Correlations between SIDIS azimuthal asymmetries in target and current fragmentation regions

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- Polarized SIDIS
  - Introduction: CFR
  - TFR, STMD fracture functions
    - M.Anselmino, V.Barone and AK, **arXiv:1102.4214; PLB 699 (2011) 108**
  - DSIDIS: TFR+CFR
    - AK, DIS2011; **arXiv:1107.2292**

M.Anselmino, V.Barone and AK, articles in preparation

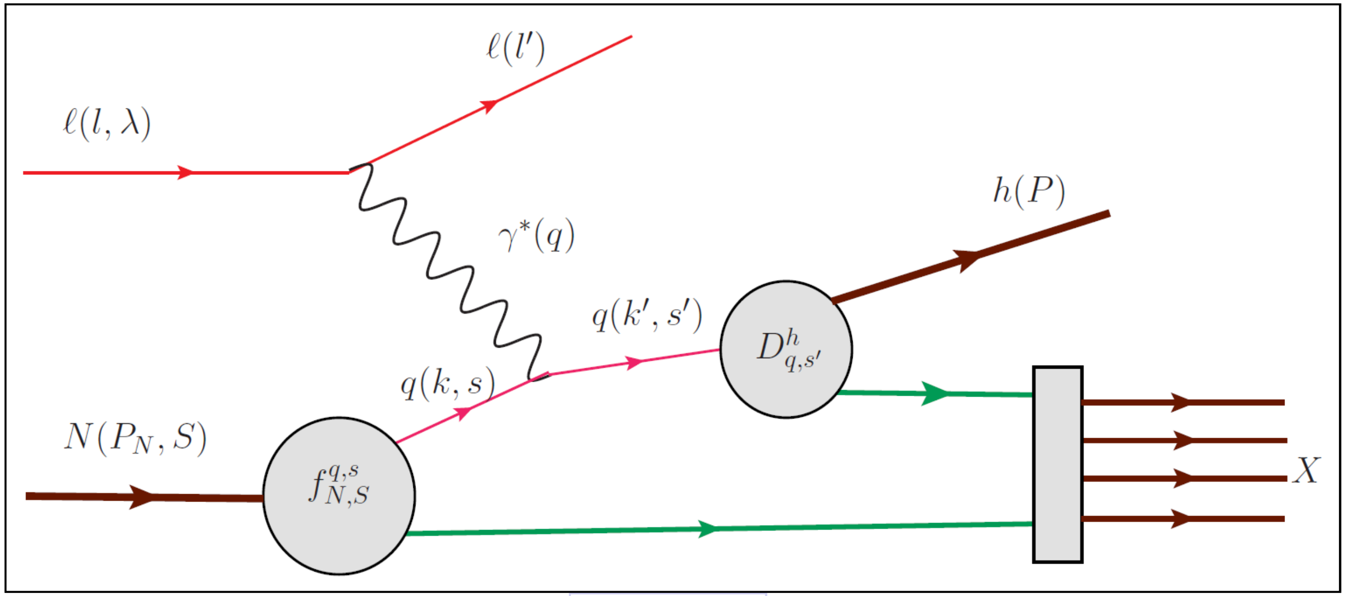
Introduction

- DIS: only lepton used as a probe
  - No access to quark k_T and transverse polarization at LO
- SIDIS: lepton and produced hadron used as a probe
  - We need excellent understanding of hadronization and factorization
    - Exploited in CFR: $x_F > 0$
    - For better understanding one have to exploit also TFR: $x_F < 0$
- New nonperturbative objects – Spin and Transverse Momentum Dependent Fracture Functions

STMD DFs

Nucleon Polarization	Quark polarization: U	L	T
U	$f_1^q(x,k_T^2)$		$\frac{\epsilon_T^{ij} k_T{}^j}{M} h_1^{\perp q}(x,k_T^2)$
L		$S_L g_{1L}^q(x,k_T^2)$	$S_L \frac{\mathbf{k}_T}{M} h_{1L}^{\perp q}(x,k_T^2)$
T	$\frac{\mathbf{k}_T \times \mathbf{S}_T}{M} f_{1T}^{\perp q}(x,k^2)$	$\frac{\mathbf{k}_T \cdot \mathbf{S}_T}{M} g_{1T}^{\perp q}(x,k_T^2)$	$\mathbf{S}_T h_{1T}^q(x,k_T^2)$ $\frac{\mathbf{k}_T}{M}\frac{(\mathbf{k}_T \cdot \mathbf{S}_T)_T}{M} h_{1T}^{\perp q}(x,k_T^2)$

SIDIS: CFR



$$x_F > 0$$

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T} = f_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z,p_T^2) + \frac{\mathbf{p}_T\times\mathbf{s}'_T}{m_h} H_1(z,p_T^2)$$

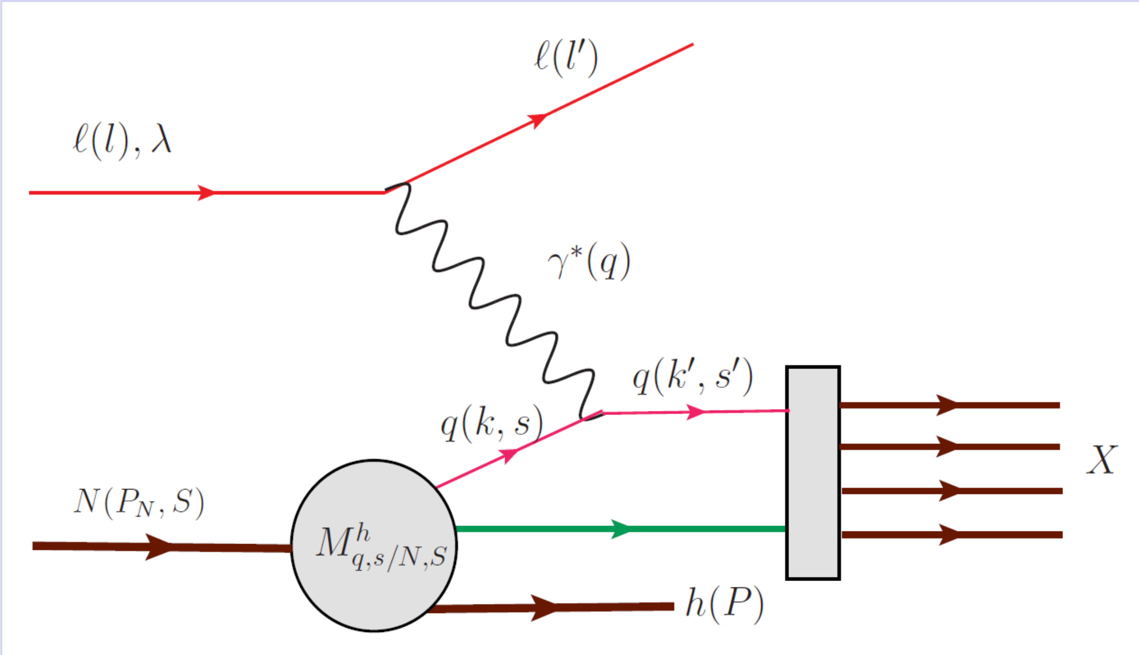
LO cross section in SIDIS CFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F>0)}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_T}=\frac{\alpha^2 x}{yQ^2}\left(1+(1-y)^2\right)\times$$

$$\times\left[\begin{array}{l} F_{UU,T}+D_{nn}(y)F_{UU}^{\cos 2\phi_h}\cos(2\phi_h)+ \\ S_L D_{nn}(y)F_{UL}^{\sin 2\phi_h}\sin(2\phi_h)+\lambda S_L D_{ll}(y)F_{LL}+ \\ S_T\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}\sin(\phi_h-\phi_S)+D_{nn}(y)\left(\begin{array}{l} F_{UT}^{\sin(\phi_h+\phi_S)}\sin(\phi_h+\phi_S)+ \\ F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S)\end{array}\right)\right)+ \\ \lambda S_T D_{ll}(y)F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S)\end{array}\right]$$

8 terms out of 18 Structure Functions, 6 azimuthal modulations

SIDIS: TFR



$x_F < 0$

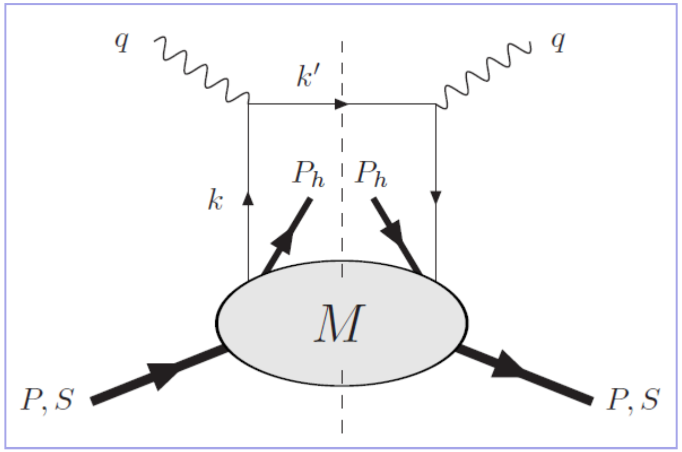
M.Anselmino, V.Barone and A.K., arXiv:1102.4214; PL B699 (2011) 108

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}}{dx\,dQ^2\,d\phi_S\,d\zeta\,d^2P_T} = M^h_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2}$$

$$\zeta = \frac{P^-}{P_N^-} \approx x_F(1-x)$$

Probabilistic interpretation at LO

Quark correlator



$$\mathcal{M}^{[\Gamma]}(x_B,\vec{k}_\perp,\zeta,\vec{P}_{h\perp}) = \frac{1}{4\zeta}\int\frac{d\xi^+ d^2\xi_\perp}{(2\pi)^6} e^{\mathrm{i}(x_B P^- \xi^+ - \vec{k}_\perp\cdot\vec{\xi}_\perp)} \sum_X \int \frac{d^3 P_X}{(2\pi)^3 2E_X} \times$$
$$\times \langle P,S|\bar{\psi}(0)\Gamma|P_h,S_h;X\rangle\langle P_h,S_h;X|\psi(\xi^+,0,\vec{\xi}_\perp)|P,S\rangle$$

$$\Gamma = \gamma^-,\quad \gamma^-\gamma_5,\quad i\sigma^{i-}\gamma_5$$

At LO 16 STMD fracture functions

Decomposition of quark correlator

$$\mathcal{M}^{[\gamma^-]} = \hat{u}_1 + \frac{\mathbf{P}_T \times \mathbf{S}_T}{m_2}\hat{u}_{1T}^h + \frac{\mathbf{k}_T \times \mathbf{S}_T}{m_N}\hat{u}_{1T}^\perp + \frac{S_L(\mathbf{k}_T \times \mathbf{P}_T)}{m_N m_2}\hat{u}_{1L}^{\perp h}$$

$$\mathcal{M}^{[\gamma^-\gamma_5]} = S_L \hat{l}_{1L} + \frac{\mathbf{P}_T \cdot \mathbf{S}_T}{m_2}\hat{l}_{1T}^h + \frac{\mathbf{k}_T \cdot \mathbf{S}_T}{m_N}\hat{l}_{1T}^\perp + \frac{\mathbf{k}_T \times \mathbf{P}_T}{m_N m_2}\hat{l}_1^{\perp h}$$

$$\mathcal{M}^{[i\sigma^{i-}\gamma_5]} = S_T^i \hat{t}_{1T} + \frac{S_L P_T^i}{m_2}\hat{t}_{1L}^h + \frac{S_L k_T^i}{m_N}\hat{t}_{1L}^\perp + \frac{(\mathbf{P}_T \cdot \mathbf{S}_T)P_T^i}{m_2^2}\hat{t}_{1T}^{hh} + \frac{(\mathbf{k}_T \cdot \mathbf{S}_T)k_T^i}{m_N^2}\hat{t}_{1T}^{\perp\perp} + \frac{(\mathbf{k}_T \cdot \mathbf{S}_T)P_T^i - (\mathbf{P}_T \cdot \mathbf{S}_T)k_T^i}{m_N m_2}\hat{t}_{1T}^{\perp h} + \frac{\epsilon_\perp^{ij} P_{Tj}}{m_2}\hat{t}_1^h + \frac{\epsilon_\perp^{ij} k_{Tj}}{m_N}\hat{t}_1^\perp$$

STMD fracture functions depend on $x, k_T^2, \zeta, P_T^2, \mathbf{k}_T \cdot \mathbf{P}_T$

$\mathbf{k}_T \cdot \mathbf{P}_T = k_T P_T \cos(\phi_h - \phi_q) -$ azimuthal dependence in fracture functions

STMD Fracture Functions for spinless hadron production

Nucleon Polarization	Quark polarization: U	L	T
U	$\hat{u}_1$	$\frac{\mathbf{k}_T \times \mathbf{P}_T}{m_N m_h} \hat{l}_1^{\perp h}$	$\frac{\epsilon_T^{ij} P_T^{\,j}}{m_h} \hat{t}_1^{h} + \frac{\epsilon_T^{ij} k_T^{\,j}}{m_N} \hat{t}_1^{\perp}$
L	$\frac{S_L (\mathbf{k}_T \times \mathbf{P}_T)}{m_N m_h} \hat{u}_{1L}^{\perp h}$	$S_L \hat{l}_{1L}$	$\frac{S_L \mathbf{P}_T}{m_h} \hat{t}_{1L}^{h} + \frac{S_L \mathbf{k}_T}{m_N} \hat{t}_{1L}^{\perp}$
T	$\frac{\mathbf{P}_T \times \mathbf{S}_T}{m_h} \hat{u}_{1T}^{h} + \frac{\mathbf{k}_T \times \mathbf{S}_T}{m_N} \hat{u}_{1T}^{\perp}$	$\frac{\mathbf{P}_T \cdot \mathbf{S}_T}{m_h} \hat{l}_{1T}^{h} + \frac{\mathbf{k}_T \cdot \mathbf{S}_T}{m_N} \hat{l}_{1T}^{\perp}$	$\mathbf{S}_T \hat{t}_{1T} + \frac{\mathbf{P}_T (\mathbf{P}_T \cdot \mathbf{S}_T)}{m_h^2} \hat{t}_{1T}^{hh} + \frac{\mathbf{k}_T (\mathbf{k}_T \cdot \mathbf{S}_T)}{m_N^2} \hat{t}_{1T}^{\perp\perp} + \frac{\mathbf{P}_T (\mathbf{k}_T \cdot \mathbf{S}_T) - \mathbf{k}_T \cdot (\mathbf{P}_T \cdot \mathbf{S}_T)}{m_N m_h} \hat{t}_{1T}^{\perp h}$

Sum Rules

$$\sum_h \int \zeta d\zeta \int d^2 P_T \hat{u}_1 = (1-x) f_1(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{u}_{1T}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{u}_{1T}^{h} \right) = -(1-x) f_{1T}^{\perp}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \hat{l}_{1L} = (1-x) g_{1L}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{l}_{1T}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{l}_{1T}^{h} \right) = (1-x) g_{1T}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{t}_{1L}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{t}_{1L}^{h} \right) = (1-x) h_{1L}^{\perp}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{t}_{1}^{\perp} + \frac{m_N}{m_h} \frac{\mathbf{k}_T \cdot \mathbf{P}}{k_T^2} \hat{t}_{1}^{h} \right) = -(1-x) h_{1}^{\perp}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{t}_{1T}^{\perp\perp} + \frac{m_N^2}{m_h^2} \frac{2\left(\mathbf{k}_T \cdot \mathbf{P}\right)^2 - k_T^2 P_T^2}{k_T^4} \hat{t}_{1T}^{hh} \right) = (1-x) h_{1T}^{\perp}(x,k_T^2)$$

$$\sum_h \int \zeta d\zeta \int d^2 P_T \left(\hat{t}_{1T} + \frac{k_T^2}{2m_N^2} \hat{t}_{1T}^{\perp\perp} + \frac{P_T^2}{2m_h^2} \hat{t}_{1T}^{hh} \right) = (1-x) h_1(x,k_T^2)$$

Nonzero fracture functions u,l,t. Useful for modeling.

LO cross-section in TFR

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h(P)+X}(x_F<0)}{dx\,dQ^2\,d\phi_S\,d\zeta\,d^2P_T}=\frac{\alpha^2 x}{yQ^4}\left(1+(1-y)^2\right)\sum_q e_q^2\times$$

$$\times\left[\tilde{u}_1\left(x,\zeta,P_T^2\right)-S_T\frac{P_T}{m_h}\tilde{u}_{1T}^h\left(x,\zeta,P_T^2\right)\sin(\phi_h-\phi_S)+\lambda y(2-y)\left(S_L\tilde{l}_{1L}\left(x,\zeta,P_T^2\right)+S_T\frac{P_T}{m_h}\tilde{l}_{1T}^h\left(x,\zeta,P_T^2\right)\cos(\phi_h-\phi_S)\right)\right]$$

$$\tilde{u}_1(x_B,\zeta_2,P_{T2}^2)=\int d^2k_T\,\hat{u}_1$$

$$\tilde{u}_{1T}^h(x_B,\zeta_2,P_{T2}^2)=\int d^2k_T\left\{\hat{u}_{1T}^h+\frac{m_2}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_{T2}}{P_{T2}^2}\hat{u}_{1T}^\perp\right\}$$

$$\tilde{l}_{1L}(x_B,\zeta_2,P_{T2}^2)=\int d^2k_T\,\hat{l}_{1L}$$

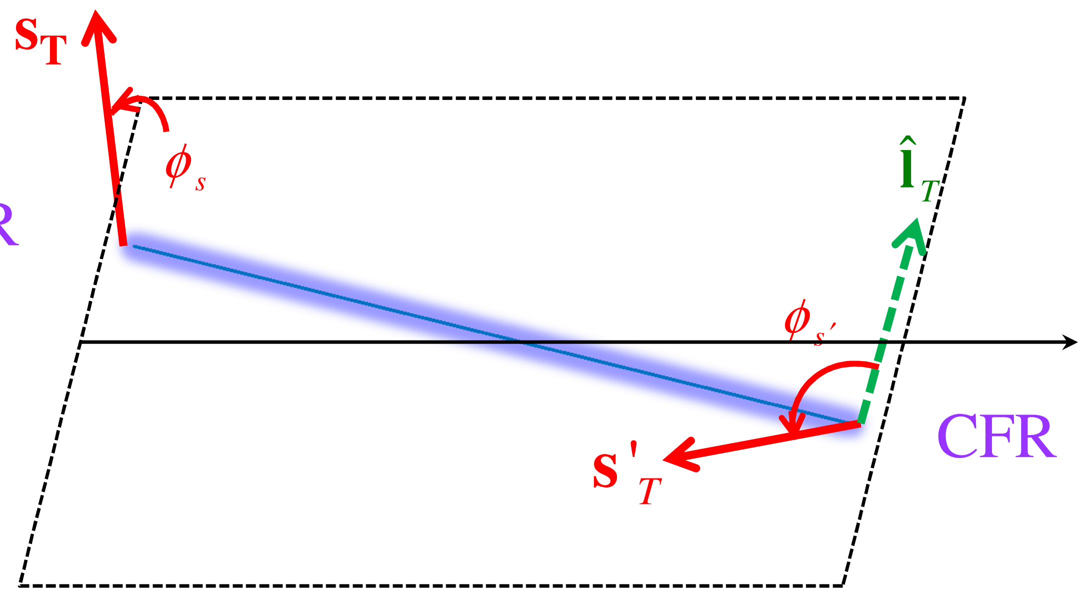
$$\tilde{l}_{1T}^h(x_B,\zeta_2,\ P_{T2}^2)=\int d^2k_T\left\{\hat{l}_{1T}^h+\frac{m_2}{m_N}\frac{\mathbf{k}_T\cdot\mathbf{P}_{T2}}{P_{T2}^2}\hat{l}_{1T}^\perp\right\}$$

At LO only 4 terms out of 18 Structure Functions,

2 azimuthal modulations

No access to quark transverse polarization

Quark spin in hard l-q scattering

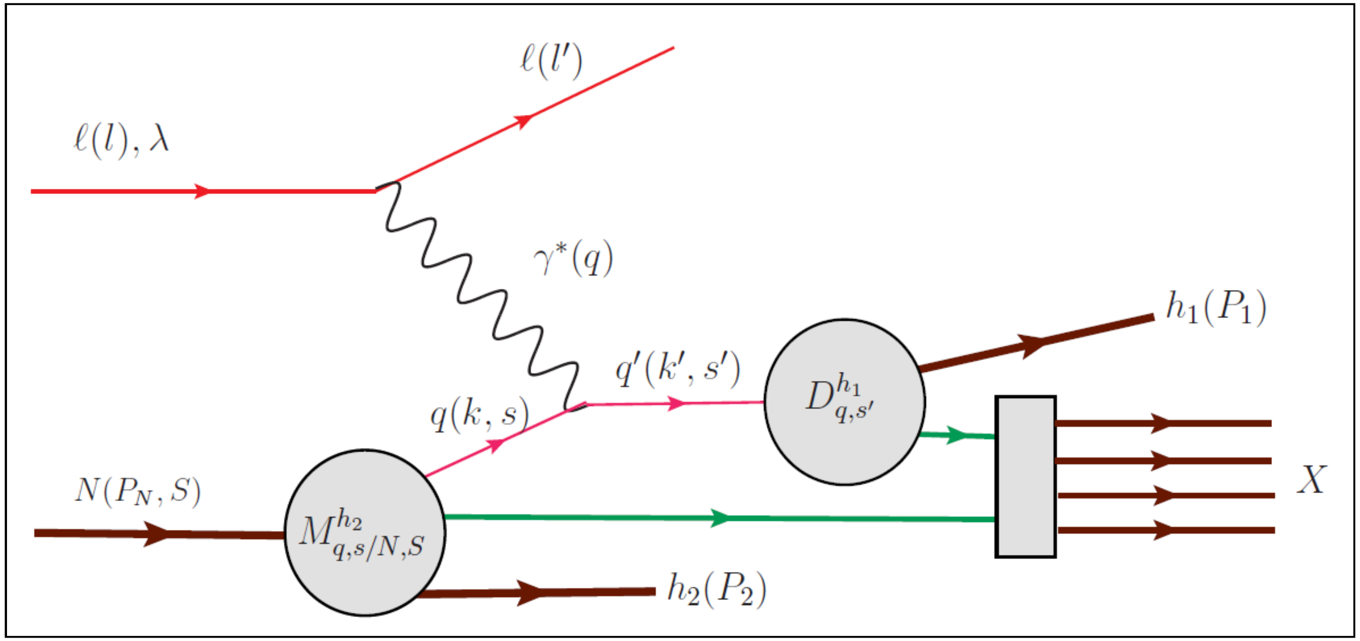


$$\frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} = e_q^2 \frac{2\pi\alpha^2}{\breve{s}^2}\frac{1}{Q^4}\Big((\breve{s}^2+\breve{u}^2)(1+s_L s'_L)+(\breve{s}^2-\breve{u}^2)\lambda(s_L+s'_L)$$
$$-2\breve{s}\breve{u}(\mathbf{s}_T\cdot\mathbf{s}'_T)-4\breve{u}(\mathbf{s}_T\cdot\mathbf{l}_T)(\mathbf{s}'_T\cdot\mathbf{l}'_T)-4\breve{s}(\mathbf{s}_T\cdot\mathbf{l}'_T)(\mathbf{s}'_T\cdot\mathbf{l}_T)\Big)$$

$\breve{s}$ and $\breve{u}$ are usual Mandelstam variables

$$s'_T = D_{nn}(y)s_T, \quad D_{nn}(y)=\frac{2(1-y)}{1+(1-y)^2}, \quad \phi_{s'}=\pi-\phi_s$$

DSIDIS: TFR & CFR



$$x_{F2} < 0, \quad x_{F1} > 0$$

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} = M^{h_2}_{q,s/N,S} \otimes \frac{d\sigma^{\ell(l,\lambda)+q(k,s)\to\ell(l')+q(k',s')}}{dQ^2} \otimes D^{h_1}_{q,s'}$$

$$D^{h_1}_{q,s'}(z,\mathbf{p}_T) = D_1(z,p_T^2) + \frac{\mathbf{p}_T \times \mathbf{s}'_T}{m_h} H_1(z,p_T^2)$$

$$\mathbf{s}'_T = D_{nn}(y)\mathbf{s}^R_T$$

Integrated over one hadron transverse momentum cross-sections

M.Anselmino, V.Barone and AK, article next week to ArXive

P_T-integrated fracture functions

$$\int d^2 P_{T2} \mathcal{M}^{[\gamma^-]} = u_1 + \frac{\mathbf{k}_T \times \mathbf{S}_\perp}{m_N} u_{1T}^\perp$$

$$\int d^2 P_{T2} \mathcal{M}^{[\gamma^- \gamma_5]} = S_L l_{1L} + \frac{\mathbf{k}_T \cdot \mathbf{S}_T}{m_N} l_{1T}$$

Only 8 $\mathbf{k}_T$-dependent "capless" fracture functions. Same prefactors as in TMDs

$$\int d^2 P_{T2} \mathcal{M}^{[i\sigma^{i-} \gamma_5]} = S_T^i t_1 + \frac{S_L k_T^i}{m_N} t_{1L}^\perp + \frac{(k_T^i k_T^j - \frac{1}{2} \mathbf{k}_T^2 \delta_{ij}) S_T^j}{m_N^2} t_{1T}^\perp + \frac{\epsilon_\perp^{ij} k_{Tj}}{m_N} t_1^\perp$$

$$t_1(x_B, k_T^2, \zeta_2) = \int d^2 P_{2T} \left\{ \hat{t}_{1T} + \frac{k_T^2}{2m_N^2} \hat{t}_{1T}^{\perp\perp} + \frac{P_{2T}^2}{2m_2^2} \hat{t}_{1T}^{hh} \right\}$$

DSIDIS cross section integrated over P_{T2}

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dy\,d\phi_S\,dz\,d\zeta\,d\phi_1 dP_{T1}^2} = \frac{\alpha_{\rm em}^2}{x_B\,y Q^2}\Bigg\{\left(1-y+\frac{y^2}{2}\right)\mathcal{F}_{UU,T} + (1-y)\cos 2\phi_1\,\mathcal{F}_{UU}^{\cos 2\phi_1}$$

$$+S_L(1-y)\sin 2\phi_1\,\mathcal{F}_{UL}^{\sin 2\phi_1} + S_L\,\lambda y\left(1-\frac{y}{2}\right)\mathcal{F}_{LL} + S_T\left(1-y+\frac{y^2}{2}\right)\sin(\phi_1-\phi_S)\,\mathcal{F}_{UT}^{\sin(\phi_1-\phi_S)}$$

$$+S_T(1-y)\sin(\phi_1+\phi_S)\,\mathcal{F}_{UT}^{\sin(\phi_1+\phi_S)} + S_T(1-y)\sin(3\phi_1-\phi_s)\,\mathcal{F}_{UT}^{\sin(3\phi_1-\phi_S)}$$

$$+S_T\,\lambda y\left(1-\frac{y}{2}\right)\cos(\phi_1-\phi_S)\,\mathcal{F}_{LT}^{\cos(\phi_1-\phi_S)}\Bigg\}$$

$$\mathcal{F}_{UU}^{\cos 2\phi_1} = \mathcal{C}\left[\frac{2(\hat{\mathbf{P}}_1\cdot\mathbf{k}_T)(\hat{\mathbf{P}}_1\cdot\mathbf{k}') - \mathbf{k}_T\cdot\mathbf{k}_T'}{m_N m_1} t_1^\perp H_1^\perp\right], \qquad \hat{\mathbf{P}}_1 = \frac{\mathbf{P}_{1T}}{|\mathbf{P}_{1T}|}$$

$$\mathcal{F}_{UT}^{\sin(\phi_1-\phi_S)} = \mathcal{C}\left[\frac{\hat{\mathbf{P}}_1\cdot\mathbf{k}_T}{m_N} u_{1T}^\perp D_1\right],$$

$$\mathcal{F}_{UT}^{\sin(\phi_1+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\mathbf{P}}_1\cdot\mathbf{k}_T'}{m_1} t_1 H_1^\perp\right],$$

$$\mathcal{C}[wuD] = \sum_a e_a^2 x_B \int d^2k_T \int d^2k_T'\,\delta^2(\mathbf{k}_T - \mathbf{k}_T' - \mathbf{P}_{1T}/z_1)\times w(\mathbf{k}_T,\mathbf{k}_T')\,u^a(x_B,k_T^2,\zeta_2)\,D^a(z_1,k_T'^2)$$

DSIDIS cross section integrated over P_{T1}

As in SIDIS in TFR only $\mathbf{k}_T$-integrated fracture functions contribute

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dy\,d\phi_S\,dz\,d\zeta\,d\phi_2\,dP_{T2}^2}=\frac{\alpha_{\rm em}^2}{yQ^2}\sum_a e_a^2 D_1(z_1)$$
$$\left\{\left(1-y+\frac{y^2}{2}\right)\left[\tilde{u}_1(x_B,\zeta_2,P_{T2}^2)-S_T\frac{P_{T2}}{m_2}\tilde{u}_{1T}^h(x_B,\zeta_2,P_{T2}^2)\sin(\phi_2-\phi_S)\right]\right.$$
$$\left.+\lambda y\left(1-\frac{y}{2}\right)\left[S_L\tilde{l}_{1L}(x_B,\zeta_2,P_{T2}^2)+S_T\frac{P_{T2}}{m_2}\tilde{l}_{1T}^h(x_B,\zeta_2,P_{T2}^2)\cos(\phi_2-\phi_S)\right]\right\}$$

Compared with one hadron production in TFR,

presence of known integrated fragmentation functions $D_1(z)$

allows quark flavor separation of "tilded" fracture functions

Unintegrated DSIDIS cross-section

$$\frac{d\sigma^{\ell(l,\lambda)+N(P_N,S)\to\ell(l')+h_1(P_1)+h_2(P_2)+X}}{dx\,dQ^2\,d\phi_S\,dz\,d^2P_{T1}\,d\zeta\,d^2P_{T2}} =$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\begin{pmatrix} \hat{u}^{h_2} \otimes D_1^{h_1} + \lambda D_{ll}(y)\hat{l}^{h_2} \otimes D_1^{h_1} \\ +\hat{t}^{h_2} \otimes \frac{\mathbf{p}_T \times \mathbf{s}'_T}{m_{h_1}} H_1^{h_1} \end{pmatrix}$$

$$= \frac{\alpha^2 x}{Q^4 y}\left(1+(1-y)^2\right)\begin{pmatrix} \sigma_{UU} + S_L\sigma_{UL} + S_T\sigma_{UT} + \\ \lambda D_{ll}\left(\sigma_{LU} + S_L\sigma_{LL} + S_T\sigma_{LT}\right) \end{pmatrix}$$

$$D_{ll}(y) = \frac{y(2-y)}{1+(1-y)^2}$$

Examples

AK @ DIS2011; **arXiv:1107.2292**

$$\sigma_{UU} = F_0^{\hat{u}\cdot D_1} - D_{nn}\left(\begin{array}{l} \frac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_1^{\perp}\cdot H_1} \cos(2\phi_1) \\ +\frac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_1^{h}\cdot H_1} \cos(\phi_1+\phi_2) \\ +\left(\frac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_1^{\perp}\cdot H_1} + \frac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_1^{h}\cdot H_1} \right)\cos(2\phi_2) \end{array} \right)$$

$$\sigma_{LU} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1} \sin(\phi_1-\phi_2)$$

$$F_{k1}^{\hat{M}\cdot D} = C\left[\hat{M}\cdot D \frac{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})(\mathbf{P}_{T2}\cdot\mathbf{k}) - (\mathbf{P}_{T1}\cdot\mathbf{k})\mathbf{P}_{T2}^2}{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})^2 - \mathbf{P}_{T1}^2\mathbf{P}_{T2}^2} \right]$$

A_{LU} asymmetry

In general structure functions $F_{...}^{\hat{u}\cdot D}$ depend on $x, z, \zeta, P_{T1}^2, P_{T2}^2$ and $(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})$

$\mathbf{P}_{T1}\cdot\mathbf{P}_{T2} = P_{T1}P_{T2}\cos(\Delta\phi)$, with $\Delta\phi = \phi_1 - \phi_2$

One can choose as independent angles $\Delta\phi$ and ϕ_2 $(\phi_1 = \Delta\phi + \phi_2)$

Integrating σ_{UU} and σ_{lU} over ϕ_2 we obtain

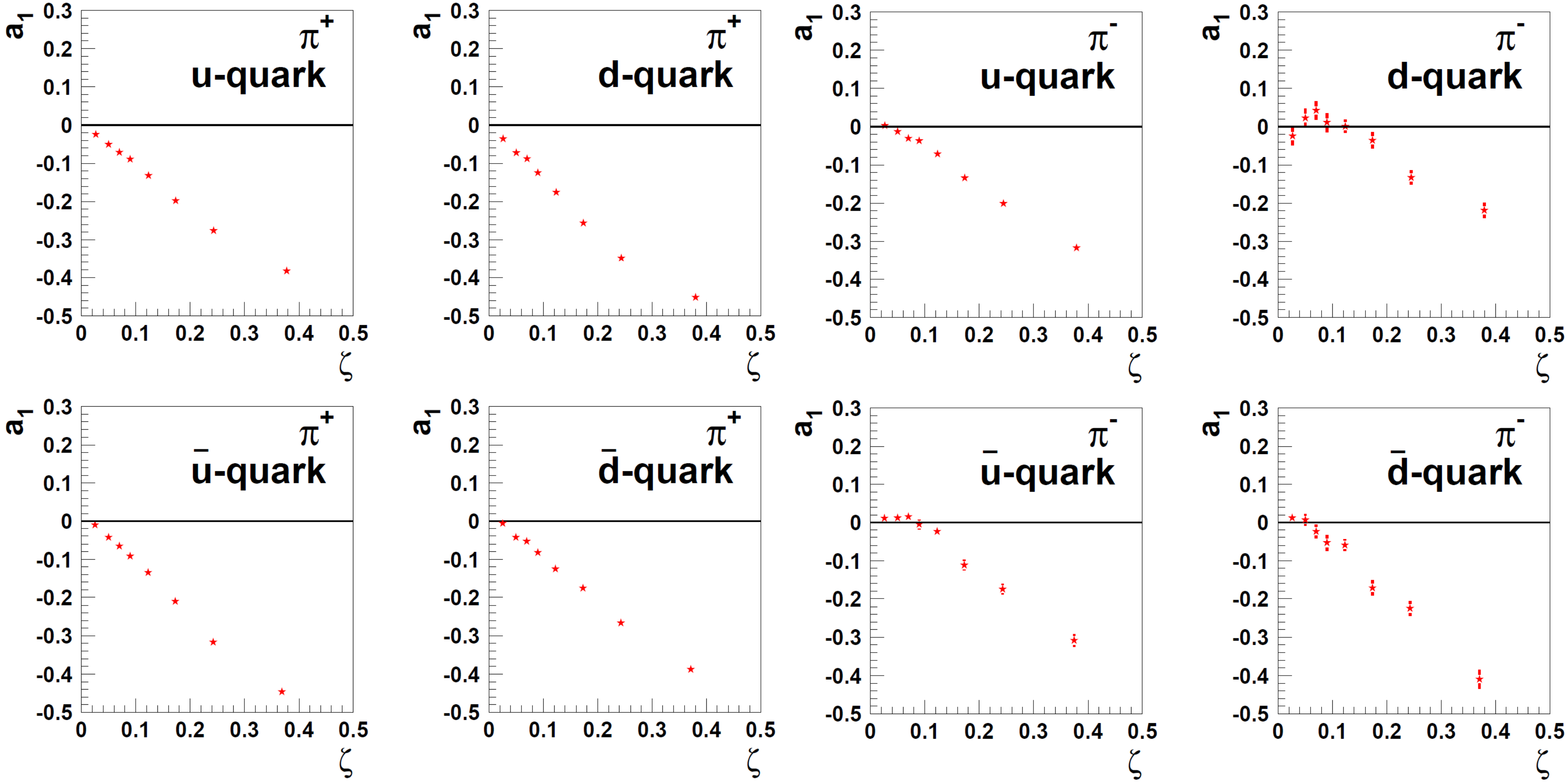
$$A_{LU} = \frac{\int d\phi_2 \sigma_{LU}}{\int d\phi_2 \sigma_{UU}} =$$

$$= \frac{-\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1}\left(x, z, \zeta, P_{T1}^2, P_{T2}^2, \cos(\Delta\phi)\right)\sin(\Delta\phi)}{F_0^{\hat{u}\cdot D_1}\left(x, z, \zeta, P_{T1}^2, P_{T2}^2, \cos(\Delta\phi)\right)}$$

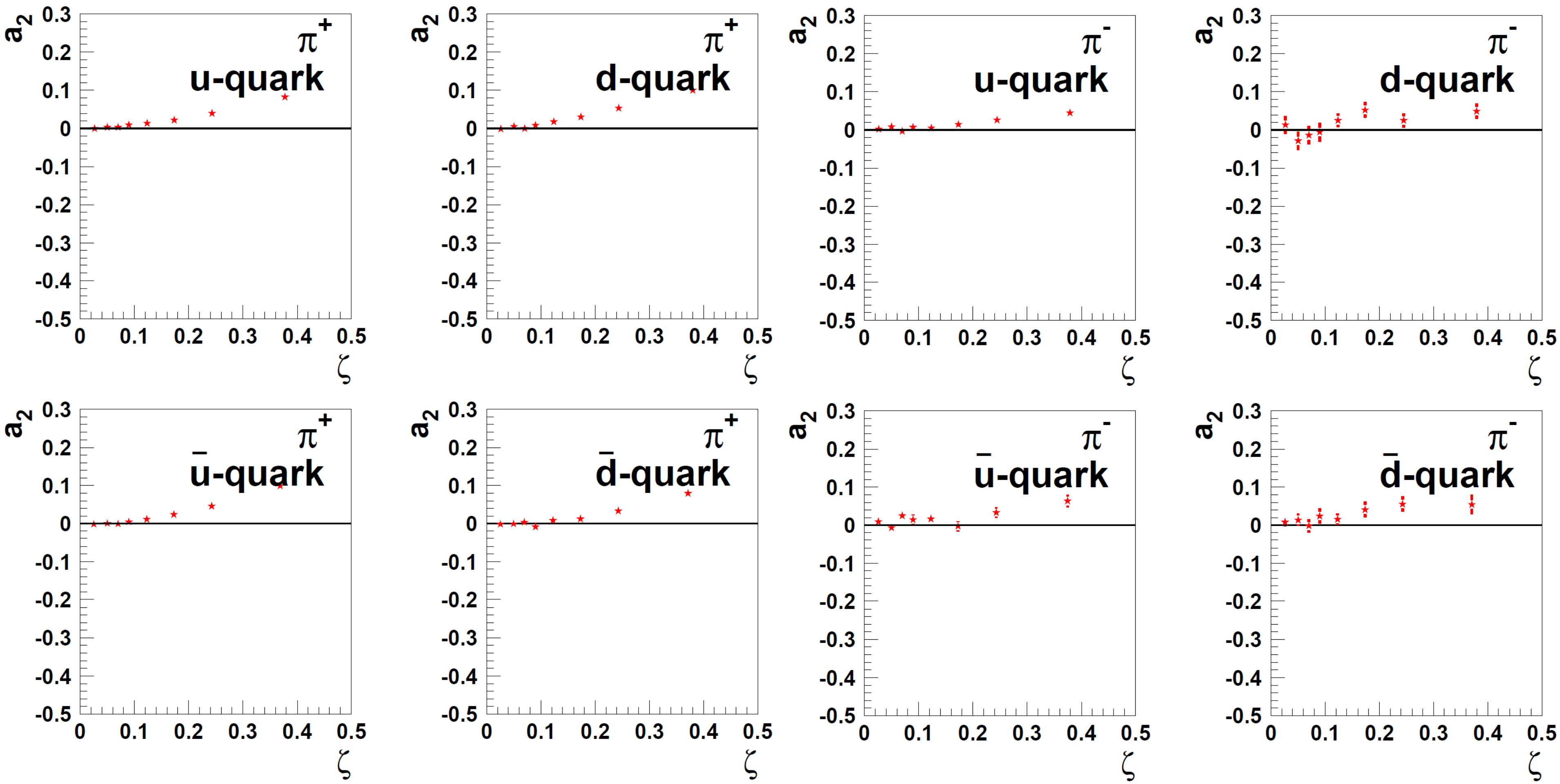
Hints from LEPTO: h-q azimuthal correlation, $a_1(\zeta)$

$$\hat{u}^{\pi^\pm}_{q/p}(x,k_T^2,\zeta,P_T^2,\mathbf{k}_T\cdot\mathbf{P}_T)=u^{\pi^\pm}_{q/p}(x,k_T^2,\zeta,P_T^2)\left(1+a_1\cos(\phi_h-\phi_q)+a_2\cos 2(\phi_h-\phi_q)+\cdots\right)$$

$$a_i=a_i(x,k_T^2,\zeta,P_T^2)$$

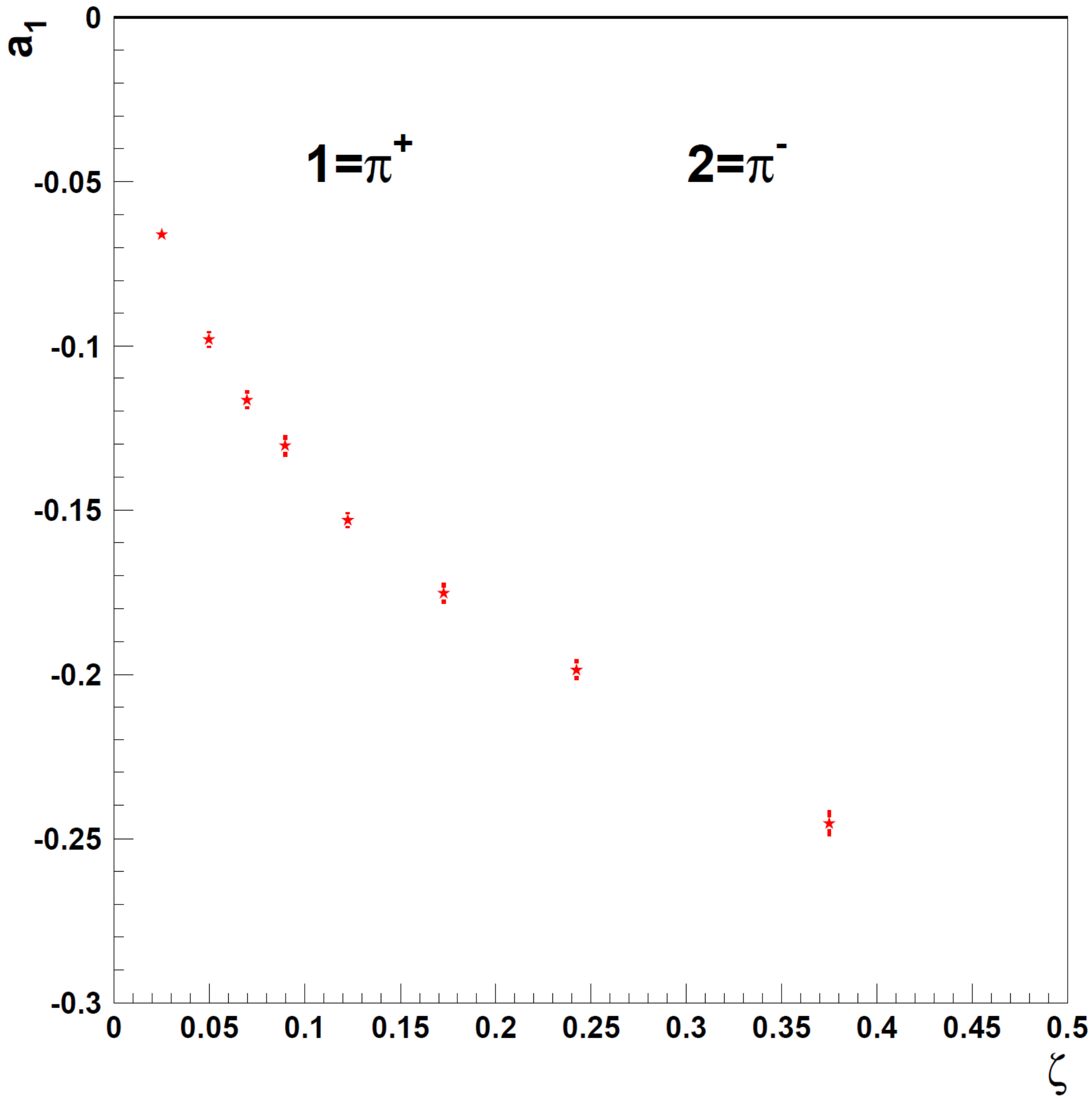


h-q azimuthal correlation, $a_2(\zeta)$



$$a_2 \ll a_1$$

h_1-h_2 azimuthal correlation, $a_1(\zeta)$

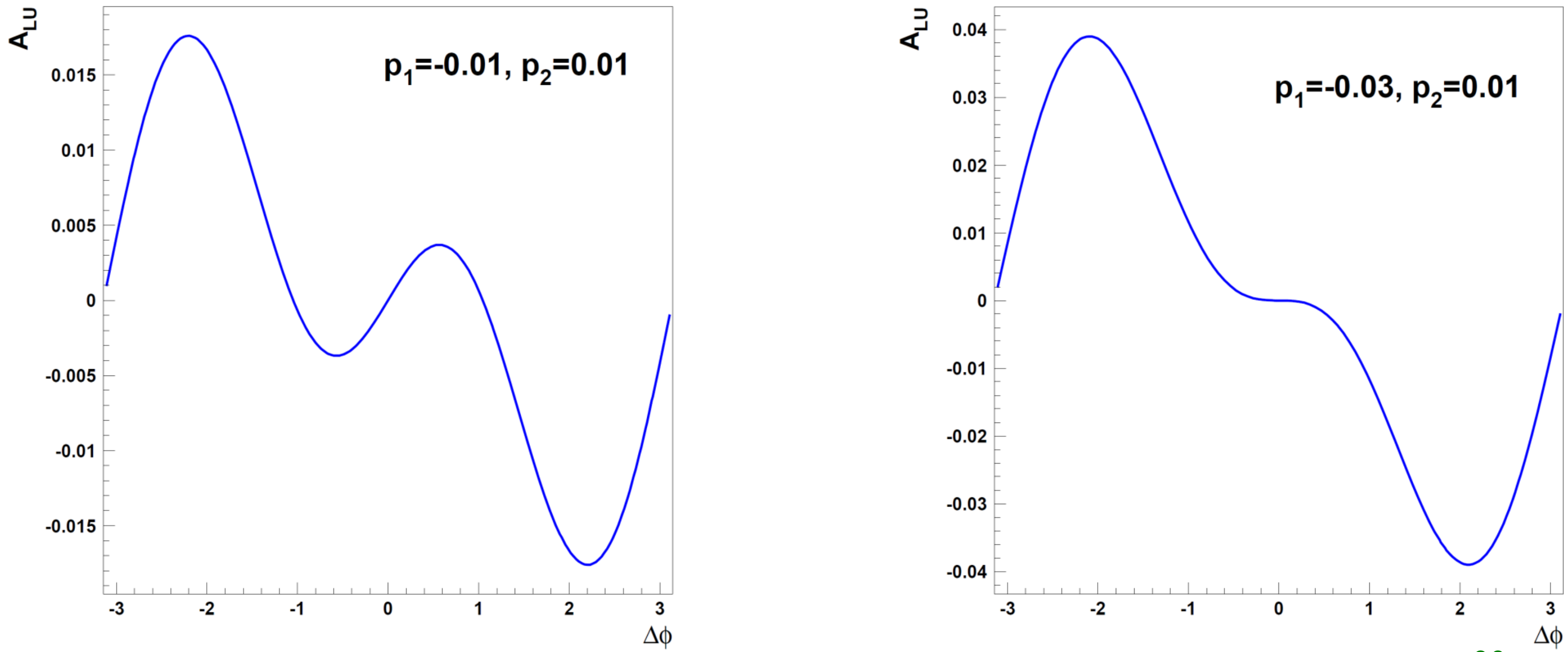


A_{LU}

$$A_{LU} = \frac{\sigma_{LU}(x,k_T^2,\zeta,P_T^2)\left(1+a_{LU1}\cos(\Delta\phi)+a_{LU2}\cos(2\Delta\phi)+\cdots\right)\sin(\Delta\phi)}{\sigma_{UU}(x,k_T^2,\zeta,P_T^2)\left(1+a_{UU1}\cos(\Delta\phi)+a_{UU2}\cos(2\Delta\phi)+\cdots\right)} \approx$$

$$\approx \frac{\sigma_{LU}(x,k_T^2,\zeta,P_T^2)}{\sigma_{UU}(x,k_T^2,\zeta,P_T^2)}\left(\sin(\Delta\phi)+\frac{1}{2}(a_{LU1}-a_{UU1})\sin(2\Delta\phi)+\cdots\right) \approx$$

$$\approx p_1\sin(\Delta\phi)+p_2\sin(2\Delta\phi)$$



CONCLUSIONS

- New members appeared in the polarized TMDs family -- 16 LO STMD fracture functions
- For SIDIS in the TFR, only 4 k_T-integrated fracture functions of unpolarized and longitudinally polarized quarks are probed.
  - SSA contains only a Sivers-type modulation $\sin(\varphi_h-\varphi_S)$ but no Collins-type $\sin(\varphi_h+\varphi_S)$ or $\sin(3\varphi_h-\varphi_S)$. The eventual observation of Collins-type asymmetry will indicate that LO factorized approach fails and long range correlations between the struck quark polarization and P_T of produced in TFR hadron might be important.
- DSIDIS cross section at LO contains 2 azimuthal independent and 20 azimuthally modulated terms.
- Integrated over one hadron transverse momentum DSIDIS cross-section expressions are rather simple. The measurements of these cross-sections allow to access transverse quark polarization and perform flavor separation.
- The ideal place to test the fracture functions factorization and measure these new nonperturbative objects are JLab12 and EIC facilities with full coverage of phase space.
- The role of STMF FracFuns in $pp \to \mu^+ \mu^- + h + X$, $pp \to h + X$. Factorization?

σ_{UL}

$$\sigma_{UL} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{u}_{1L}^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$

$$+D_{nn}\left(\frac{P_{T1}^2}{m_1 m_N} F_{kp1}^{\hat{t}_{1L}^{\perp}\cdot H_1} \sin(2\phi_1) + \frac{P_{T1}P_{T2}}{m_1 m_2} F_{p1}^{\hat{t}_{1L}^{h}\cdot H_1} \sin(\phi_1 + \phi_2) + \left(\frac{P_{T2}^2}{m_1 m_N} F_{kp2}^{\hat{t}_{1L}^{\perp}\cdot H_1} + \frac{P_{T2}^2}{m_1 m_2} F_{p2}^{\hat{t}_{1L}^{h}\cdot H_1} \right) \sin(2\phi_2) \right)$$

σ_{UT}

$$\sigma_{UT} = -\frac{P_{T1}}{m_N} F_{k1}^{\hat{u}_{1T}^{\perp}\cdot D_1} \sin(\phi_1 - \phi_S)$$

$$-\left(\frac{P_{T2}}{m_2} F_0^{\hat{u}_{1T}^{h}\cdot D_1} + \frac{P_{T2}}{m_N} F_{k2}^{\hat{u}_{1T}^{\perp}\cdot D_1} \right) \sin(\phi_2 - \phi_S)$$

$$+D_{nn}(y) \left[\begin{array}{l} \left(\frac{P_{T1}}{m_1} F_{p1}^{\hat{t}_{1T}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1 m_2^2} F_{p1}^{\hat{t}_{1T}^{hh}\cdot H_1} - \frac{P_{T1}P_{T2}^2}{2m_1 m_2 m_N} F_{kp3}^{\hat{t}_{1T}^{\perp h}\cdot H_1} + \frac{P_{T1}^3}{2m_1 m_N^2} F_{kkp1}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1 m_N^2} F_{kkp4}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T1}}{m_1 m_N^2} F_{kkp5}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_1 + \phi_S) \\ + \left(\frac{P_{T2}}{m_1} F_{p2}^{\hat{t}_{1T}\cdot H_1} + \frac{P_{T2}^3}{2m_1 m_2^2} F_{p2}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T1}^2 P_{T2}}{2m_1 m_2 m_N} F_{kp1}^{\hat{t}_{1T}^{\perp h}\cdot H_1} + \frac{P_{T2}}{m_1 m_2 m_N} F_{kp4}^{\hat{t}_{1T}^{\perp h}\cdot H_1} + \frac{P_{T1}^2 P_{T2}}{2m_1 m_N^2} F_{kkp2}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} + \frac{P_{T2}^3}{2m_1 m_N^2} F_{kkp3}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \; \frac{P_{T2}}{m_1 m_N^2} F_{kkp6}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_2 + \phi_S) \\ + \frac{P_{T1}^3}{2m_1 m_N^2} F_{kkp1}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \sin(3\phi_1 - \phi_S) \\ + \left(\frac{P_{T2}^3}{2m_1 m_2^2} F_{p2}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T2}^3}{2m_1 m_N^2} F_{kkp3}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(3\phi_2 - \phi_S) \\ + \left(\frac{P_{T1}P_{T2}^2}{2m_1 m_2^2} F_{p1}^{\hat{t}_{1T}^{hh}\cdot H_1} + \frac{P_{T1}P_{T2}^2}{2m_1 m_N^2} F_{kkp4}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \right) \sin(\phi_1 + 2\phi_2 - \phi_S) \\ - \frac{P_{T1}^2 P_{T2}}{2m_1 m_2 m_N} F_{kp1}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \sin(2\phi_1 - \phi_2 + \phi_S) \\ - \frac{P_{T1}P_{T2}^2}{2m_1 m_2 m_N} F_{kp3}^{\hat{t}_{1T}^{\perp h}\cdot H_1} \sin(\phi_1 - 2\phi_2 - \phi_S) \\ + \frac{P_{T1}^2 P_{T2}}{2m_1 m_N^2} F_{kkp2}^{\hat{t}_{1T}^{\perp\perp}\cdot H_1} \sin(2\phi_1 + \phi_2 - \phi_S) \end{array} \right]$$

σ_{LU}, σ_{LL}, σ_{LT}

$$\sigma_{LU} = -\frac{P_{T1}P_{T2}}{m_2 m_N} F_{k1}^{\hat{l}_1^{\perp h}\cdot D_1} \sin(\phi_1 - \phi_2)$$

$$\sigma_{LL} = F_0^{\hat{l}_1 \cdot D_1}$$

$$\sigma_{LT} = \frac{P_{T1}}{m_N} F_{k1}^{\hat{l}_{1T}^{\perp}\cdot D_1} \cos(\phi_1 - \phi_S) + \left(\frac{P_{T2}}{m_2} F_0^{\hat{l}_{1T}^{h}\cdot D_1} + \frac{P_{T2}}{m_N} F_{k2}^{\hat{l}_{1T}^{\perp}\cdot D_1} \right) \cos(\phi_2 - \phi_S)$$

Convolutions & tensorial decomposition

$$C[\hat{M}\cdot Dw]=\sum_a e_a^2\int d^2k_T\,d^2p_T\,\delta^{(2)}(z\mathbf{k}_T+\mathbf{p}_T-\mathbf{P}_{T1})\hat{M}_a(x,\zeta,k_T^2,P_{T2}^2,\mathbf{k}_T\cdot\mathbf{P}_{T2})D_a(z,p_T^2)w$$

$$C[\hat{M}\cdot D]=F_0^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^i]=P_{T1}^i F_{k1}^{\hat{M}\cdot D}+P_{T2}^i F_{k2}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dp^i]=P_{T1}^i F_{p1}^{\hat{M}\cdot D}+P_{T2}^i F_{p2}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^ik^j]=P_{T1}^iP_{T1}^j F_{kk1}^{\hat{M}\cdot D}+P_{T2}^iP_{T2}^j F_{kk2}^{\hat{M}\cdot D}+\delta^{ij}F_{kk3}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^ip^j]=P_{T1}^iP_{T1}^j F_{kp1}^{\hat{M}\cdot D}+P_{T2}^iP_{T2}^j F_{kp2}^{\hat{M}\cdot D}+\left(P_{T1}^iP_{T2}^j-P_{T1}^jP_{T2}^i\right)F_{kp3}^{\hat{M}\cdot D}+\delta^{ij}F_{kp4}^{\hat{M}\cdot D}$$

$$C[\hat{M}\cdot Dk^ik^jp^k]=P_{T1}^iP_{T1}^jP_{T1}^k F_{kkp1}^{\hat{M}\cdot D}+P_{T1}^iP_{T1}^jP_{T2}^k F_{kkp2}^{\hat{M}\cdot D}+P_{T2}^iP_{T2}^jP_{T2}^k F_{kkp3}^{\hat{M}\cdot D}$$

$$+P_{T2}^iP_{T2}^jP_{T1}^k F_{kkp4}^{\hat{M}\cdot D}+P_{T1}^k\delta^{ij}F_{kkp5}^{\hat{M}\cdot D}+P_{T2}^k\delta^{ij}F_{kkp6}^{\hat{M}\cdot D}$$

where $F_{\dots}^{\hat{M}\cdot D}$ depend on $x,z,\zeta,P_{T1}^2,P_{T2}^2,(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})$

Structure functions

$$F_{k1}^{\hat{M}\cdot D}=C\left[\hat{M}\cdot D\frac{(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})(\mathbf{P}_{T2}\cdot\mathbf{k})-(\mathbf{P}_{T1}\cdot\mathbf{k})\mathbf{P}_{T2}^2}{\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2}\right]$$

$$F_{k2}^{\hat{M}\cdot D}=C\left[\hat{M}\cdot D\frac{(\mathbf{P}_{T1}\cdot\mathbf{k})(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2})-(\mathbf{P}_{T2}\cdot\mathbf{k})\mathbf{P}_{T1}^2}{\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2}\right]$$

$$F_{kk1}^{\hat{M}\cdot D}=C\left[\hat{M}\cdot D\frac{\left(-2\left(\mathbf{P}_{T1}\cdot\mathbf{k}\right)^2+\mathbf{k}^2\mathbf{P}_{T1}^2\right)\mathbf{P}_{T2}^4+\left(2\left(\mathbf{P}_{T2}\cdot\mathbf{k}\right)^2-\mathbf{k}^2\mathbf{P}_{T2}^2\right)\left(2\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}{4\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2\left(\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}\right]$$

$$F_{kk2}^{\hat{M}\cdot D}=C\left[\hat{M}\cdot D\frac{\left(2\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)\left(\mathbf{P}_{T1}\cdot\mathbf{k}\right)^2+\mathbf{P}_{T1}^2\left(\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2-\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2\right)\mathbf{k}^2-\left(\mathbf{P}_{T2}\cdot\mathbf{k}\right)^2\mathbf{P}_{T1}^4}{2\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2\left(\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}\right]$$

$$F_{kk3}^{\hat{M}\cdot D}=C\left[\hat{M}\cdot D\frac{\left(\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2+\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)\mathbf{k}^2-\left(\mathbf{P}_{T2}\cdot\mathbf{k}\right)^2\mathbf{P}_{T1}^2-\left(\mathbf{P}_{T1}\cdot\mathbf{k}\right)^2\mathbf{P}_{T2}^2}{2\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2}\right]$$

$$F_{kp1}^{\hat{M}\cdot D}=C\left[\hat{M}\cdot D\left(\frac{\left(-2(\mathbf{P}_{T1}\cdot\mathbf{k})(\mathbf{P}_{T1}\cdot\mathbf{p})+(\mathbf{k}\cdot\mathbf{p})\mathbf{P}_{T1}^2\right)\mathbf{P}_{T2}^4}{4\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2\left(\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^4-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}+\frac{\left(2(\mathbf{P}_{T2}\cdot\mathbf{k})(\mathbf{P}_{T2}\cdot\mathbf{p})-(\mathbf{k}\cdot\mathbf{p})\mathbf{P}_{T2}^2\right)\left(2\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}{4\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^2\left(\left(\mathbf{P}_{T1}\cdot\mathbf{P}_{T2}\right)^4-\mathbf{P}_{T1}^2\mathbf{P}_{T2}^2\right)}\right)\right]$$