# Azimuthal distributions of pions inside a jet in hadronic collisions *

Cristian Pisano

Università di Cagliari

**Transversity 2011**

**Third International Workshop on Transverse Polarization Phenomena in Hard Scattering**

**Veli Lošinj, Croatia, 29 August – 2 September 2011**

- Study of $p^{(\uparrow)}p \to \mathrm{jet}\, \pi\, X$ within the generalized parton model
- Azimuthal asymmetries attributed to TMD pdfs and FFs
- Phenomenology for RHIC kinematics
- Test of the process dependence of the Sivers function

---

* In collaboration with U. D'Alesio (Univ. & INFN Cagliari) and F. Murgia (INFN Cagliari)

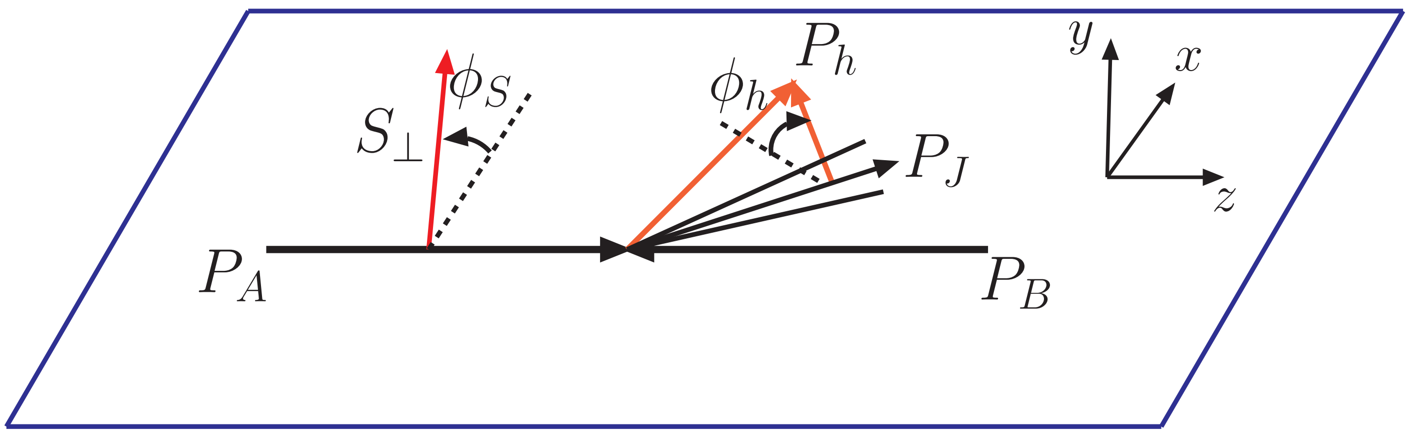
- We consider the process

$$A(P_A;S_\perp)+B(P_B)\to \text{jet}(P_\text{j})+h(P_h)\,+\,X$$

in the c.m. frame of the two spin $1/2$ hadrons $A$, $B$; with the jet in the $xz$ plane

- $A$ is polarized with transverse spin $S_\perp=(0,\cos\phi_S,\sin\phi_S,0)$

D'Alesio, Trento 2007




F. Yuan, PRL 100 (2008) 032003

$\phi_h^H$: azimuthal distribution of hadron $h$ inside the jet, around the jet axis

- $\phi_h$: azim. angle of $h$' s intrinsic transv. momentum w.r.t. the jet direction
- $\phi_h^H$: same angle, but measured in the $H$ frame, where the jet is along $z$

$$\tan\phi_h^H=\tan\phi_h\cos\theta_\text{j}$$

The two frames are related by a rotation around $y$ by $\theta_\text{j}$, polar angle of the jet

D'Alesio, Murgia, CP, PRD 83 (2011) 034021

## *The TMD generalized parton model (GPM)*

- Spin and intrinsic parton motion effects in initials hadrons and in the fragmentation
Phenomenological assumption: factorization holds for large $p_T$ jet production

- SSA and azimuthal asymmetries are generated by TMD polarized pdfs and FFs
Most relevant ones: $f_{1T}^{\perp}$ (Sivers), $h_1^{\perp}$ (Boer-Mulders), $H_1^{\perp}$ (Collins)
Anselmino, Boglione, D'Alesio, Leader, Melis, Murgia, PRD 73 (2006) 014020;
Notation: Meissner, Metz, Goeke, PRD 76 (2007) 034002

- Factorization proven in a more simplified theoretical scheme: intrinsic parton motion only in the fragmentation process. Only Collins effect for quarks is at work
F. Yuan, PRL 100 (2008) 032003;
PRD 77 (2008) 074019

- The present, more general, scheme requires a severe scrutiny by comparison with experimental results to clarify the validity of factorization and, related to this, the relevance of possible universality-breaking terms for the TMD distributions

## *Why studying the distribution of pions inside a jet?*

- SSA in $p\,p^{\uparrow} \to \pi\,X$, due to Collins and Sivers effects, cannot be disentangled
  Anselmino, Boglione, D'Alesio, Leader, Murgia, PRD 71 (2005) 014002;
  Anselmino, Boglione, D'Alesio, Leader, Melis, Murgia, PRD 73 (2006) 014020

  while in $p\,p^{\uparrow} \to \mathrm{jet}\,\pi\,X$, Collins, Sivers and other contributions involving different combinations of TMD pdfs and FFs can be singled out

- Jets coming from quark or gluon fragmentation could be identified without ambiguity, since the pion azimuthal distribution is different in the two cases:
  - symm. pion distribution for the fragmentation of an unpolarized parton jet $(D_1)$
  - $\cos\phi_\pi^H$ distribution for a transversely polarized quark parton jet $(H_1^{\perp q})$
  - $\cos 2\phi_\pi^H$ distribution for a linearly polarized gluon parton jet $(H_1^{\perp g})$

- Complex measurement, but feasible and under active consideration at RHIC
  R. Fersch [STAR Coll.], JP Conf. Ser. 295 (2011) 012048

# TMD master formula

$$E_{\rm j}\,\frac{{\rm d}\sigma}{{\rm d}^3\boldsymbol{P}_{\rm j}\,{\rm d}z\,{\rm d}^2\boldsymbol{k}_{\perp\pi}} = \sum_{a,b,c,d}\int\frac{{\rm d}x_a{\rm d}x_b}{16\pi^2 x_a x_b s}\,{\rm d}^2\boldsymbol{k}_{\perp a}{\rm d}^2\boldsymbol{k}_{\perp b}\,\Sigma(S)^{ab\to cd}\delta(\hat s+\hat t+\hat u)$$

- sum over all allowed partonic subprocesses $ab \to cd$; $\quad s = E^2_{\rm c.m.} = (P_A+P_B)^2$
- $x_{a,b}$, $\boldsymbol{k}_{\perp a,b}$: initial parton light-cone momentum fractions and intr. transv. momenta

  $z$, $\boldsymbol{k}_{\perp\pi}$: analogous variables for the observed pion w.r.t. the jet direction (parton $c$)

- Partonic kernel:

$$\Sigma(S)^{ab\to cd} = \sum_{\{\lambda\}}\rho^{a/A,S}_{\lambda_a\lambda'_a}\hat f_{a/A,S}(x_a,\boldsymbol{k}_{\perp a})\rho^{b/B}_{\lambda_b\lambda'_b}\hat f_{b/B}(x_b,\boldsymbol{k}_{\perp b}) \times \hat M_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\hat M^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b}\hat D^{\pi}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp\pi})$$

  - $\{\lambda\}$: sum over partonic helicities; $\quad\rho^{a/A,S}_{\lambda_a\lambda'_a}$: hel. density matrix of parton $a$
  - Soft terms: $\rho^{a/A,S}_{\lambda_a\lambda'_a}\hat f_{a/A,S} \longrightarrow$ leading twist-TMD pdfs

    $\hat D^{\pi}_{\lambda_c,\lambda'_c} \longrightarrow$ only two indep. leading-twist TMD FFs: $D_1$, $H_1^{\perp}$
  - Hard terms: $\hat M_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}$, pQCD LO hel. amplitudes for the process $ab \to cd$

## *Example: the $qq \to qq$ channel*

- Eight distinct partonic channels contribute to the cross section

$$qq \to qq \quad qg \to qg \quad qg \to gq \quad gq \to qg \quad gq \to gq$$
$$gg \to q\bar{q} \quad q\bar{q} \to gg \quad gg \to gg$$

in the first line $q$ stays for both quarks and antiquarks in all allowed combinations

- $qq \to qq$: max number of terms (similar structures for $gg \to gg$ with $\phi_\pi^H \to 2\phi_\pi^H$)
  - Unpolarized cross section:

$$2\mathrm{d}\sigma^{\mathrm{unp}}(\phi_\pi^H) \sim \mathrm{d}\sigma_0 + \mathrm{d}\sigma_1 \cos\phi_\pi^H$$
$$\mathrm{d}\sigma_0 \sim f_1 f_1 D_1 \qquad h_1^\perp h_1^\perp D_1$$
$$\mathrm{d}\sigma_1 \sim h_1^\perp f_1 H_1^\perp \quad f_1 h_1^\perp H_1^\perp$$

  - Numerator of the SSA: $\mathcal{N} \equiv \mathrm{d}\sigma(\phi_S, \phi_\pi^H) - \mathrm{d}\sigma(\phi_S + \pi, \phi_\pi^H)$

$$\mathcal{N} \sim \mathrm{d}\Delta\sigma_0 \sin\phi_S + \mathrm{d}\Delta\sigma_1^- \sin(\phi_S - \phi_\pi^H) + \mathrm{d}\Delta\sigma_1^+ \sin(\phi_S + \phi_\pi^H)$$
$$\mathrm{d}\Delta\sigma_0 \sim f_{1T}^\perp f_1 D_1 \quad h_1 h_1^\perp D_1 \quad h_{1T}^\perp h_1^\perp D_1$$
$$\mathrm{d}\Delta\sigma_1^- \sim h_1 f_1 H_1^\perp \quad f_{1T}^\perp h_1^\perp H_1^\perp$$
$$\mathrm{d}\Delta\sigma_1^+ \sim h_{1T}^\perp f_1 H_1^\perp \quad f_{1T}^\perp h_1^\perp H_1^\perp$$

- Neglecting intrinsic motion of initial partons, only $f_1 f_1 D_1$ and $h_1 f_1 H_1^\perp$ contribute

# Weighted cross sections

- General structure of the single transverse polarized cross section

$$\begin{aligned} 2\mathrm{d}\sigma(\phi_S, \phi_\pi^H) \quad &\sim \quad \mathrm{d}\sigma_0 + \mathrm{d}\Delta\sigma_0 \sin\phi_S + \mathrm{d}\sigma_1 \cos\phi_\pi^H + \mathrm{d}\sigma_2 \cos 2\phi_\pi^H \\ &+\mathrm{d}\Delta\sigma_1^- \sin(\phi_S - \phi_\pi^H) + \mathrm{d}\Delta\sigma_1^+ \sin(\phi_S + \phi_\pi^H) \\ &+\mathrm{d}\Delta\sigma_2^- \sin(\phi_S - 2\phi_\pi^H) + \mathrm{d}\Delta\sigma_2^+ \sin(\phi_S + 2\phi_\pi^H) \end{aligned}$$

- Unpolarized cross section:

$$\mathrm{d}\sigma(\phi_S, \phi_\pi^H) + \mathrm{d}\sigma(\phi_S + \pi, \phi_\pi^H) \equiv 2\mathrm{d}\sigma^{\mathrm{unp}}(\phi_\pi^H) \sim \mathrm{d}\sigma_0 + \mathrm{d}\sigma_1 \cos\phi_\pi^H + \mathrm{d}\sigma_2 \cos 2\phi_\pi^H$$

- Average values of appropriate functions $W(\phi_S, \phi_\pi^H) = 1,\ \cos\phi_\pi^H,\ \cos 2\phi_\pi^H$

$$\langle\, W(\phi_S, \phi_\pi^H)\,\rangle = \frac{\int \mathrm{d}\phi_S \mathrm{d}\phi_\pi^H W(\phi_S, \phi_\pi^H) \mathrm{d}\sigma(\phi_S)}{\int \mathrm{d}\phi_S \mathrm{d}\phi_\pi^H \mathrm{d}\sigma(\phi_S)}$$

single out $\mathrm{d}\sigma_0,\ \mathrm{d}\sigma_1,\ \mathrm{d}\sigma_2$ respectively

# Single spin asymmetries

- Numerator of the single spin asymmetry:

$$\begin{aligned} & \mathrm{d}\sigma(\phi_S,\phi_\pi^H) - \mathrm{d}\sigma(\phi_S+\pi,\phi_\pi^H) \\ & \quad \sim \mathrm{d}\Delta\sigma_0 \sin\phi_S + \mathrm{d}\Delta\sigma_1^- \sin(\phi_S-\phi_\pi^H) + \mathrm{d}\Delta\sigma_1^+ \sin(\phi_S+\phi_\pi^H) \\ & \qquad + \mathrm{d}\Delta\sigma_2^- \sin(\phi_S-2\phi_\pi^H) + \mathrm{d}\Delta\sigma_2^+ \sin(\phi_S+2\phi_\pi^H) \end{aligned}$$

- Appropriate azimuthal moments, with $W(\phi_S,\phi_\pi^H) = \sin\phi_S, \sin(\phi_S-\phi_\pi^H)$, ...

$$A_N^W \equiv 2\langle\, W(\phi_S,\phi_\pi^H)\,\rangle = 2\,\frac{\int \mathrm{d}\phi_S \mathrm{d}\phi_\pi^H\, W(\phi_S,\phi_\pi^H)[\mathrm{d}\sigma(\phi_S) - \mathrm{d}\sigma(\phi_S+\pi)]}{\int \mathrm{d}\phi_S \mathrm{d}\phi_\pi^H\, [\mathrm{d}\sigma(\phi_S) + \mathrm{d}\sigma(\phi_S+\pi)]}$$

will single out the different contribusions (analogy with SIDIS)

# Phenomenology at RHIC

- $\langle W \rangle \, (A_N^W)$ calculated for $p^{(\uparrow)}\, p \to \text{jet}\, \pi\, X$ at $\sqrt{s} = 200$ GeV
  Other energies ($\sqrt{s} = 62.4$, 500 GeV) considered, not shown here
  D'Alesio, Murgia, CP, PRD 83 (2011) 034021

- $\langle W \rangle$ and $A_N^W$ given as function of $P_{\text{j}T}$ and $\eta_{\text{j}}$; one can integrate over all other variables, in particular we take $0.3 \leq z \leq 1$

- Assumption for TMDs: $\mathcal{F}^{q,g}(x, \boldsymbol{k}_\perp^2) = f^{q,g}(x) g(\boldsymbol{k}_\perp^2)$, with $g(\boldsymbol{k}_\perp^2)$ being a flavor independent Gaussian-like function

- Over-maximized scenario: all TMDs are maximized in size by imposing natural positivity bounds (Soffer bound for $h_1^q$) and *the relative signs* of all active partonic contributions are chosen so that they sum up additively

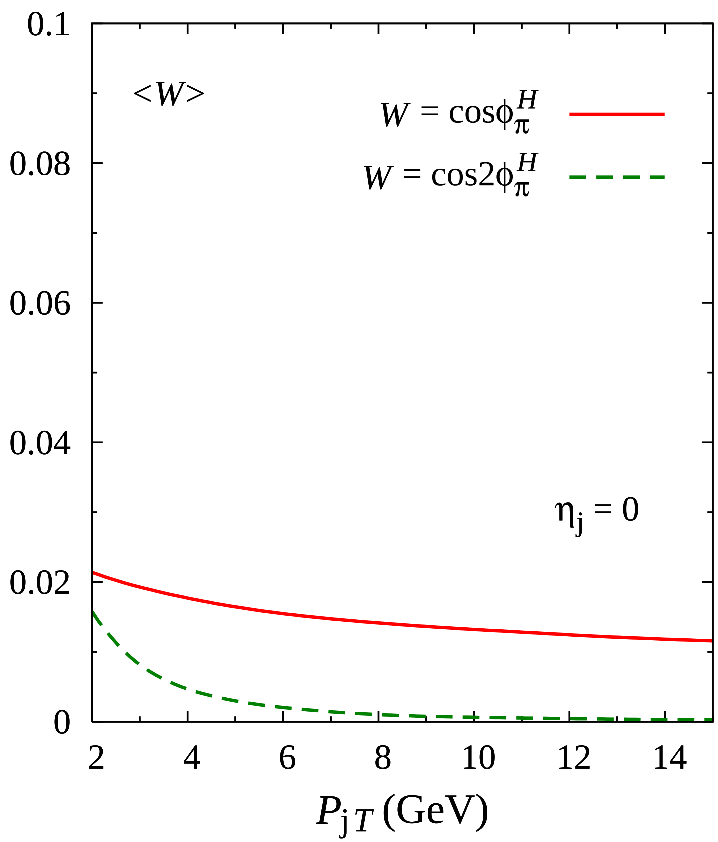
  Advantage: upper bound on the absolute value of any effect playing a role in the asymmetries. All the effects being already marginal in this scenario may be directly discarded in subsequent refined phenomenological analyses

- Parameterizations of the usual collinear LO pdfs (GRV98, GRSV2000) and FFs (Kre) evolved at the scale $\mu = P_{\text{j}T}$

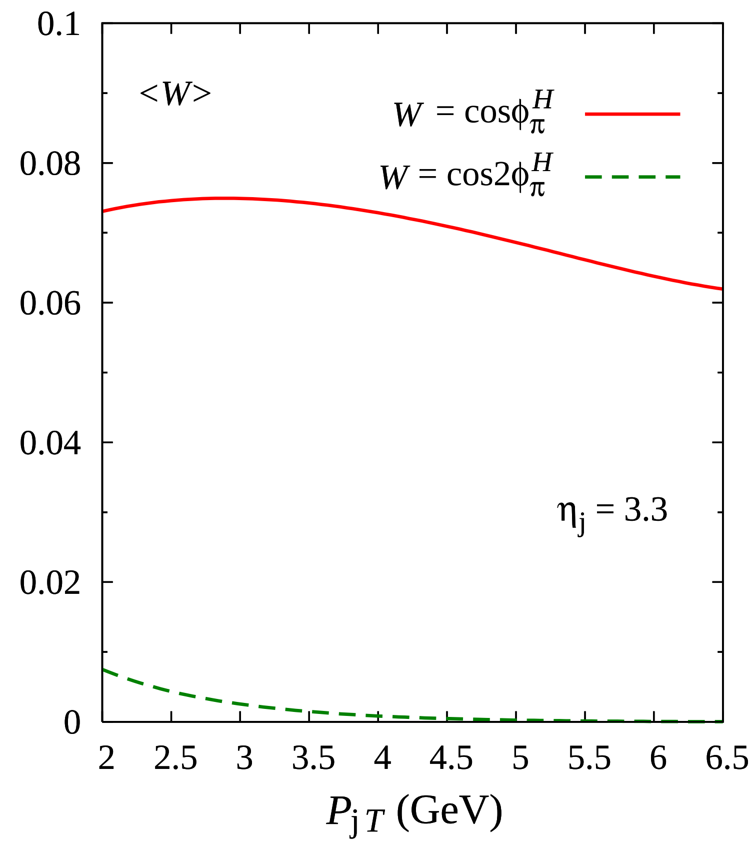
# *Weighted cross sections for $pp \to \mathrm{jet}\,\pi^+ X$: upper bounds*

$\langle \cos\phi_\pi^H \rangle \sim h_1^{\perp q} f_1 H_1^{\perp q} \oplus f_1 h_1^{\perp q} H_1^{\perp q}$  $\langle \cos 2\phi_\pi^H \rangle \sim h_1^{\perp g} f_1 H_1^{\perp g} \oplus h_1^{\perp g} f_1 H_1^{\perp g}$

$x_F = 0$




$0.27 \le x_F \le 0.88$



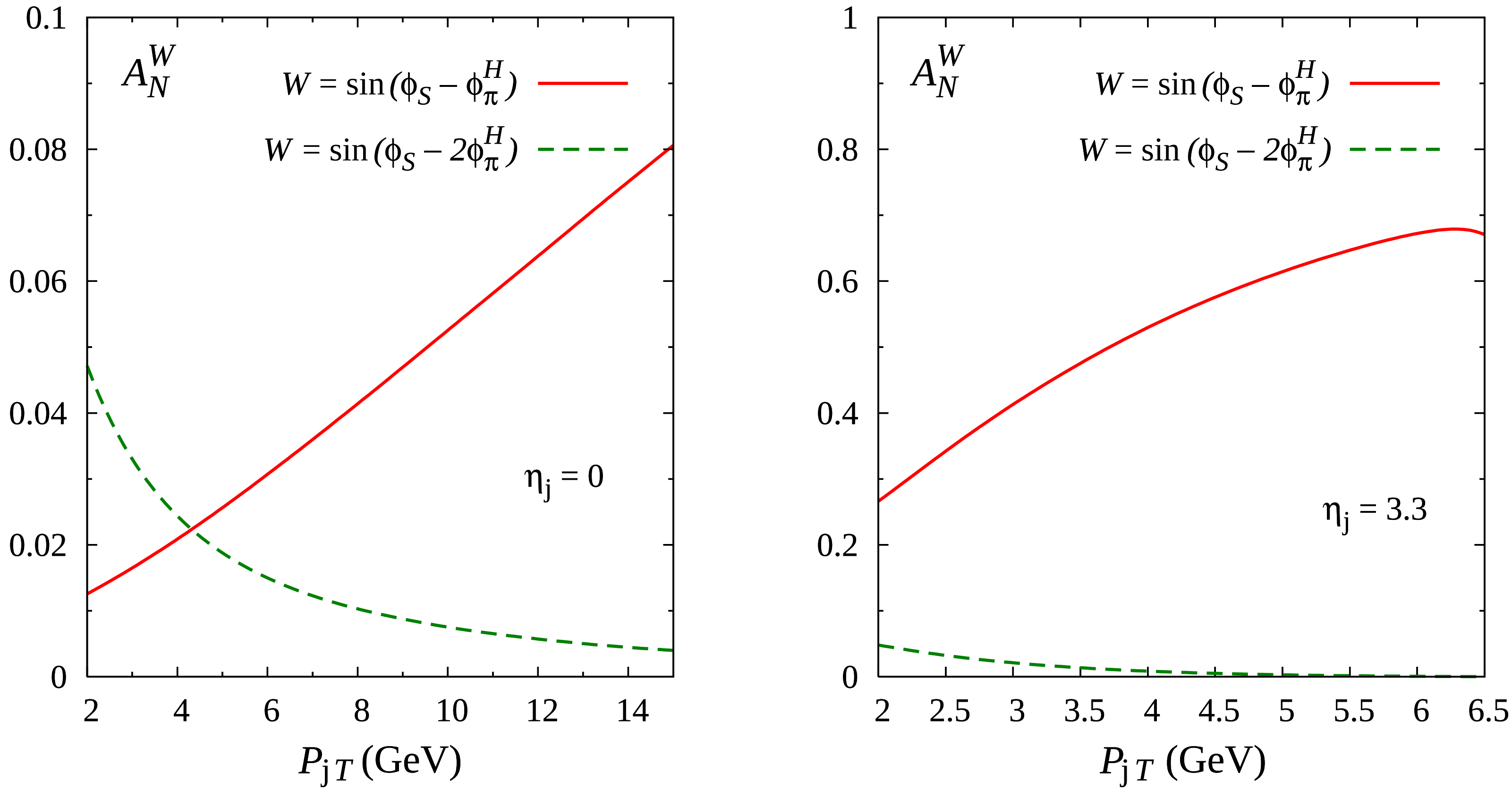

- DSS set of FF instead of kre: $\langle \cos\phi_\pi^H \rangle$ smaller, still sizeble at $\eta_\mathrm{j} = 3.3$

  $\langle \cos 2\phi_\pi^H \rangle$ slightly larger, with $\langle \cos 2\phi_\pi^H \rangle \ge \langle \cos\phi_\pi^H \rangle$ for $P_{\mathrm{j}T} \le 4$ GeV at $\eta_\mathrm{j} = 0$

# Collins(like) asymmetries in $p^\uparrow p \to \text{jet}\,\pi^+ X$: upper bounds

- $A_N^{\sin(\phi_S+\phi_\pi^H)} \sim \left[f_{1T}^{\perp q} h_1^{\perp q} \oplus h_{1T}^{\perp q} f_1\right] H_1^{\perp q}$ (and $A_N^{\sin(\phi_S+2\phi_\pi^H)}$ for gluons) $\approx 0$

$A_N^{\sin(\phi_S-\phi_\pi^H)} \sim h_1^q f_1 H_1^{\perp\, q}$ $\qquad$ $A_N^{\sin(\phi_S-2\phi_\pi^H)} \sim h_1^g f_1 H_1^{\perp\, g}$

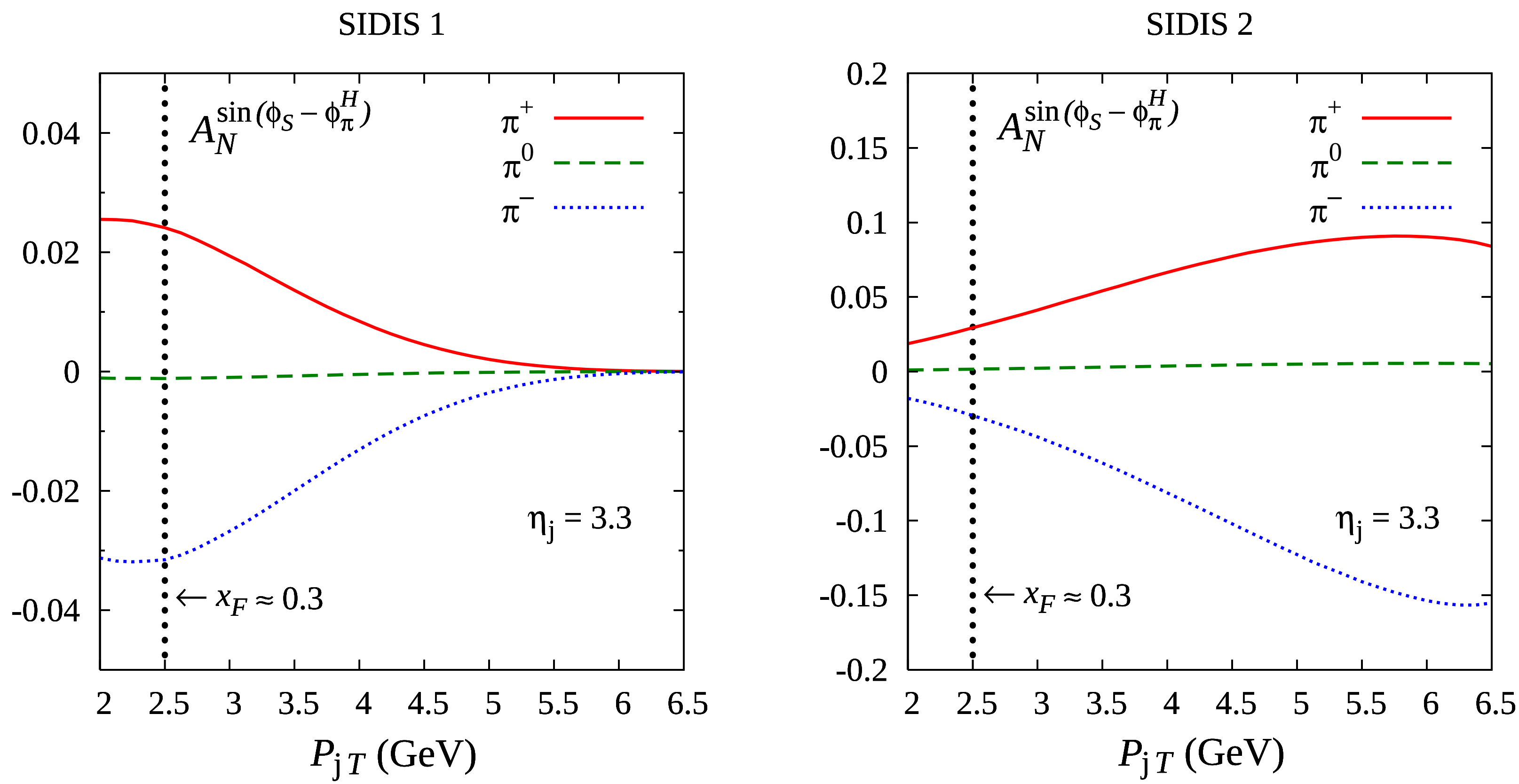



- $A_N^{\sin(\phi_S-\phi_\pi^H)}$ large at $\eta_j = 3.3$, grows with $P_{jT}$; $A_N^{\sin(\phi_S-2\phi_\pi^H)}$ at most $5\%$
- DSS set of FF: similar results, but quark (gluon) contributions to $A_N^W$ are slightly smaller (larger). The same holds true for the Sivers asymmetry

$A_N^{\sin(\phi_S-\phi_\pi^H)}$ estimated using param. of $h_1^q$, $H_1^{\perp q}$ from SIDIS and $e^+e^-$ data by Anselmino *et al*:

SIDIS 1 — PRD 75 (2007) 054032

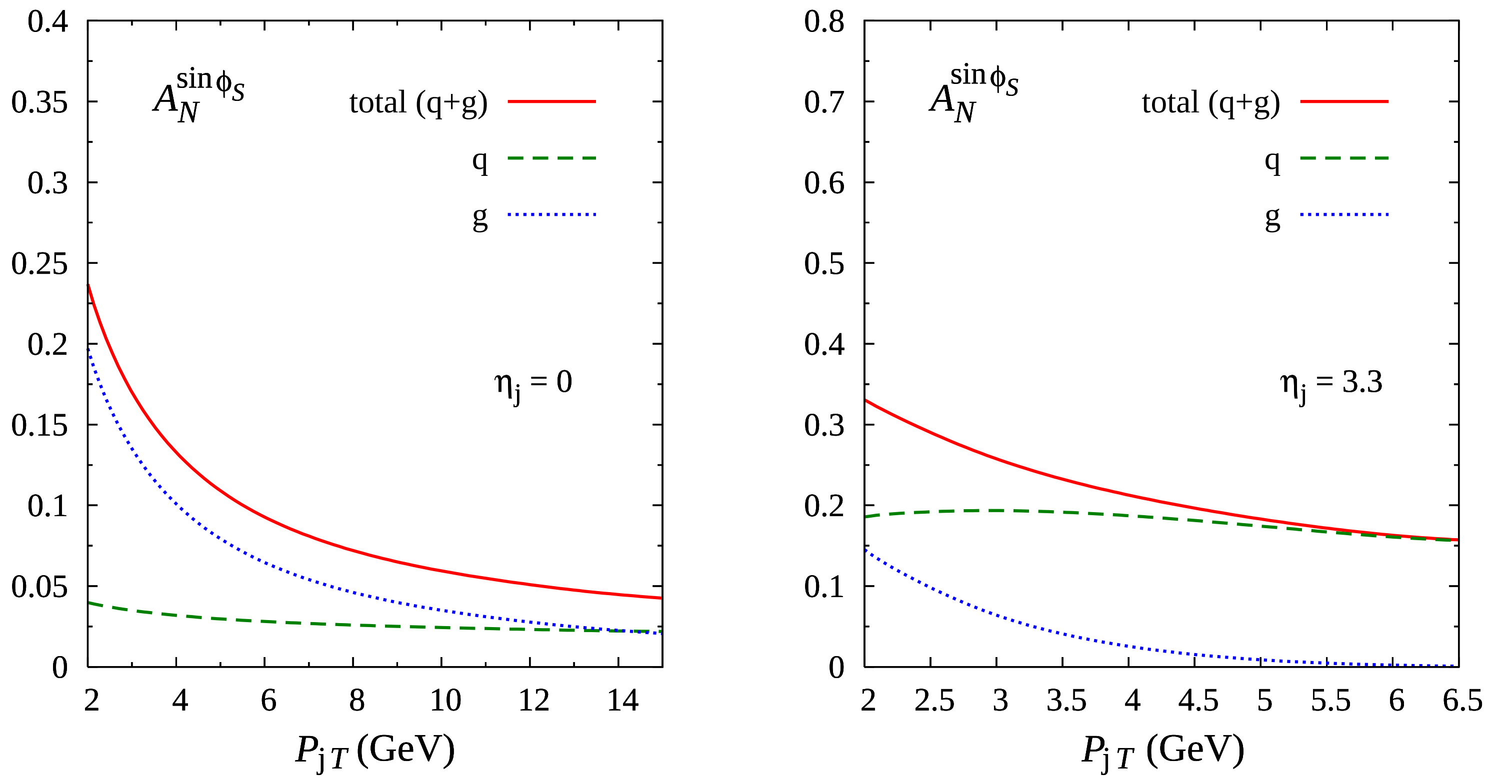
SIDIS 2 — NP (Proc. Suppl.) 191 (2009) 98




- $A_N^{\sin(\phi_S-\phi_\pi^H)} \approx 0$ at $\eta_j = 0$
- Predictions reliable only for $x_F \leq 0.3$ (region covered by present SIDIS data)
- Measurements useful to constrain $h_1^q$ and $H_1^{\perp q}$ in a new kinematic region!

# Sivers asymmetry in $p^\uparrow p \to \text{jet}\,\pi^+ X$: upper bounds

$$A_N^{\sin\phi_S} \sim f_{1T}^\perp f_1 D_1$$




- At $\eta_j = 0$, $A_N^{\sin\phi_S}$ is dominated by the gluon contribution at the lowest $P_{jT}$. A large $A_N^{\sin\phi_S}$ around $P_{jT} = 4 - 6$ GeV: indication for a sizeble gluon contribution
- At $\eta_j = 3.3$, the quark contribution dominates for $P_{jT} \geq 4$ GeV
- $A_N^{\sin\phi_S}$ has also been studied in $p^\uparrow p \to \text{jet}\,X$, similar results

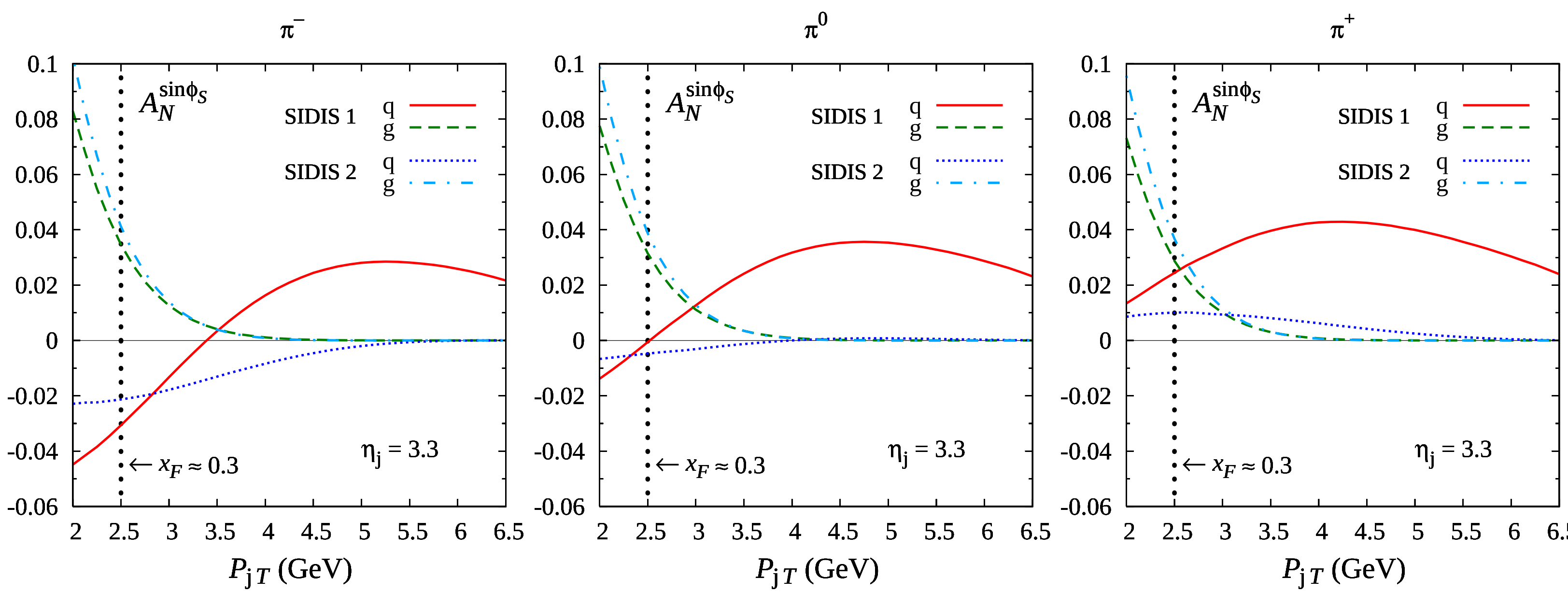
D'Alesio, Murgia, CP, PRD 83 (2011) 034021

## *Sivers asymmetry in $p^\uparrow p \to \text{jet}\, \pi\, X$: parametrizations*

- $A_N^{\sin(\phi_S-\phi_\pi^H)}$ estimated using param. of $f_{1T}^{\perp q}$ from SIDIS by Anselmino *et al*

  SIDIS 1 PRD 72 (2005) 094007

  SIDIS 2 EPJA 39 (2009) 39




- Upper bound on $f_{1T}^{\perp g}$ from analyses of SSA in $p^\uparrow p \to \pi X$ at midrapidity (updated)

  Anselmino, D'Alesio, Melis, Murgia, PRD 74 (2006) 094011

- $A_N^{\sin(\phi_S-\phi_\pi^H)}$ in $p^\uparrow p \to \text{jet}\, X$ similar to the one in $p^\uparrow p \to \text{jet}\, \pi^0\, X$

- Measurements will provide indications on the size of $f_{1T}^{\perp q}$ for $x_F \geq 0.3$

# Color gauge invariant (CGI) generalized parton model

- In the GPM: the Sivers function is assumed to be *universal*, $f_{1T}^{\perp q} \equiv f_{1T}^{\perp q,\,\mathrm{SIDIS}}$
- In the CGI (GPM): the Sivers function is *non-universal*

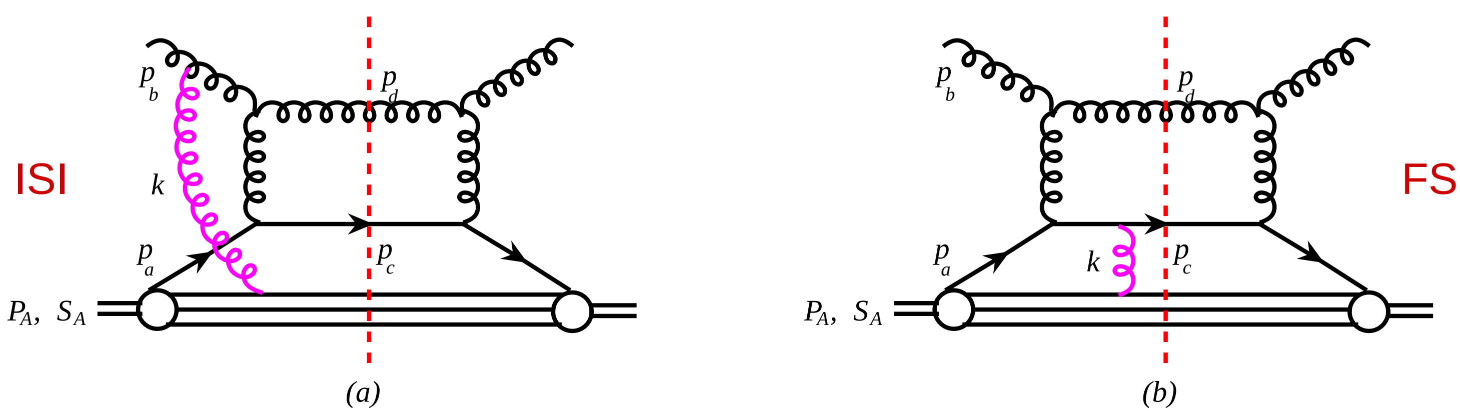
L. Gamberg, talk at Transversity 2011

  - Initial and final state interactions (ISIs/FSIs) considered between the struck parton and the spectators from the polarized hadron through gluon exchange
  - ISIs/FSIs depend on the scattering process $\Longrightarrow$ A different $f_{1T}^{\perp q}$ has to be used for each partonic subprocess $a\,b \to c\,d$ contributing to $p^\uparrow p \to \mathrm{jet}\,\pi\,X$

- The process dependent Sivers function $f_{1T}^{\perp q,\,ab\to cd}$ is known for $p^\uparrow p \to \pi\,X$

Gamberg, Kang, PLB 696 (2011) 1009

Example: $q\,g \to q\,g$

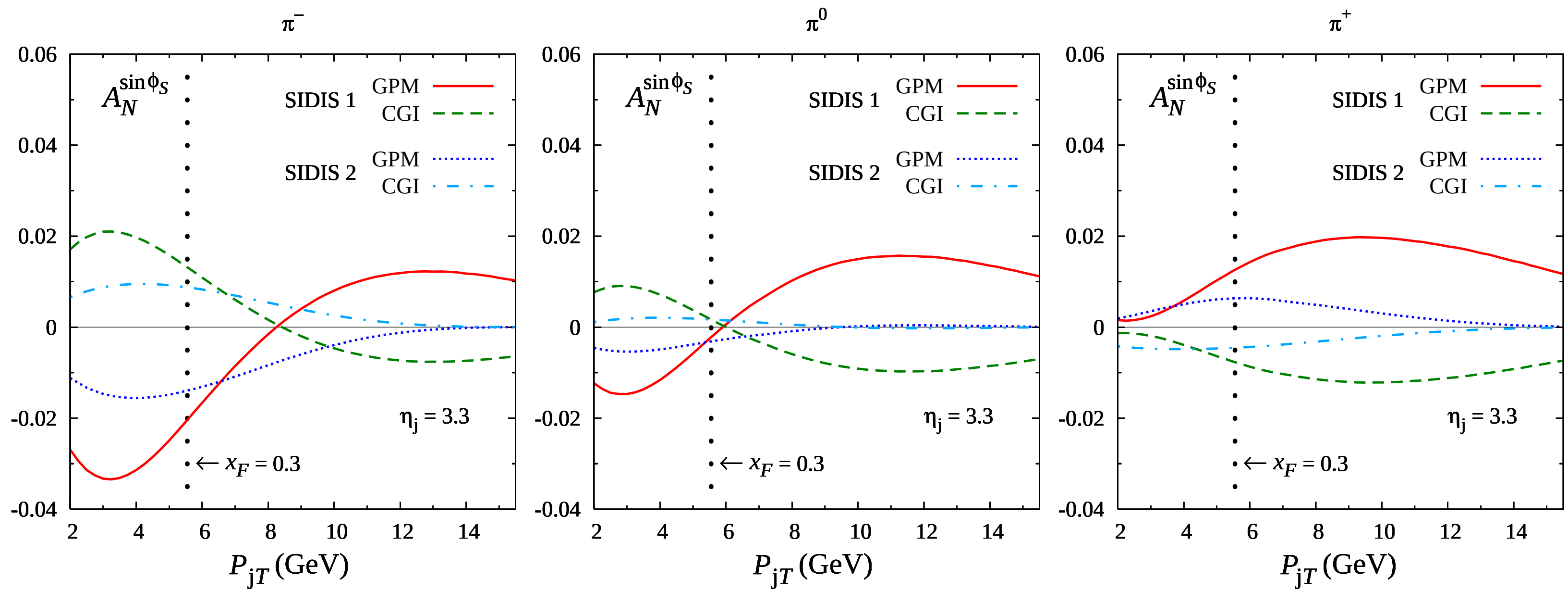



$$f_{1T}^{\perp q,\,qg\to qg} \approx -\frac{N_c^2+2}{2(N_c^2-1)}\, f_{1T}^{\perp q,\,\mathrm{SIDIS}} = -\frac{11}{16}\, f_{1T}^{\perp q,\,\mathrm{SIDIS}}$$

# Test of the process dependence of the Sivers function

GPM *vs* CGI at $\sqrt{s} = 500$ GeV

D'Alesio, Gamberg, Kang, Murgia, CP, arXiv:1108.0827




SIDIS1: $A_N^{\sin(\phi_S-\phi_\pi^H)}[\mathrm{GPM}] \approx -A_N^{\sin(\phi_S-\phi_\pi^H)}[\mathrm{CGI}]$; the same holds true for SIDIS2

- change of sign due to the dominant channel at forward rapidity, $qg \to qg$
- $x_F \leq 0.3$: optimal region to discriminate between the two approaches

At $\sqrt{s} = 200$ GeV, similar results with larger asymmetries, but narrower range of $P_{jT}$

## *Summary and conclusions*

- We have studied the process $p^{(\uparrow)} p \to \mathrm{jet}\, \pi\, X$, which is under present active investigation at RHIC, within a TMD generalized factorization scheme
- We have identified the observable leading-twist azimuthal asymmetries related to both quark and gluon-originated jets (in principle distinguishable)
- In contrast to single inclusive pion production and in analogy with SIDIS, one can discriminate among different effects by taking moments of the asymmetries
- Measurements of a sizeable Collins asymmetry could give an indication on the size and sign of transversity and the Collins function in a new kinematic region
- Test of the universality properties of the Sivers function: the Sivers asymmetry should change sign if we take into account the effects of ISIs/FSIs

- From the phenomenological point of view, the measurement of such types of asymmetries would be a crucial test for the TMD factorization approach!