Extraction of TMDs with global fits



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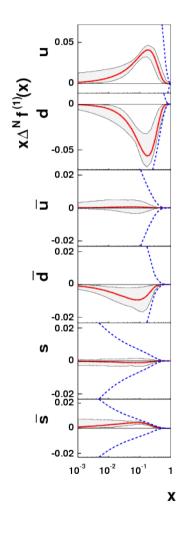
The Sivers Functions from SIDIS data

Extracted Sivers Functions

Anselmino et al. Eur. Phys. J. A39,89 (2009)

HERMES DATA 2002-2005 (proton target)
COMPASS 2003-2004 (deuteron target)

Extracted Sivers Functions



✓ Valence quarks

• $\Delta^N f_{u/p^{\uparrow}} > 0 \Longrightarrow f_{1T}^{\perp u} < 0$ • $\Delta^N f_{d/p^{\uparrow}} < 0 \Longrightarrow f_{1T}^{\perp d} > 0$

✓Sea quarks

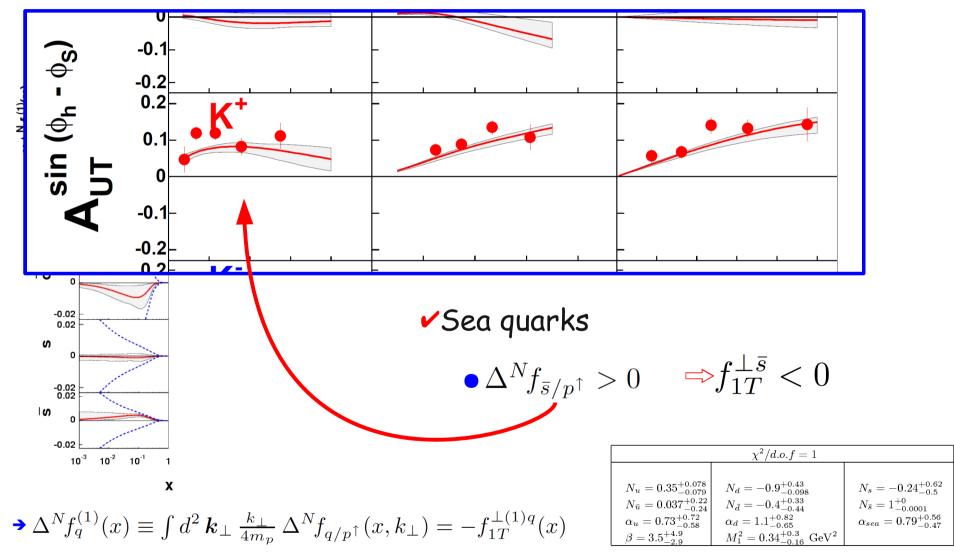
•
$$\Delta^N f_{\bar{s}/p^\uparrow} > 0 \Longrightarrow f_{1T}^{\perp \bar{s}} < 0$$

$\checkmark \Delta^N f_{q/p^{\dagger}}(x,k_{\perp}) = 2 \mathcal{N}_q(x) h(k_{\perp}) f_{q/p}(x,k_{\perp})$				
$\checkmark \mathcal{N}_q(x) = N_q x^{\alpha_q} (1-x)^{\beta_q} \frac{(\alpha_q + \beta_q)^{(\alpha_q + \beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}}$				
$\checkmark h(k_{\perp}) = \sqrt{2e} \frac{k_{\perp}}{M_1} e^{-k_{\perp}^2/M_1^2}$				
	$\chi^2/d.o.f=1$			
$N_u = 0.35^{+0.078}_{-0.079}$ $N_{\bar{u}} = 0.037^{+0.22}_{-0.24}$ $\alpha_u = 0.73^{+0.72}_{-0.58}$ $\beta = 3.5^{+4.9}_{-2.9}$	$N_d = -0.9^{+0.43}_{-0.098}$ $N_{\bar{d}} = -0.4^{+0.33}_{-0.44}$ $\alpha_d = 1.1^{+0.82}_{-0.65}$ $M_1^2 = 0.34^{+0.3}_{-0.16} \text{ GeV}^2$	$N_s = -0.24^{+0.62}_{-0.5}$ $N_{\bar{s}} = 1^{+0}_{-0.001}$ $\alpha_{sea} = 0.79^{+0.56}_{-0.47}$		

→ $\Delta^N f_q^{(1)}(x) \equiv \int d^2 \mathbf{k}_{\perp} \frac{k_{\perp}}{4m_p} \Delta^N f_{q/p^{\uparrow}}(x, k_{\perp}) = -f_{1T}^{\perp(1)q}(x)$

Extracted Sivers Functions

HERMES 2002-5

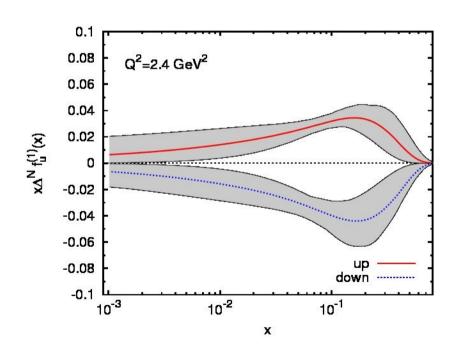


Smaller K⁺ asymmetries from HERMES
 Do we need sea guarks?

•u and d flavours only (7 parameters) •HERMES DATA(2009) $\pi^{+} \pi^{-} \pi^{0} \text{ K}^{+} \text{ K}^{-}$

•COMPASS DATA Deuteron $\pi^{+} \pi^{-} \pi^{0} K^{+} K^{-}$

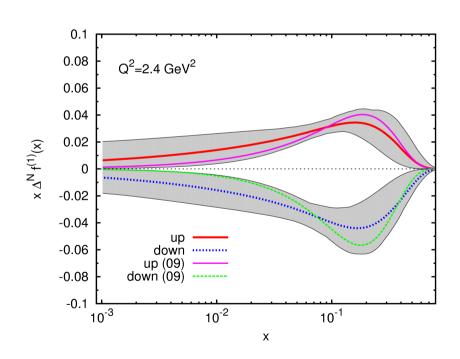
FIT u & d only



	$\chi^2_{dof} = 1.06$	
$N_u = 0.4$	$\alpha_u = 0.35$	$\beta_u = 0.26$
$N_d = -0.97$	$\alpha_d = 0.44$	$\beta_d = 0.90$
	$M_1^2 = 0.19 { m GeV}^2$	

$$\checkmark \Delta^{N} f_{q/p^{\uparrow}}(x, k_{\perp}) = 2 \mathcal{N}_{q}(x) h(k_{\perp}) f_{q/p}(x, k_{\perp})$$
$$\checkmark \mathcal{N}_{q}(x) = N_{q} x^{\alpha_{q}} (1-x)^{\beta_{q}} \frac{(\alpha_{q} + \beta_{q})^{(\alpha_{q} + \beta_{q})}}{\alpha_{q}^{\alpha_{q}} \beta_{q}^{\beta_{q}}}$$
$$\checkmark h(k_{\perp}) = \sqrt{2e} \frac{k_{\perp}}{M_{1}} e^{-k_{\perp}^{2}/M_{1}^{2}}$$

FIT u & d only



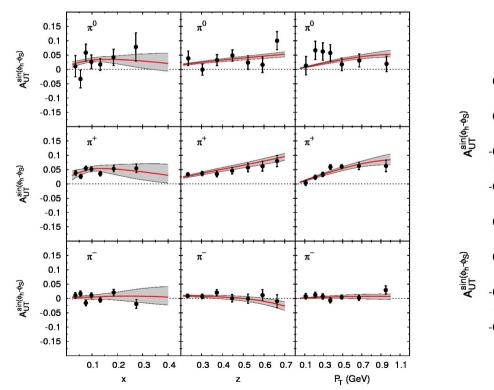
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	$M_1^2 = 0.19 \ { m GeV}^2$	

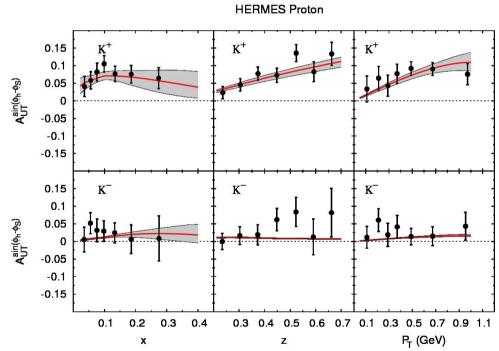
$$\checkmark \Delta^{N} f_{q/p^{\uparrow}}(x, k_{\perp}) = 2 \mathcal{N}_{q}(x) h(k_{\perp}) f_{q/p}(x, k_{\perp})$$
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$$\checkmark h(k_{\perp}) = \sqrt{2e} \frac{k_{\perp}}{M_{1}} e^{-k_{\perp}^{2}/M_{1}^{2}}$$





HERMES Proton



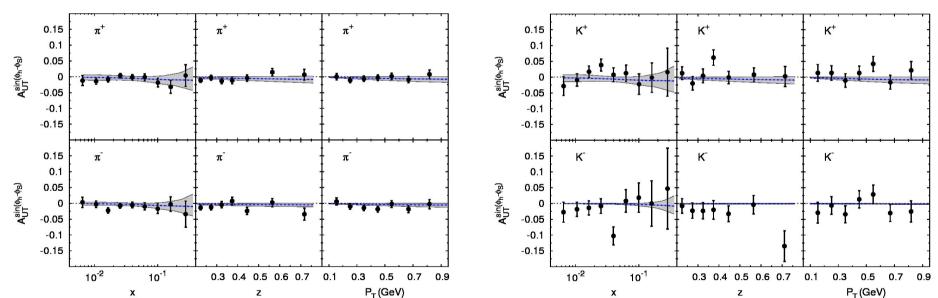


FIT u & d only

COMPASS Deuteron

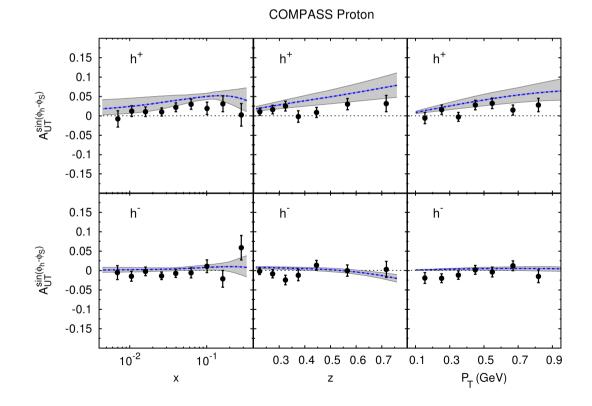


COMPASS Deuteron



FIT u & d only





Conclusions I

> A large anti-strange contribution is no more required

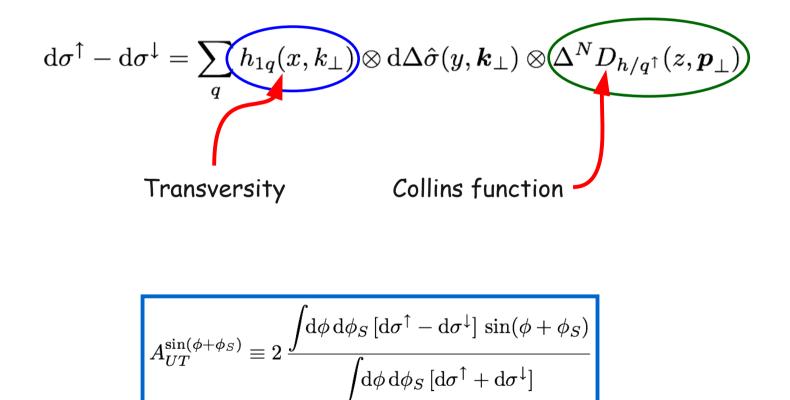
The "sizes" of the Sivers functions are well constrained in the available kinematical regions

The behavior of the Sivers functions is not constrained by data in the full x range : JLAB, EIC

Polarized SIDIS& e+e- data: Extraction of Collins function & Transversity

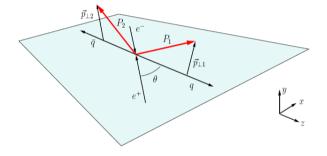
Anselmino et. al arXiv: 0812.4366v1

> Azimuthal asymmetry in polarized SIDIS

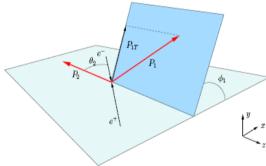


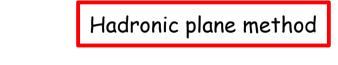
$e^+e^- \rightarrow h_1 h_2 X BELLE Data$

Thrust axis method



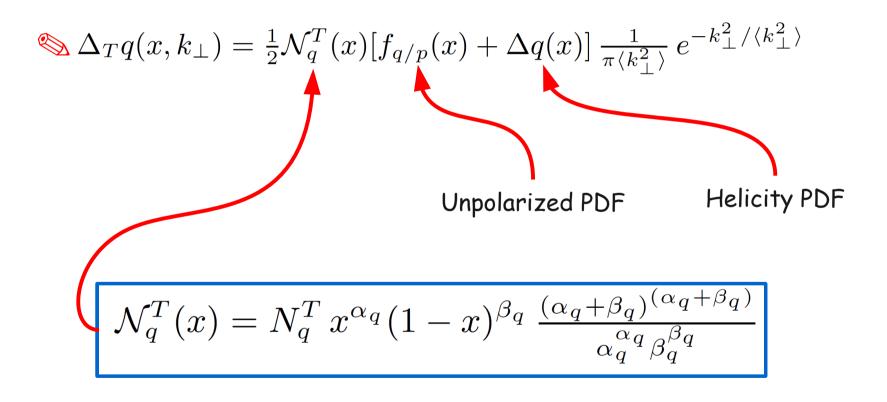
$$\begin{split} A(z_1, z_2, \theta, \varphi_1 + \varphi_2) &\equiv \frac{1}{\langle d\sigma \rangle} \frac{d\sigma^{e^+e^- \to h_1 h_2 X}}{dz_1 \, dz_2 \, d\cos\theta \, d(\varphi_1 + \varphi_2)} \\ &= 1 + \frac{1}{8} \frac{\sin^2 \theta}{1 + \cos^2 \theta} \cos(\varphi_1 + \varphi_2) \frac{\sum_q e_q^2 \Delta^N D_{h_1/q^{\uparrow}}(z_1) \, \Delta^N D_{h_2/\bar{q}^{\uparrow}}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1) \, D_{h_2/\bar{q}}(z_2)} \end{split}$$





$$A(z_1, z_2, \theta_2, \phi_1) = 1 + \frac{1}{\pi} \frac{z_1 z_2}{z_1^2 + z_2^2} \frac{\sin^2 \theta_2}{1 + \cos^2 \theta_2} \cos(2 \phi_1) \frac{\sum_q e_q^2 \Delta^N D_{h_1/q^{\dagger}}(z_1) \Delta^N D_{h_2/\bar{q}^{\dagger}}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1) D_{h_2/\bar{q}}(z_2)}$$

Parametrization of Transversity function:

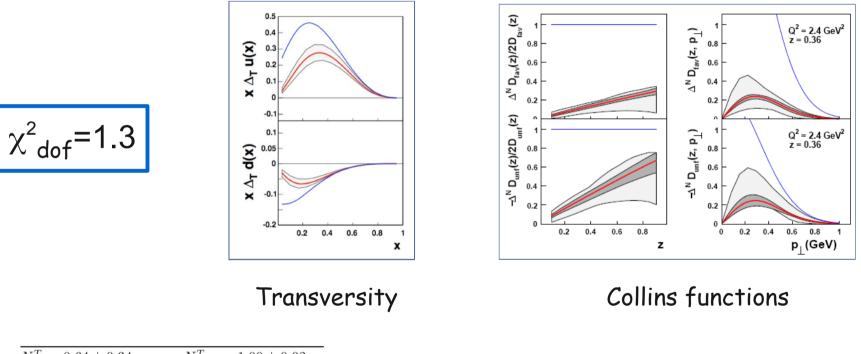


 $N_{\mathbf{q}}^{\mathsf{T}}, \boldsymbol{\alpha}, \boldsymbol{\beta}$ free parameters

Parametrization of the Collins function:

$$\begin{split} & & \Delta^{N} D_{\pi/q^{\uparrow}}(z, p_{\perp}) = 2\mathcal{N}_{q}^{C}(z) h(p_{\perp}) D_{\pi/q}(z, p_{\perp}) \\ & \bullet \mathcal{N}_{q}^{\scriptscriptstyle C}(z) = N_{q}^{\scriptscriptstyle C} z^{\gamma} (1-z)^{\delta} \frac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^{\gamma} \delta^{\delta}} \\ & \bullet h(p_{\perp}) = \sqrt{2e} \frac{p_{\perp}}{M_{h}} e^{-p_{\perp}^{2}/M_{h}^{2}} \\ & \bullet h(p_{\perp}) = \sqrt{2e} \frac{p_{\perp}}{M_{h}} e^{-p_{\perp}^{2}/M_{h}^{2}} \\ & & \mathsf{N}_{\pi/q^{\uparrow}}(z, p_{\perp}) = \frac{2p_{\perp}}{zM} H_{1}^{\perp}(z, p_{\perp}) \\ & \mathsf{V}_{\mathsf{Torino}} \text{ vs Amsterdam notation} \\ & \Delta^{N} D_{\pi/q^{\uparrow}}(z, p_{\perp}) = \frac{2p_{\perp}}{zM} H_{1}^{\perp}(z, p_{\perp}) \end{split}$$

Simultaneous fit of HERMES, COMPASS and BELLE data



$N_u^I = 0.64 \pm 0.34$	$N_d^I = -1.00 \pm 0.02$
$\alpha = 0.73 \pm 0.51$	$\beta = 0.84 \pm 2.30$
$N_{fav}^{C} = 0.44 \pm 0.07$	$N_{unf}^{C} = -1.00 \pm 0.06$
$\gamma = 0.96 \pm 0.08$	$\delta = 0.01 \pm 0.05$
$M_h^2 = 0.91 \pm 0.52 ~{\rm GeV^2}$	

•Anselmino et. al arXiv: 0812.4366v1

HERMES

2002-2005

HERMES

0.1

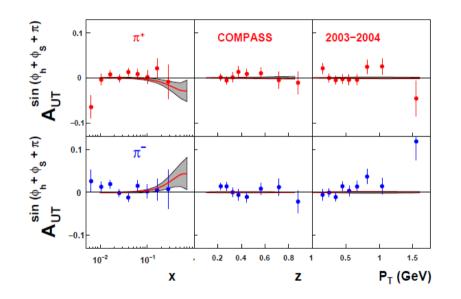
 $\begin{array}{c} {{{\rm{sin}}\left({{\varphi _h} + {\varphi _S}} \right)}_{\rm{UT}}}\,{{\rm{sin}}\left({{\varphi _h} + {\varphi _S}} \right)}\\ {{{\rm{AUT}}}} \end{array}$

 $A_{UT}^{sin (\phi_h^{} + \phi_s^{})}$

-0.0

0 0.1 0.2 0.3 0.4 0.5 0.6

Х



COMPASS

♦ M. Diefenthaler, (2007),arXiv:0706.2242

0.4

0.6

0.2

0.8

z

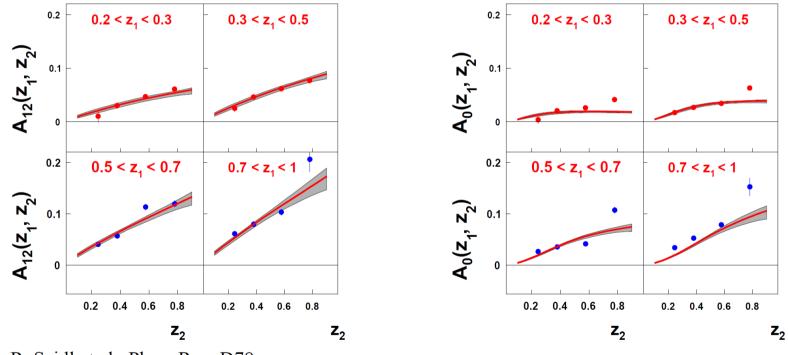
0.2 0.4 0.6 0.8

P_T (GeV)

◊ M. Alekseev et al., (2008), arXiv:0802.2160

BELLE A₁₂ (FIT)

BELLE A₀ (Predicted)



 \diamond R. Seidl et al., Phys. Rev. D78

•Anselmino et. al arXiv: 0812.4366v1

Conclusions II

>u and d transversity functions are opposite in signs

Favored and unfavored are opposite in signs

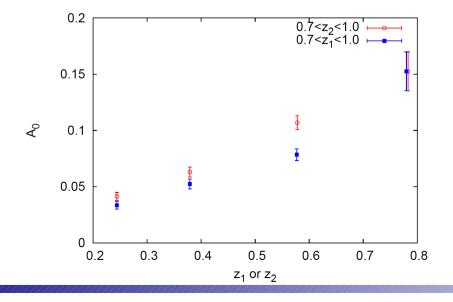
> BELLE data sets are not symmetric in $z_1 \leftrightarrow z_2$ exchange

Conclusions II

>u and d transversity functions are opposite in signs

Favored and unfavored are opposite in signs

> BELLE data sets are not symmetric in $z_1 \leftrightarrow z_2$ exchange



Unpolarized SIDIS data: Extraction of the Boer-Mulders function

Barone, Melis, Prokudin Phys. Rev. D 81,224026 (2010)

The angular distribution in the unpolarized SIDIS can be written as

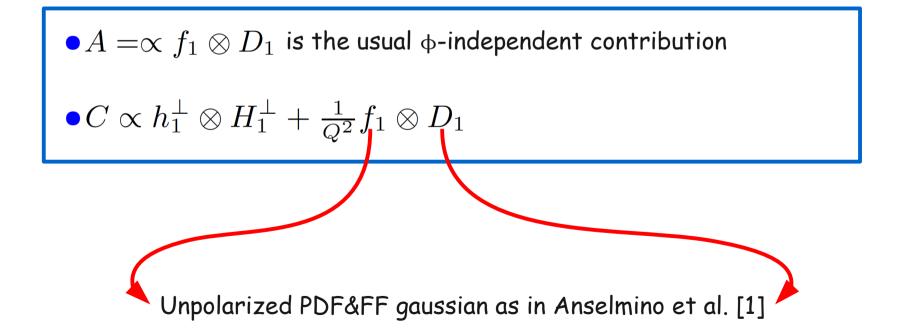
$$d\sigma = A + B\cos\phi + C\cos 2\phi$$

• $A = \propto f_1 \otimes D_1$ is the usual ϕ -independent contribution • $C \propto h_1^\perp \otimes H_1^\perp + rac{1}{Q^2} f_1 \otimes D_1$ BM effect+Twist-4 Cahn effect

$$A^{\cos 2\phi} = 2\frac{\int d\sigma \cos 2\phi}{\int d\sigma} = \frac{C}{A}$$

The angular distribution in the unpolarized SIDIS can be written as

$$d\sigma = A + B\cos\phi + C\cos 2\phi$$



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Collins function as in Anselmino et. al arXiv: 0812.4366v1

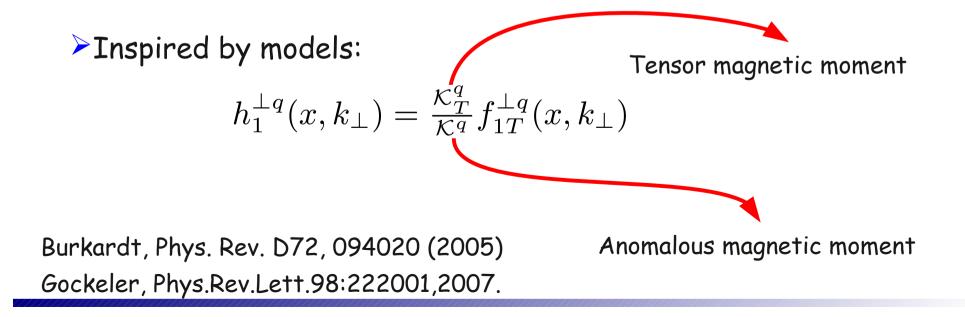
The angular distribution in the unpolarized SIDIS can be written as

$$d\sigma = A + B\cos\phi + C\cos 2\phi$$

• $A = \propto f_1 \otimes D_1$ is the usual ϕ -independent contribution • $C \propto h_1^\perp \otimes H_1^\perp + \frac{1}{Q^2} f_1 \otimes D_1$ BM that we want to extract from the fit of $A^{\cos 2\phi}$ data

Simple parametrization of the Boer-Mulders functions:

•
$$h_1^{\perp q}(x,k_{\perp}) = \lambda_q f_{1T}^{\perp q}(x,k_{\perp})$$
 for valence quarks
• $h_1^{\perp q}(x,k_{\perp}) = -|f_{1T}^{\perp q}(x,k_{\perp})|$ for sea quarks



•
$$h_1^{\perp q}(x,k_\perp) = \lambda_q \, f_{1T}^{\perp q}(x,k_\perp)$$
 for valence quarks
• $h_1^{\perp q}(x,k_\perp) = -|f_{1T}^{\perp q}(x,k_\perp)|$ for sea quarks

Models inspired:

$$h_{1}^{\perp q}(x,k_{\perp}) = \frac{\kappa_{T}^{q}}{\kappa^{q}} f_{1T}^{\perp q}(x,k_{\perp})$$

• $h_{1}^{\perp u}(x,k_{\perp}) \simeq 1.80 f_{1T}^{\perp u}(x,k_{\perp}) < 0$
• $h_{1}^{\perp d}(x,k_{\perp}) \simeq -0.94 f_{1T}^{\perp d}(x,k_{\perp}) < 0$



HERMES proton and deuteron target (x,z,P_T) charged hadrons

COMPASS deuteron target (x,z) charged hadrons

>2 free parameters:

 $\lambda_u \ \lambda_d$

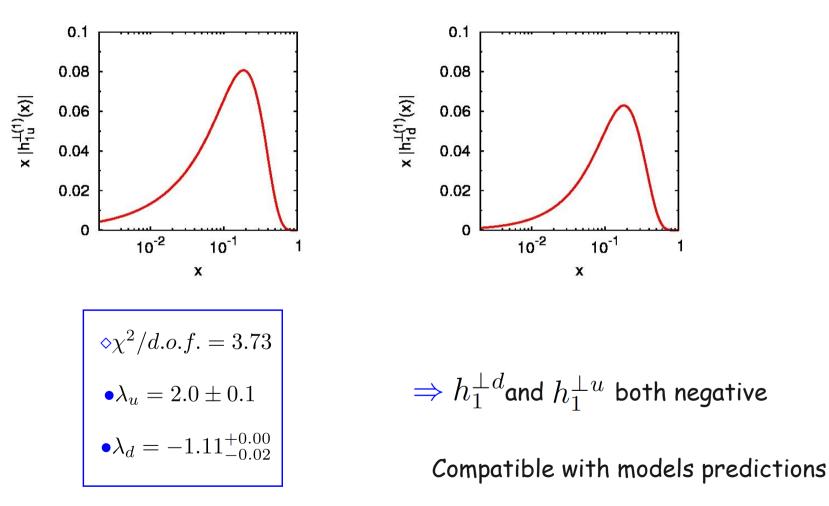
✓GRV98 PDF

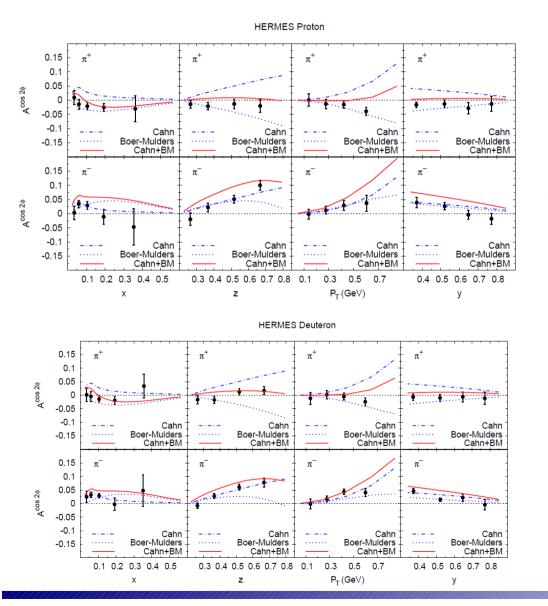
✓Gaussians: <k²→=0.25 (GeV/c)² <p²→=0.20 (GeV/c)² (from Cahn effect)

$$\checkmark h_1^{\perp q}(x,k_{\perp}) = \lambda_q f_{1T}^{\perp q}(x,k_{\perp})$$

$$\checkmark h_1^{\perp q}(x,k_\perp) = -|f_{1T}^{\perp q}(x,k_\perp)|$$

Sivers functions from Anselmino et al. Eur. Phys. J. A39,89

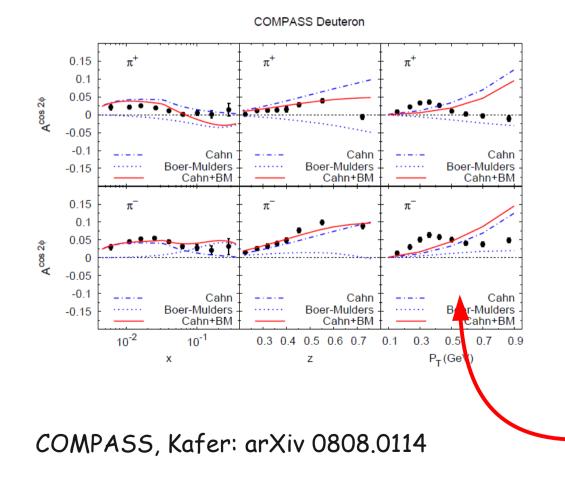






- Cahn effect (Twist-4)comparable
 to BM effect
- Same sign of Cahn contribution for positive and negative pions
- BM contribution opposite in sign for positive and negative pions

HERMES, Giordano:arXiv:0901.2438





- Cahn effect (Twist-4)comparable
 to BM effect
- Same sign of Cahn contribution for positive and negative pions
- BM contribution opposite in sign for positive and negative pions

- Data in p_T not included in the fit

> The Cahn effect is a crucial ingredient

From Ref.[*]: analysis of Cahn cos effect form EMC data

COMPASS

HERMES

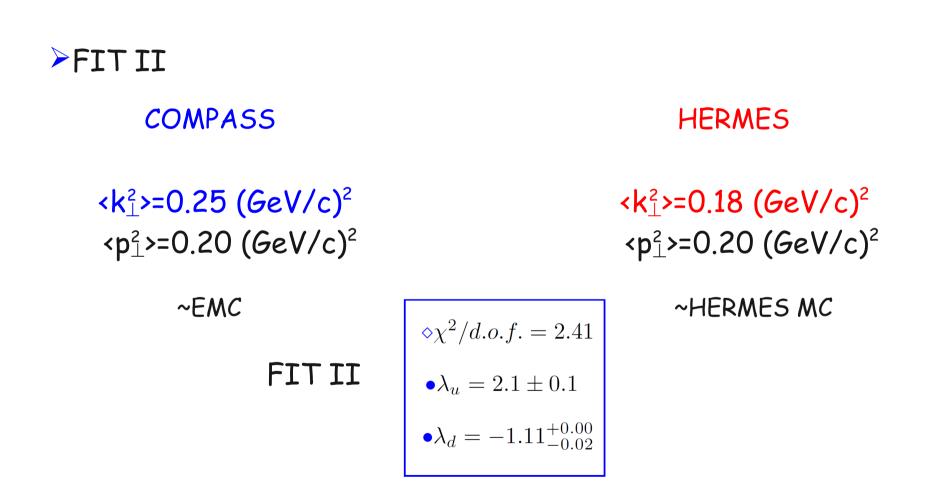
<p²/₁>=0.25 (GeV/c)² <p²/₁>=0.20 (GeV/c)²

<p²/₂>=0.18 (GeV/c)² <p²/₂>=0.20 (GeV/c)²

~HERMES MC

[*] Anselmino et al. Phys. Rev. D71 074006 (2005)

[~]EMC



Better description of HERMES but the BM is unchanged

Conclusions III

u and d BM functions have the same sign. They are compatible with models

Twist-4 Cahn effect cannot be neglected at HERMES and COMPASS.

Different average transverse momenta for different experiments?

Intrisic parton motion in unpolarized SIDIS...

Boglione, Melis, Prokudin Phys. Rev. D 84, 034033 (2011)

Why such a large Cahn effect?

The Cahn effect is suppressed by powers of Q:

$$d\sigma = A + B\cos\phi + C\cos 2\phi$$

• $A = \propto f_1 \otimes D_1$ is the usual ϕ -independent contribution

- $B\propto rac{1}{Q}\left(f_1\otimes D_1+h_1^{\perp}\otimes H_1^{\perp}
 ight)$ subleading Cahn+Boer-Mulders effect
- $ullet C \propto h_1^\perp \otimes H_1^\perp + rac{1}{Q^2} f_1 \otimes D_1 \,\,$ BM effect+Twist-4 Cahn effect

$$rac{k_\perp}{Q} \ll 1$$
 ??

Why such a large Cahn effect?

>HERMES and COMPASS: $\langle Q^2 \rangle \simeq 2~{
m GeV}^2$ $Q^2 > 1~{
m GeV}^2$

Analytical integration of the transverse momenta

$$\begin{split} f_{q/p}(x,k_{\perp}) &= f(x) \frac{e^{-k_{\perp}^2/\langle k_{\perp}^2 \rangle}}{\pi \langle k_{\perp}^2 \rangle} \\ &\int d^2 k_{\perp} \Rightarrow \int_0^{2\pi} d\varphi \int_0^{\infty} dk_{\perp} k_{\perp} \end{split} \qquad \langle k_{\perp}^2 \rangle \simeq 0.25 \; (\text{GeV}/c)^2 \end{split}$$

Bounds on the intrinic transverse momenta

The integration from 0 to infinity can be a crude assumption
 The parton model provides kinematical limits on the transverse momentum size

By requiring the energy of the parton to be smaller than the energy of its parent hadron, we have

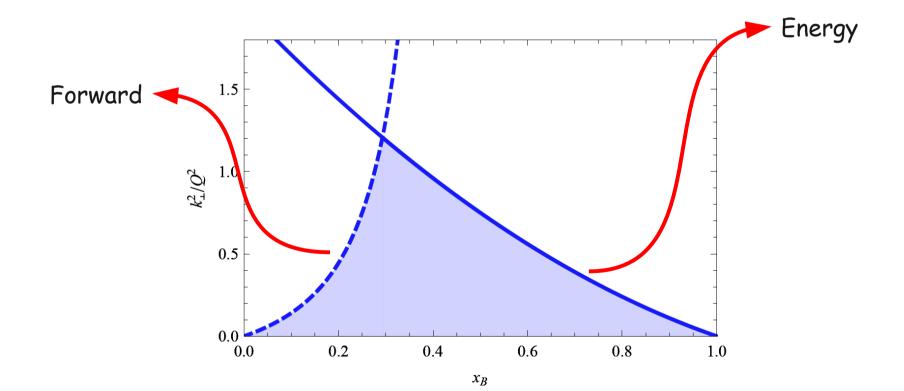
$$k_{\perp}^2 \le (2 - x_{\scriptscriptstyle B})(1 - x_{\scriptscriptstyle B})Q^2$$
 , $0 < x_{\scriptscriptstyle B} < 1$

By requiring the parton not to move backward with respect to its parent hadron, we find

$$k_{\perp}^2 \leq \frac{x_{\scriptscriptstyle B}(1-x_{\scriptscriptstyle B})}{(1-2x_{\scriptscriptstyle B})^2}Q^2 \ , \ x_{\scriptscriptstyle B} < 0.5$$

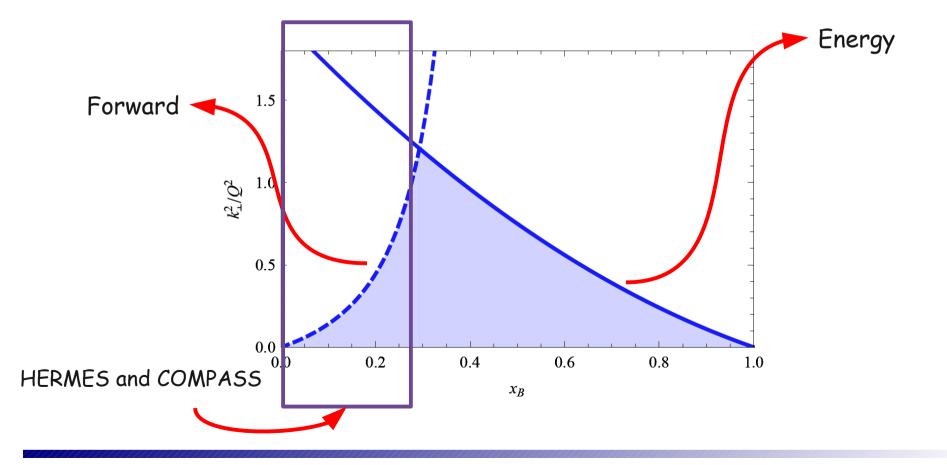
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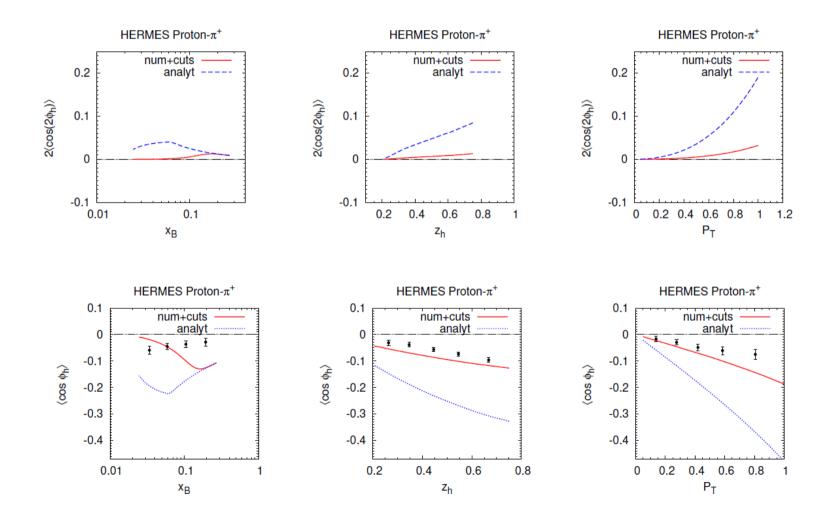


Bounds on the intrinic transverse momenta

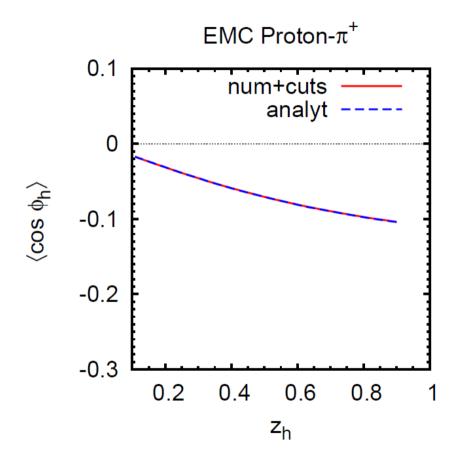
The integration from 0 to infinity can be a crude assumption
 The parton model provides kinematical limits on the transverse momentum size



Smaller Cahn effect...



No effects in "true" DIS regime...



EMC like kinematics:

 $Q^2 \ge 5 \ {\rm GeV}^2$

Conclusions IV

>In some kinematical region $k_{\perp}/Q~1$

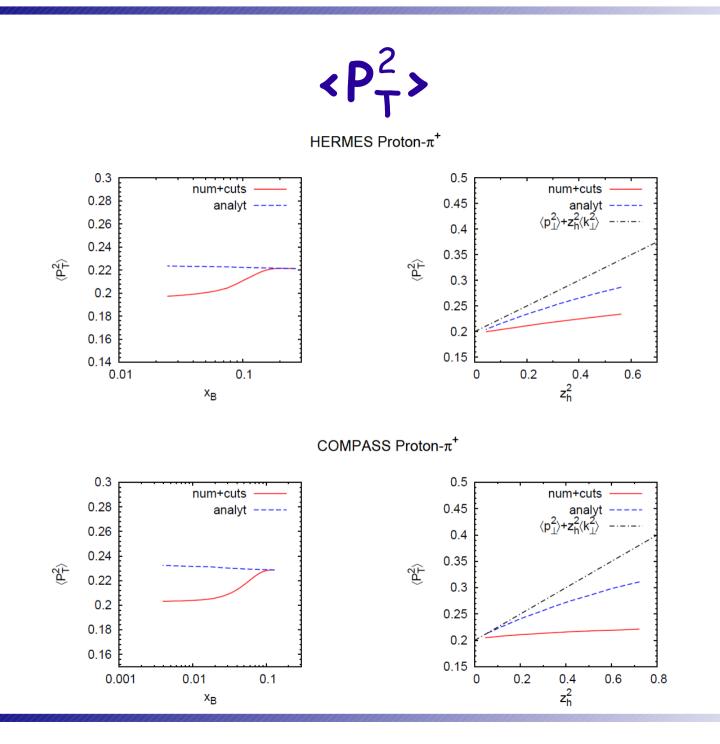
> A large Cahn effect is a consequence of that

The parton model provides constraints on the intrinsic transverse momenta

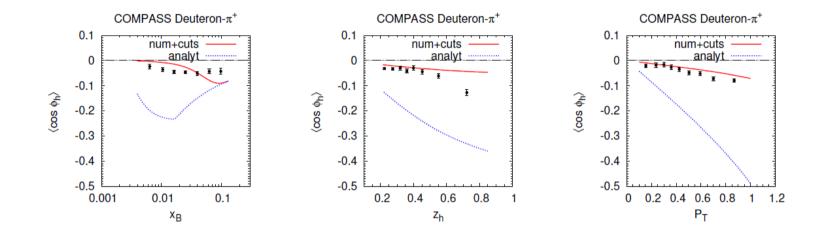
 \geq Better description of $\langle \cos \varphi \rangle$ and $\langle \cos 2\varphi \rangle$ data

> Impact in the calculation of $\langle P_T^2 \rangle$





COMPASS



Intrisic parton motion in unpolarized SIDIS...

Unpolarized PDF
$$f_{q/p}(x,k_{\perp}) = f(x) \frac{e^{-k_{\perp}^2/\langle k_{\perp}^2 \rangle}}{\pi \langle k_{\perp}^2 \rangle}$$

Unpolarized FF $D_{h/q}(z,p_{\perp}) = D_{h/q}(z) \frac{1}{\pi \langle p_{\perp}^2 \rangle} e^{-p_{\perp}^2/\langle p_{\perp}^2 \rangle}$

$$\int_0^{2\pi} d\varphi \int_0^\infty dk_\perp k_\perp f_{q/p}(x,k_\perp) = f_{q/p}(x)$$

Intrisic parton motion in unpolarized SIDIS...

 $\frac{d\sigma^{\ell+p\to\ell'hX}}{dx_B\,dy\,dz_h\,d^2\boldsymbol{P}_T} = \frac{4\pi\,\alpha^2}{x_B\,sy^2} \left\{ \frac{1+(1-y)^2}{2} F_{UU} + (2-y)\sqrt{1-y}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + (1-y)\,\cos2\phi_h\,F_{UU}^{\cos\,2\phi_h} \right\}$

$$F_{UU} = \sum_{q} e_{q}^{2} \int d^{2} \mathbf{k}_{\perp} f_{q/p}(x, k_{\perp}) D_{h/q}(z, p_{\perp})$$

$$\int d^2 \boldsymbol{k}_{\perp} \Rightarrow \int_0^{2\pi} d\varphi \int_0^{\infty} dk_{\perp} \, k_{\perp}$$

$$F_{UU} = \sum_{q} e_q^2 f_{q/p}(x_B) D_{h/q}(z_h) \frac{e^{-P_T^2/\langle P_T^2 \rangle_G}}{\pi \langle P_T^2 \rangle_G}$$

$$\langle P_T^2 \rangle_G = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle$$

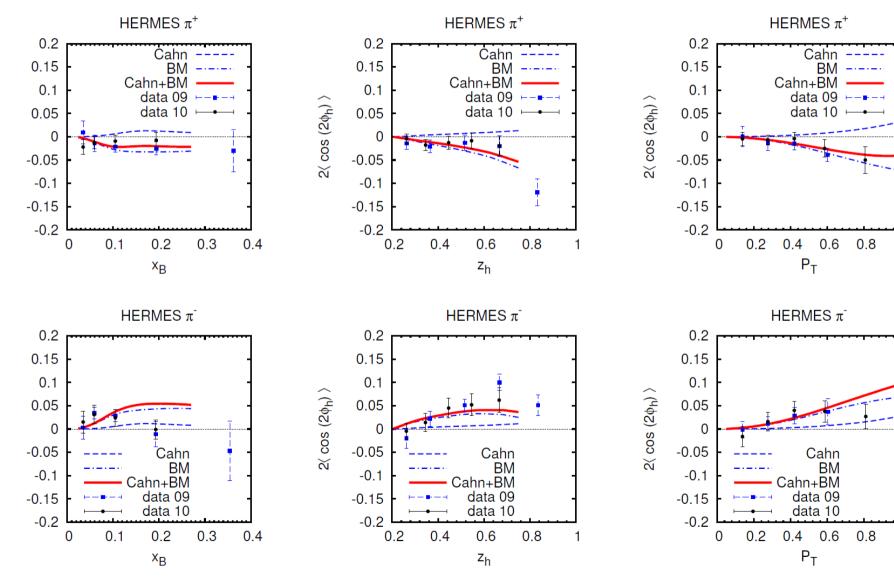
Intrisic parton motion in unpolarized SIDIS...

$$f_{q/p}(x,k_{\perp}) = f_{q/p}(x) \frac{1}{1 - e^{-(k_{\perp}^{\max})^2/\langle k_{\perp}^2 \rangle}} \frac{e^{-k_{\perp}^2/\langle k_{\perp}^2 \rangle}}{\pi \langle k_{\perp}^2 \rangle}$$

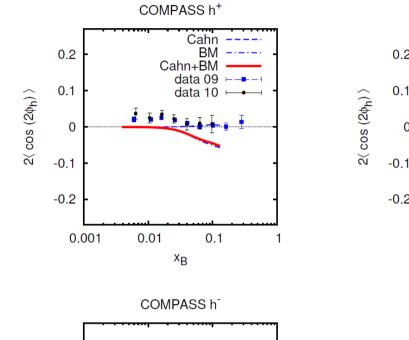
0

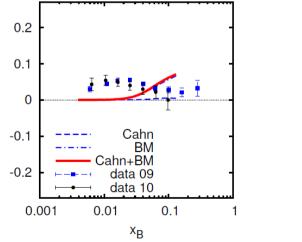
$$f_{q/p}(x) = \int_0^{2\pi} d\varphi \int_0^{k_\perp^{\max}} k_\perp dk_\perp f_{q/p}(x,k_\perp)$$

C2PHI

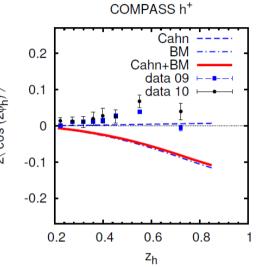


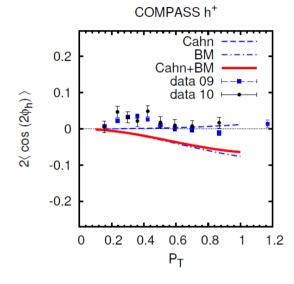
C2PHI



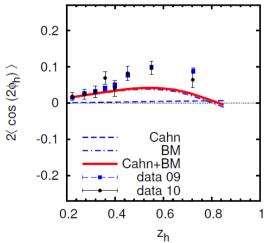


2 $\langle \cos (2\phi_h) \rangle$

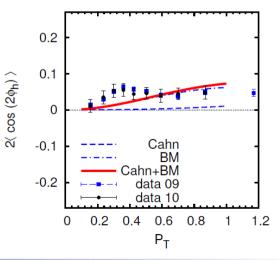






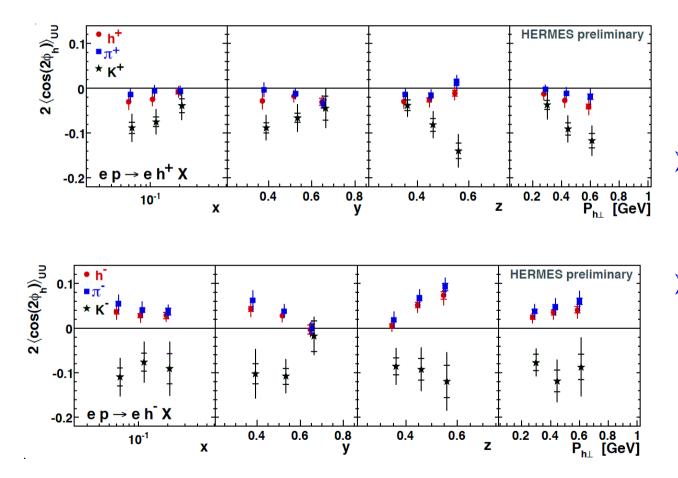


COMPASS h





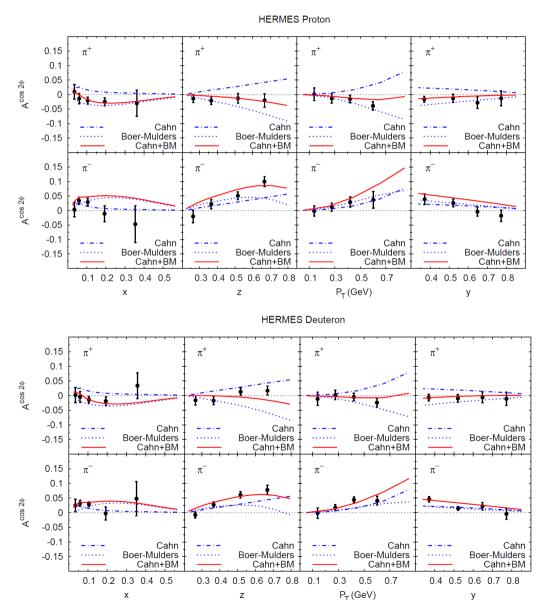
Extraction of the Boer-Mulders Function Kaons!



- Kaons production mainly driven by u quark fragmentation
- Favored & Unfavored kaon Collins functions both positive

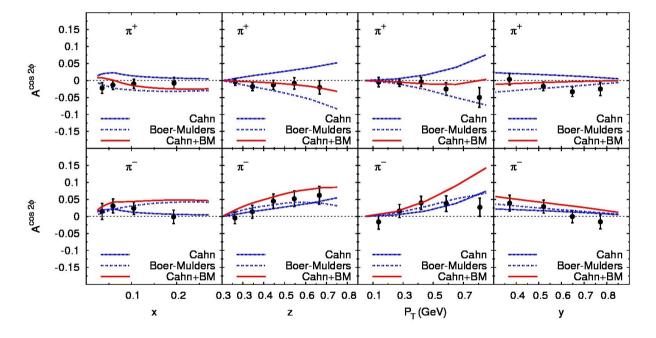


Extraction of the Boer-Mulders function





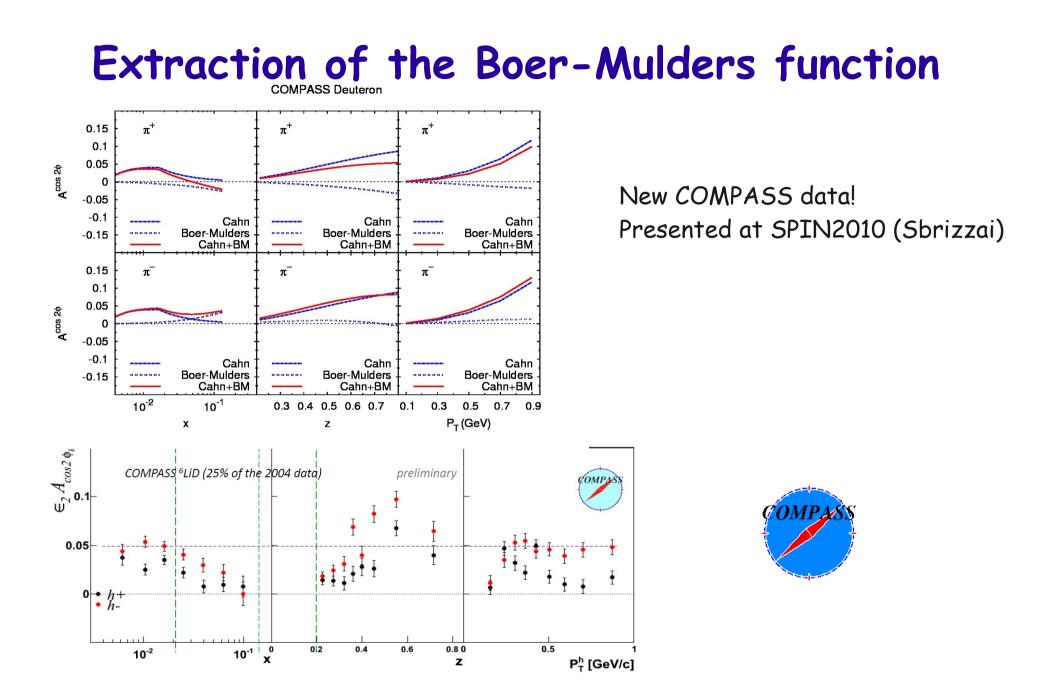
Extraction of the Boer-Mulders function



HERMES Proton



New HERMES data. Presented at SPIN2010



Extracted Sivers Functions

Simple parametrization of the Sivers function

$$\Delta^N f_{q/p^{\uparrow}}(x,k_{\perp}) = 2 \mathcal{N}_q(x) h(k_{\perp}) f_{q/p}(x,k_{\perp})$$

Unpolarized PDF $f_{q/p}(x,k_{\perp}) = f(x) \frac{e^{-k_{\perp}^2/\langle k_{\perp}^2 \rangle}}{\pi \langle k_{\perp}^2 \rangle}$

Torino vs Amsterdam notation

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$$\frac{1}{2}\Delta^N f_{q/p^{\uparrow}}(x, \mathbf{k}_{\perp}) = -\frac{\mathbf{k}_{\perp}}{m_p} f_{1T}^{\perp q}(x, \mathbf{k}_{\perp})$$

Extracted Sivers Functions

Simple parametrization of the Sivers function

$$\Delta^{N} f_{q/p^{\uparrow}}(x, k_{\perp}) = 2 \mathcal{N}_{q}(x) h(k_{\perp}) f_{q/p}(x, k_{\perp})$$
$$\mathcal{N}_{q}(x) = N_{q} x^{\alpha_{q}} (1-x)^{\beta_{q}} \frac{(\alpha_{q}+\beta_{q})^{(\alpha_{q}+\beta_{q})}}{\alpha_{q}^{\alpha_{q}} \beta_{q}^{\beta_{q}}} \leq 1$$
$$h(k_{\perp}) = \sqrt{2e} \frac{k_{\perp}}{M_{1}} e^{-k_{\perp}^{2}/M_{1}^{2}} \leq 1$$

 $N_q, \, lpha_q, \, eta_q \, \& \, M_1 \,$ free parameters