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ECSAC
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Hadron tomography through Wigner distributions

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[C.L., Pasquini, Vanderhaeghen (2011)]
[C.L., Pasquini (2011)]
[C.L., Pasquini, Xiong, Yuan (in preparation)]

Outline

- Wigner distributions
  - Parton distributions in phase space
- Model predictions
  - Light-cone wave function approach
- Orbital angular momentum
  - Comparison of different definitions

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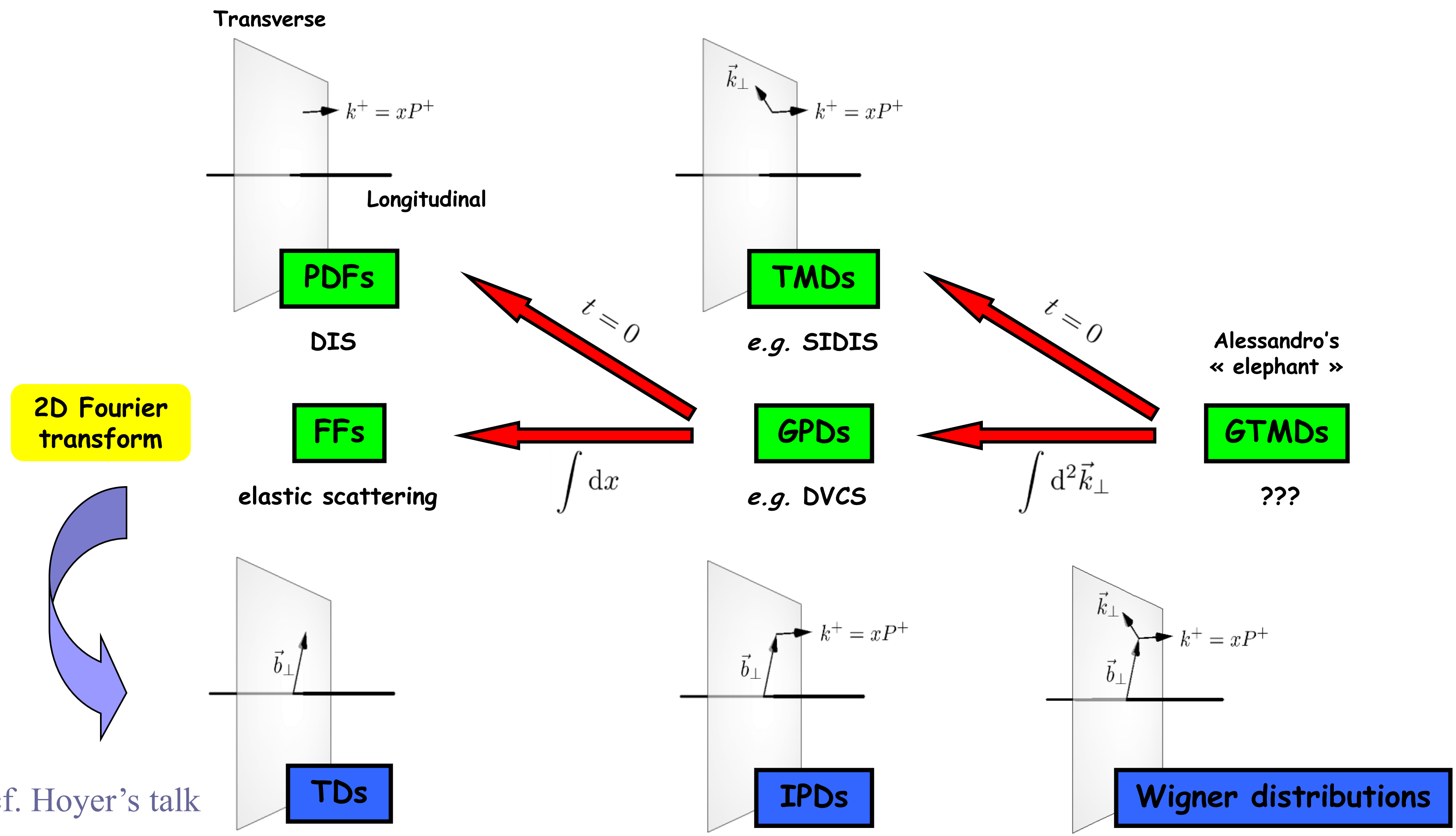
Parton distributions

cf. Bacchetta's talk



cf. Hoyer's talk

[Miller (2007)]
[Carlson, Vanderhaeghen (2008)]
[Alexandrou *et al.* (2009,2010)]

[Burkardt (2000,2003)]

[C.L., Pasquini, Vanderhaeghen (2011)]
[C.L., Pasquini (2011)]

Wigner distributions

Definition of the GTMD correlator

[Meißner, Metz, Schlegel (2009)]

cf. Schlegel's talk

Quark polarization

Gauge link

$$W^{[\Gamma]}(x,\vec{k}_\perp,\xi,\vec{\Delta}_\perp,\vec{S}) = \frac{1}{2}\int\frac{\mathrm{d}z^-\,\mathrm{d}^2z_\perp}{(2\pi)^3}\,e^{i(xP^+z^- - \vec{k}_\perp\cdot\vec{z}_\perp)}\,\langle p'^+,\tfrac{\vec{\Delta}_\perp}{2},\vec{S}|\overline{\psi}(-\tfrac{z}{2})\Gamma\mathcal{W}\psi(\tfrac{z}{2})|p^+,-\tfrac{\vec{\Delta}_\perp}{2},\vec{S}\rangle\Big|_{z^+=0}$$

Nucleon polarization

$$P^+ = \frac{p'^+ + p^+}{2} \qquad \xi = \frac{p'^+ - p^+}{p'^+ + p^+}$$

4 X 4 = 16 polarization configurations ⟷ 16 complex GTMDs

(at leading twist)

Definition of the Wigner distributions

Non-relativistic : [Ji (2003)] [Belitsky, Ji, Yuan (2004)]

Relativistic : [C.L., Pasquini, Vanderhaeghen (2011)] [C.L., Pasquini (2011)]

$$\rho_W^{[\Gamma]}(x,\vec{k}_\perp,\vec{b}_\perp,\vec{S}) = \int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\,e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\,W^{[\Gamma]}(x,\vec{k}_\perp,0,\vec{\Delta}_\perp,\vec{S})$$

16 real Wigner distributions

(at leading twist)

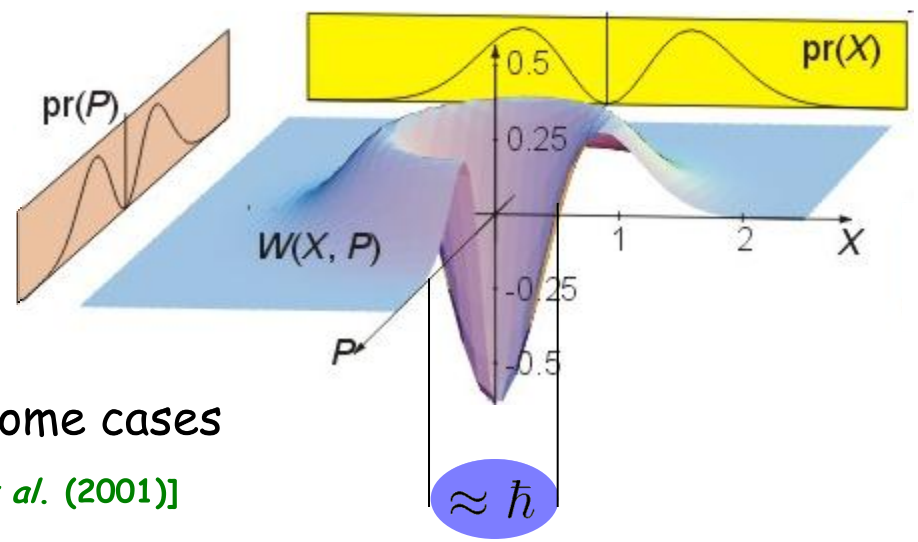
Wigner distributions

[Wigner (1932)]
[Moyal (1949)]

Wigner distributions have applications in:

- Nuclear physics
- Quantum chemistry
- Quantum molecular dynamics
- Quantum information
- Quantum optics
- Classical optics
- Signal analysis
- Image processing
- Heavy ion collisions
- …

E.g. [Antonov *et al.* (1980-1989)]



They can even be « measured » in some cases

E.g. [Lvovsky *et al.* (2001)]

Heisenberg's uncertainty relations → Quasi-probability distribution

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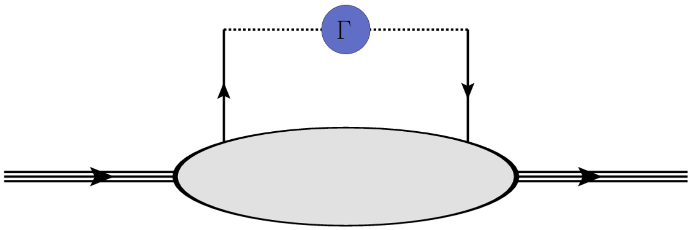
Overlap representation [C.L., Pasquini, Vanderhaeghen (2011)]

Drell-Yan-West frame $\Delta^+ = 0$



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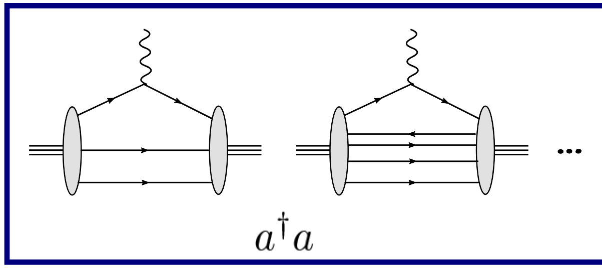
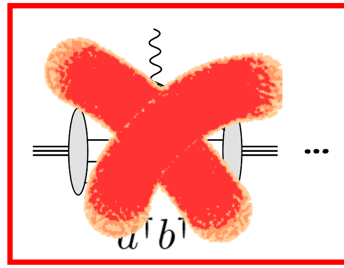
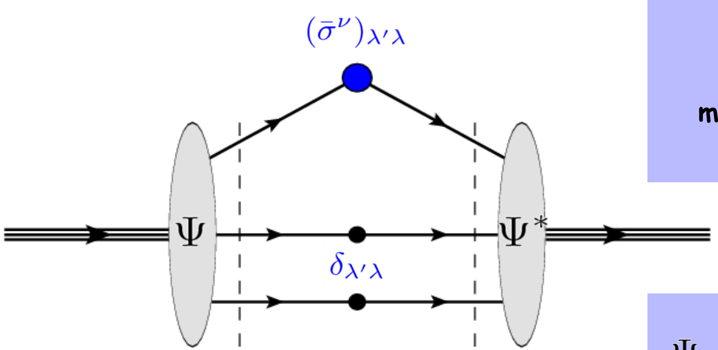




Quark-quark correlator



- σ Canonical spin
- λ LC helicity

- Light-cone constituent quark model

$$\Psi_{\rm LCCQM} = \psi \sum_{\sigma_1\sigma_2\sigma_3} \Phi^{\Lambda}_{\sigma_1\sigma_2\sigma_3} D^{(1/2)*}_{\sigma_1\lambda_1} D^{(1/2)*}_{\sigma_2\lambda_2} D^{(1/2)*}_{\sigma_3\lambda_3}$$

Symmetric momentum WF

SU(6) WF

Melosh rotation

$$D^{(1/2)*}_{\sigma\lambda} = \frac{1}{\sqrt{(m+x\mathcal{M}_0)^2 + k_\perp^2}} \begin{pmatrix} m + x\mathcal{M}_0 & -k_R \\ k_L & m + x\mathcal{M}_0 \end{pmatrix}_{\sigma\lambda}$$

- Chiral quark-soliton model

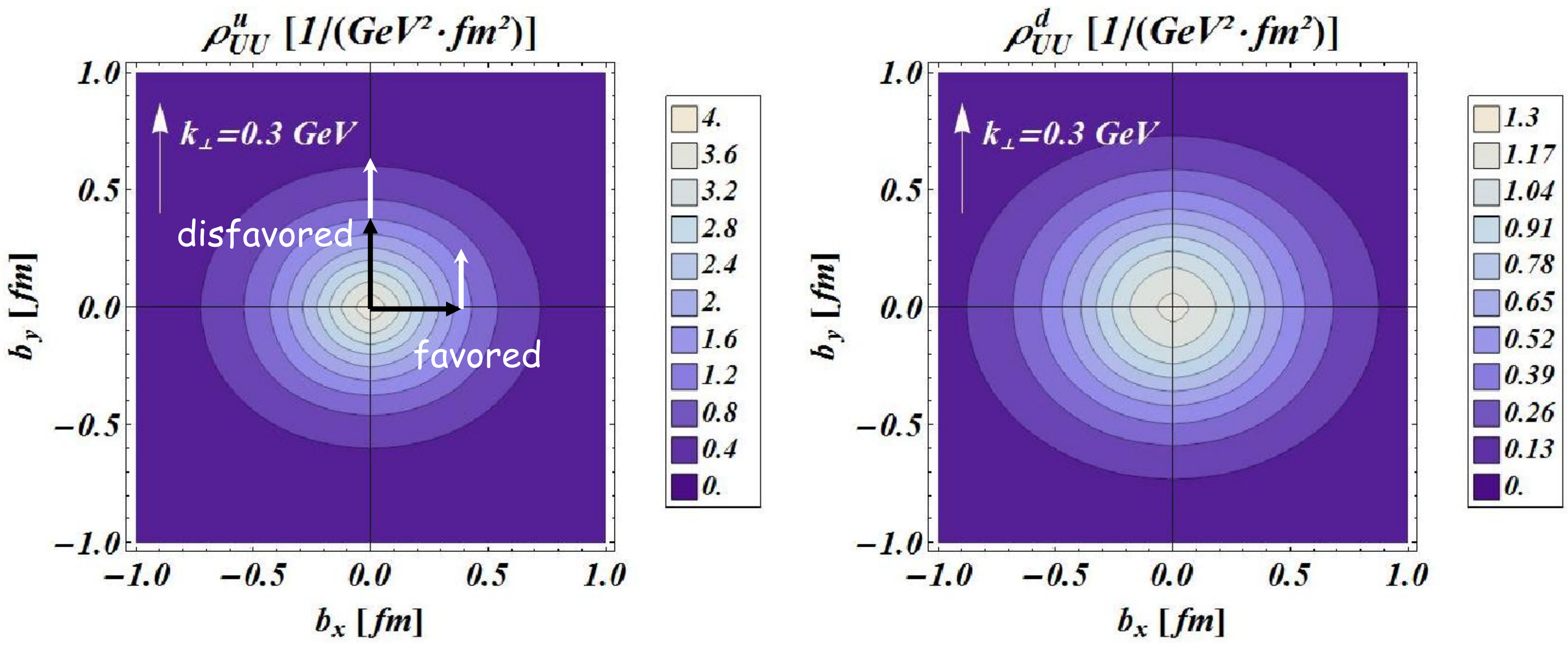
$$\Psi_{\chi{\rm QSM}} = \sum_{\sigma_1\sigma_2\sigma_3} \Phi^{\Lambda}_{\sigma_1\sigma_2\sigma_3} F_{\sigma_1\lambda_1} F_{\sigma_2\lambda_2} F_{\sigma_3\lambda_3}$$

Valence WF

$$F_{\sigma\lambda} = \frac{1}{\sqrt{2}} \begin{pmatrix} h + \frac{k_z}{k}\, j & -\frac{k_R}{k}\, j \\ \frac{k_L}{k}\, j & h + \frac{k_z}{k}\, j \end{pmatrix}_{\sigma\lambda}$$

Unpol. quark in unpol. proton

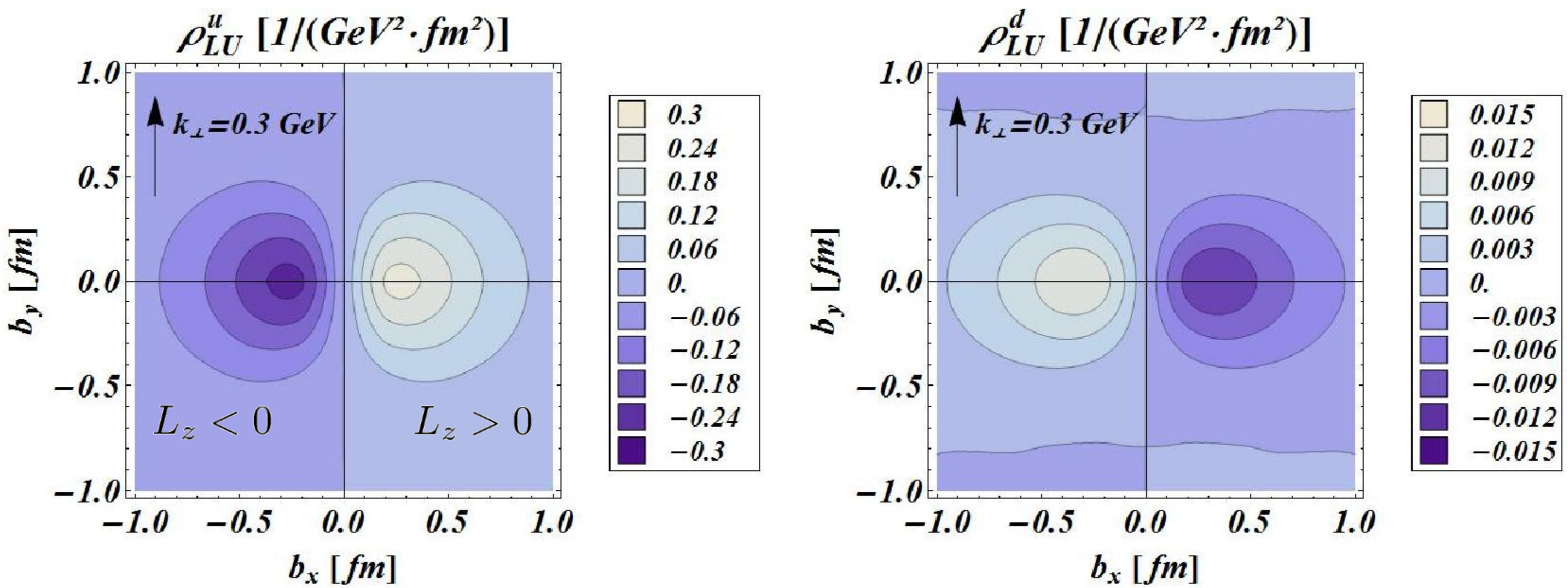
[C.L., Pasquini (2011)]

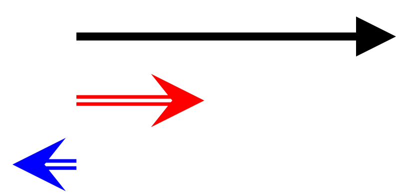


Left-right symmetry → no net quark OAM

Unpol. quark in long. pol. proton

[C.L., Pasquini (2011)]





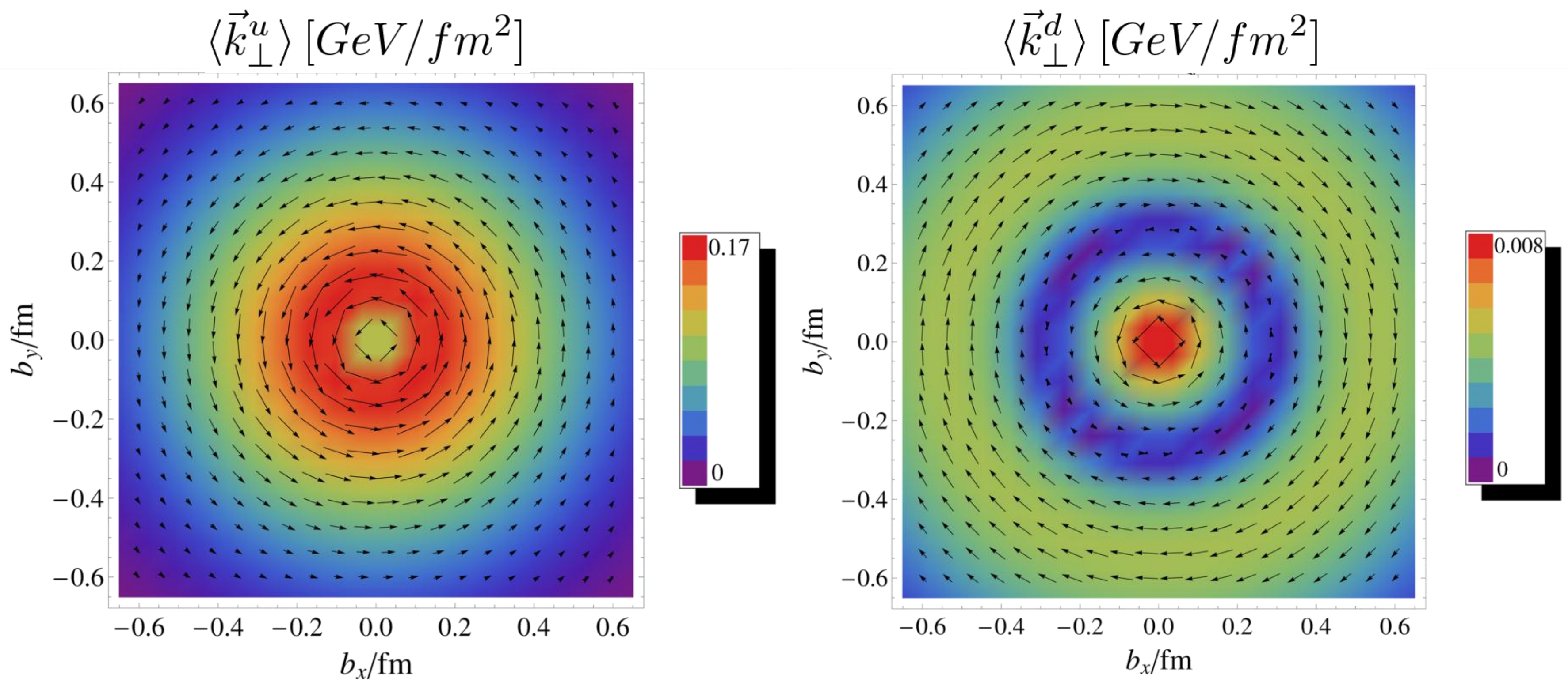
Proton spin

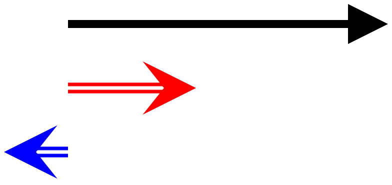
u-quark OAM

d-quark OAM

Unpol. quark in long. pol. proton

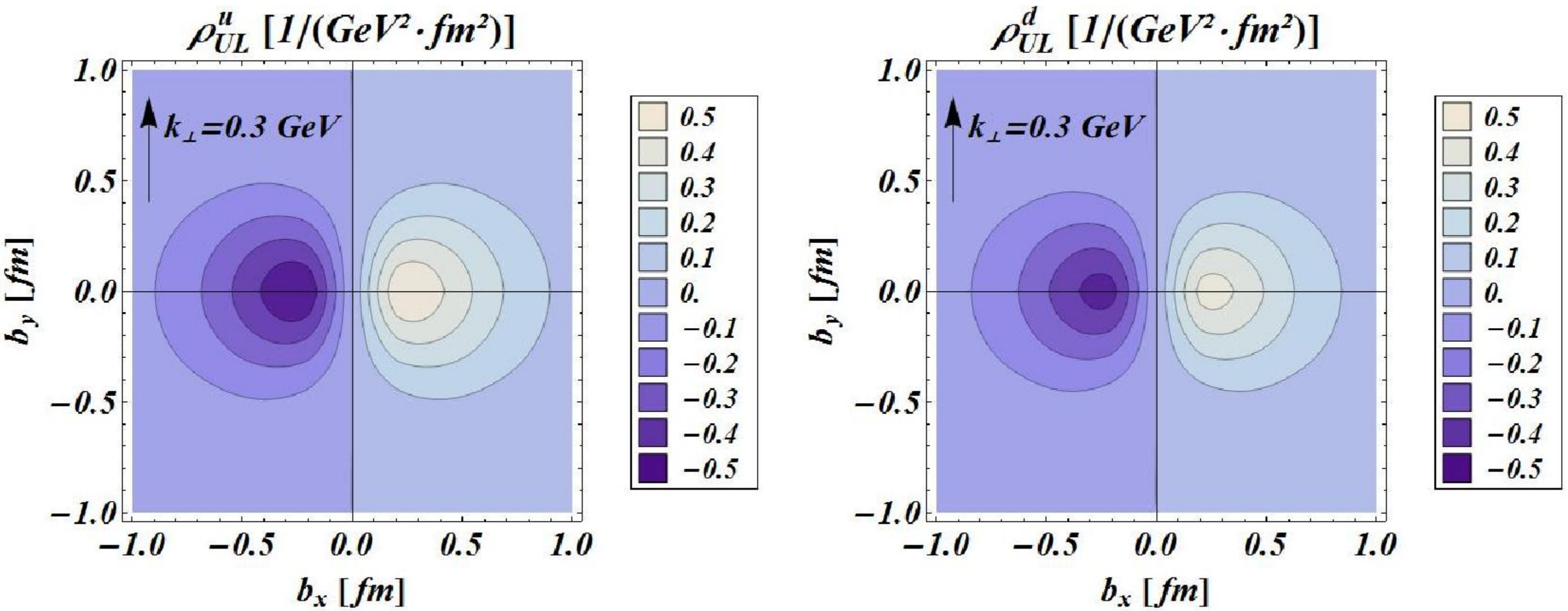
[C.L., Pasquini, Xiong, Yuan (in preparation)]

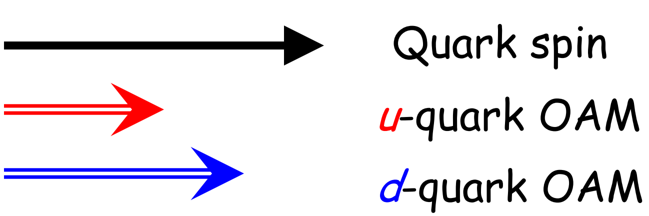




Long. pol. quark in unpol. proton

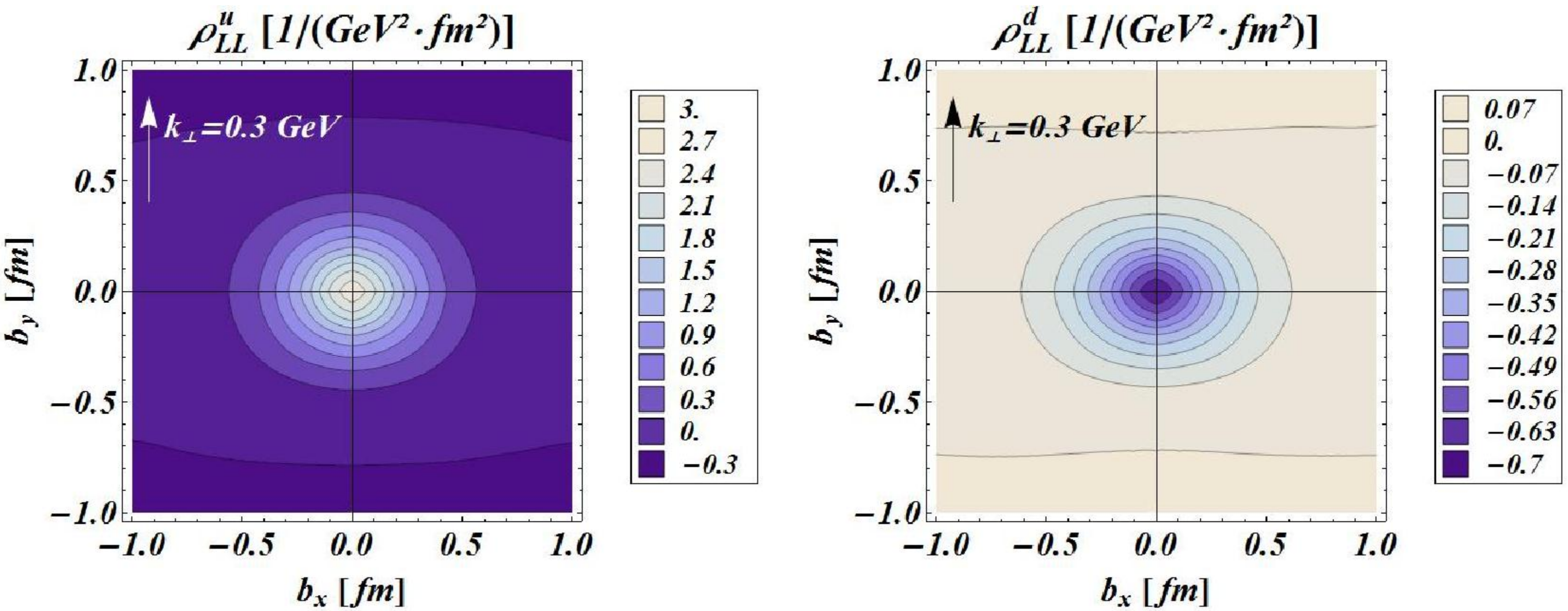
[C.L., Pasquini (2011)]

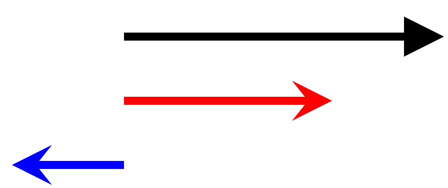




Long. pol. quark in long. pol. proton

[C.L., Pasquini (2011)]





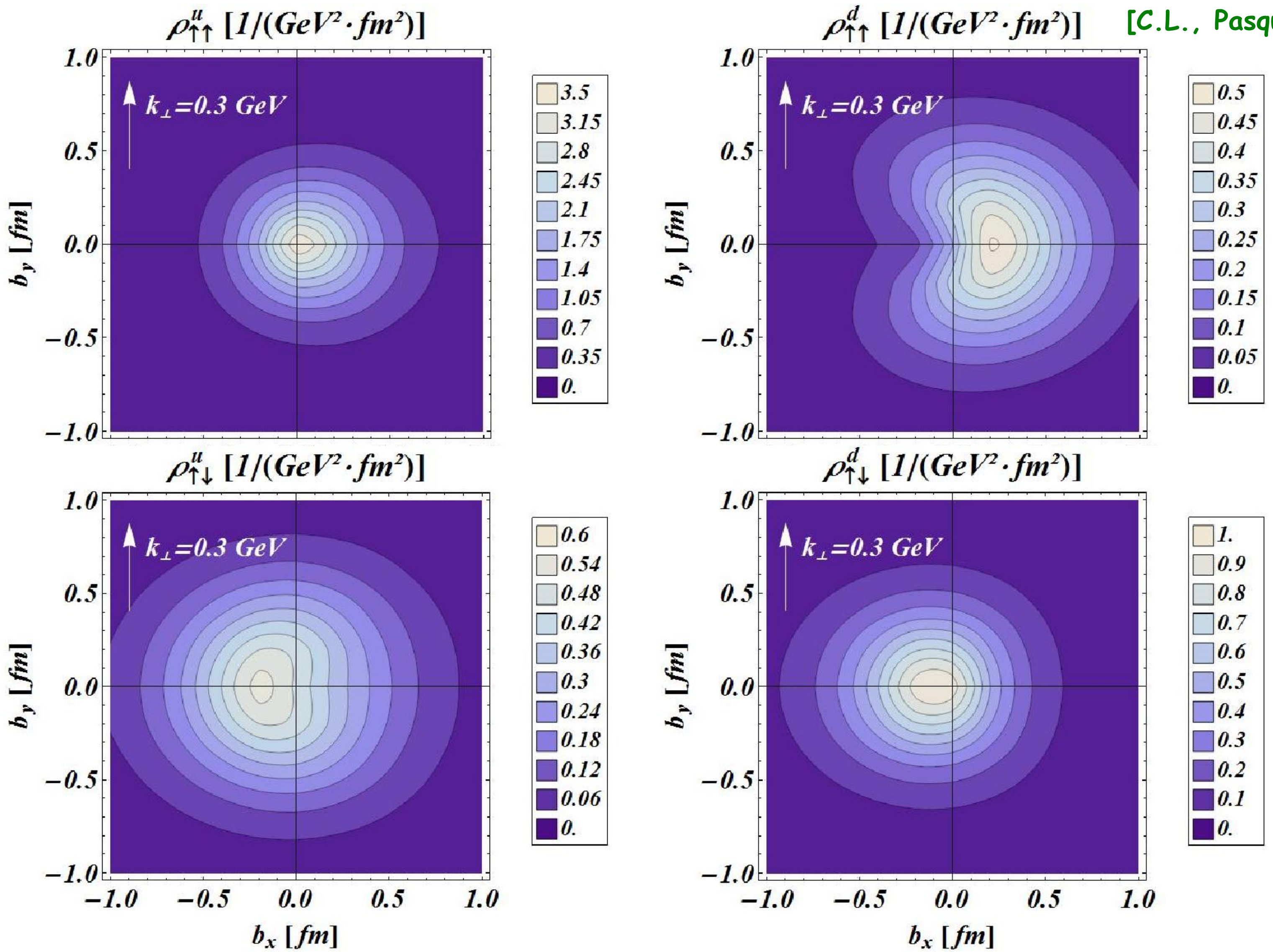
Proton spin

u-quark spin

d-quark spin

Long. pol. quark in long. pol. proton

[C.L., Pasquini (2011)]



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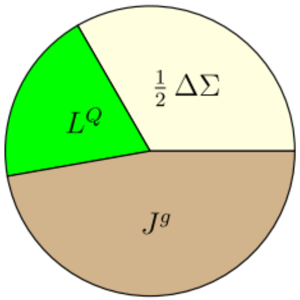
Angular momentum

cf. Burkardt, Leader and Wakamatsu’s talks

Ji



$$\vec{J}_{\rm QCD} = \frac{1}{2}\int \mathrm{d}^3r\,\psi^\dagger\vec{\Sigma}\psi + \int \mathrm{d}^3r\,\psi^\dagger\vec{r}\times(-i\vec{D})\psi + \int \mathrm{d}^3r\,\vec{r}\times(\vec{E}^a\times\vec{B}^a)$$

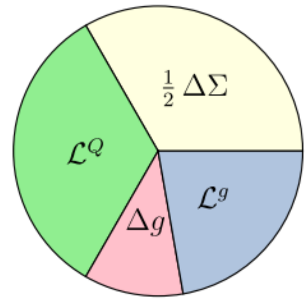
- Each term is gauge-invariant
- No decomposition of $\vec{J}^g$

Ji’s sum rule

$$L_z^Q = \underbrace{\frac{1}{2}\int \mathrm{d}x\,x\,\big[\overbrace{H^Q(x,0,0)}^{\text{Unpol. quark in trans. pol. proton}} + E^Q(x,0,0)\big]}_{J_z^Q} - \underbrace{\frac{1}{2}\int \mathrm{d}x\,\Delta q(x)}_{S_z^Q}$$

GPDs

Jaffe-Manohar



$$\vec{J}_{\rm QCD} = \frac{1}{2}\int \mathrm{d}^3r\,\psi^\dagger\vec{\Sigma}\psi + \int \mathrm{d}^3r\,\psi^\dagger\vec{r}\times(-i\vec{\nabla})\psi + \int \mathrm{d}^3r\,\vec{E}^a\times\vec{A}^a + \int \mathrm{d}^3r\,E^{ai}\vec{r}\times\vec{\nabla}A^{ai}$$

- Decomposition is gauge-dependent
- OAM in LCWFs refers to $\mathcal{L}_z^Q$ (easy)

Pretzelosity

[Avakian & *al.* (2010)]

$$\mathcal{L}_z^Q(x,\vec{k}_\perp) = -\frac{k_\perp^2}{2m^2}\,\underbrace{h_{1T}^{\perp Q}(x,\vec{k}_\perp)}_{\text{Trans. pol. quark in trans. pol. proton}}$$

Model-dependent!

TMDs

Orbital angular momentum

[C.L., Pasquini (2011)]
[C.L., Pasquini, Xiong, Yuan (in preparation)]

Definition of the OAM

OAM operator :

$$\widehat{L}_z^q = -\frac{i}{2}\int \mathrm{d}^3r\,\overline{\psi}^q\gamma^+\left(\vec{r}\times\overleftrightarrow{\nabla}_r\right)_z\psi^q$$

Unambiguous in absence of gauge fields

$$\langle p',\uparrow|\widehat{L}_z^q|p,\uparrow\rangle = \ell_z^q\,\langle p',\uparrow|p,\uparrow\rangle$$

State normalization

$$2P^+\,\delta(\Delta^+)\,(2\pi)^3\,\delta^{(2)}(\vec{\Delta}_\perp)$$

No infinite normalization constants

No wave packets

$$\ell_z^q = \int\frac{\mathrm{d}\Delta^+\,\mathrm{d}^2\Delta_\perp}{2P^+(2\pi)^3}\langle p',\uparrow|\widehat{L}_z^q|p,\uparrow\rangle$$

$$= \int \mathrm{d}x\,\mathrm{d}^2k_\perp\,\mathrm{d}^2b_\perp\left(\vec{b}_\perp\times\vec{k}_\perp\right)_z\rho^{[\gamma^+]}(x,\vec{k}_\perp,\vec{b}_\perp,\vec{e}_z)$$

Unpol. quark in long. pol. proton

Wigner distributions

Orbital angular momentum

[C.L., Pasquini (2011)]
[C.L., Pasquini, Xiong, Yuan (in preparation)]

Overlap representation

$$\ell_z^{N\beta,q} = -\frac{i}{2}\int [\mathrm{d}x]_N\,[\mathrm{d}^2k_\perp]_N \sum_{i=1}^N \delta_{qq_i} \sum_{n=1}^N (\delta_{ni}-x_n)\left[\Psi^{*\uparrow}_{N\beta}\left(\vec{k}_i\times \overleftrightarrow{\nabla}_{k_n}\right)_z \Psi^{\uparrow}_{N\beta}\right]$$

Jaffe-Manohar

Spectators are involved!

$$\mathcal{L}_z^{N\beta,q} = -\frac{i}{2}\int [\mathrm{d}x]_N\,[\mathrm{d}^2k_\perp]_N \sum_{i=1}^N \delta_{qq_i} \left[\Psi^{*\uparrow}_{N\beta}\left(\vec{k}_i\times \overleftrightarrow{\nabla}_{k_i}\right)_z \Psi^{\uparrow}_{N\beta}\right]$$

Naive

$$L_z^{N\beta,q} = \frac{1}{2}\int [\mathrm{d}x]_N\,[\mathrm{d}^2k_\perp]_N \sum_{i=1}^N \delta_{qq_i} \left\{(x_i-\lambda_i)|\Psi^{\uparrow}_{N\beta}|^2 + Mx_i\sum_{n=1}^N (\delta_{ni}-x_n)\left[\Psi^{*\uparrow}_{N\beta}\,\frac{\overleftrightarrow{\partial}}{\partial k_n^x}\,\Psi^{\downarrow}_{N\beta}\right]\right\}$$

Ji

NB : LCWFs are eigenstates of **total** OAM

$$-i\sum_{n=1}^N\left(\vec{k}_n\times\vec{\nabla}_{k_n}\right)_z \Psi^{\Lambda}_{\lambda_1\cdots\lambda_N} = l_z\,\Psi^{\Lambda}_{\lambda_1\cdots\lambda_N} \qquad l_z = \left(\Lambda - \sum_{n=1}^N \lambda_n\right)/2$$

For **total** OAM we find

$$\ell_z^N = \mathcal{L}_z^N = L_z^N = \sum_{\lambda_1\cdots\lambda_N} l_z \int [\mathrm{d}x]_N\,[\mathrm{d}^2k_\perp]_N \left|\Psi^{\uparrow}_{\lambda_1\cdots\lambda_N}\right|^2 = \sum_{l_z} l_z\ {}^{l_z}\langle P,\uparrow|P,\uparrow\rangle^{l_z}$$

Comparison using quark models

Scalar quark-diquark model : $\ell_z^q = L_z^q$ (after regularization) [Burkardt, Hikmat (2009)]

MIT Bag model : $\ell_z^q = \mathcal{L}_z^q = -\int \mathrm{d}x\,\mathrm{d}^2k_\perp\,\frac{k_\perp^2}{2M^2}\,h_{1T}^\perp$ [Avakian & *al.* (2010)]

(no spectators)

[C.L., Pasquini (2011)]

Model	LCCQM			χQSM		
q	u	d	Total	u	d	Total
ℓ_z^q	0.131	−0.005	0.126	0.073	−0.004	0.069
L_z^q	0.071	0.055	0.126	−0.008	0.077	0.069
$\mathcal{L}_z^q$	0.169	−0.042	0.126	0.093	−0.023	0.069
κ^q	1.867	−1.579	0.288	1.766	−1.551	0.215

Jaffe-Manohar — Wigner distributions

Ji — GPDs

Naive — TMDs

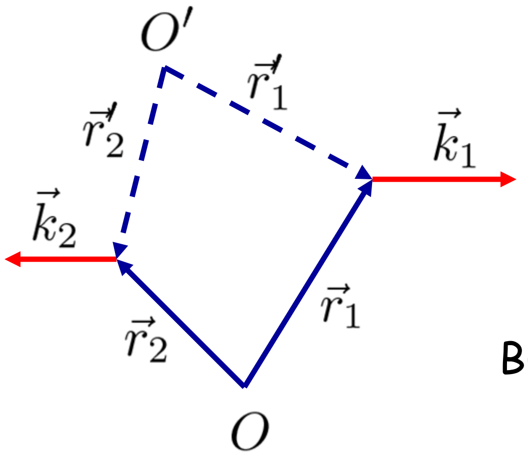
Spherically symmetric quark models : $\mathcal{L}_z^q = -\int \mathrm{d}x\,\mathrm{d}^2k_\perp\,\frac{k_\perp^2}{2M^2}\,h_{1T}^\perp$

[C.L., Pasquini, Xiong, Yuan (in preparation)]

Possible origin of the differences I

[C.L., Pasquini, Xiong, Yuan (in preparation)]

OAM depends on the origin



$$l_{z1} \neq l'_{z1} \qquad l_{z2} \neq l'_{z2}$$

But if $\vec{k}_1 + \vec{k}_2 = \vec{0}$ ⟹ $l_{z1} + l_{z2} = l'_{z1} + l'_{z2}$

Naive	~	$\left(\vec{r}_i \times \vec{k}_i\right)_z$
Jaffe-Manohar	~	$\left[(\vec{r}_i - \vec{R}) \times \vec{k}_i\right]_z$
Ji	???	

$$\vec{R}_\perp = \sum_{n=1}^{N} x_n \vec{r}_{\perp n}$$

Transverse center of momentum

Possible origin of the differences II

Jaffe-Manohar

Canonical energy-momentum tensor

$$T_C^{\mu\nu}(x)$$

OAM density operator

$$M_{\text{orb}}^{\mu\nu\lambda}(x) = x^\nu T_C^{\mu\lambda}(x) - x^\lambda T_C^{\mu\nu}(x)$$

Total AM density operator

$$J^{\mu\nu\lambda}(x) = M_{\text{orb}}^{\mu\nu\lambda}(x) + M_{\text{spin}}^{\mu\nu\lambda}(x)$$

Ji

Belifante energy-momentum tensor

$$T^{\mu\nu}(x) = \tfrac{1}{2}\left[T_C^{\mu\nu}(x) + T_C^{\nu\mu}(x)\right]$$

Total AM density operator

$$M^{\mu\nu\lambda}(x) = x^\nu T^{\mu\lambda}(x) - x^\lambda T^{\mu\nu}(x)$$

$$\boxed{M^{0ij}(x) = J^{0ij}(x) + [\text{EOM terms}] + [\text{divergence terms}]}$$

$$\langle P,\uparrow|\int \mathrm{d}^3r\, M^{0ij}(\vec{r},0)|P,\uparrow\rangle \overset{?}{=} \langle P,\uparrow|\int \mathrm{d}^3r\, J^{0ij}(\vec{r},0)|P,\uparrow\rangle$$

[Bakker, Leader, Trueman (2004)]

Summary

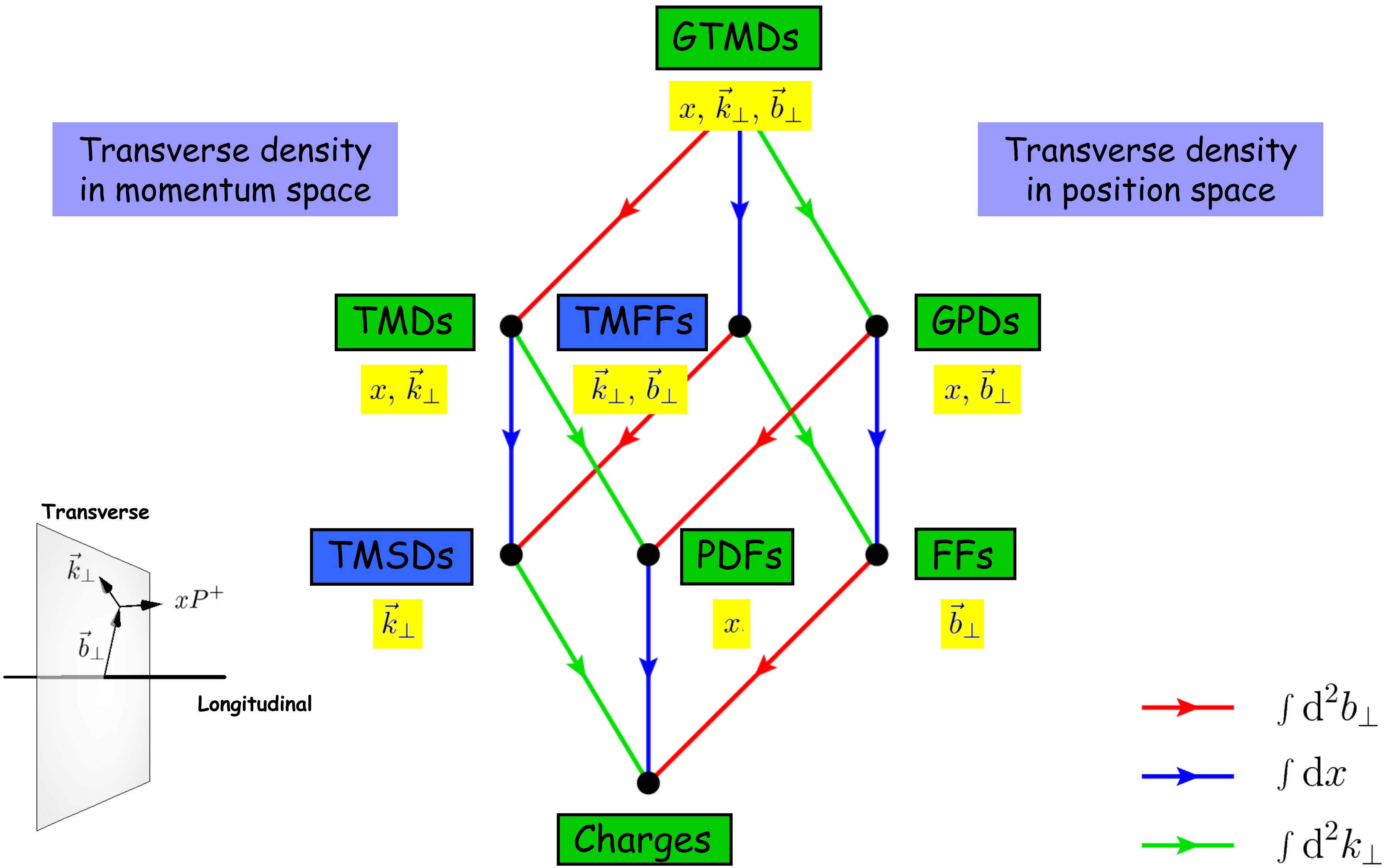
[C.L., Pasquini, Vanderhaeghen (2011)]
[C.L., Pasquini (2011)]
[C.L., Pasquini, Xiong, Yuan (in preparation)]

- Wigner distributions
  - Parton phase-space distributions
- Model calculations
  - 3Q LCWFs (χQSM, LCCQM)
  - Correlations between quark polarization/motion and nucleon polarization
- Comparison of different definitions of OAM
  - Overlap representation in non-gauge theories
  - **Total** OAM is the same but not individual contributions
  - Origin dependence, EOM?

Backup

Complete picture @ $\xi = 0$

Momentum space	$\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ $\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$	Position space



Model relations

	Linear relations	Quadratic relation
Flavor-dependent $D^u = \frac{2}{3},\ D^d = -\frac{1}{3}$	$D^q f_1^q + g_{1L}^q = 2h_1^q$ ** *	
Flavor-independent	$g_{1T}^q = -h_{1L}^{\perp q}$ ** * • • • $g_{1L}^q - h_1^q = \frac{k_\perp^2}{2M^2} h_{1T}^{\perp q}$ ** * • • •	$2h_1^q h_{1T}^{\perp q} = -(g_{1T}^q)^2$ ** * • • •

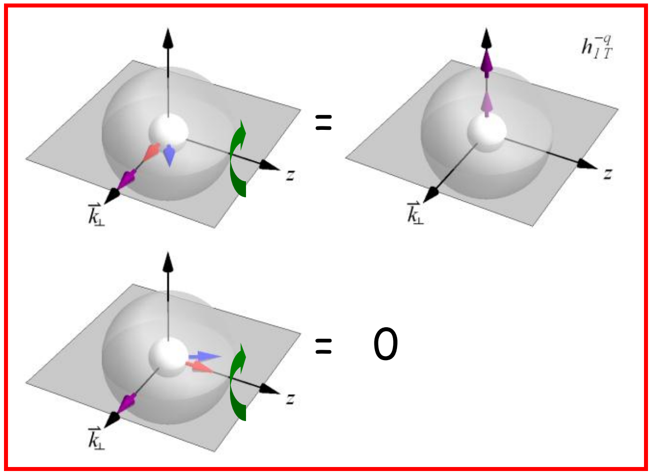
Bag	[Jaffe & Ji (1991), Signal (1997), Barone & *al.* (2002), Avakian & *al.* (2008-2010)]
χQSM	[C.L., Pasquini & Vanderhaeghen (2011)]
LCCQM	[Pasquini & *al.* (2005-2008)]
S Diquark	[Ma & *al.* (1996-2009), Jakob & *al.* (1997), Bacchetta & al. (2008)]
AV Diquark	[Ma & *al.* (1996-2009), Jakob & *al.* (1997)] [Bacchetta & *al.* (2008)]
Cov. Parton	[Efremov & *al.* (2009)]
Quark Target	[Meißner & *al.* (2007)]

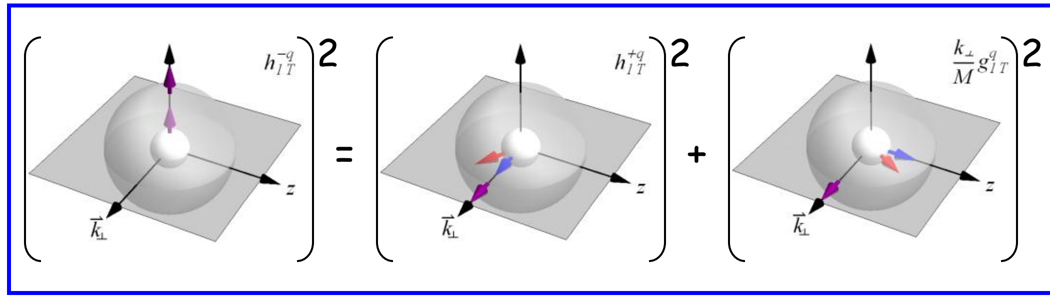
*=SU(6)

Spherical symmetry

[C.L., Pasquini (2011)]

Axial symmetry about $\hat{P}$

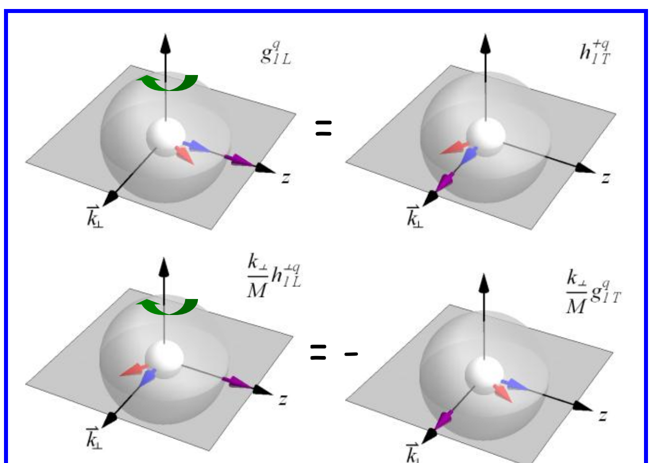




$$h_{1T}^{\pm q} \equiv h_1^q \pm \frac{k_\perp^2}{2M^2}\, h_{1T}^{\perp q}$$

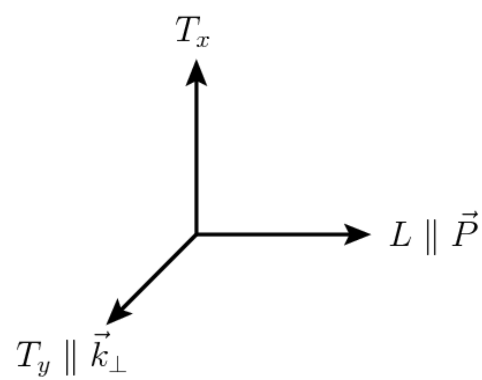
$$2h_1^q\, h_{1T}^{\perp q} = -(g_{1T}^q)^2$$

Axial symmetry about $\hat{k}_\perp \times \hat{P}$

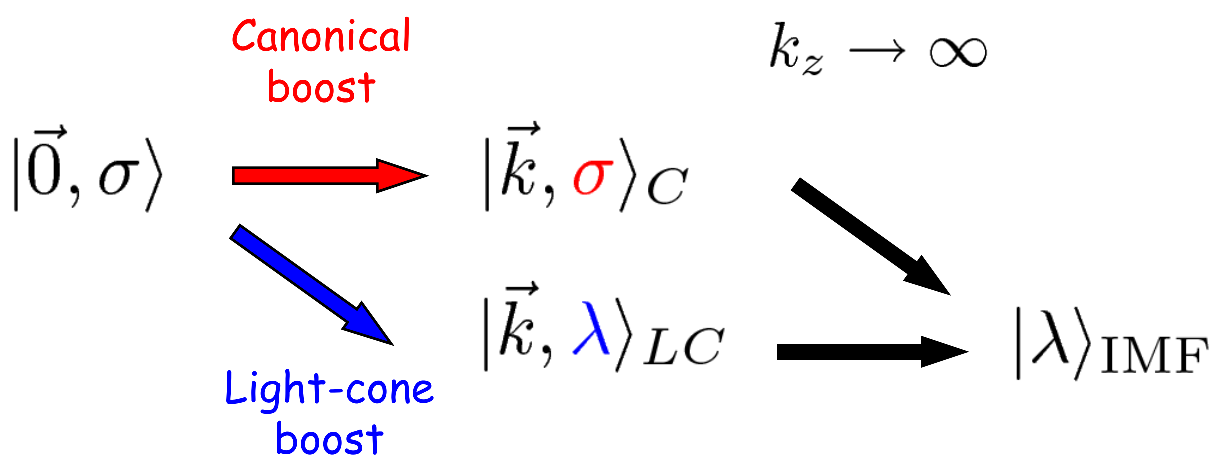


$$g_{1L}^q - h_1^q = \frac{k_\perp^2}{2M^2}\, h_{1T}^{\perp q}$$

$$g_{1T}^q = -h_{1L}^{\perp q}$$



LC helicity and canonical spin



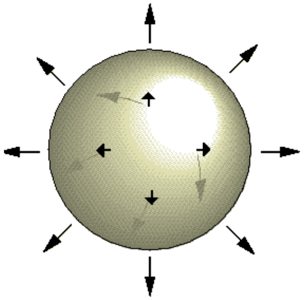
Spherical symmetry

[C.L., Pasquini (2011)]

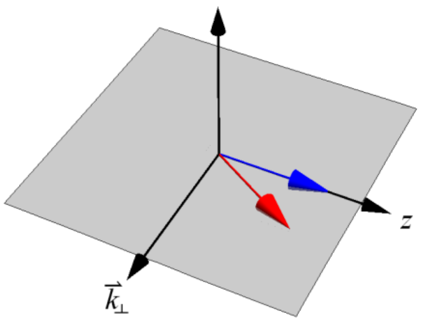
Bag Model, χQSM, LCCQM, Quark-Diquark Model (Ma) and Covariant Parton Model

Common assumption : Explicit or implicit rotational symmetry

The probability does not depend on the direction of canonical polarization



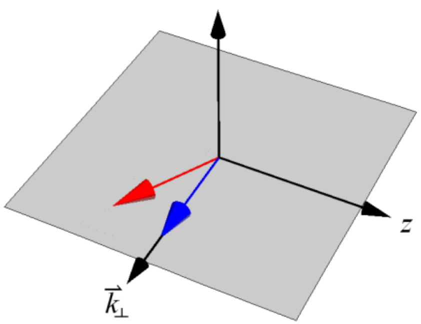
Formalism



Independent quarks → $q_\lambda^{LC}(k) = \sum_s D_{\lambda s}^{(1/2)*}(k)\, q_s^C(k)$

LC helicity
Canonical spin

$$D_{\lambda s}^{(1/2)*}(k) = \frac{1}{|\vec{K}|}\begin{pmatrix} K_z & K_L \\ -K_R & K_z \end{pmatrix}$$



Light-Cone Quark Model

$$\mathcal{M}_0^2 = \sum_i \frac{m_i^2 + \vec{k}_{i\perp}^2}{x_i}$$

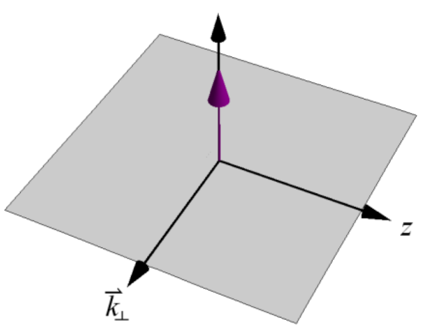
$$K_z = m + x\mathcal{M}_0 \qquad \vec{K}_\perp = \vec{k}_\perp \qquad k_z = x\mathcal{M}_0 - \sqrt{\vec{k}^2 + m^2}$$

(Melosh rotation)

Chiral Quark-Soliton Model

$$K_z = h(|\vec{k}|) + \frac{k_z}{|\vec{k}|}\, j(|\vec{k}|) \qquad \vec{K}_\perp = \frac{\vec{k}_\perp}{|\vec{k}|}\, j(|\vec{k}|) \qquad k_z = x\mathcal{M}_N - E_{\text{lev}}$$

S-wave P-wave



Bag Model

$$K_z = t_0(|\vec{k}|) + \frac{k_z}{|\vec{k}|}\, t_1(|\vec{k}|) \qquad \vec{K}_\perp = \frac{\vec{k}_\perp}{|\vec{k}|}\, t_1(|\vec{k}|) \qquad k_z = x\mathcal{M}_N - \omega/R_0$$

S-wave P-wave

Formalism

		$l_z=-1$	$l_z=0$			$l_z=+1$			$l_z=+2$
		$\Psi^{\uparrow}_{\uparrow\uparrow\uparrow}$	$\Psi^{\uparrow}_{\uparrow\uparrow\downarrow}$	$\Psi^{\uparrow}_{\uparrow\downarrow\uparrow}$	$\Psi^{\uparrow}_{\downarrow\uparrow\uparrow}$	$\Psi^{\uparrow}_{\downarrow\downarrow\uparrow}$	$\Psi^{\uparrow}_{\downarrow\uparrow\downarrow}$	$\Psi^{\uparrow}_{\uparrow\downarrow\downarrow}$	$\Psi^{\uparrow}_{\downarrow\downarrow\downarrow}$
$l_z=-1$	ψ^{+}_{+++}	$z_1z_2z_3$	$z_1z_2l_3$	$z_1l_2z_3$	$l_1z_2z_3$	$l_1l_2z_3$	$l_1z_2l_3$	$z_1l_2l_3$	$l_1l_2l_3$
	ψ^{+}_{++-}	$-z_1z_2r_3$	$z_1z_2z_3$	$-z_1l_2r_3$	$-l_1z_2r_3$	$-l_1l_2r_3$	$l_1z_2z_3$	$z_1l_2z_3$	$l_1l_2z_3$
$l_z=0$	ψ^{+}_{+-+}	$-z_1r_2z_3$	$-z_1r_2l_3$	$z_1z_2z_3$	$-l_1r_2z_3$	$l_1z_2z_3$	$-l_1r_2l_3$	$z_1z_2l_3$	$l_1z_2l_3$
	ψ^{+}_{-++}	$-r_1z_2z_3$	$-r_1z_2l_3$	$-r_1l_2z_3$	$z_1z_2z_3$	$z_1l_2z_3$	$z_1z_2l_3$	$-r_1l_2l_3$	$z_1l_2l_3$
	ψ^{+}_{--+}	$r_1r_2z_3$	$r_1r_2l_3$	$-r_1z_2z_3$	$-z_1r_2z_3$	$z_1z_2z_3$	$-z_1r_2l_3$	$-r_1z_2l_3$	$z_1z_2l_3$
$l_z=+1$	ψ^{+}_{-+-}	$r_1z_2r_3$	$-r_1z_2z_3$	$r_1l_2r_3$	$-z_1z_2r_3$	$-z_1l_2r_3$	$z_1z_2z_3$	$-r_1l_2z_3$	$z_1l_2z_3$
	ψ^{+}_{+--}	$z_1r_2r_3$	$-z_1r_2z_3$	$-z_1z_2r_3$	$l_1r_2r_3$	$-l_1z_2r_3$	$-l_1r_2z_3$	$z_1z_2z_3$	$l_1z_2z_3$
$l_z=+2$	ψ^{+}_{---}	$-r_1r_2r_3$	$r_1r_2z_3$	$r_1z_2r_3$	$z_1r_2r_3$	$-z_1z_2r_3$	$-z_1r_2z_3$	$-r_1z_2z_3$	$z_1z_2z_3$

$$z_i \equiv K^i_z, \quad l_i \equiv K^i_L, \quad r_i \equiv K^i_R$$

Assumption : ➢ $l_z = 0$ in instant form (automatic w/ spherical symmetry)

↳ More convenient to work in canonical spin basis