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Transverse Momentum Dependent PDFs/FFs, universality & factorization: *a brief overview*

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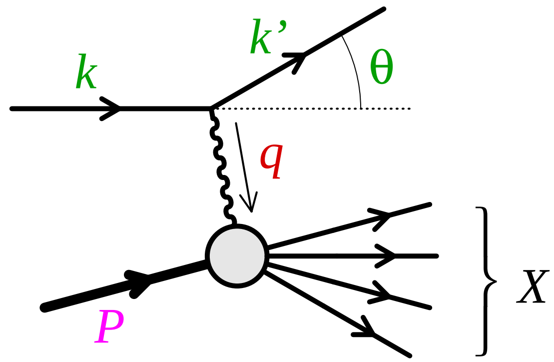
TRANSVERSE POLARIZATION PHENOMENA IN HARD SCATTERING

Outline

1. **TMDs**
   - what are TMDs?
   - partonic interpretation & properties of TMDs
   - information on TMDs
2. **Universality & factorization**
   - TMDs in QCD: gauge links, process dependence
   - divergences, soft factor, redefinition(s) of TMDs, ...
   - factorization breaking effects
   - TMDs at small x
3. **Open issues**



Four decades of DIS, $lp \to l'\,X$

$\Rightarrow$ tremendous progress in our knowledge of the nucleon structure and in testing QCD:

▶ $f_1^a(x, Q^2)$ (unpol. PDF),

and $\Delta f_a(x, Q^2)$ (helicity PDF) $\qquad (p_a = xP)$

$\Rightarrow$ one-dimensional picture: longitudinal motion/spin of partons (collinear)

Many issues/questions not addressed:

- spatial distribution of partons inside a nucleon
- parton motion in the transverse plane
- parton orbital angular momentum
- correlations between orbital motion and spin(s)

Two complementary aspects of the nucleon structure:

• description of partons in the transverse plane in *momentum* space: TMDs
• description of partons in the transverse plane in *coordinate* space: GPDs

Two faces of the same coin:

3D-imaging via the combined information from TMDs and GPDs.

TMDs: Transverse Momentum Dependent Parton Distribution Functions (this talk)
GPDs: Generalized Parton Distributions *[Burkdart, Hasch talks]*

Why are TMDs so interesting?

- *Orbital motion.*

  Most TMDs would vanish in the absence of parton orbital angular momentum.

- *Spin-orbit correlations.*

  Most TMDs are due to couplings of the transverse momentum of quarks with the nucleon (or the quark) spin.

- *QCD gauge invariance and universality.*

  The origin of some TMDs reveals fundamental properties of QCD, mainly its color gauge invariance. A stress test of QCD.

What are TMDs?

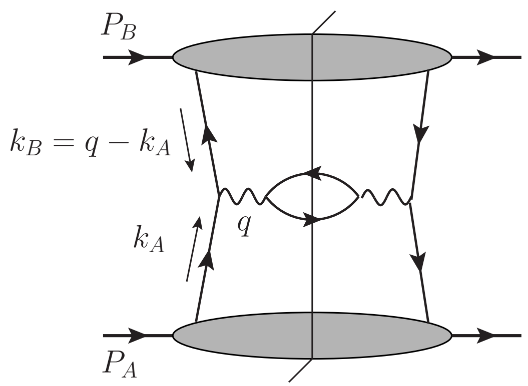
The simplest (in a parton model picture) is the unintegrated unpolarized PDF, $f_a(x, \boldsymbol{k}_\perp)$: number density of partons with a fraction x of the nucleon momentum and a transverse momentum $\boldsymbol{k}_\perp$. $\left[\int d^2\boldsymbol{k}_\perp f_a(x, \boldsymbol{k}_\perp) = f_a(x)\right]$

Motivation

Drell-Yan process: $h_A h_B \to l^+ l^- X$

$$d\sigma \simeq \sum_q f_q(x_a, \boldsymbol{k}_{\perp A}) \otimes f_{\bar q}(x_b, \boldsymbol{q}_T - \boldsymbol{k}_{\perp A})\, d\hat\sigma$$

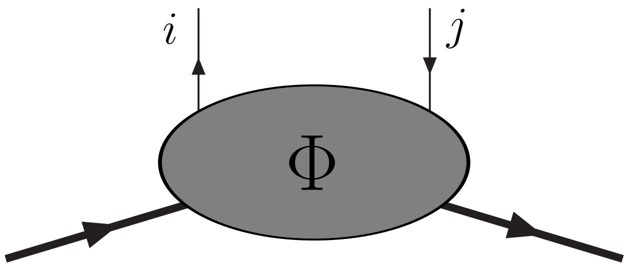
where $\boldsymbol{q}_T$ is the transverse momentum of the lepton pair.



▶ without $\boldsymbol{k}_\perp$'s $\Rightarrow \boldsymbol{q}_T = 0$

In general: TMDs crucial in less inclusive processes.

In a more formal way, TMDs defined via the unintegrated quark-quark correlator



$$\Phi^q_{ij}(x,\boldsymbol{k}_\perp) = \int \frac{dz^- d^2 z_\perp}{(2\pi)^3}\, \mathrm{e}^{ik\cdot z} \langle \boldsymbol{P},\boldsymbol{S}\,|\,\bar\psi^q_j(0)\, W(0,z)\, \psi^q_i(z)\,|\boldsymbol{P},\boldsymbol{S}\rangle\Big|_{z^+=0}\,,$$

$W(0,z)$: gauge link operator to ensure color gauge invariance (I'll be back on this).

► For a spin-1/2 particle (a nucleon) there are 8 leading-twist TMDs.

• The most general TMD, $f_1^q(x,\boldsymbol{k}_\perp;\boldsymbol{s}_q,\boldsymbol{S})$, may depend only on 8 combinations of the pseudo-vectors $\boldsymbol{s}_q, \boldsymbol{S}$ and the vectors $\boldsymbol{k}_\perp, \boldsymbol{P}$ [**parity invariance**].

By appropriate Dirac projections one gets

$$
\begin{aligned}
\langle\, |\bar{\psi}^q \gamma^+ \psi^q|\,\rangle \quad &\sim \quad f_1^q - \frac{\varepsilon^{ij}\, k_\perp^i\, S_T^j}{M}\, f_{1T}^{\perp q} \\
\lambda \langle\, |\bar{\psi}^q \gamma^+ \gamma_5\, \psi^q|\,\rangle \quad &\sim \quad \lambda S_L\, g_1^q + \lambda \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} g_{1T}^q \\
s_T^i \langle\, |\bar{\psi}^q i\sigma^{i+} \gamma_5\, \psi^q|\,\rangle \quad &\sim \quad \boldsymbol{s}_T \cdot \boldsymbol{S}_T\, h_1^q + S_L\, \frac{\boldsymbol{s}_T \cdot \boldsymbol{k}_\perp}{M}\, h_{1L}^{\perp q} \\
&+ \quad \frac{2(\boldsymbol{s}_T \cdot \boldsymbol{k}_\perp)(\boldsymbol{S}_T \cdot \boldsymbol{k}_\perp) - \boldsymbol{k}_\perp^{\,2}(\boldsymbol{s}_T \cdot \boldsymbol{S}_T)}{2M^2}\, h_{1T}^{\perp q} + \frac{\varepsilon^{ij}\, s_T^i k_\perp^j}{M}\, h_1^{\perp q}
\end{aligned}
$$

By appropriate Dirac projections one gets

quark pol.

nucleon pol.	U	L	T	
U	$\boldsymbol{f_1}$		$h_1^\perp$	
L		$\boldsymbol{g_1}$	$h_{1L}^\perp$	
T	$f_{1T}^\perp$	g_{1T}	$\boldsymbol{h_1}$	$h_{1T}^\perp$

$$\langle\, |\bar{\psi}^q \gamma^+ \psi^q|\,\rangle \;\sim\; f_1^q - \frac{\varepsilon^{ij}\, k_\perp^i\, S_T^j}{M}\, f_{1T}^{\perp q}$$

$$\lambda \langle\, |\bar{\psi}^q \gamma^+ \gamma_5\, \psi^q|\,\rangle \;\sim\; \lambda S_L\, g_1^q + \lambda \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} g_{1T}^q$$

$$s_T^i \langle\, |\bar{\psi}^q i\sigma^{i+} \gamma_5\, \psi^q|\,\rangle \;\sim\; \boldsymbol{s}_T \cdot \boldsymbol{S}_T\, h_1^q + S_L\, \frac{\boldsymbol{s}_T \cdot \boldsymbol{k}_\perp}{M}\, h_{1L}^{\perp q}$$

$$+\; \frac{2(\boldsymbol{s}_T \cdot \boldsymbol{k}_\perp)(\boldsymbol{S}_T \cdot \boldsymbol{k}_\perp) - \boldsymbol{k}_\perp^{\,2}(\boldsymbol{s}_T \cdot \boldsymbol{S}_T)}{2M^2}\, h_{1T}^{\perp q} + \frac{\varepsilon^{ij}\, s_T^i k_\perp^j}{M}\, h_1^{\perp q}$$

► Analogous pattern for gluon TMDs (transverse polariz. → linear polariz.)

Partonic interpretation and properties

- $f_1^a(x, k_\perp)$: unpolarized, $k_\perp$-dependent parton distribution
- $g_1^a(x, k_\perp)$: $k_\perp$-dependent helicity distribution
(longitudinal polarized partons in longitudinal polarized nucleon)
- $h_1^q(x, k_\perp)$: analogue of the helicity distribution, for transverse nucleon spin, *i.e.* the transversity distribution, chiral-odd, decouples from DIS, hard to measure.

► These TMD distributions survive in the collinear limit, $\boldsymbol{k}_\perp = 0$.

- $f_{1T}^{\perp a}$: Sivers function [1990], unpolarized partons inside a polarized proton.

$$f_1^a(x, \boldsymbol{k}_\perp; \boldsymbol{0}, \boldsymbol{S}) = f_1^a(x, k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp a}(x, k_\perp)\, \boldsymbol{S} \cdot (\hat{\boldsymbol{P}} \times \hat{\boldsymbol{k}}_\perp)\,.$$

Chiral-even, T-odd, window on parton orbital motion; its origin and expected process dependence related to fundamental QCD effects.

Also $\Delta^N f_{a/p^\uparrow}(x, k_\perp) = -(2k_\perp/M)\, f_{1T}^{\perp a}(x, k_\perp)$

- $h_1^{\perp q}$: Boer-Mulders funct. [1998], polarized quarks inside an unpol. proton.

$$f_1^q(x, \boldsymbol{k}_\perp; \boldsymbol{s}_q, \boldsymbol{0}) = \frac{1}{2}\left[f_1^q(x, k_\perp) - \frac{k_\perp}{M} h_1^{\perp q}(x, k_\perp)\, \boldsymbol{s}_q \cdot (\hat{\boldsymbol{P}} \times \hat{\boldsymbol{k}}_\perp)\right].$$

Chiral-odd, T-odd.

- $h_{1T}^{\perp q}$: transverse polarization of quarks orthogonal to the nucleon transverse polarization (quadrupole shape, some models $\rightarrow h_{1T}^{\perp} = g_1 - h_1$)

- g_{1T}^a, $h_{1L}^{\perp q}$: *worm-gear* functions, linked to OAM, real parts of interference amplitudes whose imaginary parts are $f_{1T}^{\perp}, h_1^{\perp}$

Quantitative relations between TMDs and OAM *[Bacchetta, Burkardt talks]*.

TMD Fragmentation functions

► For a spin-1/2 hadron (e.g., Λ): same pattern as for TMD PDFs (8 TMD FFs)

► For a spinless (or unpolarized) hadron (e.g., π) [2 TMD FFs]:

- D_1^a: unpolarized, $p_\perp$-dependent, parton fragmentation function into a hadron.
- $H_1^{\perp q}$: Collins function [1993], a polarized quark fragmenting into an unpol. hadron

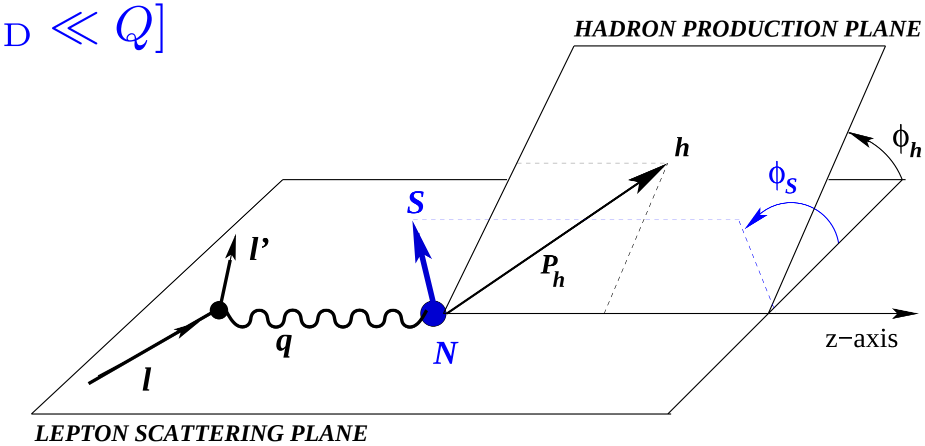
$$D_1^q(z, \boldsymbol{p}_{\perp h}; \boldsymbol{s}_q) = D_1^q + \frac{p_{\perp h}}{z M_h} H_1^{\perp q}(z, p_{\perp h}) \, \boldsymbol{s}_q \cdot (\hat{\boldsymbol{p}}_q \times \hat{\boldsymbol{p}}_{\perp h}) \; .$$

Chiral-odd, T-odd, universal; also $\Delta^N D_{h/q^\uparrow} = 2p_{\perp h}/(zM_h)\, H_1^{\perp q}$
ideal partner of h_1 ($h_1^\perp$): first SIDIS extraction [Anselmino et al. 2007].

Information on TMDs

SIDIS: $l(\boldsymbol{s})\,N(\boldsymbol{S}) \to l' h X \quad [P_T \simeq \Lambda_{\rm QCD} \ll Q]$

Leading-twist, tree level



$\sigma_{UU} \to f_1 \otimes D_1 \qquad \underline{h_1^\perp \otimes H_1^\perp \cos 2\phi_h}$

$\sigma_{LL} \to g_1 \otimes D_1$

$\sigma_{UL} \to h_{1L}^\perp \otimes H_1^\perp \sin 2\phi_h$

$\sigma_{LT} \to g_{1T} \otimes D_1 \cos(\phi_h - \phi_S)$

$\sigma_{UT} \to \underline{f_{1T}^\perp \otimes D_1 \sin(\phi_h - \phi_S)} \quad \underline{h_1 \otimes H_1^\perp \sin(\phi_h + \phi_S)} \quad h_{1T}^\perp \otimes H_1^\perp \sin(3\phi_h - \phi_S)$

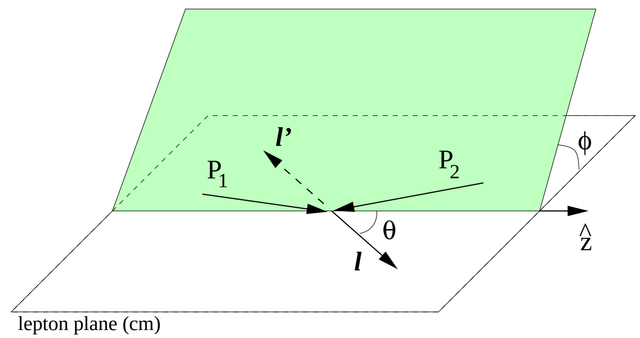
► complete access to TMDs: HERMES, COMPASS, JLAB, EIC

[talks: Sozzi, Aghasyan, Litvinenko; σ_{UU}: Barone, Sbrizzai, Schnell;
σ_{UT}: Melis, Bradamante, Rostomyan]

Complementary and crucial information from

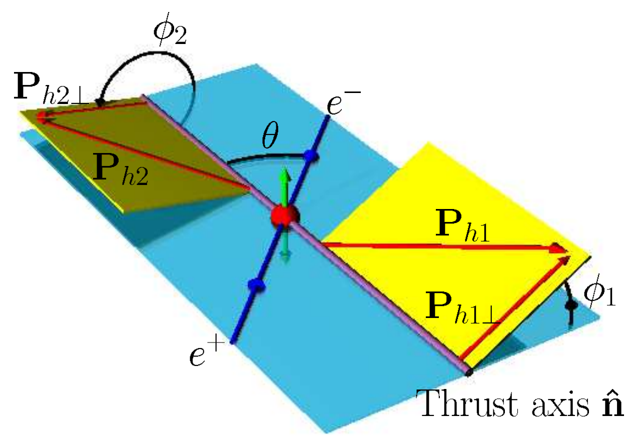
▶ $h_1 h_2 \to l^+ l^- X$ (DY): $h_1, f_{1T}^{\perp}, h_1^{\perp}$
J-PARC, RHIC (pp), COMPASS(πp) PAX($\bar{p}p$)

[Reimer, Lorenzon, Denisov talks]



▶ $e^+ e^- \to h_1 h_2 X$: $D_1, H_1^{\perp}$
Belle, BaBar

[Vossen, Garzia talks]



For gluon TMDs

▶ $pp \to \gamma\gamma X$: $f_{1T}^{\perp}, h_1^{\perp}$ [Qiu-Schlegel-Vogelsang 2011]

▶ $lp \to l' Q\bar{Q} X$: $h_1^{\perp}$ [Boer et al. 2011]

Universality and factorization

- QCD: local gauge invariant under SU(3)
- gauge invariant definition of TMDs requires a gauge link (Wilson line) (true also for ordinary PDFs)
- two unique features for TMDs:
  1. some TMDs are non-zero only if the Wilson line is included $(f_{1T}^{\perp}, h_1^{\perp})$
  2. the Wilson line depends on the process: modified universality

Let's start with the standard integrated/collinear quark PDF:

- parton model: hadron expectation value of the number density of partons, $\langle P|b^\dagger b|P\rangle$
- QCD:
  1. $A^+ = 0$ (light-cone gauge) $\longrightarrow$ number density (as in parton model)
     BUT $n \cdot A = 0$ (axial gauge), $n = (0, 1, \mathbf{0}_t)$ $[A^+ = 0] \longrightarrow$
     singularity in the gluon propagator $1/(k \cdot n)$ $\longrightarrow$ problems in factorization.
  2. Alternatively: Feynman (covariant) gauge;
     Price: extra regions to be considered (collinear gluons)

▶ Gauge invariant definition of PDFs: a Wilson-line between $\bar{\psi}(w)$ and $\psi(0)$:

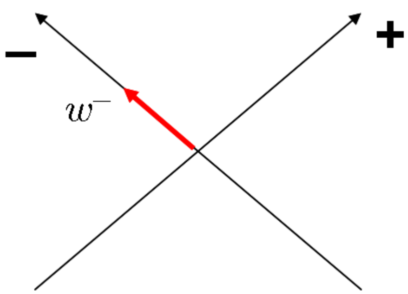
$$f(x;\mu) = \mathrm{F.T.}\,\langle P|\,\bar{\psi}(0,w^-,\mathbf{0}_t)\gamma^+ W(w^-,0)\,\psi(0)|P\rangle\ ,$$

$\mathrm{F.T.} \equiv$ Fourier transform from coordinate space to momentum space.

μ: renormalization scale to remove UV divergences ($\Rightarrow$ DGLAP evolution)

$W(w^-,0)$: path ordered exponential of the gauge field

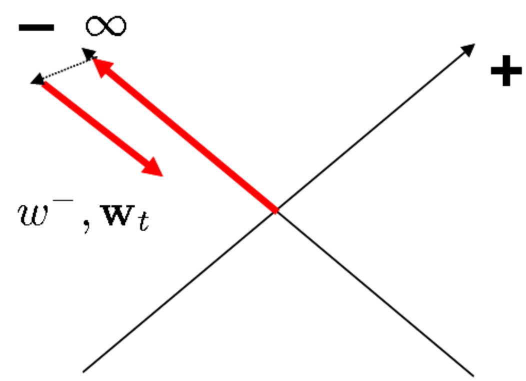
$$W(w^-,0) = P\exp\left(-ig\int_0^{w^-} dy^-\, n\cdot A(0,y^-,\mathbf{0}_t)\right).$$



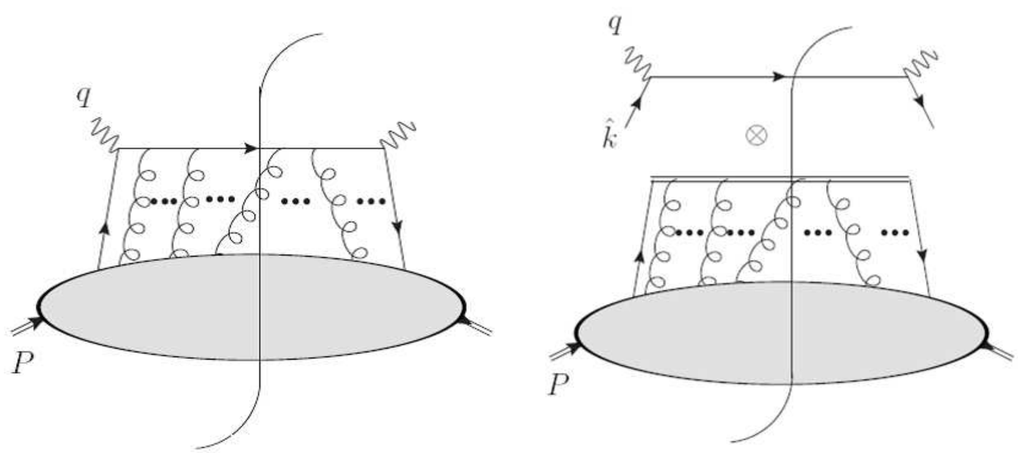
The gauge-link follows a straight path connecting 0 and $(0,w^-,\mathbf{0}_t)$ along the light-like *minus* direction.

▶ TMD PDFs

- parton fields are no longer at light-like separation
- the Wilson line has to make a small detour at infinity in the transverse direction.



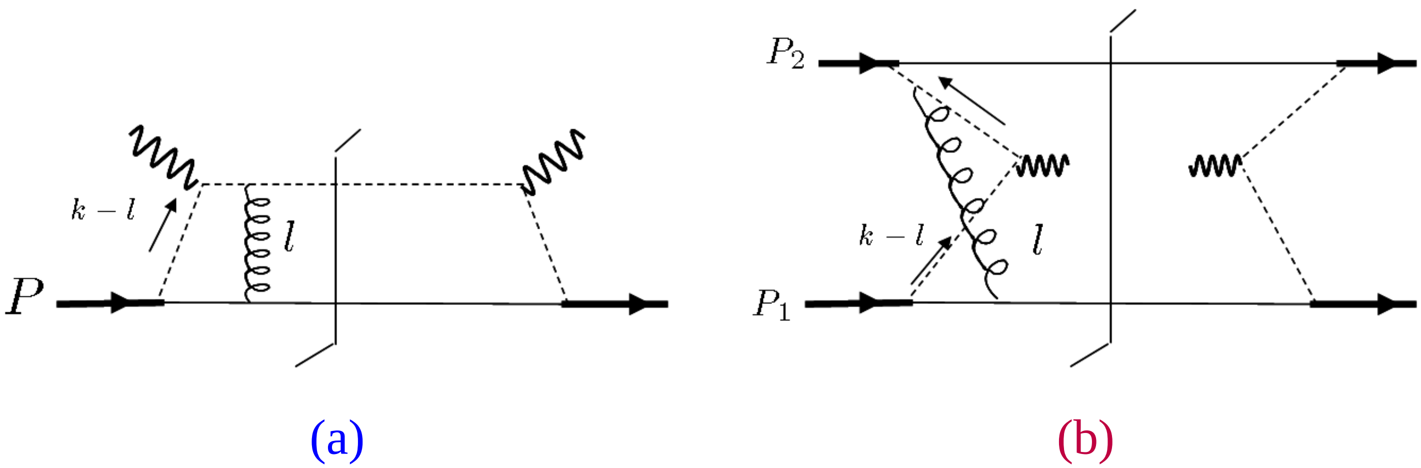
$$f(x, \boldsymbol{k}_\perp) = \text{F.T.}\,\langle P|\,\bar{\psi}(0, w^-, \boldsymbol{w}_t)\,W(w,\infty)\gamma^+ W_{\text{transv.}}(\infty) W(0,\infty)\,\psi(0)\,|P\rangle.$$



Wilson lines: resummation of extra gluons in the derivation of factorization

- in $lN \to l'h\,X$ (SIDIS), $h_1 h_2 \to l^+ l^-\,X$ (DY) and $l^+ l^- \to h_1 h_2\,X$:
  ONE hard process (a virtual photon coupling to a parton line)
  $\Rightarrow$ simple color flow from the hard to the soft part

▶ for TMD PDFs: if the color flows after the hard scattering $\Rightarrow$ the gauge link is *future* pointing;
if the color is annihilated before the hard scattering $\Rightarrow$ the gauge link is *past* pointing.



Gluon attachment (a) after the hard scattering (SIDIS); (b) before the hard scattering (DY).

- Parity and time-reversal invariance allow to related these different paths
- 6 (T-even) TMDs come out to be universal; 2 (T-odd) TMDs are *non-universal*

$$f_{1T}^{\perp}|_{\rm DY} = -f_{1T}^{\perp}|_{\rm DIS} \qquad\qquad h_1^{\perp}|_{\rm DY} = -h_1^{\perp}|_{\rm DIS}$$

[Collins 2002]

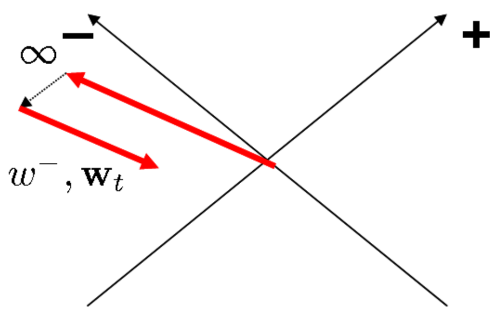
- no effects for TMD FFs: Collins function is expected to be universal.

[Collins-Metz 2004; Yuan 2008]

Extra complications: rapidity divergences [Collins 2008]

- light-like Wilson lines → divergences at $l^+_{\rm gluon} = 0$, $[y \equiv 1/2\ln(l^+/l^-)]$;
- gluons moving with negative infinity rapidity w.r.t. the parent nucleon;
- collinear PDFs: safe! cancelation between real and virtual gluon contributions;

▶ Regularized by tilting the gauge link out of the light-like direction



A new parameter (the tilt) must be introduced and a generalization of renormalization is needed
→ Collins Soper Sterman (CSS) evolution equations
[Collins-Soper 1982; CSS 1985]

TMD factorization in QCD

Drell-Yan processes [CSS 1985], extension to SIDIS [Ji-Ma-Yuan 2005]

$$d\sigma_{\rm DY} \sim H \otimes A \otimes B \otimes S \qquad q_T \ll Q$$

H: hard scattering factor A, B: TMD PDFs S: Soft factor (soft gluons, $y \sim 0$)

$-$ S non perturbative, *spoiling* the picture of 2 hadrons $\leftrightarrow$ 2 soft functions (A, B)

- *TMDs in covariant gauge* [Collins (*book*) 2011; Aybat-Rogers 2011]:
  - redefinition of TMDs with proper inclusion/manipulation of the soft factor.
  - Factorization with maximal universality for the TMDs, without soft factor.
  - Rapidity (and Wilson-line self-energy) divergences cancel.
  - Homogeneous Collins-Soper (CS) evolution equations for $\tilde{f}(x, \boldsymbol{b}_T, \zeta, \mu)$ [$\boldsymbol{b}_T$: F.T.[$\boldsymbol{k}_\perp$]; $\zeta \leftrightarrow$ rapidity cutoff].
  - TMDs with spin: still to be considered *[Aybat talk]*

- *TMDs in light-like axial gauge,* $A^+ = 0$ [Cherednikov-Stefanis 2008,09,11]:
  - one-loop order approximation, regulated gluon propagator
  - transverse gauge link (absent in covariant gauges) responsible for T-odd TMDs
  - evolution equations; no complete proof of TMD factorization

  Still some controversy (i.e. cancelation of rapidity divergences).

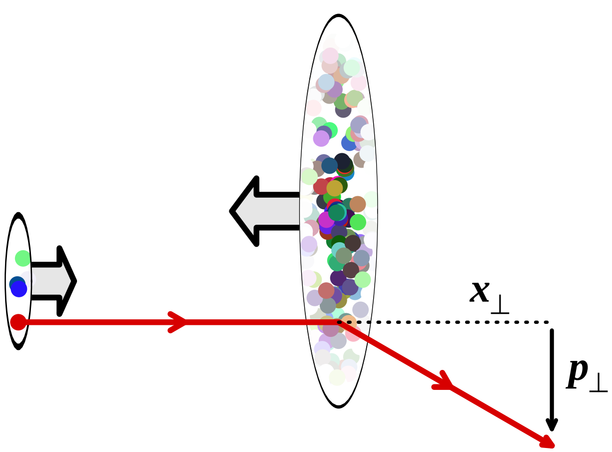
▶ **TMD factorization breaking** in $h_A h_B \to h_C h_D X$ [Rogers-Mulders 2010]

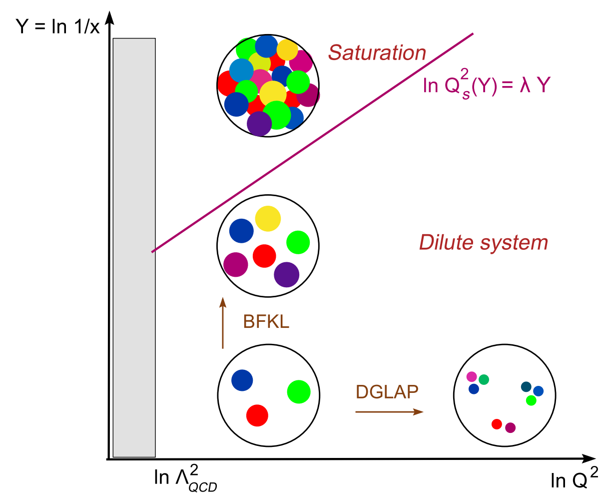
- failure of Ward identities (these allow to factorize gluons into a Wilson line)
- more color partons involved: more complicated gauge link structure
- violation of universality

▶ **TMDs at small x**

- small-x collisions of dilute probe (B) off dense nuclei (A)
- saturation effects, Color Glass Condensate: effective field theory with separation between fast frozen color sources and slow dynamical color fields

[McLerran-Venugopalan 1994, Gelis et al. 2010]





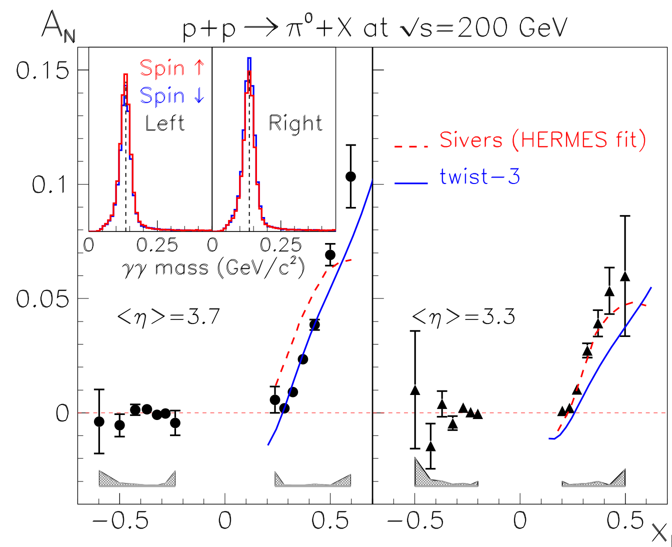
- two unintegrated gluon PDFs are used (*not always properly*)
  - Weizsäcker-Williams PDF (number density, $A^+ = 0$): $G^{(WW)}(x, k_\perp)$
  - F.T. of the color-dipole cross section (e.g., BFKL), $G^{(CD)}(x, k_\perp)$

► Latest achievements:

1. negligible soft gluon exchanges from the probe to the hard part: *effective* TMD factorization for almost back-to-back hadrons in $BA \to h_C h_D$
2. equivalence of TMD and CGC approaches in their overlapping domain
3. $G^{(WW)}(x, k_\perp)$ probed in $\gamma^* A \to q\bar{q}\, X$, $G^{(CD)}(x, k_\perp)$ in $pA \to \gamma q X$; [Dominguez-Marquet-Xiao-Yuan 2011]
4. linearly polarized gluons inside a nucleus [Metz-Zhou 2011]

More on hadron-hadron processes

▶ $p^{\uparrow}p \rightarrow hX$: $A_N \equiv \frac{\sigma^{\uparrow}-\sigma^{\downarrow}}{\sigma^{\uparrow}+\sigma^{\downarrow}}$ **still challenging**



- sizable A_N at large rapidity, increasing with x_F and P_T (RHIC) *[Koster, Poljak talks]*.

- collinear factorization at twist-3: universal quark-gluon-quark correlators, $T_q(x,x)$ [Efremov-Teryaev 82,85; Qiu-Sterman 91,92,98; Kouvaris et al. 06; Kanazawa-Koike 00,10; Kang et al. 11]

- Generalized Parton Model (GPM): TMDs (*assuming* factorization) [Anselmino-Boglione-Murgia 95, Anselmino et al. 06; D'Alesio-Murgia 04,08] Description of A_N by TMDs from SIDIS? [Boglione-D'Alesio-Murgia 08]

★ Link between the Sivers and Qiu-Sterman functions [Boer-Mulders-Piljman 03]

$$\int d^2\boldsymbol{k}_\perp \left(\frac{\boldsymbol{k}_\perp^2}{M}\right) f_{1T}^{\perp q}(x, \boldsymbol{k}_\perp^2)|_{\rm SIDIS} = -g\, T_q(x,x)$$

sign *mismatch* between phenom. extractions (other effects?, x, $k_\perp$ regions?, a node?) [Boer 2011]

- GPM with color gauge links: process dependence of $f_{1T}^\perp$ *[Gamberg talk]*

▶ $p^\uparrow p \to$ jet πX: azimuthal distribution of pions inside the jet *[Pisano talk]*

- within a GPM scheme: $h_1 \otimes H_1^\perp$ and access to gluon TMDs
- test of process dependence for $f_{1T}^\perp$ (color gauge invariant GPM vs. GPM)

Open issues

- Different QCD definitions of TMDs: different *schemes* or something deeper?
- Phenomenology with proper evolution of TMDs;
- check of the sign change of the Sivers function (DY measurement): outstanding;
- TMDs and orbital angular momentum: quantitative relations;
- factorization breaking effects: precise TMDs needed; GPM calculations;
- Twist-3 and TMDs: role/interplay in $pp \to \pi X$;
- ...answers and more questions during the workshop.

THANKS