Universality and Evolution of TMDs

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Based on:

Phys.Rev.D83:114042,2011 [arXiv: 1101.5057]

Transversity 2011

Collinear and TMD Factorization

- QCD gains its predictive power through factorization theorems.
- Collinear factorization: well understood and tested.
- Enormous effort for extracting PDFs and FFs: CTEQ, MRST, HERA PDF, NNPDF ...
- Status for TMD Phenomenology
  - CSS Approach : No direct link with TMDs, fits process dependent.
  - Generalized Parton Model : Gaussian approximations at fixed scales.
  - No TMD repositories.
  - No agreed upon definitions of TMDs...

Universal ?
Predictive ?

Want: Analogous to collinear factorization

Main Philosophy and the Goal

Extend collinear factorization methodology to TMD factorization.

- TMD Factorization

From ***Foundations of Perturbative QCD***, J. Collins

$$W^{\mu\nu} = \sum_f |\mathcal{H}_f(Q^2,\mu)|^{\mu\nu} \int d^2\mathbf{k}_{1T} d^2\mathbf{k}_{2T} \tilde{F}_{f/P_1}(x_1,\mathbf{k}_{1T},\mu,\zeta_F)\tilde{F}_{\bar{f}/P_2}(x_2,\mathbf{k}_{2T},\mu,\zeta_F)\delta^{(2)}(\mathbf{k}_{1T}+\mathbf{k}_{2T}-\mathbf{q}_T)$$

$$+ \text{corrections}$$

- Repositiory of well defined TMDs with QCD evolution included

https://projects.hepforge.org/tmd

Evolution for TMDs

Energy evolution from Collins-Soper equation

$$\frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T, \mu)$$

Renormalization group equations

$$\frac{d\tilde{K}}{d\ln\mu} = -\gamma_K(g(\mu))$$

and

$$\frac{d \ln \tilde{F}(x, b_T, \mu, \zeta)}{d \ln\mu} = -\gamma_F(g(\mu), \zeta/\mu^2)$$

Important: Evolution equations are in configuration space!

Evolution for TMDs

- Use collinear factorization treatment for small b_T .

$$\tilde{F}_{f/P}(x, b_T, \mu, \zeta_F) \quad = \quad \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_T, \zeta_F, \mu, g(\mu)) \, f_{j/P}(\hat{x}, \mu) + \mathcal{O}((\lambda_{\rm QCD} b_T)^a)$$

- Implement matching procedure.

$$\mathbf{b}_*(\mathbf{b}_T) = \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\rm max}^2}}$$

Collins and Soper (1982)

- Apply evolution equations.

Evolved TMDs

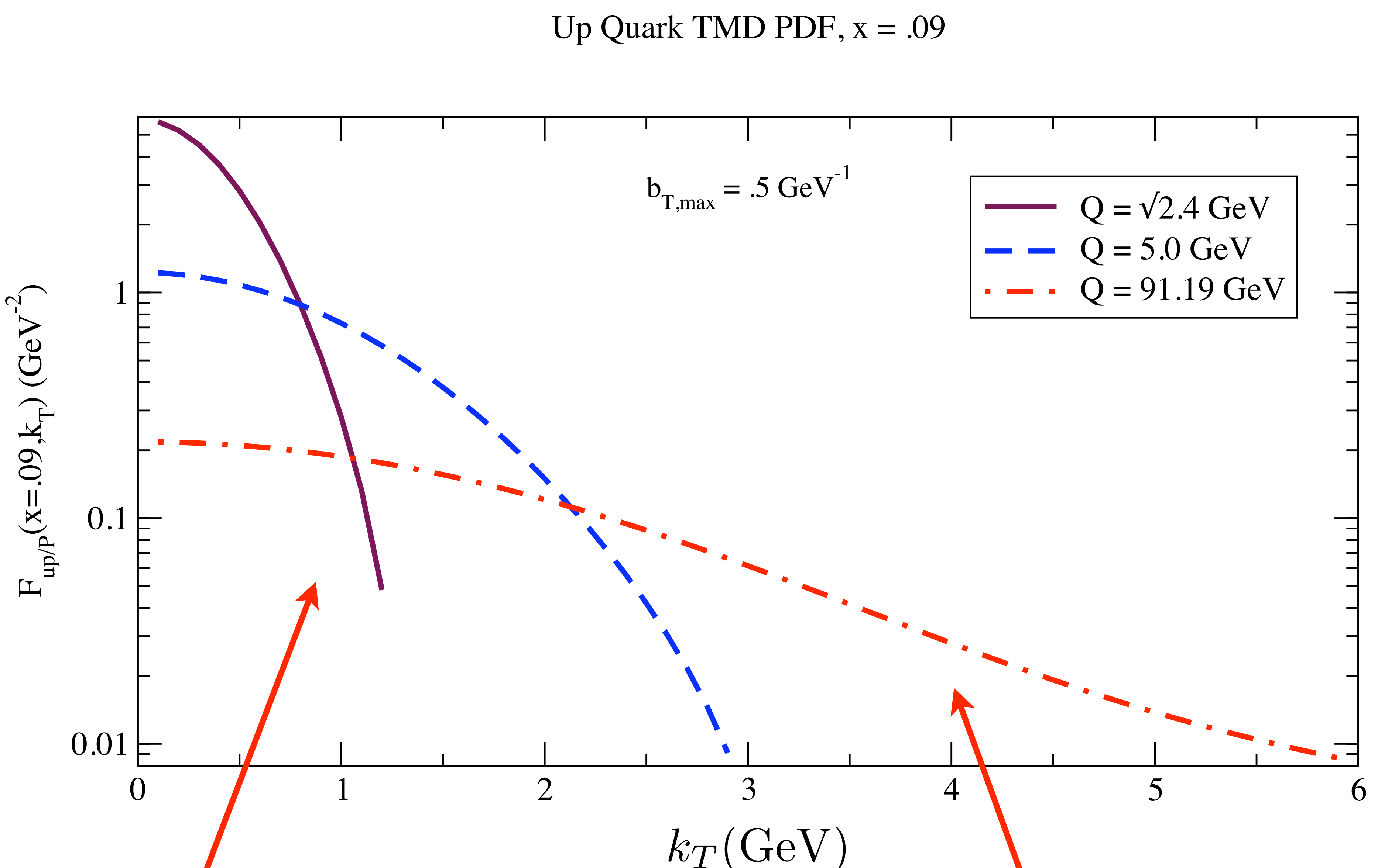
$$\tilde{F}_{f/P}(x, b_T, \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*, \zeta_F, \mu, g(\mu))\, f_{j/P}(\hat{x}, \mu_b)$$

$$\times \exp\left\{ \ln \frac{\sqrt{\zeta_F}}{\mu_b} \tilde{K}(b_*, \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'), 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times \exp\left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\}$$

$$\tilde{C}_{j'/j}(x, b_T, \mu, \zeta_F/\mu^2) = \delta_{j'j}\delta(1-x) + \delta_{j'j} \frac{\alpha_s C_F}{2\pi} \left\{ 2 \left[\ln\left(\frac{2}{\mu b_T} \right) - \gamma_E \right] \left[\left(\frac{2}{1-x} \right)_+ - 1 - x \right] + 1 - x \right.$$

$$\left. + \delta(1-x) \left[-\frac{1}{2} [\ln(b_T^2 \mu^2) - 2(\ln 2 - \gamma_E)]^2 - [\ln(b_T^2 \mu^2) - 2(\ln 2 - \gamma_E)] \ln\left(\frac{\zeta_F}{\mu^2} \right) \right] \right\} + \mathcal{O}(\alpha_s^2)$$

Some Results



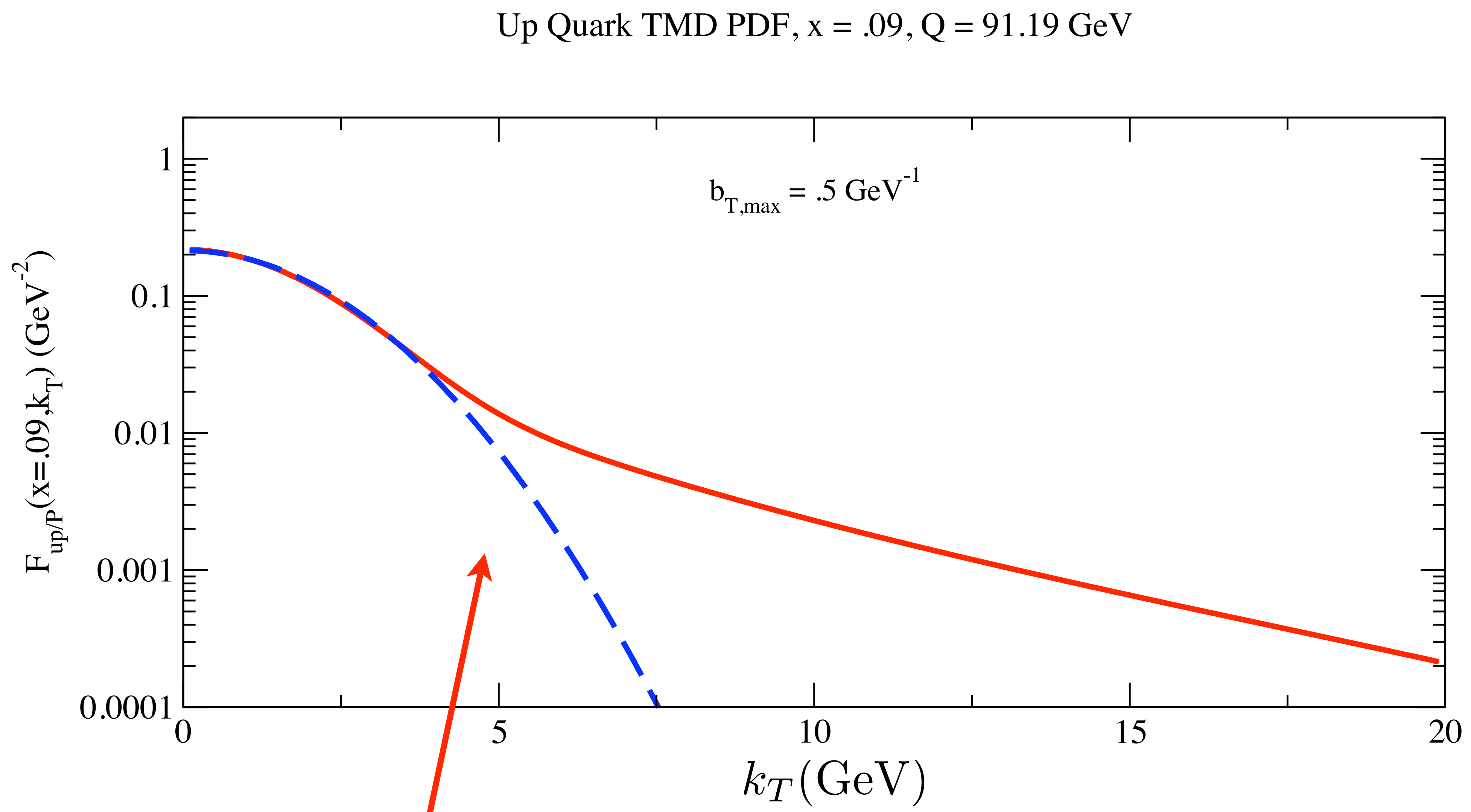
JLab Energies
matches STM fit

Schweitzer, Teckentrup and Metz (2010) (STM)

Tevatron Energies
matches BNLY fit

Brock, Landry, Nadolsky, Yuan (2003) (BLNY)

Some Results



gaussian fit does not capture the effects of evolution quite well

Unambiguous Hard Part

- Definition:

$$|\mathcal{H}_f(Q,\mu/Q)^2|^{\mu\nu} = \frac{W^{\mu\nu}}{F_{f/P_1} \otimes F_{f/P_2}}$$

- Drell-Yan:

$$|\mathcal{H}_f(Q,\mu/Q)^2|^{\mu\nu} = e_f^2 |H_0^2|^{\mu\nu} \left(1 + \frac{C_F \alpha_s}{\pi}\left[\frac{3}{2}\ln(Q^2/\mu^2) - \frac{1}{2}\ln^2(Q^2/\mu^2) - 4 + \frac{\pi^2}{2}\right]\right) + \mathcal{O}(\alpha_s^2)$$

- SIDIS:

$$|\mathcal{H}_f(Q,\mu/Q)^2|^{\mu\nu} = e_f^2 |H_0^2|^{\mu\nu} \left(1 + \frac{C_F \alpha_s}{\pi}\left[\frac{3}{2}\ln(Q^2/\mu^2) - \frac{1}{2}\ln^2(Q^2/\mu^2) - 4\right]\right) + \mathcal{O}(\alpha_s^2)$$

Using spin decomposition

$$F_{f/P^{\uparrow}}(x, k_T, S; \mu, \zeta_F) = F_{f/P}(x, k_T; \mu, \zeta_F) - F_{1T}^{\perp\, f}(x, k_T; \mu, \zeta_F)\frac{\epsilon_{ij} k_T^i S^j}{M_p}$$

unpolarized TMD

Sivers function

- Written in momentum space.
- Evolution equations in configuration space.
- Extra momentum in prefactor will complicate things.

Evolution for the Sivers Function

- It is $\tilde{F}_{1T}^{\perp}{}'(b)$ not $\tilde{F}_{1T}^{\perp}(b)$ that appears in the physically relevant formulas.

- $\tilde{F}_{1T}^{\perp}(b)$ and $\tilde{F}_{1T}^{\perp}{}'(b)$ satisfies the same evolution equations.

- Anomalous dimensions for the unpolarized TMDs and the polarized TMDs are the same.

$$\frac{\partial \ln \tilde{F}'^{\perp\, f}_{1T}(x, b_T; \mu, \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(b_T; \mu) \qquad \frac{d\tilde{K}}{d\ln\mu} = -\gamma_K(g(\mu))$$

$$\frac{d \ln \tilde{F}'^{\perp\, f}_{1T}(x, b_T; \mu, \zeta_F)}{d \ln \mu} = \gamma_F(g(\mu); \zeta_F/\mu^2)$$

Evolution for the Sivers Function

Using analogous steps to the unpolarized case

$$\tilde{F}'^{\perp f}_{1T}(x, b_T; \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\hat{x}_1 d\hat{x}_2}{\hat{x}_1 \hat{x}_2} \tilde{C}_{f/j}(\hat{x}_1, \hat{x}_2, b_*; \mu_b^2, \mu_b, g(\mu_b)) T_{Fj/P}(\hat{x}_1, \hat{x}_2, \mu_b)$$

$$\times \exp\left\{ \ln \frac{\sqrt{\zeta_F}}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times \exp\left\{ g^{\text{Sivers}}_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{Q_0} \right\}$$

recall for the unpolarized TMD PDF

$$\tilde{F}_{f/P}(x, b_T, \mu, \zeta_F) = \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*, \zeta_F, \mu, g(\mu)) \, f_{j/P}(\hat{x}, \mu_b)$$

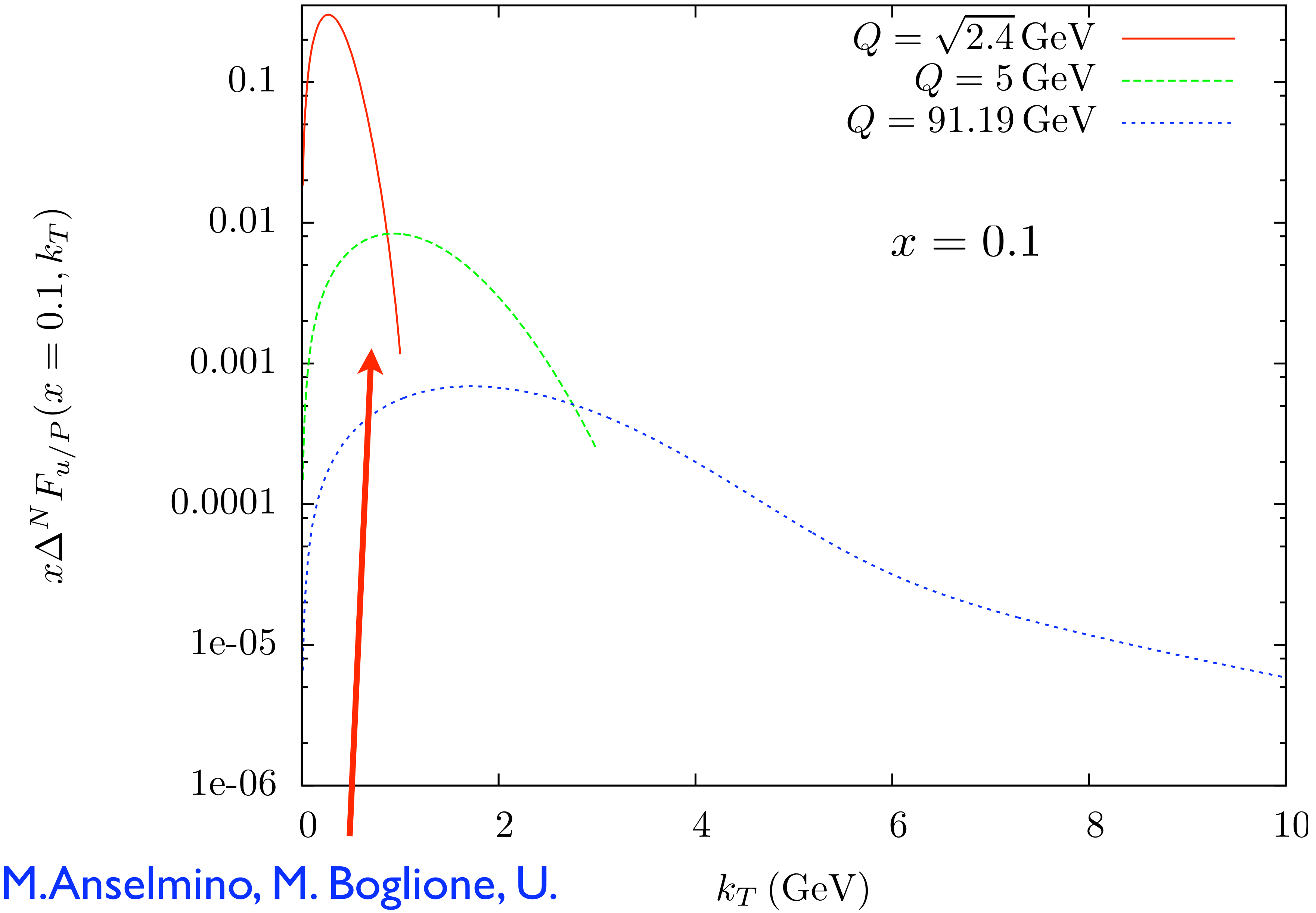
$$\times \exp\left\{ \ln \frac{\sqrt{\zeta_F}}{\mu_b} \tilde{K}(b_*, \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'), 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times \exp\left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\}$$

Twist 2 Large b Behaviour

- Small b_T region is twist-three (Qiu-Sterman).
- Tail of the momentum space Sivers function is power suppressed relative to the unpolarized TMD.
- Starting point: neglect twist-3 tail and study the large b_T behavior, which is the experimentally interesting region.
- Use already existing gaussian fits to determine the non-perturbative information.

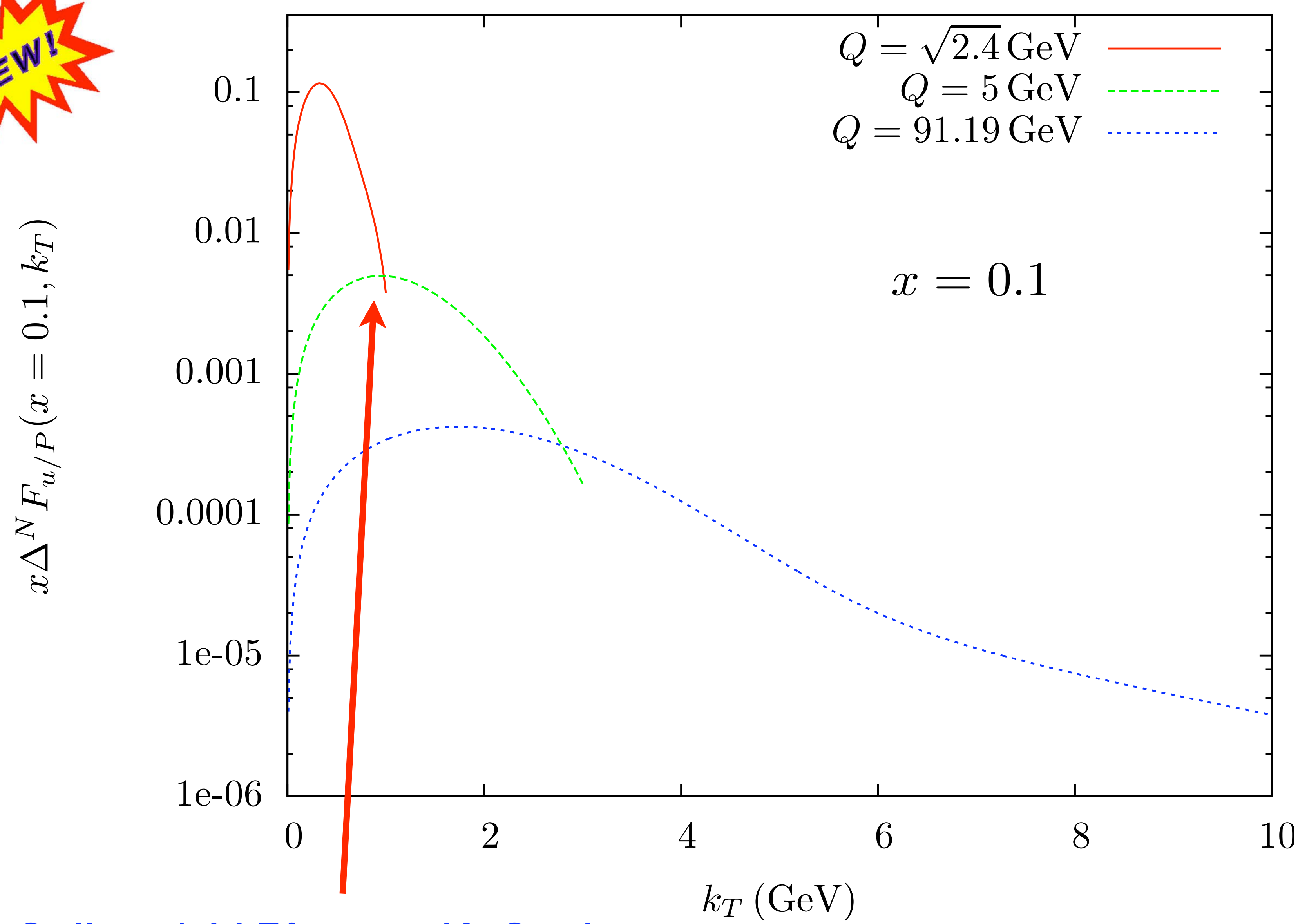
Some Results: Torino Fits

$$\Delta^N F_{f/P^\uparrow}(x, k_T) = -\frac{2k_T}{M_p} F_{1T}^{\perp f}(x, k_T; \mu, \zeta_F)$$



M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, S. Melis, F. Murgia, A. Prokudin, C. Turk; 2009

Some Results: Bochum Fits



J. Collins, A.V. Efremov, K. Goeke, M. Grosse Perdekamp, S. Menzel, B. Meredith, A. Metz, P. Schweitzer; 2006

Epilogue: What is next ?

Ultimate Goal: Global Fits!

- Polarized TMDs: twist 3 large transverse momentum region - NLO coefficient function.
- Y term corrections: matching to large transverse momentum collinear factorization formalism.
- Global Fits for unpolarized and polarized TMDs!
- Higher orders in anomalous dimensions.

Stay tuned for new and improved results
at

https://projects.hepforge.org/tmd

Don't forget to subscribe to the mailing list!

Backup Slides

QCD gains its predictive power through factorization

Consider Drell-Yan process: $P_1 + P_2 \to l\bar{l}(Q^2) + X$

$$\frac{d\sigma_{P_1 P_2 \to l l'(Q^2)+X}(s, Q^2)}{dQ^2} = \sum_{i,j} \int_0^1 dx_1\, dx_2\, f_{i/P_1}(x_1, \mu^2)\, f_{j/P_2}(x_2, \mu^2)\, \mathcal{H}_{ij}\left(\frac{Q^2}{x_1 x_2 s}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$$

Collins, Soper and Sterman (1985,1988)

PDFs: Long distance dynamics, non-perturbative, universal, evolution through DGLAP

Hard scattering function: short distance dynamics, pQCD calculations

Not complete description $\rightarrow$ transverse momentum of incoming partons are important

examples: DY, SIDIS, hadron production at e^+e^- collisions...

CSS Formalism: Typical Implementation

Collins-Soper-Sterman formalism for DY gives

(1985)

$$d\sigma \sim \int d^2\mathbf{b} e^{-i\mathbf{b}\cdot\mathbf{q}_T}$$

$$\int_{x_1}^{1} \frac{d\hat{x}_1}{\hat{x}_1} \tilde{C}_{f/j}(x_1/\hat{x}_1, b_*, \mu_b^2, \mu_b, g(\mu_b))\, f_{j/P_1}(\hat{x}_1, \mu_b)$$

$$\int_{x_2}^{1} \frac{d\hat{x}_2}{\hat{x}_2} \tilde{C}_{f/j}(x_2/\hat{x}_2, b_*, \mu_b^2, \mu_b, g(\mu_b))\, f_{j/P_2}(\hat{x}_2, \mu_b)$$

$$\exp\left[\int_{1/b^2}^{Q^2} \frac{d\mu'^2}{\mu'^2}\left\{\mathcal{A}(\alpha_s(\mu'))\ln\frac{Q^2}{\mu'^2} + \mathcal{B}(\alpha_s(\mu'))\right\}\right]$$

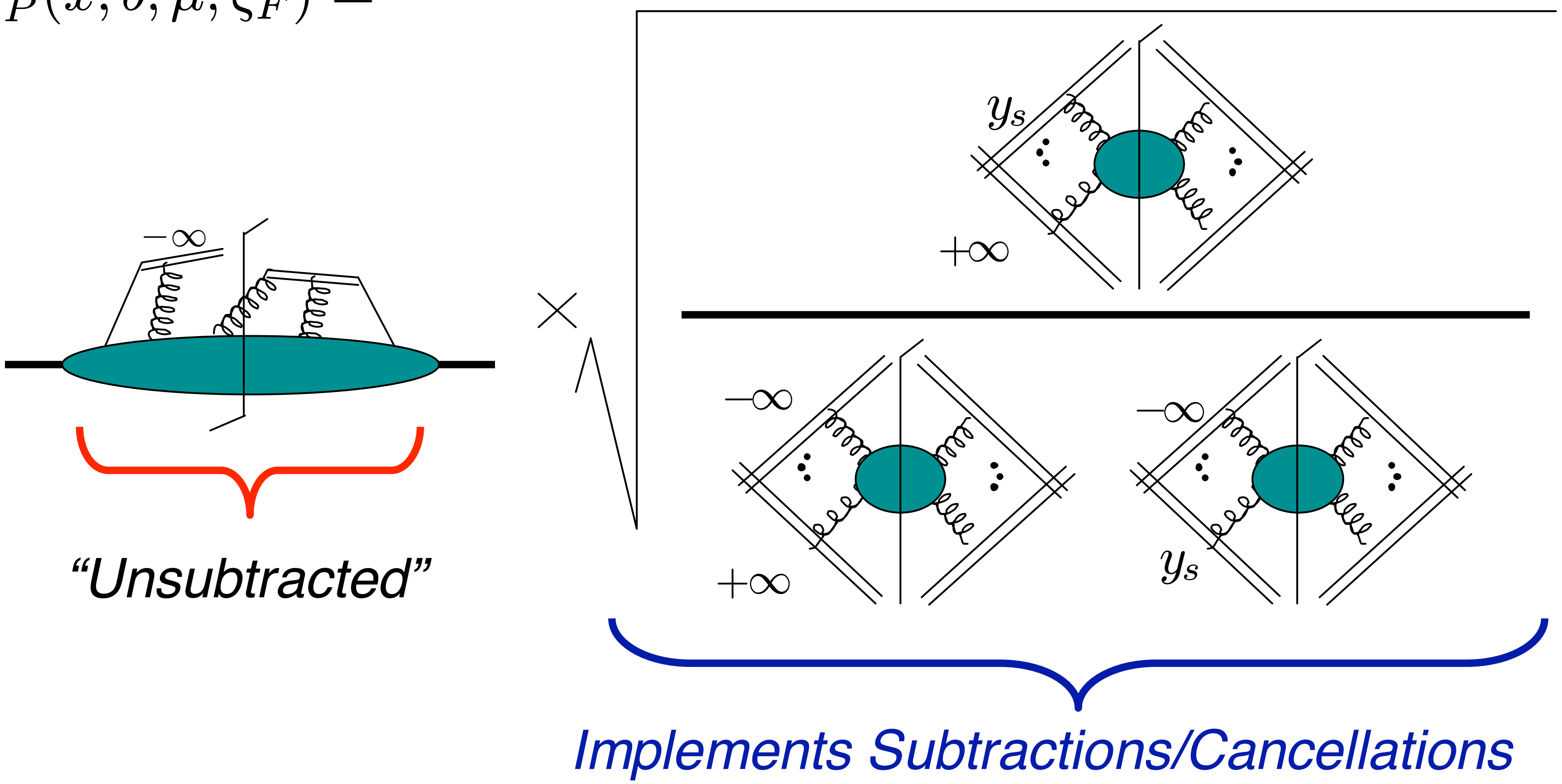
$$\exp\left[-g_K(b)\ln\frac{Q^2}{Q_0^2} - g_1(x_1, b) - g_2(x_2, b)\right]$$

$$+\,\mathrm{Large}\; q_T\; \mathrm{term}$$

Where is the TMD PDF ?

New TMD Definitions

$F_{f/P}(x, b; \mu; \zeta_F) =$



From ***Foundations of Perturbative QCD***, J. Collins

Fourier Transform is Complicated

$$F_{1T}^{\perp\, f}(x, k_T; \mu, \zeta_F) = \frac{-1}{2\pi k_T} \int_0^\infty db_T \, b_T \, J_1(k_T b_T) \tilde{F}'^{\perp\, f}_{1T}(x, b_T; \mu, \zeta_F)$$

$$\tilde{F}'^{\perp\, f}_{1T}(x, b_T; \mu, \zeta_F) = -2\pi \int_0^\infty dk_T \, k_T^2 J_1(k_T b_T) F_{1T}^{\perp\, f}(x, k_T; \mu, \zeta_F)$$

with

$$\tilde{F}'^{\perp\, f}_{1T}(x, b_T; \mu, \zeta_F) \equiv \frac{\partial \tilde{F}_{1T}^{\perp\, f}(x, b_T; \mu, \zeta_F)}{\partial b_T}$$

It is $\tilde{F}'(b_T)$, not $\tilde{F}(b_T)$ that appears in the physically relevant formulas.