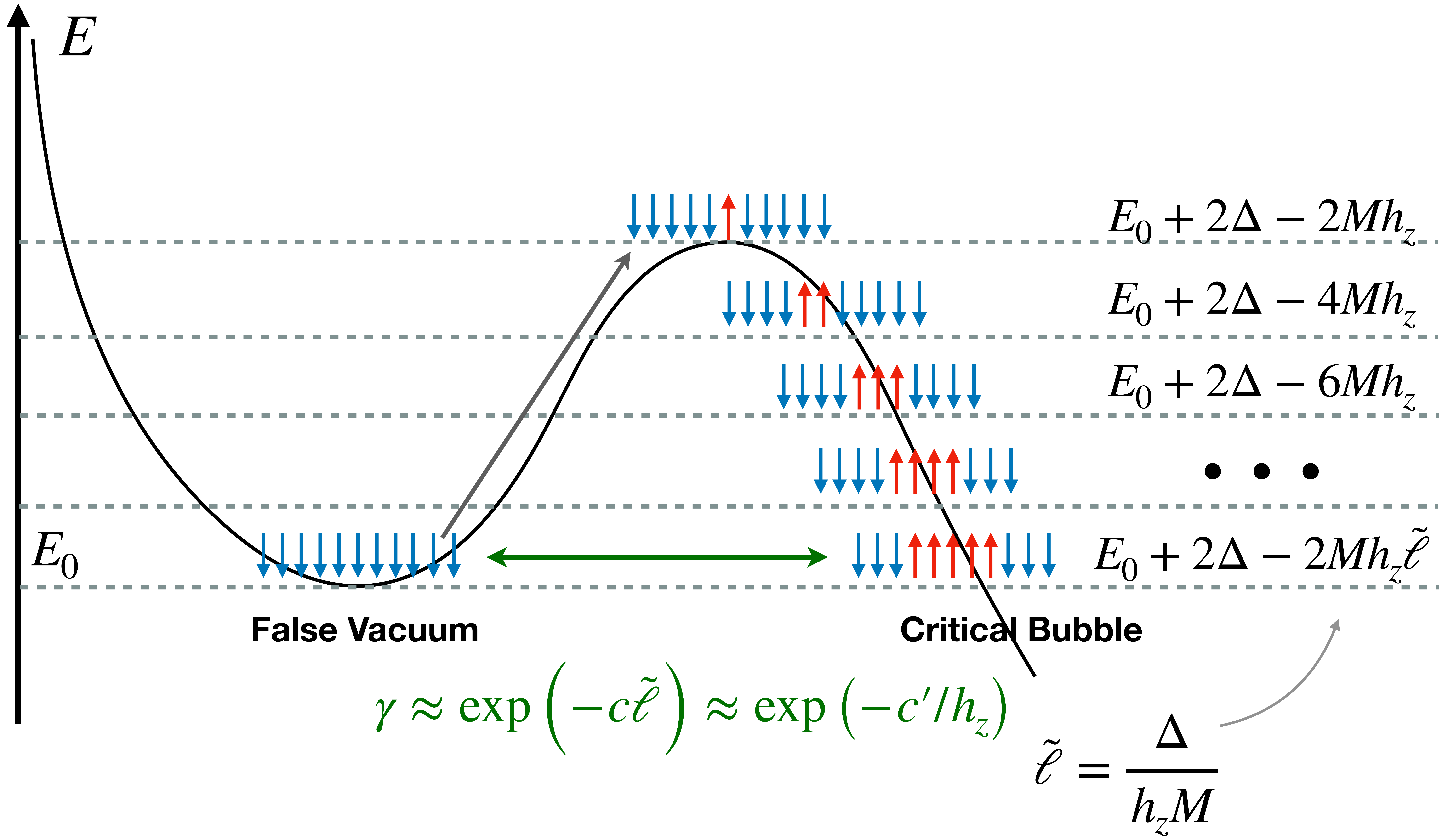
$E_0 + 2\Delta - 2Mh_z\tilde{\ell}$
$E_0$
False Vacuum
Critical Bubble
$\tilde{\ell} = \frac{\Delta}{h_z M}$



$E$
$E_0 + 2\Delta - 2Mh_z$
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$E_0$
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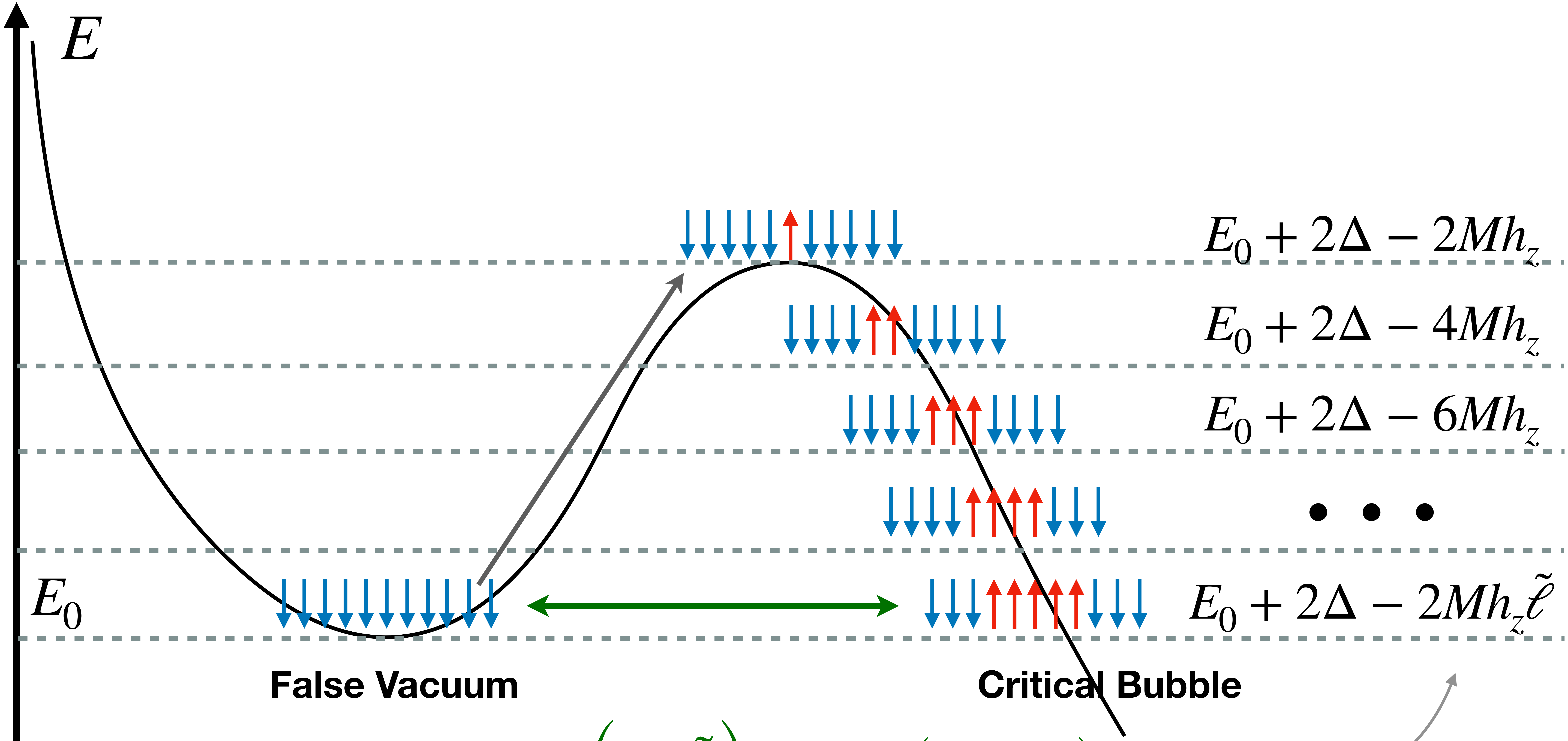
False Vacuum
Critical Bubble
$\gamma \approx \exp\left(-c\tilde{\ell}\right) \approx \exp\left(-c'/h_z\right)$
$\tilde{\ell} = \frac{\Delta}{h_z M}$




$$\gamma \approx \exp\left(-c\tilde{\ell}\right) \approx \exp\left(-c'/h_z\right)$$

$$\tilde{\ell} = \frac{\Delta}{h_z M}$$

$h_z M \ll \Delta$ Rutkevich 1999 PRB

# The Quench Protocol

**1 -** Prepare $|\psi_\downarrow\rangle = \otimes_i |\downarrow\rangle_i$ : Ground state of $H(h_x = 0, h_z = 0)$

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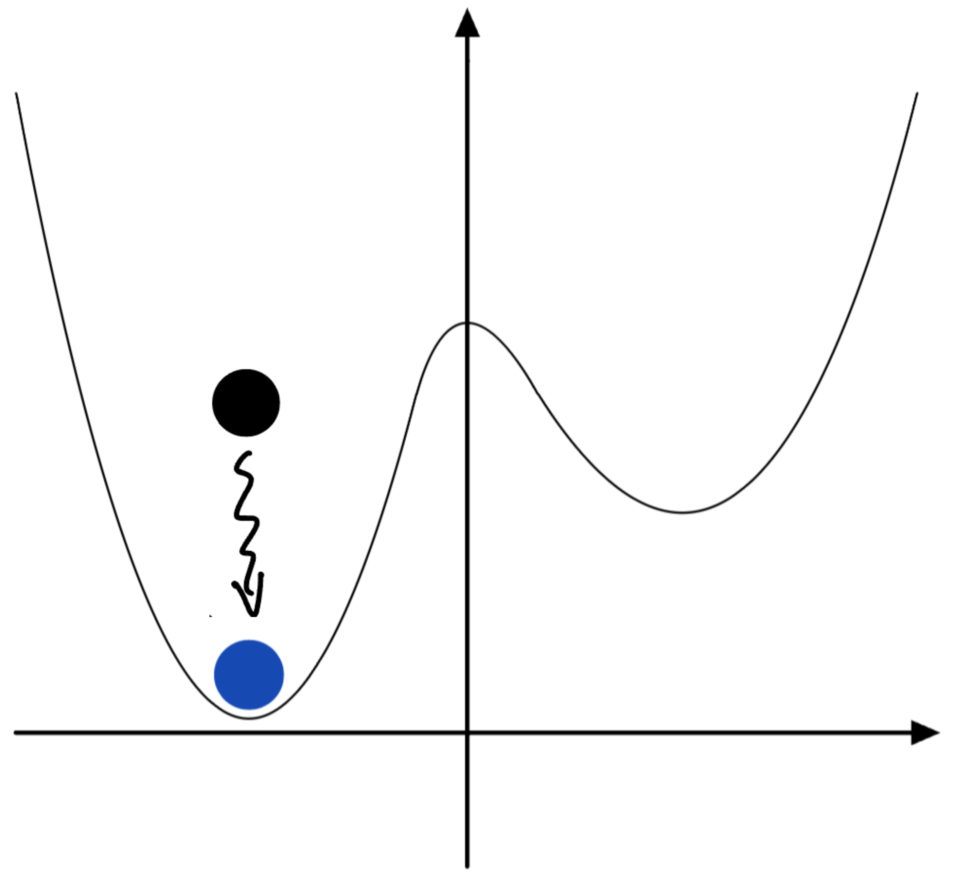
$$|0\rangle = exp\left[-\tau H(h_x, -h_z)\right]|\psi_{\downarrow}\rangle$$



# The Quench Protocol

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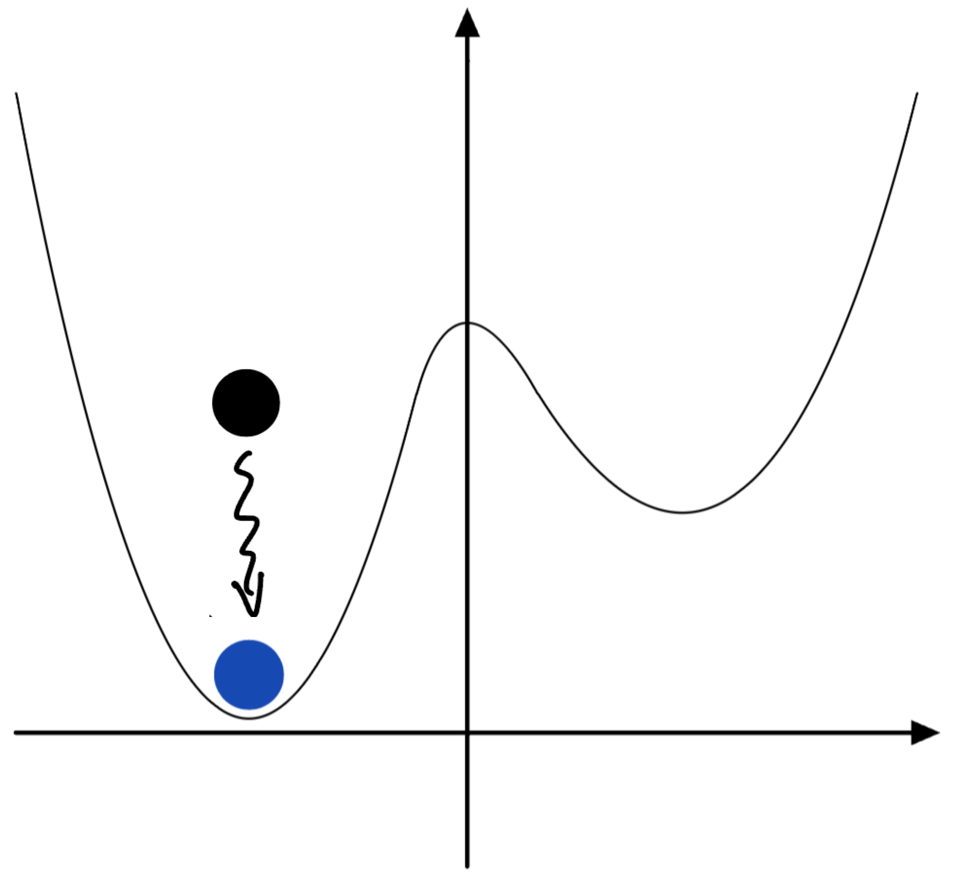

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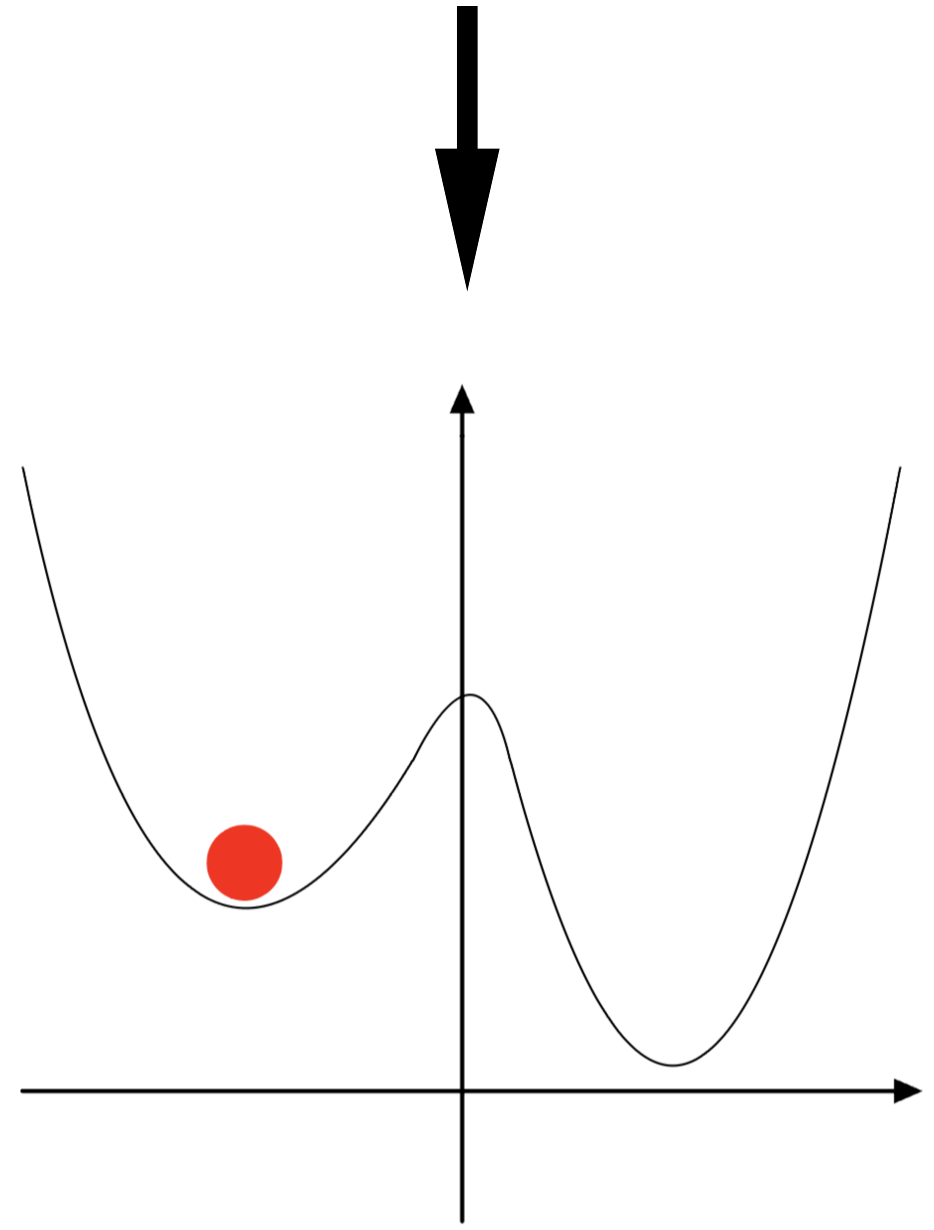
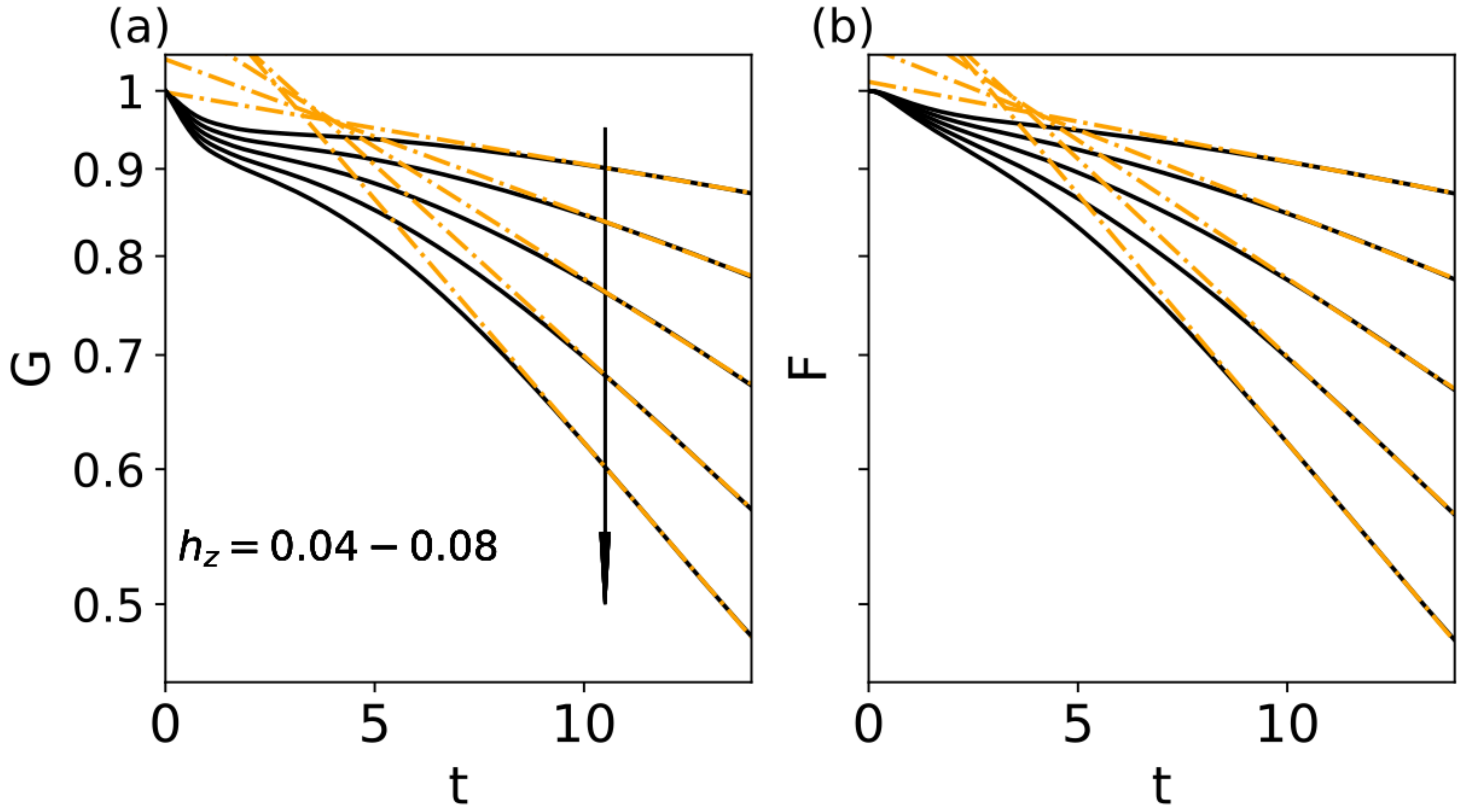
**3 - Switch -$h_z$ to $+h_z$** and evolve with $exp\left[-i\, H(h_x, +h_z)\, t\right]$:



# Rate extraction

Two-sites observable

$$G(t) = 1 - ||\rho(t) - \rho(0)||_1$$




Rescaled Magnetisation

$$F(t) = \frac{\langle \sigma^z(t) \rangle + \langle \sigma^z(0) \rangle}{2\langle \sigma^z(0) \rangle}$$

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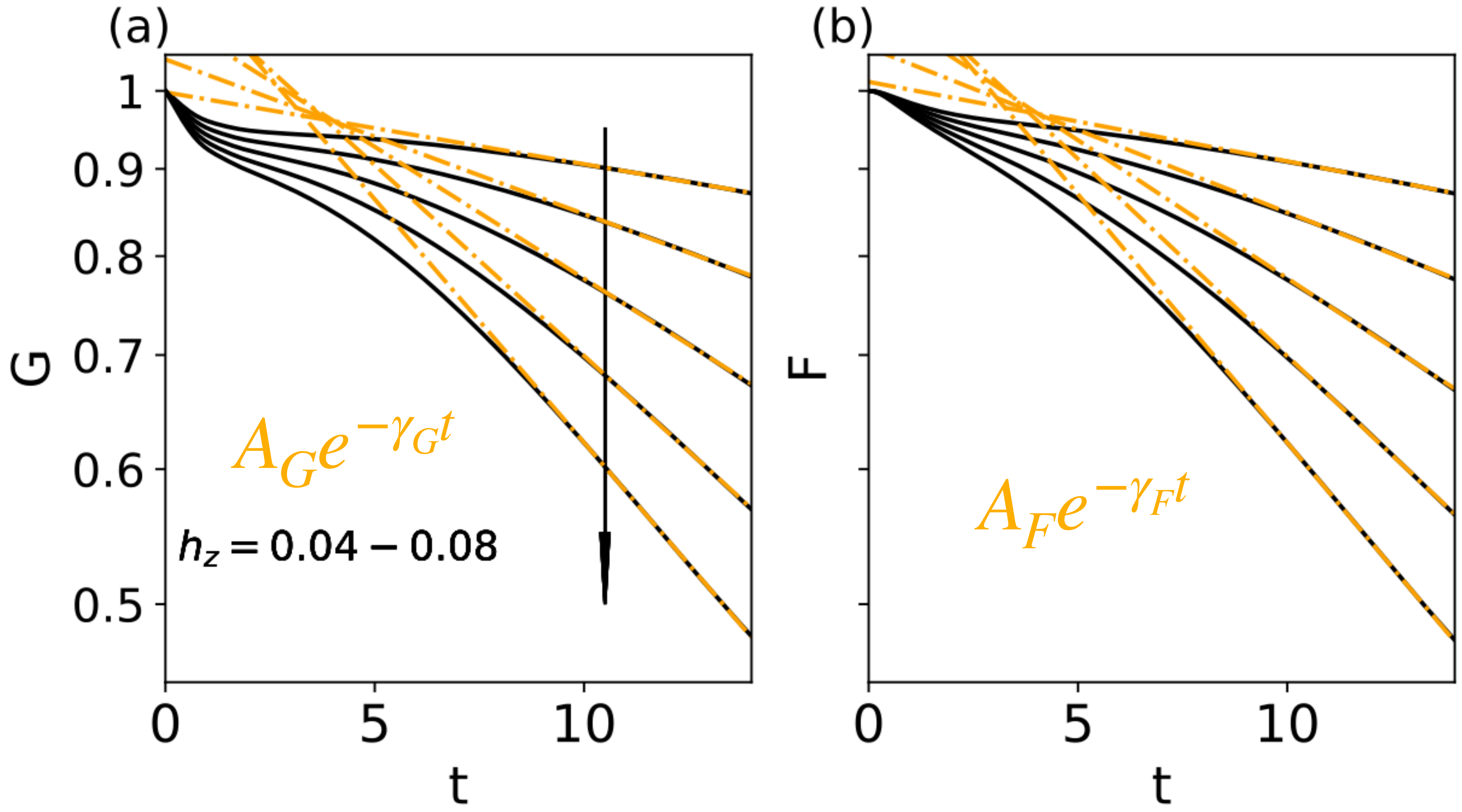


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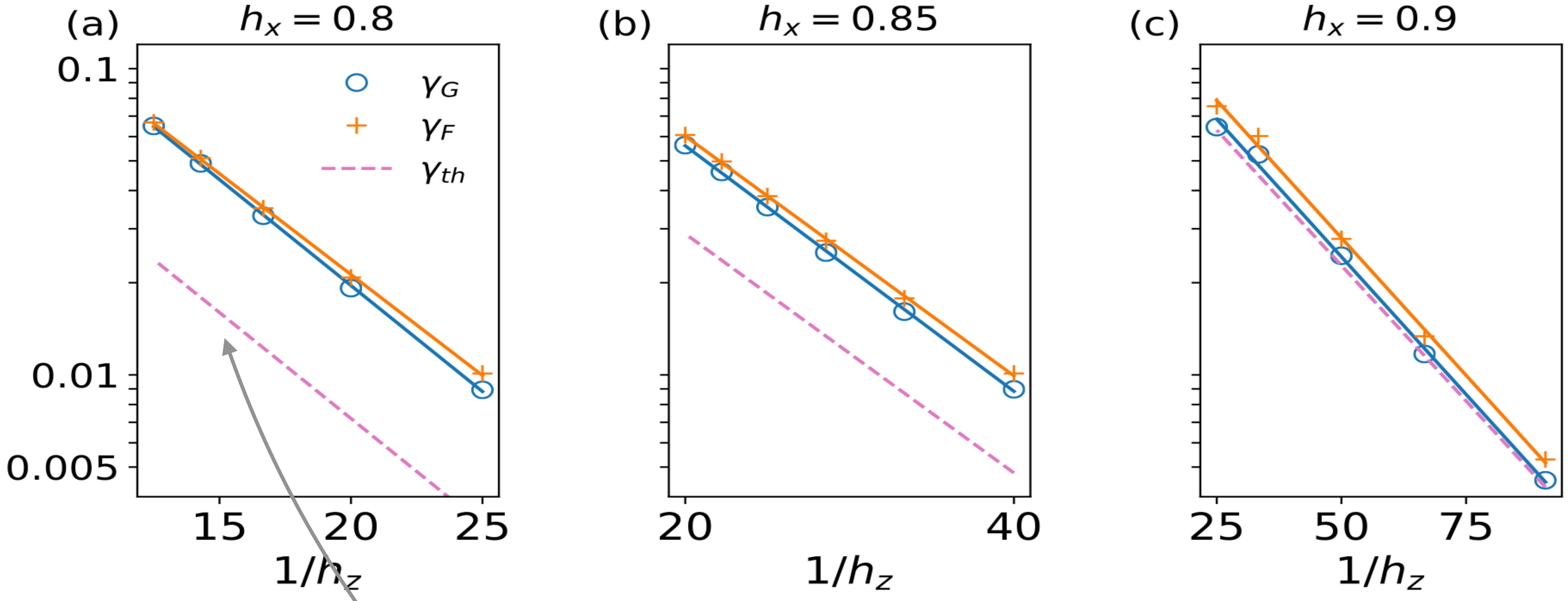
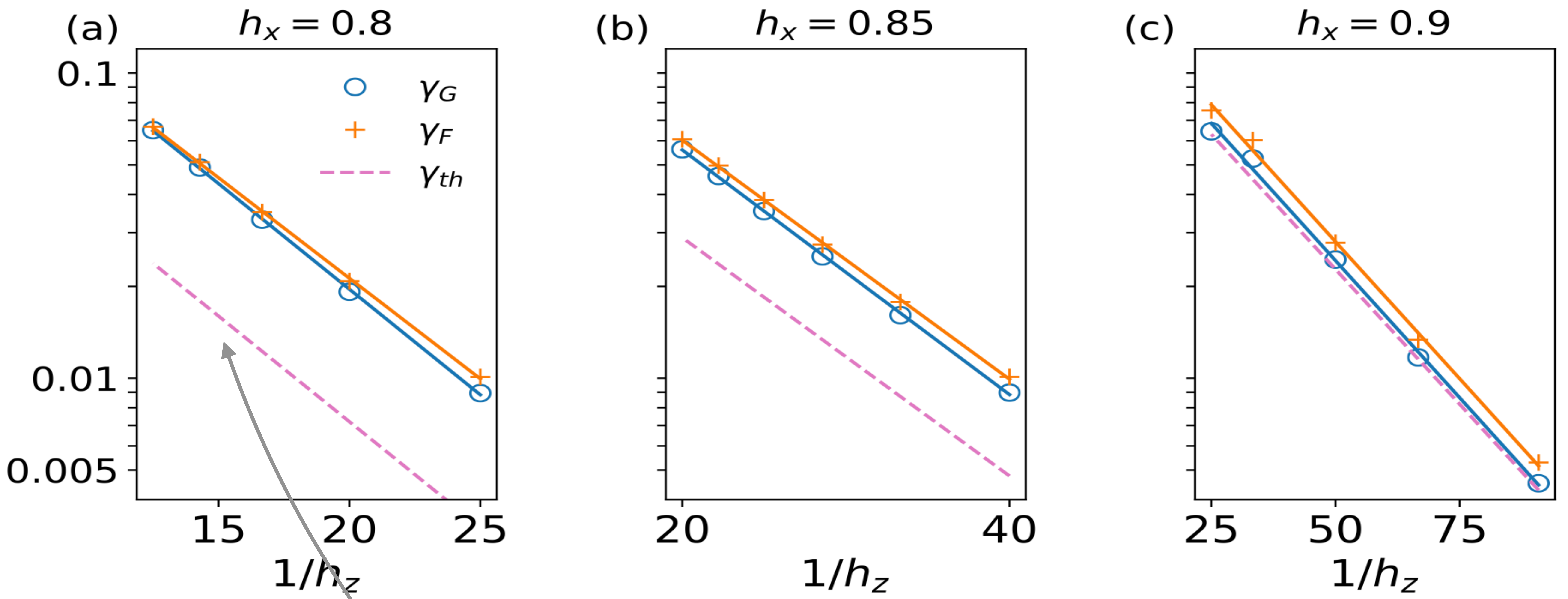
# Comparison with theoretical prediction




$$\gamma_{th} = c(h_x)\exp\left(-\frac{q_h}{h_z}\right)$$

Rutkevich 1999 PRB

# Comparison with theoretical prediction

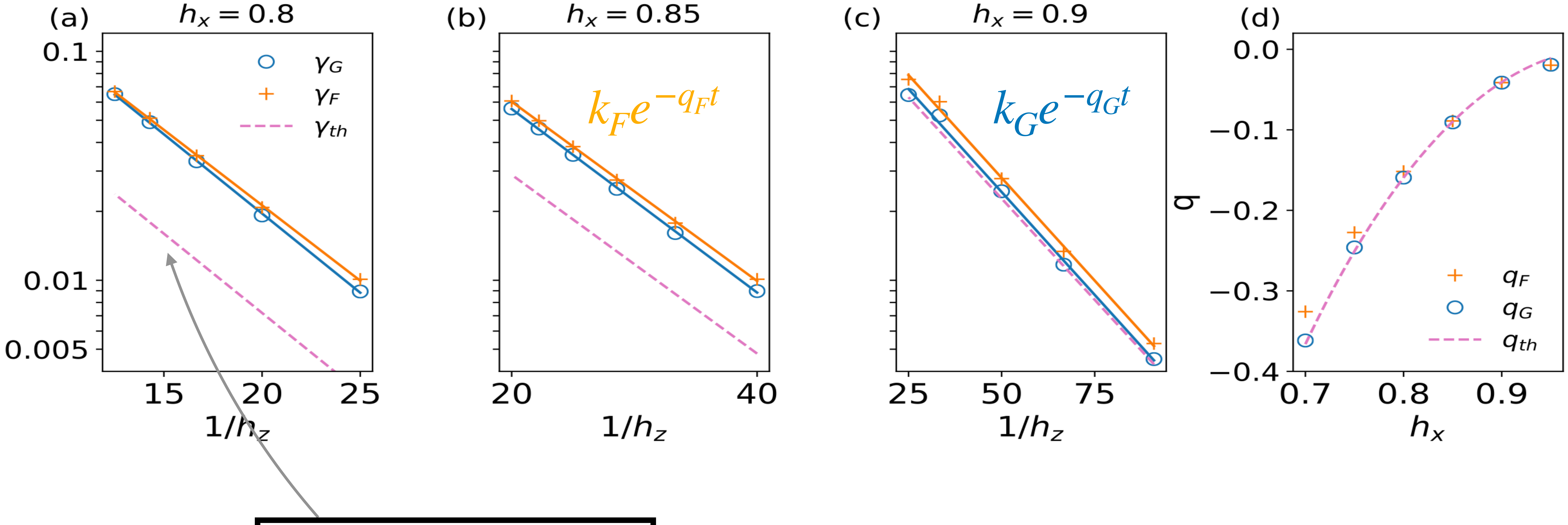



$$\gamma_{th} = c(h_x)\exp\left(-\frac{q_h}{h_z}\right)$$

Rutkevich 1999 PRB

Prefactor is off !

# Comparison with theoretical prediction

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Rutkevich 1999 PRB

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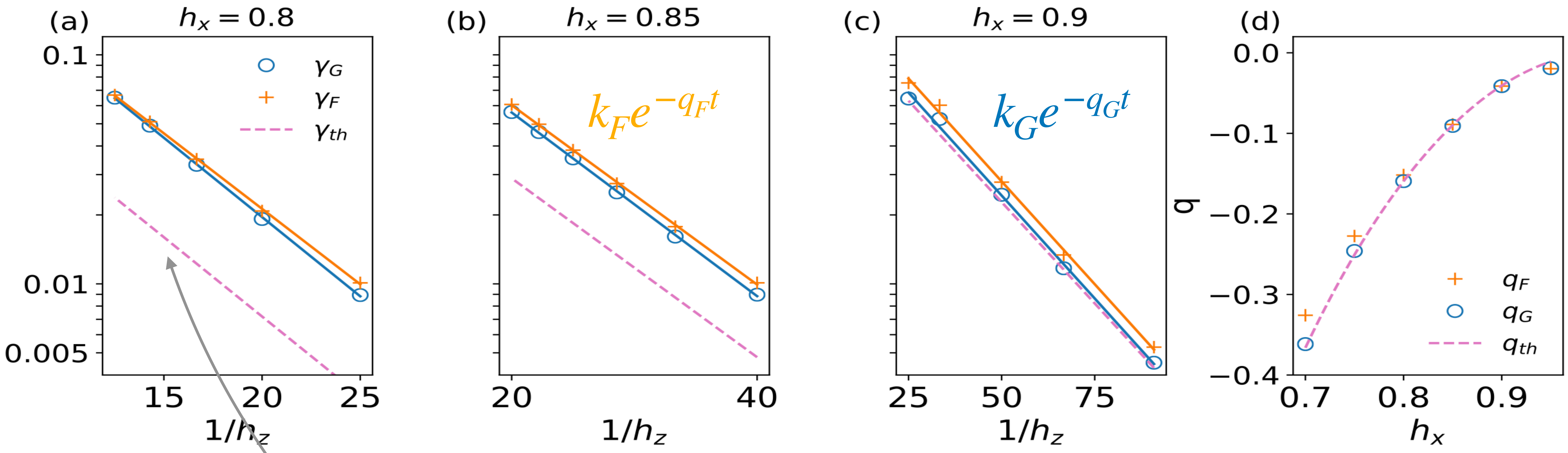
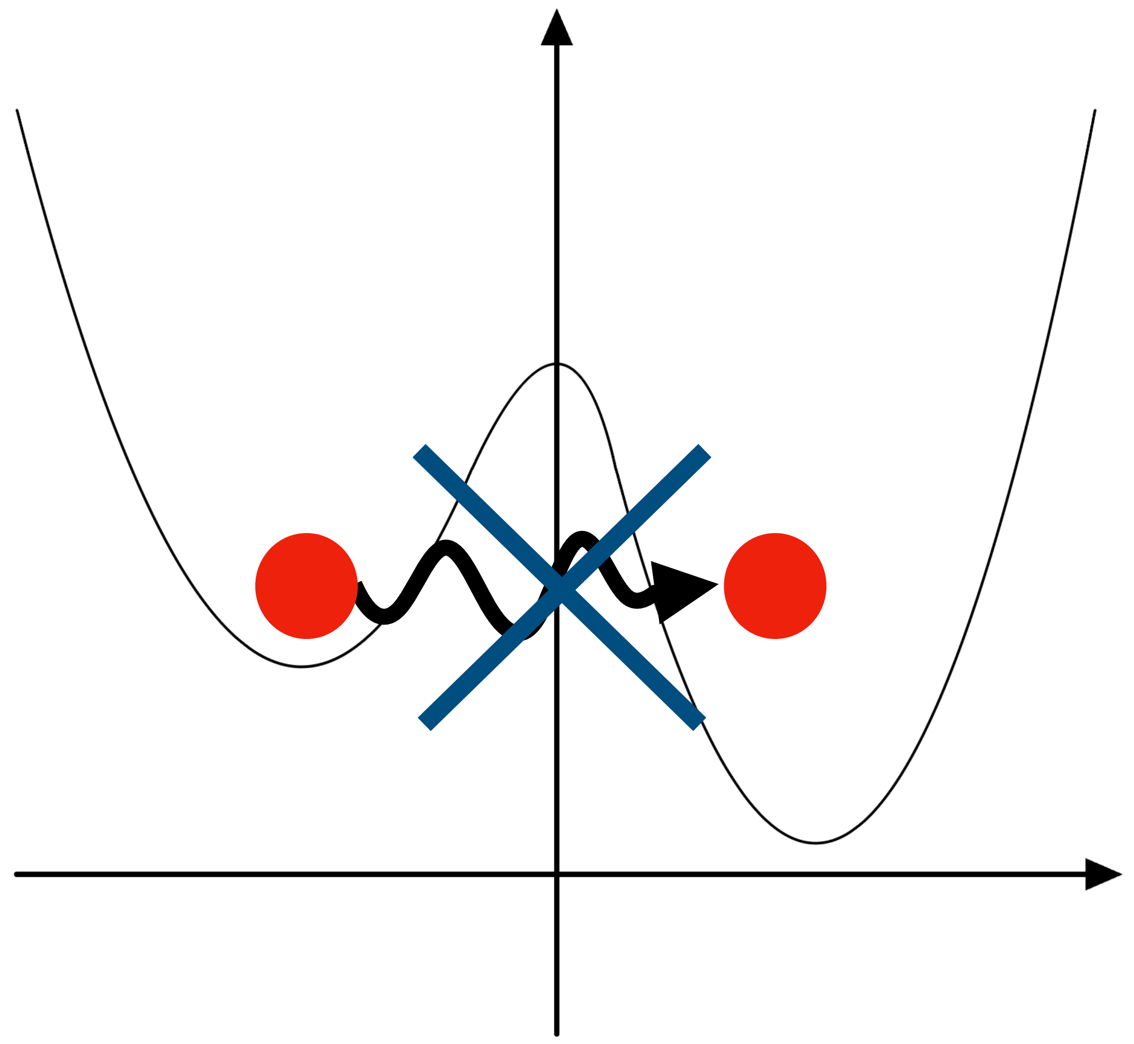



$$\gamma_{th} = c(h_x)\exp\left(-\frac{q_h}{h_z}\right)$$

Rutkevich 1999 PRB

Exponential coefficient is good!

# More on the oscillations



Lagnese, Surace, Morampudi, Wilczek, arXiv:2308.08340

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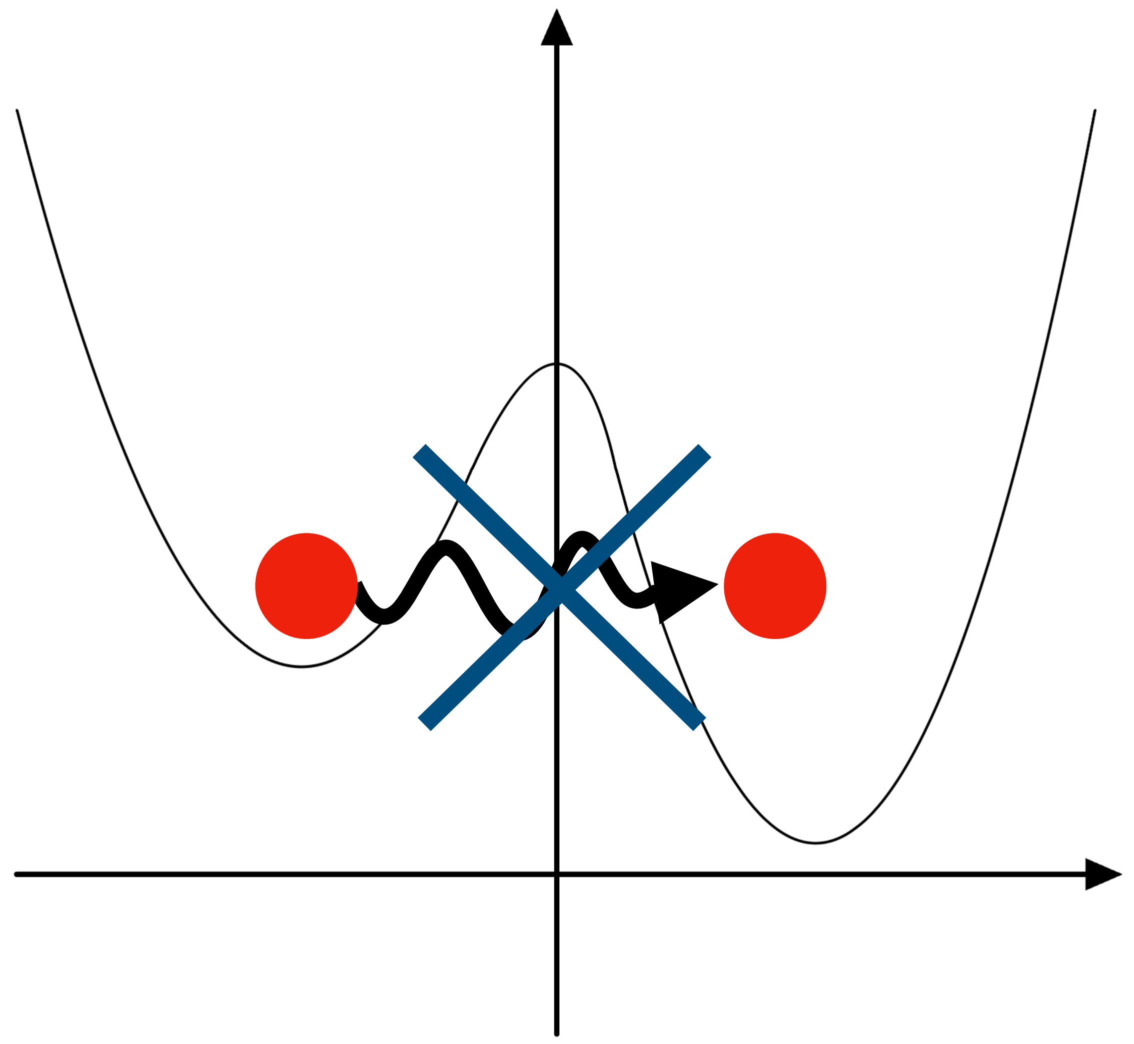

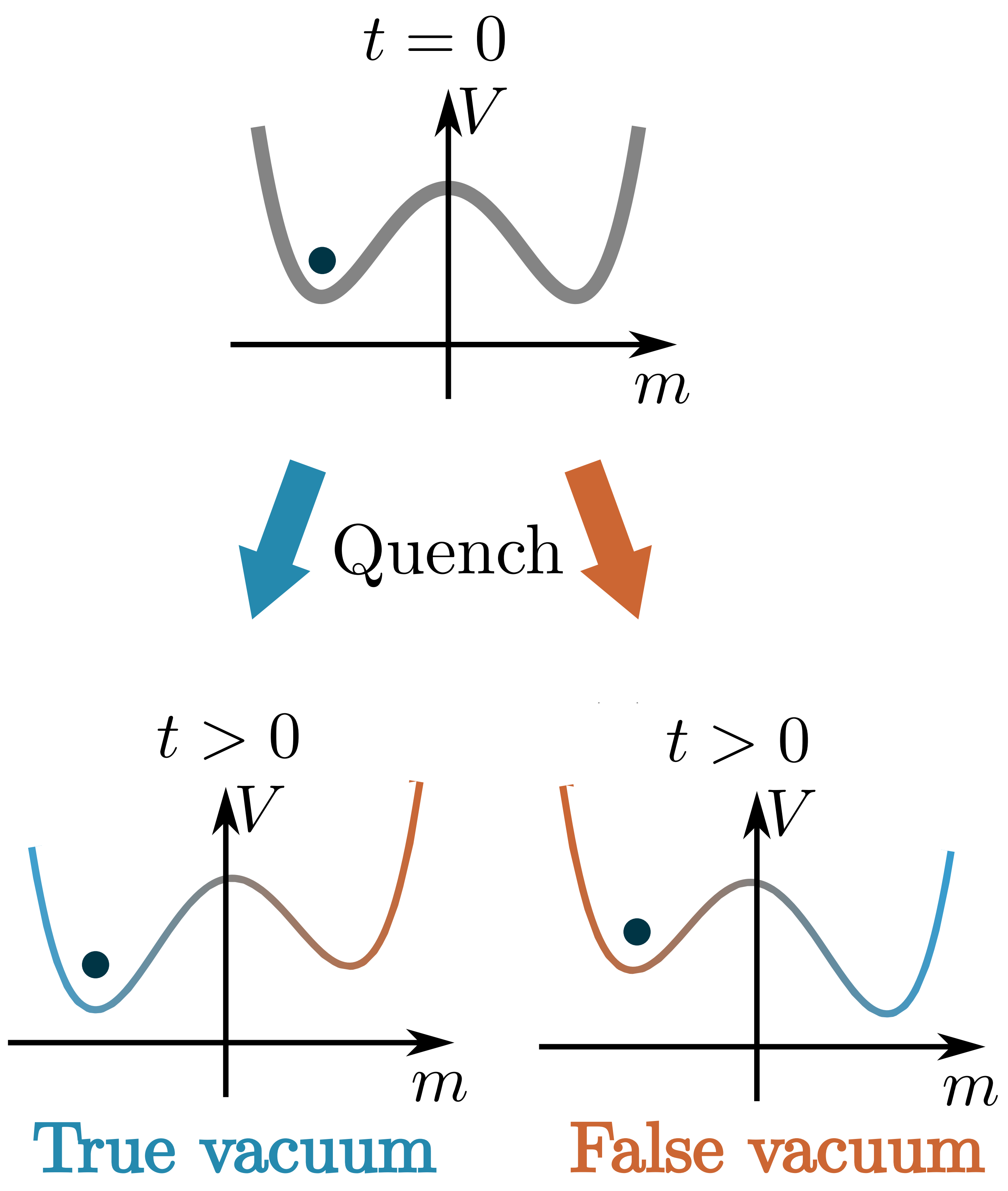


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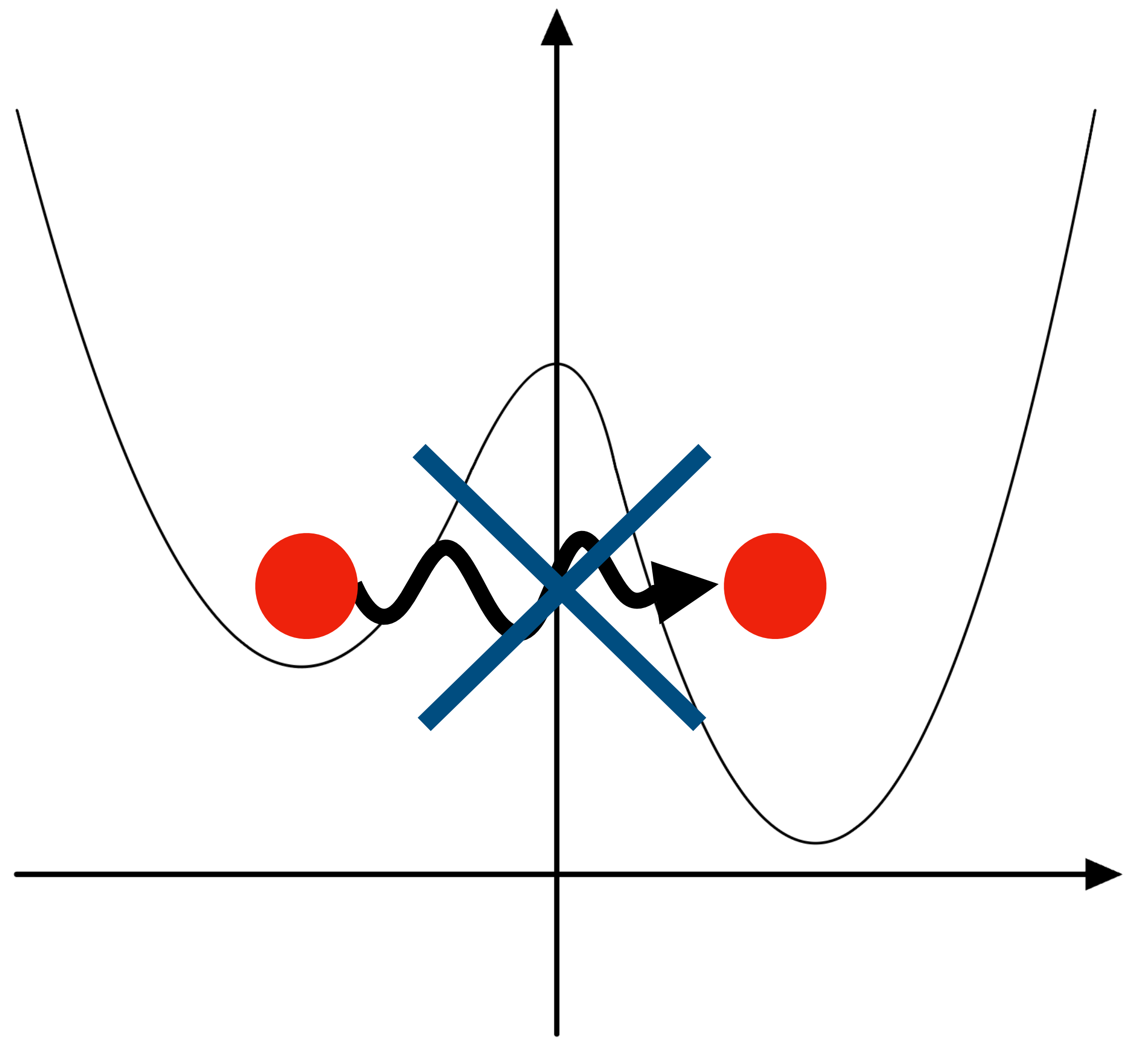

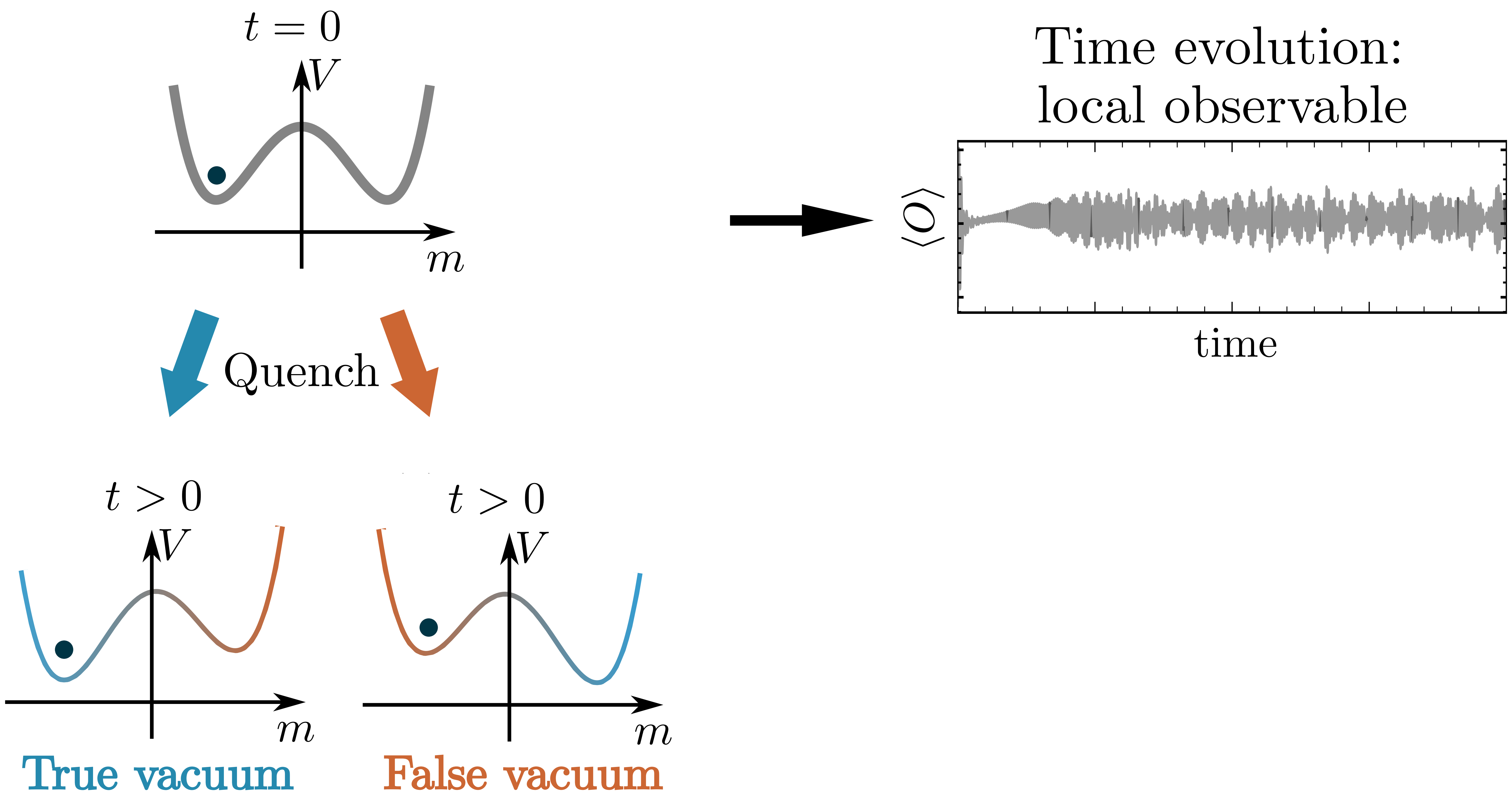


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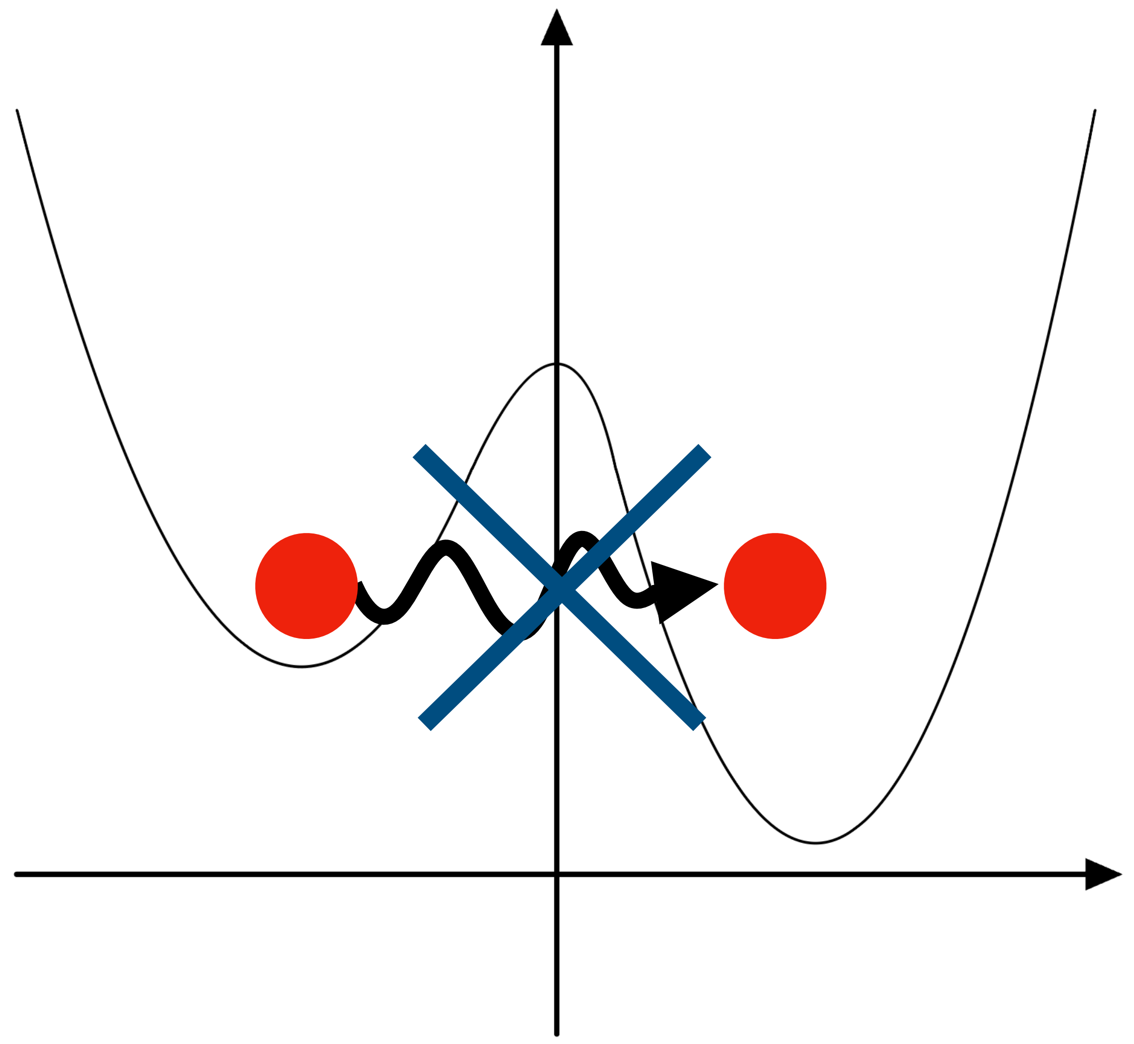
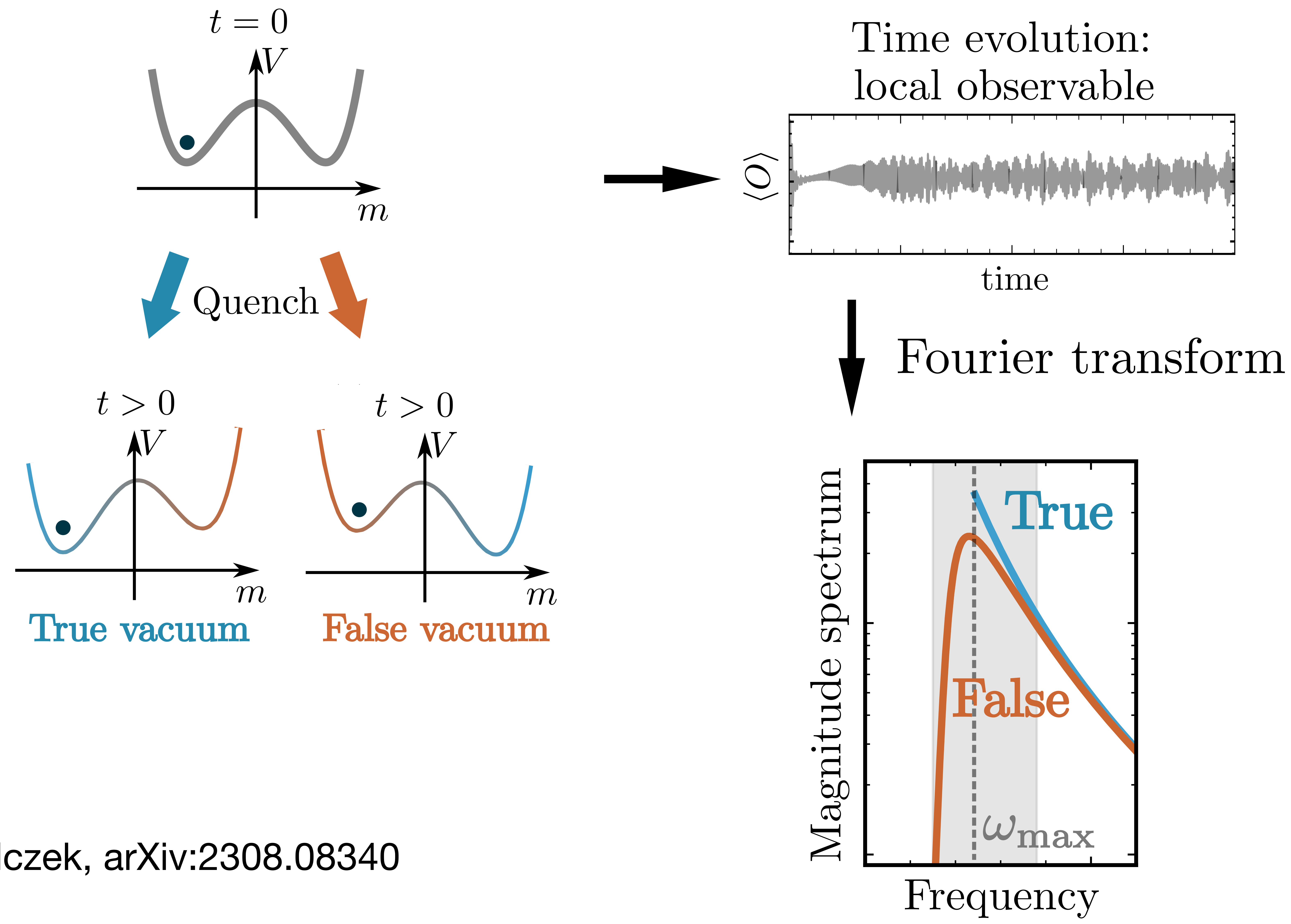


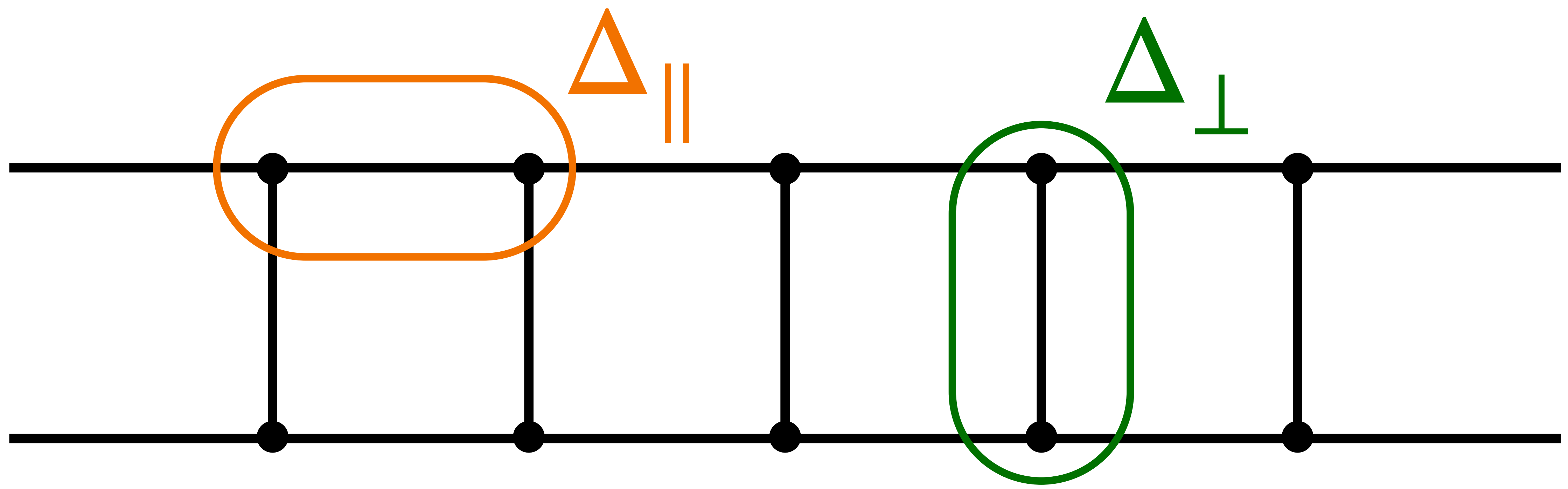



Lagnese, Surace, Morampudi, Wilczek, arXiv:2308.08340

# Another Model

# XXZ Ladder

$$H = \frac{1}{2} \sum_{j} \sum_{\alpha=1,2} \left( \sigma^x_{j,\alpha}\sigma^x_{j+1,\alpha} + \sigma^y_{j,\alpha}\sigma^y_{j+1,\alpha} + \Delta_{\parallel}\sigma^z_{j,\alpha}\sigma^z_{j+1,\alpha} \right) + \Delta_{\perp}\sigma^z_{j,1}\sigma^z_{j+1,2}$$



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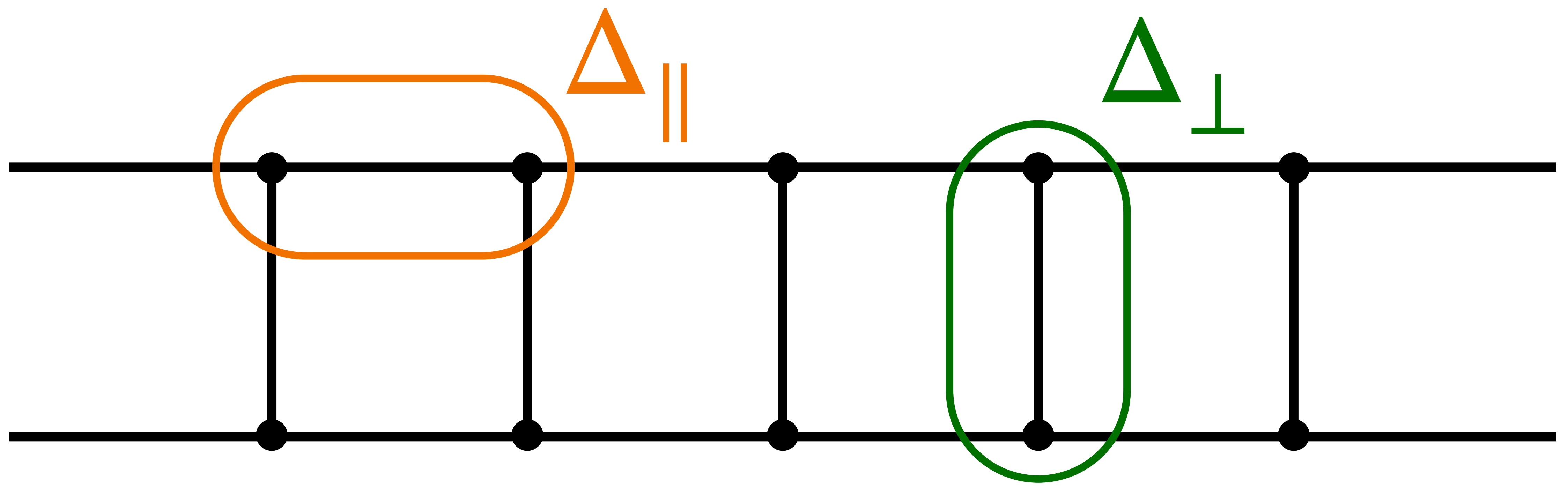
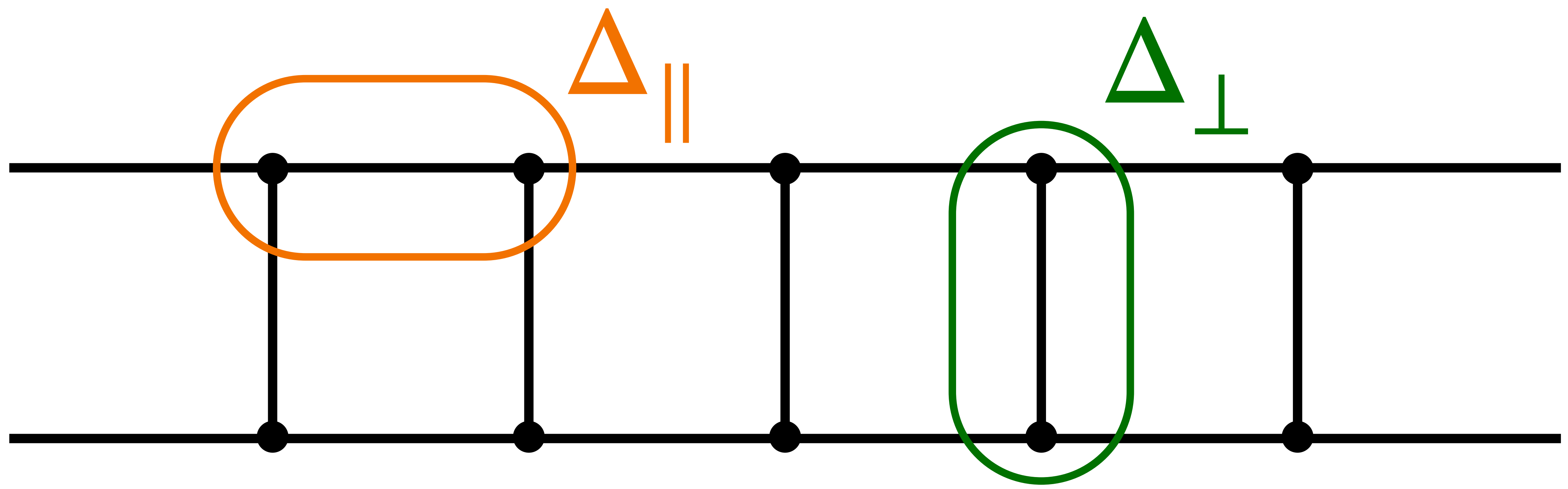



$$\Delta_{\perp} \leftrightarrow h_z$$

Interacting ↔ No Free Fermion Mapping

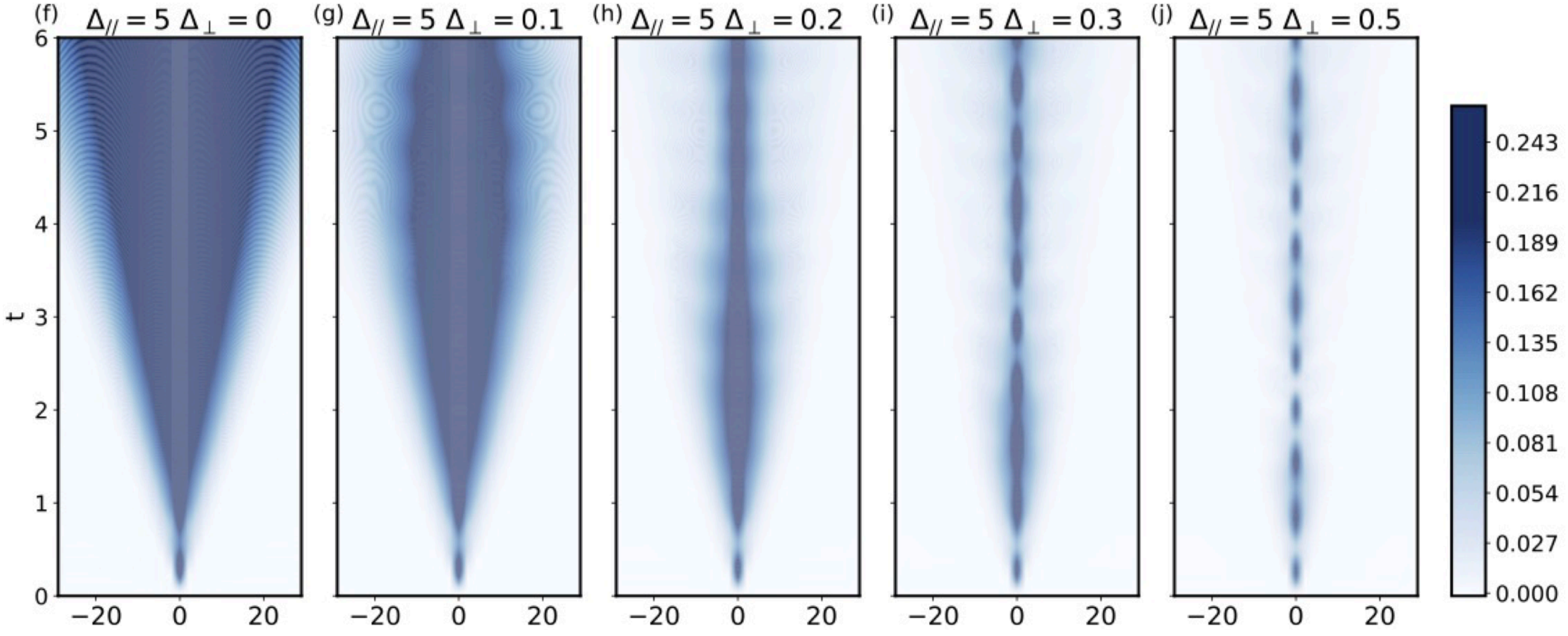
# XXZ Ladder

$$H = \frac{1}{2}\sum_j \sum_{\alpha=1,2} \left(\sigma^x_{j,\alpha}\sigma^x_{j+1,\alpha} + \sigma^y_{j,\alpha}\sigma^y_{j+1,\alpha} + \Delta_\parallel \sigma^z_{j,\alpha}\sigma^z_{j+1,\alpha}\right) + \Delta_\perp \sigma^z_{j,1}\sigma^z_{j+1,2}$$




Lagnese, Surace, Kormos, Calabrese J. Phys A 2022

Lagnese, Surace, Kormos, Calabrese J.  Stat. Mech. 2022



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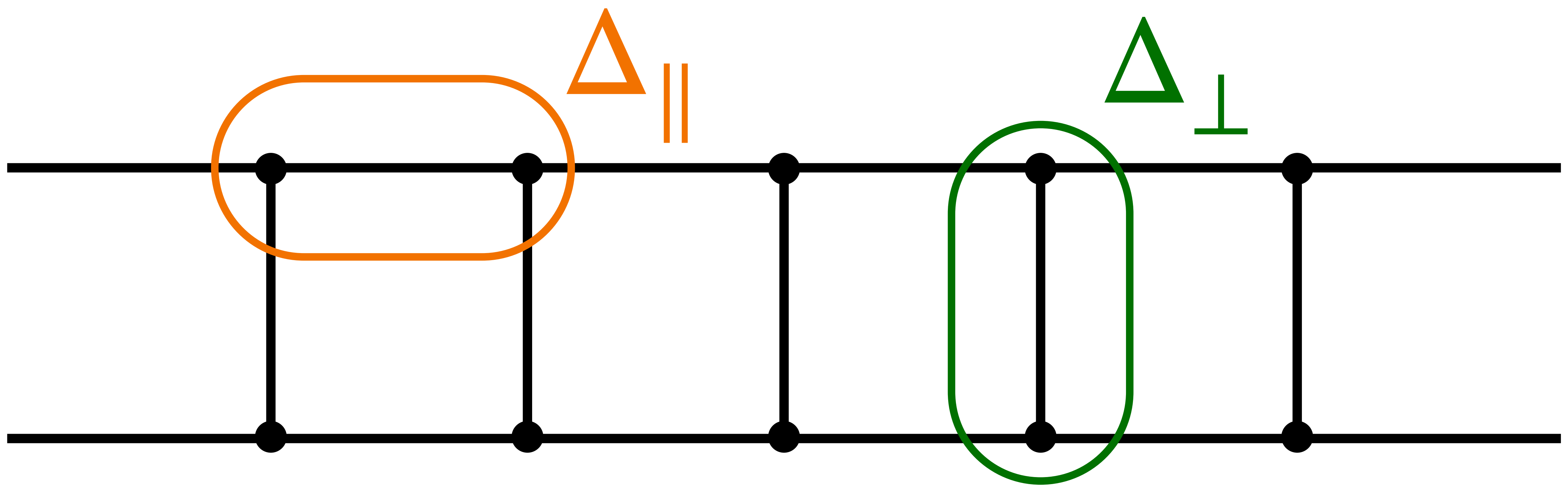


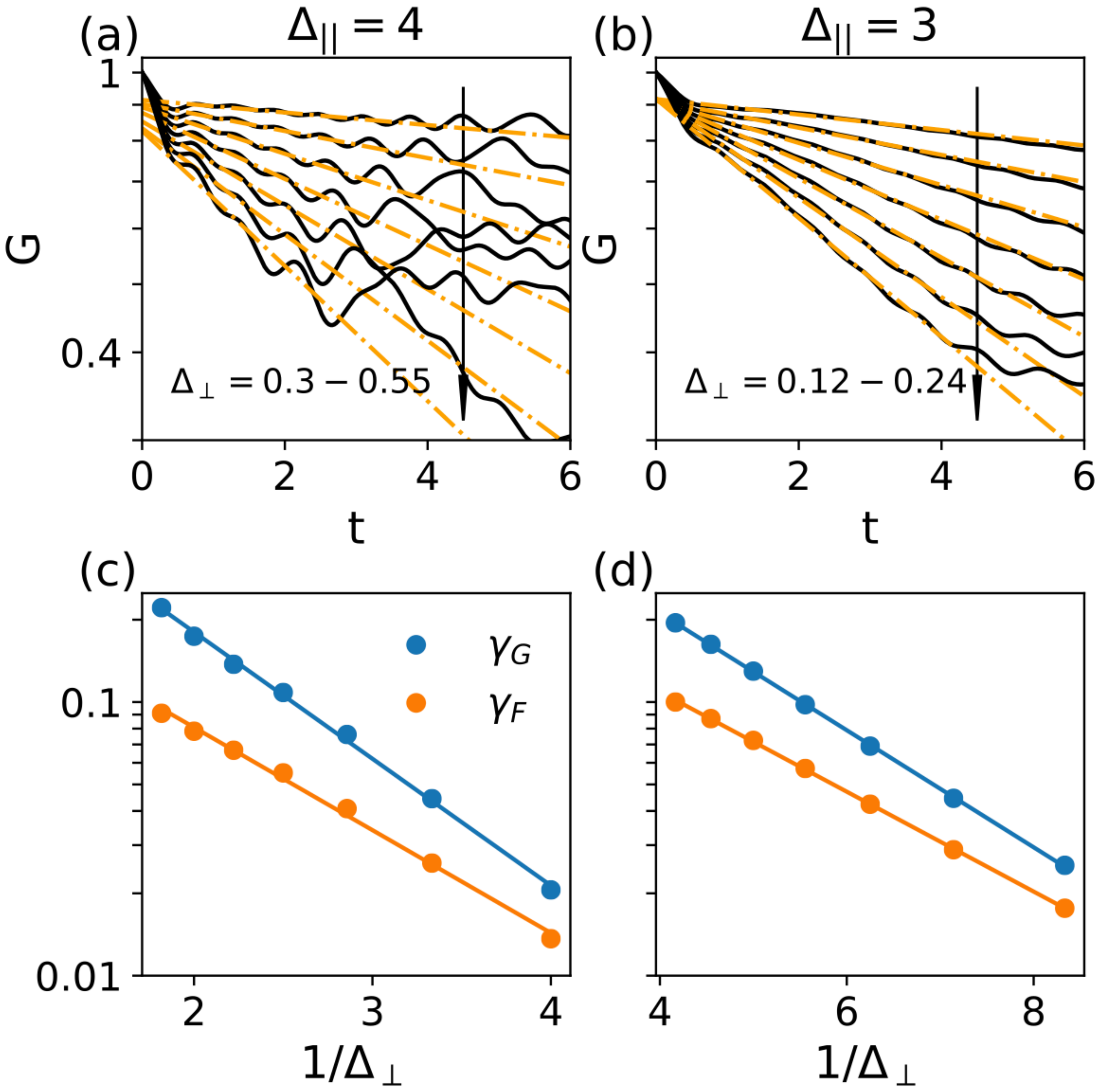

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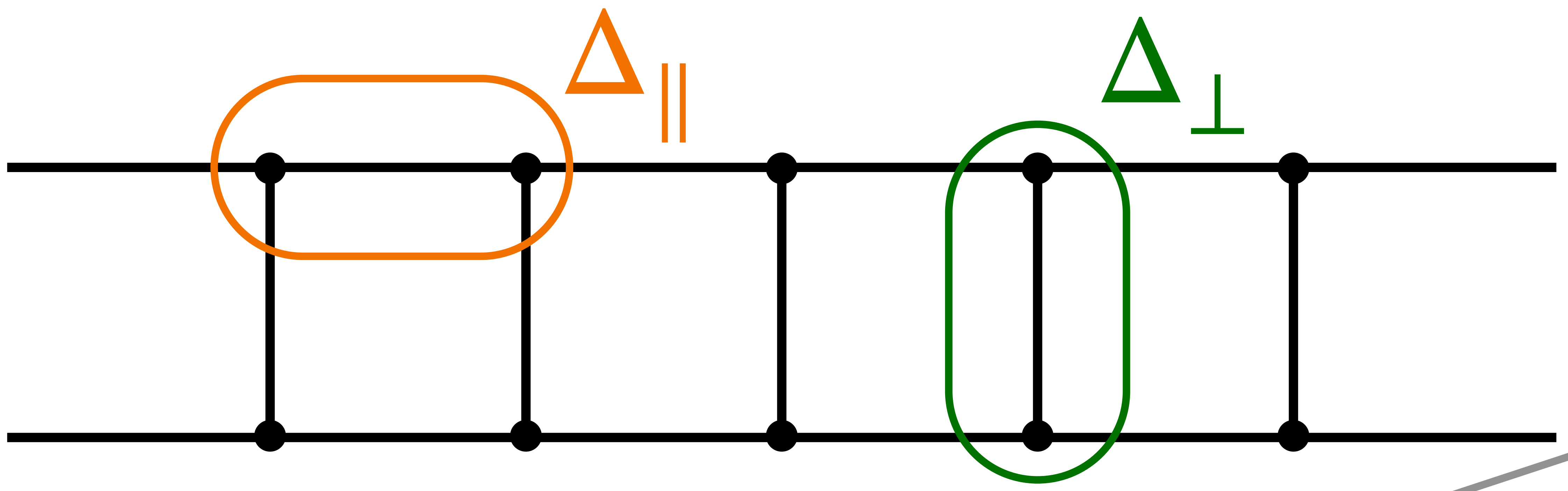
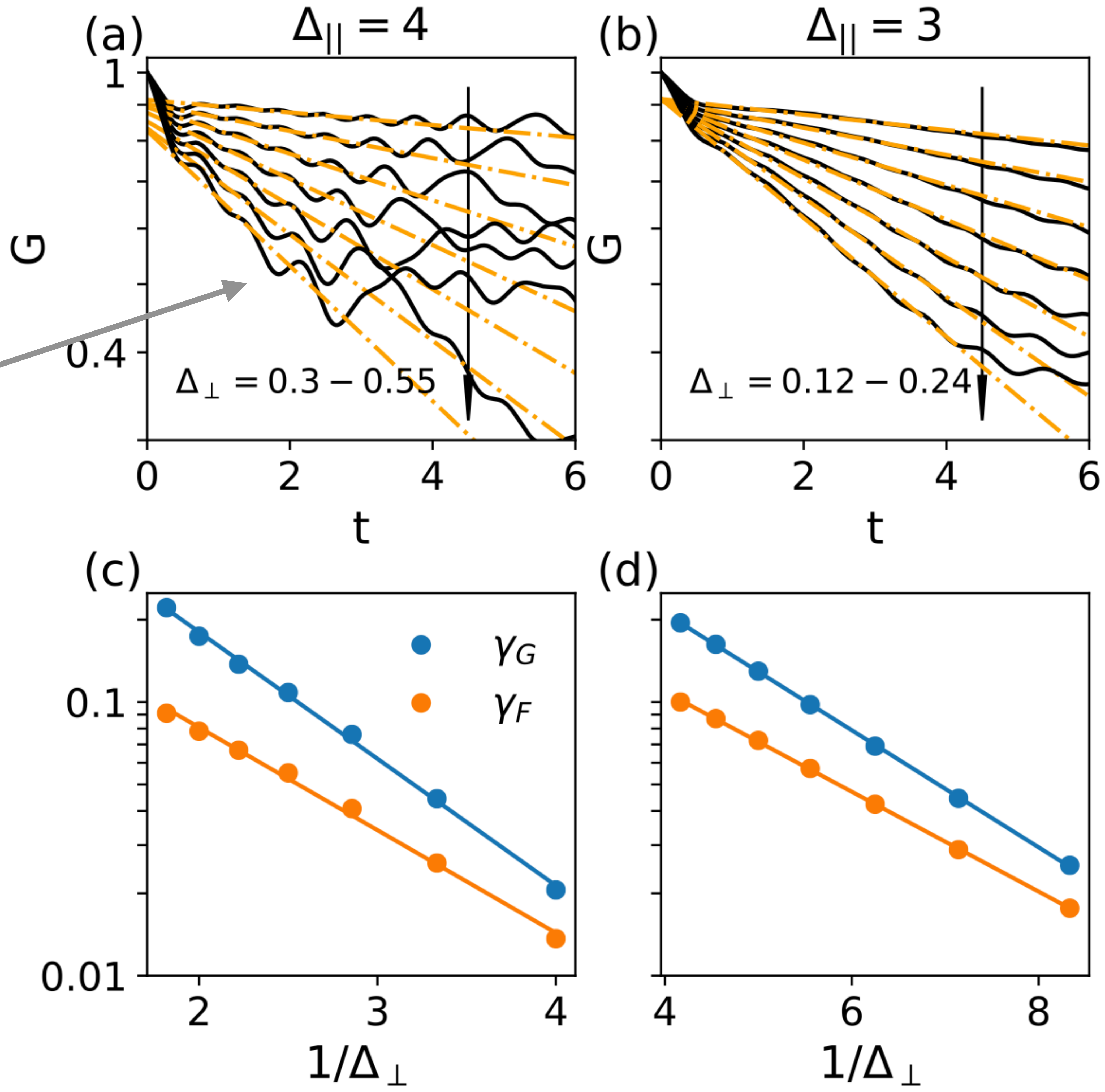



Extract decay rates



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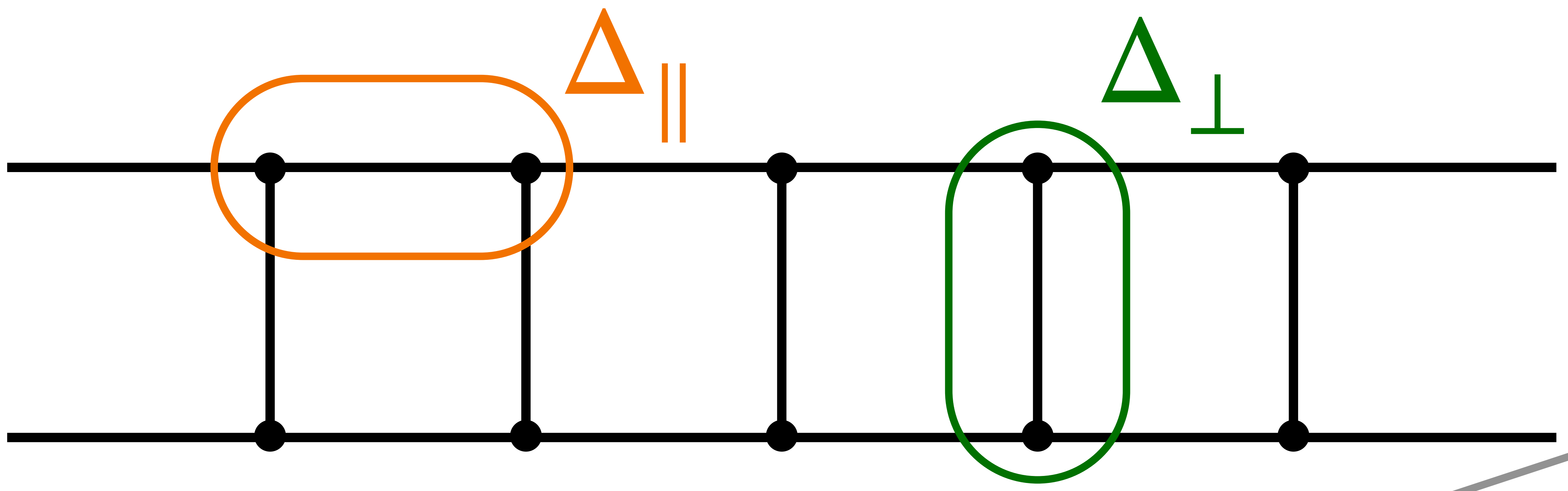
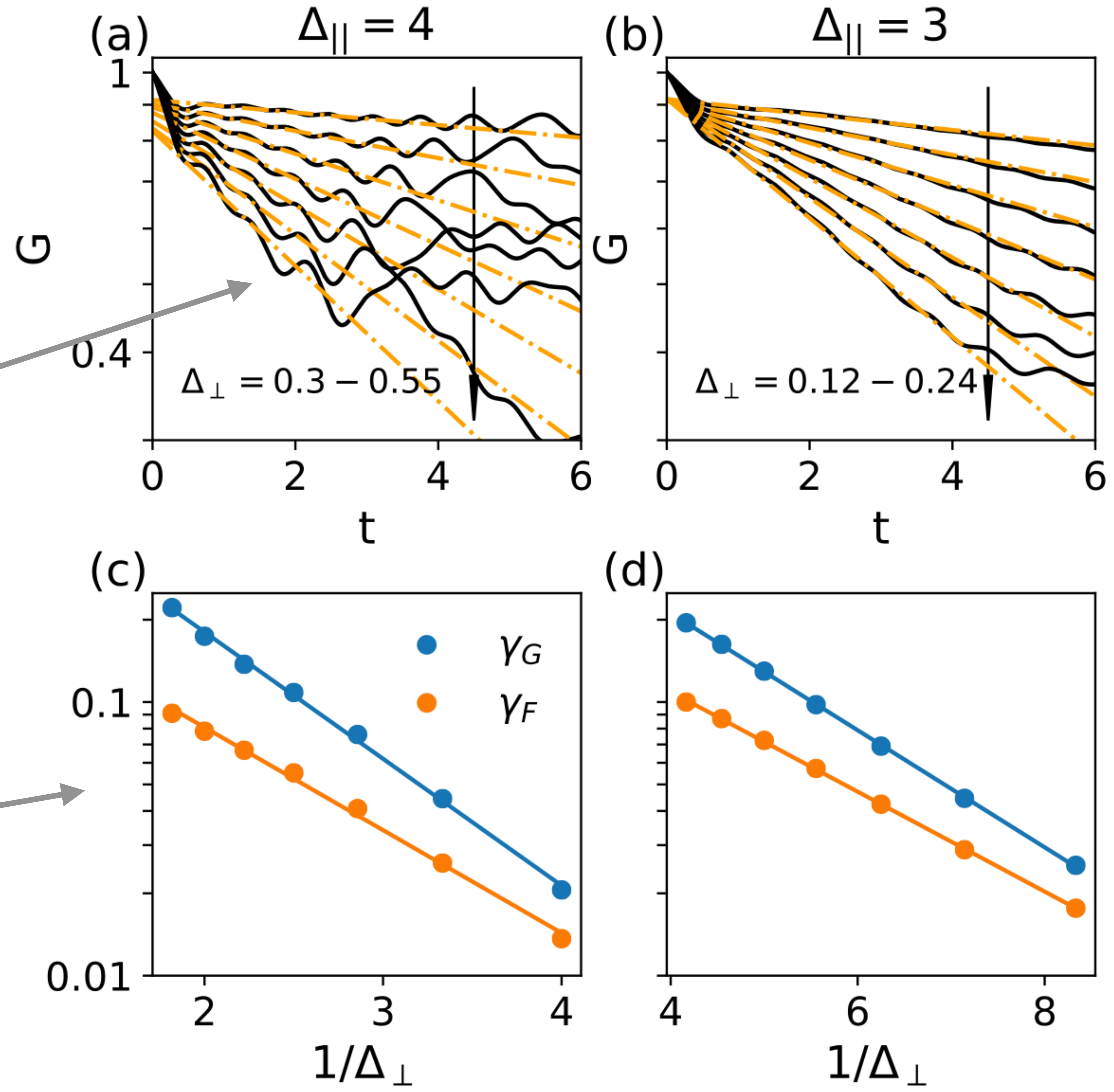


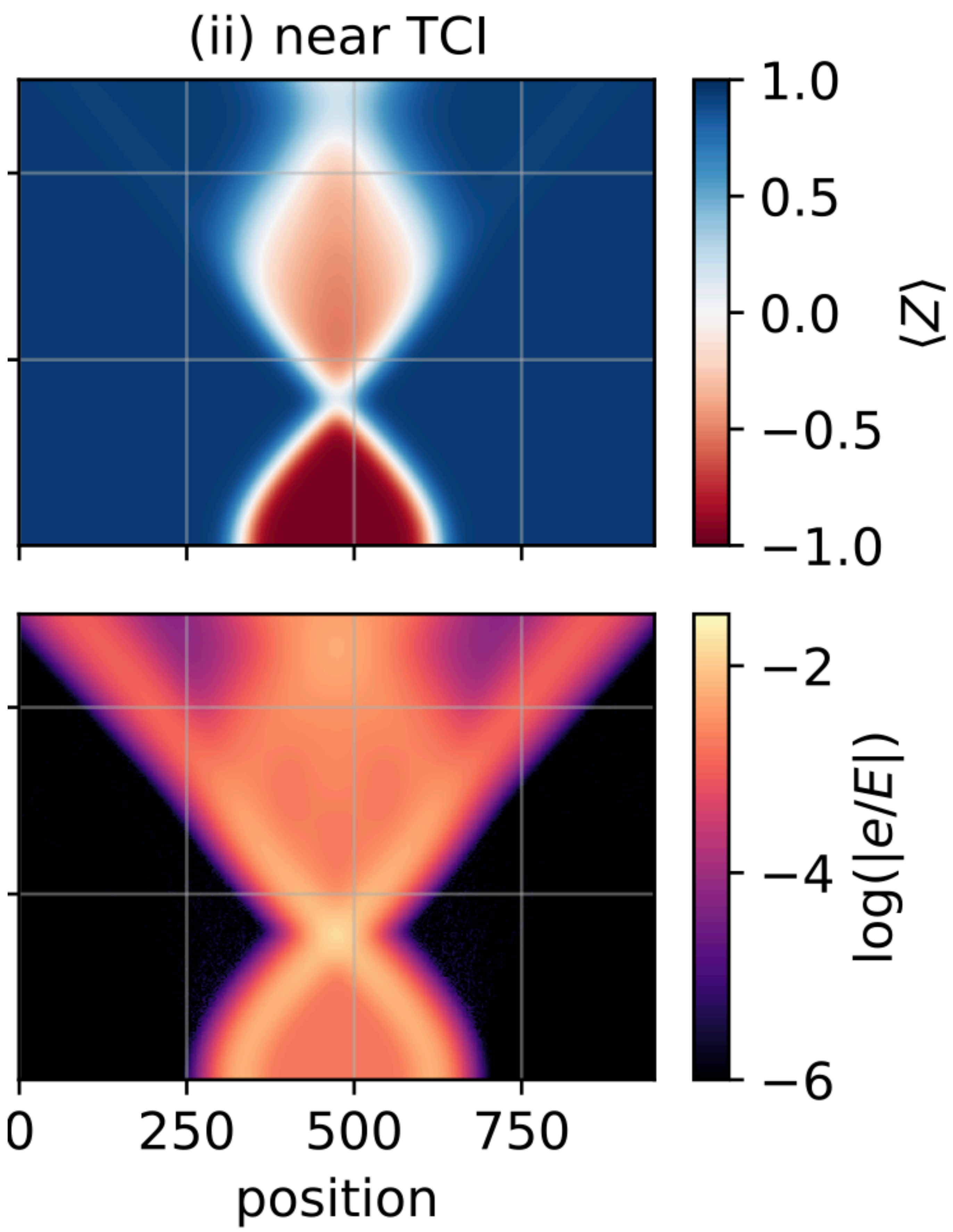

Extract decay rates

Test functional form

$$\gamma = A e^{-c/\Delta_{\perp}}$$

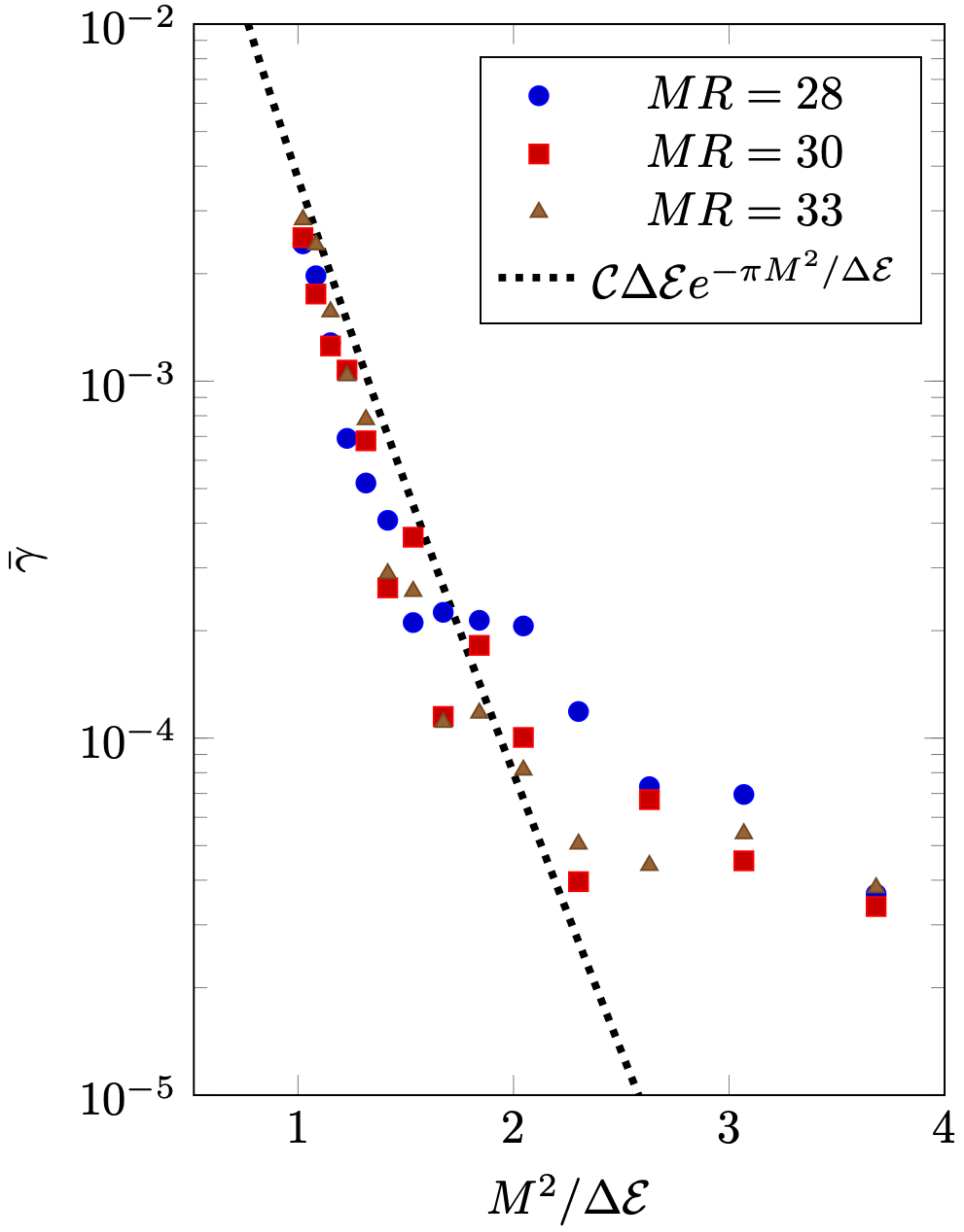
# Outlook

## More on Bubble Dynamics



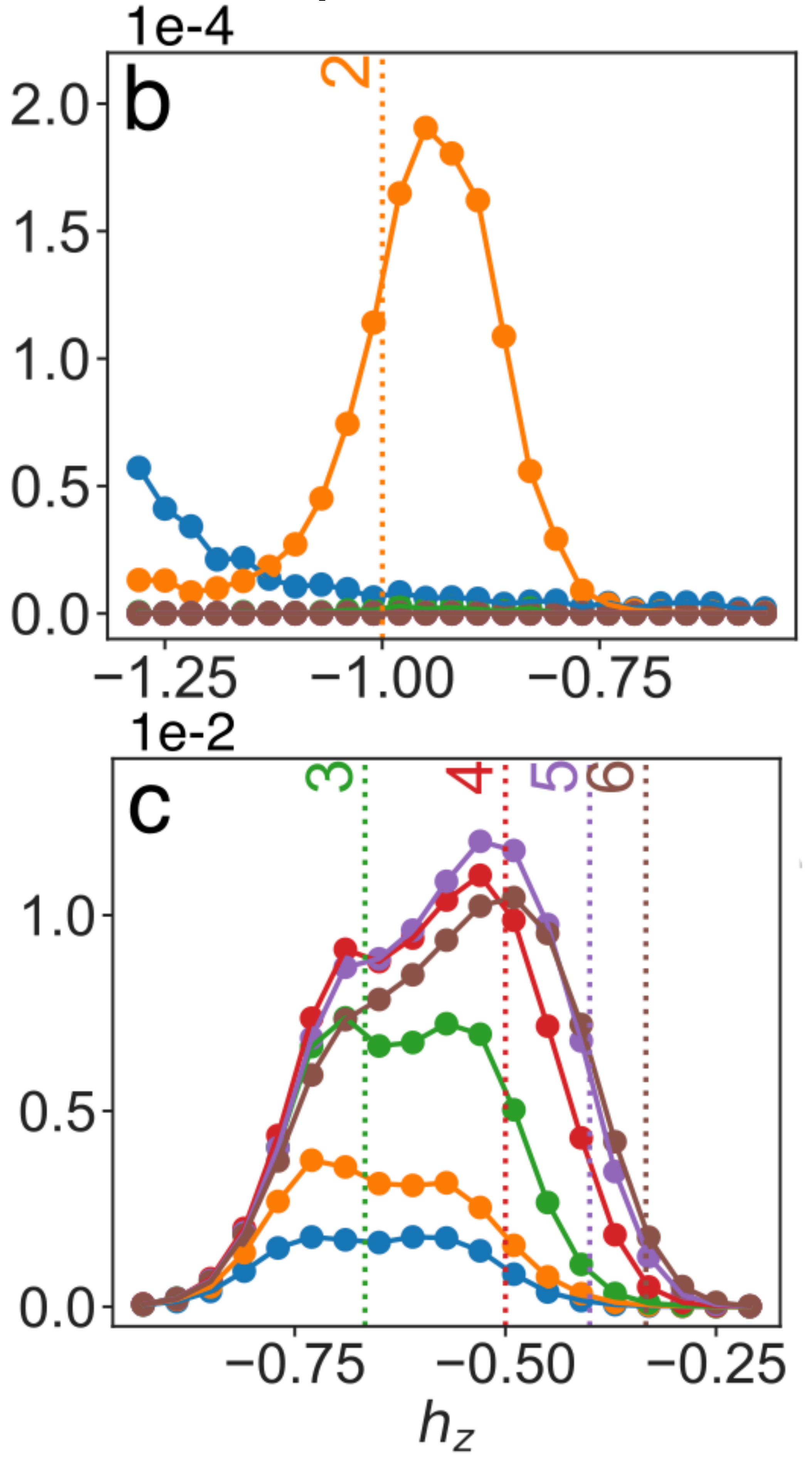

Milsted, Liu, Preskill, Vidal
PRX Quantum 2022

## Prefactor in the decay rate




Lencses, Mussardo, Tacaks PRD 2022

## Experiments

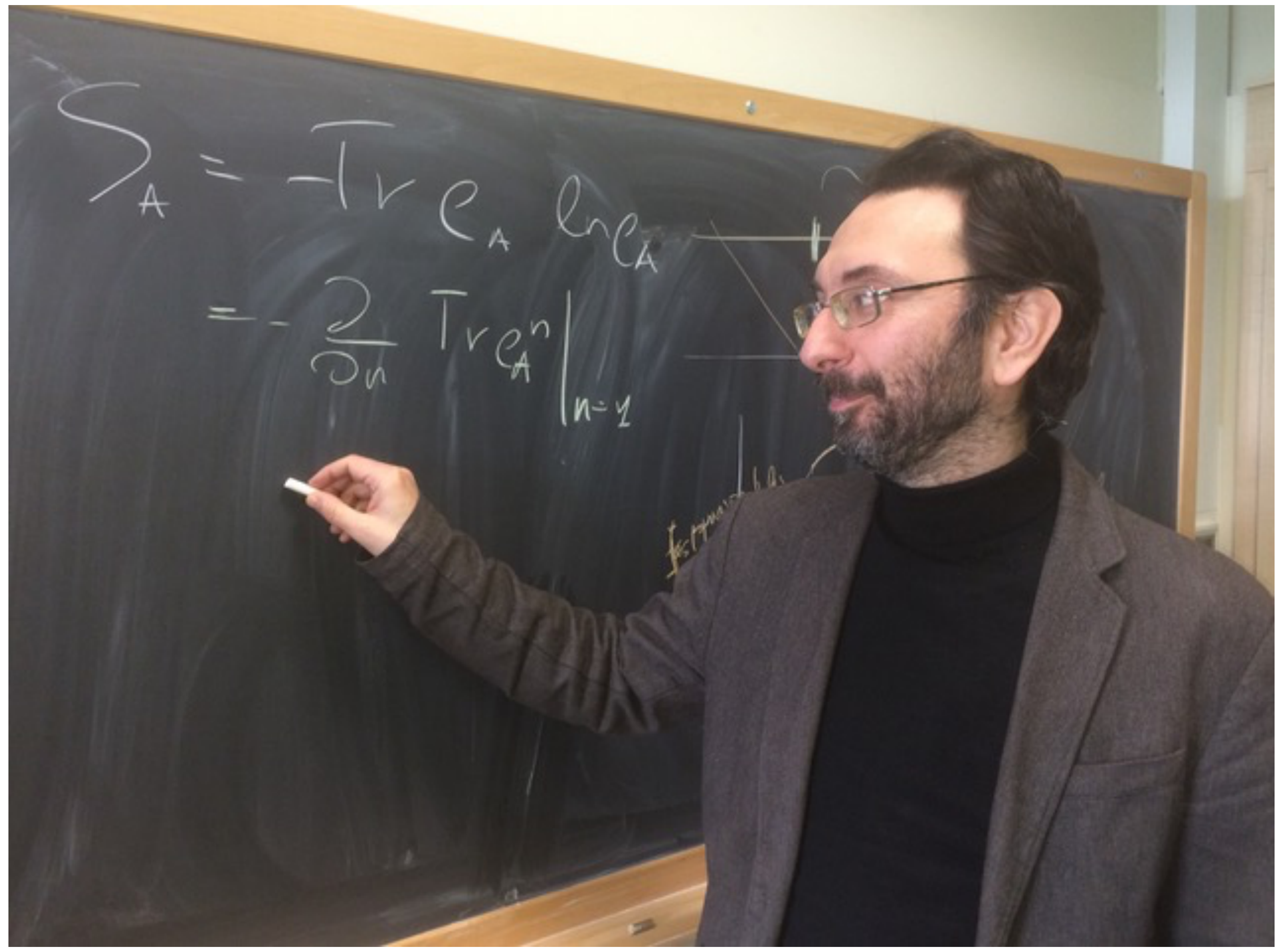



Jaka Vodeb et al. arXiv:2406.14718

# Thanks for the attention !

Federica M. Surace

Pasquale Calabrese




Marton Kormos