Gauge Links & Process dependence in Hadronic Reactions

30 August 2011

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Phys.Lett. B696 2011 w/ Zhongbo Kang ***BNL***

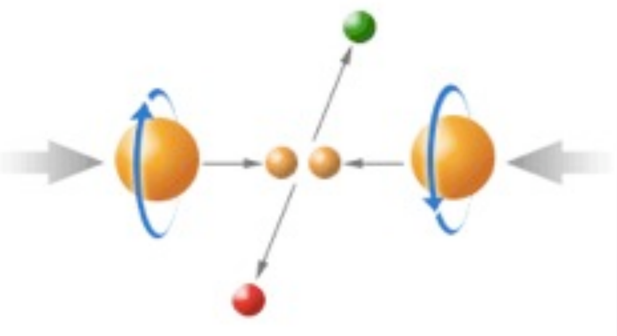
Outline

- **Reivew transverse spin Effects - TSSAs**
- **Gauge links-Color Gauge Inv.-“T-odd” TMDs**
- **T-odd PDFs via ISI/FSIs ... Phases & gauge link**

  “QCD calc “ FSIs Gauge Links-Color Gauge Inv. “T-odd” TMDs

- Generalizing the Generalized Parton Model (GPM) color gauge invariance   CGI-GPM
- Some pheno results
- Connection w/ twist three approach

Comments I



- Single inclusive hadron production in hadronic collisions largest/oldest observed TSSAs
- From theory view most challenging understand-best theory description-twist 3 power suppressed (compared w/ SIDIS, DY, e^+e^-)
- Can we use twist 2: There are connections w/ twist 2 approach
  - Operator level ETQS fnct 1[st] moment of Sivers

$$gT_F(x,x) = -\int d^2p_T \frac{|p_T^2|}{M} f_{1T}^{\perp}(x,p_T^2) \quad + \quad \text{“UV”} \ldots$$

$$= -2M f_{1T}^{\perp(1)}$$

$$gT_F(x,x;|\boldsymbol{b}_T|) = -2M \int d^2p_T \frac{|p_T|}{|\boldsymbol{b}_T|M} J_1(|\boldsymbol{b}_T||p_T|) f_{1T}^{\perp}(x,p_T^2)$$

Boer, LG, Musch, Prokudin arXiv:1107.5294 see also Abyat's talk

  - Connection btwn twist 3 approach and twist 2 in overlap regime
    Ji, Qiu, Vogelsang, Yuan PRL 2006 ...
  - Same mechanism in both approaches ISI/FSI
  - Explore role parton model processes in tw-2&3 approaches
    LG & Z. Kang PLB 2011 and for Collins in prep

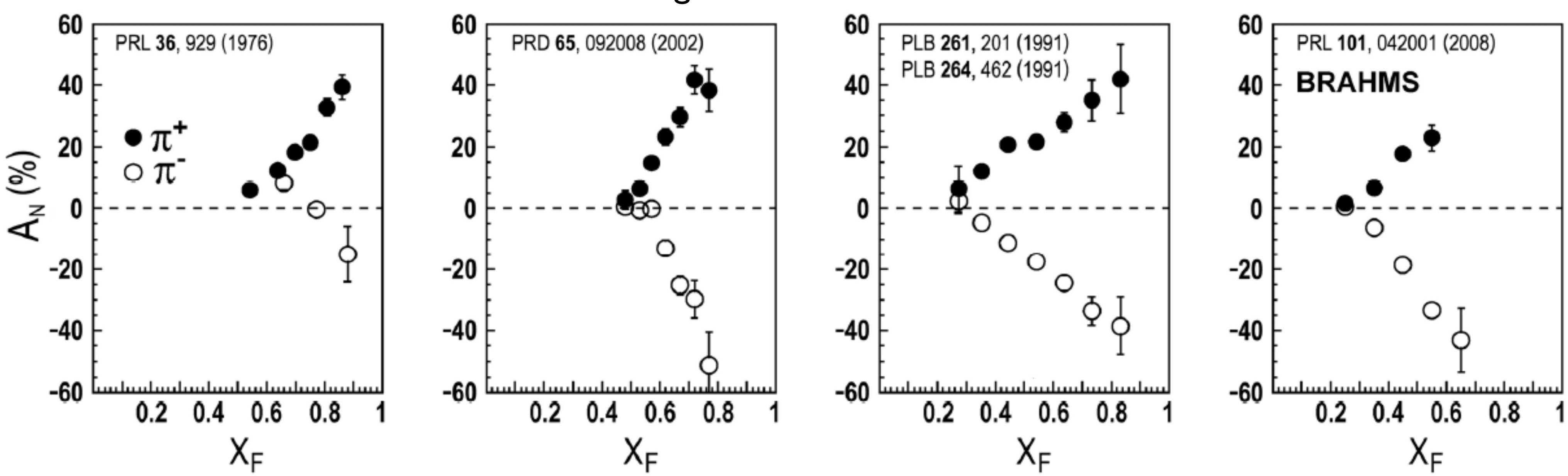
Large Transverse Polarization in Inclusive Reactions

Transverse Single-Spin Asymmetries: From Low to High Energies!

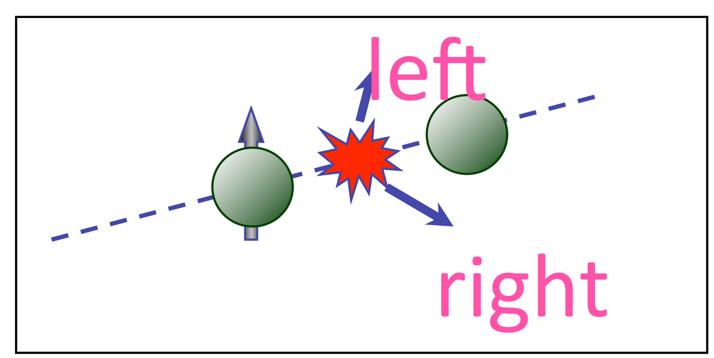
BRAHMS

p_{beam}=12 Gev/c	p_{beam}=22 Gev/c	p_{beam}=200 Gev/c	
ANL	BNL	FNAL	RHIC
$\sqrt{s}$=4.9 GeV	$\sqrt{s}$=6.6 GeV	$\sqrt{s}$=19.4 GeV	$\sqrt{s}$=62.4 GeV

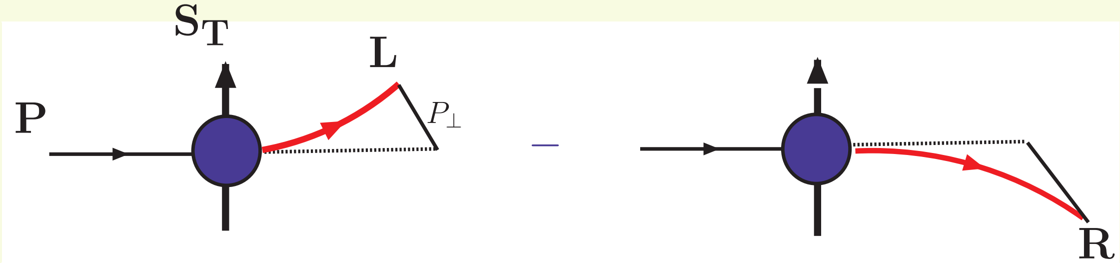
← Fixed target → Collider

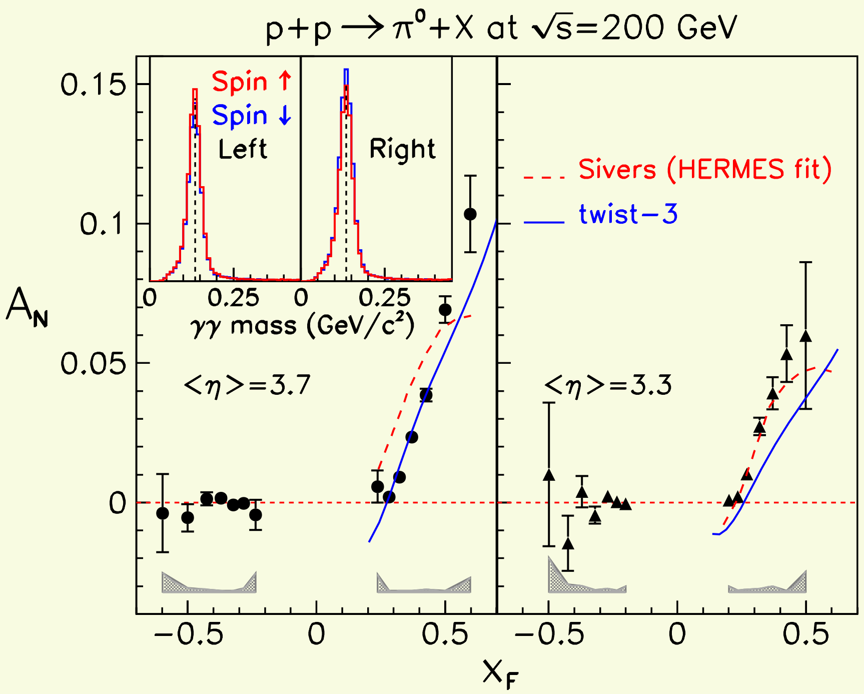


$$x_F = 2p_{long}/\sqrt{s}$$



Modern Era Transverse SSA's at $\sqrt{s}$ = 62.4 & 200 GeV at RHIC





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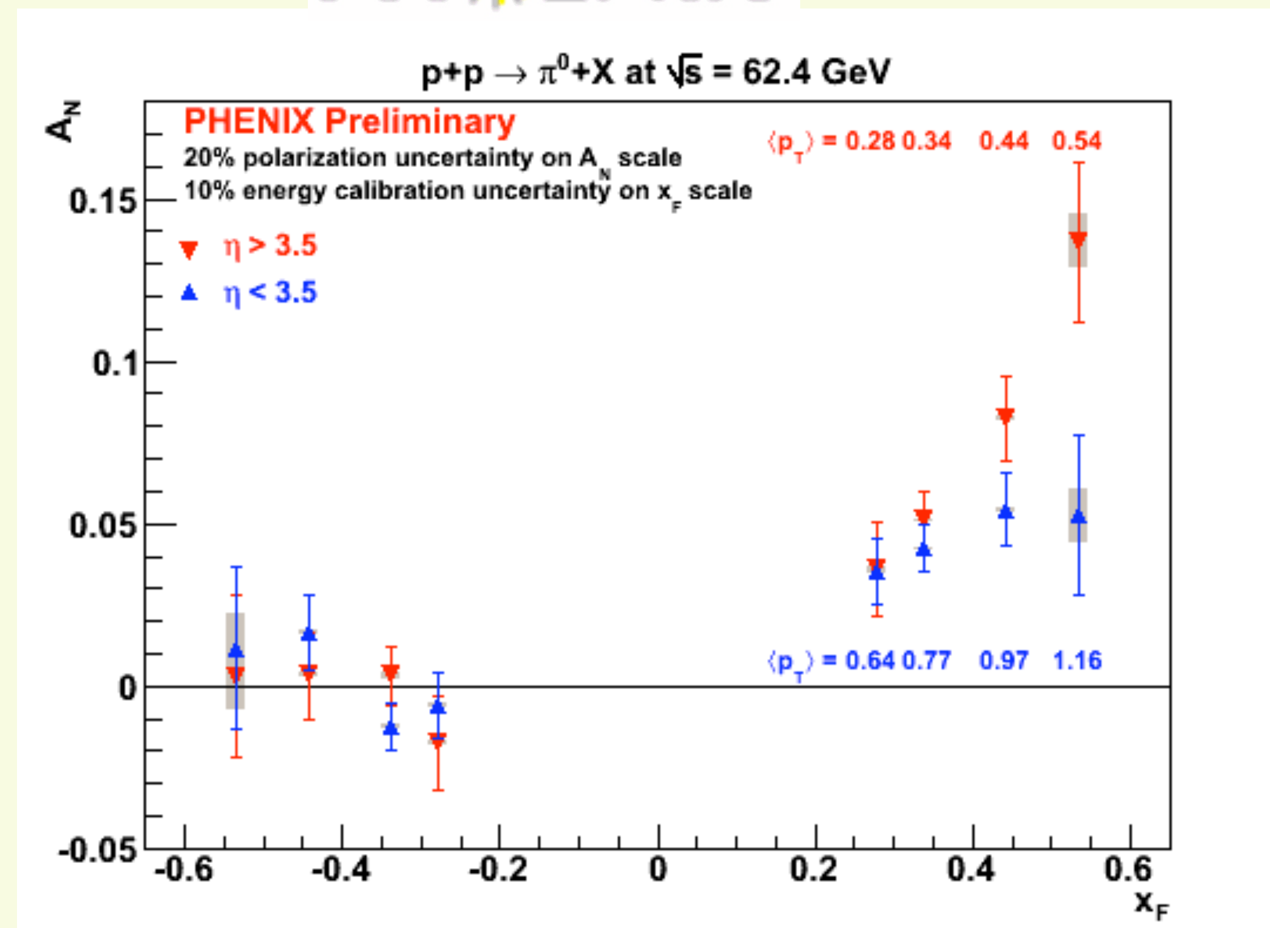
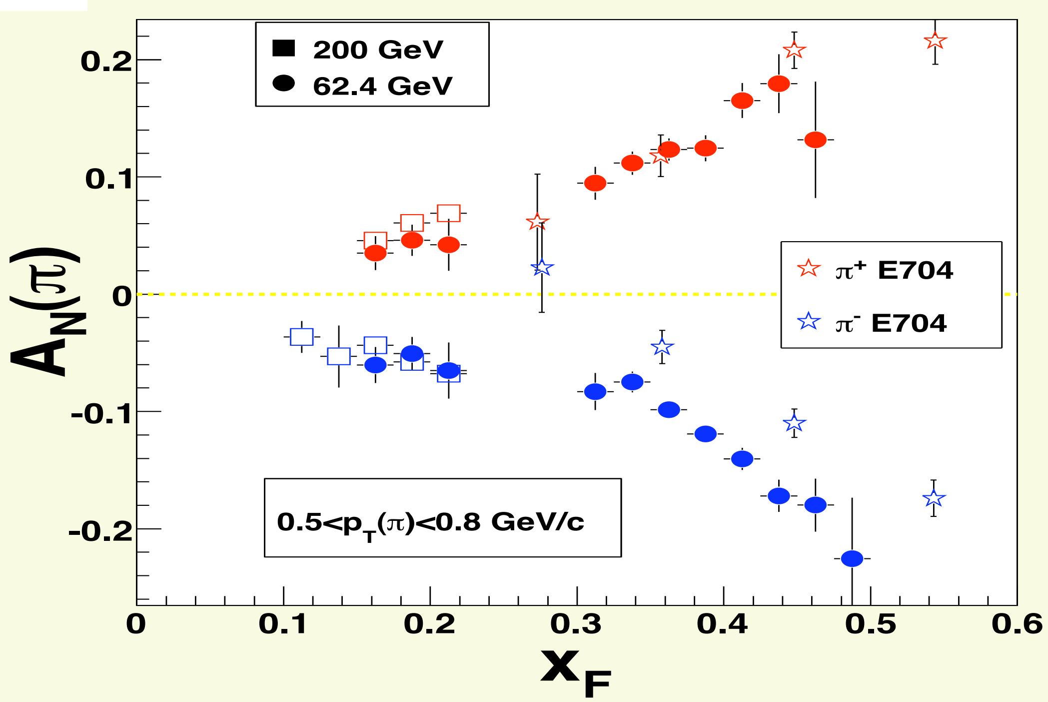


BRAHMS

PRL101, 042001 (2008)



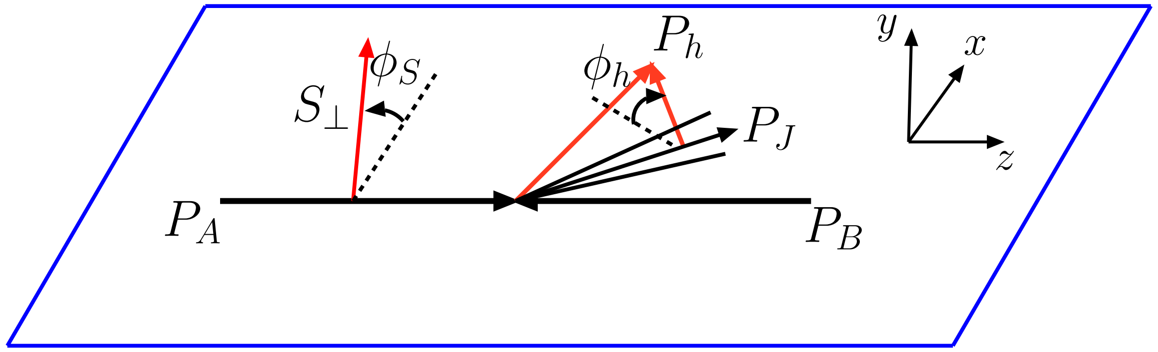
{\em Model Assumptions}

- "WTIM" consider hadronic processes taking into account ISI/FSI in gen. parton model GPM
- Consider impact in three cases
  - Inclusive pion production at forward rapidity- Both Collins and Sivers can contribute
  - Direct photon - Sivers only, can be used to test sign change as in DY
  - and ...

cont ...

Azimuthal asymmetric distribution of hadrons inside a high energy jet in transverse polarized nucleon-nucleon scattering

$$p^{\uparrow}\, p \longrightarrow h_1\, \text{jet}\, X$$



- Collins effect Yuan PRL 2008
- Pion about jet-Can disentangle Collins & Sivers
-

w/o ISI/FSI- D'Alesio, Murgia, Pisano PRD10,
w/ ISI/FSI- D'Alesio, LG, Kang, Murgia, Pisano w/ ISI/FSI-arXiv:1108.0827 [hep-ph]

  - Inclusive jet - Only Sivers, can be used to test sign change as in DY
  - Talk of Cristian Pisano Wednesday

Caution !!! Comments

Similar studies performed for weighted k_T and unweighted

- photon Jet $p^{\uparrow}\, p \longrightarrow \gamma\, \text{jet}\, X$ Bacchetta Bomhof, D'Alesio, Mulders, Murgia PRL 09
- 2-particle inclusive hadron production $p^{\uparrow}\, p \longrightarrow h_1\, h_2\, X$

Bacchetta Bomhof, Mulders, Piljman PRD05, Qiu Vogelsang Yuan PRD2007, Vogelsang Yuan PRD 2007

Merits "Pre-Collins Qiu Mulders Rogers" "PCQMR period"

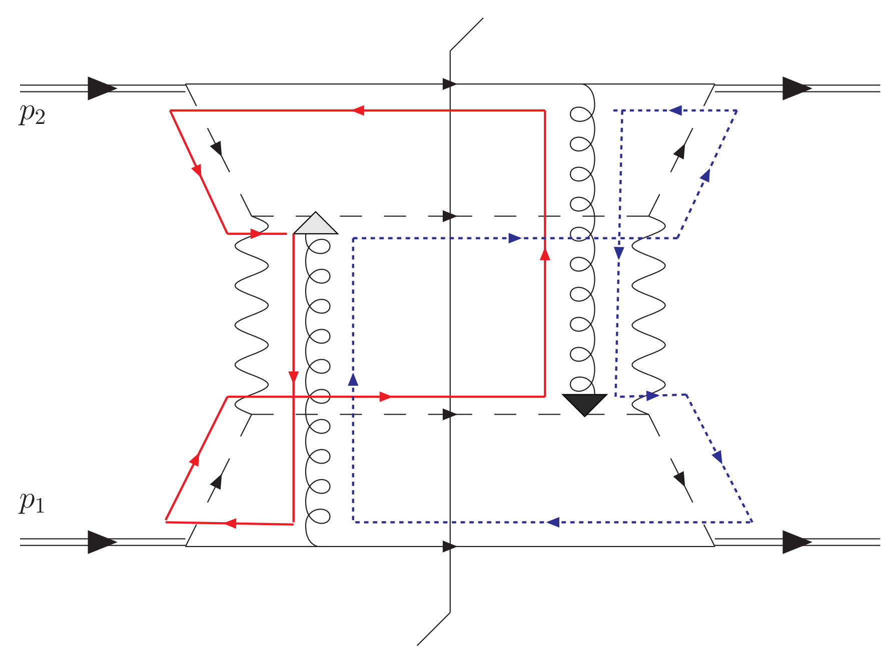
1) two scale problem--TMD factorization

2) weighted submits to transverse moments leads to gluonic pole factors & gluonic pole matrix elements--connection to twist three formalism

Problems/Challenges-"post CQMR period"

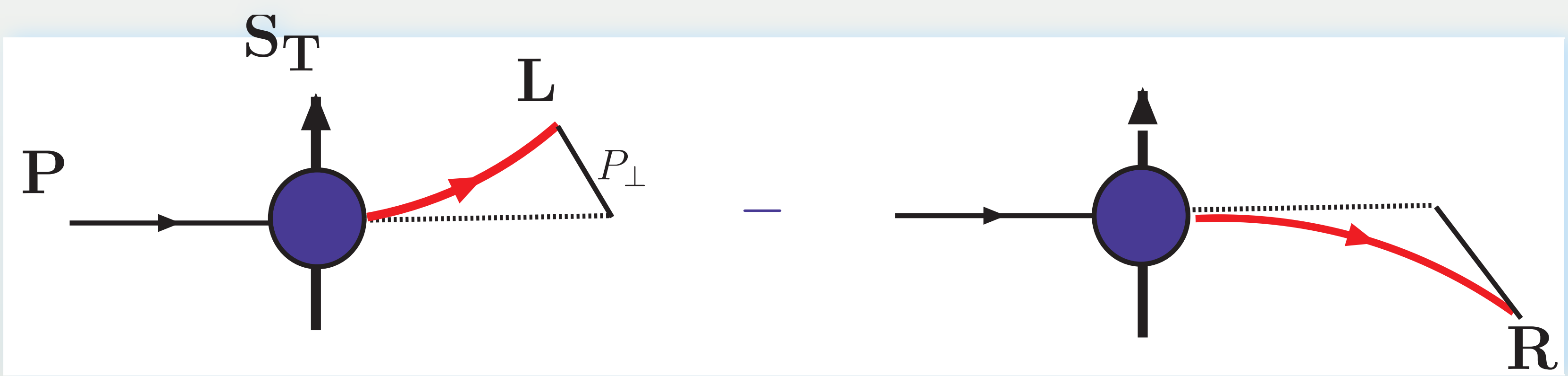
Collins Qiu PRD 2007 & Mulders Rogers 2010

*) factorization violated cannot define even a generalized gauge link



Transverse SPIN Observable kinematics (TSSA) $P^{\uparrow} P \to \pi X$

- **Single Spin Asymmetry**



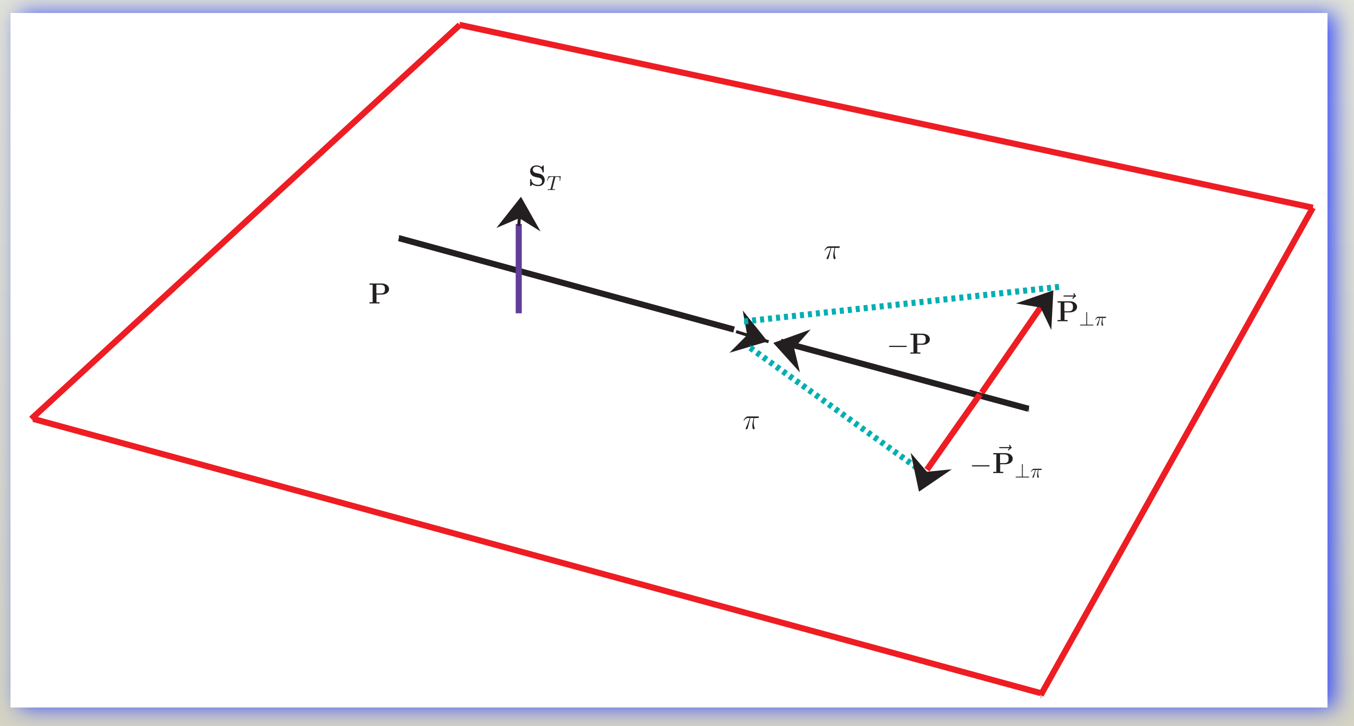
Parity Conserving interactions: SSAs Transverse Scattering plane

$$\Delta\sigma \sim i S_T \cdot (\mathbf{P} \times P_\perp^\pi)$$

- **Rotational invariance** $\sigma^{\downarrow}(x_F, \boldsymbol{p}_\perp) = \sigma^{\uparrow}(x_F, -\boldsymbol{p}_\perp)$

  $\Rightarrow$ ***Left-Right Asymmetry***

$$\boldsymbol{A}_N = \frac{\sigma^{\uparrow}(x_F, \boldsymbol{p}_\perp) - \sigma^{\uparrow}(x_F, -\boldsymbol{p}_\perp)}{\sigma^{\uparrow}(x_F, \boldsymbol{p}_\perp) + \sigma^{\uparrow}(x_F, -\boldsymbol{p}_\perp)} \equiv \Delta\sigma$$



Reaction Mechanism w/ Partonic Description $P^{\uparrow}P \to \pi X$

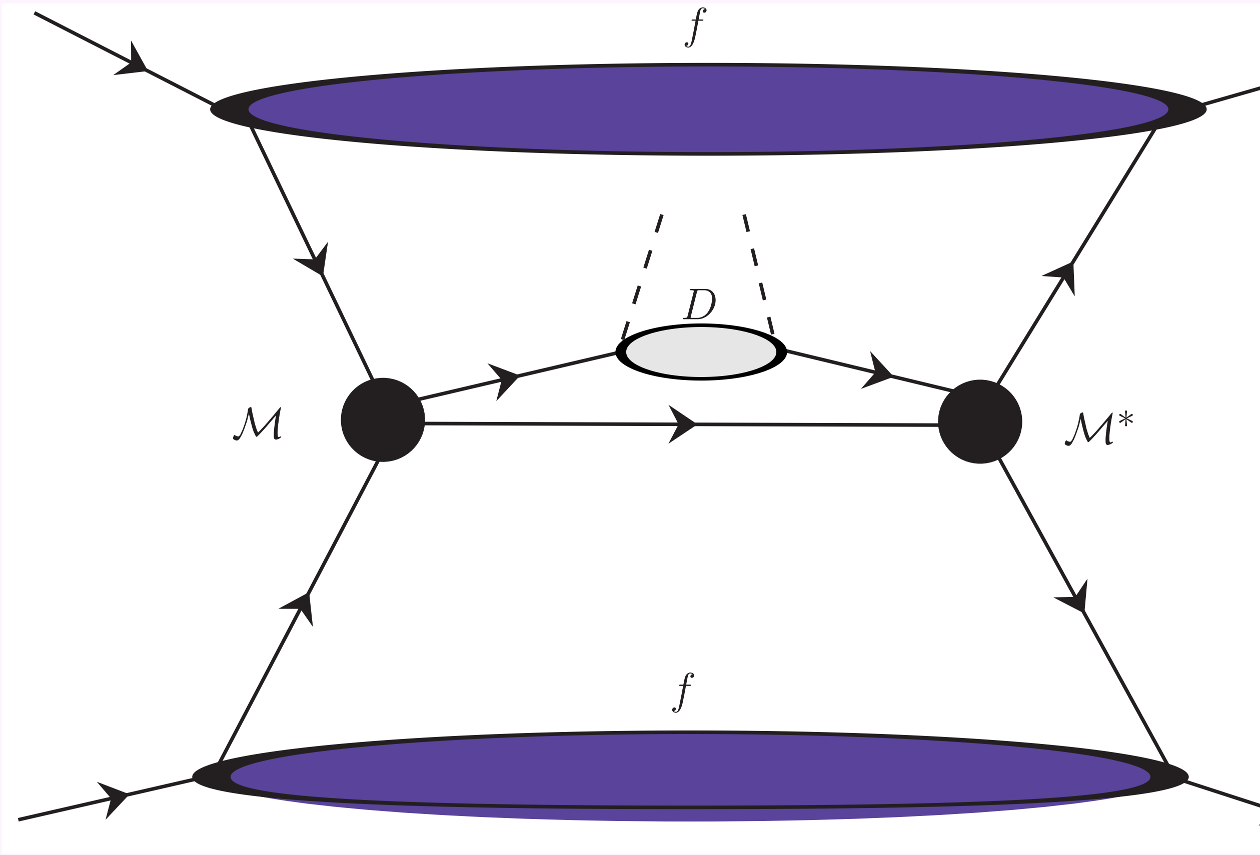
Collinear factorized QCD parton dynamics

$$\Delta\sigma^{pp^{\uparrow}\to\pi X} \sim f_a \otimes f_b \otimes \Delta\hat{\sigma} \otimes D^{q\to\pi}$$

$$\Delta\hat{\sigma} \equiv \hat{\sigma}^{\uparrow} - \hat{\sigma}^{\downarrow}$$

$$|\uparrow/\downarrow\rangle = (|+\rangle \pm i|-\rangle)$$

$$\hat{a}_N = \frac{\hat{\sigma}^{\uparrow} - \hat{\sigma}^{\downarrow}}{\hat{\sigma}^{\uparrow} + \hat{\sigma}^{\downarrow}} \sim \frac{\mathrm{Im}\left(\mathcal{M}^{+*}\mathcal{M}^{-}\right)}{|\mathcal{M}^{+}|^2 + |\mathcal{M}^{-}|^2}$$



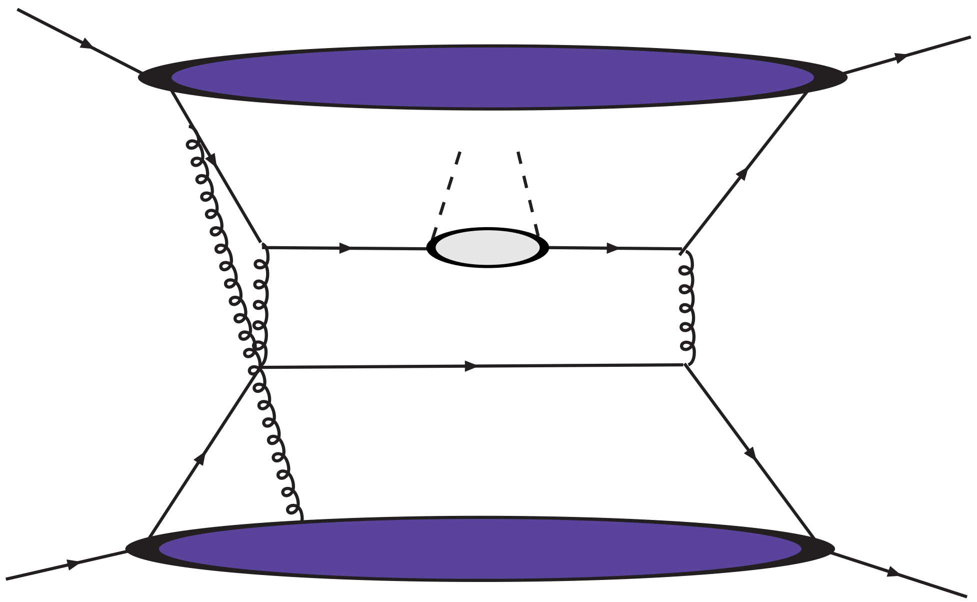
Transv. polarization cross section "interference" of helicity flip and non-flip amps.

Interference of helicity flip and non-flip amps
1) requires breaking of chiral symmetry m_q/E
2) relative phases require higher order corrections

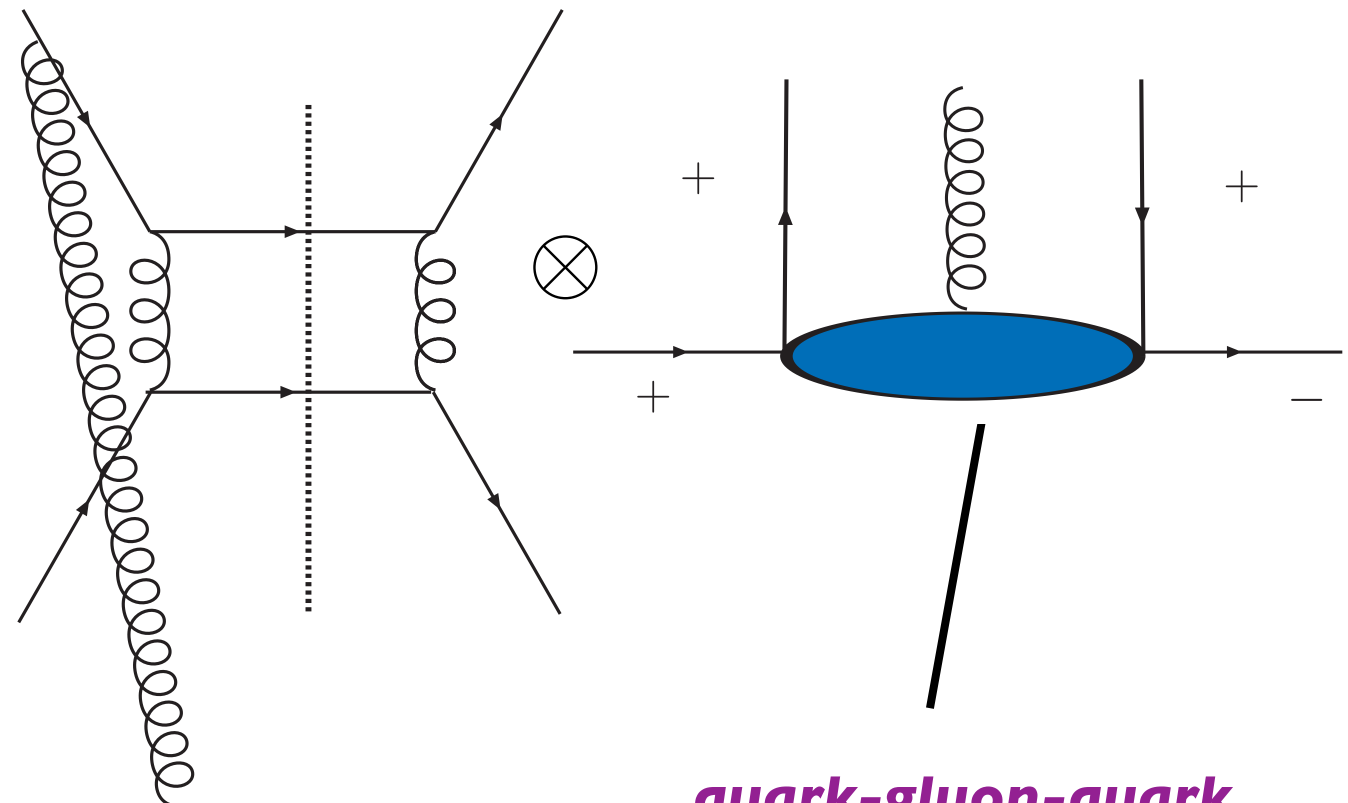
Not the full story @ Twist 3 approach ETQS approach

$Q \sim P_T >> \Lambda_{\rm qcd}$ One scale Collinear fact Twist 3

Phases in soft poles of prop in hard processes Efremov & Teryaev PLB 1982



- $\Delta\sigma \sim f_a \otimes T_F \otimes H_{ETQS} \otimes D^{q\to\pi}$



quark-gluon-quark correlator

$$\frac{1}{xs + i\epsilon} = \mathcal{P}\left(\frac{1}{xs}\right) \pm i\pi\delta(xs)$$

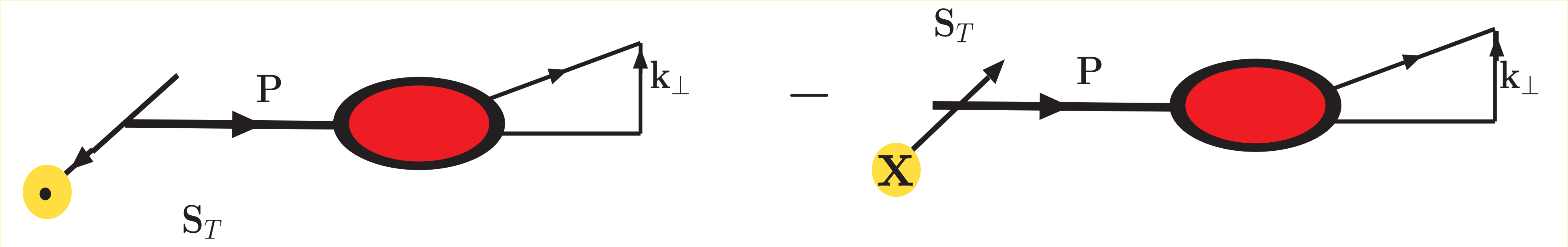
- Phases come from interference of two parton and three parton scattering amplitudes

Factorization and Pheno: Qiu, Sterman 1991,1999..., Koike et al, 2000, ... 2010, Ji, Qiu, Vogelsang, Yuan, 2005 ... 2008 ..., Yuan, Zhou 2008, 2009, Kang, Qiu, 2008, 2009 ... Kouvaris Ji, Qiu, Vogelsang! 2006, Vogelsang and Yuan PRD 2007

TSSAs thru "T-odd" non-pertb. **spin-orbit** *correlations....*

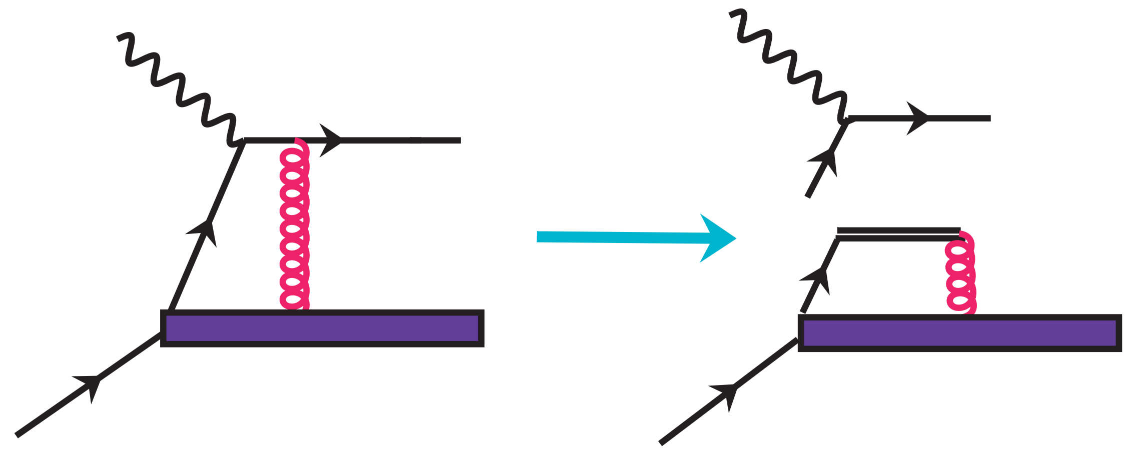
Sensitivity to $p_T \sim \mathbf{k}_T << \sqrt{Q^2}$

- Sivers PRD: 1990 TSSA is associated w/ correlation *transverse* spin and momenta in initial state hadron



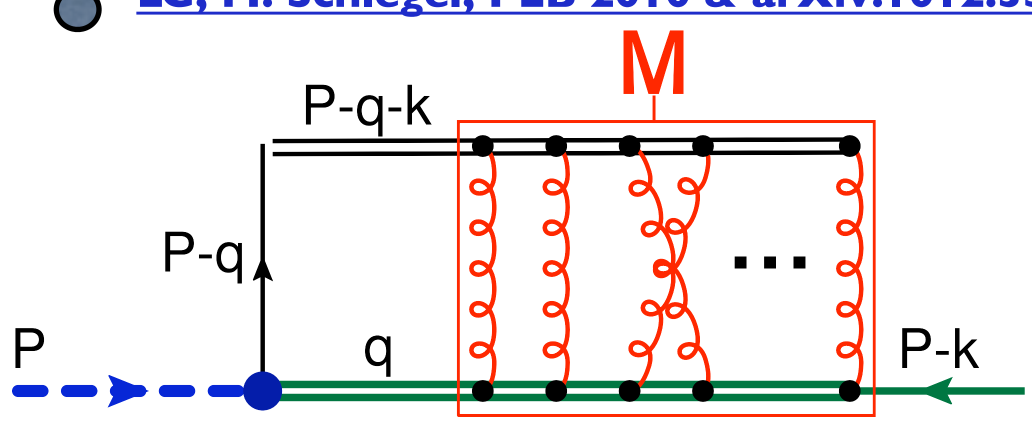
$$\Delta\sigma^{pp^{\uparrow}\to\pi X} \sim D \otimes f \otimes \Delta f^{\perp} \otimes \hat{\sigma}_{Born} \Longrightarrow \boldsymbol{\Delta f^{\perp}(x, k_{\perp}) = i S_T \cdot (P \times k_{\perp})}\; f_{1T}^{\perp}(x, \boldsymbol{k}_{\perp})$$

FSI phases in TSSAs unsuppressed



$$\Delta f^{\perp}(x,k_{\perp}) = iS_T \cdot (P \times k_{\perp})\ f_{1T}^{\perp}(x,\boldsymbol{k}_{\perp})$$

- **Unsuppressed reaction mech. Boer PRD 1999 context of DY @ RHIC**
- ***Brodsky Hwang Schmidt PLB 2002- SIDIS w/ transverse polarized target***
- ***Collins PLB 2002- Gauge link Sivers function doesn't vanish***
- Ji, Yuan PLB: 2002 -Sivers fnct. FSI emerge from Color Gauge-links
- **LG, Goldstein, Oganessyan, Schlegel 2002, 2003 2008** Boer-Mulders Fnct, and Sivers -spectator model
- **Burkardt** Sivers chromdynamic lensing NPA 2004
- **Bacchetta, Schaefer, Yang, PLB 2004, Bacchetta Conti Radici ... 2008,2010,2011 PRD**
- **LG, M. Schlegel, PLB 2010 & arXiv:1012.3395** B-M, Sivers sum FSIs w/color Chromo Lensing M. Schegel



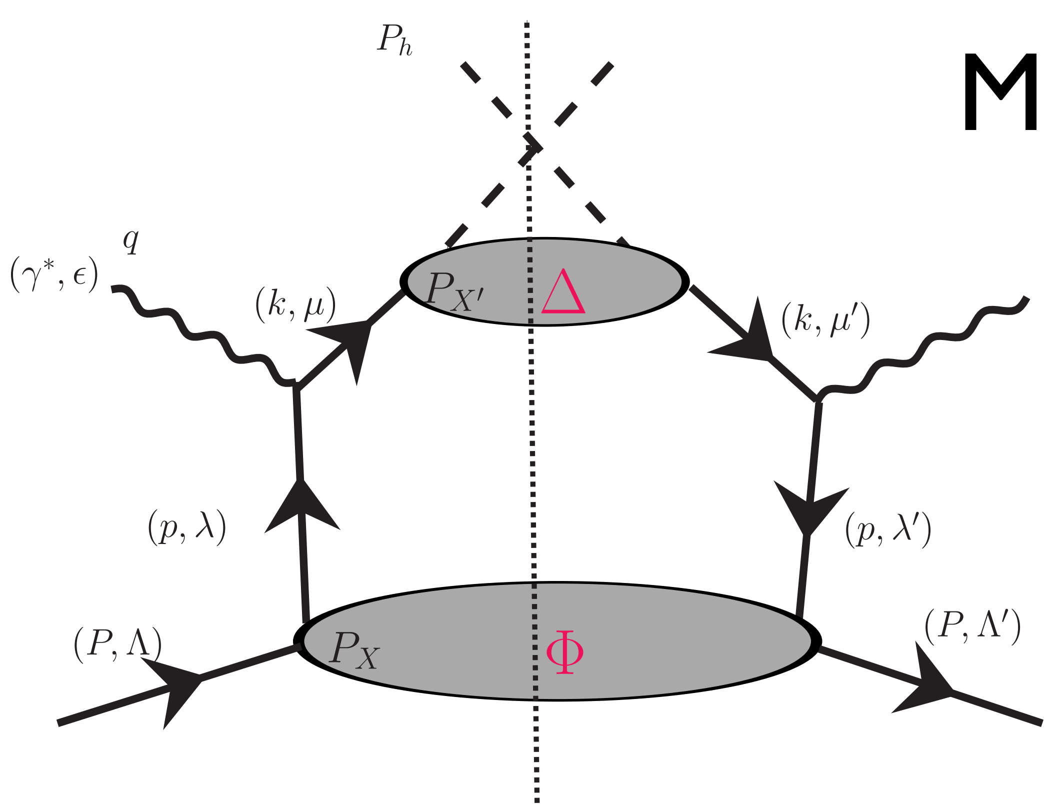
Many more model calcs.
talk of A. Bacchetta

Factorization parton model when P_T of the hadron small

$$W^{\mu\nu}(q,P,S,P_h) = \int \frac{d^2\mathbf{p}_T}{(2\pi)^4} \int \frac{d^2\mathbf{k}_T}{(2\pi)^4} \delta^2\left(\mathbf{p}_T - \frac{\mathbf{P}_{h\perp}}{z} - \mathbf{k}_T\right) \mathrm{Tr}\left[\left(\int dp^- \Phi\right)\gamma^\mu \left(\int dk^+ \Delta\right)\gamma^\nu\right]$$

Small transverse momentum !!!

$$\Phi(x,\mathbf{p}_T,S) \equiv \int dp^- \Phi(p,P,S)\Big|_{p^+=x_B P^+}, \qquad \Delta(z,\mathbf{k}_T) \equiv \int dk^+ \Delta(k,P_h)\Big|_{k^-=\frac{P^-}{z_h}}$$



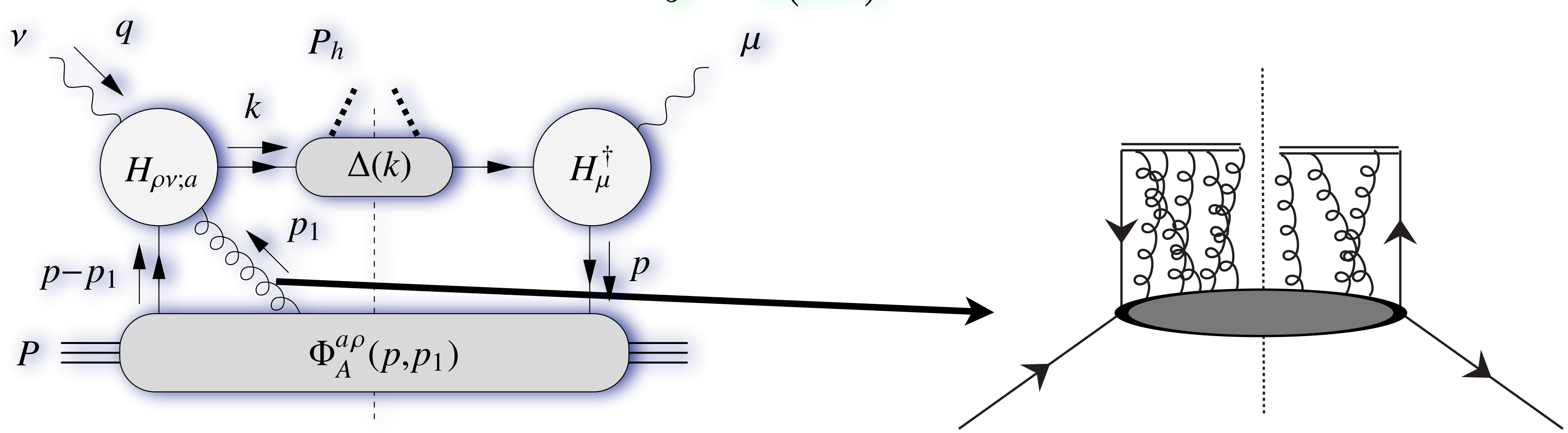
Minimal requirement satisfy color gauge invariance

"T-Odd" Effects From Color Gauge Inv. Via Gauge links

Gauge link determined re-summing leading gluon interactions btwn soft and hard

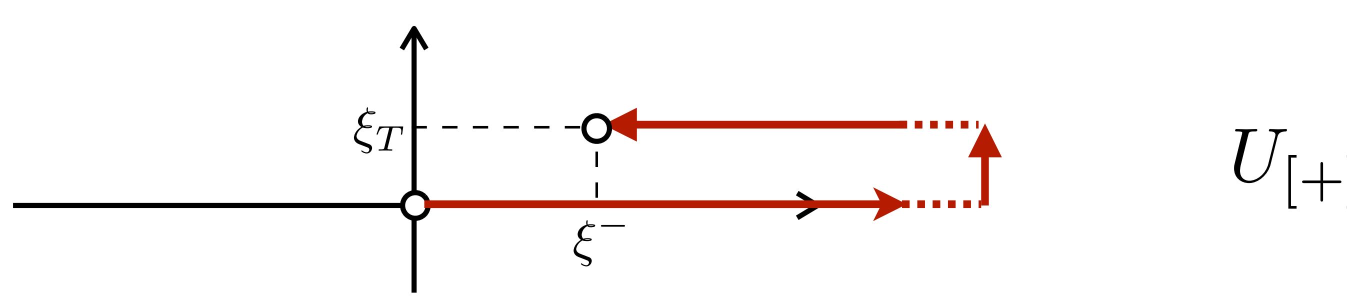
Efremov,Radyushkin Theor. Math. Phys. 1981,Belitsky, Ji, Yuan NPB 2003,
Boer, Bomhof, Mulders Pijlman, et al. 2003 - 2008- NPB, PLB, PRD

$$\Phi^{[\mathcal{U}[\mathcal{C}]]}(x,p_T) = \int \frac{d\xi^- d^2\xi_T}{2(2\pi)^3} e^{ip\cdot\xi} \langle P|\overline{\psi}(0)\mathcal{U}^{[C]}_{[0,\xi]}\psi(\xi^-,\xi_T)|P\rangle|_{\xi^+=0}$$



- **The path $[C]$ is fixed by hard subprocess within hadronic process.**

$$\int d^4p d^4k \delta^4(p+q-k) \mathrm{Tr}\left[\Phi^{[U^{C}_{[\infty;\xi]}]}(p) H^\dagger_\mu(p,k)\Delta(k)H_\nu(p,k)\right]$$

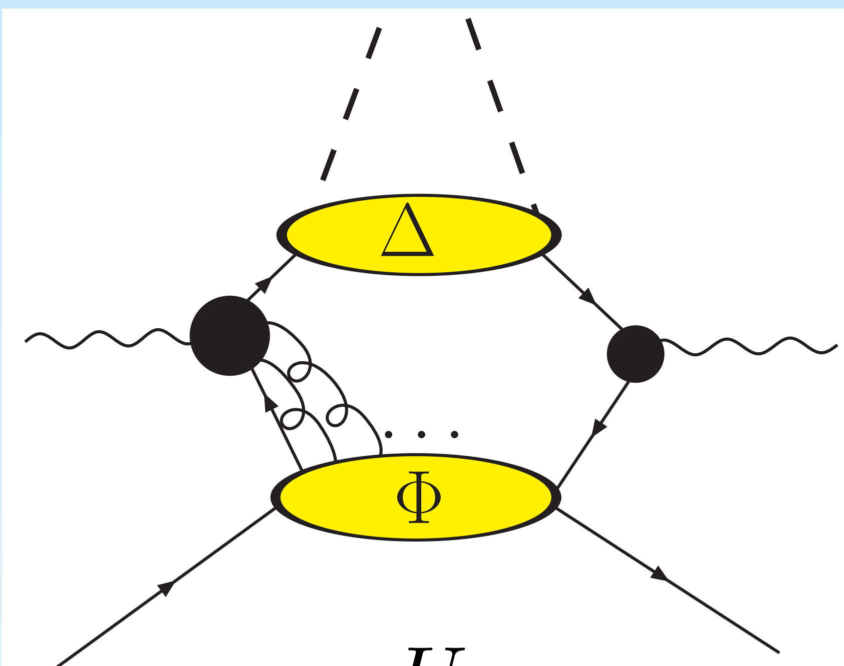


"Generalized Universality" Fund. Prediction of QCD Factorization

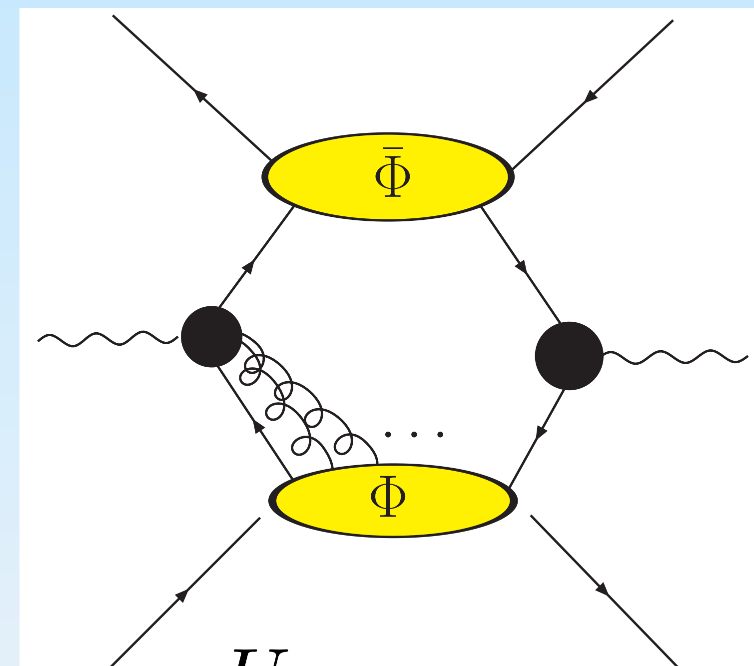
$$f_{1T\,sidis}^{\perp}(x,k_T) = -f_{1T\,DY}^{\perp}(x,k_T) \qquad p_T \sim \mathbf{k}_T << \sqrt{Q^2}$$

EIC conjunction with DY exp. E906-Fermi, RHIC II, Compass, JPARC

Process Dependence, Collins PLB 02, Brodsky et al. NPB 02, Boer Mulders Pijlman Bomhoff 03, 04 ...

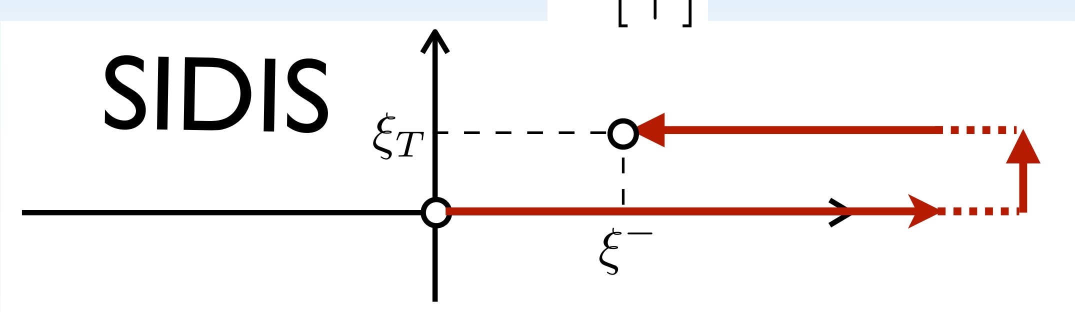


$$d\sigma = L_{\mu\nu} W^{\mu\nu} \Rightarrow$$

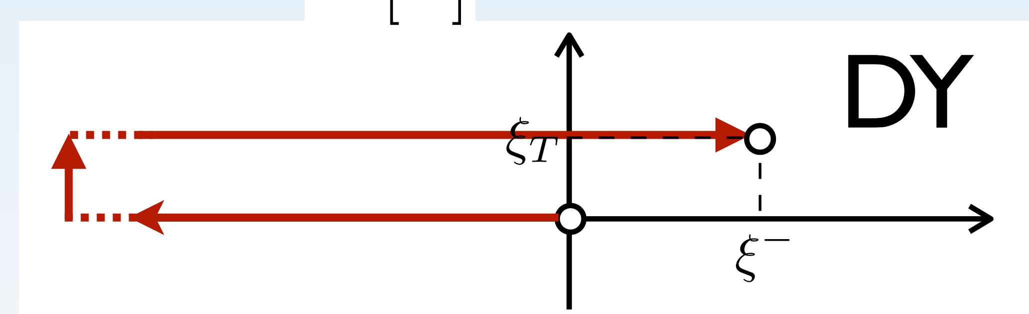


$U_{[+]}$

$U_{[-]}$



P&T



$$\Phi^{[+]*}(x,p_T) = i\gamma^1\gamma^3\Phi^{[-]}(x,p_T)i\gamma^1\gamma^3$$

Summary of Trans polz effects in QCD

- Realization that FSI and ISI btwn struck parton and target remnant provide necessary phases that lead to non-vanishing TSSAs

- Two scale factorization in terms TMDs twist 2

$$p_T \sim \mathbf{k}_T << \sqrt{Q^2}$$

- One large scale factorization in terms twist 3 approach $Q \sim P_T >> \Lambda_{\rm qcd}$

- Connection btwn two approaches overlap region for DY and SIDIS Unified picture Ji,Qiu,Vogelsang,Yuan PRL 2006 ...

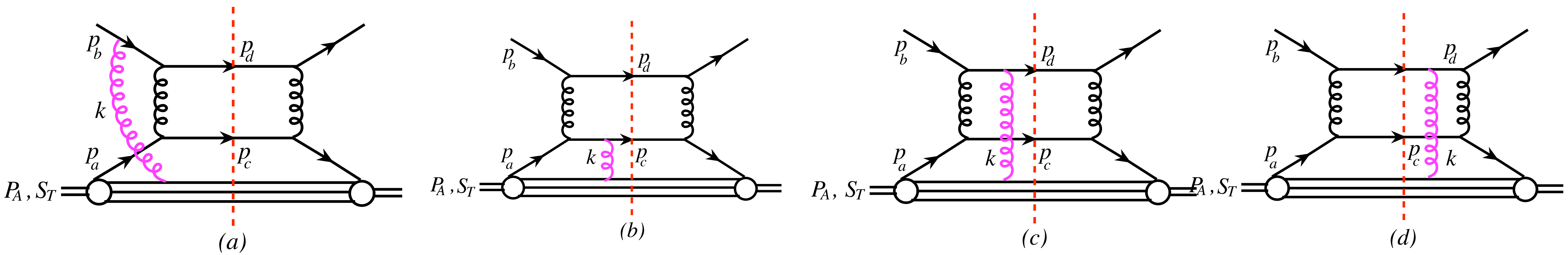
$$\Lambda_{\rm QCD} << q_T << Q$$

Generalizing the GPM CGI-GPM

- Feynman, Field, Fox (PRD 77 & 78)-incorporate intrisic k_T
- Include Transverse spin pol. w/ intrisic k_t --Anselmino, Boglione, Murgia, ... et al. *PLB 95* see talk of D'Alesio
- Pheno, Torino Cagliari group .... 1995-2011 inclusive processes
- Inclusive processes studied tw-3 formalism Kouvaris, Qiu, Vogelsang, Yuan PRD 2006, $pp \to \pi X$ & $pp \to \gamma X$
- What happens when you adopt ansatz of GPM including dynamical reaction mechanism of FSI/ISI in inclusive processes
- Take into account ISI/FSI process dependent Sivers function
- Since this one scale process-twist three connection w/ twist 3 ?

Observation

- Crucial point: Sivers function in inclusive single particle production contains both ISI and FSI
- Color factors entirely due to color structure of the partonic subprocess
- consider channel $qq' \to qq'$



Method

- Use diagramatic rather than helicity approach
  Bacchetta Bomhoff Mulders Pijlman 2005 PRD
- Has advantage of directly connecting to matrix elements of quark and gluon fields
- Allows inclusion of effects of ISI/FSI to determine color structure

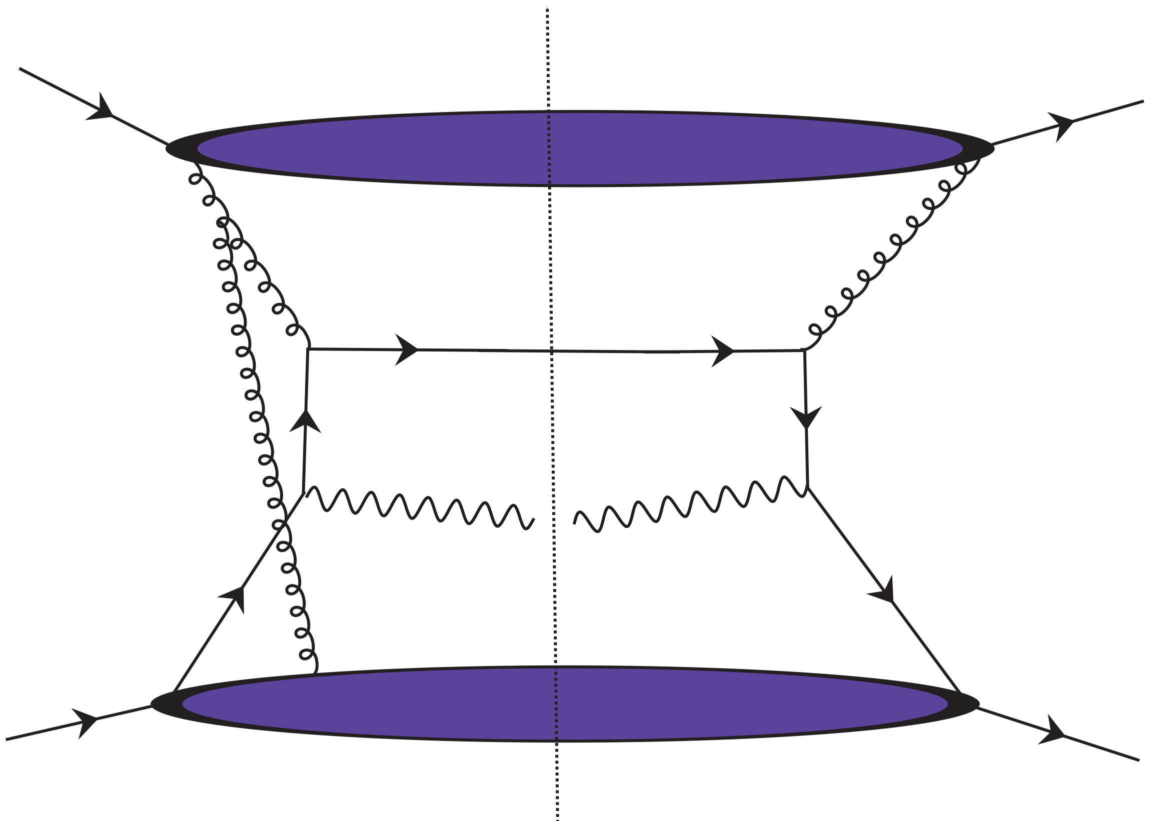
Consider direct Photon in GPM

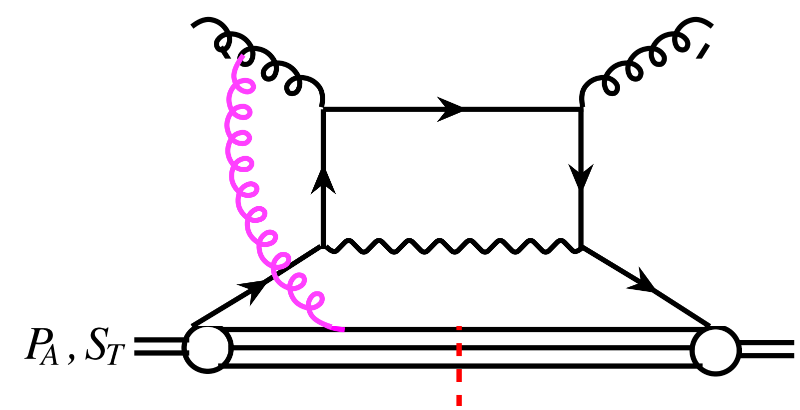
GPM w/color
LG & Z. Kang
Phys.Lett. B696 2011

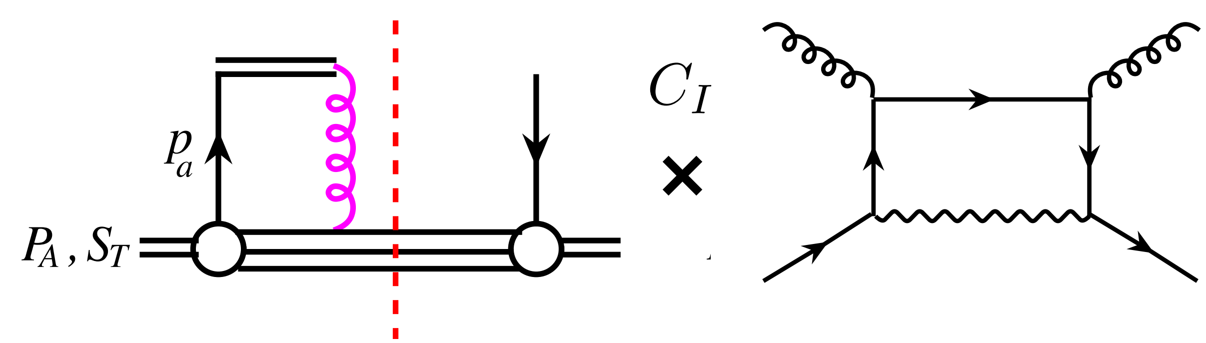
$$\Delta\sigma^{pp^{\uparrow}\to\gamma X} \sim \Delta f_a \otimes f_b \otimes \Delta\hat{\sigma}$$

Factorize w/ leading 1 gluon exchange get color phase

Vogelsang & Yuan PRD 2007 & agrees w/ "color flow" approach Bomhoff, Mulders, Pijlman







Get Sivers function for this process to use in GPM

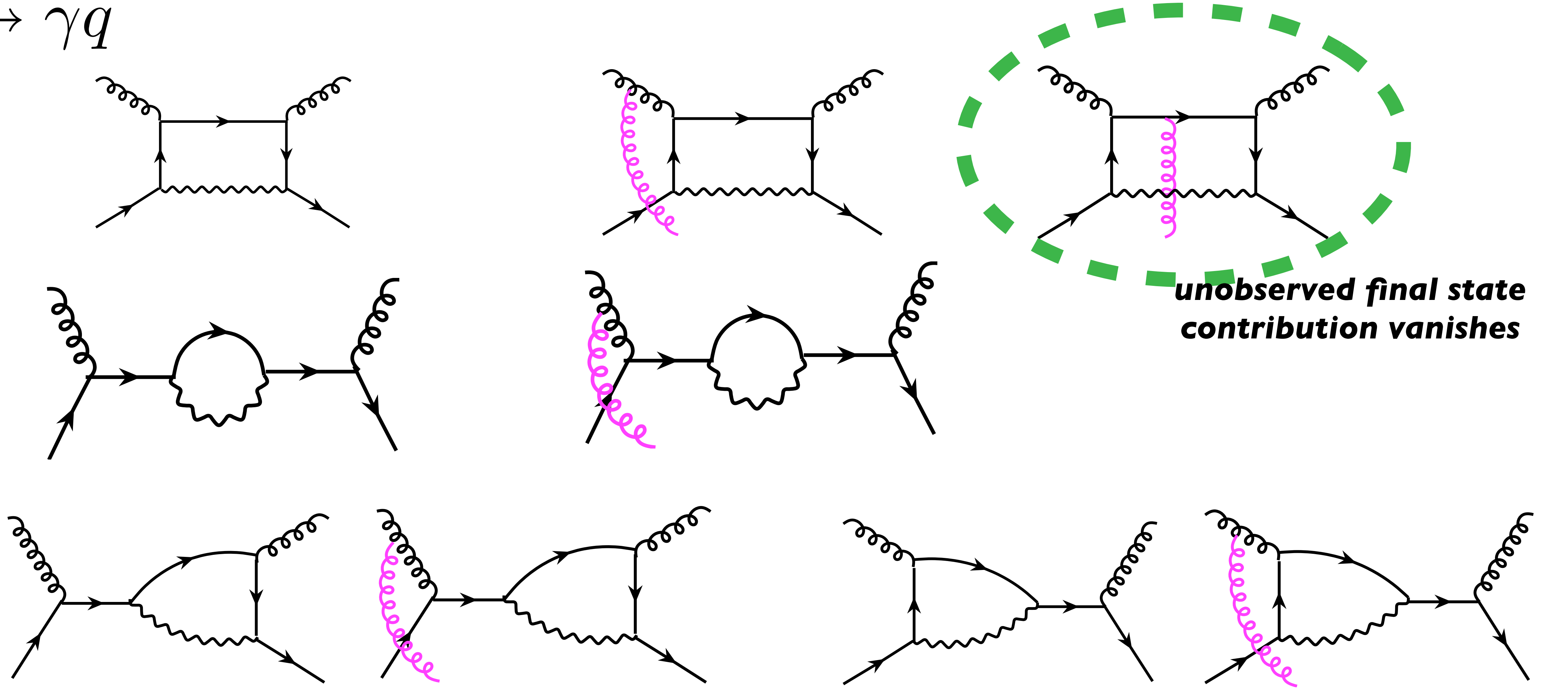
Color modification of hard cross sections due to "phases"

$qg \to \gamma q$

t-channel

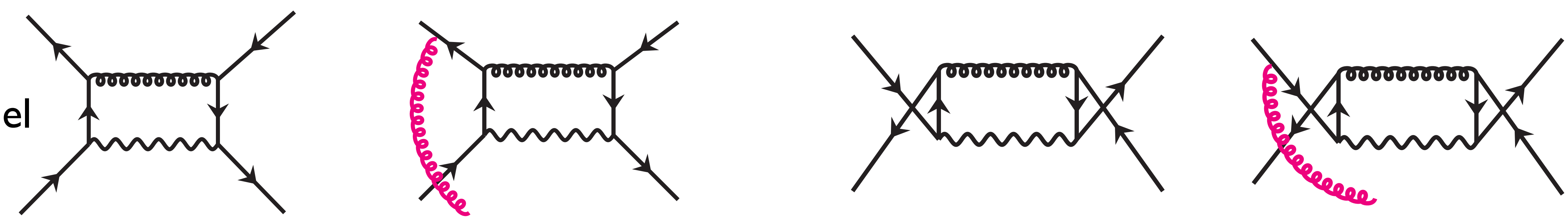
s-channel

s-t interference



$\bar{q}q \to \gamma g$

t & u-channel



t-u interference etc ....

Spin Dependent Cross Section in GPM $pp \to \gamma X$

$$f_{q/A^\uparrow}(x, \vec{k}_T) = f_{q/A}(x, k_T^2) + \frac{1}{2}\Delta^N f_{q/A^\uparrow}(x, k_T^2)\vec{S}\cdot(\hat{P}\times\vec{k}_T)$$

A_N is defined by the ratio $A_N = E_\gamma \frac{d\Delta\sigma}{d^3P_\gamma} \Big/ E_\gamma \frac{d\sigma}{d^3P_\gamma}.$

$$E_\gamma \frac{d\Delta\sigma}{d^3P_\gamma} = \frac{\alpha_{em}\alpha_s}{S}\sum_{a,b}\int\frac{dx_a}{x_a}d^2k_{aT}\Delta^N f_{a/A}^{\rm DIS}(x_a, k_{aT})\frac{1}{2}S_A\cdot(\hat{P}_A\times\hat{k}_{aT})$$
$$\times\int\frac{dx_b}{x_b}d^2k_{bT}f_{b/B}(x_b, k_{bT})H^U_{ab\to\gamma}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u}).$$

GPM Anselmino et al.

$$E_\gamma \frac{d\Delta\sigma}{d^3P_\gamma} = \frac{\alpha_{em}\alpha_s}{S}\sum_{a,b}\int\frac{dx_a}{x_a}d^2k_{aT}\Delta^N f_{a/A}^{ab\to\gamma}(x_a, k_{aT})\frac{1}{2}S_A\cdot(\hat{P}_A\times\hat{k}_{aT})$$
$$\times\int\frac{dx_b}{x_b}d^2k_{bT}f_{b/B}(x_b, k_{bT})H_{ab\to\gamma}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u}),$$

GPM w/color

LG & Z. Kang

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process-dependent Sivers function denoted as $\Delta^N f_{a/A}^{ab\to c}(x_a, k_{aT})$

Spin Dependent Cross Section in GPM $pp \to \pi X$

A_N is defined by the ratio $A_N \equiv E_h \frac{d\Delta\sigma}{d^3P_h} \Big/ E_h \frac{d\sigma}{d^3P_h}.$

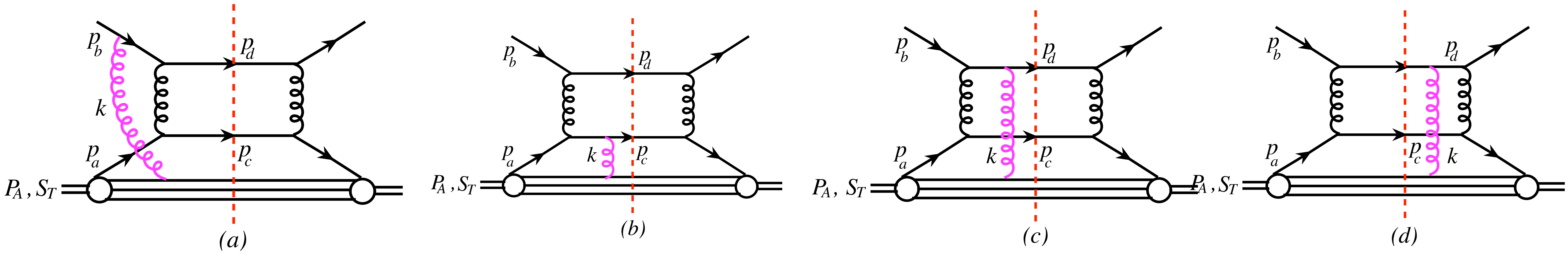
$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f_{a/A}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT}) \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b, k_{bT}) \times \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) H^U_{ab\to c}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}),$$

GPM Anselmino et al.

$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f^{ab\to c}_{a/A}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT}) \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b, k_{bT}) \times \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) H^U_{ab\to c}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}),$$

process-dependent Sivers function denoted as $\Delta^N f^{ab\to c}_{a/A}(x_a, k_{aT})$

One gluon exchange approx for ISI and FSI



$$\left[\frac{-g}{-k^+ - i\epsilon}T^a\right]$$

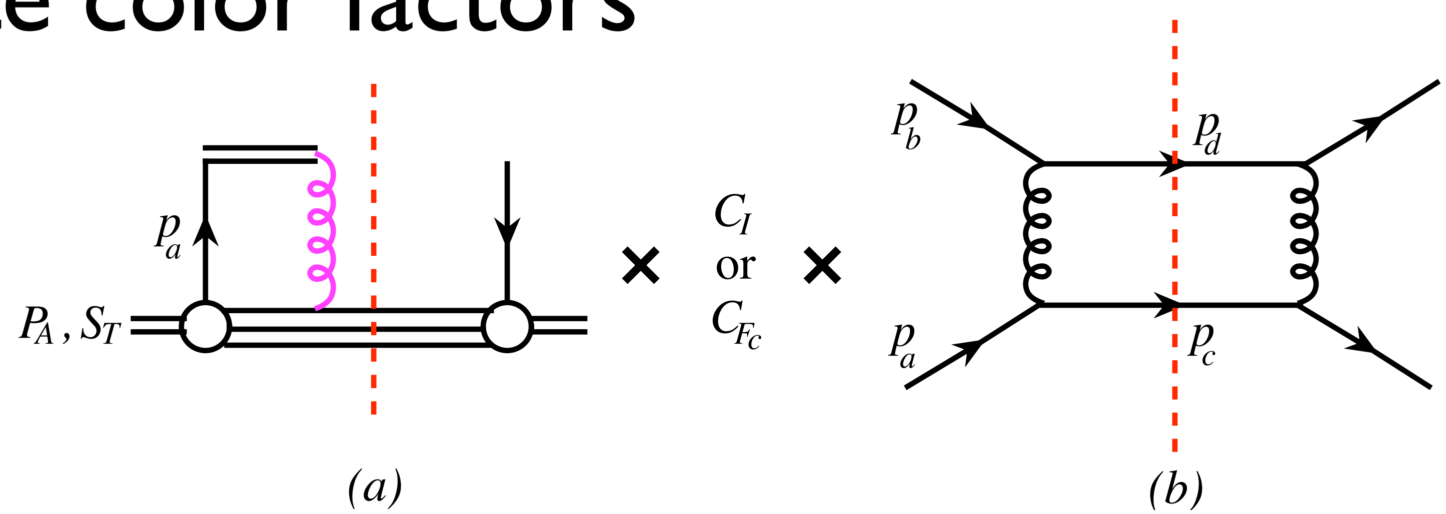
$$\rightarrow C_I = -\frac{1}{2N_c^2},$$

$$\left[\frac{g}{-k^+ + i\epsilon}T^a\right]$$

$$\rightarrow C_{F_c} = -\frac{1}{4N_c^2},$$

interaction w/unobserved particle "d" vanishes after summing over both cuts

calculate color factors

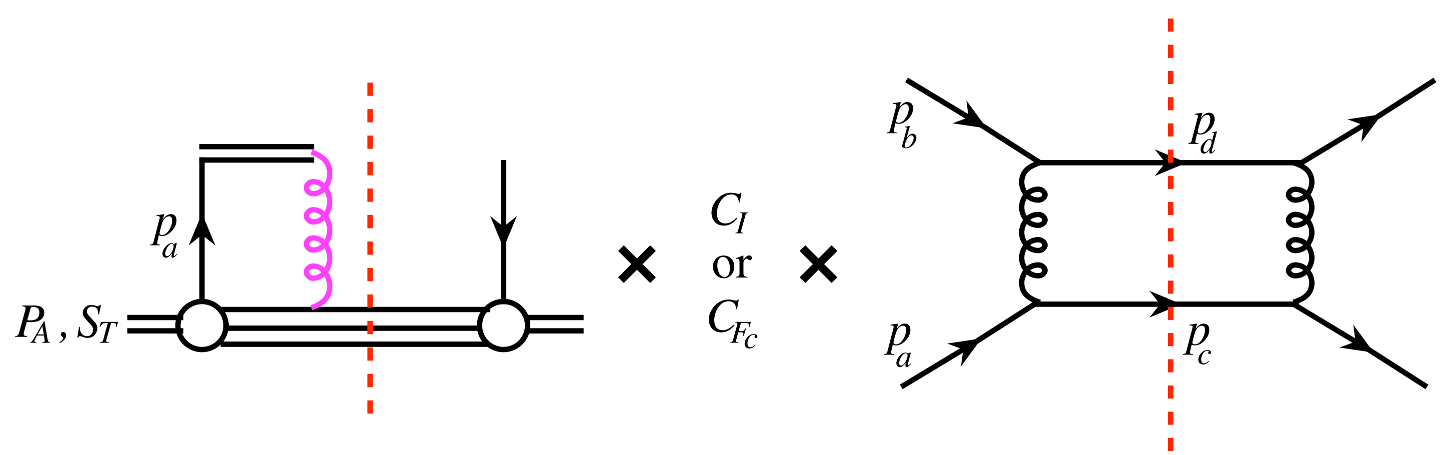


Note unpolarized color factor

$$C_u = \frac{N_c^2 - 1}{4N_c^2}$$

Comparing imag. pt of eikonal propagators for subprocess in SIDIS and inclusive single particle production

Sivers function probed in $qq' \to qq'$ process is related to those in SIDIS



(a) *(b)*

$$\Delta^N f_{a/A}^{qq'\to qq'} = \frac{C_I + C_{F_c}}{C_u} \Delta^N f_{a/A}^{\rm SIDIS}.$$

Alternatively one can move color factors "process dependence" to hard parts

$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f_{a/A}^{ab\to c}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT}) \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b, k_{bT})$$
$$\times \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) H_{ab\to c}^{U}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}),$$

$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f_{a/A}^{\rm SIDIS}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT}) \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b, k_{bT})$$
$$\times \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) H_{ab\to c}^{\rm Inc}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}),$$

In spirit of twist 3 approach, color factors from hard part

In spirit of twist 3 approach

That is rearrange

$$\Delta^N f_{a/A}^{qq'\to qq'} H^U_{qq'\to qq'} = \frac{C_I + C_{F_c}}{C_u} \Delta^N f_{a/A}^{\rm SIDIS} H^U_{qq'\to qq'} = \Delta^N f_{a/A}^{\rm SIDIS} \left[C_I h_{qq'\to qq'} + C_{F_c} h_{qq'\to qq'} \right],$$

where hard partonic c.s. w/o color factors

$$h_{qq'\to qq'} = 2\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}.$$

Then “modified” GPM is

$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f_{a/A}^{\rm SIDIS}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT}) \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b, k_{bT})$$
$$\times \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) H_{ab\to c}^{\rm Inc}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}),$$

$$H_{qq'\to qq'}^{\rm Inc} \equiv H_{qq'\to qq'}^{\rm Inc-I} + H_{qq'\to qq'}^{\rm Inc-F},$$

where,

$$H_{qq'\to qq'}^{\rm Inc-I} = C_I h_{qq'\to qq'}, \qquad H_{qq'\to qq'}^{\rm Inc-F} = C_{F_c} h_{qq'\to qq'}$$

The contributions for $pp \to \pi X$

the various contributing partonic subprocesses are given by

$$H^{\rm Inc-I}_{qq'\to qq'} = -H^{\rm Inc-I}_{\bar q\bar q'\to \bar q\bar q'} = -\frac{1}{N_c^2}\left[\frac{\hat s^2+\hat u^2}{\hat t^2}\right]$$

$$H^{\rm Inc-F}_{qq'\to qq'} = -H^{\rm Inc-F}_{\bar q\bar q'\to \bar q\bar q'} = -\frac{1}{2N_c^2}\left[\frac{\hat s^2+\hat u^2}{\hat t^2}\right]$$

$$H^{\rm Inc-I}_{q\bar q'\to q\bar q'} = -H^{\rm Inc-I}_{\bar q q'\to \bar q q'} = -\frac{N_c^2-2}{2N_c^2}\left[\frac{\hat s^2+\hat u^2}{\hat t^2}\right]$$

$$H^{\rm Inc-F}_{q\bar q'\to q\bar q'} = -H^{\rm Inc-F}_{\bar q q'\to \bar q q'} = -\frac{1}{2N_c^2}\left[\frac{\hat s^2+\hat u^2}{\hat t^2}\right]$$

$$H^{\rm Inc-I}_{qq'\to q'q} = -H^{\rm Inc-I}_{\bar q\bar q'\to \bar q'\bar q} = -\frac{1}{N_c^2}\left[\frac{\hat s^2+\hat t^2}{\hat u^2}\right]$$

$$H^{\rm Inc-F}_{qq'\to q'q} = -H^{\rm Inc-F}_{\bar q\bar q'\to \bar q'\bar q} = \frac{N_c^2-2}{2N_c^2}\left[\frac{\hat s^2+\hat t^2}{\hat u^2}\right]$$

$$H^{\rm Inc-I}_{q\bar q'\to \bar q' q} = -H^{\rm Inc-I}_{\bar q q'\to q'\bar q} = -\frac{N_c^2-2}{2N_c^2}\left[\frac{\hat s^2+\hat t^2}{\hat u^2}\right]$$

$$H^{\rm Inc-F}_{q\bar q'\to \bar q' q} = = -H^{\rm Inc-F}_{\bar q q'\to q'\bar q} = \frac{1}{N_c^2}\left[\frac{\hat s^2+\hat t^2}{\hat u^2}\right]$$

$$H^{\rm Inc-I}_{qq\to qq} = -H^{\rm Inc-I}_{\bar q\bar q\to \bar q\bar q} = -\frac{1}{N_c^2}\left[\frac{\hat s^2+\hat u^2}{\hat t^2}+\frac{\hat s^2+\hat t^2}{\hat u^2}\right]+\frac{N_c^2+1}{N_c^3}\frac{\hat s^2}{\hat t\hat u}$$

$$H^{\rm Inc-F}_{qq\to qq} = -H^{\rm Inc-F}_{\bar q\bar q\to \bar q\bar q} = -\frac{1}{2N_c^2}\left[\frac{\hat s^2+\hat u^2}{\hat t^2}\right]+\frac{N_c^2-2}{2N_c^2}\left[\frac{\hat s^2+\hat t^2}{\hat u^2}\right]+\frac{1}{N_c^3}\frac{\hat s^2}{\hat t\hat u}$$

$$H^{\rm Inc-I}_{q\bar q\to q'\bar q'} = -H^{\rm Inc-I}_{\bar q q\to \bar q' q'} = \frac{1}{2N_c^2}\left[\frac{\hat t^2+\hat u^2}{\hat s^2}\right]$$

$$H^{\rm Inc-F}_{q\bar q\to q'\bar q'} = -H^{\rm Inc-F}_{\bar q q\to \bar q' q'} = \frac{N_c^2-2}{2N_c^2}\left[\frac{\hat t^2+\hat u^2}{\hat s^2}\right]$$

$$H^{\rm Inc-I}_{q\bar q\to \bar q' q'} = -H^{\rm Inc-I}_{\bar q q\to q'\bar q'} = \frac{1}{2N_c^2}\left[\frac{\hat t^2+\hat u^2}{\hat s^2}\right]$$

$$H^{\rm Inc-F}_{q\bar q\to \bar q' q'} = -H^{\rm Inc-F}_{\bar q q\to q'\bar q'} = \frac{1}{N_c^2}\left[\frac{\hat t^2+\hat u^2}{\hat s^2}\right]$$

$$H^{\rm Inc-I}_{q\bar q\to q\bar q} = -H^{\rm Inc-I}_{\bar q q\to \bar q q} = -\frac{N_c^2-2}{2N_c^2}\left[\frac{\hat s^2+\hat u^2}{\hat t^2}\right]+\frac{1}{2N_c^2}\left[\frac{\hat t^2+\hat u^2}{\hat s^2}\right]-\frac{1}{N_c^3}\frac{\hat u^2}{\hat s\hat t}$$

$$H^{\rm Inc-F}_{q\bar q\to q\bar q} = -H^{\rm Inc-F}_{\bar q q\to \bar q q} = -\frac{1}{2N_c^2}\left[\frac{\hat s^2+\hat u^2}{\hat t^2}\right]+\frac{N_c^2-2}{2N_c^2}\left[\frac{\hat t^2+\hat u^2}{\hat s^2}\right]+\frac{1}{N_c^3}\frac{\hat u^2}{\hat s\hat t}$$

$$H^{\rm Inc-I}_{q\bar q\to \bar q q} = -H^{\rm Inc-I}_{\bar q q\to q\bar q} = -\frac{N_c^2-2}{2N_c^2}\left[\frac{\hat s^2+\hat t^2}{\hat u^2}\right]+\frac{1}{2N_c^2}\left[\frac{\hat t^2+\hat u^2}{\hat s^2}\right]-\frac{1}{N_c^3}\frac{\hat t^2}{\hat s\hat u}$$

$$H^{\rm Inc-F}_{q\bar q\to \bar q q} = -H^{\rm Inc-F}_{\bar q q\to q\bar q} = \frac{1}{N_c^2}\left[\frac{\hat s^2+\hat t^2}{\hat u^2}+\frac{\hat t^2+\hat u^2}{\hat s^2}\right]-\frac{N_c^2+1}{N_c^3}\frac{\hat t^2}{\hat s\hat u}$$

$$H^{\rm Inc-I}_{qg\to qg} = -H^{\rm Inc-I}_{\bar q g\to \bar q g} = \frac{1}{2(N_c^2-1)}\left[-\frac{\hat s}{\hat u}-\frac{\hat u}{\hat s}\right]+\frac{N_c^2}{2(N_c^2-1)}\left[\frac{\hat s^2+\hat u^2}{\hat t^2}\frac{\hat u}{\hat s}\right]$$

$$H^{\rm Inc-F}_{qg\to qg} = -H^{\rm Inc-F}_{\bar q g\to \bar q g} = \frac{1}{2N_c^2(N_c^2-1)}\left[-\frac{\hat s}{\hat u}-\frac{\hat u}{\hat s}\right]-\frac{1}{N_c^2-1}\left[\frac{\hat s^2+\hat u^2}{\hat t^2}\right]$$

$$H^{\rm Inc-I}_{qg\to gq} = -H^{\rm Inc-I}_{\bar q g\to g\bar q} = \frac{1}{2(N_c^2-1)}\left[-\frac{\hat s}{\hat t}-\frac{\hat t}{\hat s}\right]+\frac{N_c^2}{2(N_c^2-1)}\left[\frac{\hat s^2+\hat t^2}{\hat u^2}\frac{\hat t}{\hat s}\right]$$

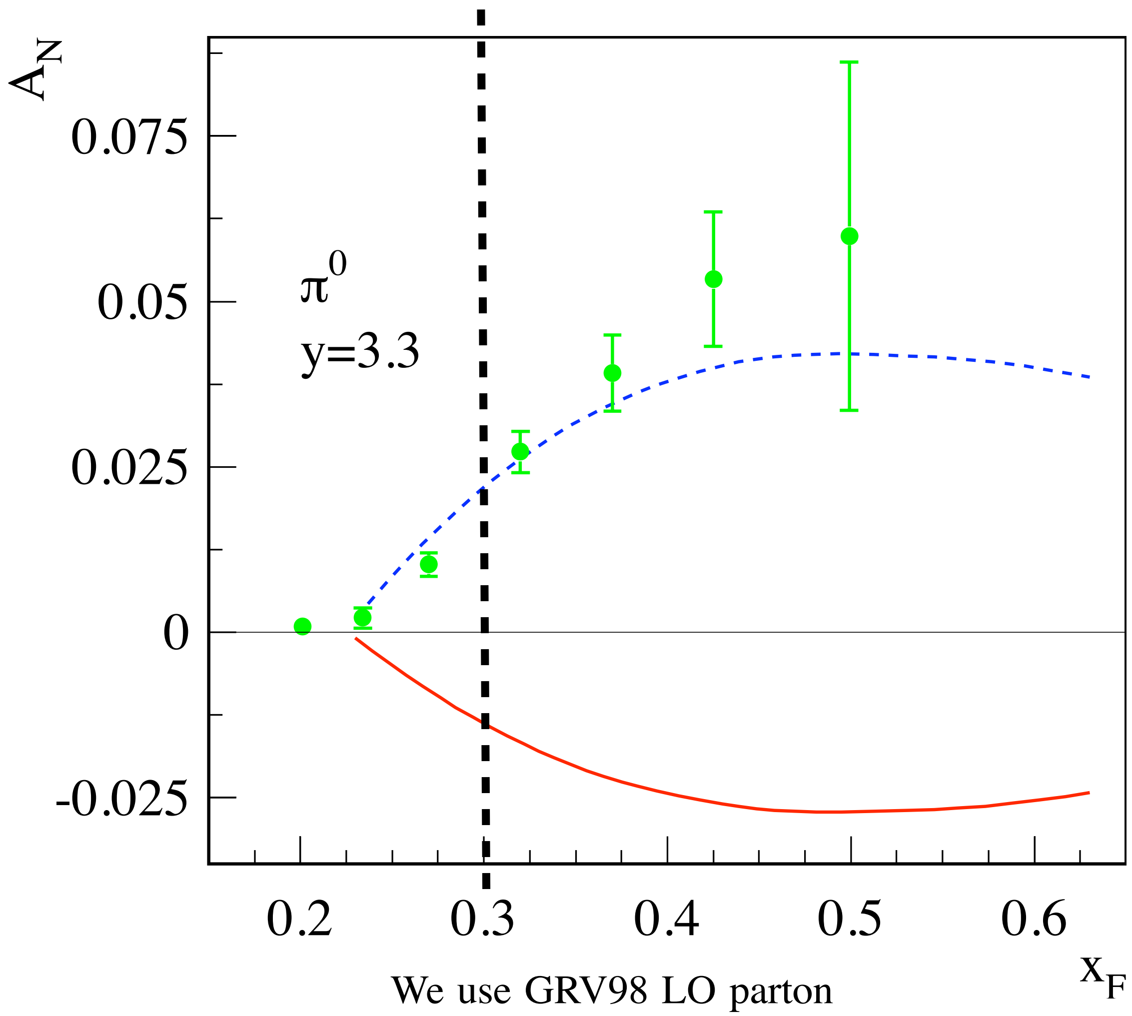
$$H^{\rm Inc-F}_{qg\to gq} = -H^{\rm Inc-F}_{\bar q g\to g\bar q} = -\frac{1}{2(N_c^2-1)}\left[-\frac{\hat s}{\hat t}-\frac{\hat t}{\hat s}\right]-\frac{N_c^2}{2(N_c^2-1)}\left[\frac{\hat s^2+\hat t^2}{\hat u^2}\frac{\hat s}{\hat t}\right]$$

$$H^{\rm Inc-I}_{q\bar q\to gg} = -H^{\rm Inc-I}_{\bar q q\to gg} = -\frac{1}{2N_c^3}\left[\frac{\hat u}{\hat t}+\frac{\hat t}{\hat u}\right]-\frac{1}{N_c}\left[\frac{\hat t^2+\hat u^2}{\hat s^2}\right]$$

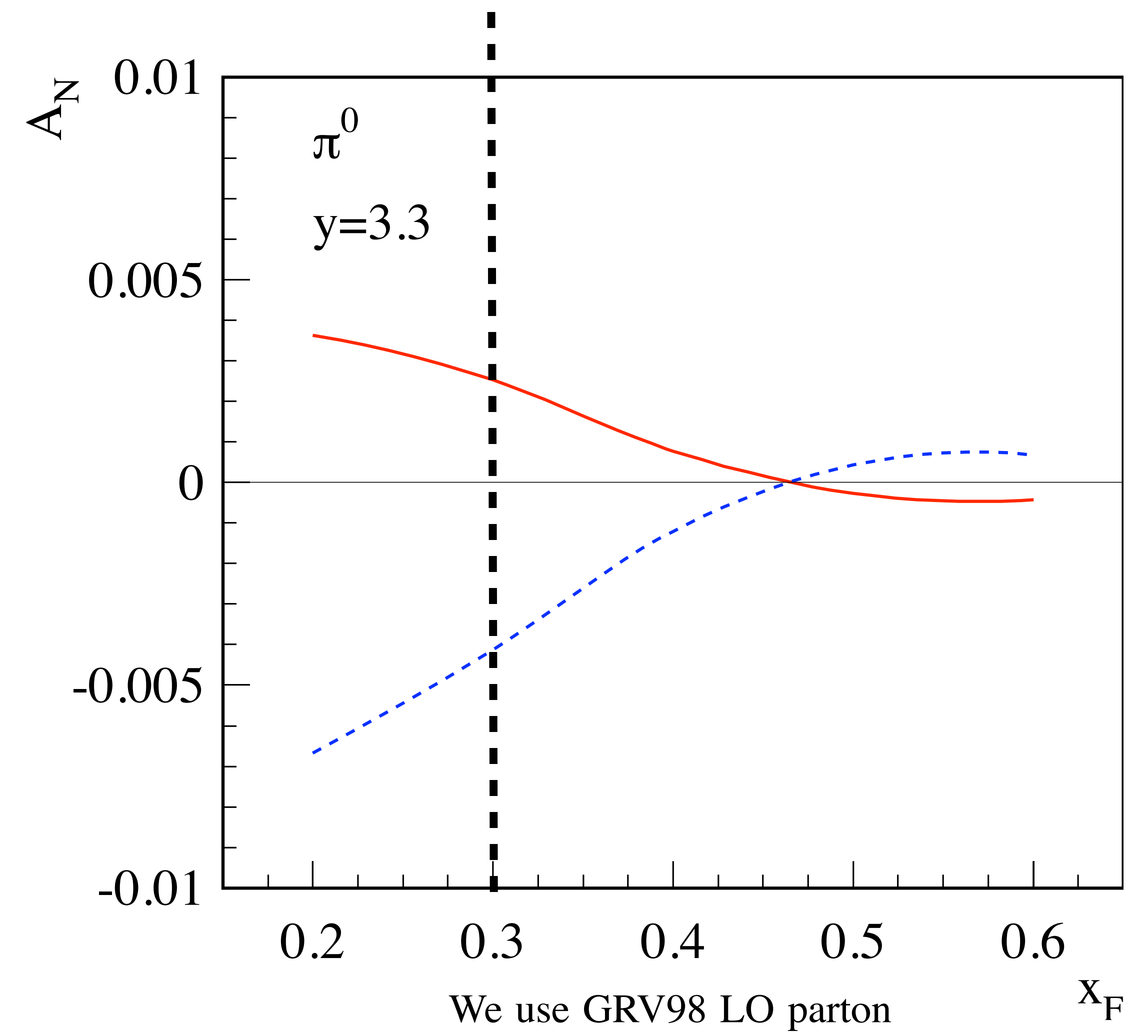
$$H^{\rm Inc-F}_{q\bar q\to gg} = -H^{\rm Inc-F}_{\bar q q\to gg} = -\frac{1}{2N_c}\left[\frac{\hat u}{\hat t}+\frac{\hat t}{\hat u}\right]+\frac{N_c}{2}\left[\frac{\hat t^2+\hat u^2}{\hat s^2}\frac{\hat u}{\hat t}\right]$$

Based on old parameterization

Based on new parameterization



Sivers from Anselmino et al PRD72 (2005)
Fragmentation from Kretzer PRD62 (2000)



Sivers from Anselmino et al EPJA (2009)
Fragmentation from DSS PRD75 (2007)

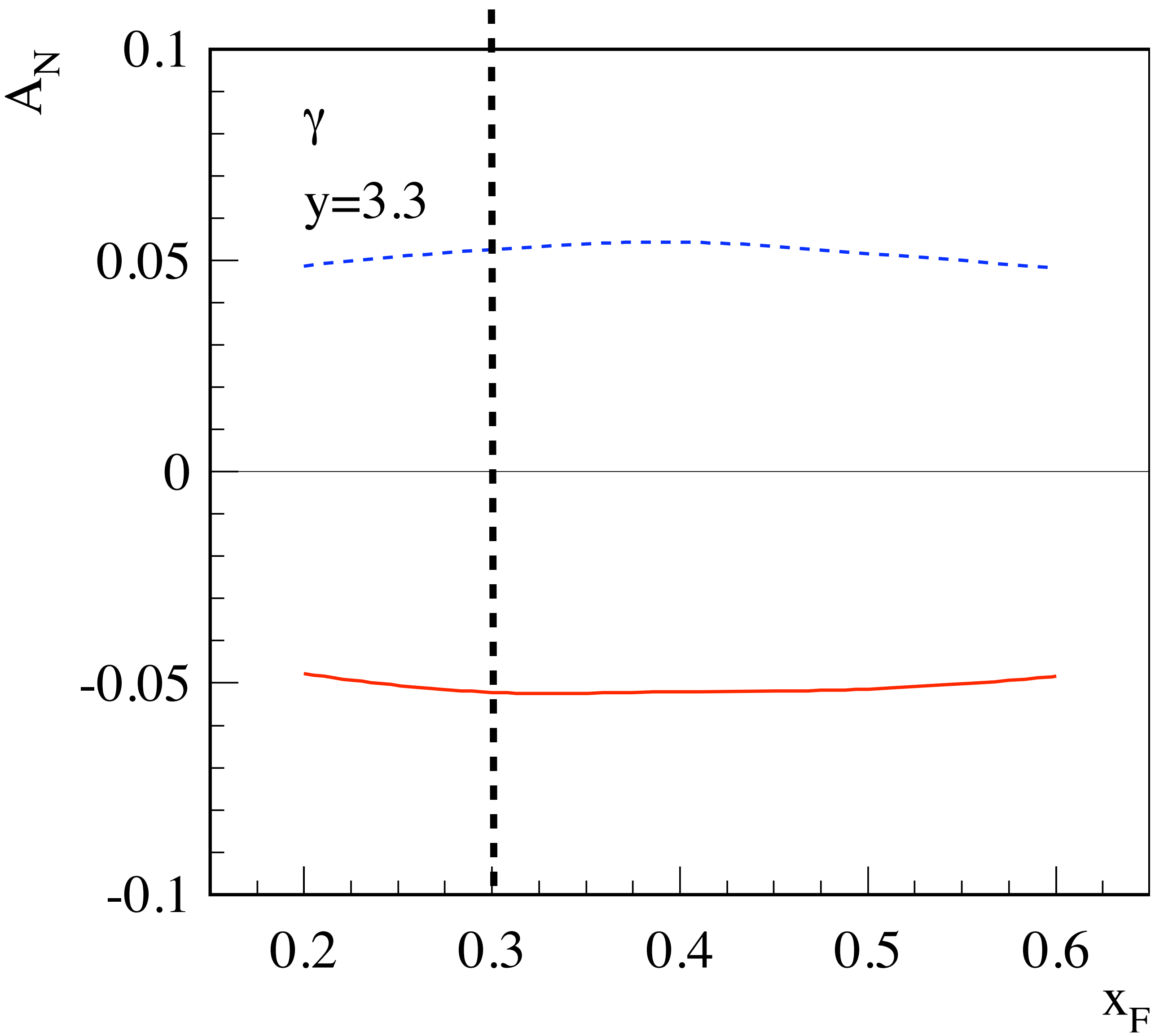
$$H^{\text{Inc}}_{qg\to qg} = H^{\text{Inc-I}}_{qg\to qg} + H^{\text{Inc-F}}_{qg\to qg} = -\frac{N_c^2+2}{N_c^2-1}\frac{\hat{s}^2}{\hat{t}^2},$$

vs.

forward direction, $\hat{t}$ is small, while $\hat{u} \sim -\hat{s}$,

$$H^{U}_{qg\to qg} = \frac{2\hat{s}^2}{\hat{t}^2}.$$

Based on old parameterization

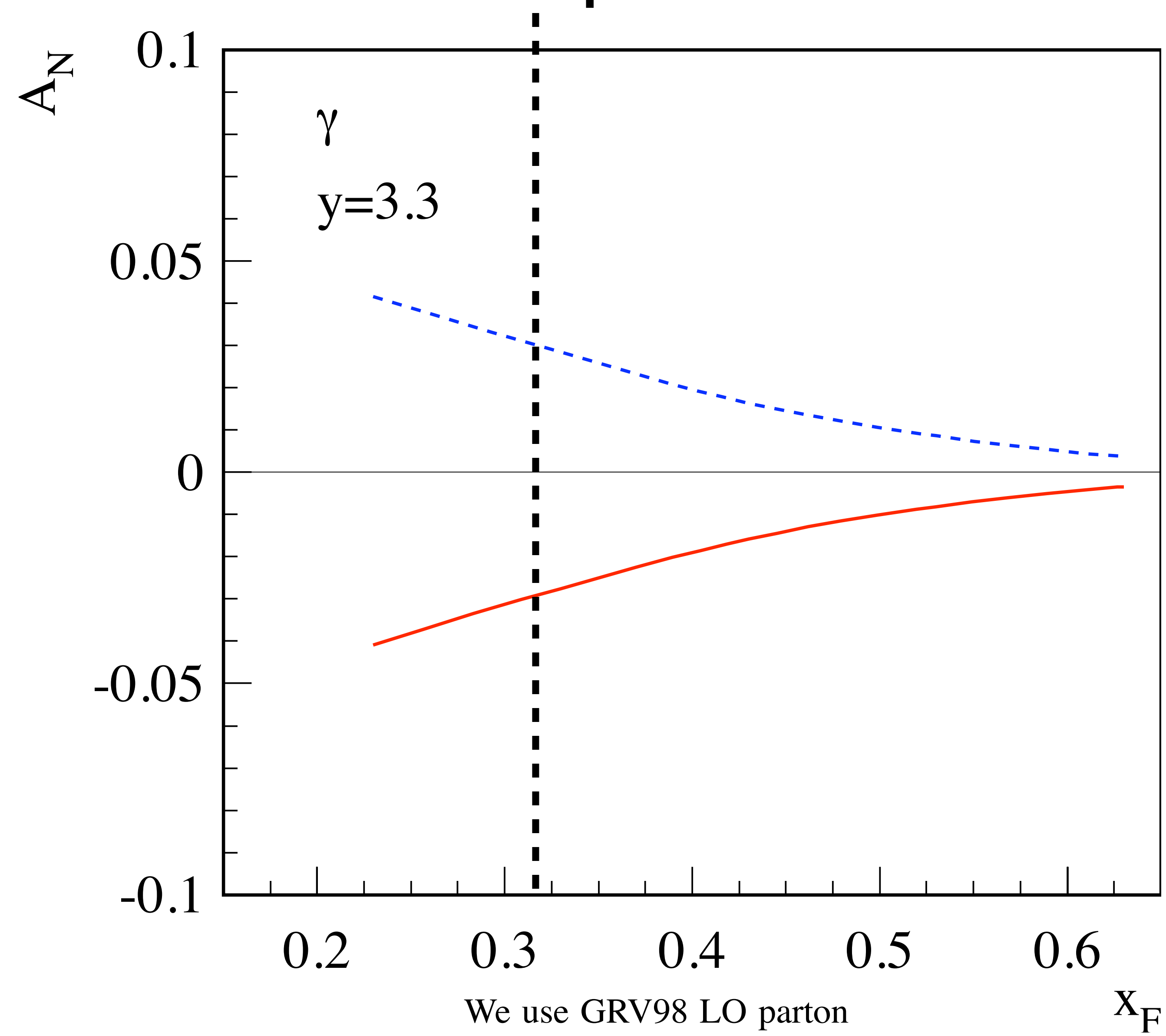


We use GRV98 LO parton

Sivers from Anselmino et al PRD72 (2005)

Fragmentation from Kretzer PRD62 (2000)

Based on new parameterization

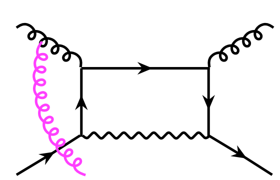


We use GRV98 LO parton

Sivers from Anselmino et al EPJA (2009)

Fragmentation from DSS PRD75 (2007)

$$H^{U}_{qg\to\gamma q} = \frac{1}{N_c} e_q^2 \left[-\frac{\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} \right]$$



ISI drives result

$$H^{\rm Inc}_{qg\to\gamma q} = -\frac{N_c}{N_c^2-1} e_q^2 \left[-\frac{\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} \right]$$

Observations

- Hard amplitudes squared have same form in Mandelstam variables as twist-3 $\hat{s},\ \hat{t},\ \hat{u}$

see Kouvaris,Qiu ,Vogelsang, and Yuan PRD 2006

- However $\hat{s},\ \hat{t},\ \hat{u}$ depend on k_T in GPM whereas in twist-3 approach there has been collinear expansion on hard and soft factors

- We have shown that GPM expanded with respect to k_T results in twist-3 result {\em almost }

Collinear Expansion in GPM

- Here $\hat{s},\ \hat{t},\ \hat{u}$ depend on k_{aT}
- Implement delta function $\delta(\hat{s}+\hat{t}+\hat{u}) = \frac{1}{x_b S + T/z_c}\delta\left(x_a - x - \frac{2P_{hT}\cdot k_{aT}}{z_c x_b S + T}\right)$
- expand k_{aT} and study contribution from Sivers function and hard cross section

$$E_h \frac{d\Delta\sigma}{d^2P_h} = \frac{\alpha_s}{s}\sum_{abc}\int d^2k_{aT}\frac{1}{M}\epsilon^{\alpha S_T n\hat{n}}k_{aT\alpha}\frac{1}{x_a}f_{1T}^{\perp\,\mathrm{SIDIS}}(x_a,k_{aT}^2)\Big|_{x_a=X+\frac{2P_{hT}\cdot k_{aT}/z_c}{x_b s+T/z_c}}$$
$$\times\int\frac{dx_b}{x_b}f_{b/B}(x_b)\int\frac{dz_c}{z_c^2}H^{\mathrm{Inc}}_{ab\to c}(\hat{s},\hat{t},\hat{u})\frac{1}{x_b s+T/z_c}$$

That is.... in GPM

$$\hat{s} = (p_a + p_b)^2 = x_a x_b S + \quad \mathcal{O}(k_T^2)$$

$$\hat{t} = (x_a P_A + k_{aT} - \frac{P_h}{z})^2 = \frac{x_a}{z} T - \frac{2 P_{hT} \cdot k_{aT}}{z}$$

$$\hat{u} = (p_b - p_c)^2 = (x_b P_B - \frac{P_h}{z})^2 = \frac{x_b}{z} U$$

$$\delta(\hat{s} + \hat{t} + \hat{u}) = \frac{1}{x_b s + \frac{T}{z_c}} \delta(x_a - x - \frac{2 P_{hT} \cdot k_{aT}}{x_b s + \frac{T}{z_c}})$$

$$x = -x_b U / (z_c x_b S + T)$$

Collinear twist three

$$E_h \frac{d\Delta\sigma^{(a)}}{d^3 P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) \frac{\epsilon^{P_{hT} S_A n \bar{n}}}{z_c \tilde{u}} \frac{1}{x} \left[T_{a,F}(x,x) - x\frac{d}{dx} T_{a,F}(x,x) \right] \int \frac{dx_b}{x_b} f_{b/B}(x_b) H^{\mathrm{Inc}}_{ab\to c}(\tilde{s},\tilde{t},\tilde{u}) \frac{1}{x_b S + T/z_c}$$

Almost same as Kouvaris,Qiu ,Vogelsang, and Yuan PRD 2006

$$H^{\mathrm{Inc}}_{ab\to c}(\hat{s},\hat{t},\hat{u}) = H^{\mathrm{Inc\text{-}I}}_{ab\to c}(\hat{s},\hat{t},\hat{u}) + H^{\mathrm{Inc\text{-}F}}_{ab\to c}(\hat{s},\hat{t},\hat{u}),$$ CGI GPM

$$H^{\mathrm{twist\text{-}3}}_{ab\to c}(\hat{s},\hat{t},\hat{u}) = H^{\mathrm{twist\text{-}3\text{-}I}}_{ab\to c}(\hat{s},\hat{t},\hat{u}) + H^{\mathrm{twist\text{-}3\text{-}F}}_{ab\to c}(\hat{s},\hat{t},\hat{u})\left(1 + \frac{\hat{u}}{\hat{t}}\right)$$ Kouvaris et al.

- Another term from k_{aT}-dependence from $H^{\rm Inc}_{ab\to c}(\hat{s},\hat{t},\hat{u})$

Thus to the leading order (linear in k_{aT} terms).

$$E_h\frac{d\Delta\sigma}{d^3P_h} = E_h\frac{d\Delta\sigma^{(a)}}{d^3P_h} + E_h\frac{d\Delta\sigma^{(b)}}{d^3P_h},$$

small

$$E_h\frac{d\Delta\sigma^{(a)}}{d^3P_h} = \frac{\alpha_s^2}{S}\sum_{a,b,c}\int\frac{dz_c}{z_c^2}D_{h/c}(z_c)\frac{\epsilon^{P_{hT}S_A n\bar{n}}}{z_c\tilde{u}}\frac{1}{x}\left[T_{a,F}(x,x) - x\frac{d}{dx}T_{a,F}(x,x)\right]\int\frac{dx_b}{x_b}f_{b/B}(x_b)H^{\rm Inc}_{ab\to c}(\tilde{s},\tilde{t},\tilde{u})\frac{1}{x_bS+T/z_c}$$

small

$$E_h\frac{d\Delta\sigma^{(b)}}{d^3P_h} = \frac{\alpha_s^2}{S}\sum_{a,b,c}\int\frac{dz_c}{z_c^2}D_{h/c}(z_c)\frac{\epsilon^{P_{hT}S_A n\bar{n}}}{z_c\tilde{u}}\frac{1}{x}T_{a,F}(x,x)\int\frac{dx_b}{x_b}f_{b/B}(x_b)\left[-\tilde{s}\frac{\partial}{\partial\tilde{s}}H^{\rm Inc}_{ab\to c}(\tilde{s},-\tilde{s}-\tilde{u},\tilde{u})\right]\frac{1}{x_bS+T/z_c}$$

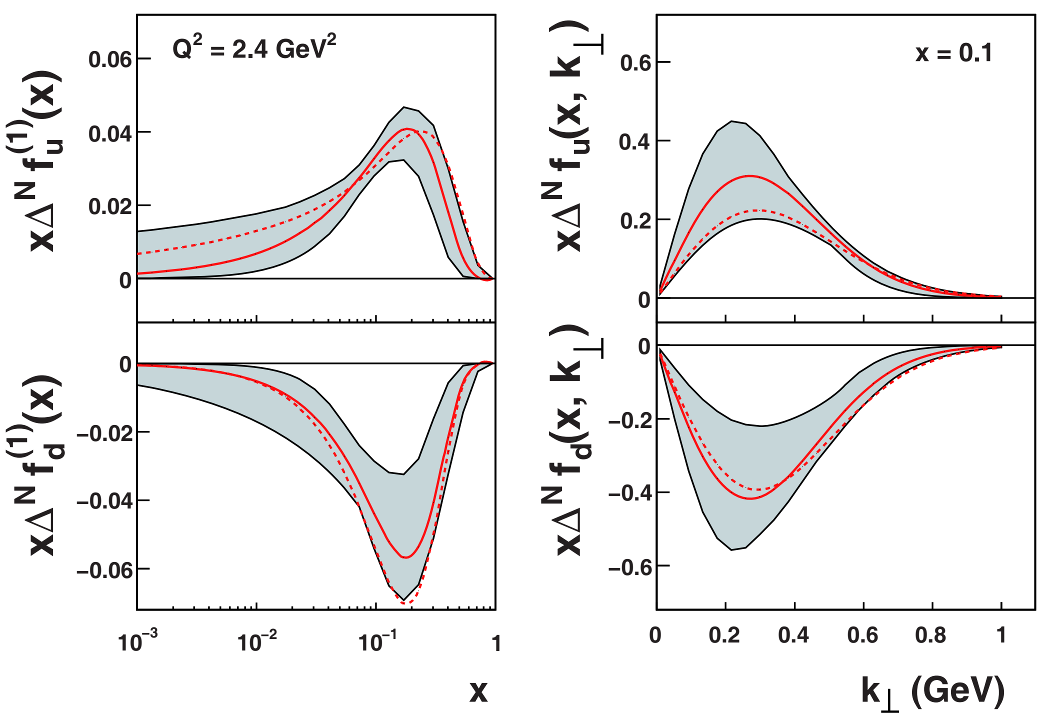
small

Conclusions

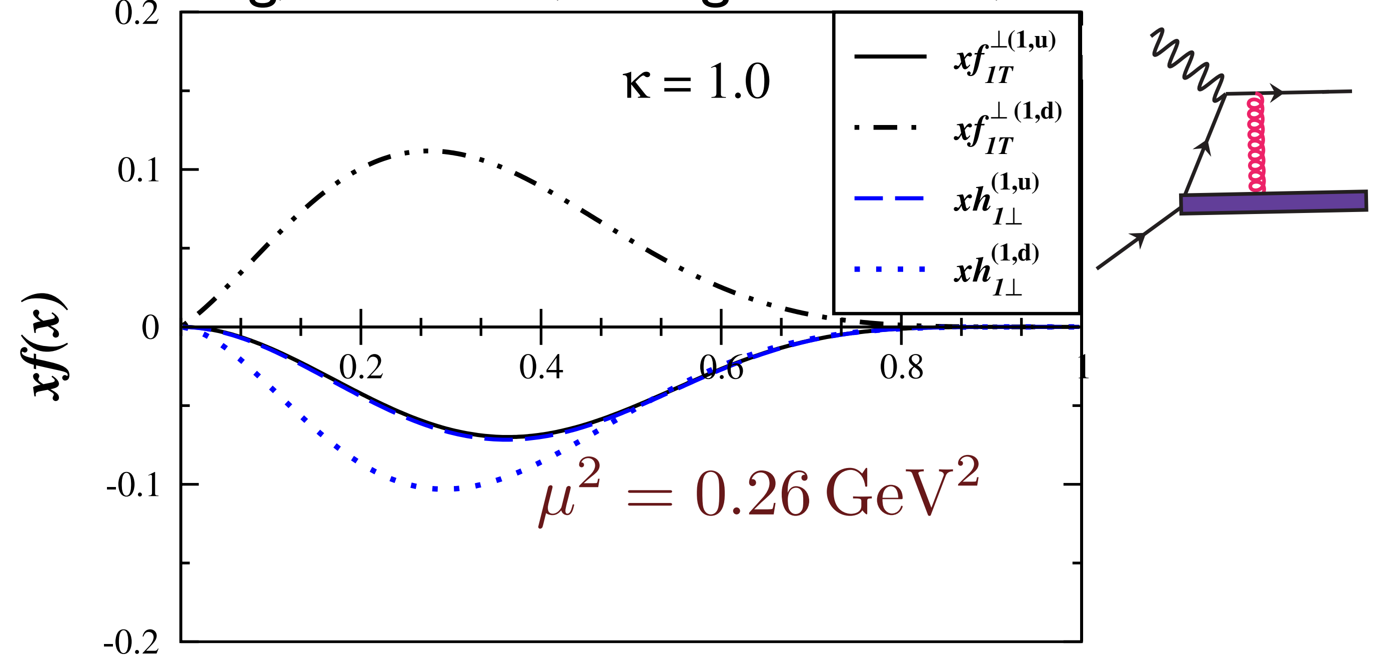
- Generalize GPM w/ color--can then perform global analysis
- Elephant in the room is break down of factorization for these processes
- Appears to be connection between generalized parton model at twist 3 and twist 3 approach
- Estimate mismatch-investigating LG Z. Kang
- TMD fact. is assumed in both GPM and GGPM is this a reasonable pheno. approximation?
- Direct photon driven by same ISI factor as in DY

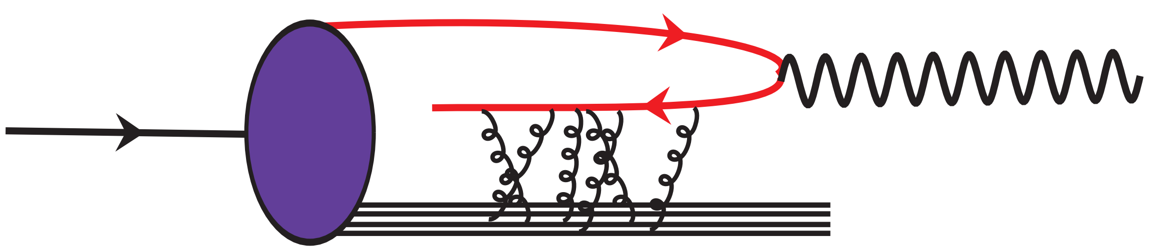
Sivers

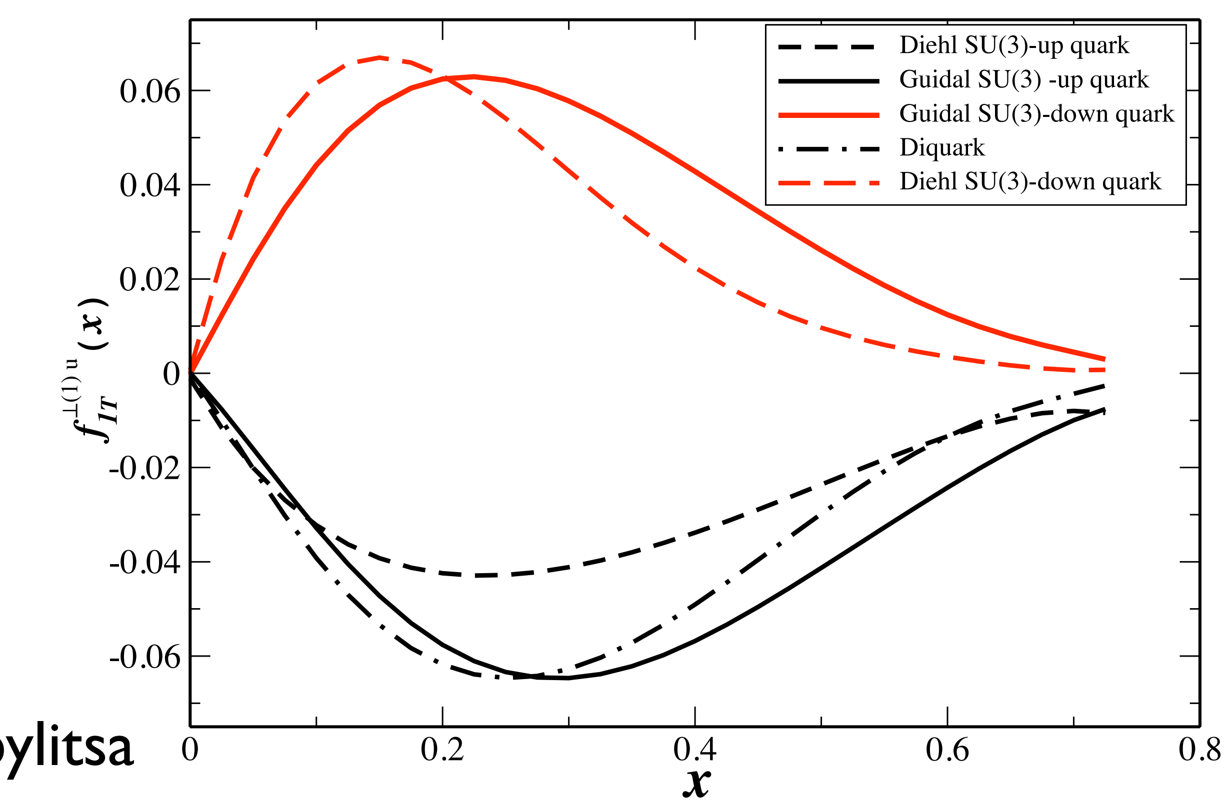
Anselmino et al. PRD 05, EPJA 08



Gamberg, Goldstein,Schlegel PRD 77, 2008







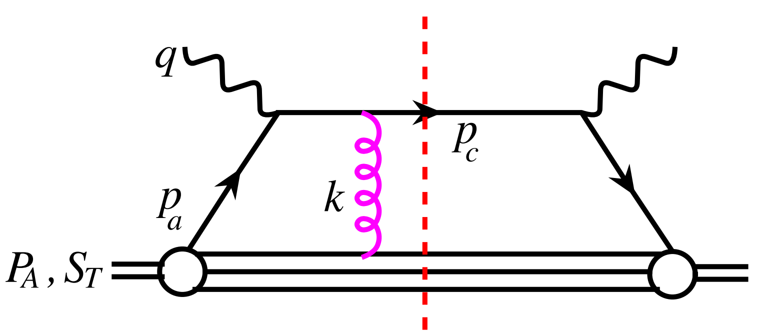
- Relations produce a Sivers effect ~0.05 N_c=3
- Torino extraction ~ 0.05
  SU(3)Chromo-lensing (Burkardt NPA 2003)
- Sivers effect increases with color
- Color tracing gives result of N_c counting of Pobylitsa

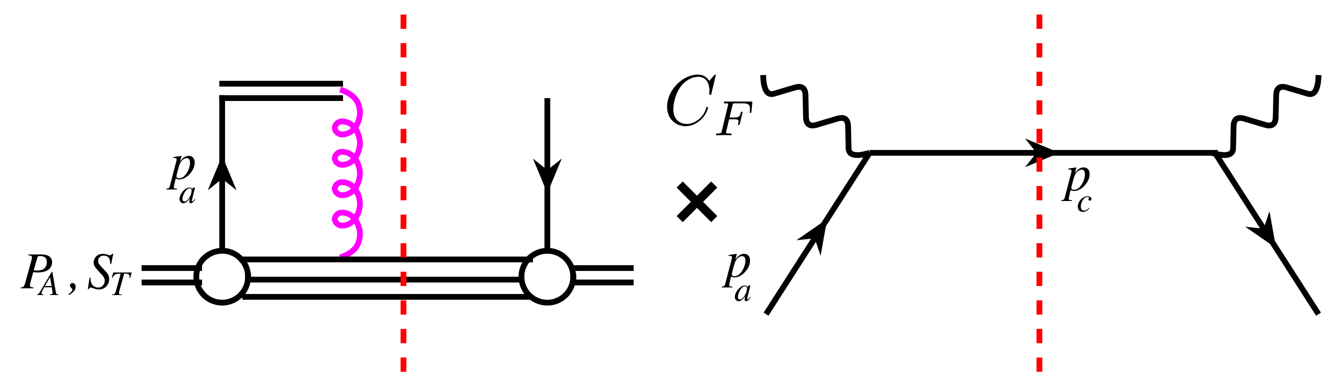
L.G. & Marc Schlegel

Phys.Lett.B685:95-103,2010 & Mod.Phys.Lett.A24:2960-2972,2009.

Classic example-same real pts opposite imaginary pts

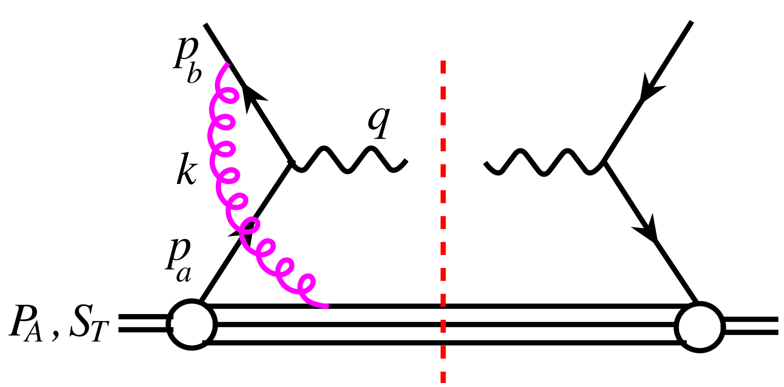
Final-state interaction in SIDIS

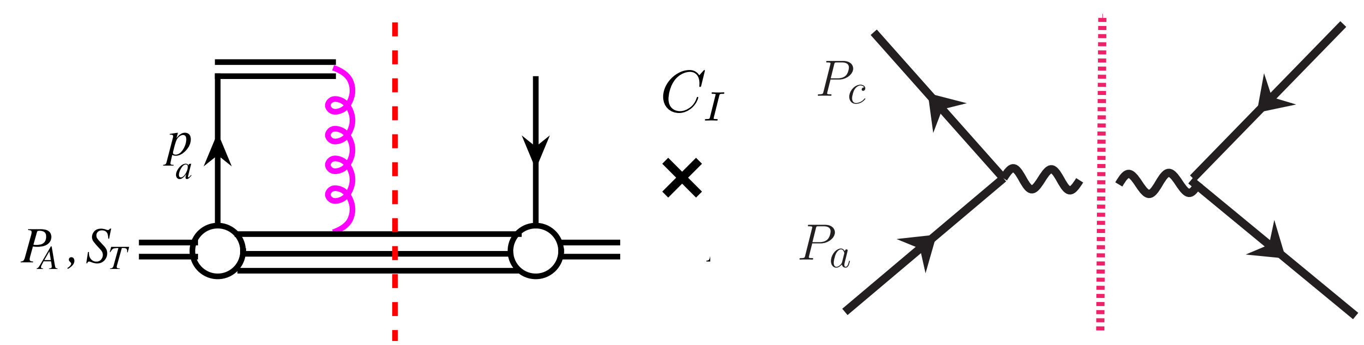




$$\bar{u}(p_c)(-ig)\gamma^- T^a \frac{i(\not{p}_c - \not{k})}{(p_c - k)^2 + i\epsilon} \approx \bar{u}(p_c)\left[\frac{g}{-k^+ + i\epsilon} T^a\right] \quad \to -\bar{u}(p_c) T^a i\pi\delta(k^+)$$

and initial-state interaction in DY

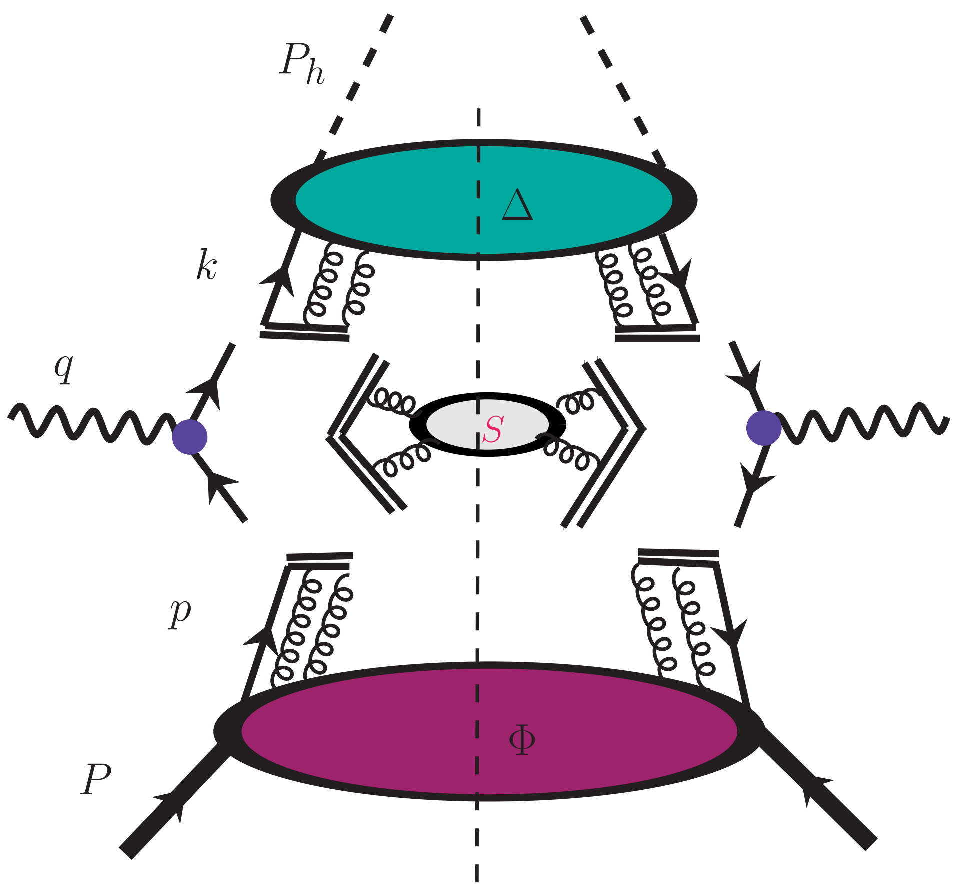




$$\bar{v}(p_b)(-ig)\gamma^- T^a \frac{-i(\not{p}_b + \not{k})}{(p_b + k)^2 + i\epsilon} \approx \bar{v}(p_b)\left[\frac{g}{-k^+ - i\epsilon} T^a\right], \quad \to v(p_b) T^a i\pi\delta(k^+)$$

Beyond "tree level" factorization

CS NPB 81,CSS NPB 1985 Collins Hautman PLB 00, Ji Ma Yuan PRD 05,
Cherednikov Karanikas Stefanis NPB 10, Collins Oxford Press 2011,
Collins Oxford Press 2011 & Abyat & Rogers arXiv: 2011



Hard

$$\mathcal{C}[H; wfSD] \equiv x_B H(Q^2, \mu^2, \rho) \sum_a e_a^2 \int d^2p_T\, d^2K_T\, d^2\ell_T\, \delta^{(2)}(zp_T + K_T + \ell_T - P_{h\perp})\, w\left(p_T, -\frac{K_T}{z}\right)$$
$$\times f^a(x, p_T^2, \mu^2, x\zeta, \rho)\, S(\ell_T^2, \mu^2, \rho)\, D^a(z, K_T^2, \mu^2, \hat{\zeta}/z, \rho)$$

TMD

Soft

FF