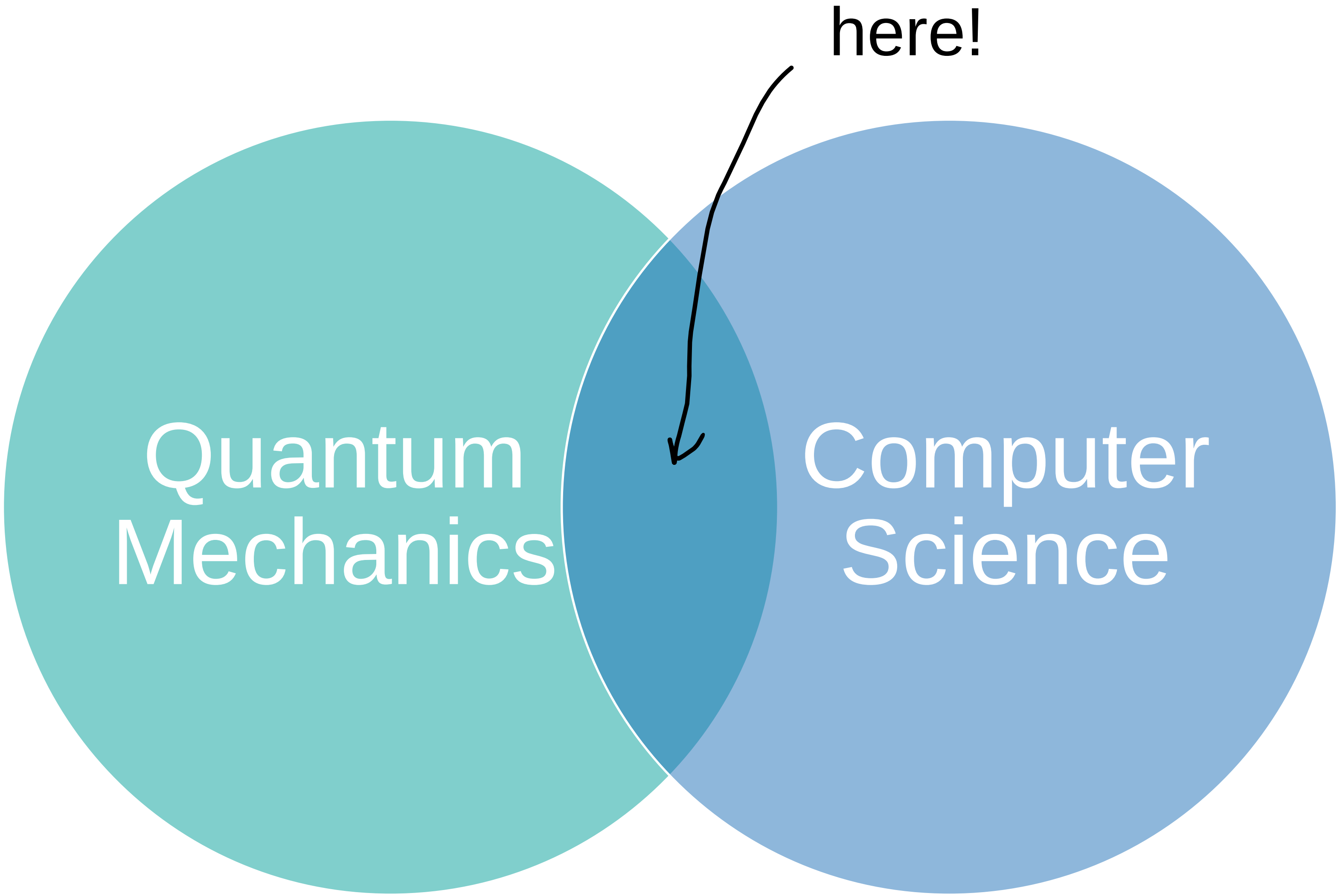
Basics of quantum computing

openlab Summer Student Lectures

QUANTUM
TECHNOLOGY
INITIATIVE

Quantum computing ?



Superposition : What is a Qubit ?

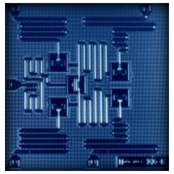
0 or 1

Classical bit

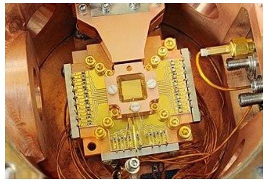
0 / 1

Quantum bit

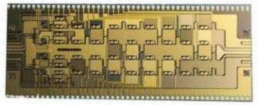
Superposition



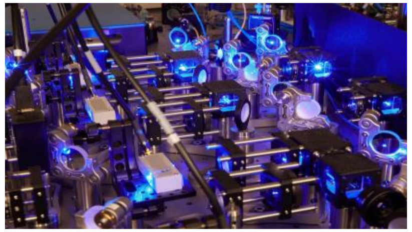
superconducting



ion trap



photonics



neutral atom

Each "classical state" is an "axis"

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |\uparrow\rangle$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad |\downarrow\rangle$$

Each Quantum state is a vector in that space

$$|+\rangle := \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \qquad \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$|\alpha\rangle := \frac{3}{5}|0\rangle - \frac{4}{5}|1\rangle \qquad \begin{bmatrix} 3/5 \\ 4/5 \end{bmatrix}$$

Measurement : Born rule

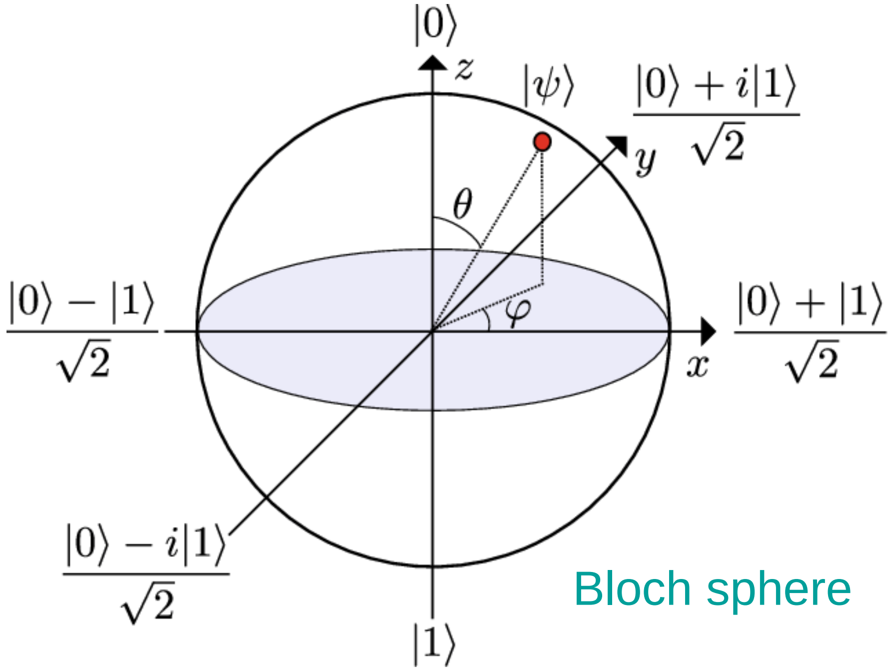
Born Rule

$$p_0 = |\langle\psi|0\rangle|^2$$

Each Quantum state is a 2D vector

$$|+\rangle := \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \qquad \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$|\alpha\rangle := \frac{3}{5}|0\rangle - \frac{4}{5}|1\rangle \qquad \begin{bmatrix} 3/5 \\ 4/5 \end{bmatrix}$$



Bloch sphere

The coordinates tell us the probabilities of getting a result.
The sum of probabilities is always equal to 1.

$$|\psi\rangle = \cos(\theta)|0\rangle + e^{i\varphi}\sin(\theta)|1\rangle$$

Entanglement : Several qubits

What happens when I have two qubits next to each other ?

$|1\rangle_a \otimes |0\rangle_b = |10\rangle_{ab} = |2\rangle_{ab}$ $\quad \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

In quantum the bits can be entangled, always giving the same results

$\frac{|00\rangle}{\sqrt{2}} + \frac{|11\rangle}{\sqrt{2}}$ Bell state

For two qubits there are four “axis”

$|00\rangle \; , |01\rangle \; , |10\rangle \; , |11\rangle$

How many “axis” for three qubits ? 8!

$\otimes$ Tensor product

$\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} a+2b & 3b \\ 2c & 3c \end{pmatrix}$

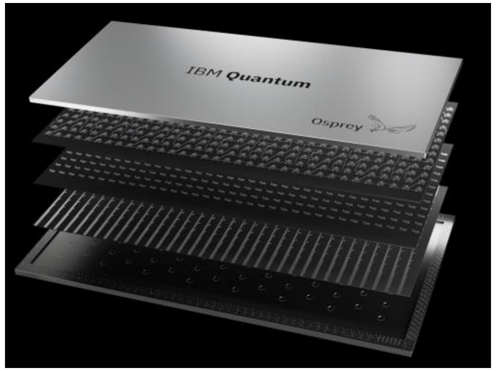
$\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$ a+2b

$\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix} = \left(\begin{array}{cc|cc} a & b & 0 & 0 \\ 0 & c & 0 & 0 \\ \hline 2a & 2b & 3a & 3b \\ 0 & 2c & 0 & 3c \end{array}\right)$

What would be the state for always different results ?

$\frac{|01\rangle}{\sqrt{2}} + \frac{|10\rangle}{\sqrt{2}}$

Big dimension – Small result



IBM Osprey 433 qubits

For n qubits :

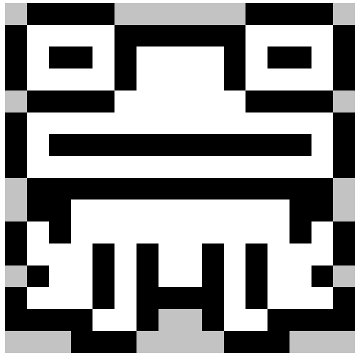
How many dimensions (“axis”) ? 2^n numbers

How many bits per answer ? n bits

For 260 qubits :

Dimension
~ number of atoms in the known universe

Answer
= 260 bits of information
~16x16 B&W image



Gates : Manipulating qubits

Properties : Conserve norms (more generally dot products)

Can you think of something that conserve dot products ? Rotations !

Unitary matrices : $UU^{+} = U^{+}U = I$

Pauli X $-\boxed{X}-$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} : \quad \begin{aligned} |0\rangle &\to |1\rangle \\ |1\rangle &\to |0\rangle \end{aligned}$$

"NOT" gate

Pauli Z $-\boxed{Z}-$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} : \quad \begin{aligned} &|1\rangle \to -|1\rangle \\ &|+\rangle = |0\rangle + |1\rangle \to |0\rangle - |1\rangle \to |-\rangle \end{aligned}$$

Pauli Y $-\boxed{Y}-$

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} : \quad |0\rangle \to -i|1\rangle$$

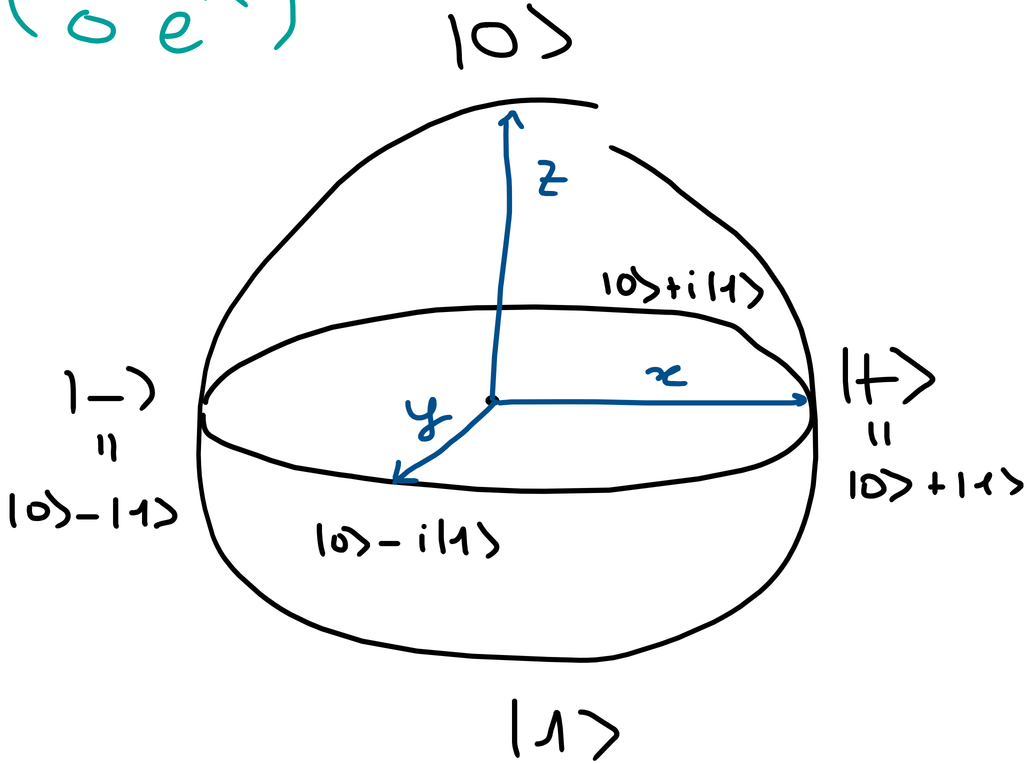
Hadamard $-\boxed{H}-$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} : \quad \begin{aligned} |0\rangle &\to |+\rangle \\ |1\rangle &\to |-\rangle \end{aligned}$$

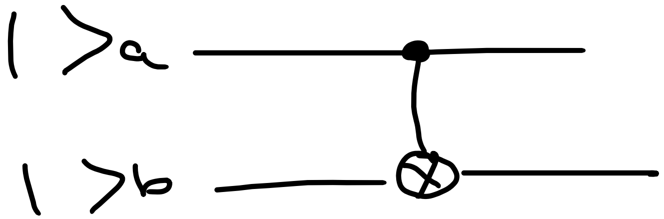
Parameterized gates :

$$\begin{pmatrix} c\theta & s\theta \\ -s\theta & c\theta \end{pmatrix}$$

$$\begin{pmatrix} e^{i\varphi} & 0 \\ 0 & e^{-i\varphi} \end{pmatrix}$$



CNOT



when qubit a is in the $|1\rangle$ state
apply a NOT (Pauli X) gate to qubit b

$$|0\rangle_a|0\rangle_b \rightarrow |0\rangle_a|0\rangle_b$$

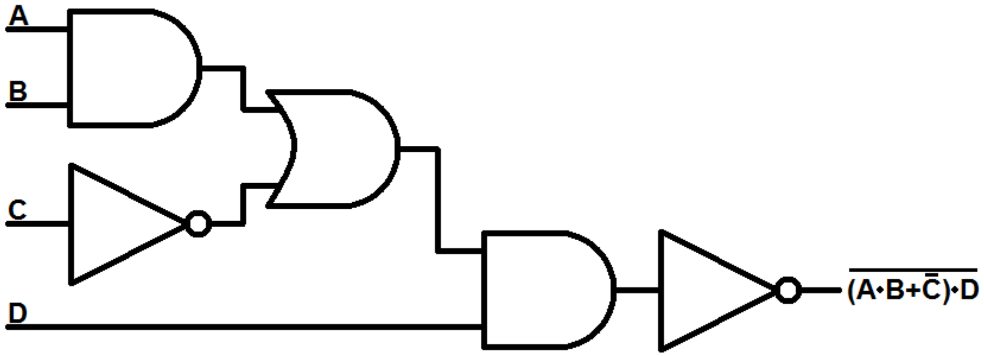
$$|0\rangle_a|1\rangle_b \rightarrow |0\rangle_a|1\rangle_b$$

$$|1\rangle_a|0\rangle_b \rightarrow |1\rangle_a|1\rangle_b$$

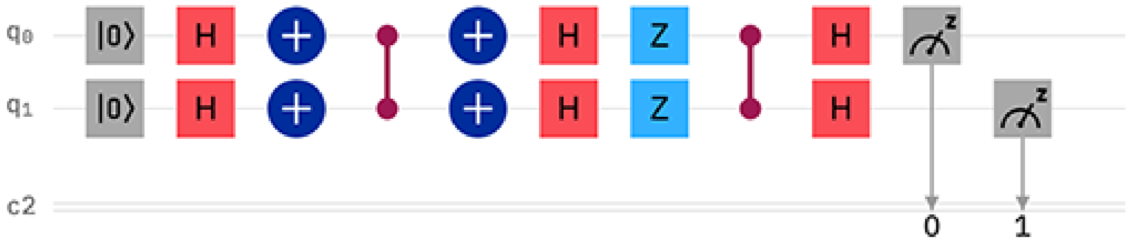
$$|1\rangle_a|1\rangle_b \rightarrow |1\rangle_a|0\rangle_b$$

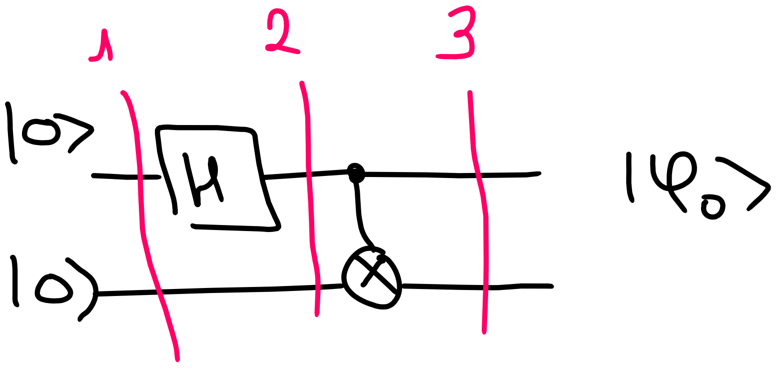
$$\left[\begin{array}{c|c} \begin{matrix}1 & 0\\ 0 & 1\end{matrix} & 0 \\ \hline 0 & \begin{matrix}0 & 1\\ 1 & 0\end{matrix}\end{array}\right]$$

Quantum circuits : Bell state preparation



$\longrightarrow$





Bell state preparation

①	②	③
$\lvert 0\rangle \otimes \lvert 0\rangle$	$\lvert +\rangle \otimes \lvert 0\rangle$	$\left(\lvert 0\rangle\lvert 0\rangle + \lvert 1\rangle\lvert 1\rangle\right)\frac{1}{\sqrt{2}}$
$\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$	$\begin{bmatrix}1\\0\\1\\0\end{bmatrix}/\sqrt{2}$	$\begin{bmatrix}1\\0\\0\\1\end{bmatrix}/\sqrt{2}$

$$\begin{bmatrix}|00\rangle\\|01\rangle\\|10\rangle\\|11\rangle\end{bmatrix}$$

Bra-Ket notation

$+$: conjugate transpose = dagger

ket : $|\cdot\rangle$ $\qquad |\psi\rangle = \begin{bmatrix} a+ib \\ c+id \end{bmatrix}$

bra : $\langle\cdot| = |\cdot\rangle^+$ $\qquad \langle\psi| = [a-ib, \; c-id]$

dot product : $\langle\cdot|\cdot\rangle$ $\qquad \langle\psi|\psi\rangle = a^2+b^2+c^2+d^2 = \|\psi\|_2^2$

Matrix : $|\cdot\rangle\langle\cdot|$

$$|0\rangle\langle 0| + |1\rangle\langle 1| = \begin{bmatrix} 1 \\ 0 \end{bmatrix}\overset{[1\;0]}{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\overset{[0\;1]}{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Orthonormal basis : $\{|k\rangle\}_{1\leq k\leq n}$ $\qquad \langle i|j\rangle = \delta_{ij} \begin{cases} 1 & i=j \\ 0 & i\neq j \end{cases}$

$\{|0\rangle, |1\rangle\}$

$$|\psi\rangle = \sum_{k=1}^{n} \langle k|\psi\rangle \, |k\rangle$$

$$I = \sum_{k=1}^{n} |k\rangle\langle k|$$

Closing question

Who here thinks that the following statement is true :

“Quantum Computers can compute things that Classical Computers cannot compute”

Closing question

In fact the following statement is true :

"Quantum Computers can **efficiently** compute things that Classical Computers cannot compute **efficiently**"

Thank you for your attention