Measuring transverse size with virtual photons

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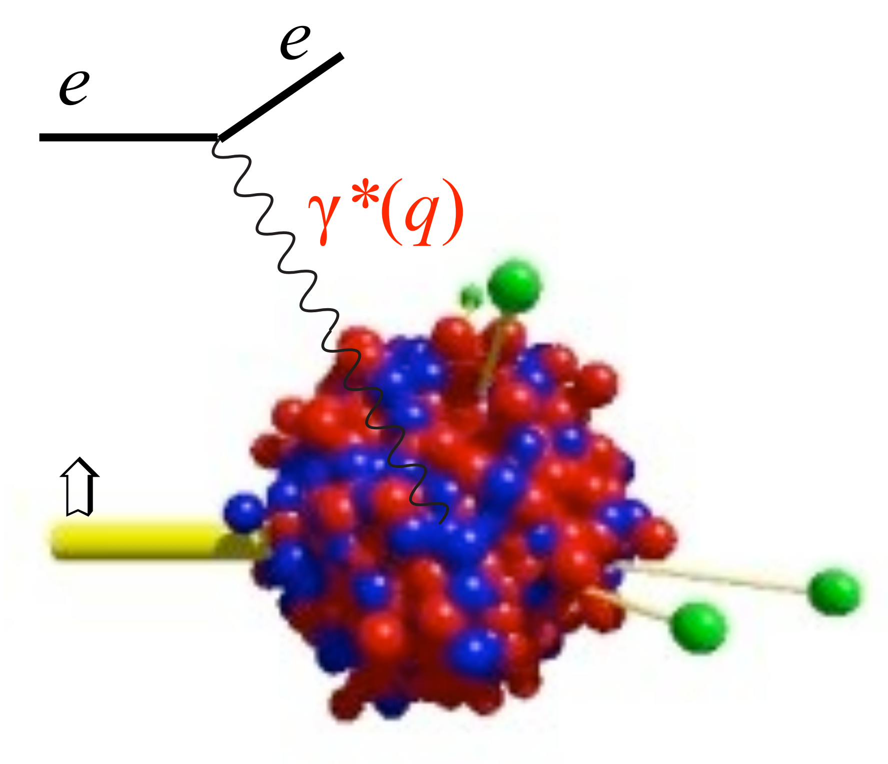


Learning about dynamics in transverse coordinate space from data on

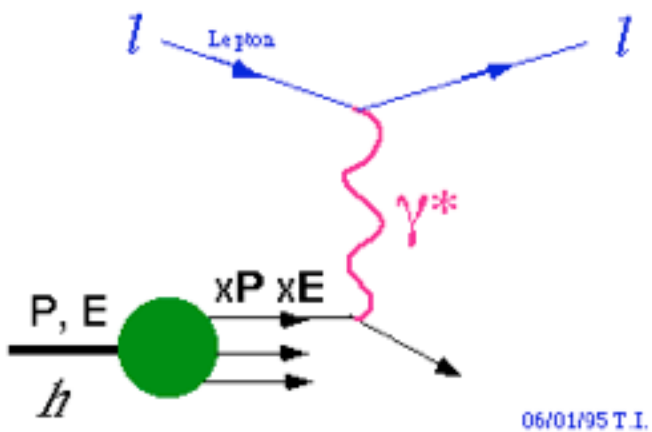
$\gamma^* N \to \pi N,\ \pi\pi N$, etc.

Work with Samu Kurki:

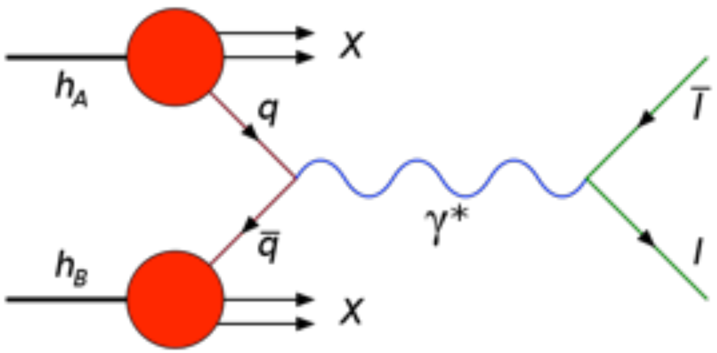
Phys.Rev. D83 (2011) 114012
[arXiv:1101.4810 [hep-ph]]

The photon is a precise probe of QCD dynamics

Deep Inelastic Scattering



Drell-Yan

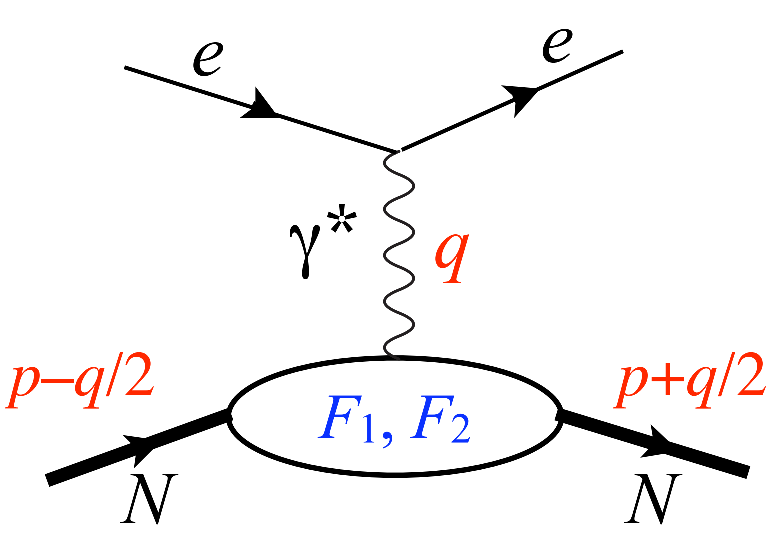


At leading twist $(Q^2 \to \infty)$ QCD factorization allows to measure parton distributions (PDF, GPD, TMD,...)

Photons are perturbative probes of strong dynamics at any Q^2.

How is the Q^2 distribution related to the size of the scattering region?

Example: Electromagnetic Form Factors



A Fourier transform of $F(Q^2)$ determines the charge density in coordinate space

$$F(Q^2) = \int d^3\boldsymbol{r}\,\rho(r)\exp(-i\boldsymbol{q}\cdot\boldsymbol{r})$$

$$\langle r^2 \rangle = -6\,\frac{dF_1}{dQ^2}\bigg|_{Q^2=0}$$

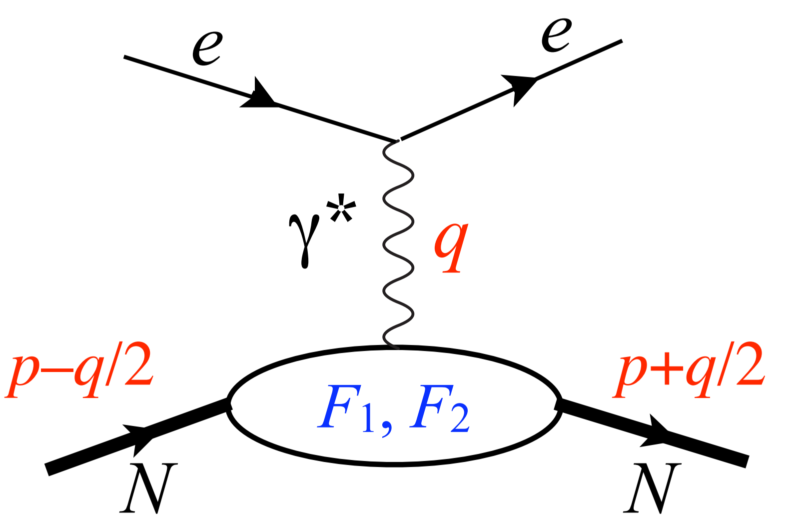
Only (too) recently was it realized that *these relations are inappropriate* when the target or its constituents are in relativistic motion.

Soper, **Burkardt**, Diehl, Ralston et al, Miller, Carlson et al...
Rocha et al, Eur. J. Phys. **A44** (2010) 411

Two reasons why relativity is relevant

I. The photon probe couples to quarks, which move nearly at the velocity of light.

We cannot get a sharp picture of objects that move during the time that the shutter is open.



II. The nucleon momentum is different in the initial and final states.

Boosting equal-time states is a dynamic process, which conserves neither particle number nor shape.

Even though these are fundamental obstacles, they can be circumvented.

Boosting to the Infinite Momentum Frame (I)

A photon moving along the $-z$ axis probes the target at fixed $x^+ = t+z$

The Light Front (LF) ≈ Infinite Momentum Frame (IMF)

Quark motion in the transverse direction vanishes in the IMF:

$$v_\perp = \frac{p_\perp}{xE_h} \to 0 \;\; as \;\; E_h \to \infty$$

This removes objection I: The speed of the quarks.

But it restricts the analysis to the transverse plane.

Boosting to the Infinite Momentum Frame (II)

A hadron state of momentum $P^+ = P^0 + P^3$ can at fixed $x^+ = x^0 + x^3$ be expanded in terms its quark and gluon Fock states as

$$|P^+, \boldsymbol{P}_\perp, \lambda\rangle_{x^+=0} = \sum_{n,\lambda_i} \prod_{i=1}^{n} \Big[\int_0^1 \frac{dx_i}{\sqrt{x_i}} \int \frac{d^2\boldsymbol{k}_i}{16\pi^3}\Big] 16\pi^3 \delta(1 - \sum_i x_i)\, \delta^{(2)}(\sum_i \boldsymbol{k}_i)$$

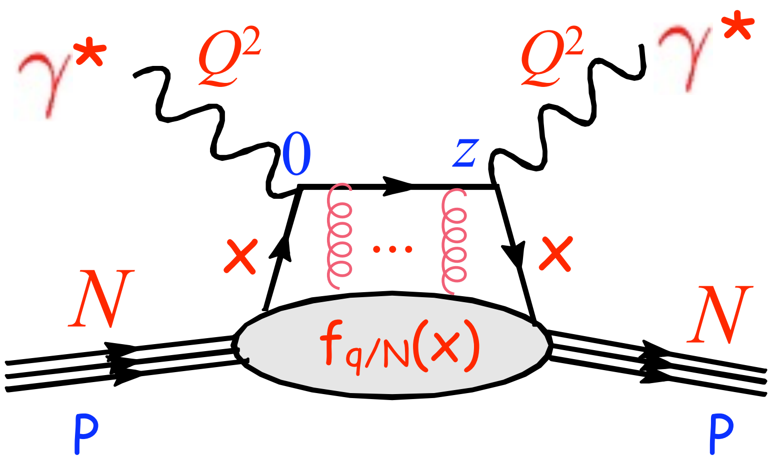
$$\times \psi_n(x_i, \boldsymbol{k}_i, \lambda_i) |n;\, x_i P^+, x_i \boldsymbol{P}_\perp + \boldsymbol{k}_i, \lambda_i\rangle_{x^+=0}$$

The LF wave functions $\psi_n(x_i, \boldsymbol{k}_i, \lambda_i)$ are independent of P^+, $P_\perp$. Hadrons can be (trivially) boosted.

This removes objection II: Boosting hadron wave functions.

PDF's in terms of LF wave functions

The probability interpretation of PDF's is expressed in terms of LF wave functions:



$$f_{q/N}(x) = \sum_{n,\lambda_i,k} \prod_{i=1}^{n} \Big[\int \frac{dx_i\, d^2\boldsymbol{k}_i}{16\pi^3} \Big] 16\pi^3 \delta(1 - \sum_i x_i)\, \delta^{(2)}(\sum_i \boldsymbol{k}_i)$$

$$\times \delta(x - x_k) |\psi_n(x_i, \boldsymbol{k}_i, \lambda_i)|^2$$

Note: 1. Parton distributions factorize at leading twist ($Q^2 \to \infty$).

2. The above expression is approximate, since rescattering of the struck parton (the Wilson line) is neglected.

Charge density in terms of LF wave functions

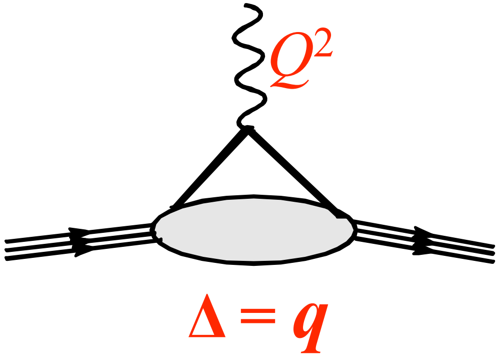
The Fourier transform of an elastic EM form factor in transverse space

$$\rho_0(\boldsymbol{b}) = \int_0^\infty \frac{dQ}{2\pi}\, Q\, J_0(b\,Q) F_1(Q^2)$$

gives the charge density in impact parameter space:

$$\rho_0(\boldsymbol{b}) = \sum_{n,\lambda_i,k} e_k \Big[\prod_{i=1}^{n} \int dx_i \int 4\pi d^2\boldsymbol{b}_i\Big] \delta(1-\sum_i x_i)\frac{1}{4\pi}\delta^{(2)}(\sum_i x_i \boldsymbol{b}_i)$$

$$\times\ \delta^{(2)}(\boldsymbol{b}-\boldsymbol{b}_k)\,|\psi_n^\lambda(x_i,\boldsymbol{b}_i,\lambda_i)|^2$$

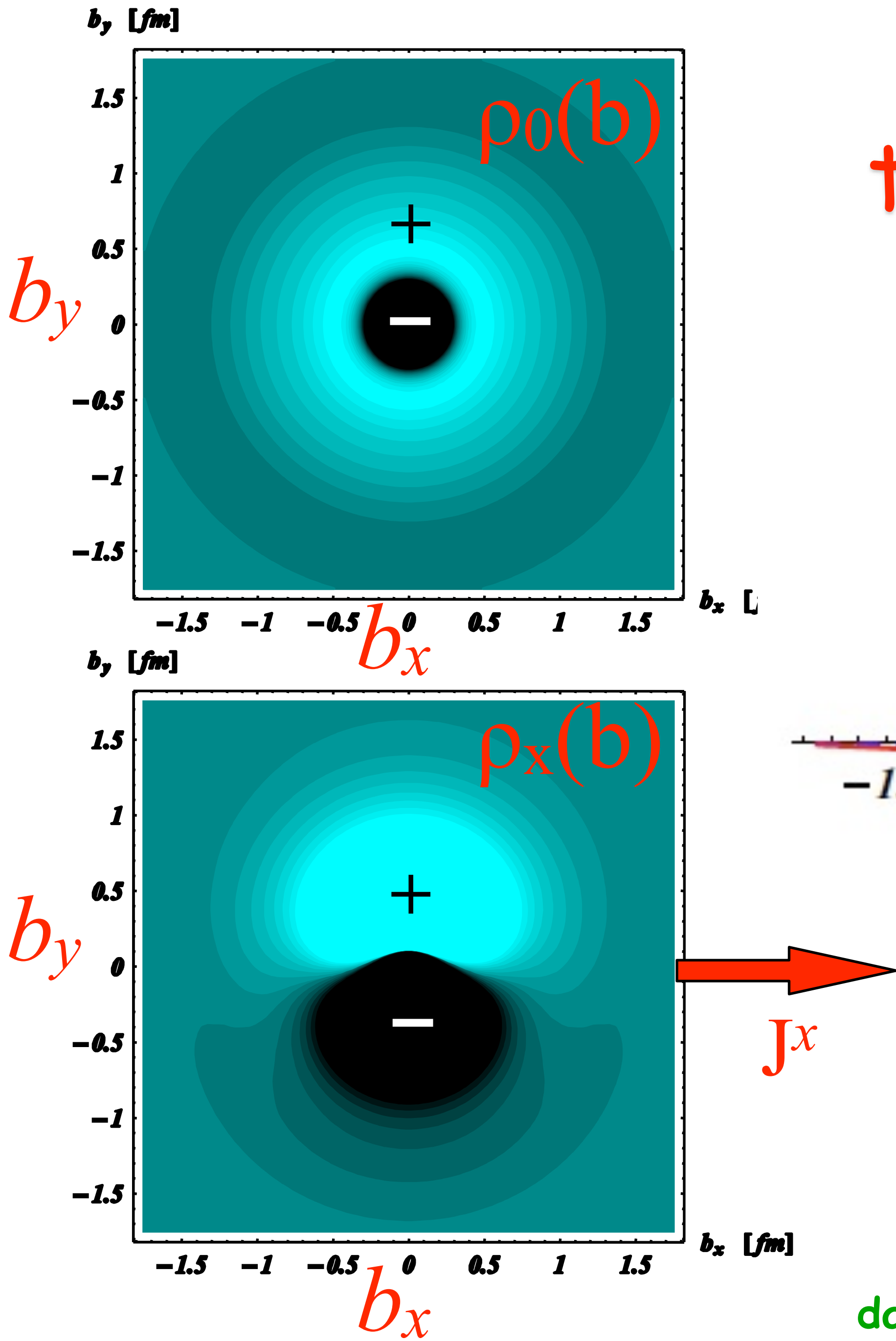


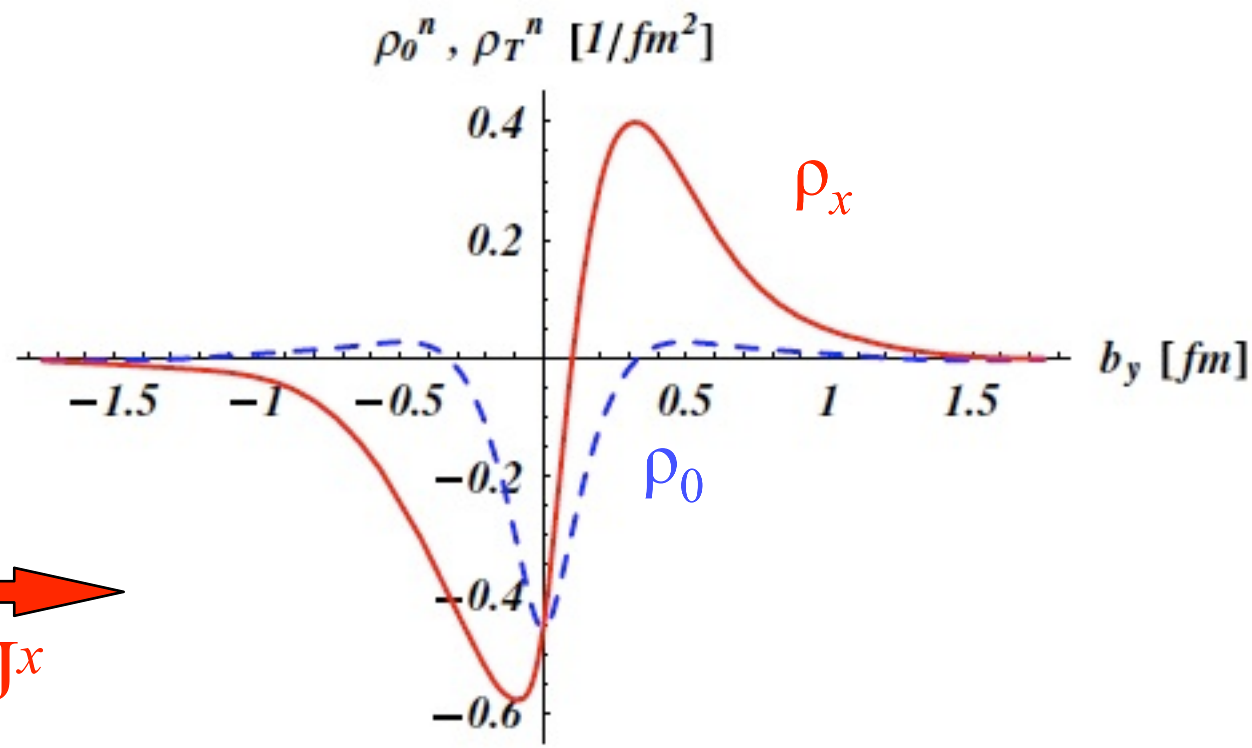
No "leading twist": Resolution $\Delta b \sim 1/Q_{max}$

No Wilson line: Fock expansion is "exact"

Important extension of the applications of virtual photons!

Using measured form factors, find the empirical quark transverse densities in neutron





Miller (2007)
Carlson and Vanderhaeghen (2008)

data : Bradford, Bodek, Budd, Arrington (2006)

Game Over?

The transverse charge densities of polarized protons and neutrons have been determined using existing form factor data.

Is there anything else to do?

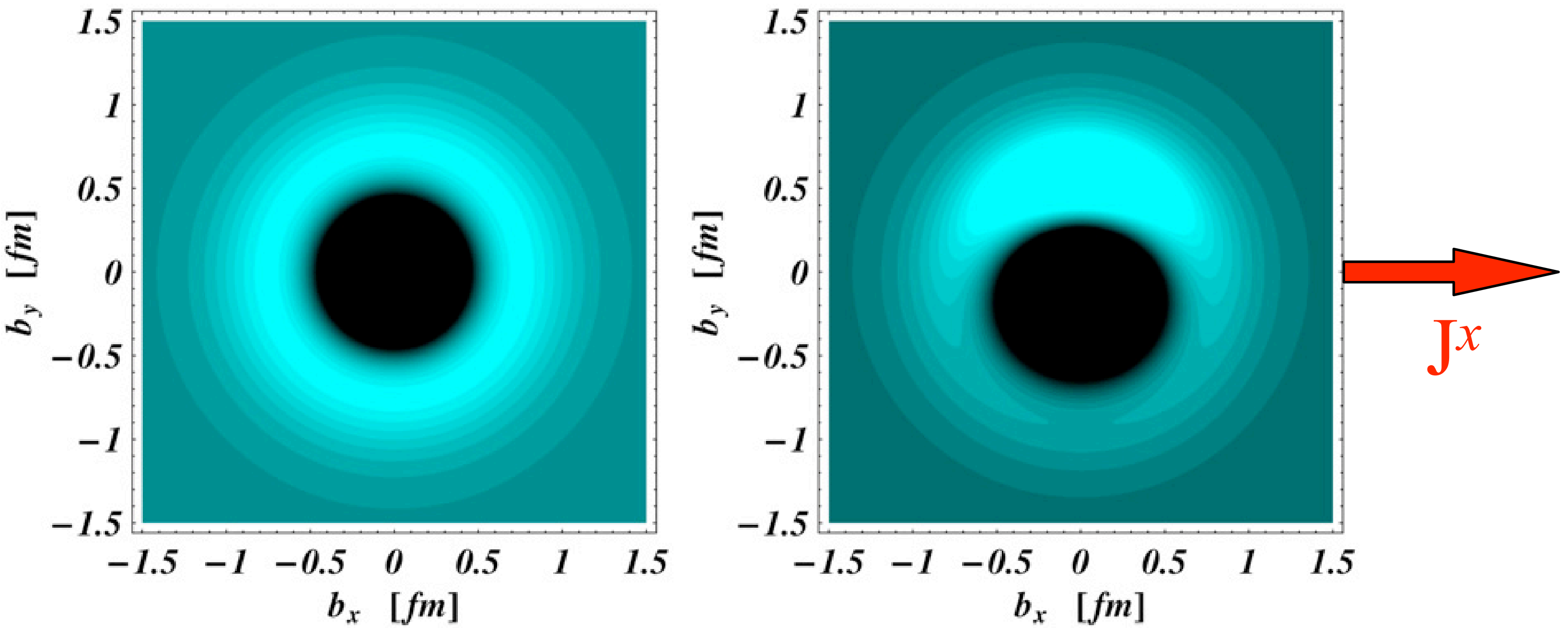
Extension to transition Form Factors: p → N*

$$\rho(b) \sim \delta^{(2)}(\boldsymbol{b}-\boldsymbol{b}_k)\,\psi^*_{N^*}(x_i,\boldsymbol{b}_i)\psi_p(x_i,\boldsymbol{b}_i)$$

"It is found that the transition from the proton to its first radially excited state is dominated by up quarks in a central region of around 0.5 fm and by down quarks in an outer band which extends up to about 1 fm."

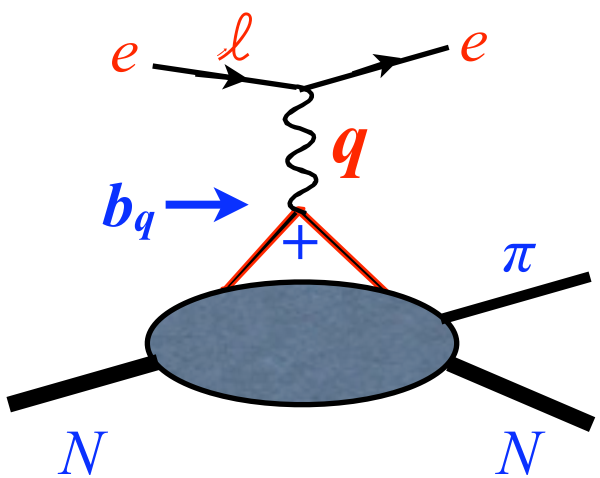
Tiator and Vanderhaeghen (2009)

$N \to P_{11}(1440)$



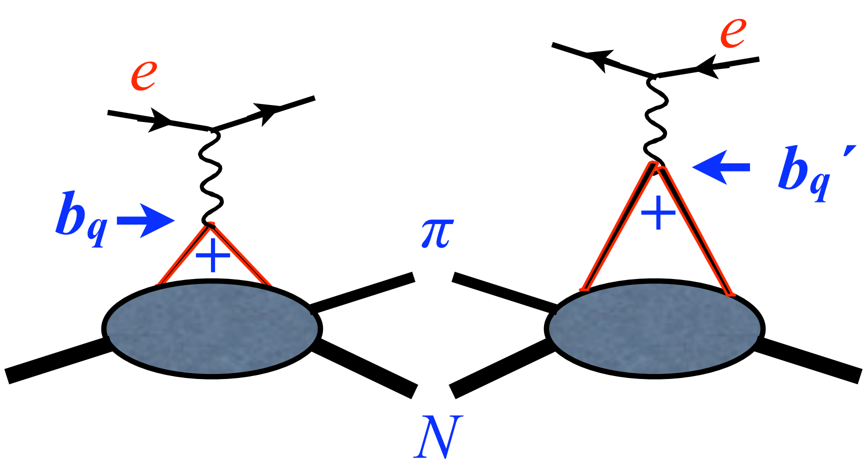
Charge density of inelastic processes: $\gamma^* N \to \pi N$,...

Any inelastic process initiated by a virtual photon can be similarly analyzed in transverse space:



Fourier transform of amplitude measures

$b_q - b_N$



Fourier transform of cross section measures $b_q - b_q'$

Interpretation requires to isolate contribution of j^+ photon current.

j^+ dominates in the high energy limit, $\ell^- \to \infty$ at fixed q

Example: f = π (p1) N(p2)

In order to conform with the Lorentz covariance of LF states, at any p_f :

$$|\pi N(p_f^+,\boldsymbol{p}_f;\Psi^f)\rangle \;\equiv\; \int_0^1 \frac{dx}{\sqrt{x(1-x)}}\int\frac{d^2\boldsymbol{k}}{16\pi^3}\,\Psi^f(x,\boldsymbol{k})|\pi(p_1)N(p_2)\rangle$$

where $\Psi^f(x,\boldsymbol{k})$ is a freely chosen function of the relative variables x, $\boldsymbol{k}$:

$$p_1^+ = xp_f^+ \qquad\qquad \boldsymbol{p}_1 = x\boldsymbol{p}_f + \boldsymbol{k}$$

$$p_2^+ = (1-x)p_f^+ \qquad\qquad \boldsymbol{p}_2 = (1-x)\boldsymbol{p}_f - \boldsymbol{k}$$

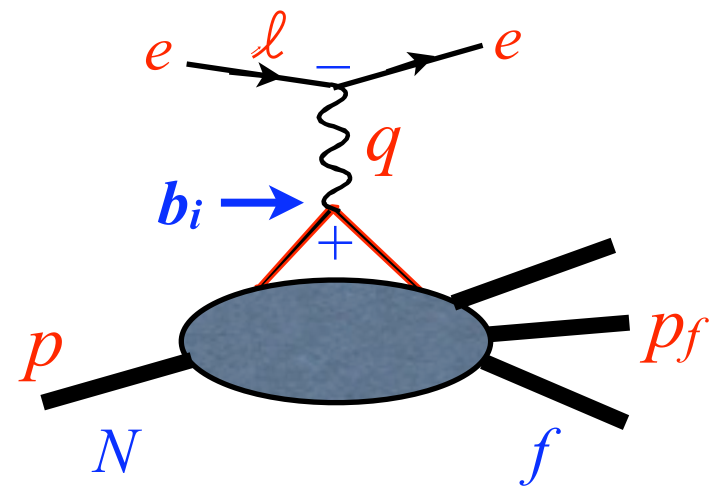
With x, $\boldsymbol{k}$ being independent of p_f, this defines the pion and nucleon momenta p_1, p_2 at all photon momenta q.

The $|\pi N(p_f^+,\boldsymbol{p}_f;\Psi^f)\rangle$ state has an LF Fock expansion of standard form.

Fourier transform of γ*N → f

In the frame:

$$\begin{aligned} p &= (p^+, p^-, -\tfrac{1}{2}\boldsymbol{q}) \\ q &= (0^+, q^-, \boldsymbol{q}) \\ p_f &= (p^+, p^- + q^-, \tfrac{1}{2}\boldsymbol{q}) \end{aligned}$$



define

$$\mathcal{A}_{fN}(\boldsymbol{b}) \equiv \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \frac{1}{2p^+} \langle f(p_f)|J^+(0)|N(p)\rangle$$

then $\mathcal{A}_{fN}(\boldsymbol{b})$ is given by an overlap of Fock amplitudes:

$$\mathcal{A}_{fN}(\boldsymbol{b}) = \frac{1}{4\pi} \sum_n \Big[\prod_{i=1}^{n} \int_0^1 dx_i \int 4\pi d^2\boldsymbol{b}_i\Big] \delta(1 - \sum_i x_i)\delta^2(\sum_i x_i \boldsymbol{b}_i)$$

$$\times \boxed{\psi_n^{f\,*}(x_i, \boldsymbol{b}_i)\psi_n^N(x_i, \boldsymbol{b}_i)} \sum_k e_k \delta^2(\boldsymbol{b}_k - \boldsymbol{b})$$

Illustration (1): $\gamma^* + \mu \to \mu + \gamma$

The QED matrix element $\mathcal{A}^{\mu\gamma}_{\lambda_1,\lambda_2} = \frac{1}{2p^+}\langle \mu(p_1,\lambda_1)\gamma(p_2,\lambda_2)|J^+(0)|\mu(p,\lambda=\tfrac{1}{2})\rangle$

expressed in terms of the relative variables *x*, ***k*** is:

$$\mathcal{A}^{\mu\gamma}_{+\frac{1}{2}+1}(\boldsymbol{q};x,\boldsymbol{k}) = 2e\sqrt{x}\left[\frac{\boldsymbol{e}_-\cdot\boldsymbol{k}}{(1-x)^2m^2+\boldsymbol{k}^2} - \frac{\boldsymbol{e}_-\cdot(\boldsymbol{k}-(1-x)\boldsymbol{q})}{(1-x)^2m^2+(\boldsymbol{k}-(1-x)\boldsymbol{q})^2}\right]$$

where $\boldsymbol{e}_\lambda\cdot\boldsymbol{k} = -\lambda e^{i\lambda\phi_k}|\boldsymbol{k}|/\sqrt{2}$. The Fourier transform gives:

$$\mathcal{A}^{\mu\gamma}_{+\frac{1}{2}+1}(\boldsymbol{b};x,\boldsymbol{k}) = 2e\sqrt{x}\left[\frac{\boldsymbol{e}_-\cdot\boldsymbol{k}}{(1-x)^2m^2+\boldsymbol{k}^2}\delta^2(\boldsymbol{b}) - \frac{i}{2\sqrt{2}\pi}\frac{m\,e^{-i\phi_b}}{1-x}K_1(mb)\exp\left(-i\frac{\boldsymbol{k}\cdot\boldsymbol{b}}{1-x}\right)\right]$$

In the first term the γ^* interacts with the initial muon, which by definition is at b = 0. The second term reflects the distribution of the final muon in transverse space.

This expression conforms exactly with the wave function overlap formula.

Fourier transform of the cross section

The $\gamma^* + N \to f$ amplitudes have dynamical phases (resonances,...).
$\Rightarrow$ Calculating their Fourier transforms requires a partial wave analysis.

However, one can FT the cross section itself.

Then the $\boldsymbol{b}$-distribution reflects the difference between the impact parameters of the photon vertex in the amplitude and its complex conjugate:

$$\int \frac{d^2\boldsymbol{q}}{(2\pi)^2} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \left| \frac{1}{2p^+} \langle f(p_f)|J^+(0)|N(p)\rangle \right|^2 = \int d^2\boldsymbol{b}_q \, \mathcal{A}_{fN}(\boldsymbol{b}_q) \, \mathcal{A}^*_{fN}(\boldsymbol{b}_q - \boldsymbol{b})$$

Illustration (3): $\sigma(\gamma^* + \mu \to \mu + \gamma)$

For the QED example considered above the Fourier transform of the cross section can be done analytically:

$$\mathcal{S}^{\mu\gamma}(\boldsymbol{b};x,\boldsymbol{k}) = 4e^2x\Big\{\frac{\boldsymbol{k}^2/2}{[(1-x)^2m^2+\boldsymbol{k}^2]^2}\delta^{(2)}(\boldsymbol{b}) \;-\; \frac{|\boldsymbol{k}|\cos(\phi_b-\phi_k)}{(1-x)^2m^2+\boldsymbol{k}^2}\,\frac{im}{2\pi}\,\frac{\exp\big(-i\frac{\boldsymbol{k}\cdot\boldsymbol{b}}{1-x}\big)}{1-x}K_1(mb)$$

$$+\;\frac{1}{4\pi}\,\frac{\exp\big(-i\frac{\boldsymbol{k}\cdot\boldsymbol{b}}{1-x}\big)}{(1-x)^2}\Big[K_0(mb)-\tfrac{1}{2}mb\,K_1(mb)\Big]\Big\}$$

The 3 terms correspond to 2, 1 and 0 of the γ^* interactions occurring on the initial muon.

The imaginary part arises from an angular correlation between $\boldsymbol{b}$ and $\boldsymbol{k}$.

Summary

We obtain information on the transverse space dynamics of any γ^*-induced process by Fourier transforming wrt. ***q***.

The analysis can be done directly on the cross section.

Requires identification of the j^+ photon current (*e.g.*, high energy limit).

There is no twist expansion.

A Fourier range $0 \le Q \le Q_{max}$ provides a resolution $\Delta b \sim 1/Q_{max}$

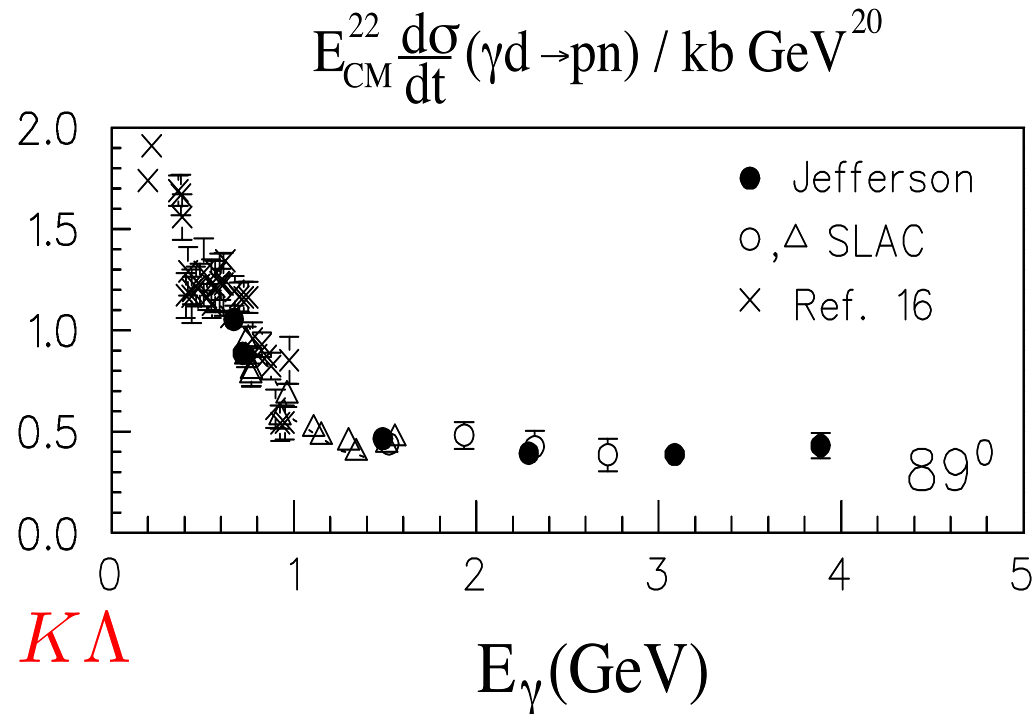
Applications to basic QED processes work as advertized.

This type of analysis awaits application to real data.

Remarks

In $\gamma^* N \to \pi N$, expect the b-distribution to narrow with the relative transverse momentum k between the π and the N.

$\sigma(\gamma D \to pn) \propto E^{-22}$ at large angles, suggesting compact states. A measurement of the q^2-dependence would allow a direct measurement of the transverse size.



In heavy quark production:

$$\gamma^* N \to K \Lambda$$
$$\gamma^* N \to D \Lambda_c$$

the b-distribution should narrow with the quark mass if the photon couples directly to the heavy quarks.

Orbital angular momentum in the target may be reflected in the final momenta.

One may study the b-distribution of diffractive events.