Probing the multi-gluon correlations through single spin asymmetries

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⋆ QCD-mechanisms for SSA in the two regions of P_T

- Naively "T-odd" distribution/fragmentation functions (Sivers, Collins *etc*)

  · Describes SSA in the region of $\Lambda_{\rm QCD} \ll P_T \ll Q$ in the framework of TMD factorization.

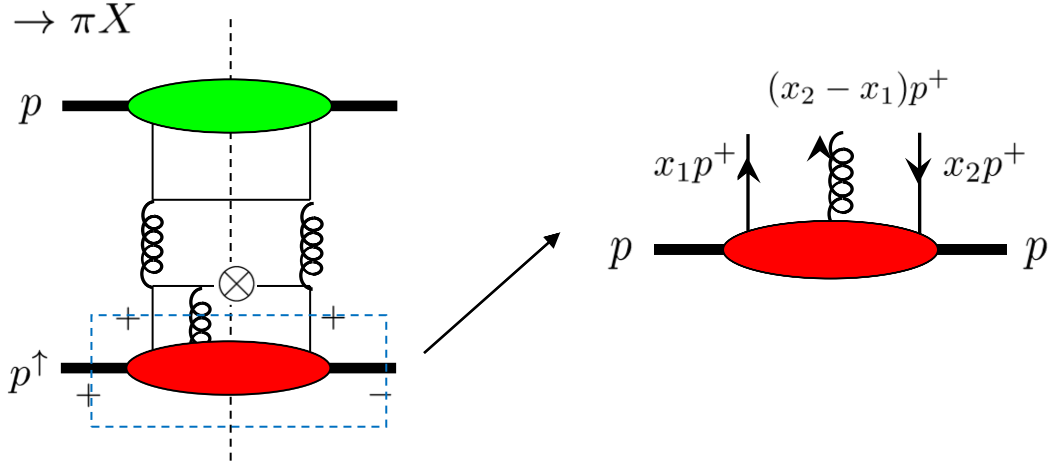
  · TMD distributions are process dependent!

- Multi-parton correlation functions (quark-gluon, purely gluonic *etc*)
  (Efremov-Teryaev, Qiu-Sterman, Eguchi-YK-Tanaka,..)

  · Describes SSA in the region of $P_T \sim Q \gg \Lambda_{\rm QCD}$ as a twist-3 observable in the framework of collinear factorization.

  · Multi-parton correlation functions are process-independent!

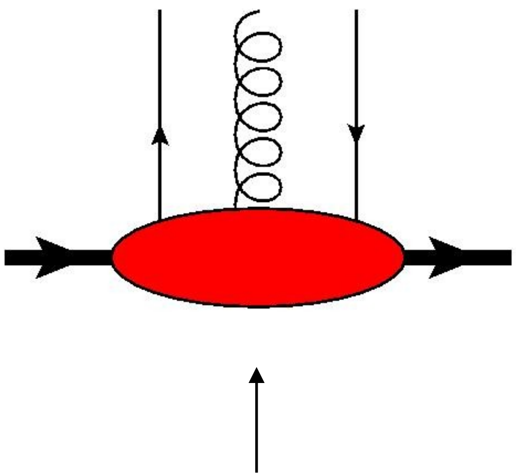
$p^\uparrow p \to \pi X$



$\sim \langle \bar{\psi} g F^{+\perp} \psi \rangle$

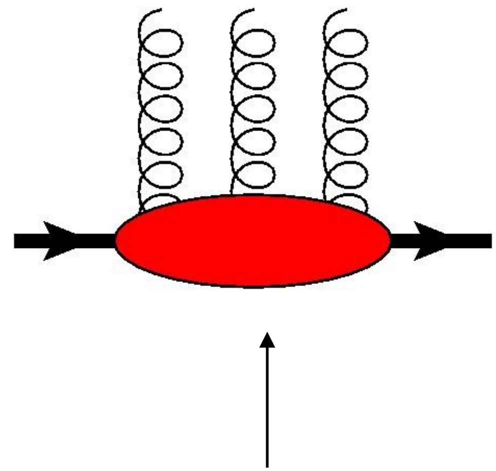
⋆ Multi-parton correlation functions in the nucleon.

Quark-gluon correlation functions



Many works on this!

Three-gluon correlation functions



This talk.

Early works:
Ji, PLB 289('92)137; Kang-Qiu, PRD 78('08)034005;
Kang-Qiu-Vogelsang-Yuan, PRD 78('08)114013.

Contents:

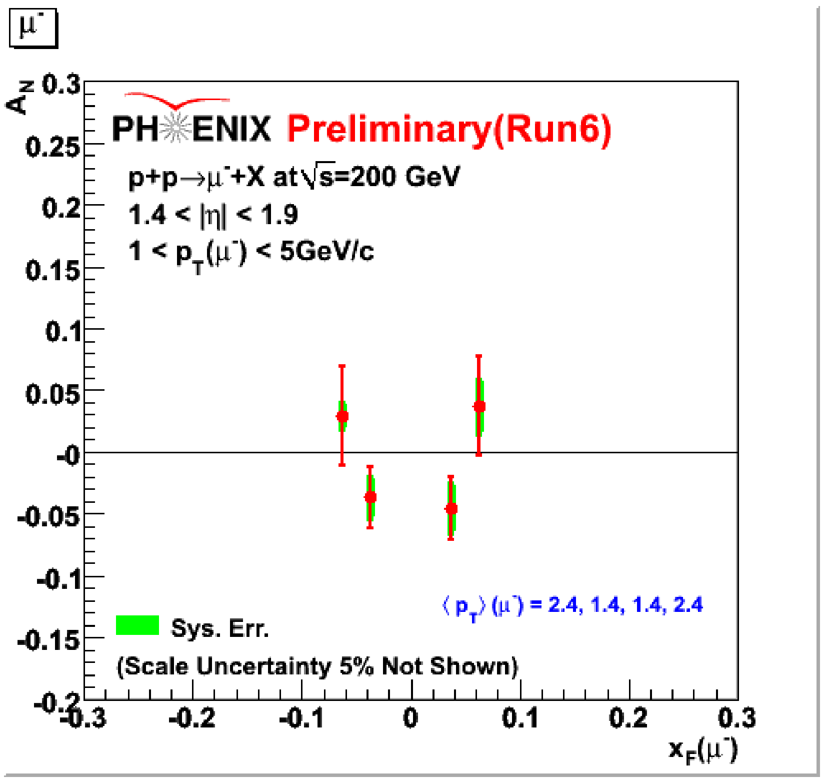
1. $p^{\uparrow}p \to DX$
   - Twist-3 three-gluon correlation functions.
   - Formalism and the cross section formula.
   - Model calculation and comparison with RHIC (preliminary) data.
2. Drell-Yan and Direct-photon: $p^{\uparrow}p \to \gamma^{(*)}X$
3. SIDIS, $ep^{\uparrow} \to eDX$, at the EIC energy
4. Summary

$p^{\uparrow}p \to DX$

YK, S.Yoshida, PRD84(2011)014026 (arXiv:1104.3943 [hep-ph]).

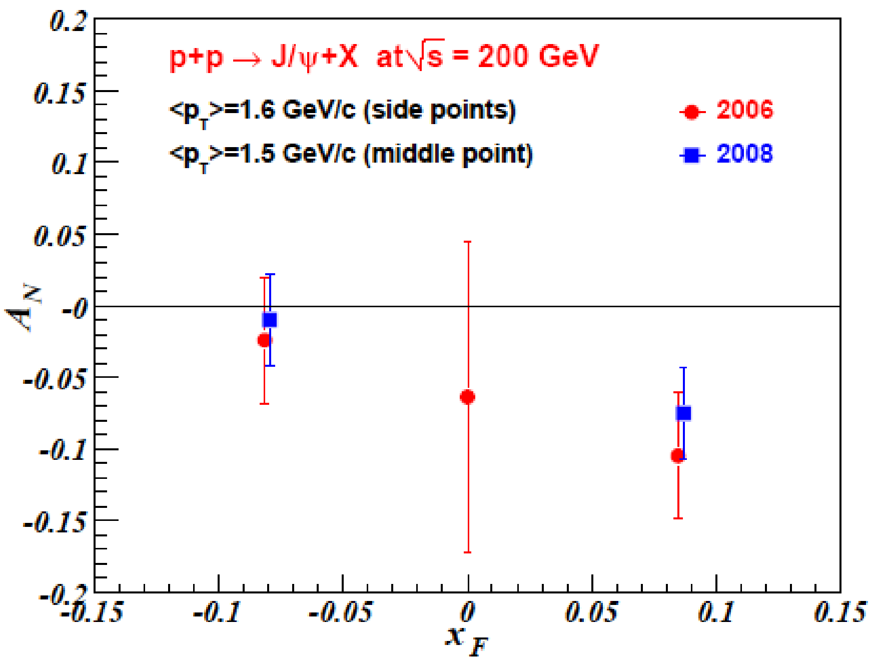
Cf. $ep^{\uparrow} \to eDX$:
H.Beppu, YK, T. Tanaka, S. Yoshida, PRD 82(2010)054005,(arXiv:1007.2034)
YK, T. Tanaka, S.Yoshida, PRD 83(2011)114014 (arXiv:1104.0798) .

- $p^{\uparrow}p \to DX$ at RHIC(PHENIX)

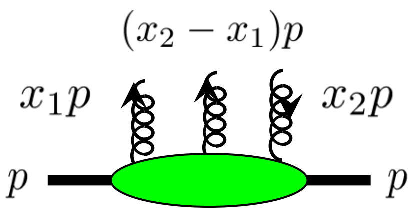


- $p^{\uparrow}p \to J/\psi X$ at RHIC(PHENIX)

arXiv:1009.4864[hep-ex]



⋆ Twist-3 "three-gluon" correlation functions



cf. Beppu-Koike-Tanaka-Yoshida (PRD 82('10)054005)

See also, Belitsky-Ji-Lu-Osborne, PRD63,094012(2001)
Braun-Manashov-Pirnay, PRD80,114002(2009).

-Two independent correlation functions $O(x_1, x_2)$ and $N(x_1, x_2)$ due to Hermiticity, PT-invariance and Permutation symmetry

$$O^{\alpha\beta\gamma}(x_1,x_2) = -gi^3 \int \frac{d\lambda}{2\pi}\int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle pS| d^{bca} F_b^{\beta n}(0) F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n)|pS\rangle$$

$$= 2iM_N \left[O(x_1,x_2) g^{\alpha\beta}\epsilon^{\gamma pnS} + O(x_2, x_2 - x_1) g^{\beta\gamma}\epsilon^{\alpha pnS} + O(x_1, x_1 - x_2) g^{\gamma\alpha}\epsilon^{\beta pnS}\right]$$

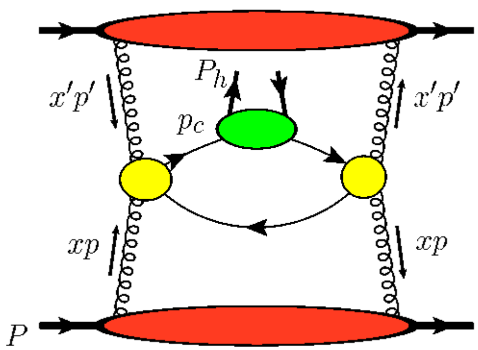
$$N^{\alpha\beta\gamma}(x_1,x_2) = -gi^3 \int \frac{d\lambda}{2\pi}\int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle pS| i f^{bca} F_b^{\beta n}(0) F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n)|pS\rangle$$

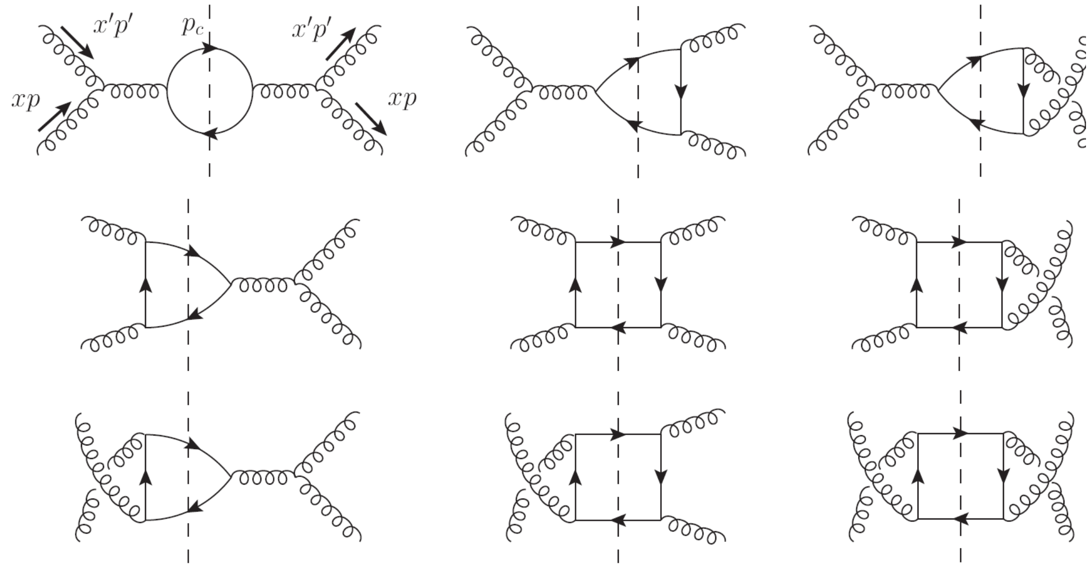
$$= 2iM_N \left[N(x_1,x_2) g^{\alpha\beta}\epsilon^{\gamma pnS} - N(x_2, x_2 - x_1) g^{\beta\gamma}\epsilon^{\alpha pnS} - N(x_1, x_1 - x_2) g^{\gamma\alpha}\epsilon^{\beta pnS}\right].$$

$F_a^{\alpha n} \equiv F_a^{\alpha\mu} n_\mu$ $\quad$ n: lightlike vector satisfying $p \cdot n = 1$.

$\epsilon^{\gamma pnS} \equiv \epsilon^{\gamma\mu\nu\lambda} p_\mu n_\nu S_\lambda$ etc. $\quad$ Gauge-links suppressed above.

$\star$ Gluon contribution to twist-2 unpolarized cross section for $pp \to DX$.





$c \to D$ fragmentation function gluon density

$$P_h^0 \frac{d\sigma^{\rm unpol}}{d^3P_h} = \frac{\alpha_s^2}{S} \sum_{f=c,\bar{c}} \int \frac{dz}{z^2} D_f(z) \int \frac{dx'}{x'} G(x') \int \frac{dx}{x} G(x) \delta(\tilde{s}+\tilde{t}+\tilde{u}) \hat{\sigma}_{gg\to c}$$

$$\hat{\sigma}_{gg\to c} = \frac{1}{2N}\left(\frac{1}{\tilde{t}\tilde{u}} - \frac{N}{C_F}\frac{1}{\tilde{s}}\right)\left(\tilde{t}^2+\tilde{u}^2+4m_c^2\tilde{s} - \frac{4m_c^4 s^2}{\tilde{t}\tilde{s}}\right) \qquad C_F = \frac{N^2-1}{2N}$$

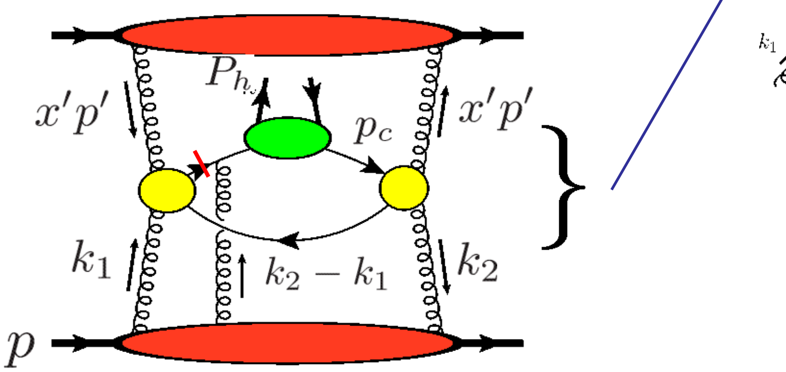
$$\tilde{s} = (xp+x'p')^2 \qquad \tilde{t} = (p_c - xp)^2 - m_c^2 \qquad \tilde{u} = (p_c - x'p')^2 - m_c^2,$$

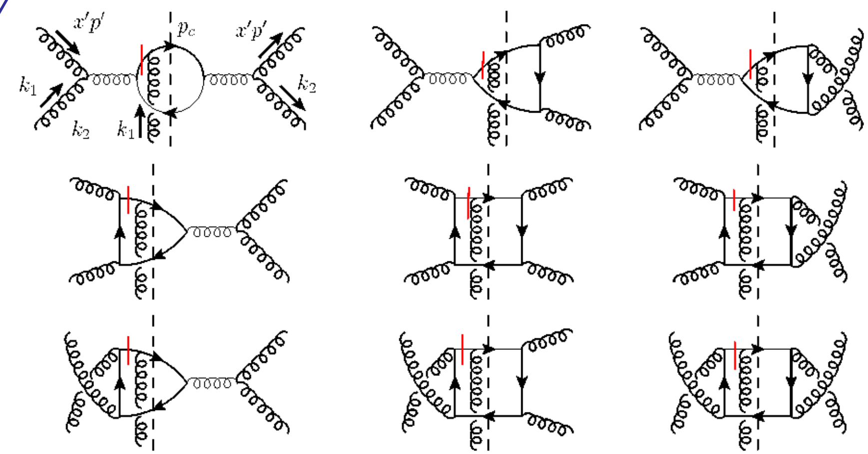
⋆ Three-gluon contribution to twist-3 cross section for $p^\uparrow p \to DX$

· SSA occurs as a pole contribution at $x_1 = x_2 \to$ soft-gluon-pole (SGP).

• Final state interaction

$S^{abc}_{\mu\nu\lambda}(k_1, k_2, x'p', p_c)$: hard part

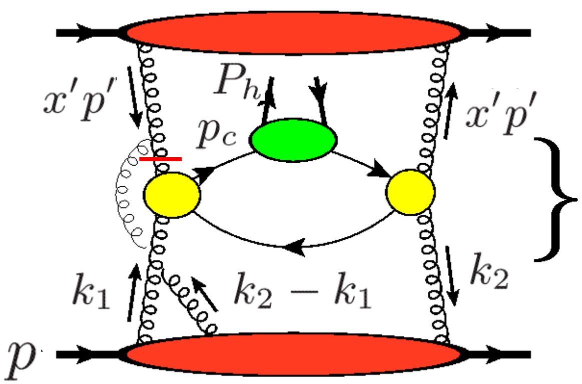


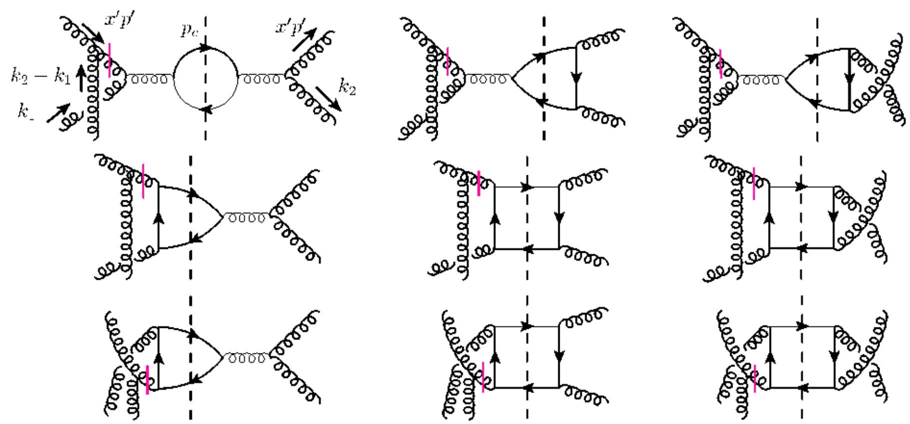


$\langle A^b_\nu A^c_\lambda A^a_\mu \rangle$

+ mirrors

• Initial state interaction





⋆ Twist-3 cross section from the soft-gluon-pole (SGP) at $x_1 = x_2$

(Feynman gauge calculation)

$$P_h^0 \frac{d\sigma^{3\text{gluon}}}{d^3 P_h} = \frac{\alpha_s}{S} \sum_{a=c,\bar{c}} \int \frac{dx'}{x'} G(x') \int \frac{dz}{z^2} D_a(z) \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \times \left. \frac{\partial S^{abc}_{\mu\nu\lambda}(k_1, k_2, x'p', p_c) p^\lambda}{\partial k_2^\sigma} \right|_{k_i = x_i p} \omega^\mu{}_\alpha \, \omega^\nu{}_\beta \, \omega^\sigma{}_\gamma \, \mathcal{M}^{\alpha\beta\gamma}_{F,abc}(x_1, x_2),$$

$$\omega^\mu{}_\alpha = g^\mu{}_\alpha - p^\mu n_\alpha$$

where

$$\mathcal{M}^{\alpha\beta\gamma}_{F,abc}(x_1, x_2) = -g i^3 \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \langle pS | F_b^{\beta n}(0) F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n) | pS \rangle$$
$$= \frac{3}{40} d^{abc} O^{\alpha\beta\gamma}(x_1, x_2) - \frac{i}{24} f^{abc} N^{\alpha\beta\gamma}(x_1, x_2),$$

· $\left. \frac{\partial S^{abc}_{\mu\nu\lambda}(k_1, k_2, x'p', p_c) p^\lambda}{\partial k_2^\sigma} \right|_{k_i = x_i p}$ produces $\delta(x_1 - x_2)$ and $\delta'(x_1 - x_2)$.

$$\longrightarrow \quad O^{\alpha\beta\gamma}(x, x) = 2iM_N \left[O(x, x) g^{\alpha\beta} \epsilon^{\gamma p n S} + O(x, 0) \left(g^{\beta\gamma} \epsilon^{\alpha p n S} + g^{\gamma\alpha} \epsilon^{\beta p n S} \right) \right]$$

$$N^{\alpha\beta\gamma}(x, x) = 2iM_N \left[N(x, x) g^{\alpha\beta} \epsilon^{\gamma p n S} - N(x, 0) \left(g^{\beta\gamma} \epsilon^{\alpha p n S} + g^{\gamma\alpha} \epsilon^{\beta p n S} \right) \right]$$

· Different cross sections for $O(x, x)$ and $O(x, 0)$ (Likewise for $N(x, x)$ and $N(x, 0)$).

($\leftrightarrow$ Kang-Qiu('08), Kang-Qiu-Vogelsang-Yuan('08))

⋆ Three-gluon contribution to $p^{\uparrow}(p) + p(p') \to D(P_h) + X$

$$P_h^0 \frac{d\sigma^{3\text{gluon}}}{d^3P_h} = \frac{\alpha_s^2 M_N \pi}{S} \epsilon^{P_h p n S_\perp} \sum_{f=c\bar{c}} \int \frac{dx'}{x'} G(x') \int \frac{dz}{z^3} D_a(z) \int \frac{dx}{x} \delta\left(\tilde{s}+\tilde{t}+\tilde{u}\right) \frac{1}{\tilde{u}}$$

$$\times \left[\delta_f \left\{ \left(\frac{d}{dx} O(x,x) - \frac{2O(x,x)}{x} \right) \hat{\sigma}^{O1} + \left(\frac{d}{dx} O(x,0) - \frac{2O(x,0)}{x} \right) \hat{\sigma}^{O2} + \frac{O(x,x)}{x} \hat{\sigma}^{O3} + \frac{O(x,0)}{x} \hat{\sigma}^{O4} \right\} \right.$$

$$\left. + \left\{ \left(\frac{d}{dx} N(x,x) - \frac{2N(x,x)}{x} \right) \hat{\sigma}^{N1} + \left(\frac{d}{dx} N(x,0) - \frac{2N(x,0)}{x} \right) \hat{\sigma}^{N2} + \frac{N(x,x)}{x} \hat{\sigma}^{N3} + \frac{N(x,0)}{x} \hat{\sigma}^{N4} \right\} \right].$$

$$\tilde{s} = (xp + x'p')^2 \qquad \tilde{t} = (p_c - xp)^2 - m_c^2 \qquad \tilde{u} = (p_c - x'p')^2 - m_c^2,$$

$$\delta_c = 1 \ (D\text{-meson}) \text{ and } \delta_{\bar{c}} = -1 \ (\bar{D}\text{-meson}) \qquad\qquad p_c = \frac{P_h}{z}$$

· Receives derivative contributions from $O(x,x), O(x,0), N(x,x), N(x,0)$.

· O-function contributes to D and $\bar{D}$ with opposite signs.

· Differs from Kang-Qiu-Vogelsang-Yuan (PRD 78('08)114013)!

-Setting $\hat{\sigma}^{O2} = \hat{\sigma}^{O4} = \hat{\sigma}^{N2} = \hat{\sigma}^{N4} \equiv 0$, and $O(x,x) \to O(x,x) + O(x,0)$ and $N(x,x) \to N(x,x) - N(x,0)$, they agree.

· 4 hard cross sections for the derivative terms are dominant and have similar magnitude (at RHIC energy): $\hat{\sigma}^{O1} \simeq \hat{\sigma}^{O2} \sim \hat{\sigma}^{N1} \simeq -\hat{\sigma}^{N2}$ ($\gg \hat{\sigma}^{O3}, \hat{\sigma}^{N3}, \hat{\sigma}^{O4}, \hat{\sigma}^{N4}$).

· At $m_c \to 0$, one has $\hat{\sigma}^{O1} = \hat{\sigma}^{O2}$ and $\hat{\sigma}^{N1} = -\hat{\sigma}^{N2}$ and $\hat{\sigma}^{O3,O4,N3,N4} = 0$.

$\star$ Model calculation of A_N^D

$$\hat{\sigma}^{O1} \simeq \hat{\sigma}^{O2} \sim \hat{\sigma}^{N1} \simeq -\hat{\sigma}^{N2}$$

$\longrightarrow$ $O(x,x)+O(x,0)$ and $N(x,x)-N(x,0)$ can be taken as effective 3-gluon functions.

$\longrightarrow$ Ansatz: $O(x,x) = O(x,0) = N(x,x) = -N(x,0)$

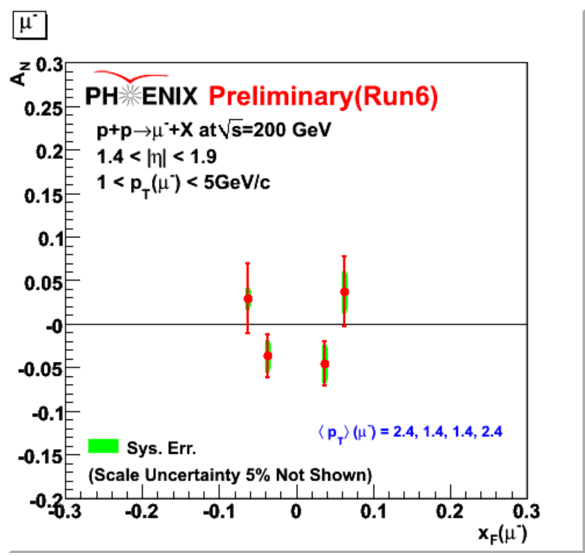
Two models for the functions with constant K_G, K'_G.

$$\begin{cases} \text{Model 1: } O(x,x) = K_G \times x\,G(x) \\ \text{Model 2: } O(x,x) = K'_G \times \sqrt{x}\,G(x) \end{cases}$$

· $G(x)$:unpolarized gluon distribution function
Gluck, Jimenez-Delgado, Reya Eur.phys.J.C53(2008)

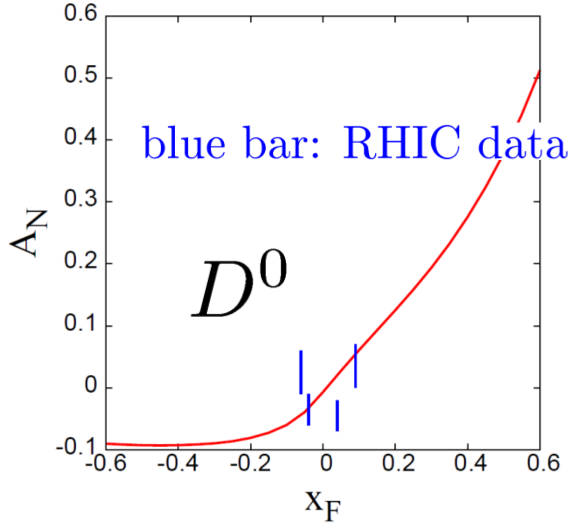
· D-meson fragmentation function by
KKKS (Kneesh *et al.* NPB799('08)34)

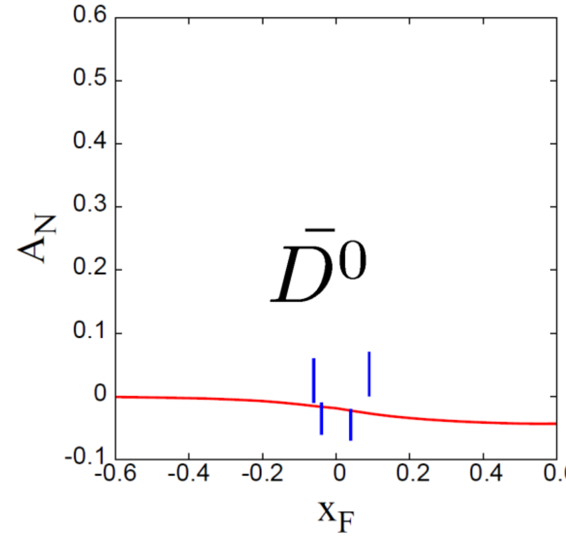
$\star$ We determine K_G, K'_G so that A_N is consistent with the RHIC preliminary data.



Model 1:
$O(x,x) = 0.002xG(x)$

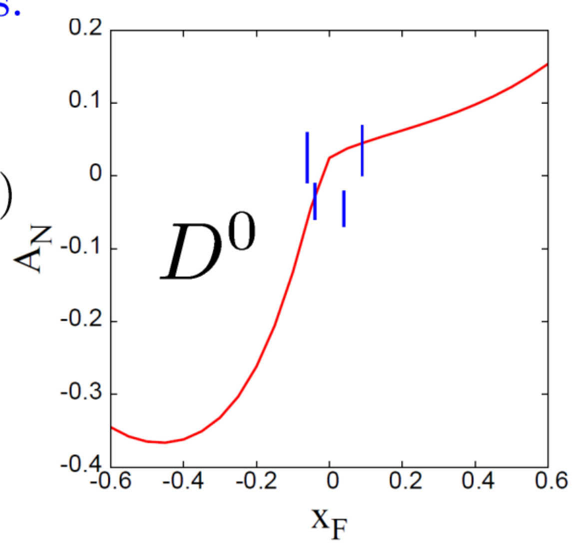
@ $\sqrt{S} = 200$ GeV, $P_T = 2$ GeV

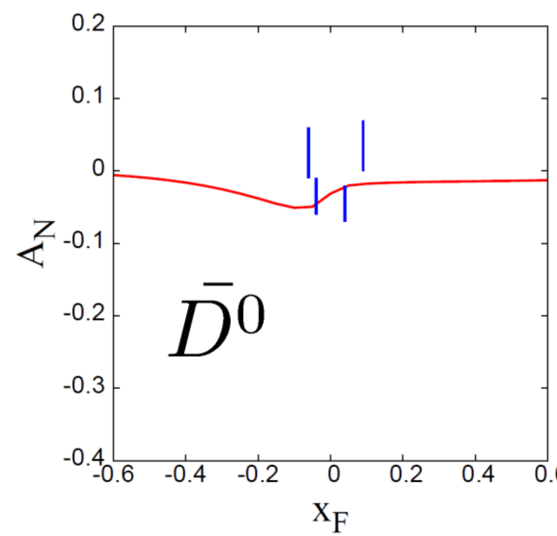




$\cdot\{O(x,x), O(x,0)\}$ and $\{N(x,x), N(x,0)\}$ contributes constructively (destructively) for D ($\bar{D}$). $\rightarrow$ Change of relative signs between O and N leads to opposite prediction for D and $\bar{D}$ mesons.

Model 2:
$O(x,x) = \frac{1}{4} \times 0.002\sqrt{x}G(x)$





$\cdot$ A_N at $x_F < 0$ strongly depends on the small-x behavior of 3-gluon correlation function.

Direct photon production and Drell-Yan

$$p^{\uparrow} p \to \gamma^{(*)} X$$

(YK, S.Yoshida, in preparation)

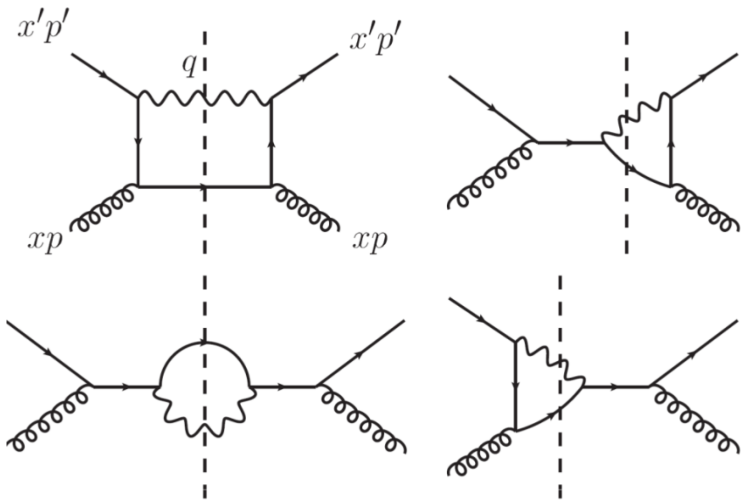
⋆ Twist-2 unpolarized cross section for direct photon production

$$p(p) + p(p') \to \gamma(q) + X$$

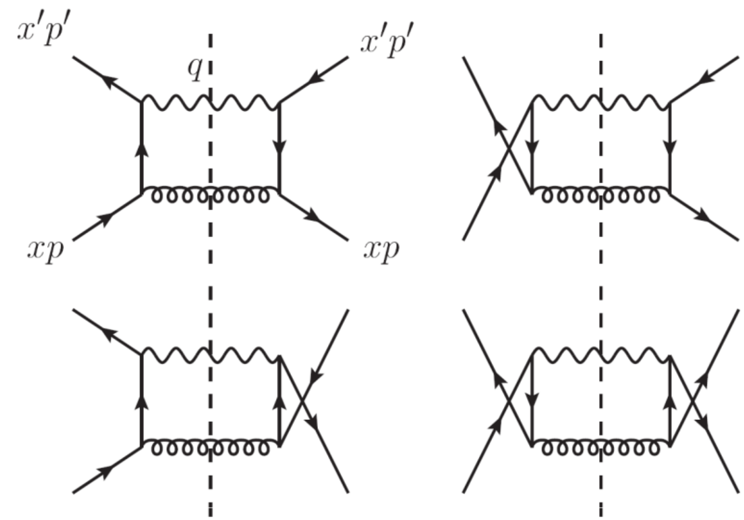
$$E_\gamma \frac{d\sigma}{d^3q} = \frac{\alpha_{em}\alpha_s}{S} \sum_a \int \frac{dx'}{x'} \int \frac{dx}{x} \left[f_a(x) G(x') \hat{\sigma}_{qg\to\gamma q} + f_a(x')\{G(x)\hat{\sigma}_{gq\to\gamma q} + f_{\bar{a}}(x)\hat{\sigma}_{q\bar{q}\to\gamma g}\}\right] \delta\left(\hat{s}+\hat{t}+\hat{u}\right)$$

$$\hat{\sigma}_{qg\to\gamma q} = -\frac{1}{N}(\frac{\hat{t}}{\hat{s}} + \frac{\hat{s}}{\hat{t}}), \quad \hat{\sigma}_{gq\to\gamma q} = -\frac{1}{N}(\frac{\hat{u}}{\hat{s}} + \frac{\hat{s}}{\hat{u}}), \quad \hat{\sigma}_{q\bar{q}\to\gamma g} = \frac{2C_F}{N}(\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}}),$$

$$\hat{s} = (xp + x'p')^2, \quad \hat{t} = (xp - q)^2, \quad \hat{u} = (x'p' - q)^2$$

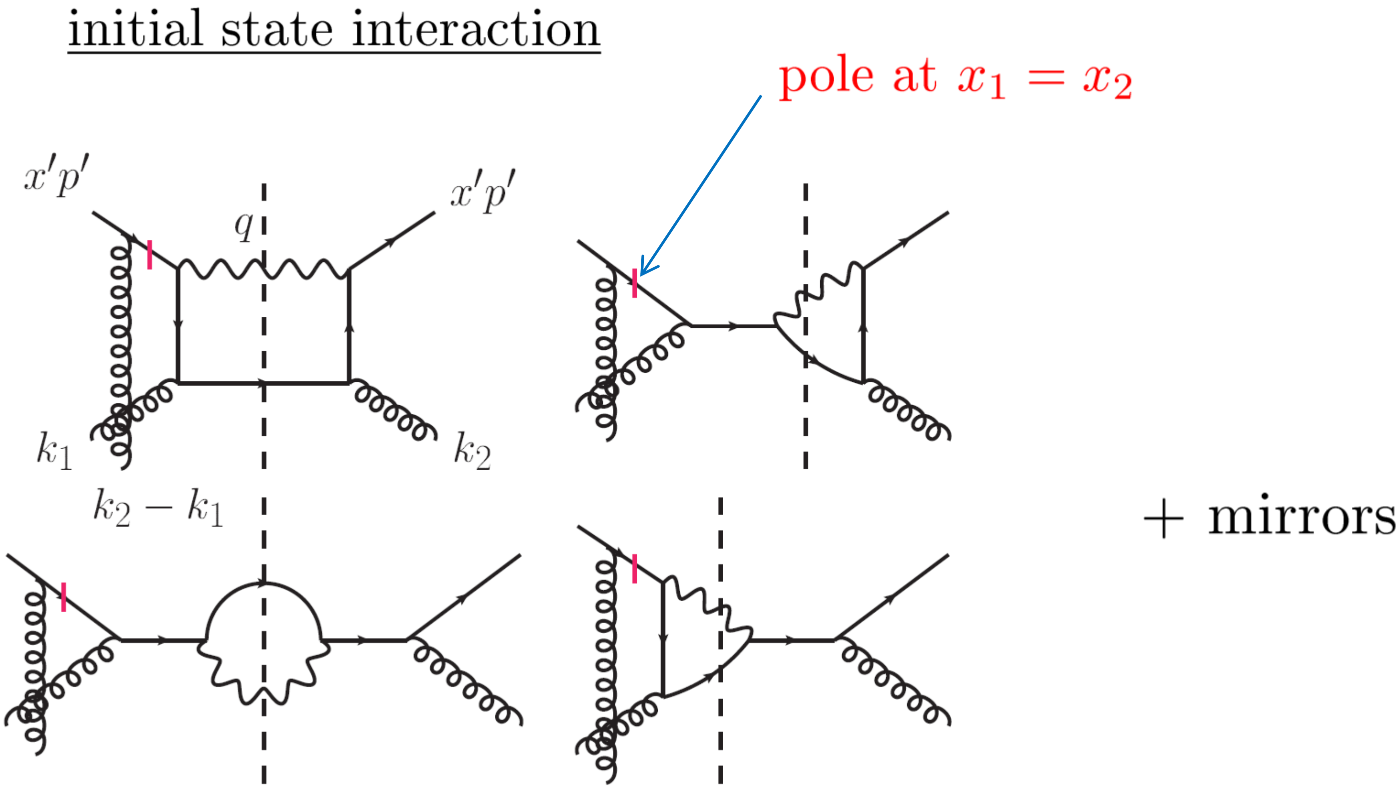


$gq \to \gamma q$



$q\bar{q} \to \gamma g$

Twist-3(polarized cross section)



These diagrams give pole contribution at the $x_1 = x_2$

$\star$ Three-gluon contribution to $p^{\uparrow}(p) + p(p') \to \gamma(q) + X$.

$$E_\gamma \frac{d\sigma}{d^3q} = \frac{4\alpha_{em}\alpha_s M_N \pi}{S} \sum_a e_a^2 \int \frac{dx'}{x'} f_a(x') \int \frac{dx}{x} \delta(\hat{s}+\hat{t}+\hat{u}) \epsilon^{qpnS_\perp} \frac{1}{\hat{u}}$$
$$\times \left[\delta_a \left(\frac{d}{dx} O(x) - \frac{2O(x)}{x} \right) - \left(\frac{d}{dx} N(x) - \frac{2N(x)}{x} \right) \right] \left(\frac{1}{N} \left(\frac{\hat{s}}{\hat{u}} + \frac{\hat{u}}{\hat{s}} \right) \right)$$

$$O(x) \equiv O(x,x) + O(x,0)$$
$$N(x) \equiv N(x,x) - N(x,0)$$

$\uparrow$ The same as twist-2 cross section (also from master formula!)

$\delta_a = 1$ for a =quark, $\delta_a = -1$ for a =anti-quark.

$\hat{s} = (xp + x'p')^2$, $\hat{t} = (xp - q)^2$, $\hat{u} = (x'p' - q)^2$

· This differs from the previous study (X. Ji, Phys.lett.B289 ('92)137).

· Three-gluon correlations contribute in the combination of $O(x) \equiv O(x,x) + O(x,0)$ and $N(x) \equiv N(x,x) - N(x,0)$ as in the $m_c \to 0$ case for $p^{\uparrow}p \to DX$.

· If $O(x) \simeq -N(x)$, quarks in the unpolarized nucleon are active. $\to$ Large A_N^γ.

· If $O(x) \simeq N(x)$, quarks in the unpolarized nucleon are NOT active. $\to$ Small A_N^γ.

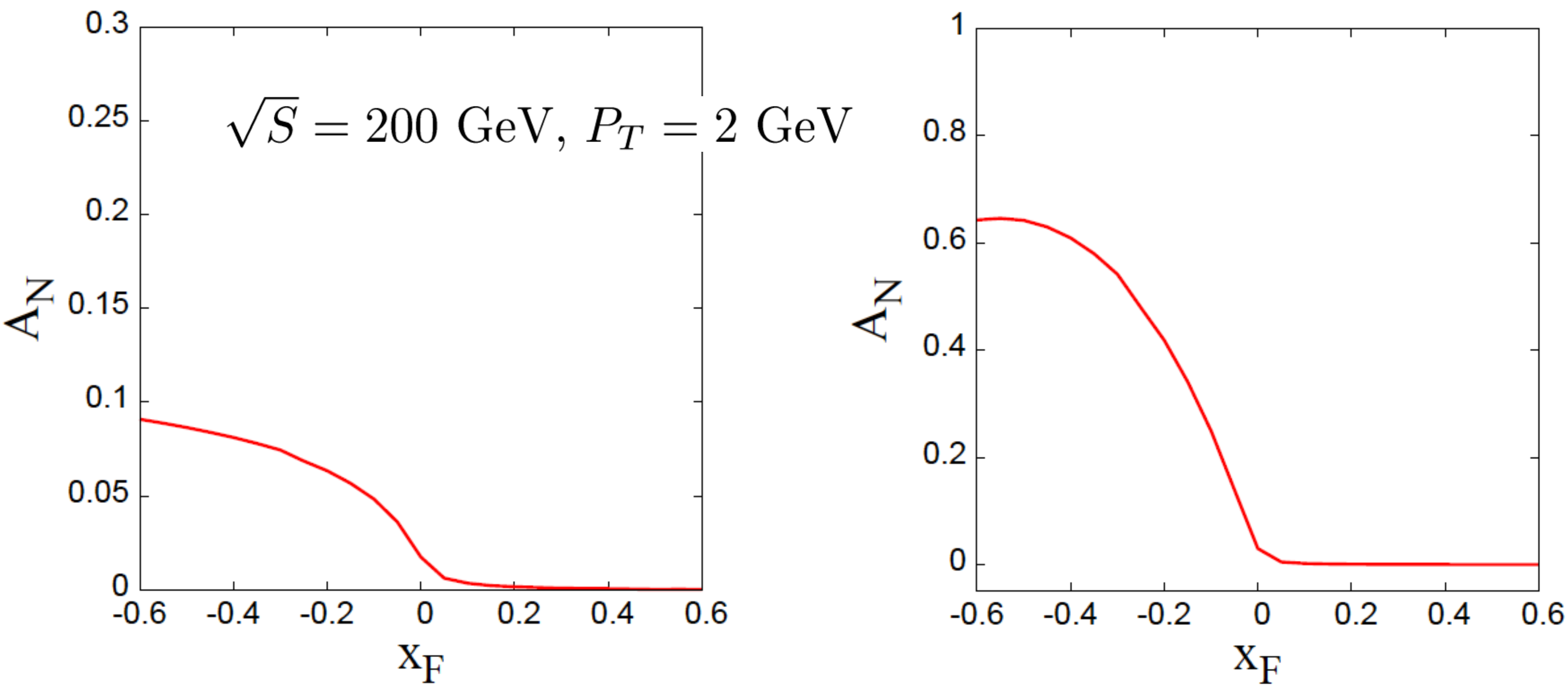
⋆ Estimate of A_N for $p^\uparrow p \to \gamma X$ by the models determined from $p^\uparrow p \to DX$.

(i) $O(x) = -N(x)$ case:

$\to$ Quark contribution from the unpolarized nucleon is active, while anti-quark contribution is cancelled.

Model 1': $O(x,x) = 0.002 \times xG(x)$
$O(x,x) = O(x,0) = -N(x,x) = N(x,0)$

Model 2': $O(x,x) = 0.0005\sqrt{x}G(x)$
$O(x,x) = O(x,0) = -N(x,x) = N(x,0)$



· $A_N \sim 0$ at $x_F > 0$ regardless of magnitude of the 3-gluon correlation functions.

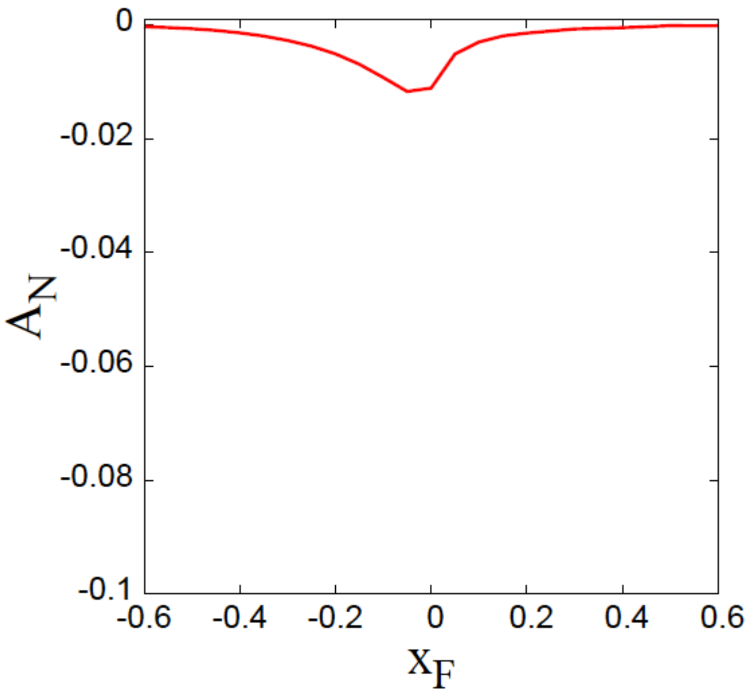
· Behavior at $x_F < 0$ is sensitive to small x behavior similarly to $pp \to DX$.

(ii) $O(x) = N(x)$ case:

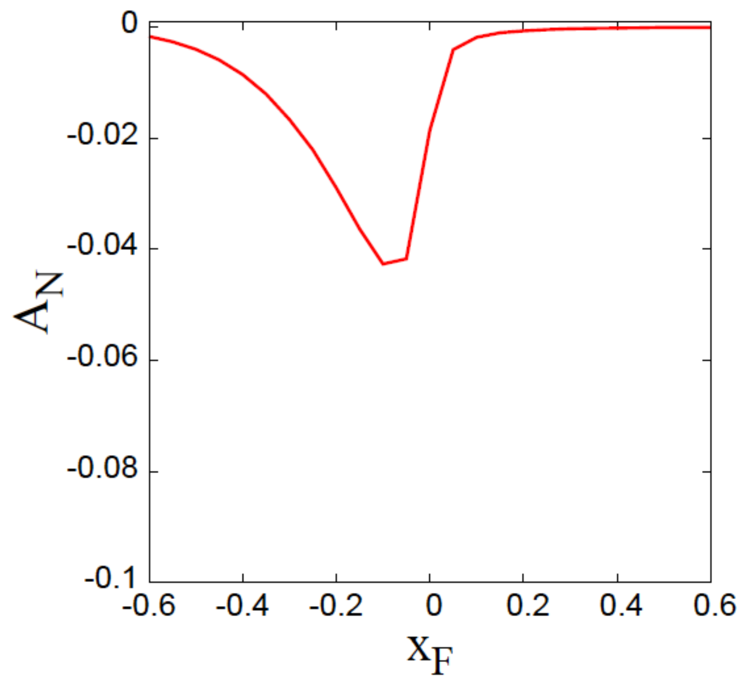
· Only anti-quark contribution from the unpolarized nucleon are active for the polarized cross section.

@ $\sqrt{S} = 200$ GeV, $P_T = 2$ GeV

Model 1: $O(x,x) = 0.002 \times xG(x)$
$O(x,x) = O(x,0) = N(x,x) = -N(x,0)$



Model 2: $O(x,x) = 0.0005\sqrt{x}G(x)$
$O(x,x) = O(x,0) = N(x,x) = -N(x,0)$



⋆ Three-gluon contribution to Drell-Yan: $p^\uparrow p \to \gamma^* X$.

$$\frac{d\sigma}{dQ^2 dy d^2q_\perp} = \frac{2\pi M_N \alpha_{em}^2 \alpha_s}{3\pi S Q^2} \int \frac{dx}{x} \int \frac{dx'}{x'} \delta(\hat{s}+\hat{t}+\hat{u}-Q^2) \epsilon^{qpnS_\perp} \frac{1}{\hat{u}} \sum_a e_a^2 f_a(x')$$

$$\times \Big[\delta_a \Big(\frac{d}{dx} O(x,x) - \frac{2O(x,x)}{x} \Big) \hat{\sigma}_1 + \Big(\frac{d}{dx} O(x,0) - \frac{2O(x,0)}{x} \Big) \hat{\sigma}_2 + \frac{O(x,x)}{x} \hat{\sigma}_3 + \frac{O(x,0)}{x} \hat{\sigma}_4$$

$$- \Big(\frac{d}{dx} N(x,x) - \frac{2N(x,x)}{x} \Big) \hat{\sigma}_1 + \Big(\frac{d}{dx} N(x,0) - \frac{2N(x,0)}{x} \Big) \hat{\sigma}_2 - \frac{N(x,x)}{x} \hat{\sigma}_3 + \frac{N(x,0)}{x} \hat{\sigma}_4 \Big]$$

$$\hat{\sigma}_1 = \frac{2}{N} \Big(\frac{\hat{u}}{\hat{s}} + \frac{\hat{s}}{\hat{u}} + \frac{2Q^2 \hat{t}}{\hat{s}\hat{u}} \Big) \qquad \hat{\sigma}_3 = -\frac{1}{N} \frac{4Q^2(Q^2+\hat{t})}{\hat{s}\hat{u}}$$

$$\hat{\sigma}_2 = \frac{2}{N} \Big(\frac{\hat{u}}{\hat{s}} + \frac{\hat{s}}{\hat{u}} + \frac{4Q^2 \hat{t}}{\hat{s}\hat{u}} \Big) \qquad \hat{\sigma}_4 = -\frac{1}{N} \frac{4Q^2(3Q^2+\hat{t})}{\hat{s}\hat{u}}$$

· At $Q^2 \neq 0$, hard cross sections for $\{O(x,x), N(x,x)\}$ differ from those for $\{O(x,0), N(x,0)\}$ as in $ep^\uparrow \to eDX$.

· As $Q^2 \to 0$, this agrees with the result for the direct-photon production.

· Sum of the above result and that from the quark-gluon correlation functions gives the complete twist-3 cross section for Drell-Yan and direct-photon processes.

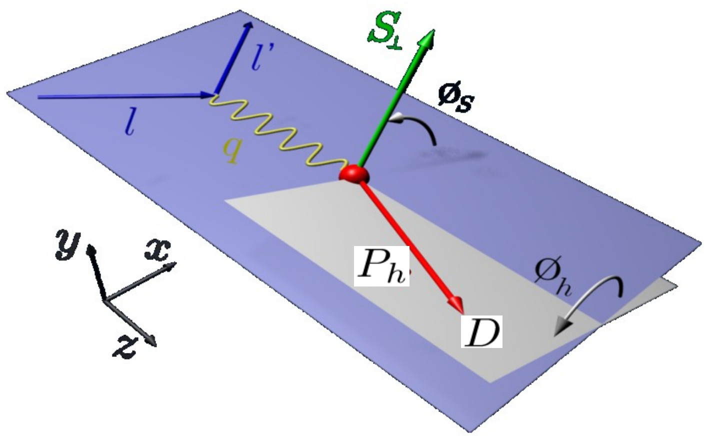
For q-g correlations, see SGP: Ji-Qiu-Vogelsang-Yuan, PRD73('06), YK-Tanaka,PLB646('07)
Hard pole+SFP: Kanazawa-YK, PLB701(2011)576 (arXiv:1105.1036).

SIDIS: $ep^{\uparrow} \to eDX$ at the EIC energy

(Beppu-YK-Tanaka-Yoshida, in preparation)

H.Beppu, YK, T. Tanaka, S. Yoshida, PRD 82(2010)054005,(arXiv:1007.2034)
YK, T. Tanaka, S.Yoshida, PRD 83(2011)114014 (arXiv:1104.0798) .

$\star$ Kinematics for $e(\ell) + p(p, S_\perp) \to e(\ell') + D(P_h) + X$



"Trento convention"

$$S_{ep} = (\ell + p)^2$$
$$q = \ell - \ell'$$
$$x_{bj} = \frac{Q^2}{2p\cdot q}$$
$$z_f = \frac{p\cdot P_h}{p\cdot q}$$

$P_{h\perp} = z_f q_T$: $\perp$-mom. of final π.

ϕ_h: azimuth. angle of hadron plane

ϕ_S: azimuth. angle of $\vec{S}_\perp$

$$\Phi_S \equiv \phi_h - \phi_S$$
$$\phi \equiv \phi_h$$

- **Unpol. cross section**

$$\frac{d^5\sigma^U}{dQ^2 dx_{bj} dz_f dq_T{}^2 d\phi_h} = \sigma_1^U + \sigma_2^U \cos(\phi_h) + \sigma_3^U \cos(2\phi_h)$$

$\star$ Final cross section formula for $ep^\uparrow \to eDX$ generated from three-gluon correlation functions

$$\frac{d^6\Delta\sigma}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\Phi_S}$$

$$= \frac{\alpha_{em}^2\alpha_s e_c^2 M_N}{16\pi^2 x_{bj}^2 S_{ep}^2 Q^2}\left(\frac{-\pi}{2}\right)\sum_{k=1,\cdots,4,8,9}\mathcal{A}_k\mathcal{S}_k\int\frac{dx}{x}\int\frac{dz}{z}\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)+\frac{m_c^2}{\hat{z}^2Q^2}\right)$$

$$\times\sum_{a=c,\bar{c}}D_a(z)\left[\delta_a\left\{\left(\frac{d}{dx}O(x,x)-\frac{2O(x,x)}{x}\right)\Delta\hat{\sigma}_k^1+\left(\frac{d}{dx}O(x,0)-\frac{2O(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{O(x,x)}{x}\Delta\hat{\sigma}_k^3+\frac{O(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right.$$

$$\left.+\left\{\left(\frac{d}{dx}N(x,x)-\frac{2N(x,x)}{x}\right)\Delta\hat{\sigma}_k^1-\left(\frac{d}{dx}N(x,0)-\frac{2N(x,0)}{x}\right)\Delta\hat{\sigma}_k^2+\frac{N(x,x)}{x}\Delta\hat{\sigma}_k^3-\frac{N(x,0)}{x}\Delta\hat{\sigma}_k^4\right\}\right],$$

$\delta_c = 1,\ \delta_{\bar{c}} = -1$

$\mathcal{A}_1 = 1+\cosh^2\psi,\quad \mathcal{A}_2 = -2,\quad \mathcal{A}_3 = -\cos\phi\sinh 2\psi,\quad \mathcal{A}_4 = \cos 2\phi\sinh^2\psi,$

$\mathcal{A}_8 = -\sin\phi\sinh 2\psi,\quad \mathcal{A}_9 = \sin 2\phi\sinh^2\psi.$

$\cosh\psi = \frac{2x_{bj}S_{ep}}{Q^2} - 1$

$\mathcal{S}_k = \sin\Phi_S\quad (k=1,2,3,4),\qquad \mathcal{S}_k = \cos\Phi_S\quad (k=8,9)$

$\Phi_S \equiv \phi_h - \phi_S \qquad \phi \equiv \phi_h$

-5 structure functions with different azimuthal dependence

$$= \sin\Phi_S\left(\mathcal{F}_1 + \mathcal{F}_2\cos\phi + \mathcal{F}_3\cos 2\phi\right) + \cos\Phi_S\left(\mathcal{F}_4\sin\phi + \mathcal{F}_5\sin 2\phi\right).$$

$$= \sin(\phi_h-\phi_S)\,F^{\sin(\phi_h-\phi_S)} + \sin(2\phi_h-\phi_S)\,F^{\sin(2\phi_h-\phi_S)} + \sin\phi_S\,F^{\sin\phi_S} + \sin(3\phi_h-\phi_S)\,F^{\sin(3\phi_h-\phi_S)} + \sin(\phi_h+\phi_S)\,F^{\sin(\phi_h+\phi_S)},$$

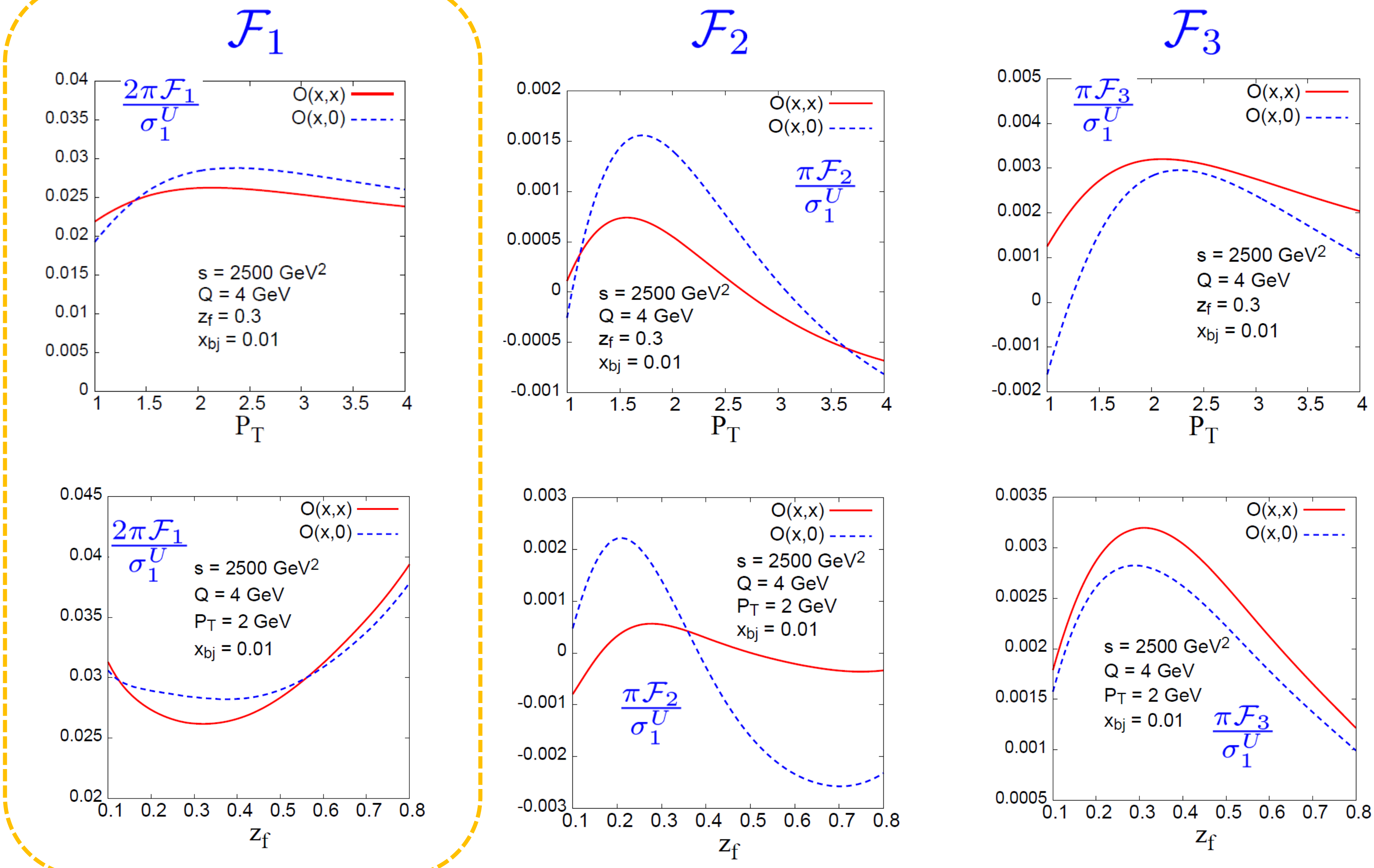
The same as TMD factorization approach.

⋆ Asymmetry at the EIC energy

Model 1: $O(x,x) = 0.004 \times xG(x)$

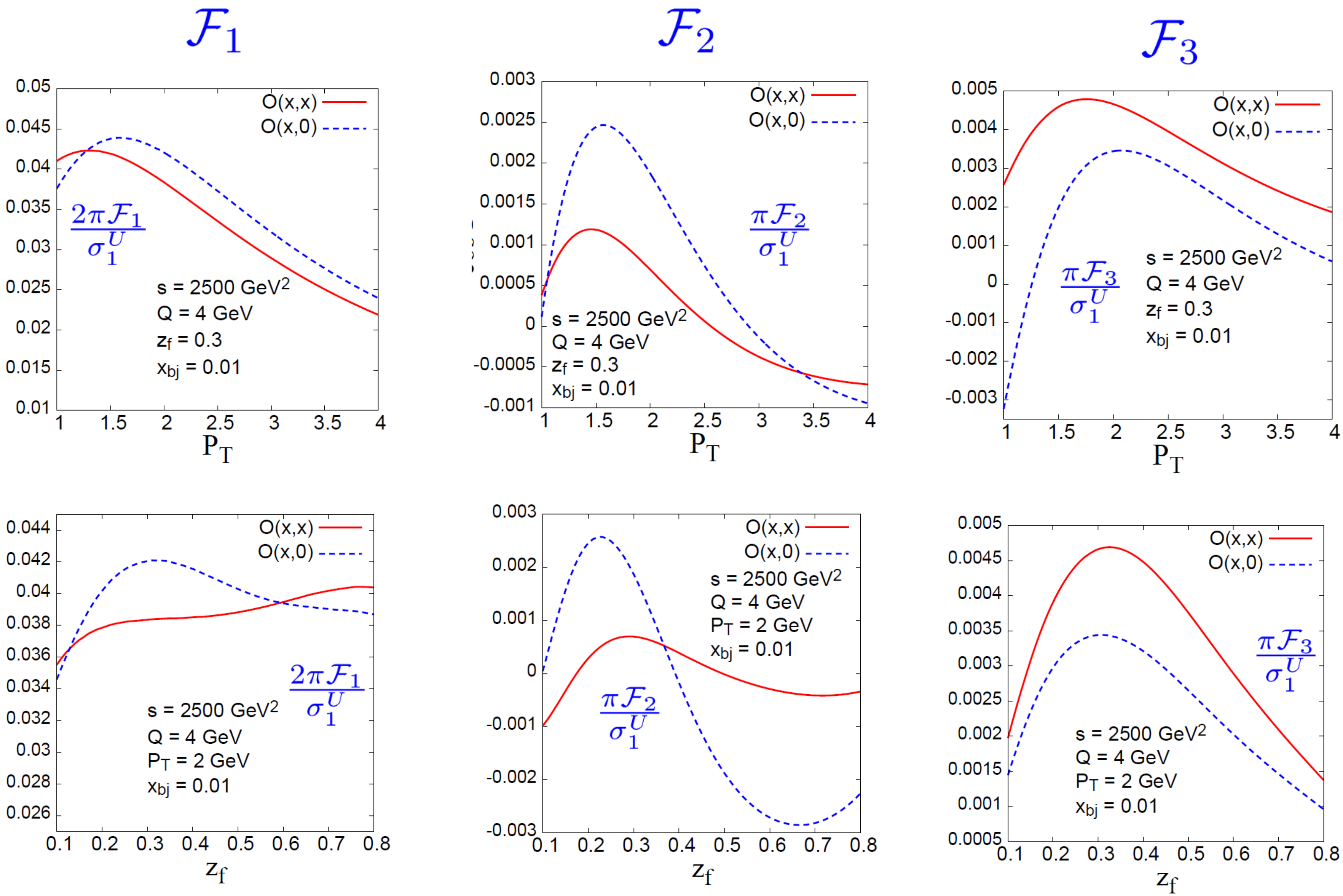
$(\text{CTEQ6L}, \mu^2 = Q^2 + P_T^2 + m_c^2)$

- SSA for $\mathcal{F}_1 \sim O(5-10)$ %.
- SSA for others $\leq O(1)$ %.
- Big difference between $O(x,x)$ and $O(x,0)$.



Model 2: $O(x,x) = 0.001\sqrt{x}G(x)$

· Large dependence on the form of $O(x,x)$ etc.



$\star$ Summary and outlook

· We have studied the SSA for $ep^{\uparrow} \to eDX$, $p^{\uparrow}p \to DX$, $p^{\uparrow}p \to \gamma X$ and $p^{\uparrow}p \to \ell^+\ell^- X$ induced by the 3-gluon correlation functions in the polarized nucleon.

· There are two independent twist-3 three-gluon correlation functions in the polarized nucleon due to the different color contractions; $O(x_1, x_2)$ and $N(x_1, x_2)$.

· SSA occurs as a pole contribution which is written in terms of four independent functions $O(x, x)$, $N(x, x)$, $O(x, 0)$ and $N(x, 0)$.

- Numerical calculation for $p^{\uparrow}p \to DX$ and $p^{\uparrow}p \to \gamma X$.

· Rising behavior of A_N for $p^{\uparrow}p \to DX$ at $x_F > 0$ was observed similarly to the SGP contribution from the quark-gluon correlation function for $p^{\uparrow}p \to \pi X$.

· For $p^{\uparrow}p \to \gamma X$, $A_N \simeq 0$ at $x_F > 0$ regardless of the magnitude of the 3-gluon correlation functions due to the smallness of hard cross section.

$\to$ Quark-gluon correlation functions are dominant at $x_F > 0$.

(Kanazawa-YK, in preparation.)

· A_N at $x_F < 0$ is sensitive to small-x behavior of 3-gluon correlation function for the two processes.

$\star$ Two processes are useful to get constraint on the magnitude and shape of 3-gluon correlation functions.

- Numerical calculation for $ep^{\uparrow} \to eDX$ at the EIC energy.

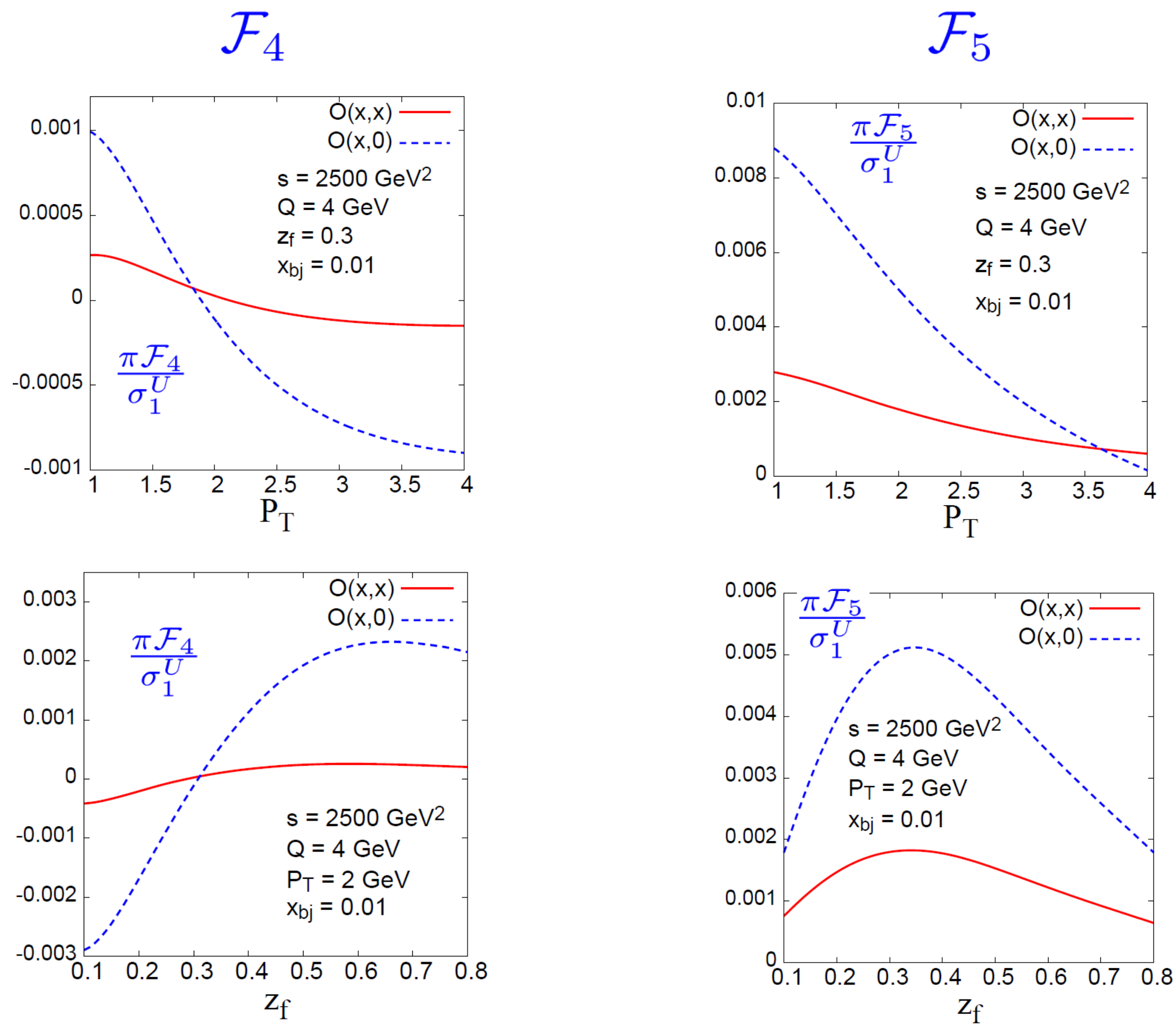
· $\sin(\phi_h - \phi_S)$-asymmetry could be as large as $O(5-10)$ %, while others may be $< O(1)$ %.

- Outlook

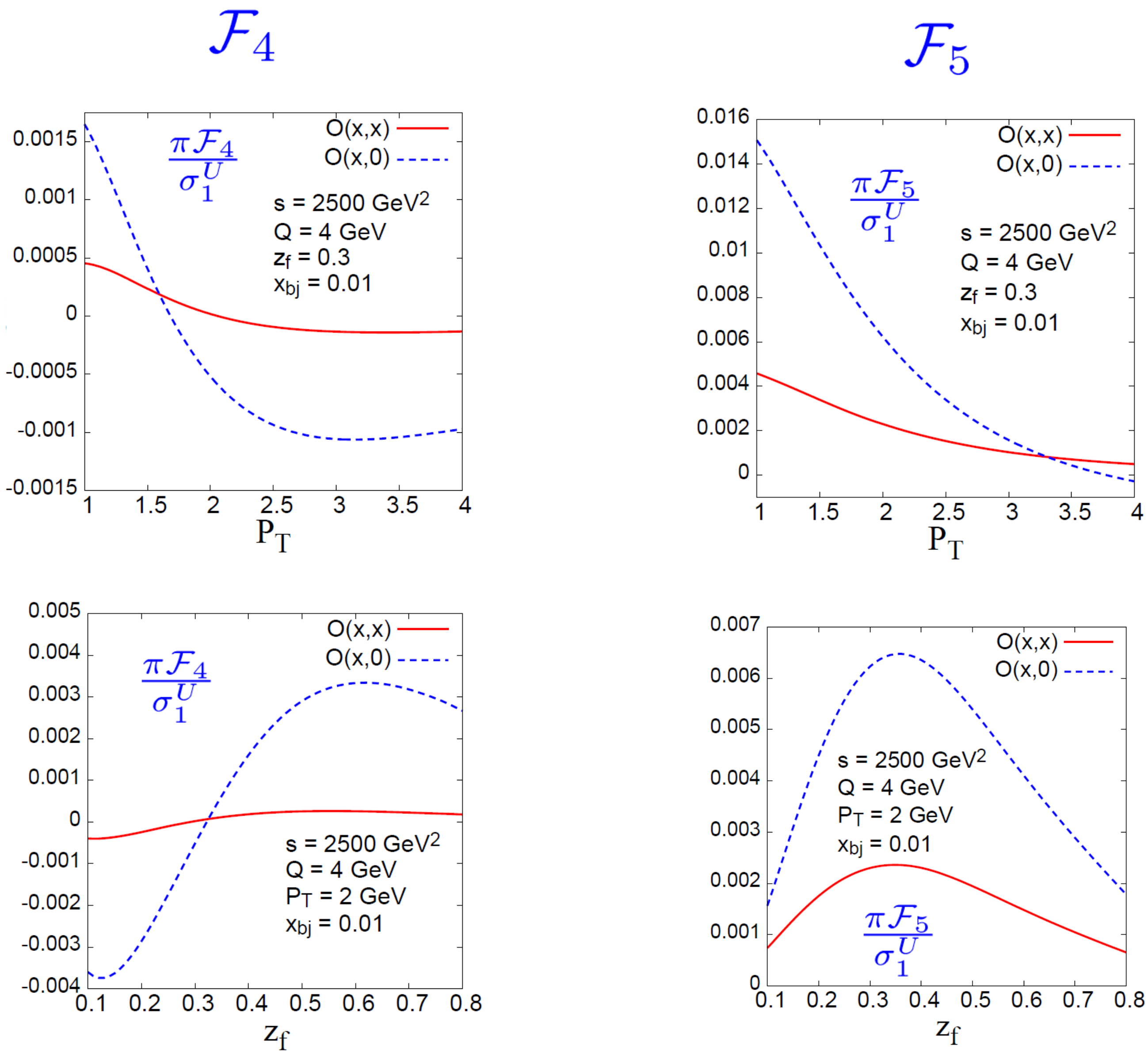
· Extention to $p^{\uparrow}p \to J/\psi X$.
· Combined analysis of $p^{\uparrow}p \to hX$ with quark-gluon, three-gluon correlation functions and twist-3 fragmentation functions.
· Retation between the three-gluon functions and the gluon-Sivers function.

Backup

Model 1: $O(x,x) = 0.004 \times xG(x)$



Model 2: $O(x,x) = 0.001\sqrt{x}G(x)$



“Master formula” for the three-gluon contribution

YK, K. Tanaka, S.Yoshida, arXiv:1104.0798 [hep-ph], PRD in press.
YK, S.Yoshida, arXiv:1104.3943 [hep-ph].

Twist-3 cross section can be obtained from the twist-2 $gg \to c\bar{c}$ scattering!

-Extention of the master formula for the SGP contribution from the quark-gluon correlation function (YK-Tanaka('07))

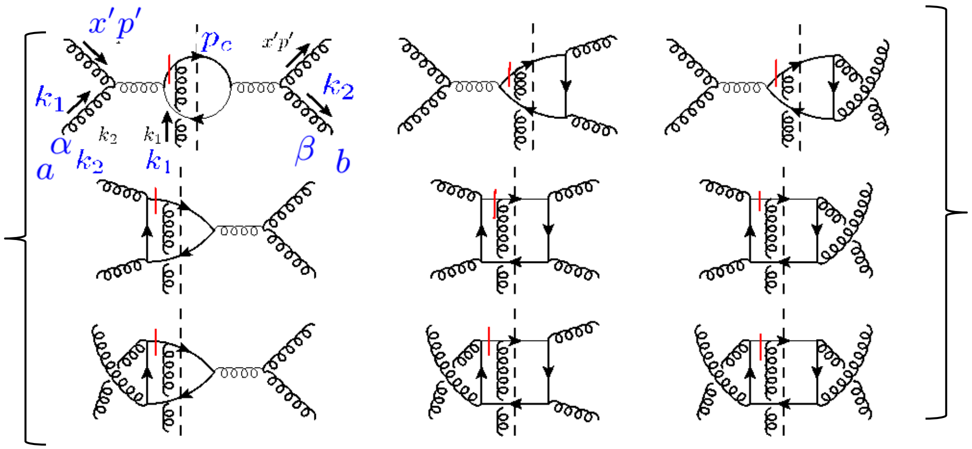
⋆ Master formula for the FSI 3-gluon correlation functions

$$\left.\frac{\partial S^{F,abc}_{\alpha\beta\lambda}(k_1,k_2,x'p',p_c)p^\lambda}{\partial k_2^\gamma}\right|^{\rm SGP}_{k_i=x_ip} = \frac{1}{x_1-x_2+i\epsilon}\frac{d}{dp_c^\gamma}H^{F,abc}_{\alpha\beta}(x_2p,x'p',p_c)$$

$(\alpha,\ \beta,\ \gamma=\perp)$

$$p_c^\mu = \left(\frac{m_c^2+\vec{p}_{c\perp}^{\,2}}{2p^-}, p^-, \vec{p}_{c\perp}\right)$$

$$\frac{\partial}{\partial k_2^\gamma}\times$$

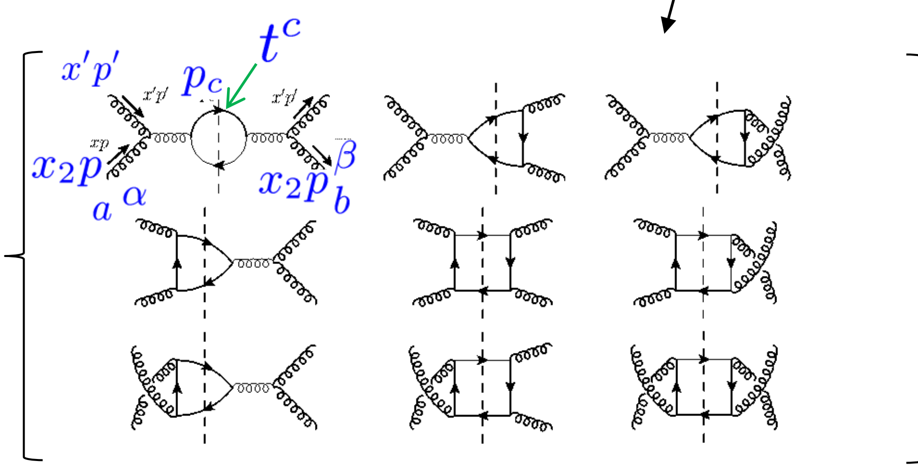


$\Big|^{\rm SGP}_{k_i=x_ip}$

(+ mirror)

2→2 ($gg\to c\bar{c}$) scattering cross section with an extra color insertion

$$= \frac{1}{x_1-x_2+i\epsilon}\frac{d}{dp_c^\gamma}\times$$



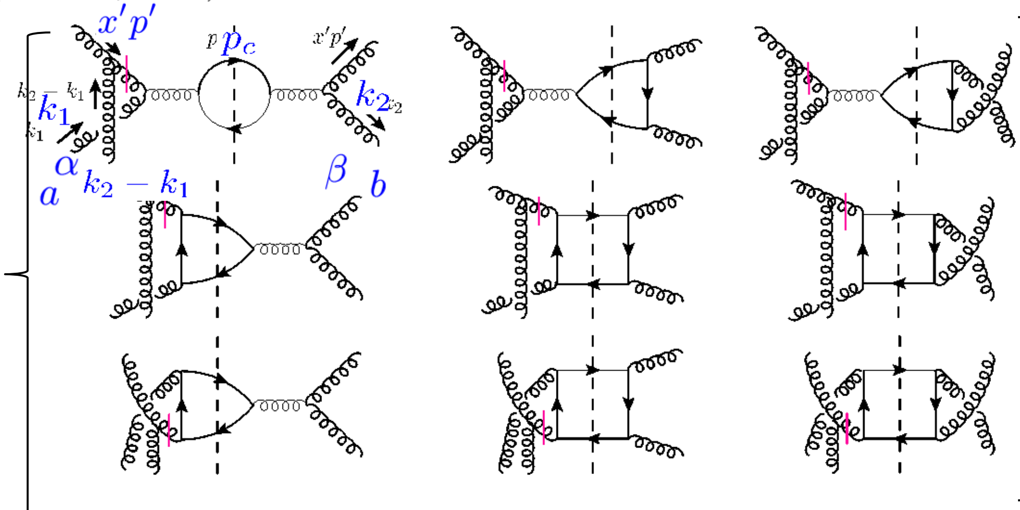
⋆ Master formula for the ISI 3-gluon correlation functions

$$\left.\frac{\partial S^{I,abc}_{\alpha\beta\lambda}(k_1,k_2,x'p',p_c)p^\lambda}{\partial k_2^\gamma}\right|^{\rm SGP}_{k_i=x_ip} = \frac{-1}{x_2-x_1+i\epsilon}\,\frac{d}{d(x'p'^\gamma)}H^{I,abc}_{\alpha\beta}(x_2p,x'p',p_c)$$

$(\alpha,\ \beta,\ \gamma=\perp)$

$$p'^\mu = \left(\frac{\vec{p'}_\perp^{\,2}}{2p'^-},p'^-,\vec{p'}_\perp\right)$$

$$\frac{\partial}{\partial k_2^\gamma}\times\Bigg\{$$

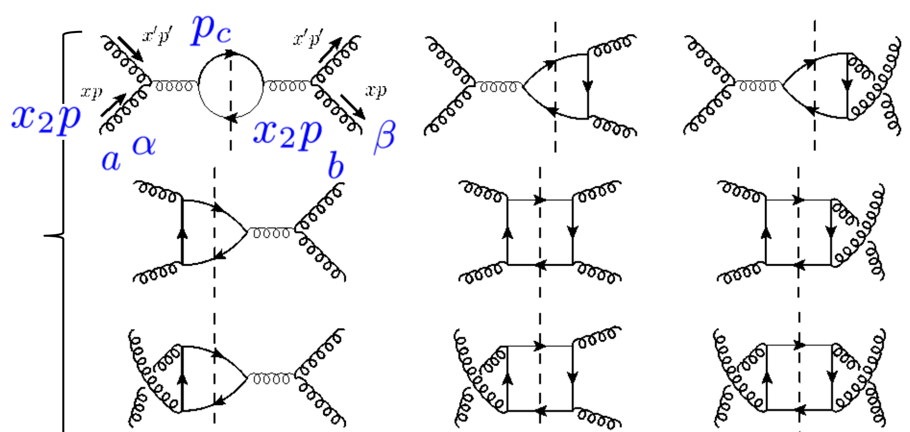


$$\Bigg\}\Bigg|^{\rm SGP}_{k_i=x_ip}$$

(+ mirror)

2→2 scattering cross section with an extra color insertion

$$=\frac{-1}{x_2-x_1+i\epsilon}\,\frac{d}{d(x'p'^\gamma)}\times\Bigg\{$$



$$\Bigg\}$$

⋆ Master formula (cont'd)

$$P_h^0 \frac{d\sigma^{3\text{gluon}}}{d^3P_h} = \frac{\alpha_s^2 M_N \pi}{S}\epsilon^{P_h p n S_\perp} \sum_{f=c\bar{c}} \int \frac{dx'}{x'} G(x') \int \frac{dz}{z^3} D_a(z) \int \frac{dx}{x} \delta\left(\tilde{s}+\tilde{t}+\tilde{u}\right)\frac{1}{\tilde{u}}$$

$$\times\left[\delta_f\left\{\left(\frac{d}{dx}O(x,x) - \frac{2O(x,x)}{x}\right)\hat{\sigma}^{O1} + \left(\frac{d}{dx}O(x,0) - \frac{2O(x,0)}{x}\right)\hat{\sigma}^{O2} + \frac{O(x,x)}{x}\hat{\sigma}^{O3} + \frac{O(x,0)}{x}\hat{\sigma}^{O4}\right\}\right.$$

$$\left. + \left\{\left(\frac{d}{dx}N(x,x) - \frac{2N(x,x)}{x}\right)\hat{\sigma}^{N1} + \left(\frac{d}{dx}N(x,0) - \frac{2N(x,0)}{x}\right)\hat{\sigma}^{N2} + \frac{N(x,x)}{x}\hat{\sigma}^{N3} + \frac{N(x,0)}{x}\hat{\sigma}^{N4}\right\}\right].$$

$$\tilde{s} = (xp + x'p')^2 \qquad \tilde{t} = (p_c - xp)^2 - m_c^2 \qquad \tilde{u} = (p_c - x'p')^2 - m_c^2,$$

- Unpol: $\hat{\sigma}^U = \sum_i C_i^U \xi_i$ (← color fac.: C_i^U) ⟶ $\hat{\sigma}^{O1} = \frac{-\tilde{s}}{\tilde{t}} \sum_i C_i^{FO}\xi_i + \sum_i C_i^{IO}\xi_i$, $\hat{\sigma}^{N1} = \frac{-\tilde{s}}{\tilde{t}} \sum_i C_i^{FN}\xi_i + \sum_i C_i^{IN}\xi_i$

  ξ_i: cross section from diagram "i".

- From the master formula and the scale invariance;

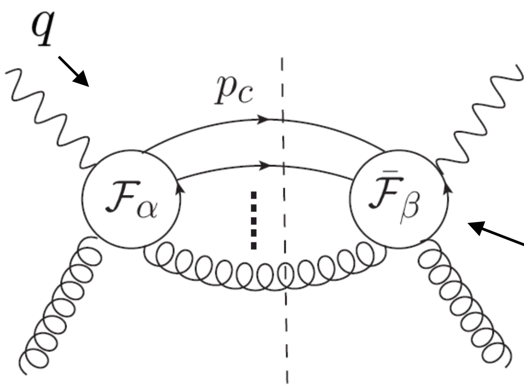
$$\left(\tilde{s}\frac{\partial}{\partial\tilde{s}} + \tilde{t}\frac{\partial}{\partial\tilde{t}} + \tilde{u}\frac{\partial}{\partial\tilde{u}} + m_c{}^2\frac{\partial}{\partial m_c^2}\right)\hat{\sigma}^{O1,N1} = 0 \quad \Rightarrow \quad \hat{\sigma}^{O3,N3} = -m_c^2\frac{\partial}{\partial m_c^2}\hat{\sigma}^{O1,N1}$$

(Also noticed by Kang *et al.* by the explicit calculation)

- $\hat{\sigma}^{O2,N2,O4,N4}$ also obtained from $gg \to c\bar{c}$ scattering.

⋆ Extention to higher order corrections

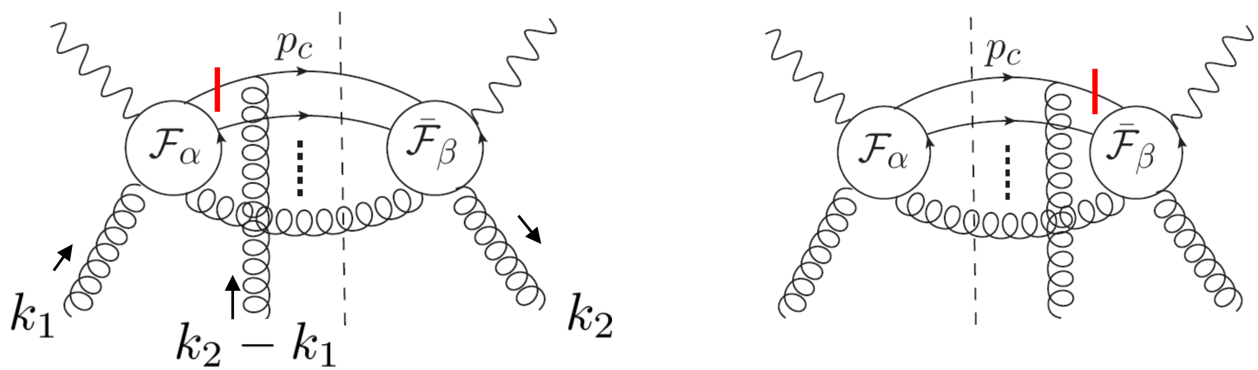
Ex. $ep^{\uparrow} \to eDX$ (YK, K. Tanaka, S.Yoshida, arXiv:1104.0798 [hep-ph])



Generic diagrams for twist-2 $\gamma^* g \to c\bar{c}$ scattering including higher order corrections

$\mathcal{F}_\alpha$ and $\bar{\mathcal{F}}_\beta$ contains loop corrections inside.

-Twist-3 SGP contribution is obtained by attaching the extra gluon onto the final parton line fragmenting into D.



$\to$ Master formula holds at higher order as well.

⋆ Comparison with the previous works

Kang-Qiu (PRD 78('08)034005), Kang-Qiu-Vogelsang-Yuan (PRD 78('08)114013)

These authors defined a particular projection

$$T_G^{(\pm)}(x,x) = \int \frac{dy_1^- dy_2^-}{2\pi} e^{ixp^+y_1^-} \frac{1}{xp^+} g_{\beta\alpha}\epsilon_{S\gamma np} \langle pS | C_\pm^{bca} F_b^{\beta +}(0) F_c^{\gamma +}(y_2^-) F_a^{\alpha +}(y_1^-) | pS \rangle .$$

which are related to our functions as

$$\frac{xg}{2\pi} T_G^{(+)}(x,x) = -4M_N \left(N(x,x) - N(x,0) \right),$$
$$\frac{xg}{2\pi} T_G^{(-)}(x,x) = -4M_N \left(O(x,x) + O(x,0) \right).$$

- Formula by Kang-Qiu:

$$\sigma_{3\text{gluon}}^{KQ} \sim \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \left. \frac{\partial S_{\mu\nu\lambda}^{abc}(k_1, k_2, x'p', p_c) p^\lambda g_\perp^{\mu\nu}}{\partial k_{2\perp}^\gamma} \right|_{k_i = x_i p} g_{\alpha\beta}^\perp \mathcal{M}_{F,abc}^{\alpha\beta\gamma}(x_1, x_2),$$

$T_G^{(\pm)}(x,x)$

ad-hoc projections

- Our formula:

$$\sigma_{3\text{gluon}} \sim \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \left. \frac{\partial S_{\mu\nu\lambda}^{abc}(k_1, k_2, x'p', p_c) p^\lambda}{\partial k_2^\sigma} \right|_{k_i = x_i p} \omega^\mu{}_\alpha\, \omega^\nu{}_\beta\, \omega^\sigma{}_\gamma\, \mathcal{M}_{F,abc}^{\alpha\beta\gamma}(x_1, x_2),$$

$$\omega^\mu{}_\alpha = g^\mu{}_\alpha - p^\mu n_\alpha$$

$\star$ Twist-3 description of A_N for $p^{\uparrow}p \to hX$ at RHIC

Soft-gluon-pole (SGP) functions

$$d\sigma = (G_F(x,x) - x\frac{dG_F(x,x)}{dx}) \otimes f(x') \otimes D(z) \otimes \hat{\sigma}_{SGP}$$

Qiu-Sterman('98), Kouvaris-Qiu-Vogelsang-Yuan ('06), YK-Tanaka('07)

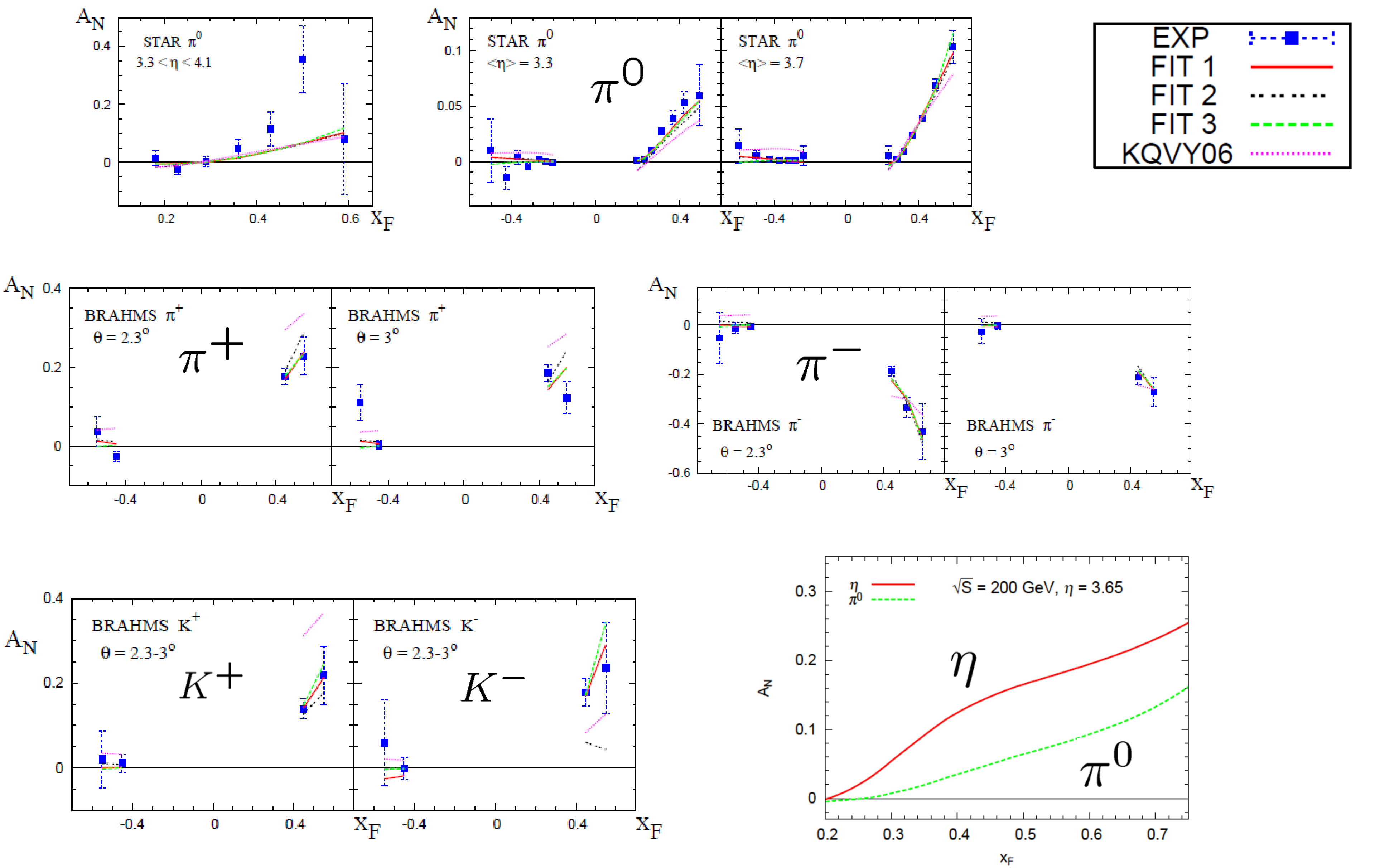
$$+(G_F(x,0) + \widetilde{G}_F(x,0)) \otimes f(x') \otimes D(z) \otimes \hat{\sigma}_{SFP}$$

YK-Tomita('09), YK-Kanazawa ('10,'11)

Soft-fermion-pole (SFP) functions

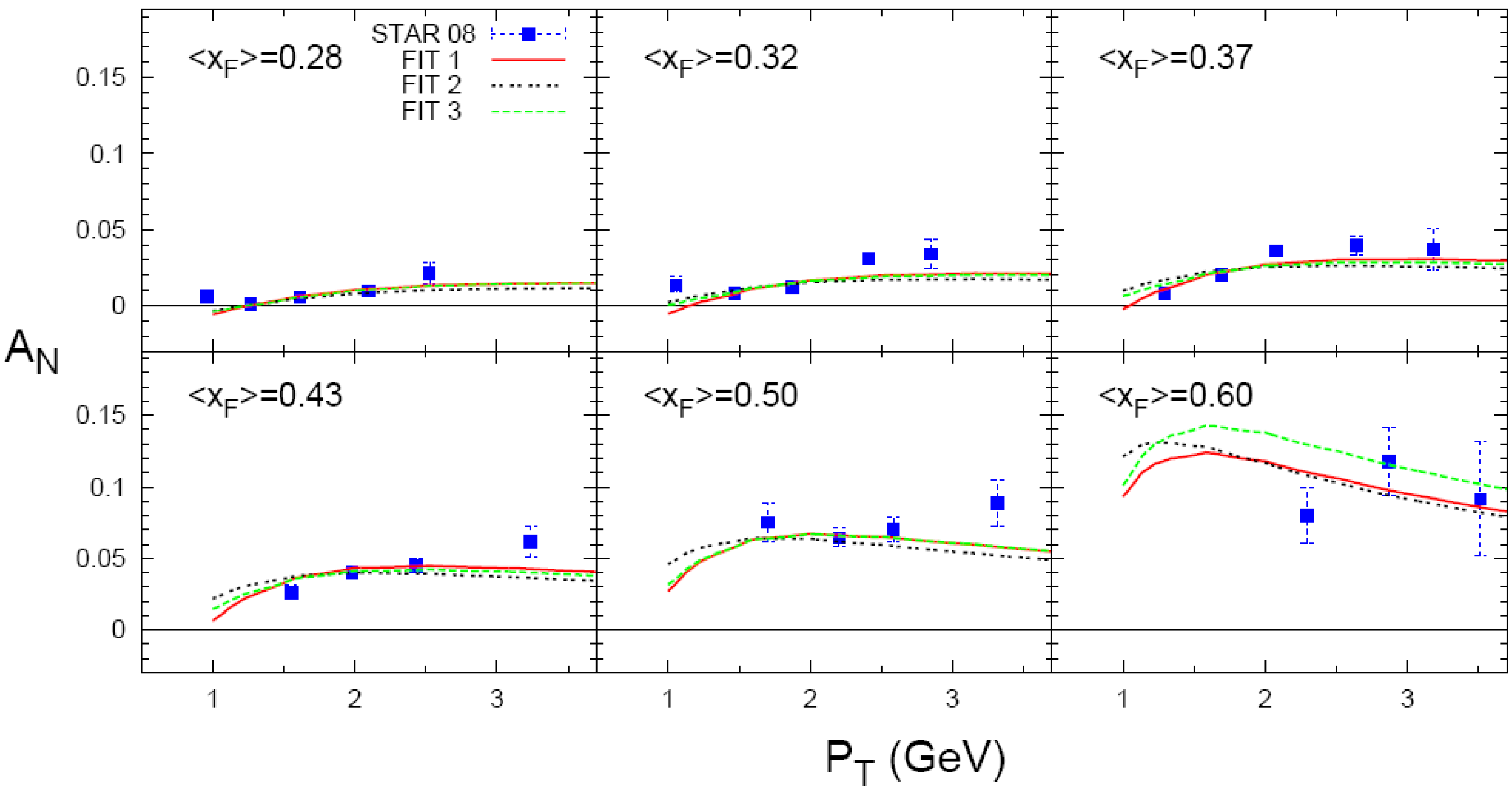
⋆ Analysis of RHIC A_N data for $p^\uparrow p \to hX$ $(h = \pi, K, \eta)$

From YK-Kanazawa, PRD82('10) and arXiv:1104.0117 [hep-ph]



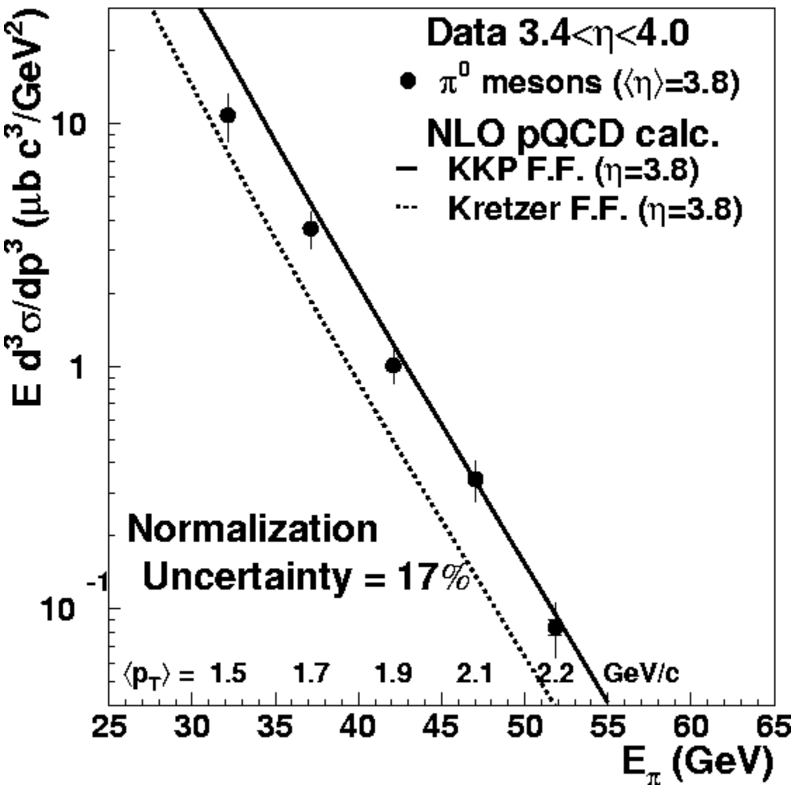
⋆ P_T-dependence of $A_N^{\pi^0}$ in comparison to STAR data.

YK-Kanazawa, PRD82('10) and arXiv:1104.0117 [hep-ph]

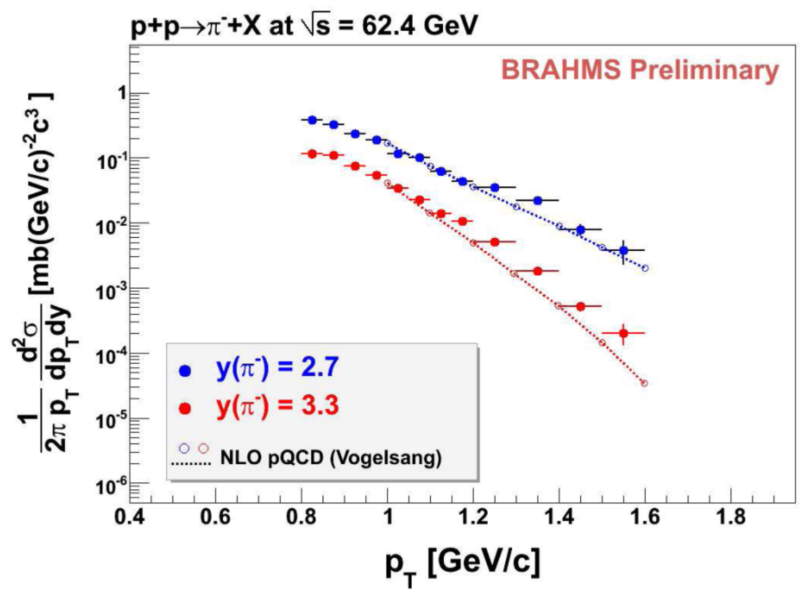


• Analysis of unpolarized cross section in terms of the collinear factorization (NLO).

★ RHIC-STAR ('04)

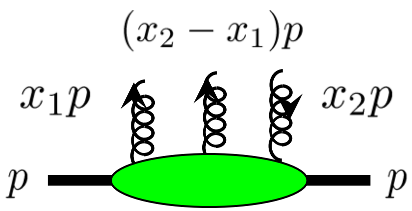


★ RHIC-BRAHMS ('07)



NLO works perfectly well at RHIC energy!

⋆ Twist-3 “three-gluon” correlation functions



cf. Beppu-Koike-Tanaka-Yoshida (PRD 82('10)054005)

See also, Belitsky-Ji-Lu-Osborne, PRD63,094012(2001)
Braun-Manashov-Pirnay, PRD80,114002(2009).

- Hermiticity, PT-invariance, Permutation symmetry

$$O^{\alpha\beta\gamma}(x_1,x_2) = -gi^3 \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle pS| d^{bca} F_b^{\beta n}(0) F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n)|pS\rangle$$

$$= 2iM_N \left[O(x_1,x_2) g^{\alpha\beta} \epsilon^{\gamma pnS} + O(x_2, x_2 - x_1) g^{\beta\gamma} \epsilon^{\alpha pnS} + O(x_1, x_1 - x_2) g^{\gamma\alpha} \epsilon^{\beta pnS} \right]$$

$$N^{\alpha\beta\gamma}(x_1,x_2) = -gi^3 \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle pS| i f^{bca} F_b^{\beta n}(0) F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n)|pS\rangle$$

$$= 2iM_N \left[N(x_1,x_2) g^{\alpha\beta} \epsilon^{\gamma pnS} - N(x_2, x_2 - x_1) g^{\beta\gamma} \epsilon^{\alpha pnS} - N(x_1, x_1 - x_2) g^{\gamma\alpha} \epsilon^{\beta pnS} \right].$$

$F_a^{\alpha n} \equiv F_a^{\alpha\mu} n_\mu$ $\quad$ n: lightlike vector satisfying $p \cdot n = 1$.

$\epsilon^{\gamma pnS} \equiv \epsilon^{\gamma\mu\nu\lambda} p_\mu n_\nu S_\lambda$ etc. $\quad$ Gauge-links suppressed above.

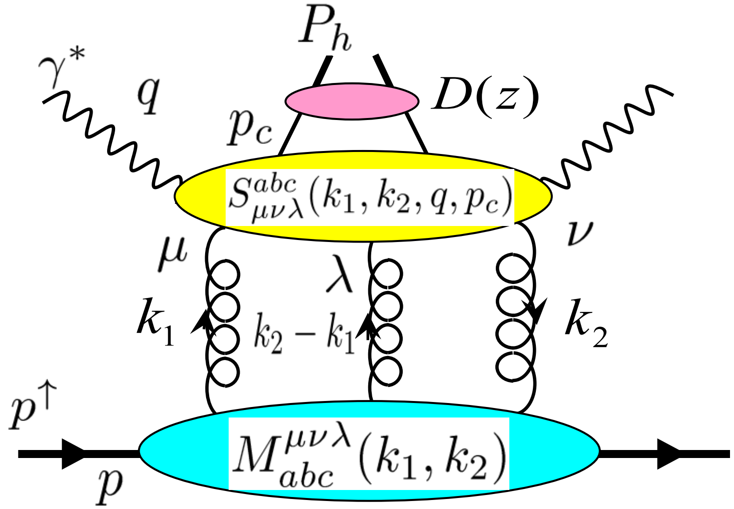
- Only two independent scalar functions due to the different color structures:

$$O(x_1,x_2) = O(x_2,x_1), \qquad O(x_1,x_2) = O(-x_1,-x_2),$$

$$N(x_1,x_2) = N(x_2,x_1), \qquad N(x_1,x_2) = -N(-x_1,-x_2).$$

⋆ Formalism

Ex: $ep^{\uparrow} \to eDX$



Hadronic tensor after factorizing $D(z)$

-Lorentz indices for γ^* suppressed

$$w\,(p, q, p_c) = \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4}\, S^{abc}_{\mu\nu\lambda}\,(k_1, k_2, q, p_c)\, M^{\mu\nu\lambda}_{abc}(k_1, k_2),$$

Partonic hard part　　Nucleon matrix element

$$M^{\mu\nu\lambda}_{abc}(k_1, k_2) = g \int d^4\xi \int d^4\eta\, e^{ik_1\xi} e^{i(k_2-k_1)\eta} \langle pS| A^{\nu}_b(0) A^{\lambda}_c(\eta) A^{\mu}_a(\xi) |pS\rangle.$$

- spin-dependent part of $C^{abc}_{\pm} M^{\mu\nu\lambda}_{abc}(k_1, k_2)$ ($C^{abc}_{+} = if^{abc}$, $C^{abc}_{-} = d^{abc}$) is pure imaginary.

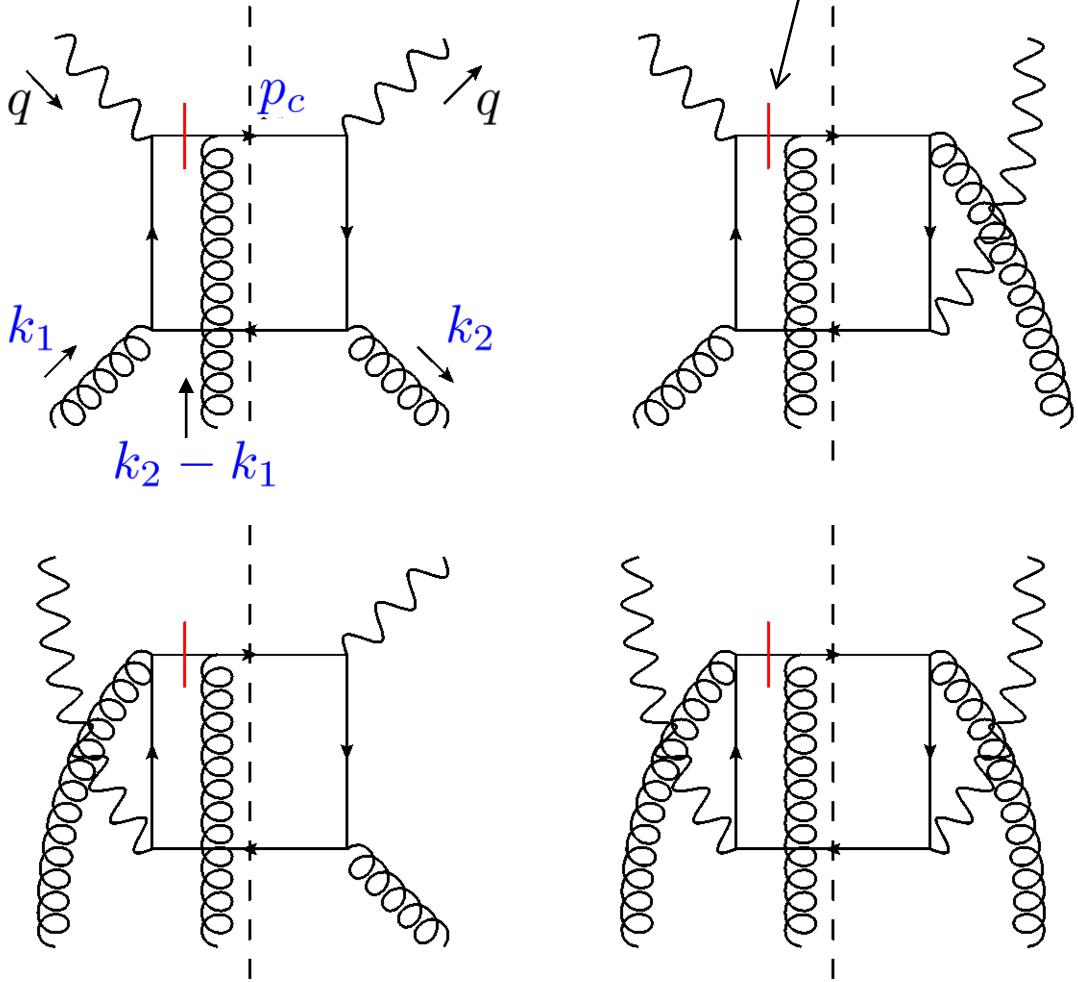
$\to$ Only imaginary (pole) contribution from $C^{abc}_{\pm} S^{abc}_{\mu\nu\lambda}$ can give rise to $w(p, q, p_c)$.

$\to$ Soft-gluon-pole (SGP)

⋆ Leading order diagrams for the SGPs

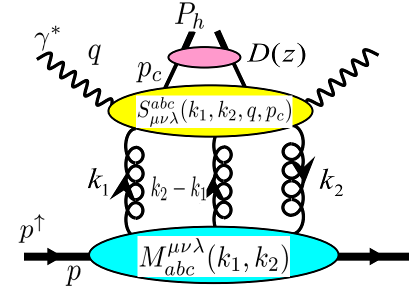
SGP: $\delta\left((p_c - k_2 + k_1)^2\right) \rightarrow \propto \delta(x_1 - x_2)$

in the collinear limit $(k_i \rightarrow x_i p)$



+ mirrors

Other pole contributions cancel among them!



⋆ Formalism

- Apply collinear expansion to $S^{abc}_{\mu\nu\lambda}(k_1,k_2,q,p_c)$ in Feynman gauge.

  · Write $k_i^\mu = x_i p + \omega^\mu{}_\nu k_i^\nu$ with $x_i = k_i \cdot n$ and $\omega^\mu{}_\nu = g^\mu{}_\nu - p^\mu n_\nu$.

  · Also $A^\mu = p^\mu n \cdot A + \omega^\mu{}_\kappa A^\kappa$ in $M^{\mu\nu\lambda}_{abc}$. Note $A^\perp$ is suppressed compared to A^+ by a hard scale.

- One has to keep up to third-order derivatives of $S^{abc}_{+++}(k_1,k_2,q,p_c)$ with respect to $k_{1,2}$, while only the leading term is necessary for $S^{abc}_{\alpha\beta\gamma}(k_1,k_2,q,p_c)$ (for $\alpha,\beta,\gamma=\perp$).

  · Each term is not gauge invariant.

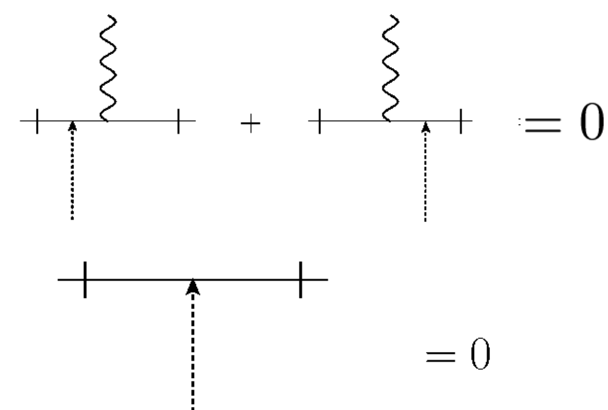
  · Can the result be expressed in terms of only gauge invariant $\langle F_a^{\alpha+} F_b^{\beta+} F_c^{\gamma+}\rangle$?

- Ward identities satisfied by the pole part of $S^{abc}_{\mu\nu\lambda}(k_1,k_2,q,p_c)$ are crucial:

$$k_1^\mu S^{abc}_{\mu\nu\lambda}(k_1,k_2,q,p_c) = 0,$$

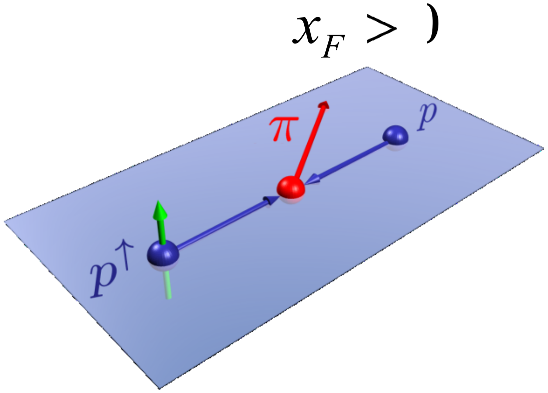
$$k_2^\nu S^{abc}_{\mu\nu\lambda}(k_1,k_2,q,p_c) = 0,$$

$$(k_2-k_1)^\lambda S^{abc}_{\mu\nu\lambda}(k_1,k_2,q,p_c) = 0.$$



⋆ Single spin asymmetry (SSA)

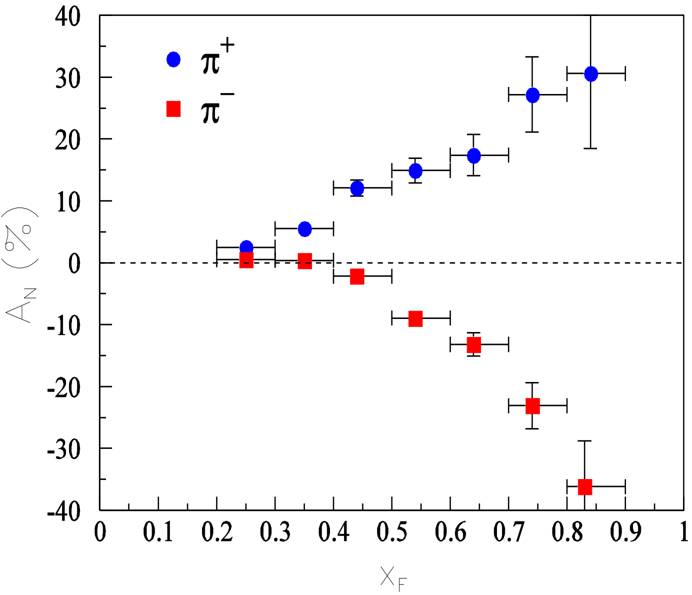
- $p^{\uparrow}p \to \pi X$ $A_N = \dfrac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$



- FNAL-E704('91)($\sqrt{s} = 20$ GeV), RHIC($\sqrt{s} = 200$, 62 GeV): $A_N \sim 0.3$ at large $x_F = 2p_{\parallel}/\sqrt{s}$.

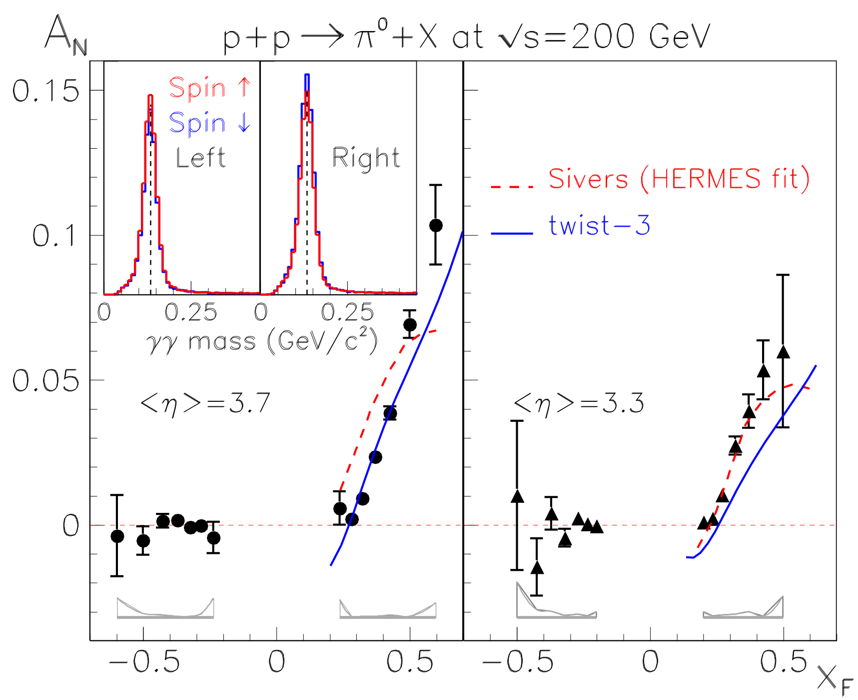
FNAL-E704

$\sqrt{s} = 20$ GeV

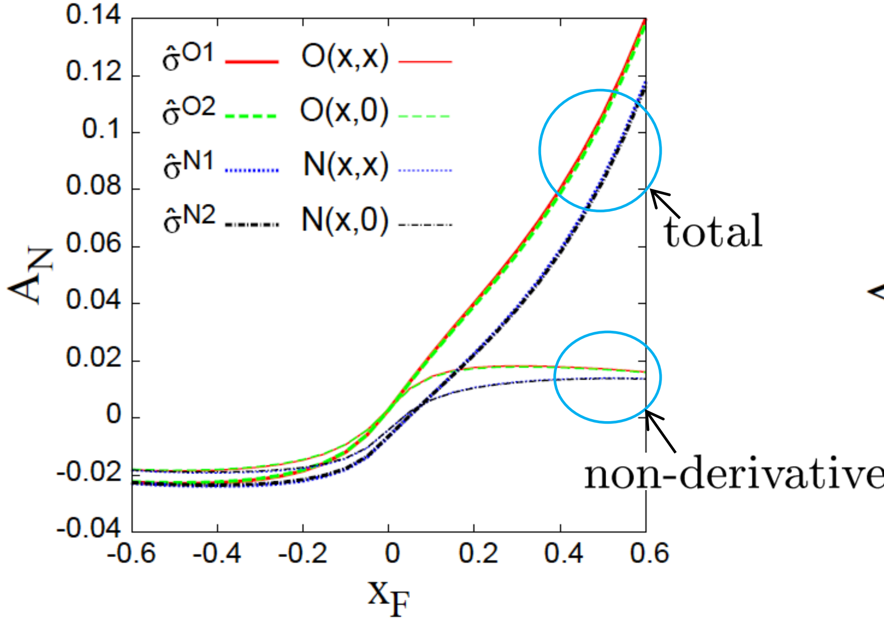


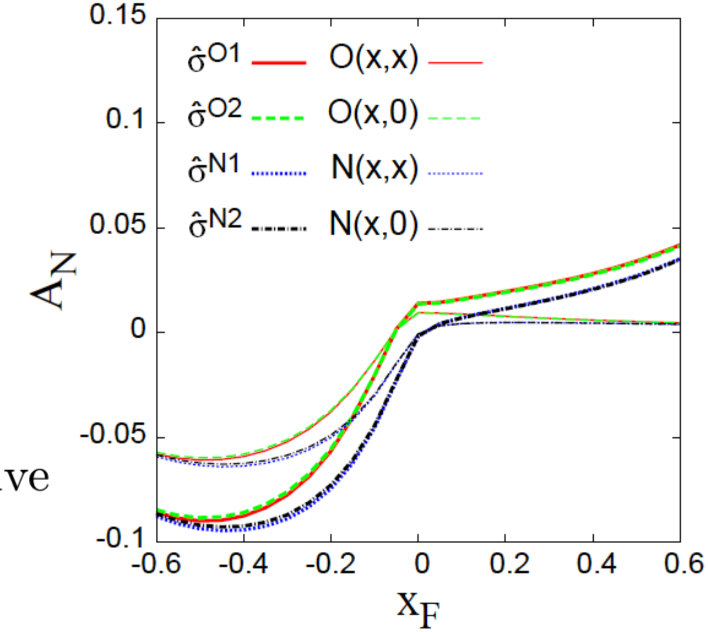
P.L. B264 ('91) 462
P.L. B261 ('91) 201

RHIC-STAR, PRL92('04),
PRL 101(2008)
$\sqrt{s} = 200$ GeV



Model 1 @ $\sqrt{S} = 200$ GeV, $P_T = 2$ GeV Model 2

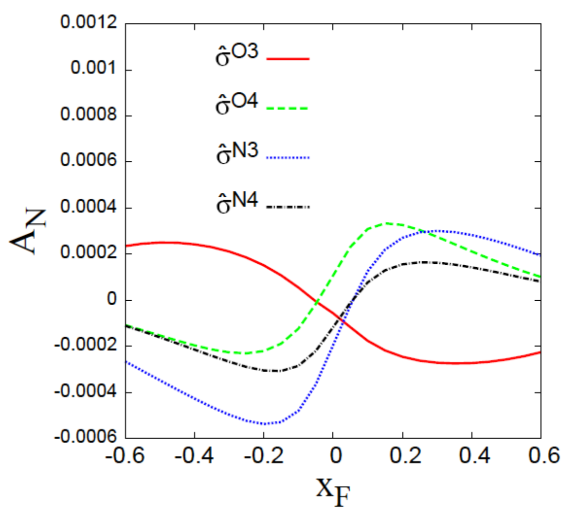


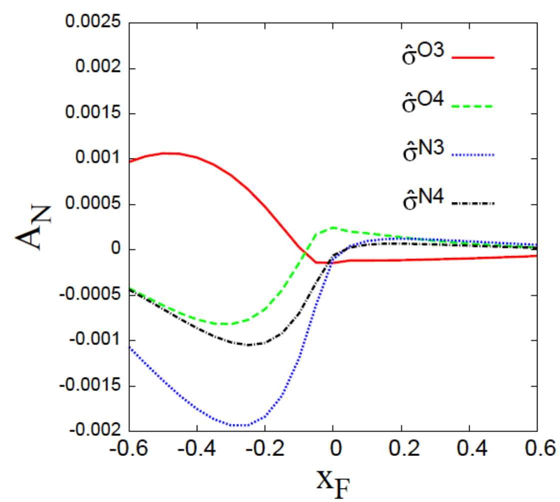


· Derivative term dominates at large $x_F > 0$.

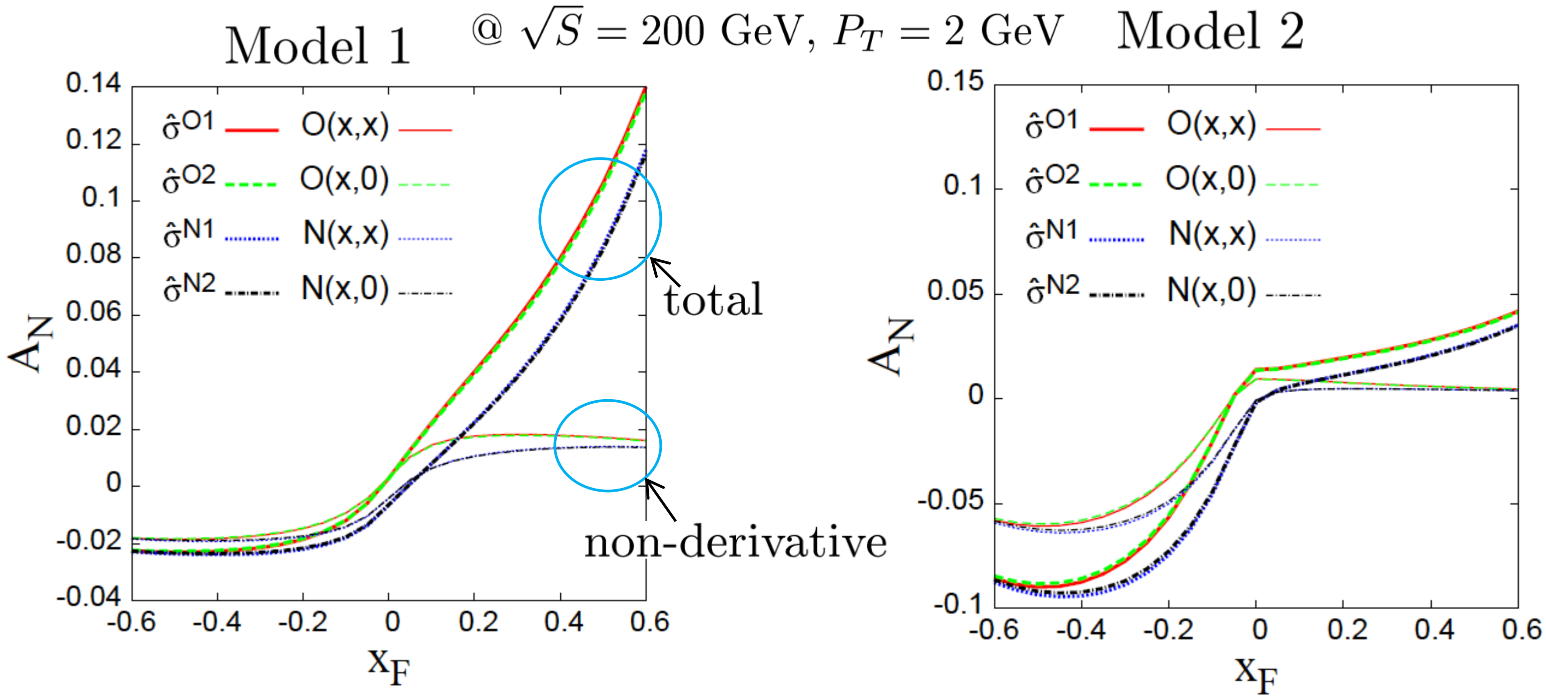
· Non-derivative terms become important at $x_F < 0$.

· Terms with $\hat{\sigma}^{O3,O4,N3,N4}$ are negligible.





@ $\sqrt{S} = 200$ GeV, $P_T = 2$ GeV



· Derivative term dominates at large $x_F > 0$.

· Non-derivative terms become important at $x_F < 0$.

· Terms with $\hat{\sigma}^{O3,O4,N3,N4}$ are negligible.

· $O(x,x)+O(x,0)$ and $N(x,x)-N(x,0)$ can be regarded as "effective" three-gluon correlation functions for $p^\uparrow p \to DX$. (But for $ep^\uparrow \to eDX$.)