From double-slits to silly drawings

Feynman's *mathemagic*

Luis Roberto Flores Castillo

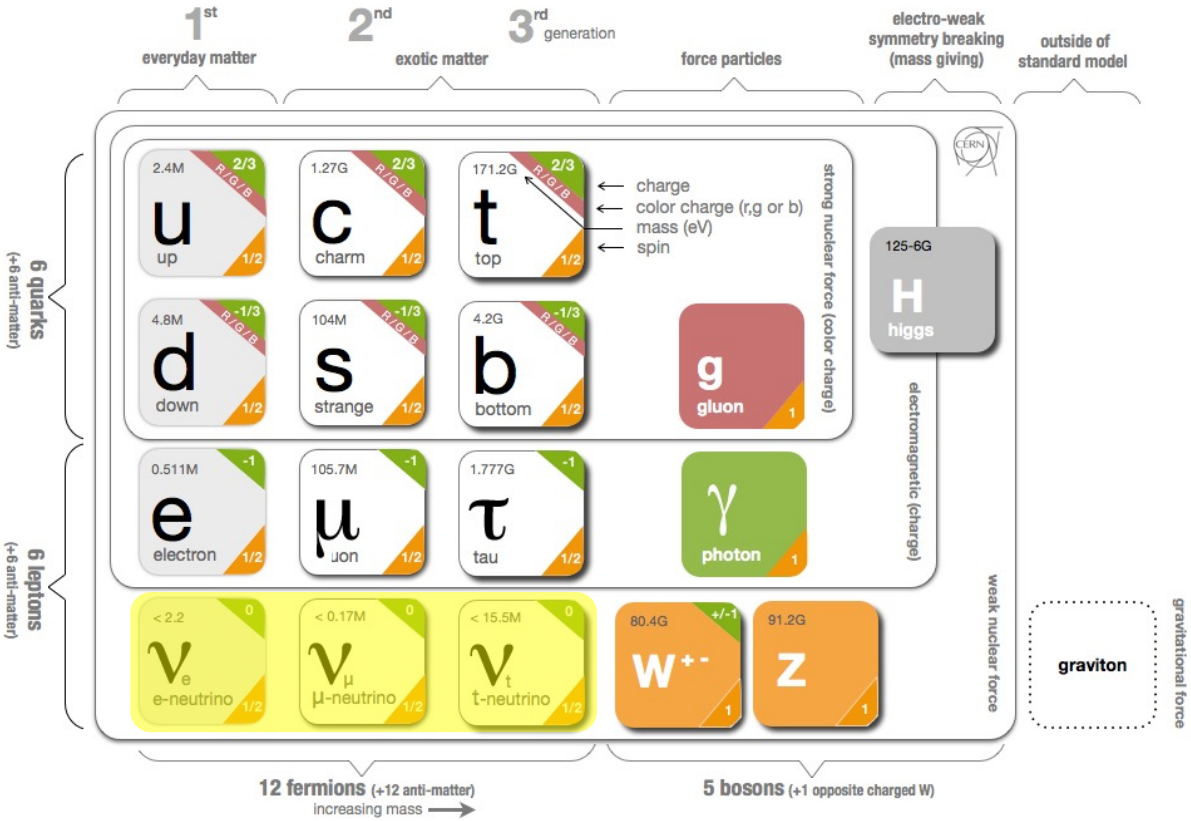
Chinese University of Hong Kong

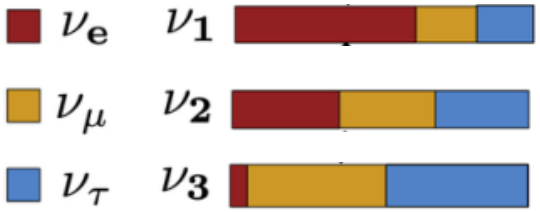
CERN International Teacher Weeks

August 16 2023

First: a few words about neutrinos

What we know





- They come in three *flavors* (one per lepton; all have been detected)
- As they travel, they *oscillate* among these three!!
- Only *left-handed* neutrinos have ever been observed

Detection

- Neutrino interactions:

  no electric charge

  no color

  almost no mass

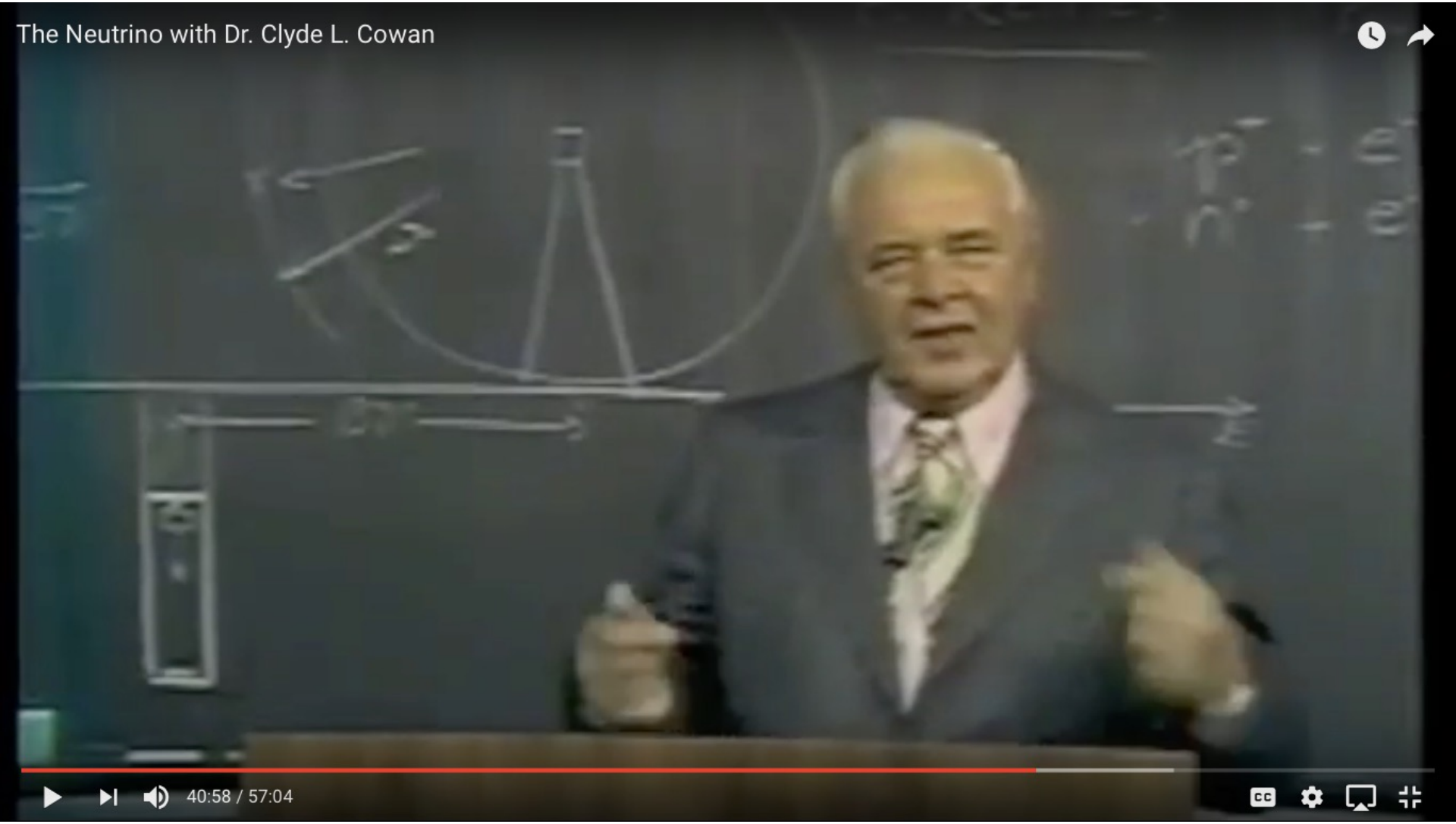
  they interact *only through the weak nuclear force*

- Detection:

  intense sources

  huge detectors

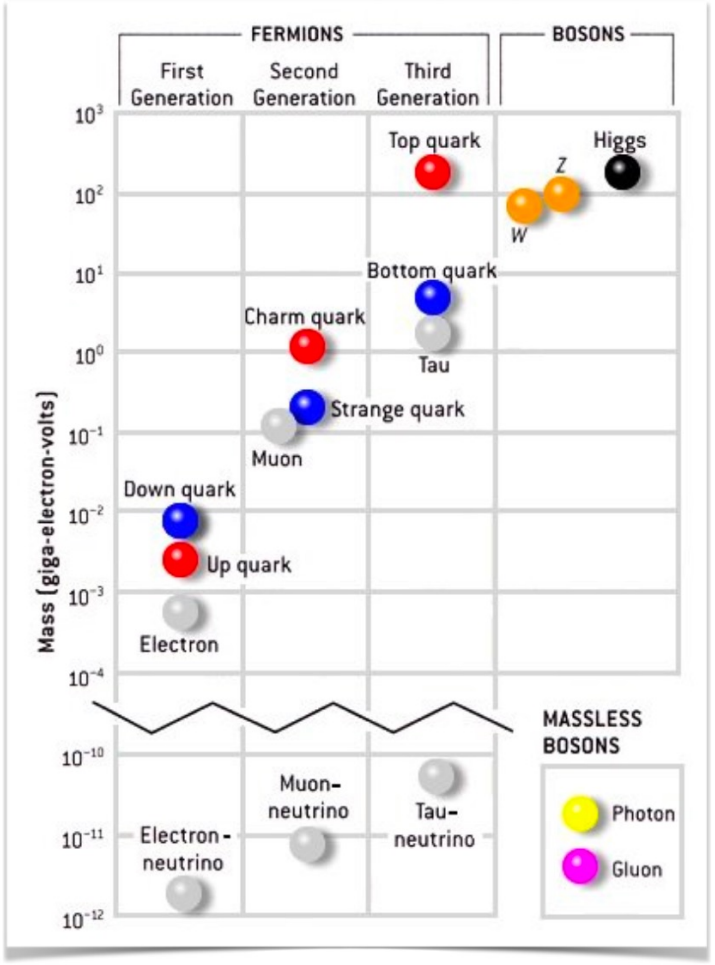
  a LOT of patience



https://www.youtube.com/watch?v=AYqEtm0X2Sc

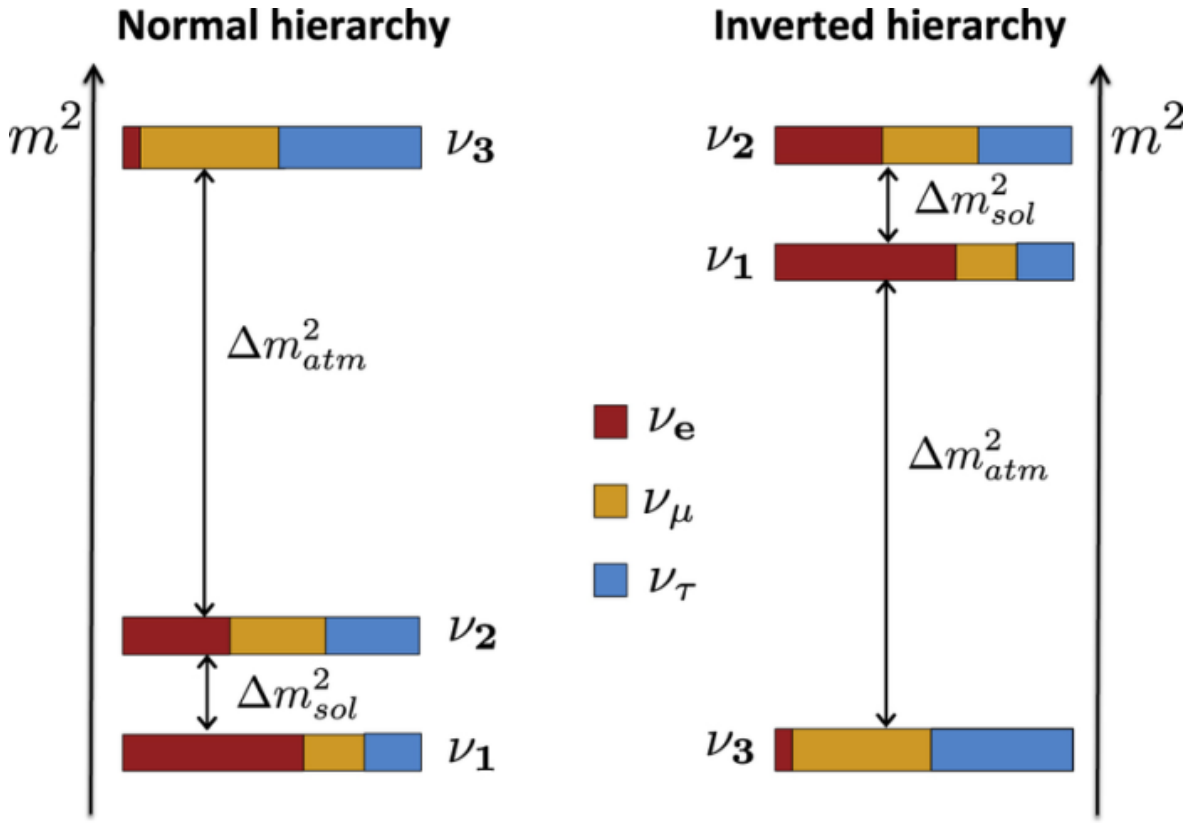
What we don't know

- Why are their masses so small?
- Why do they break the 3-generation pattern?
- Do they violate CP?
- Are there more than 3?
- Are they their own antiparticles?



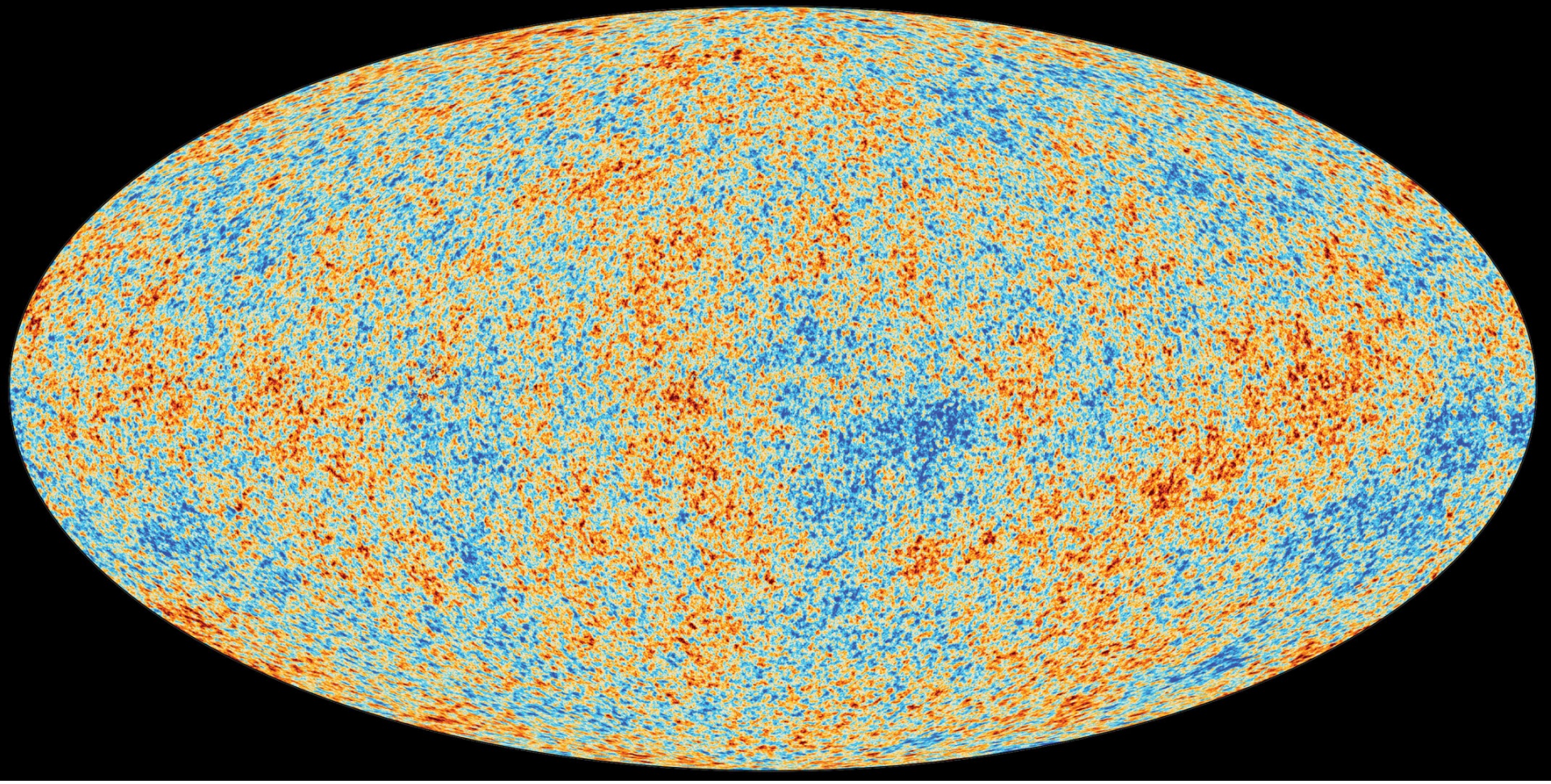
What we don't know

- Oscillations give us the $\Delta\left(m_{ij}^2\right)$ … but, how are they ordered?



What we don't know

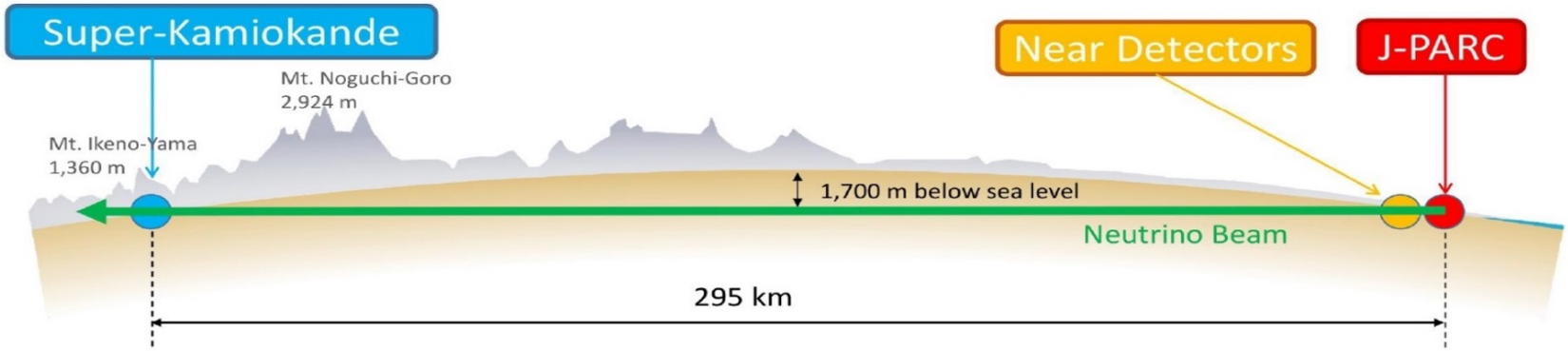
- Also, the way the Higgs boson gives mass to fermions is through mixing their left and right handed components.
- But right handed neutrinos have never been observed
- … so their masses may need a different mechanism!

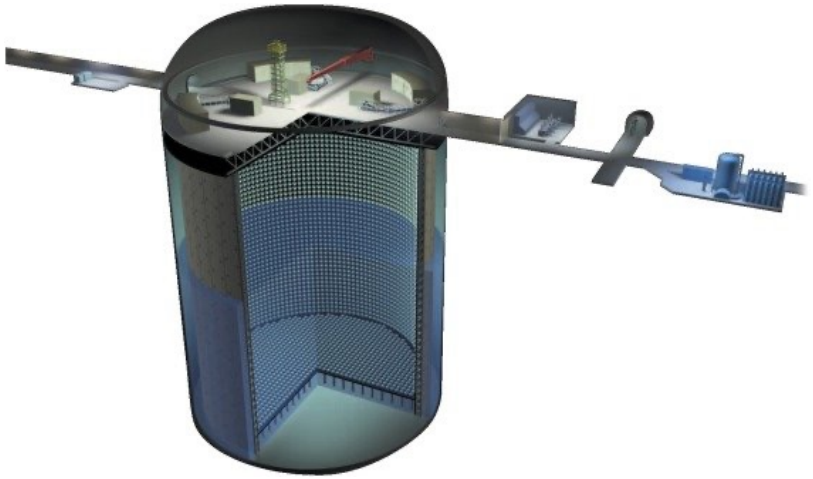


- Photons from about 380,000 years after the BB
- Instead, neutrinos decoupled from matter ~ 1 s after the BB!
- This Cosmic Neutrino Background (CNB), is yet to be observed

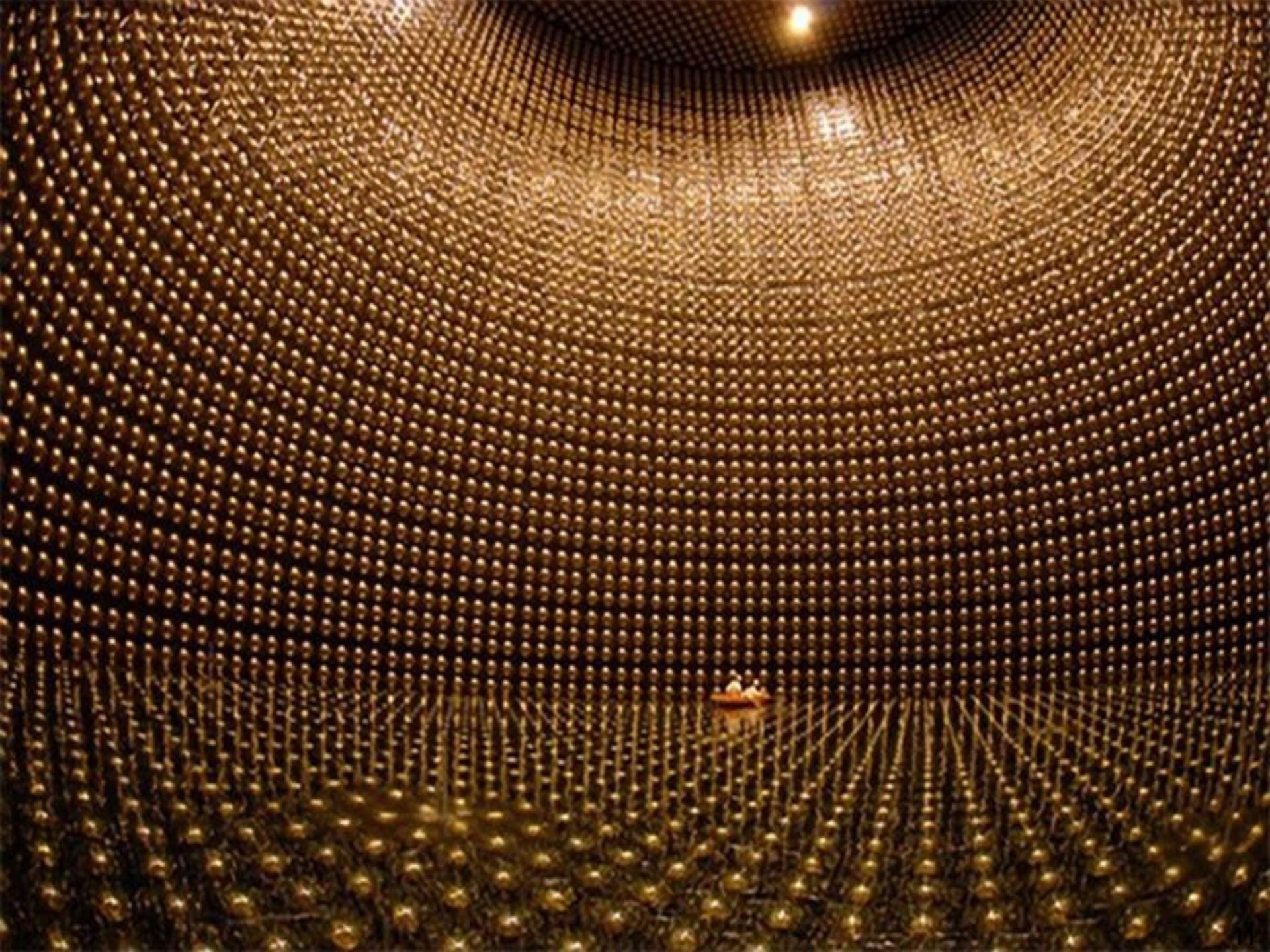
T2K - Tokai to Kamioka

- Measure oscillations: $\bar{\nu}_\mu/\nu_\mu$ disappearance and $\bar{\nu}_e/\nu_e$ appearance.
- Measure the oscillation parameters $\theta_{13}, \theta_{23}, \delta_{CP}$ and Δm^2_{32}

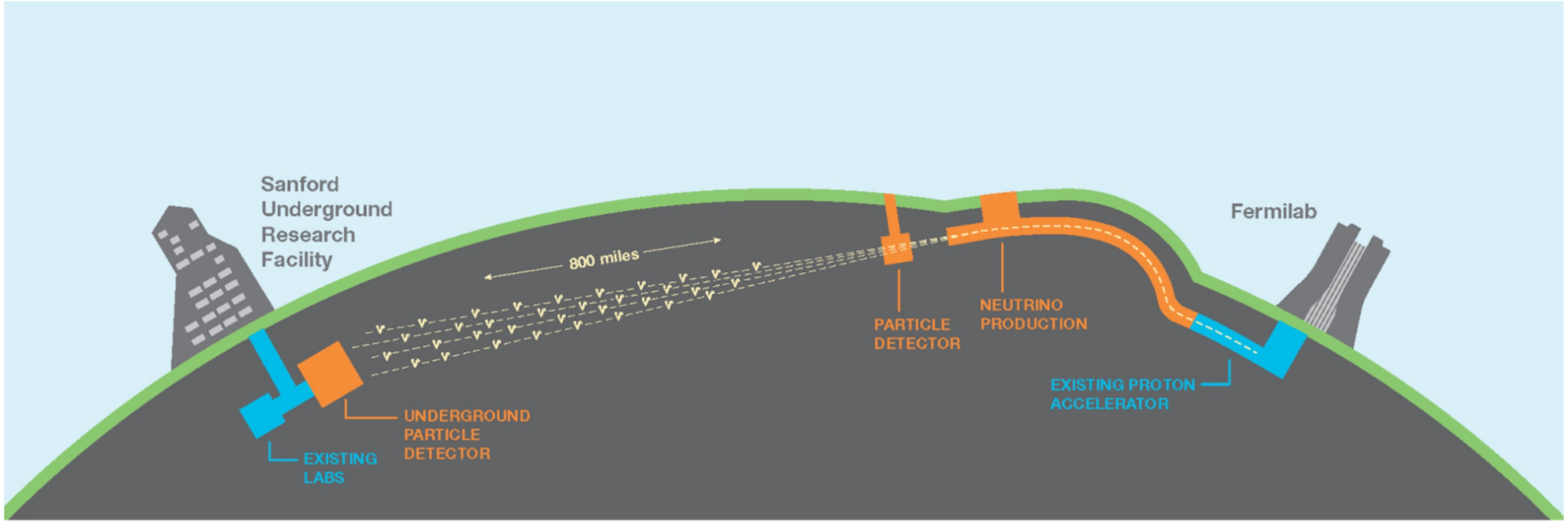




Water Cherenkov detector
- 50 kton of ultra-pure water
- 40 m diameter, ~50 m deep
- ~11,000 20 inch PMTs
- 1 km underground
- Operational since 1996

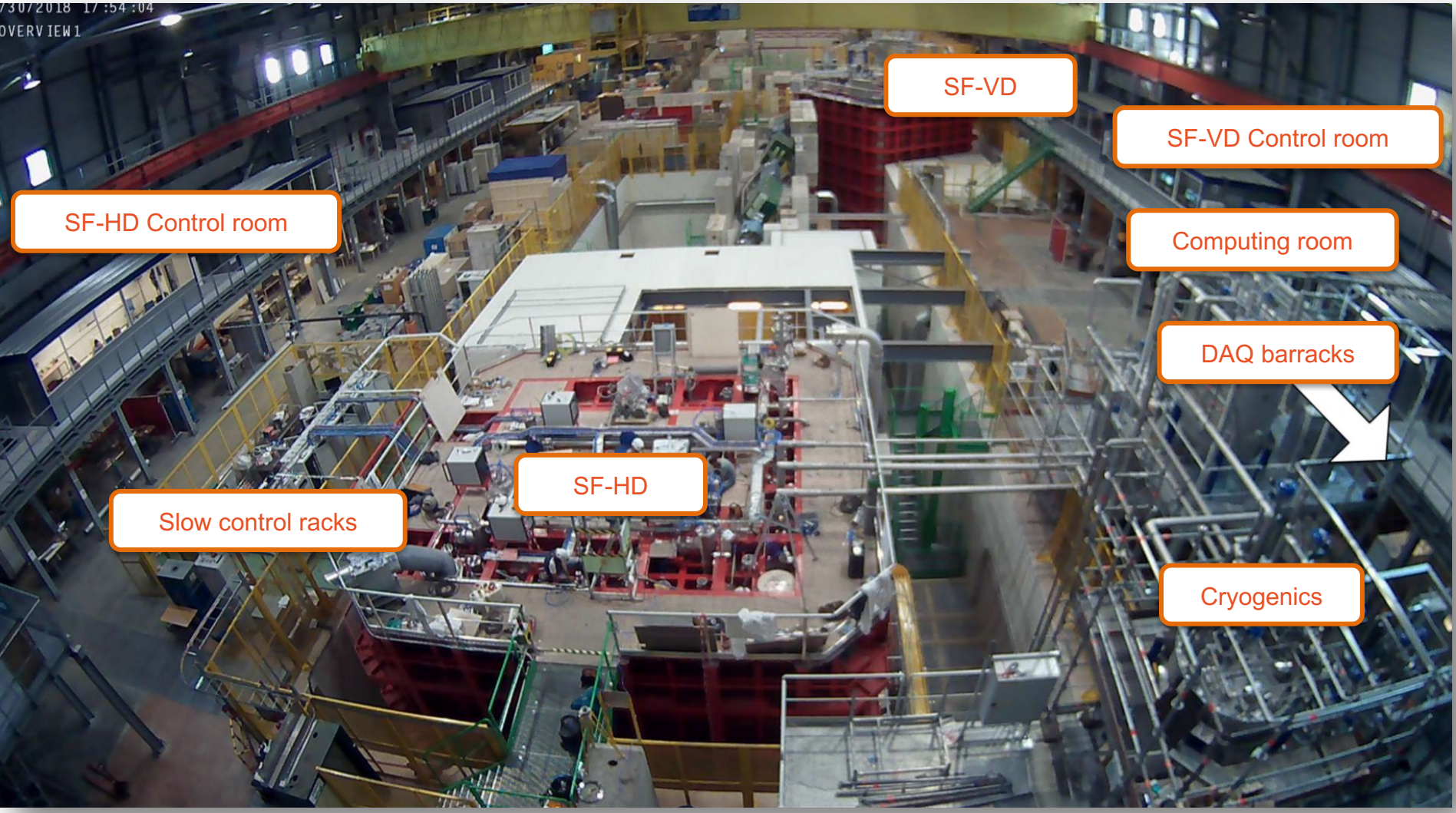


DUNE (Deep Inderground Neutrino Experiment)



- $\bar{\nu}_\mu / \nu_\mu$ beams generated by a proton accelerator at Fermilab, propagate for 1300 km, detected at SURF 1500m underground
- Far detector: four Liquid Argon detectors (TPC's) of 17k Tons each

CERN Neutrino Platform



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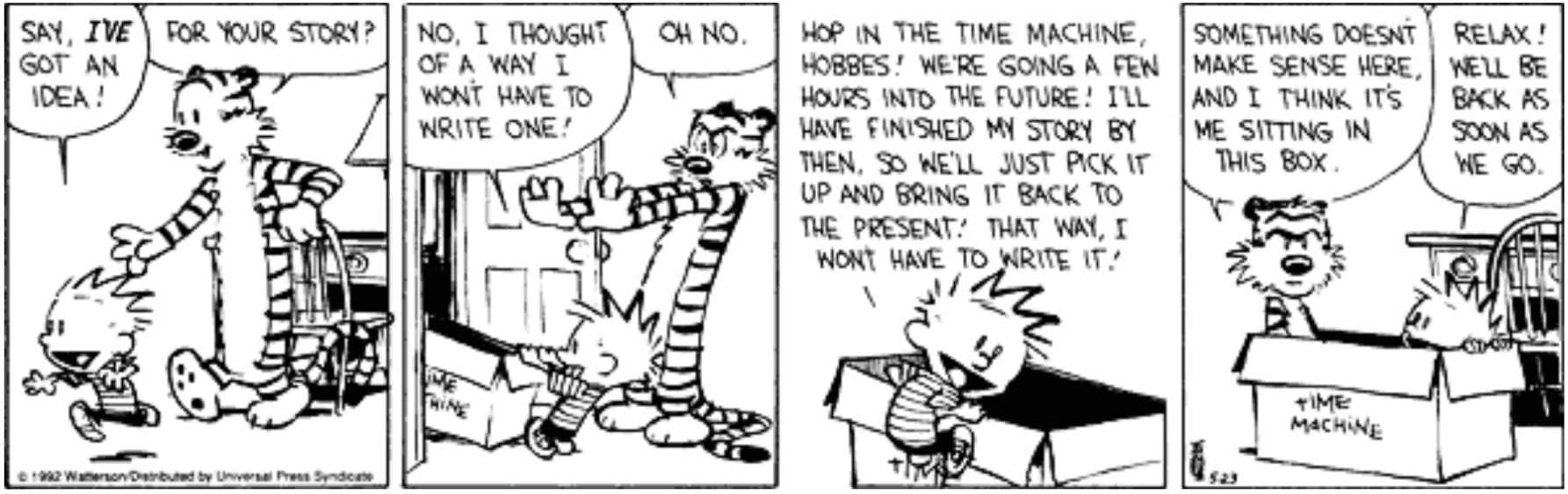
What are *Feynman diagrams*?

Preliminaries …


SAY, I'VE GOT AN IDEA!
FOR YOUR STORY?
© 1992 Watterson/Distributed by Universal Press Syndicate
NO, I THOUGHT OF A WAY I WON'T HAVE TO WRITE ONE!
OH NO.
HOP IN THE TIME MACHINE, HOBBES! WE'RE GOING A FEW HOURS INTO THE FUTURE! I'LL HAVE FINISHED MY STORY BY THEN, SO WE'LL JUST PICK IT UP AND BRING IT BACK TO THE PRESENT! THAT WAY, I WON'T HAVE TO WRITE IT!
SOMETHING DOESN'T MAKE SENSE HERE, AND I THINK IT'S ME SITTING IN THIS BOX.
RELAX! WE'LL BE BACK AS SOON AS WE GO.
TIME MACHINE
5-23

$z \leftarrow z^2 + c$

c: a fixed number,

Always starting with $z = 0$

$c = -1$

$z_0 = 0$

$z_1 = 0^2 + (-1) = -1$

$z_2 = (-1)^2 + (-1) = 0$

$z_3 = 0^2 + (-1) = -1$

$z_4 = (-1)^2 + (-1) = 0$

.
.
.

BLACK

BOUND

$c = 1$

$z_0 = 0$

$z_1 = 0^2 + (1) = 1$

$z_2 = 1^2 + (1) = 2$

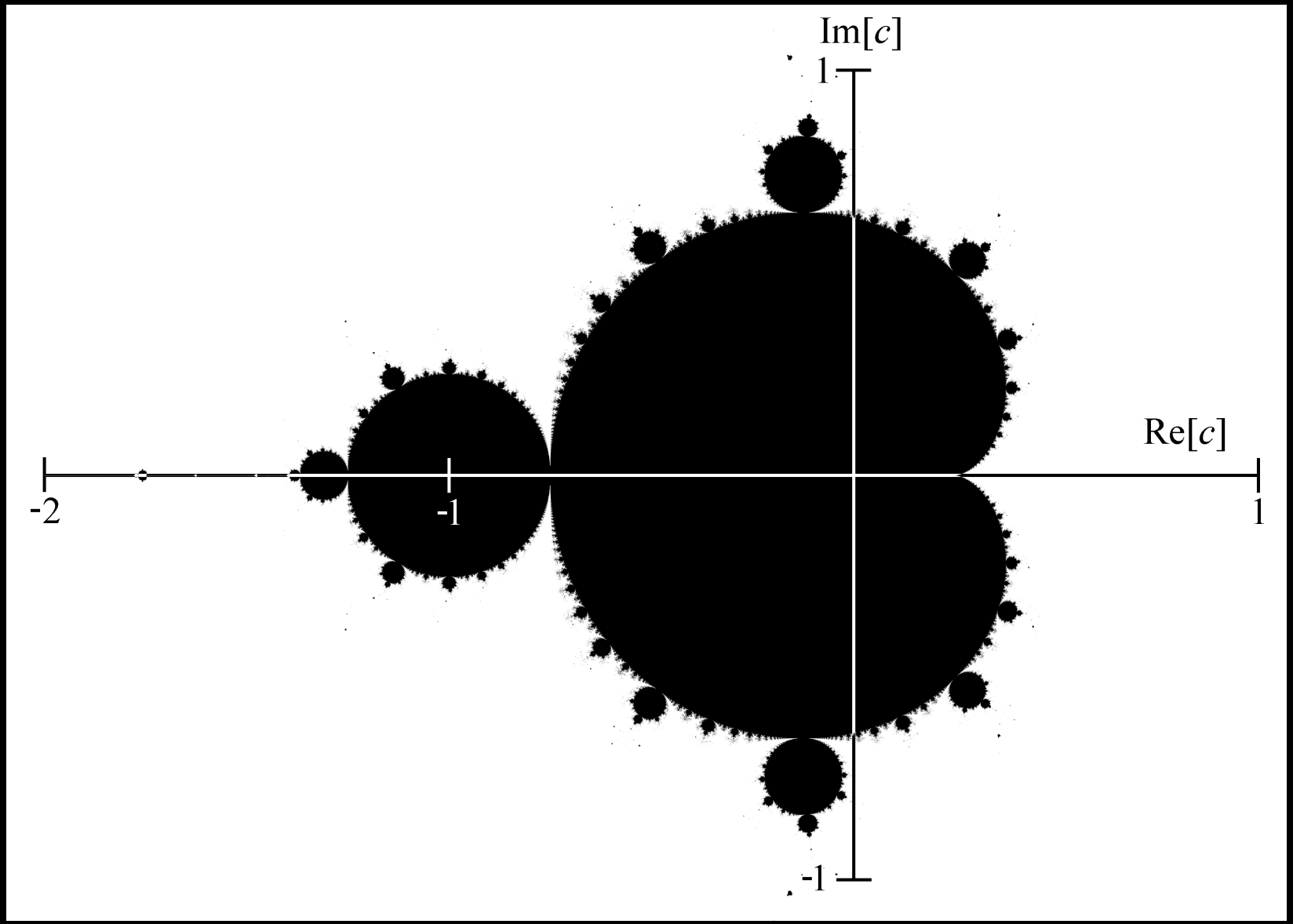
$z_3 = 2^2 + (1) = 5$

$z_4 = 5^2 + (1) = 26$

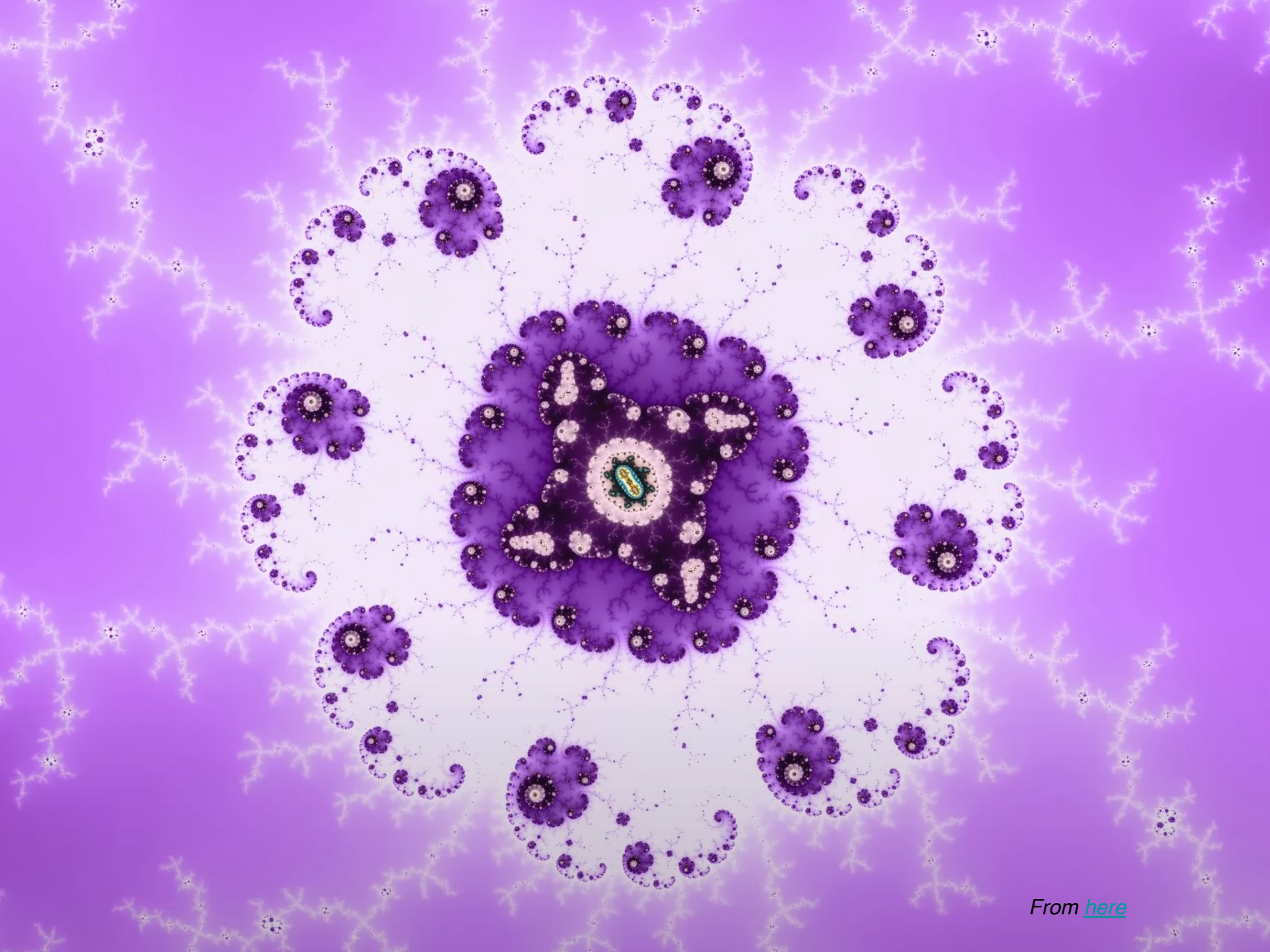
.
.
.

COLORS
(according to divergence speed)

DIVERGENT



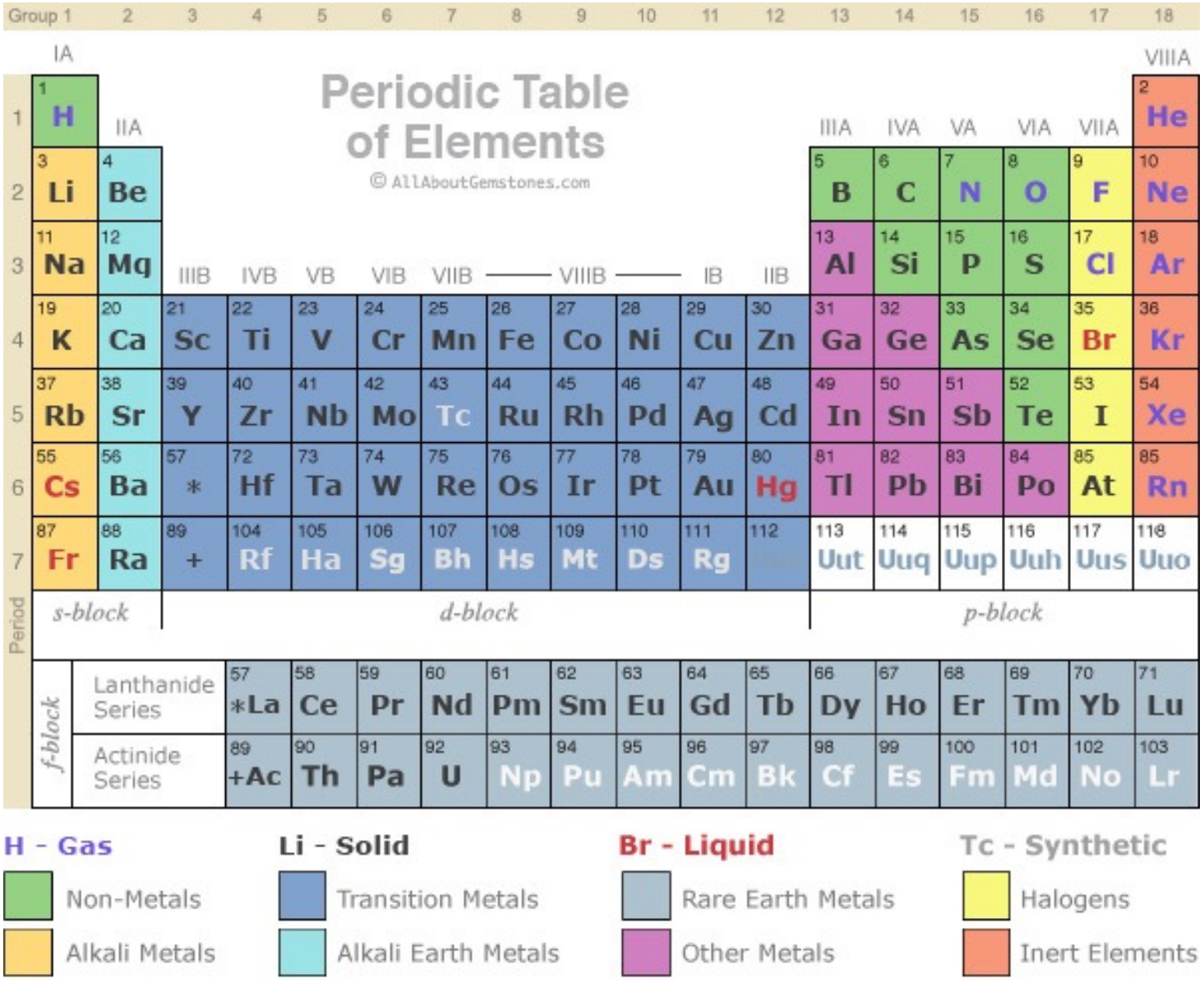
https://www.youtube.com/watch?v=pCpLWbHVNhk

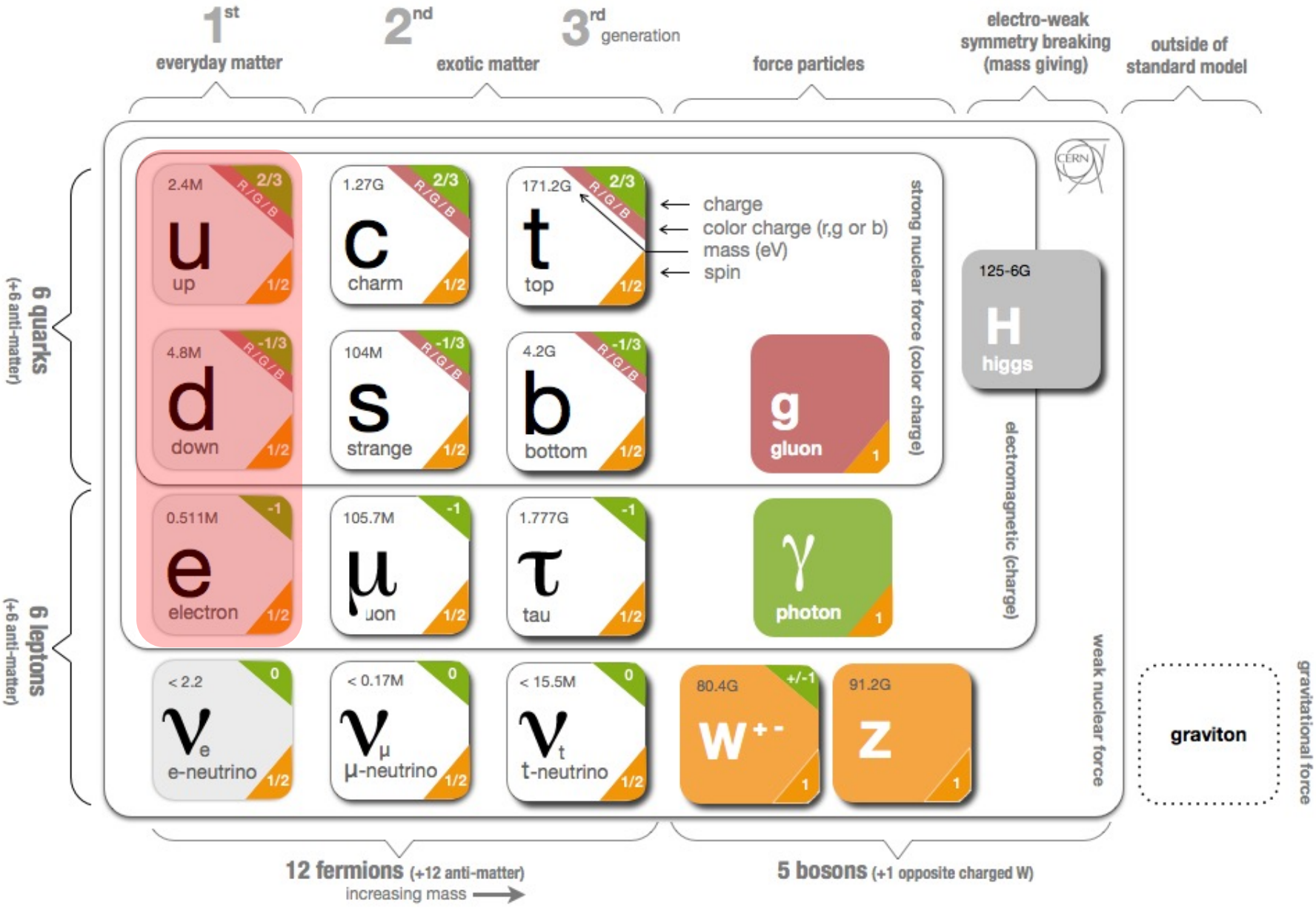


From here

A proof …

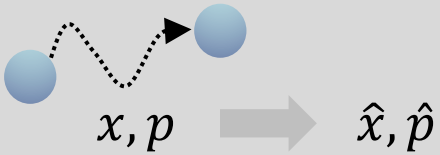
“Simplifying” our representations

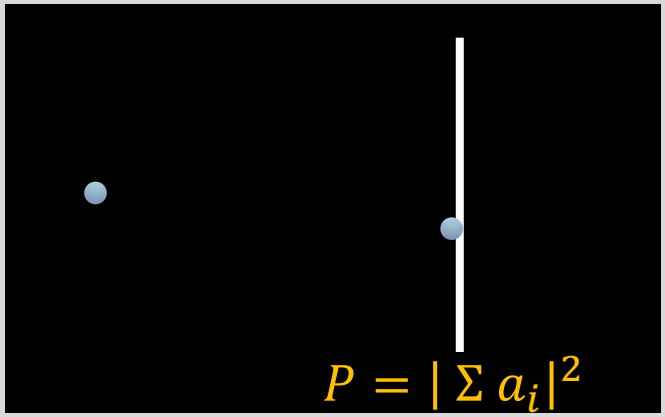

Periodic Table of Elements
© AllAboutGemstones.com
Group 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
IA IIA IIIB IVB VB VIB VIIB VIIIB IB IIB IIIA IVA VA VIA VIIA VIIIA
Period
1 H He
2 Li Be B C N O F Ne
3 Na Mg Al Si P S Cl Ar
4 K Ca Sc Ti V Cr Mn Fe Co Ni Cu Zn Ga Ge As Se Br Kr
5 Rb Sr Y Zr Nb Mo Tc Ru Rh Pd Ag Cd In Sn Sb Te I Xe
6 Cs Ba * Hf Ta W Re Os Ir Pt Au Hg Tl Pb Bi Po At Rn
7 Fr Ra + Rf Ha Sg Bh Hs Mt Ds Rg Uut Uuq Uup Uuh Uus Uuo
s-block
d-block
p-block
f-block
Lanthanide Series *La Ce Pr Nd Pm Sm Eu Gd Tb Dy Ho Er Tm Yb Lu
Actinide Series +Ac Th Pa U Np Pu Am Cm Bk Cf Es Fm Md No Lr
H - Gas
Li - Solid
Br - Liquid
Tc - Synthetic
Non-Metals
Alkali Metals
Transition Metals
Alkali Earth Metals
Rare Earth Metals
Other Metals
Halogens
Inert Elements


1st
everyday matter
2nd
3rd generation
exotic matter
force particles
electro-weak
symmetry breaking
(mass giving)
outside of
standard model
6 quarks
(+6 anti-matter)
6 leptons
(+6 anti-matter)
2.4M
2/3
R/G/B
u
up
1/2
1.27G
c
charm
171.2G
t
top
charge
color charge (r,g or b)
mass (eV)
spin
4.8M
-1/3
d
down
104M
s
strange
4.2G
b
bottom
g
gluon
1
125-6G
H
higgs
strong nuclear force (color charge)
electromagnetic (charge)
0.511M
-1
e
electron
105.7M
μ
uon
1.777G
τ
tau
γ
photon
< 2.2
0
νe
e-neutrino
< 0.17M
νμ
μ-neutrino
< 15.5M
νt
t-neutrino
80.4G
+/-1
W+-
91.2G
Z
weak nuclear force
graviton
gravitational force
12 fermions (+12 anti-matter)
increasing mass
5 bosons (+1 opposite charged W)

… and then what?

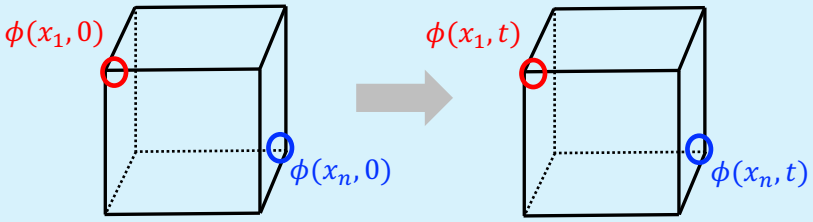
CLASSICAL MECHANICS:





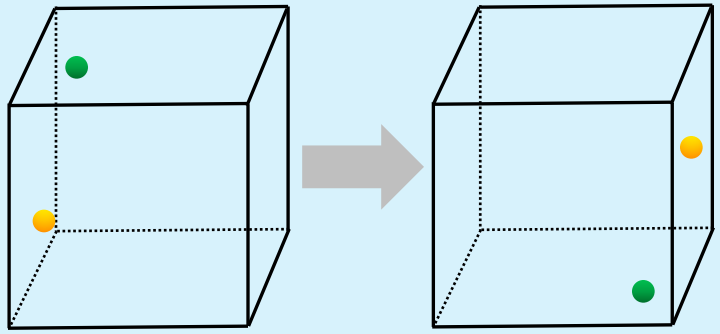
AT HIGH ENERGIES (SHORT DISTANCES):

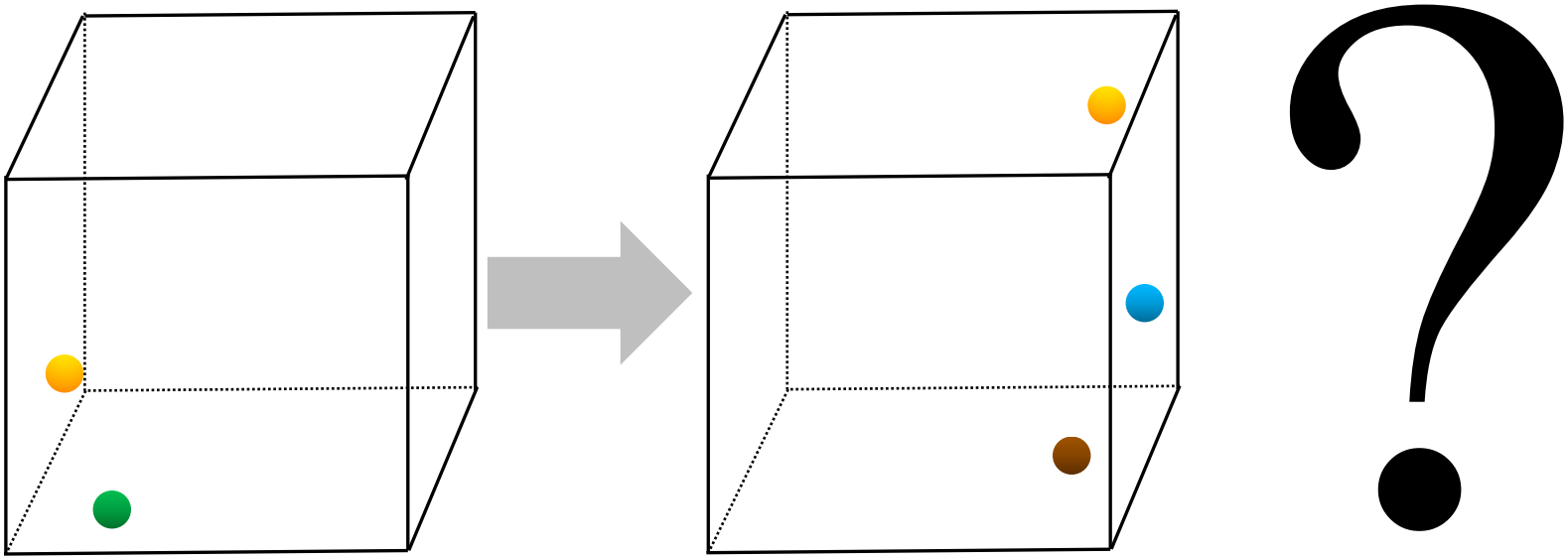
- We cannot ignore particle production



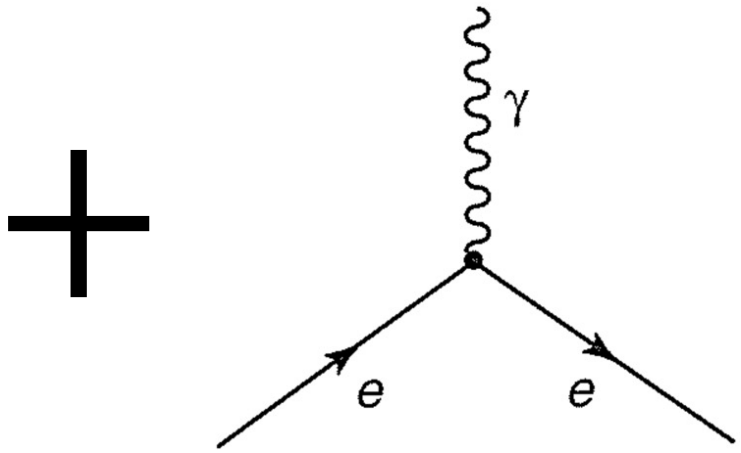
Each $\phi(x) \rightarrow \hat{\phi}(x)$

Excitations at each point: ***quanta of that field***

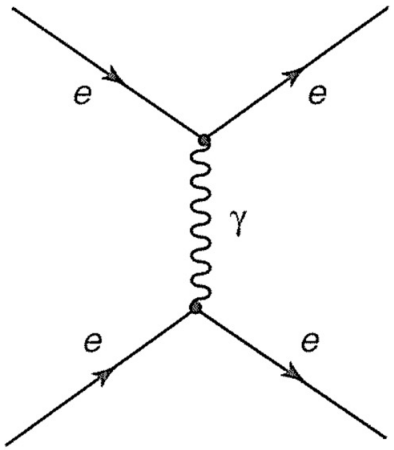


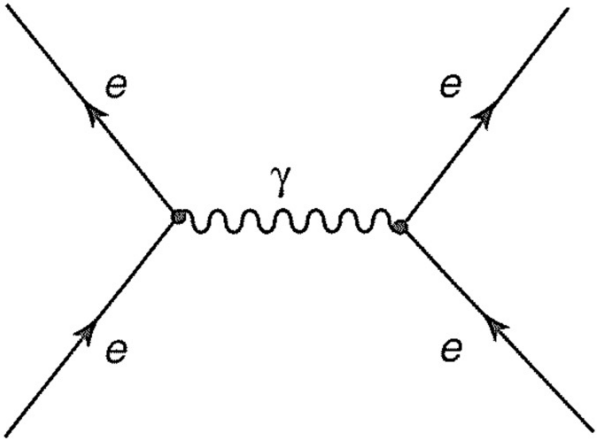


"All possible histories" are those that can be built by combining a few "basic vertices" that represent fundamental interactions.



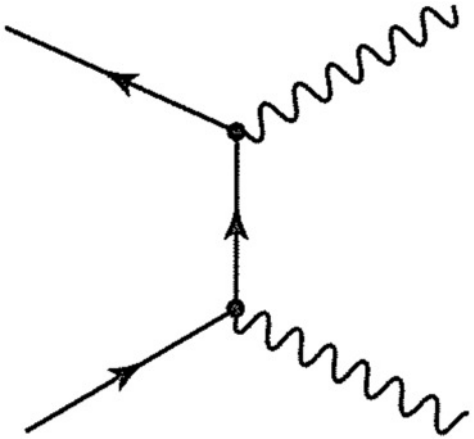
time →





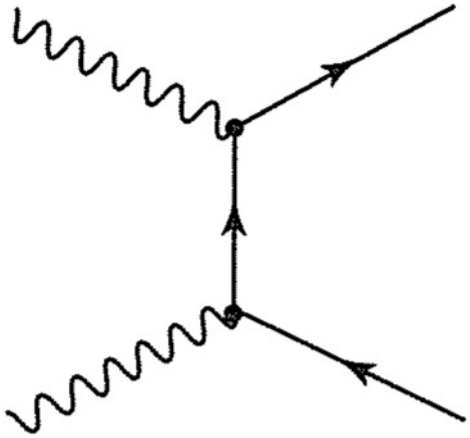
Other combinations

time



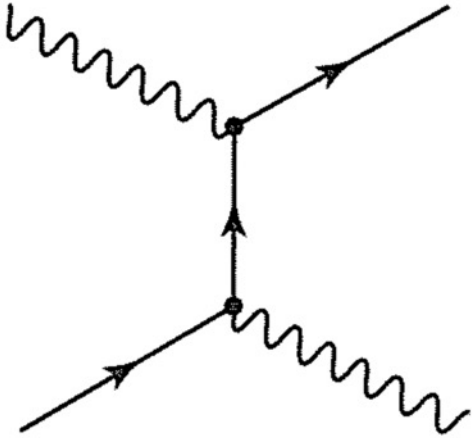
Pair annihilation

$e^+ + e^- \rightarrow \gamma + \gamma$



Pair production

$\gamma + \gamma \rightarrow e^+ + e^-$



Compton scattering

$e^- + \gamma \rightarrow e^- + \gamma$

What is $\frac{1}{0.98}$?

$$\frac{1}{1-0.02} \approx 1 + 0.02$$

$$\approx 1 + 0.02 + 0.02^2$$

$$\approx 1 + 0.02 + 0.02^2 + 0.02^3$$

$$\cdots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

- It is an infinite series, but we only may need two terms.
- We know how to obtain the series, but **not "by inspection"**.
- Maybe a supersmart alien species calls this function the *"all ones"*

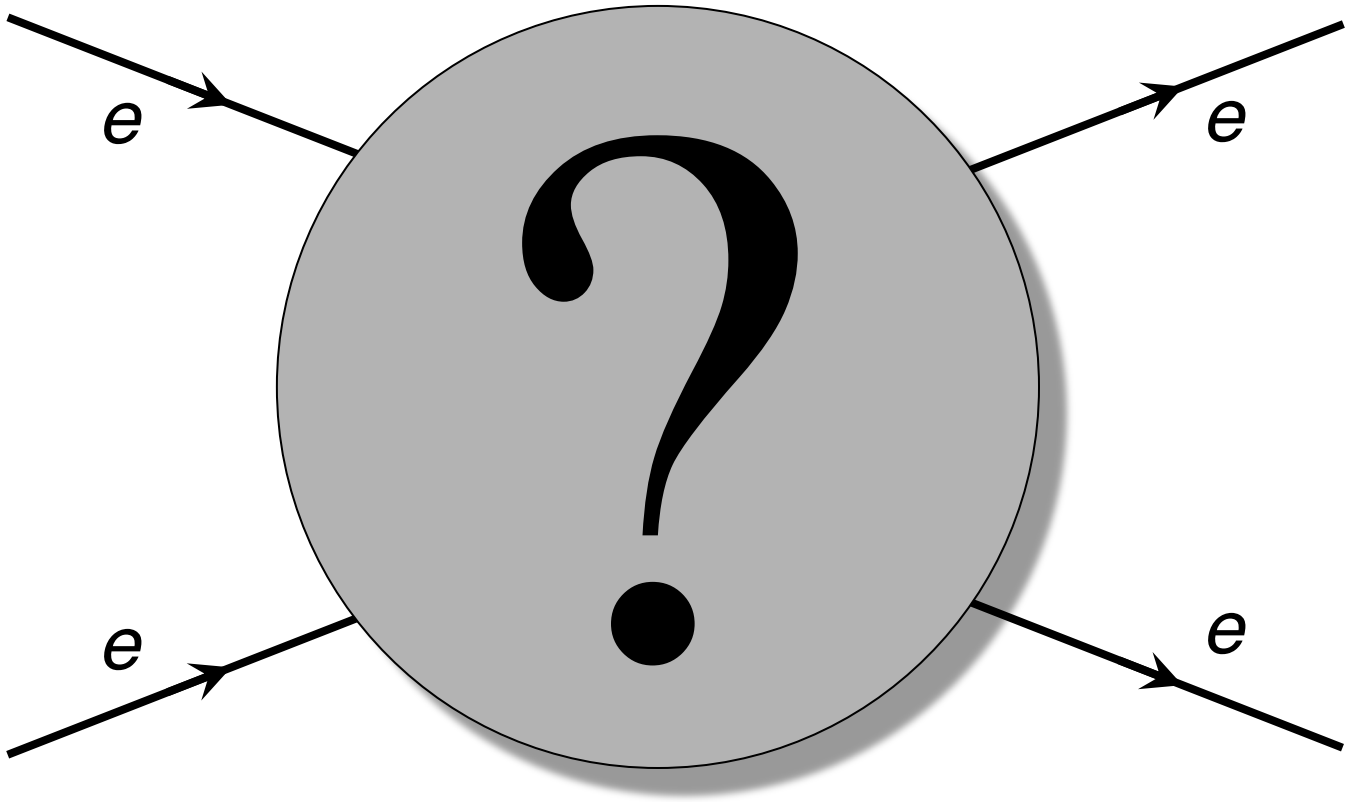
Each Feynman diagram stands for an *expression* that produces *one complex number*.

They are terms in a series; they are just written in an exceedingly intuition-friendly notation.

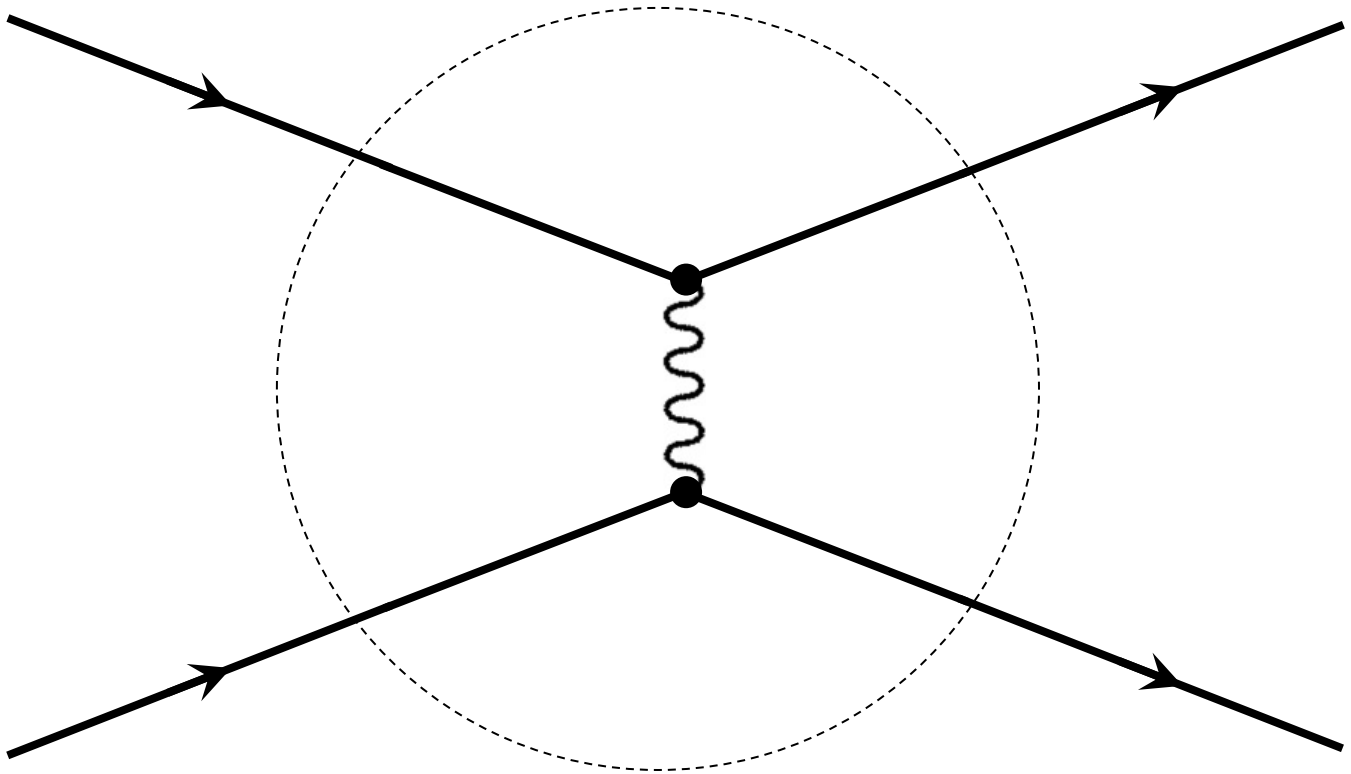
Once we know the valid vertices and what particles **enter** and **exit** a process, *we can write 'by inspection' all the terms needed!!!*

The sum of those expressions is the probability amplitude for the process.

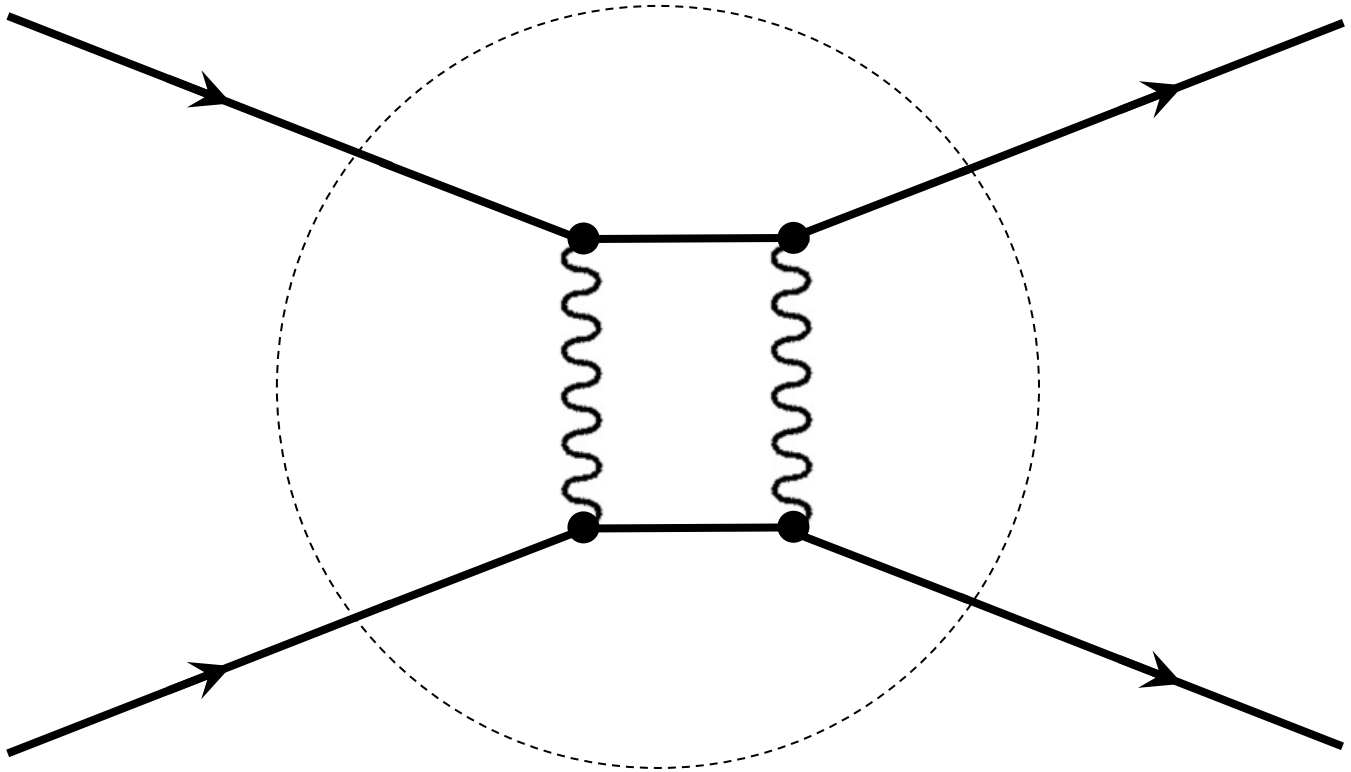
For example,



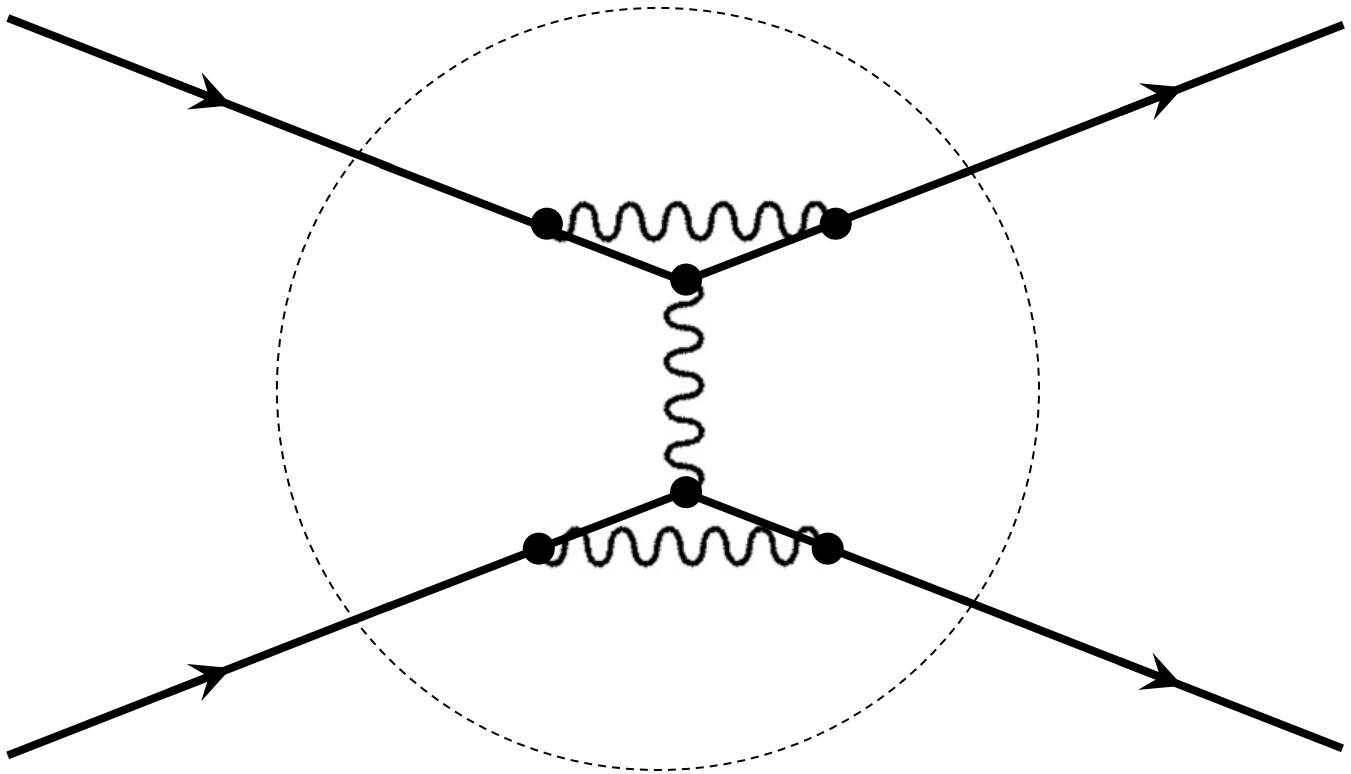
For example,



For example,



For example,



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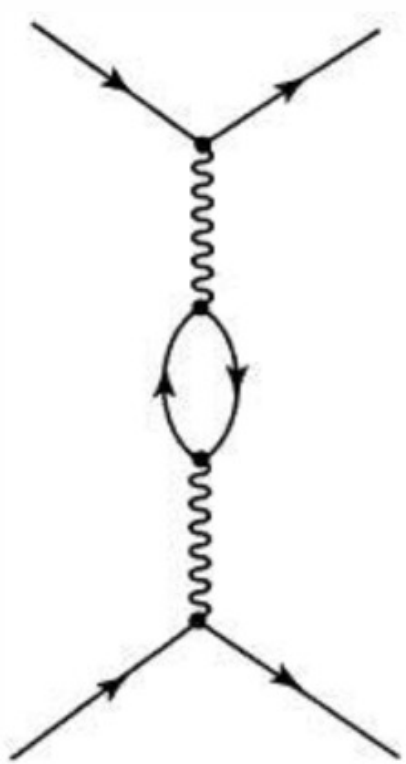
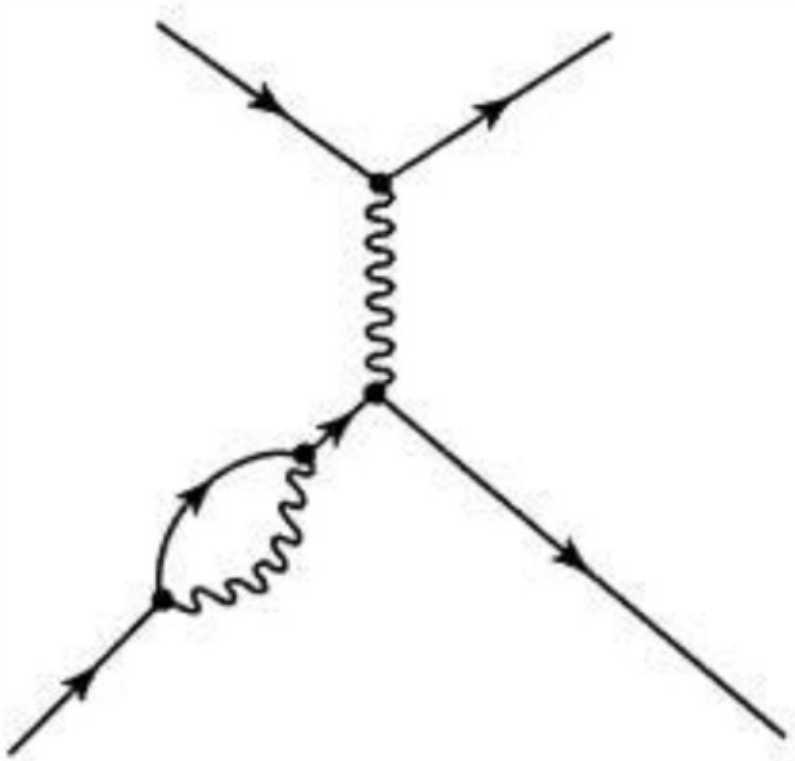
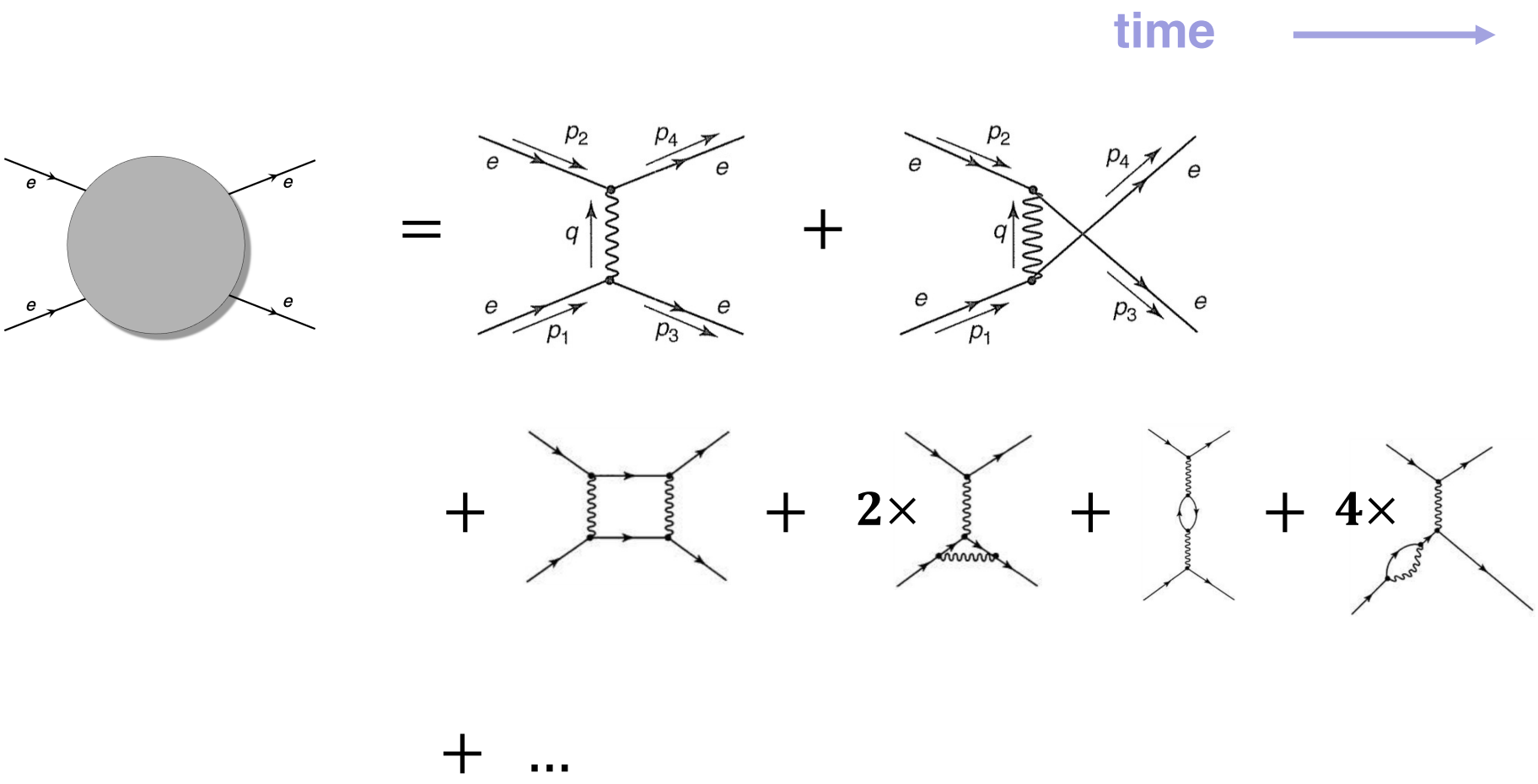
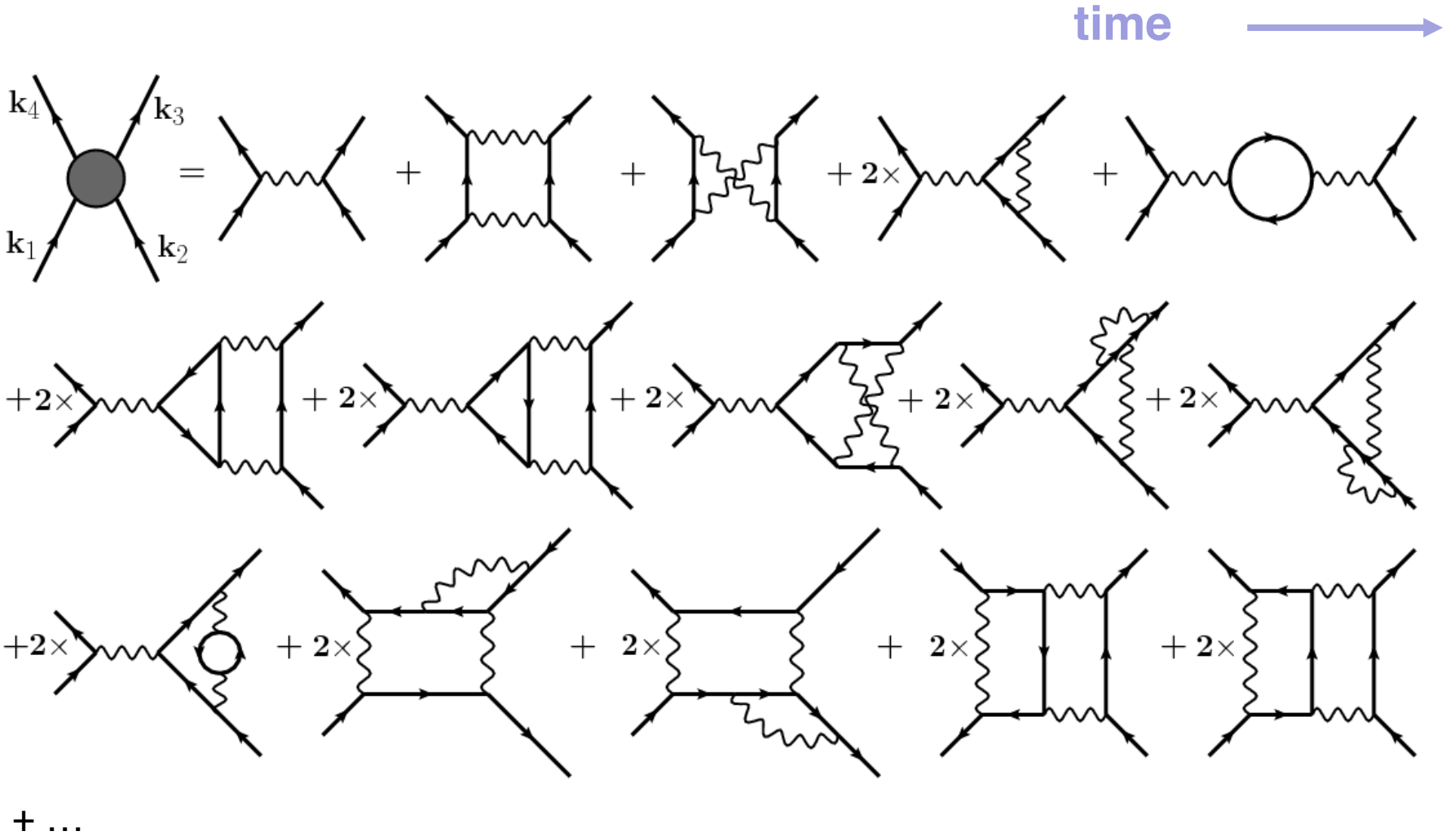
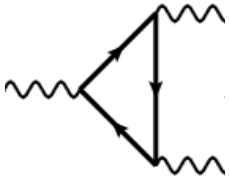


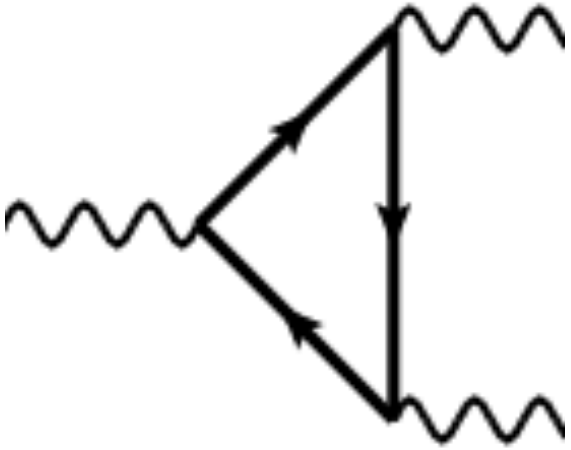




- In QED, for each diagram: **more vertices → smaller value**
- Only the connections are important (angles are irrelevant)
- For very high precision, thousands of terms may be needed


time
k_4
k_3
k_1
k_2
=
+
+
+ 2×
+
+2×
+ 2×
+ 2×
+ 2×
+ 2×
+2×
+ 2×
+
2×
+
2×
+ 2×
+ …





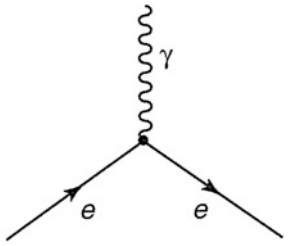
Also, remember that “proof” at the beginning?

- The two “90 degree rotations” that make it “work” are absurd because expressions and what they represent do not share rotational properties

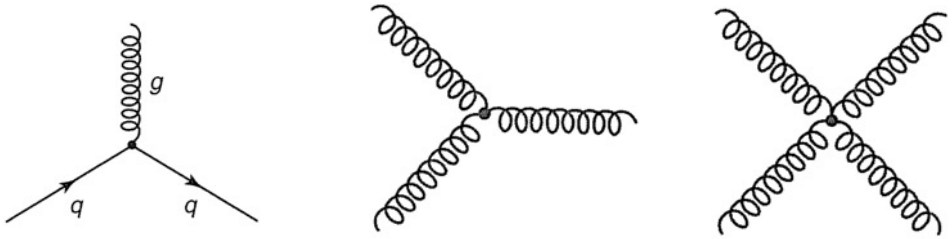
- On the other hand, when we rotate a Feynman diagram (i.e., we “rotate an expression”), we get valid expressions!

Interaction summary (w/o the Higgs boson)

QED:

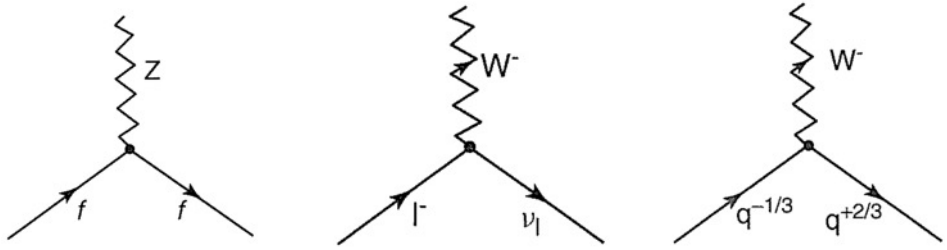


QCD:



	Fermions			Bosons	
Quarks	u up	c charm	t top	γ photon	Force carriers
	d down	s strange	b bottom	Z Z boson	
Leptons	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	
	e electron	μ muon	τ tau	g gluon	

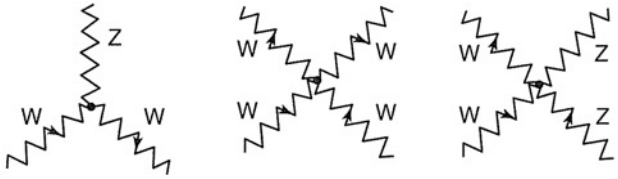
Weak:



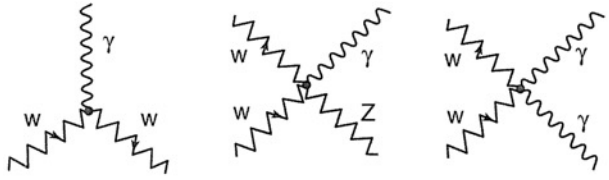
Cabibbo-Kobayashi-Maskawa

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

W/Z:

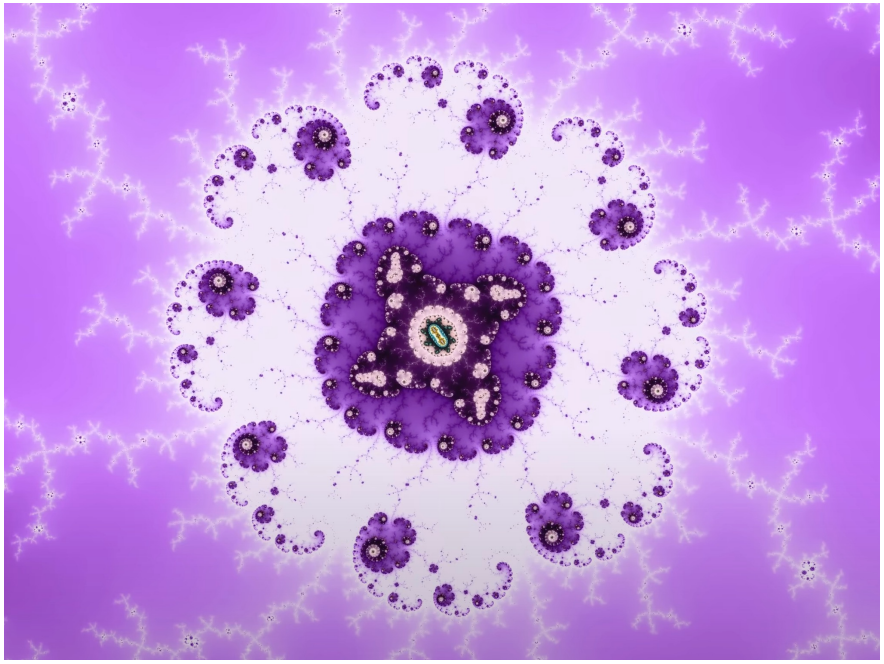


(W\|Z)/γ:



- Each valid vertex corresponds to a term in the Standard Model Lagrangian.

- i.e., Feynman diagrams are *terms in the perturbative expansion of the probability amplitude of a process*.



$$z \leftarrow z^2 + c$$

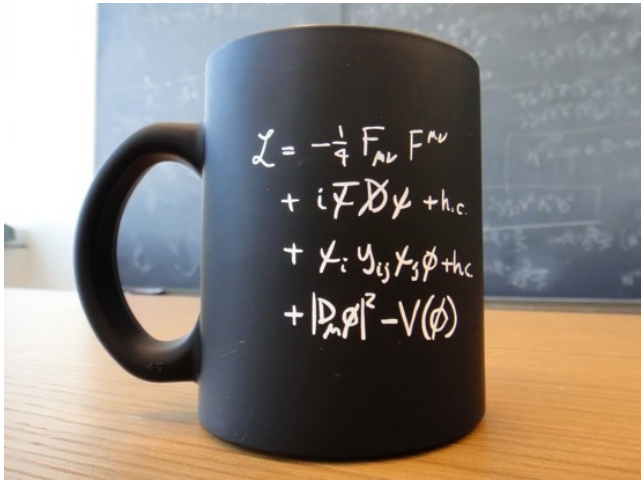


Image: NASA / STScI