Relation between TMDs and PDFs in the covariant parton model approach

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(based on collaboration and discussions
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Outline

- 3D covariant parton model
- PDF-TMD relations
- TMDs: numerical predictions
- general comment on DIS kinematics
- summary

3D covariant parton model

- Model of non-interacting quarks fulfils the requirements of **Lorentz invariance & rotational symmetry** of (3D) quark momentum distribution in the nucleon rest frame.
- Model implies relations and rules:
  - between 3D distributions and structure functions
  - between structure functions themselves
- For example: WW relation, sum rules WW, BC, ELT; helicity↔transversity, transversity↔pretzelosity,...

Relations between different TMDs, recently also TMDs↔PDFs

See our recent paper and citations therein:
A.Efremov, P.Schweitzer, O.Teryaev and P.Z., Phys.Rev.D **83**, 054025(2011)

TMDs

(Transverse Momentum Dependent parton distributions)

$\phi(x, \mathbf{p}_T)_{ij}$ light-front correlators

$$\frac{1}{2}\,\mathrm{tr}[\gamma^+\ \phi(x,\mathbf{p}_T)] = f_1(x,\mathbf{p}_T) - \frac{\varepsilon^{jk} p_T^j S_T^k}{M} f_{1T}^\perp(x,\mathbf{p}_T)$$

$$\frac{1}{2}\,\mathrm{tr}[\gamma^+\gamma_5\ \phi(x,\mathbf{p}_T)] = S_L\, g_1(x,\mathbf{p}_T) + \frac{\mathbf{p}_T\cdot\mathbf{S}}{M} g_{1T}^\perp(x,\mathbf{p}_T)$$

$$\frac{1}{2}\,\mathrm{tr}[i\sigma^{j+}\gamma_5\ \phi(x,\mathbf{p}_T)] = S_T^j\, h_1(x,\mathbf{p}_T) + S_L\frac{p_T^j}{M} h_{1L}^\perp(x,\mathbf{p}_T)$$

$$+ \frac{(p_T^j p_T^k - \frac{1}{2}\mathbf{p}_T^2\delta^{jk})S_T^k}{M^2} h_{1T}^\perp(x,\mathbf{p}_T) + \frac{\varepsilon^{jk} p_T^k}{M} h_1^\perp(x,\mathbf{p}_T)$$

[A.Efremov, P.Schweitzer, O.Teryaev and P.Z. Phys.Rev.D **80**, 014021(2009)]

PDF-TMD relations

1. UNPOLARIZED

$$f_1^a(x,\mathbf{p}_T) = -\frac{1}{\pi M^2}\frac{d}{dy}\left[\frac{f_1^a(y)}{y}\right]_{y=\xi(x,\mathbf{p}_T^2)}$$

$$\xi(x,\mathbf{p}_T^2) = x\left(1+\frac{\mathbf{p}_T^2}{x^2M^2}\right)$$

For details see:
P.Z. Phys.Rev.D **83**, 014022 (2011)
A.Efremov, P.Schweitzer, O.Teryaev and P.Z. Phys.Rev.D **83**, 054025(2011)

The same relation was obtained indepedently:
U. D'Alesio, E. Leader and F. Murgia, Phys.Rev. D 81, 036010 (2010)

In this talk we assume m→0

PDF-TMD relations

2. POLARIZED

$$g_1^a(x,\mathbf{p}_T) = \frac{2x-\xi}{2} K^a(x,\mathbf{p}_T) \ ,$$

$$h_1^a(x,\mathbf{p}_T) = \frac{x}{2} K^a(x,\mathbf{p}_T) \ ,$$

$$g_{1T}^{\perp a}(x,\mathbf{p}_T) = K^a(x,\mathbf{p}_T) \ ,$$

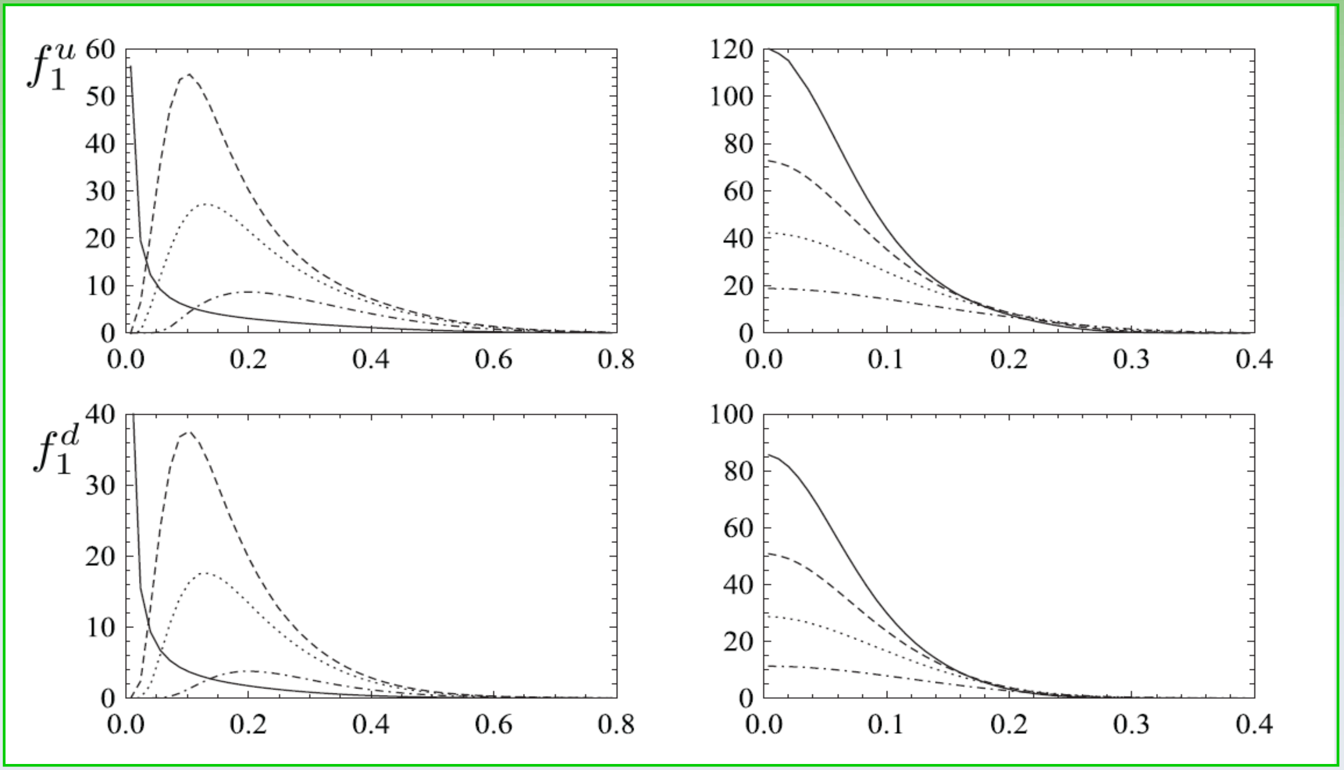
$$h_{1L}^{\perp a}(x,\mathbf{p}_T) = -K^a(x,\mathbf{p}_T) \ ,$$

$$h_{1T}^{\perp a}(x,\mathbf{p}_T) = -\frac{1}{x} K^a(x,\mathbf{p}_T) \ .$$

Known $f_1(x)$, $g_1(x)$ allow us to predict unknown TMDs

$$K^a(x,\mathbf{p}_T) = \frac{2}{\pi \xi^3 M^2}\left(2\int_\xi^1 \frac{dy}{y}\, g_1^a(y) + 3\, g_1^a(\xi) - x\, \frac{dg_1^a(\xi)}{d\xi}\right)$$

Numerical results:

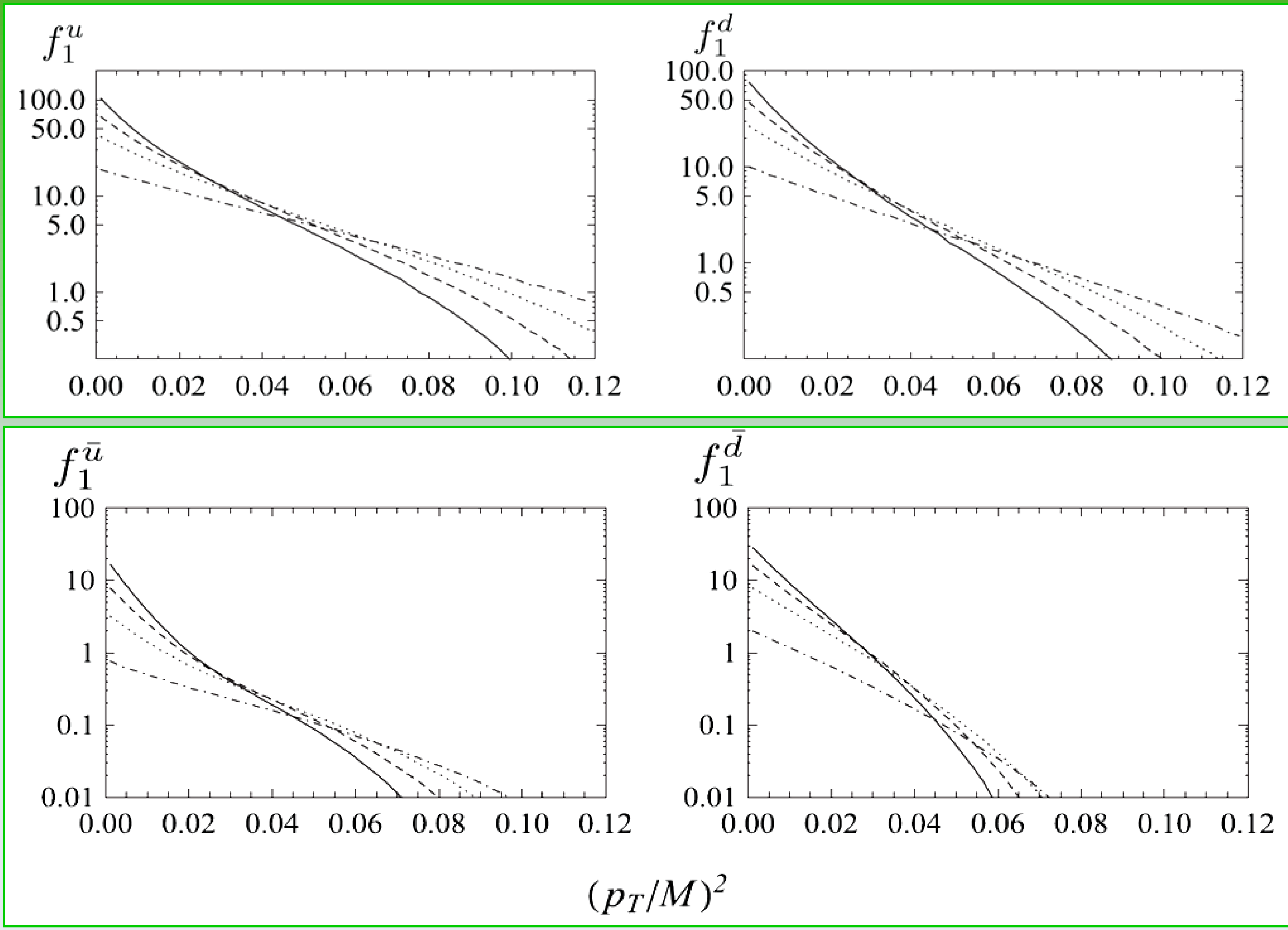


x **p_T/M**

p_T/M		***x***
q(x)	———	*0.15*
0.10	- - - -	*0.18*
0.13	········	*0.22*
0.20	—·—·—	*0.30*

Input for $f_1(x)$
MRST LO at *4 GeV*^{2}

Another model approaches to TMDs give compatible results:
1. U. D'Alesio, E. Leader and F. Murgia, Phys.Rev. D 81, 036010 (2010)
2. C.Bourrely, F.Buccellla, J.Soffer, Phys.Rev. D 83, 074008 (2011)



x
0.15
0.18
0.22
0.30

- Gaussian shape – is supported by phenomenology
- $\langle p_T^2 \rangle$ depends on x , is smaller for sea quarks

...corresponds to our former results on momentum distributions in the rest frame, see
PZ, Eur.Phys.J. C52, 121 (2007)

$$f_1^q(x) \rightarrow P_q(p_T)$$

Input for $f_1(x)$
MRST LO at $4\ GeV^2$

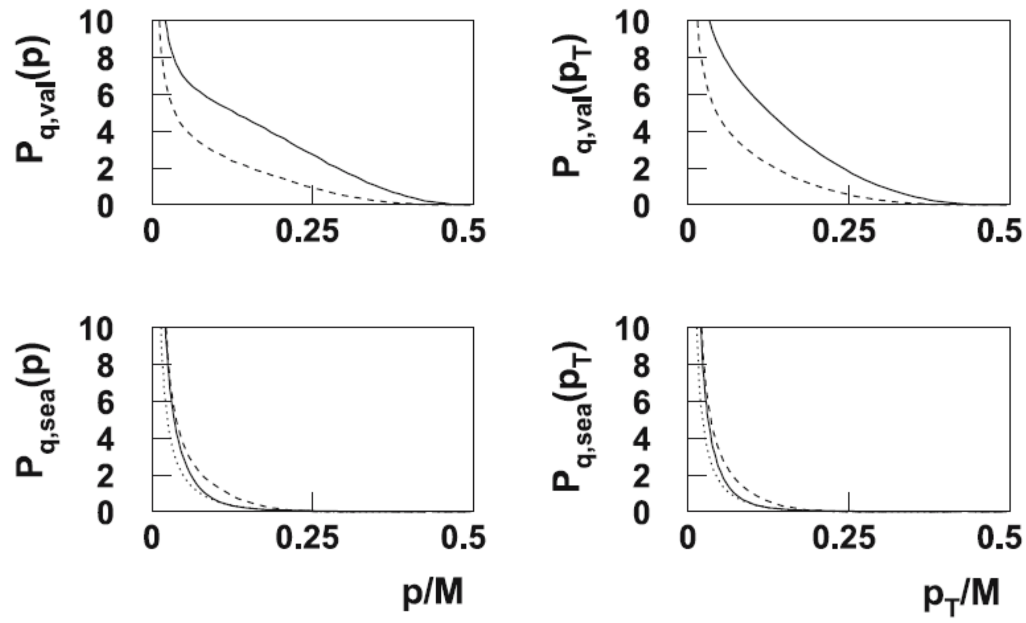


Fig. 1. The quark momentum distributions in the rest frame of the proton: the p and p_{T} distributions for valence quarks $P_{q,\mathrm{val}} = P_q - P_{\bar{q}}$ and sea quarks $P_{\bar{q}}$ at $Q^2 = 4\,\mathrm{GeV}^2$. Notation: $u, \bar{u}$ is indicated by a *solid line*, $d, \bar{d}$ by a *dashed line* and $\bar{s}$ by a *dotted line*

Calculation of $\langle p\rangle_{q,\mathrm{val}}$ gives roughly $0.11\,\mathrm{GeV}/c$ for u and $0.083\,\mathrm{GeV}/c$ for d quarks. Since $G_q(p)$ has rotational symmetry, the average transversal momentum can be calculated to be $\langle p_{\mathrm{T}}\rangle = \pi/4 \cdot \langle p\rangle$.

What do we know about intrinsic motion?

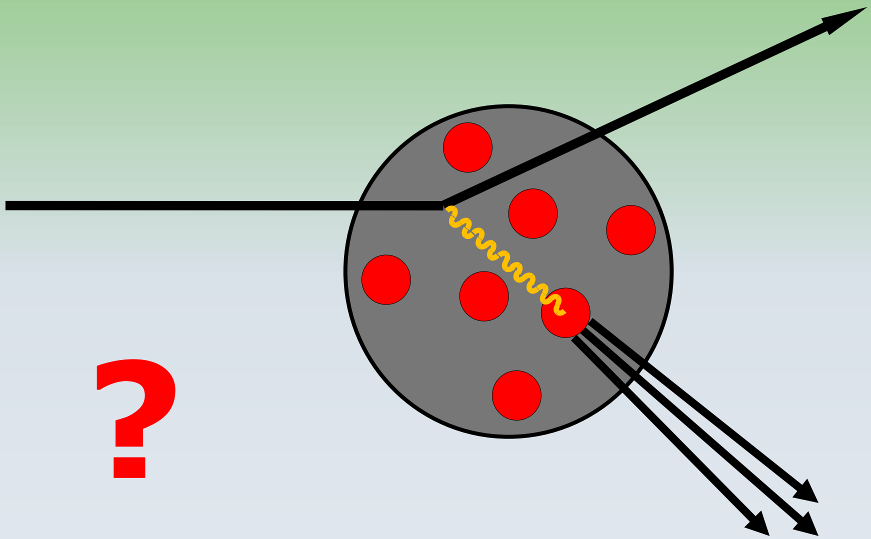
Leptonic data:

Models: statistical, covariant

$\langle p_T \rangle \approx 0.1$ **GeV/c**

R. S. Bhalerao, N. G. Kelkar, and B. Ram, Phys. Lett. B **476**, 285 (2000).
J. Cleymans and R. L. Thews, Z. Phys. C **37**, 315 (1988).
C. Bourrely, J. Soffer, and F. Buccella, Eur. Phys. J. C **23**, 487 (2002); Mod. Phys. Lett. A **18**, 771 (2003); Eur. Phys. J. C **41**, 327 (2005); Mod. Phys. Lett. A **21**, 143 (2006); Phys. Lett. B **648**, 39 (2007).

J. D. Jackson, G. G. Ross, and R. G. Roberts, Phys. Lett. B **226**, 159 (1989).
P. Zavada, Phys. Rev. D **83**, 014022 (2011).



?

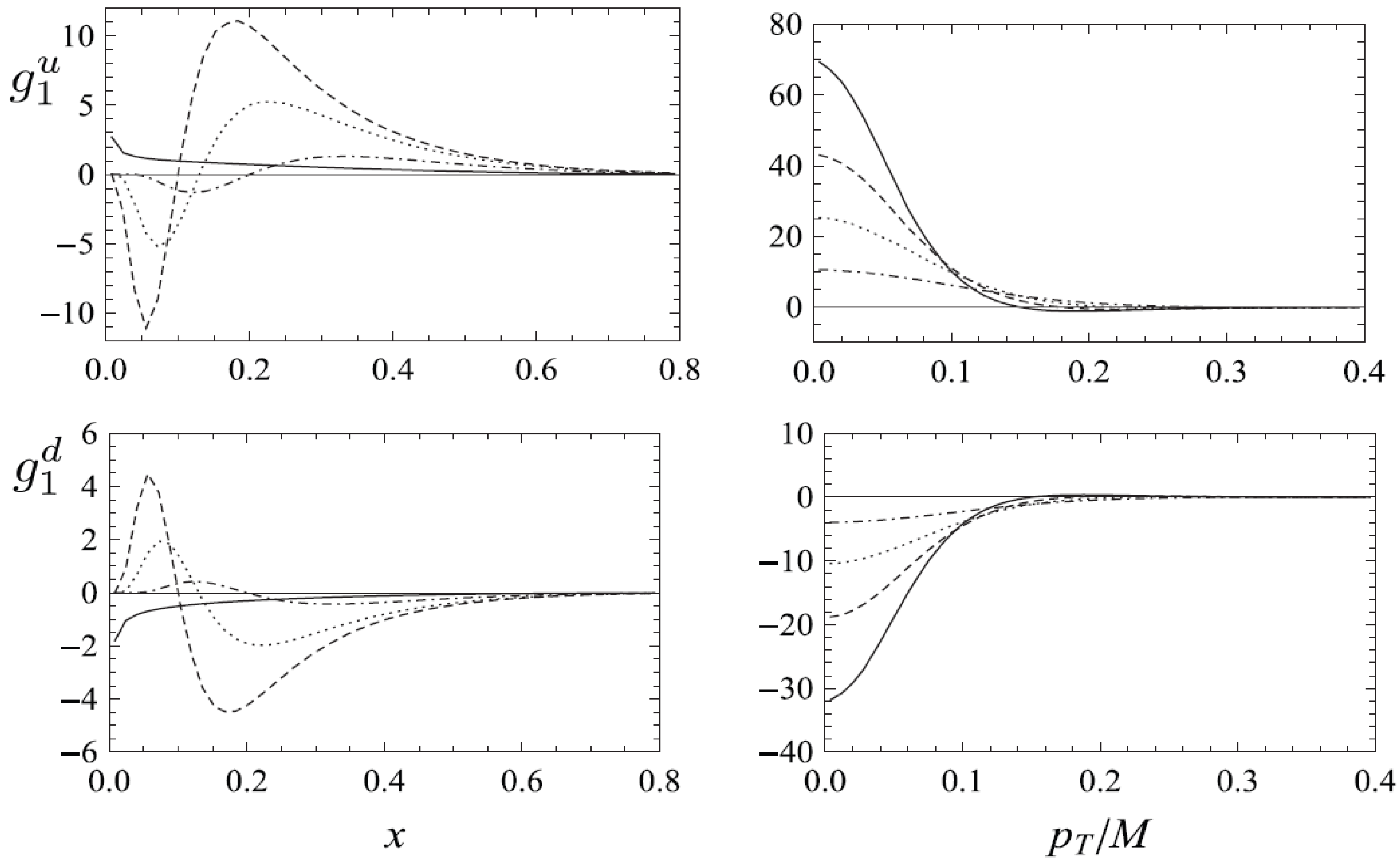
P. Schweitzer, T. Teckentrup, and A. Metz, Phys. Rev. D **81**, 094019 (2010).
M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, and A. Prokudin, Phys. Rev. D **71**, 074006 (2005).
J. C. Collins, A. V. Efremov, K. Goeke, S. Menzel, A. Metz, and P. Schweitzer, Phys. Rev. D **73**, 014021 (2006).

Hadronic data:

SIDIS, Cahn effect

$\langle p_T \rangle \approx 0.6$ **GeV/c**

Further study is needed!



p_T/M		X
$g_{1q}(x)$	———	0.15
0.10	- - - - -	0.18
0.13	·········	0.22
0.20	—·—·—	0.30

Input for g_1 :
LSS LO at *4 GeV*^{2}

Comment

In general $g_{1q}\ (x,\mathbf{p_T})$ changes sign at $p_T=Mx$. It is due to the factor:

$$2x-\xi = x\left(1-\left(\frac{p_T}{Mx}\right)^2\right) = -2\bar{p}^1/M$$

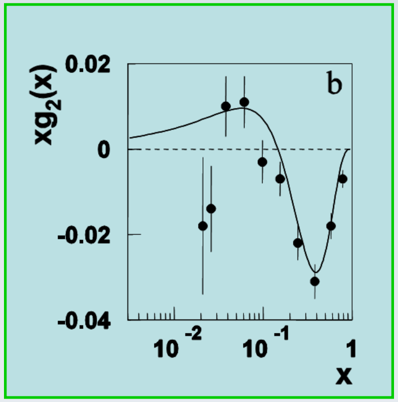
The situation is similar to the $\boldsymbol{g_2\ (x)}$ case:

$$g_2(x) = \frac{1}{2}\int H(p^0)\left(p^1 - \frac{(p^1)^2 - p_T^2/2}{p^0+m}\right)\delta\left(\frac{p^0-p^1}{M}-x\right)\frac{d^3p}{p^0}$$

With our choice of the light-cone direction:

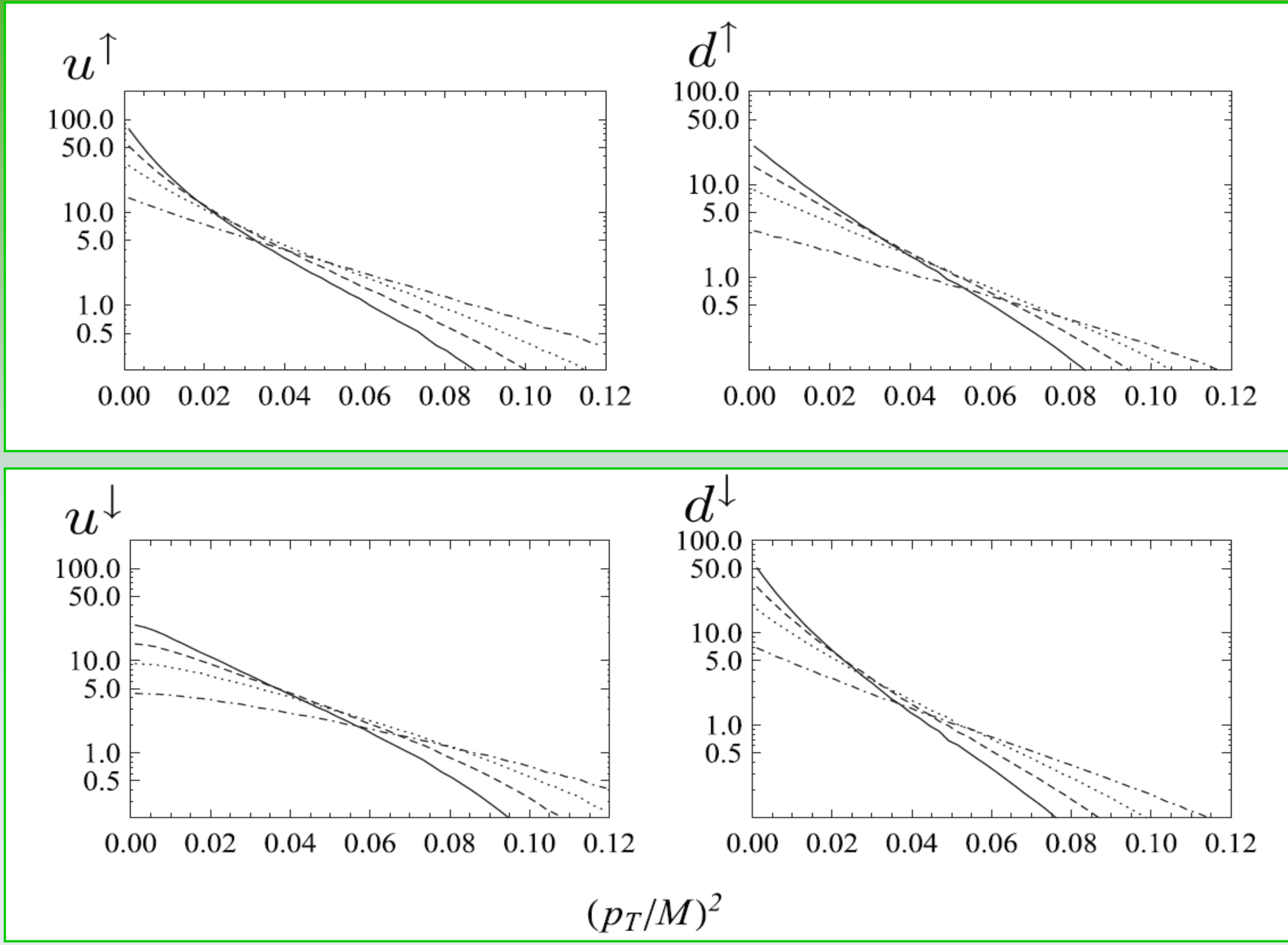
- large ***x*** are correlated with large negative $\boldsymbol{p^1}$
- low ***x*** are correlated with large positive $\boldsymbol{p^1}$

⇒ *both expressions have opposite signs for large and low **x***

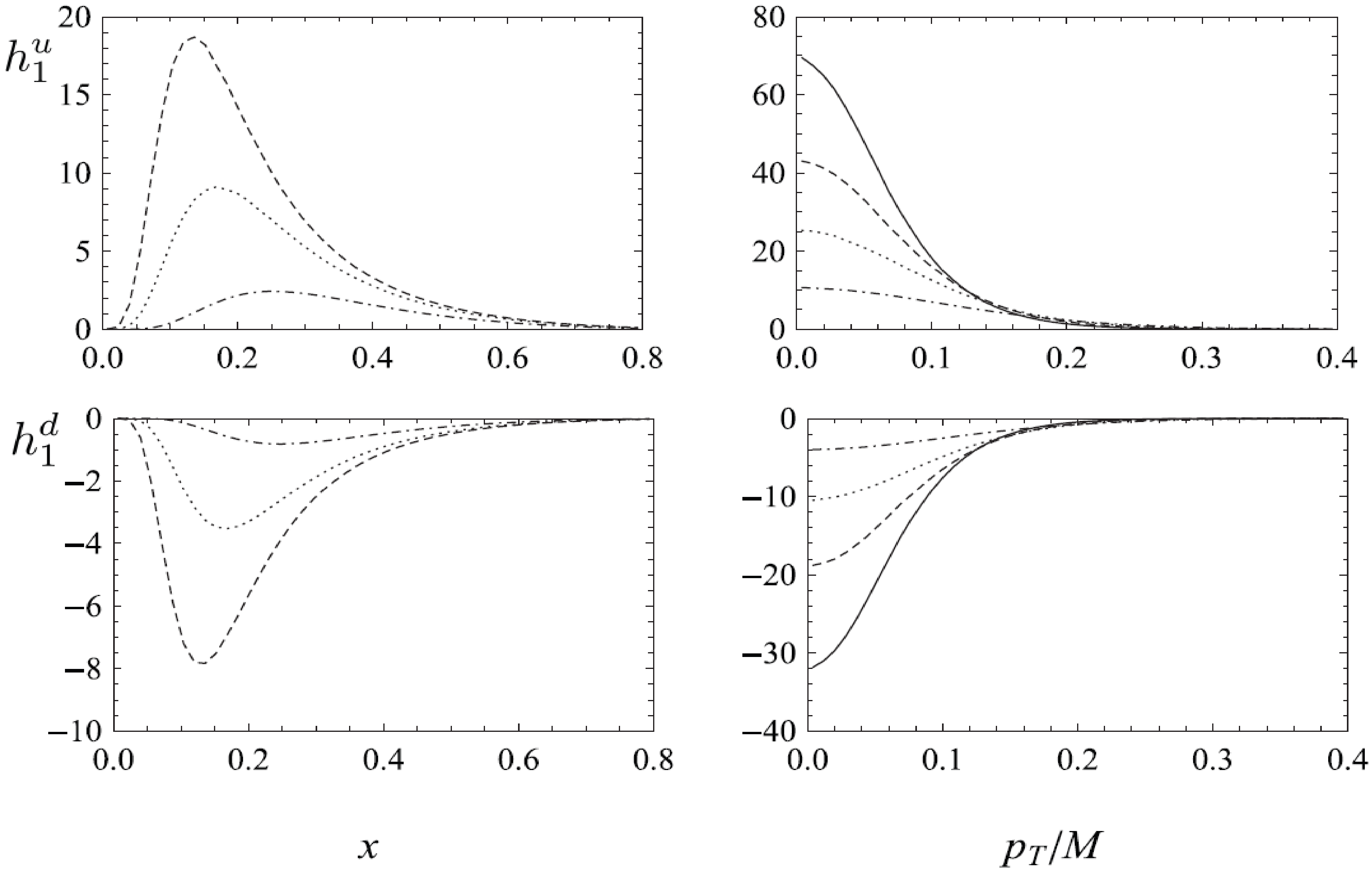


E155 experiment

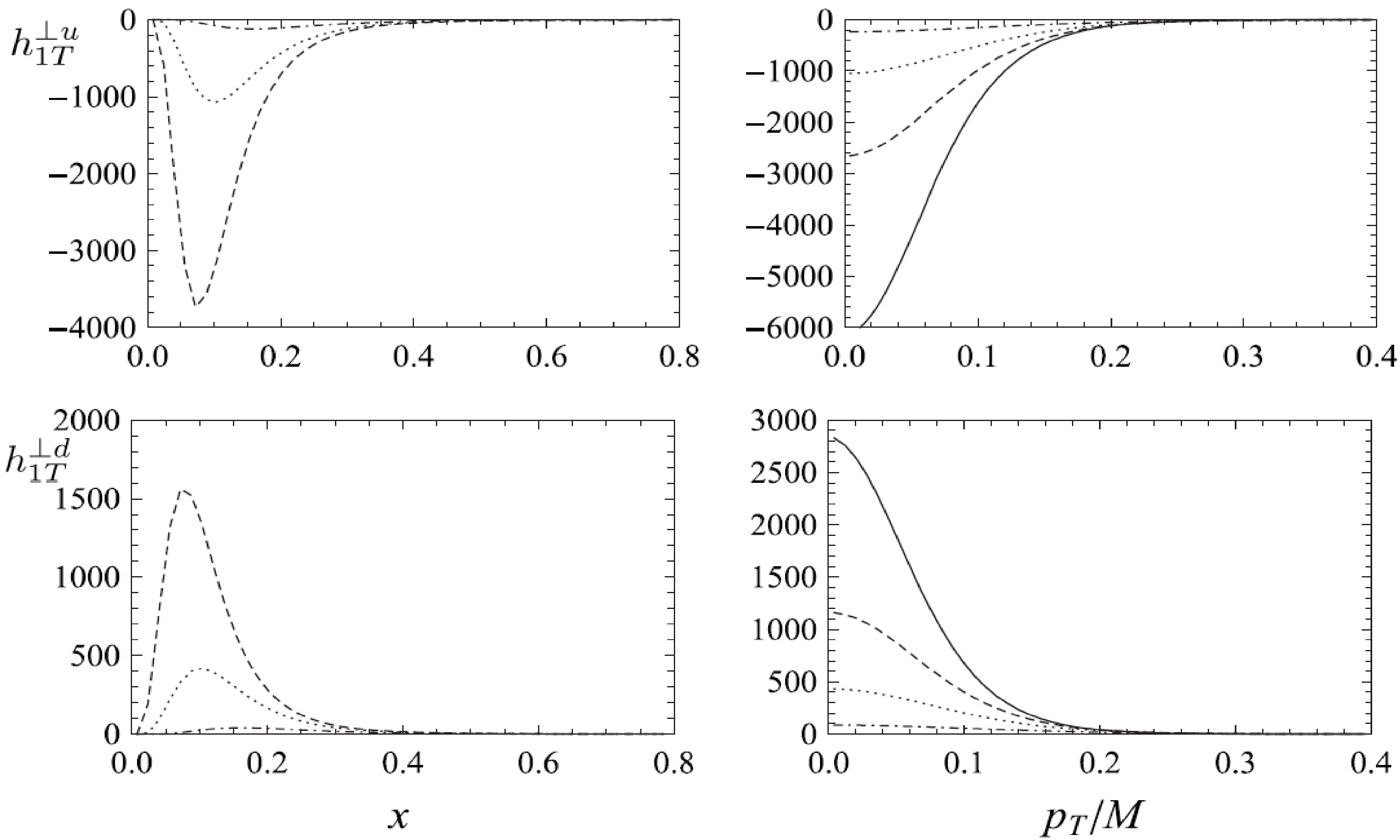
$$q^{\uparrow}(x, \mathbf{p_T}) = \frac{1}{2}(f_1^q + g_1^q)$$



$$q^{\downarrow}(x, \mathbf{p_T}) = \frac{1}{2}(f_1^q - g_1^q)$$

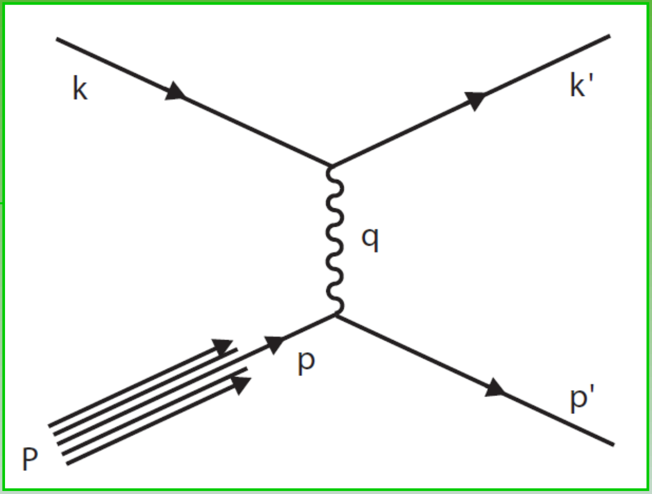


h_1^u
20
15
10
5
0
0.0
0.2
0.4
0.6
0.8
80
60
40
20
0
0.0
0.1
0.2
0.3
0.4
h_1^d
0
−2
−4
−6
−8
−10
0.0
0.2
0.4
0.6
0.8
0
−10
−20
−30
−40
0.0
0.1
0.2
0.3
0.4
x
p_T/M



$h_{1T}^{\perp u}$
0
−1000
−2000
−3000
−4000
0.0
0.2
0.4
0.6
0.8
0
−1000
−2000
−3000
−4000
−5000
−6000
0.0
0.1
0.2
0.3
0.4
$h_{1T}^{\perp d}$
2000
1500
1000
500
0
0.0
0.2
0.4
0.6
0.8
3000
2500
2000
1500
1000
500
0
0.0
0.1
0.2
0.3
0.4
x
p_T/M

Kinematic constraints



Bjorken variable satisfies:

$$x_B = \frac{Q^2}{2Pq}$$

$$0 \leq x_B \leq 1$$

For sufficiently large Q^2,

$$Q^2 \gg \delta m^2 \equiv p'^2 - p^2$$

AND

$$Q^2 \gg 4M^2 x_B^2$$

one can replace (in *any* reference frame):

$$x_B \simeq x \equiv \frac{p_0 - p_1}{P_0 - P_1}$$

For details see P.Z. arXiv:1106.5607[hep-ph]

Rest frame:

$$x = \frac{p_0 - p_1}{M}$$

AND

$$0 \leq \frac{p_0 - p_1}{M} \leq 1$$

rot. sym. $\Rightarrow$

$$0 \leq \frac{p_0 + p_1}{M} \leq 1$$

Combinations (+,-) of both imply:

$$0 \leq |p_1| \leq p_0 \leq M, \qquad |p_1| \leq \frac{M}{2}$$

rot. sym. $\Rightarrow$

$$0 \leq p_T \leq p_0 \leq M, \qquad p_T \leq \frac{M}{2}$$

$$p_T = \sqrt{p_2^2 + p_3^2}$$

$$0 \leq |p| \leq p_0 \leq M, \qquad |p| \leq \frac{M}{2}$$

$$|p| = \sqrt{p_1^2 + p_2^2 + p_3^2}$$

Shortly:

$$x_B \simeq x \equiv \frac{p_0 - p_1}{P_0 - P_1} \Rightarrow p_T < M/2$$

Conditions for equality x, x_B are satisfied

OR:

$$p_T > M/2 \Rightarrow x_B \neq x$$

x, x_B cannot be identified!

We still assume rotational symmetry in the rest frame

Remarks:

- $x \neq x_B$ would imply experimentally measured structure functions (x_B) cannot be compared with the light cone calculations (x)
- $x = x_B$ and $p_T > M/2$ - are contradictory statements
- **Obtained constraints are model-independent**
- Our covariant model assumes $x = x_B$ and we observe that p_T, p obtained from corresponding analysis of structure functions are always less then $M/2$. It is only consequence and illustration of general conditions above.

Summary

1. ***We discussed some aspects of quark motion inside nucleon within 3D covariant parton model:***

- We derived the relations between TMDs and PDFs.
- With the use of these relations we calculated the set of unpolarized and polarized TMDs.
- We again demonstrated **Lorentz invariance + rotational symmetry** represent powerful tool for obtaining new (approximate) relations among distribution functions, including PDFs ↔ TMDs.

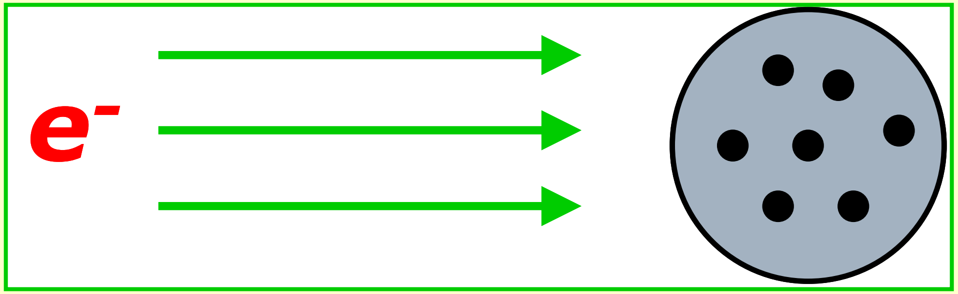
2. ***We discussed kinematic constraints due to rotational symmetry (model independent)***

Thank you !

Backup slides

3D covariant parton model

- General framework



$$\Delta\sigma(x, Q^2) \sim |A|^2$$

$$|A|^2 = L_{\alpha\beta} W^{\alpha\beta}$$

The quarks are represented by the quasifree fermions, which are in the proton rest frame described by the set of distribution functions with spheric symmetry

$$G_q^{\pm}(p_0) d^3p; \qquad p_0 = \sqrt{m^2 + \mathbf{p}^2},$$

which are expected to depend effectively on Q^2. These distributions measure the probability to find a quark in the state

$$u(p, \lambda\mathbf{n}) = \frac{1}{\sqrt{N}} \begin{pmatrix} \phi_{\lambda\mathbf{n}} \\ \frac{\mathbf{p}\sigma}{p_0+m} \phi_{\lambda\mathbf{n}} \end{pmatrix}; \qquad \frac{1}{2}\mathbf{n}\sigma\phi_{\lambda\mathbf{n}} = \lambda\phi_{\lambda\mathbf{n}},$$

where m and p are the quark mass and momentum, $\lambda = \pm 1/2$ and $\mathbf{n}$ coincides with the direction of target polarization $\mathbf{J}$.

$W^{\alpha\beta} \Rightarrow$

$$F_1(x, Q^2)$$
$$F_2(x, Q^2)$$
$$g_1(x, Q^2)$$
$$g_2(x, Q^2)$$

Structure functions

- **Input:**

  3D distribution functions in the proton rest frame (starting representation)

The distributions allow to define the generic functions G and ΔG:

$$G(p_0) = \sum_q e_q^2 G_q(p_0), \quad G_q(p_0) \equiv G_q^+(p_0) + G_q^-(p_0),$$

$$\Delta G(p_0) = \sum_q e_q^2 \Delta G_q(p_0), \quad \Delta G_q(p_0) \equiv G_q^+(p_0) - G_q^-(p_0)$$

from which the structure functions can be obtained.

- **Result:**

  structure functions

  (x=Bjorken x_B !)

If one assumes $Q^2 \gg 4M^2x^2$, then:

$$F_2(x) = Mx^2 \int G(p_0)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{d^3p}{p_0}$$

$$g_1(x) = \frac{1}{2}\int \Delta G(p_0)\left(m + p_1 + \frac{p_1^2}{p_0+m}\right)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{d^3p}{p_0},$$

$$g_2(x) = -\frac{1}{2}\int \Delta G(p_0)\left(p_1 + \frac{p_1^2 - p_T^2/2}{p_0+m}\right)\delta\left(\frac{p_0+p_1}{M} - x\right)\frac{d^3p}{p_0}$$

F_1, F_2 - manifestly covariant form:

$$F_1(x) = \frac{M}{2}\left(\frac{B}{\gamma} - A\right), \qquad F_2(x) = \frac{Pq}{2M\gamma}\left(\frac{3B}{\gamma} - A\right),$$

where

$$A = \frac{1}{Pq}\int G\left(\frac{Pp}{M}\right)[m^2 - pq]\delta\left(\frac{pq}{Pq} - x\right)\frac{d^3p}{p_0},$$

$$B = \frac{1}{Pq}\int G\left(\frac{pP}{M}\right)\left[\left(\frac{Pp}{M}\right)^2 + \frac{(Pp)(Pq)}{M^2} - \frac{pq}{2}\right]\delta\left(\frac{pq}{Pq} - x\right)\frac{d^3p}{p_0},$$

$$\gamma = 1 - \left(\frac{Pq}{Mq}\right)^2.$$

g_1, g_2 - manifestly covariant form:

$$g_1 = Pq\left(G_S - \frac{Pq}{qS}G_P\right), \qquad g_2 = \frac{(Pq)^2}{qS}G_P,$$

where

$$G_P = \frac{m}{2Pq}\int \Delta G\left(\frac{pP}{M}\right)\left[\frac{pS}{pP+mM}1 + \frac{1}{mM}\left(pP - \frac{pu}{qu}Pq\right)\right] \times \delta\left(\frac{pq}{Pq} - x\right)\frac{d^3p}{p_0},$$

$$G_S = \frac{m}{2Pq}\int \Delta G\left(\frac{pP}{M}\right)\left[1 + \frac{pS}{pP+mM}\frac{M}{m}\left(pS - \frac{pu}{qu}qS\right)\right] \times \delta\left(\frac{pq}{Pq} - x\right)\frac{d^3p}{p_0};$$

$$u = q + (qS)S - \frac{(Pq)}{M^2}P.$$

Comments

- In the limit of usual approach assuming $p = xP$, (i.e. intrinsic motion is completely supressed) one gets known relations between the structure and distribution functions:

$$F_2(x) = x \sum_q e_q^2 q(x)$$

$$g_1(x) = \frac{1}{2} \sum_q e_q^2 (q^+(x) - q^-(x))$$

- We work with a 'naive' 3D parton model, which is based on covariant kinematics (and not infinite momentum frame). Main potential: implication of some old and new sum rules and relations among PDF's and TMDs.

ROLE OF QUARKS

IN PROTON SPIN

Intrinsic motion

1) electrons in atom:

$$d \approx 10^{-10}m, \quad p \approx 10^{-3}MeV, \quad m_e \approx 0.5MeV, \quad \beta \approx 0.002$$

2) nucleons in nucleus:

$$d \approx 10^{-15}m, \quad p \approx 10^{2}MeV, \quad m_N \approx 940MeV, \quad \beta \approx 0.1$$

3) quarks in nucleon:

$$d \approx 10^{-15}m, \quad p \approx 10^{2}MeV, \quad m_e \approx 5MeV, \quad \beta \approx 1$$

Angular momentum

- Total angular momentum consists of ***j=l+s***.
- In relativistic case ***l,s*** are not conserved separately, only ***j*** is conserved. So, we can have pure states of ***j*** ***(j²,j_z)*** only, which are represented by the bispinor spherical waves:

$$\psi_{kjlj_z}(\mathbf{p}) = \frac{\delta(p-k)}{p\sqrt{2p_0}} \begin{pmatrix} i^{-l}\sqrt{p_0+m}\,\Omega_{jlj_z}(\boldsymbol{\omega}) \\ i^{-\lambda}\sqrt{p_0-m}\,\Omega_{j\lambda j_z}(\boldsymbol{\omega}) \end{pmatrix},$$

where $\boldsymbol{\omega} = \mathbf{p}/p$, $l = j \pm \frac{1}{2}$, $\lambda = 2j - l$ (l defines the parity) and

$$\Omega_{j,l,j_z}(\boldsymbol{\omega}) = \begin{pmatrix} \sqrt{\frac{j+j_z}{2j}}\, Y_{l,j_z-1/2}(\boldsymbol{\omega}) \\ \sqrt{\frac{j-j_z}{2j}}\, Y_{l,j_z+1/2}(\boldsymbol{\omega}) \end{pmatrix}; \quad l = j - \frac{1}{2},$$

$$\Omega_{j,l,j_z}(\boldsymbol{\omega}) = \begin{pmatrix} -\sqrt{\frac{j-j_z+1}{2j+2}}\, Y_{l,j_z-1/2}(\boldsymbol{\omega}) \\ \sqrt{\frac{j+j_z+1}{2j+2}}\, Y_{l,j_z+1/2}(\boldsymbol{\omega}) \end{pmatrix}; \quad l = j + \frac{1}{2}.$$

[P.Z. Eur.Phys.J. C52, 121 (2007)]

j=1/2

For $j = j_z = 1/2$ and $l = 0$:

$$Y_{00} = \frac{1}{\sqrt{4\pi}}, \qquad Y_{10} = i\sqrt{\frac{3}{4\pi}}\cos\theta, \qquad Y_{11} = -i\sqrt{\frac{3}{8\pi}}\sin\theta\exp(i\varphi),$$

$$\psi_{kjlj_z}(\mathbf{p}) = \frac{\delta(p-k)}{p\sqrt{8\pi p_0}}\begin{pmatrix} \sqrt{p_0+m}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ -\sqrt{p_0-m}\begin{pmatrix} \cos\theta \\ \sin\theta\exp(i\varphi) \end{pmatrix} \end{pmatrix}.$$

For the superposition

$$\Psi(\mathbf{p}) = \int a_k \psi_{kjlj_z}(\mathbf{p})dk; \qquad \int a_k^\star a_k dk = 1$$

the average spin contribution to the total angular momentum is calculated as

$$\langle s \rangle = \int \Psi^\dagger(\mathbf{p})\Sigma_z\Psi(\mathbf{p})d^3p; \qquad \Sigma_z = \frac{1}{2}\begin{pmatrix} \sigma_z & \cdot \\ \cdot & \sigma_z \end{pmatrix}.$$

Spin & orbital motion

$$\langle s_z \rangle = \int a_p^\star a_p \frac{(p_0 + m) + (p_0 - m)(\cos^2\theta - \sin^2\theta)}{16\pi p^2 p_0} d^3p$$

$$= \frac{1}{2} \int a_p^\star a_p \left(\frac{1}{3} + \frac{2m}{3p_0} \right) dp.$$

$$\langle l_z \rangle = \frac{1}{3} \int a_p^\star a_p \left(1 - \frac{m}{p_0} \right) dp.$$

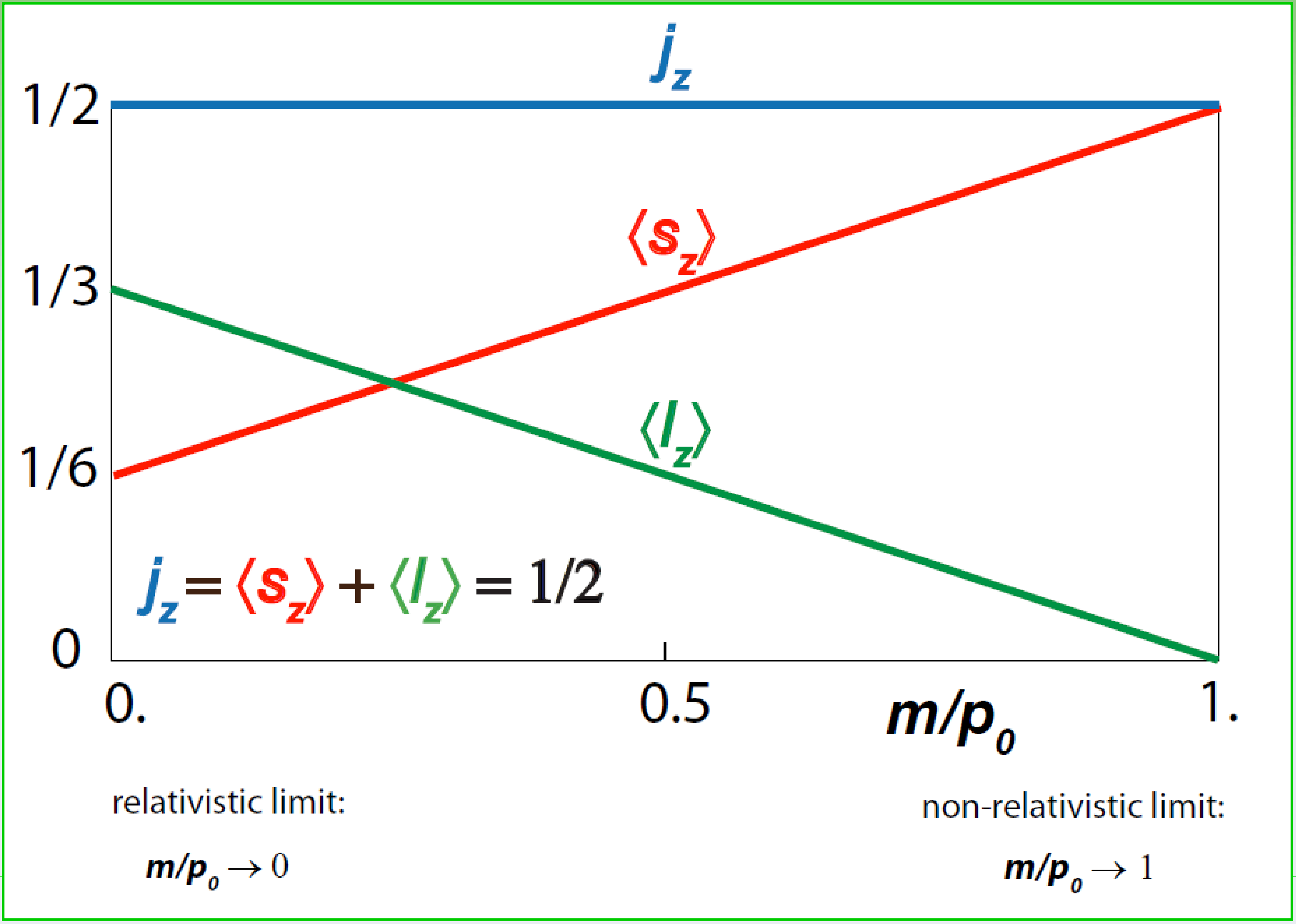
In relativistic limit:

$$m \ll p_0 \quad \Rightarrow \quad \langle s_z \rangle \to 1/6, \quad \langle l_z \rangle \to 1/3.$$

… in general: $\langle l_z \rangle = 2\langle s_z \rangle.$

$\Longrightarrow$ **only 1/3 of j contributes to Σ**

Interplay of spin and orbital motion



Spin and orbital motion from PDF's

$$\langle s^q \rangle = \int g_1^q(x)dx.$$

$$\langle l^q \rangle = -\int h_{1T}^{\perp(1)q}(x)dx.$$

H. Avakian, A. V. Efremov, P. Schweitzer and F. Yuan
Phys.Rev.D81:074035(2010).

J. She, J. Zhu and B. Q. Ma
Phys.Rev.D79 054008(2009).

Our model:

$$\int g_1^q(x)dx = \frac{1}{2}\int \Delta G_q(p_0)\left(\frac{1}{3}+\frac{2m}{3p_0}\right)d^3p.$$

$$-\int h_{1T}^{\perp(1)q}(x)dx = \frac{1}{3}\int \Delta G(p_0)\left(1-\frac{m}{p_0}\right)d^3p.$$

Two pictures:

1. wavefunctions (bispinor spherical waves) & operators

$\langle s^q \rangle$	$\langle l^q \rangle$
$\frac{1}{2}\int a_p^* a_p \left(\frac{1}{3} + \frac{2m}{3p_0}\right) dp$	$\frac{1}{3}\int a_p^* a_p (1 - \frac{m}{p_0}) dp$

2. probabilistic distributions & structure functions (in our model)

$\int g_1^q(x) dx$	$-\int h_{1T}^{\perp(1)q}(x) dx$
$\frac{1}{2}\int \Delta G_q(p_0) \left(\frac{1}{3} + \frac{2m}{3p_0}\right) d^3p$	$\frac{1}{3}\int \Delta G_q(p_0)(1 - \frac{m}{p_0}) d^3p$

$$a_p^* a_p dp \Leftrightarrow \Delta G_q(p_0) d^3p; \quad \Delta G_q(p_0) = G_q^+(p_0) - G_q^-(p_0)$$

$\Rightarrow$ ***Also in our model OAM can be identified with pretzelosity!***