Accessing Orbital Angular Momentum from TMDs and GPDs

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- Probabilistic interpretation of GPDs as Fourier transforms of impact parameter dependent PDFs
    - $H(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow q(x,\mathbf{b}_\perp)$
    - $\tilde{H}(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow \Delta q(x,\mathbf{b}_\perp)$
    - $E(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow \perp$ deformation of PDFs when the target is $\perp$ polarized
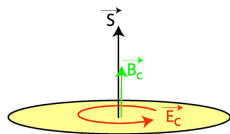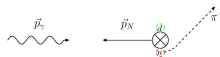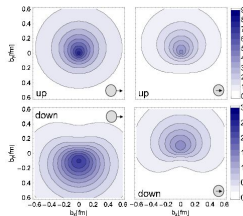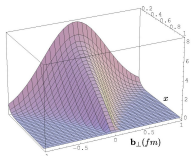
↪ Ji relation

- Chromodynamik lensing and $\perp$ single-spin asymmetries (SSA)

$$\left. \begin{array}{c} \text{transverse distortion of PDFs} \\ + \text{ final state interactions} \end{array} \right\} \quad \Rightarrow$$

↪ SSA in $\quad \gamma N \longrightarrow \pi + X$

- quark-gluon correlations $\to \perp$ force on q in DIS
- Summary

- form factors: $\xleftrightarrow{FT} \rho(\vec{r})$
- $GPDs(x, \vec{\Delta})$: form factor for quarks with momentum fraction x
- $\hookrightarrow$ suitable FT of $GPDs$ should provide spatial distribution of quarks with momentum fraction x
- careful: cannot measure longitudinal momentum (x) and longitudinal position simultaneously (Heisenberg)
- $\hookrightarrow$ consider purely transverse momentum transfer

Impact Parameter Dependent Quark Distributions

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, \xi = 0, -\boldsymbol{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp}$$

$q(x, \mathbf{b}_\perp)$ = parton distribution as a function of the separation $\mathbf{b}_\perp$ from the transverse center of momentum $\mathbf{R}_\perp \equiv \sum_{i \in q,g} \mathbf{r}_{\perp,i} x_i$
MB, Phys. Rev. D62, 071503 (2000)

- No relativistic corrections (Galilean subgroup!)
- $\hookrightarrow$ corollary: interpretation of 2d-FT of $F_1(Q^2)$ as charge density in transverse plane also free of relativistic corrections
- probabilistic interpretation

$q(x, \mathbf{b}_\perp)$ for unpol. p
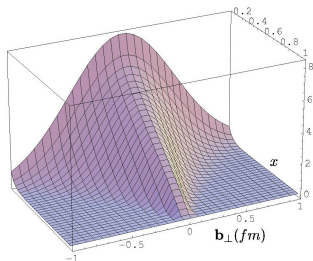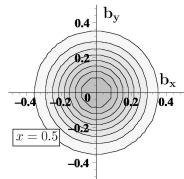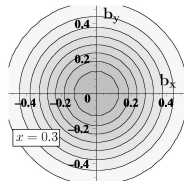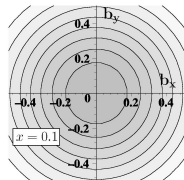
unpolarized proton

- $q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H(x, 0, -\boldsymbol{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp}$
- $x =$ momentum fraction of the quark
- $\vec{b} = \perp$ distance of quark from $\perp$ center of momentum
- small x: large 'meson cloud'
- larger x: compact 'valence core'
- $x \to 1$: active quark becomes center of momentum

$\hookrightarrow$ $\vec{b}_\perp \to 0$ (narrow distribution) for $x \to 1$

$u(x, \mathbf{b}_\perp)$ $d(x, \mathbf{b}_\perp)$

$\vec{p}_{\gamma^*}$ $\hat{z}$ $\hat{y}$

$j^z > 0$

$j^z < 0$
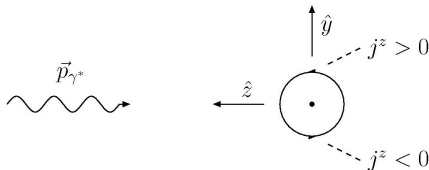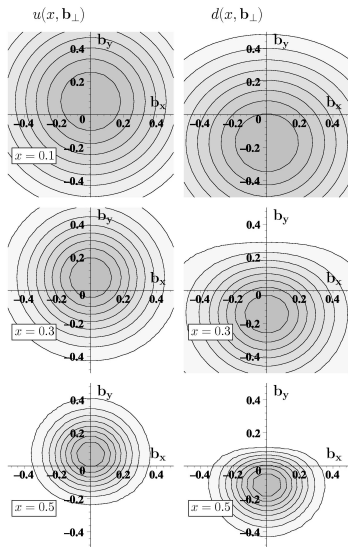
proton polarized in $+\hat{x}$ direction

no axial symmetry!

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta_\perp^2) e^{-i \mathbf{b}_\perp \cdot \Delta_\perp}$$

$$-\frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta_\perp^2) e^{-i \mathbf{b}_\perp \cdot \Delta_\perp}$$

Physics: relevant density in DIS is $j^+ \equiv j^0 + j^3$ and left-right asymmetry from j^3

$u(x, \mathbf{b}_\perp)$ $d(x, \mathbf{b}_\perp)$

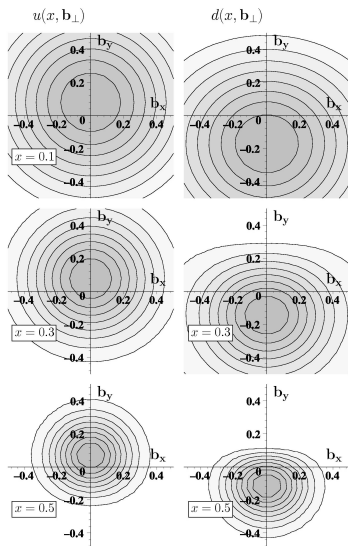**proton polarized in $+\hat{x}$ direction**

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sign & magnitude of the average shift

model-independently related to p/n
anomalous magnetic moments:

$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp \, q(x, \mathbf{b}_\perp) b_y$$

$$= \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa_q}{2M}$$

$u(x, \mathbf{b}_\perp)$ $d(x, \mathbf{b}_\perp)$
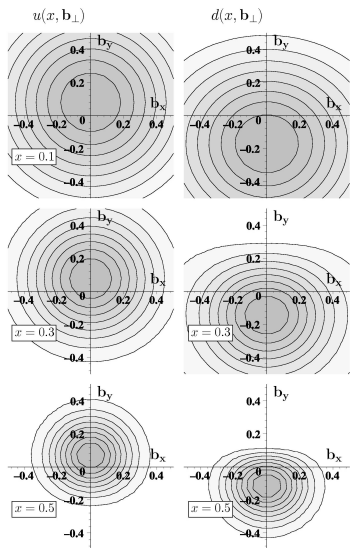
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$\kappa^p = 1.913 = \frac{2}{3}\kappa_u^p - \frac{1}{3}\kappa_d^p + ...$

- u-quarks: $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673$
$\hookrightarrow$ shift in $+\hat{y}$ direction
- d-quarks: $\kappa_d^p = 2\kappa_n + \kappa_p = -2.033$
$\hookrightarrow$ shift in $-\hat{y}$ direction
- $\langle b_y^q \rangle = \mathcal{O}\left(\pm 0.2 fm\right)$!!!!

$u(x, \mathbf{b}_\perp)$ $d(x, \mathbf{b}_\perp)$
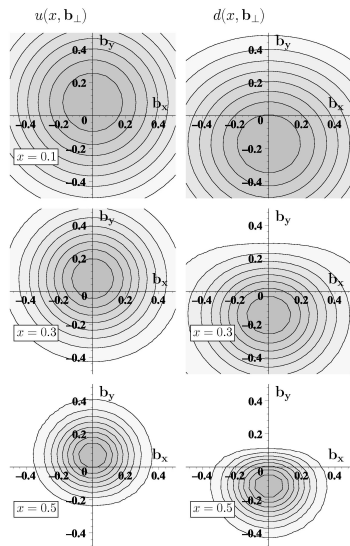
anomalous gravito-magnetic moment

- $B(0) \equiv \sum_{i \in q,g} B_i(0) = 0$
  ($\to$ S.J.Brodsky)

- $B_i(0) =$ shift of CoM for flavor i in $\perp$ pol. nucleon relative to nucleon CoM

$\hookrightarrow$ $B(0) = 0$ $\leftrightarrow$ CoM of quarks and gluons sum up to CoM of nucleon

- lattice (LHPC, QCDSF) u and d quarks seem to almost saturate $\sum_{i \in q,g} B_i(0) = 0$

example: semi-inclusive deep-inelastic scattering (SIDIS) $\gamma p \to \pi X$
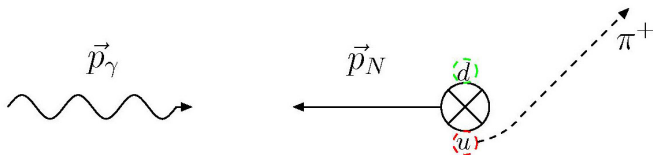
- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign 'determined' by κ_u & κ_d
- attractive FSI deflects active quark towards the CoM
↪ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction → 'chromodynamic lensing'

$\Rightarrow$ κ_p , κ_n ⟷   sign of SSA!!!!!!!! (MB,2004)

- confirmed by HERMES (and recent COMPASS) p data; consistent with vanishing isoscalar Sivers (COMPASS)

- treat FSI to lowest order in g

$\hookrightarrow$
$$\langle k_q^i \rangle = -\frac{g}{4p^+} \int \frac{d^2 \mathbf{b}_\perp}{2\pi} \frac{b^i}{|\mathbf{b}_\perp|^2} \left\langle p, s \left| \bar{q}(0)\gamma^+ \frac{\lambda_a}{2} q(0)\rho_a(\mathbf{b}_\perp) \right| p, s \right\rangle$$

  with $\rho_a(\mathbf{b}_\perp) = \int dr^- \rho_a(r^-, \mathbf{b}_\perp)$ summed over all quarks and gluons

$\hookrightarrow$ SSA related to dipole moment of parton density-density correlations

- similar density-density correlations as appear in multiparton correlations ($\to$ M.Diehl)
- GPDs (N polarized in $+\hat{x}$ direction): $u \longrightarrow +\hat{y}$ and $d \longrightarrow -\hat{y}$

$\hookrightarrow$ expect density density correlation to show same asymmetry
$\langle b^y \bar{u}(0)\gamma^+ \frac{\lambda_a}{2} u(0)\rho_a(\mathbf{b}_\perp)\rangle > 0$

$\hookrightarrow$ sign of SSA opposite to sign of distortion in position space

'Chromodynamic lensing' mechanism for $\perp$ SSA requires long range coherence of color field!
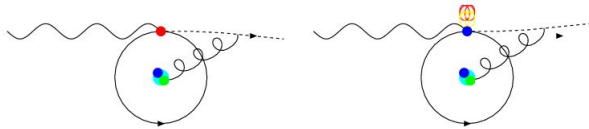
before 'dressing'   active quark 'dressed' with glue

QCD-evolution: long-range color decoherence:

- after 'dressing' itself with a gluon, previously red quark more likely to be blue or green
- $\hookrightarrow$ attraction to far-away spectators mostly gone
- only attracted to close-by (high Q^2) g from dressing
- high Q^2: q at low x likely to have dressed itself with perturbative gluon!
- $\hookrightarrow$ 'Chromodynamic lensing' mechanism suppressed for high Q^2 & small x?

sea quarks

- consider pert. $\bar{q}$ from splitting of *glue*
- $f_{1T,g}^{\perp}$ expected to be small due to sum rule

$$\sum_{i \in q,g} \int dx \int d^2\mathbf{k}_\perp \mathbf{k}_\perp^2 f_{1T}^{\perp,i}(x, \mathbf{k}_\perp) = 0$$

- expect $f_{1T,\bar{q}}^{\perp}$ also to be small for pert. $\bar{q}$

Total (Spin+Orbital) Quark Angular Momentum

$$J_q^x = L_q^x + S_q^x = \int d^3r \left[y T_q^{0z}(\vec{r}) - z T_q^{0y}(\vec{r}) \right]$$

- $T_q^{\mu\nu}(\vec{r})$ energy momentum tensor $(T_q^{\mu\nu}(\vec{r}) = T_q^{\nu\mu}(\vec{r}))$
- $T_q^{0i}(\vec{r})$ momentum density $[P_q^i = \int d^3r \, T_q^{0i}(\vec{r})\,]$
- think: $(\vec{r} \times \vec{p})^x = y p^z - z p^y$

relate to impact parameter dependent quark distributions $q(x, \mathbf{r}_\perp)$:

Consider spherically symmetric wave packet with nucleon polarized in $+\hat{x}$ direction

- eigenstate under rotations about x-axis
- $\hookrightarrow$ both terms in J_q^x equal:
  $$J_q^x = 2 \int d^3r \, y T_q^{0z}(\vec{r}) = \int d^3r \, y \left[T_q^{0z}(\vec{r}) + T_q^{z0}(\vec{r}) \right]$$
- $\int d^3r \, y T_q^{00}(\vec{r}) = 0 = \int d^3r \, y T_q^{zz}(\vec{r})$

$$\Rightarrow \quad J_q^x = \int d^3r \, y T_q^{++}(\vec{r}) \quad \text{with} \quad T^{++} \equiv T^{00} + T^{0z} + T^{z0} + T^{zz}$$

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- $\int dx\, x q(x, \mathbf{r}_\perp) = \frac{1}{2m_N} \int dz\, T^{++}(\vec{r})$
  (note: here x is momentum fraction and not r^x)
- $\hookrightarrow$ $J_q^x = m_N \int dx\, x r^y q(x, \mathbf{r}_\perp)$
- before applying this result to $\perp$ shifted PDFs, need to consider 'overall $\perp$ shift' of CoM for $\perp$ polarized target...

spherically symmetric wave packet has center of momentum off-center:

- relativistic effect $\longrightarrow$ use Dirac wave packet for nucleon

$$\psi = \left(\begin{array}{c} f(r) \\ \frac{\vec{\sigma} \cdot \vec{p}}{E + M_N} f(r) \end{array} \right) \chi \qquad \text{with} \qquad \chi = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 1 \\ 1 \end{array} \right)$$

$\int d^3 r f^2(r) = 1$, take limit of large 'radius' for wave packet

- evaluate $T_q^{0z} = \frac{i}{2} \bar{q} \left(\gamma^0 \partial^z + \gamma^z \partial^0 \right) q$ in this state
- $\psi^\dagger \partial_z \psi$ even under $y \to -y$, i.e. no contribution to $\langle y T_q^{0z} \rangle$
- use $i \psi^\dagger \gamma^0 \gamma^z \partial^0 \psi = E \psi^\dagger \gamma^0 \gamma^z \psi$

$$\begin{aligned} \langle T^{0z} y \rangle &= E \int d^3 r \psi^\dagger \gamma^0 \gamma^z \psi y = E \int d^3 r \psi^\dagger \left(\begin{array}{cc} 0 & \sigma^z \\ \sigma^z & 0 \end{array} \right) \psi y \\ &= \frac{2E}{E + M_N} \int d^3 r \chi^\dagger \sigma^z \sigma^y \chi f(r) (-i) \partial^y f(r) y \\ &= \frac{E}{E + M_N} \int d^3 r f^2(r) \stackrel{R \to \infty}{\longrightarrow} \frac{1}{2} \end{aligned}$$

$\hookrightarrow$ p pol. in $+\hat{x}$ direction has CoM shifted by $\frac{1}{2 M_N}$ in $+\hat{y}$ direction!

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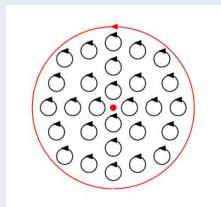
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$\hookrightarrow$ p pol. in $+\hat{x}$ direction has CoM shifted by $\frac{1}{2M_N}$ in $+\hat{y}$ direction!

origin of 'shift' of CoM

- nucleon polarization: $\odot$
- counterclockwise momentum density from lower component
- $p \sim \frac{1}{R}$, but $y \sim R$

$\hookrightarrow$ $\langle T^{++} y \rangle = \mathcal{O}(1)$

relate to impact parameter dependent quark distributions $q(x, \mathbf{b}_\perp)$:

- Thus $J_q^x = m_N \int dx\, x r^y q(x, \mathbf{r}_\perp)$ with $b^y = r^y - \frac{1}{2m_N}$, where $q(x, \mathbf{r}_\perp)$ is distribution relative to CoM of whole nucleon
- recall: $q(x, \mathbf{b}_\perp)$ for nucleon polarized in $+\hat{x}$ direction

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

$$-\frac{1}{2M_N} \frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}$$

$$\Rightarrow J_q^x = M_N \int dx\, x r^y q(x, \mathbf{r}_\perp) = \int dx\, x \left(m_N b^y + \frac{1}{2} \right) q(x, \mathbf{r}_\perp)$$

$$= \frac{1}{2} \int dx\, x \left[H(x, 0, 0) + E(x, 0, 0) \right]$$

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- X.Ji (1996): rotational invariance $\Rightarrow$ apply to all components of $\vec{J}$
- partonic interpretation exists only for $\perp$ components!

Ji decomposition

'pizza tre stagioni'
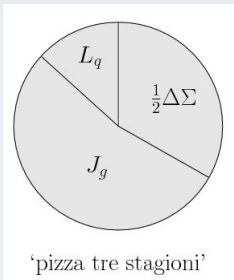
$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + L_q + J_g$$

$$\frac{1}{2}\Delta q = \frac{1}{2}\int d^3x \, \langle P,S| \, q^\dagger(\vec{x})\Sigma^3 q(\vec{x}) \, |P,S\rangle$$

$$L_q = \int d^3x \langle P,S| q^\dagger(\vec{x})\left(\vec{x}\times i\vec{D}\right)^3 q(\vec{x})|P,S\rangle$$

$$J_g = \int d^3x \, \langle P,S| \left[\vec{x}\times\left(\vec{E}\times\vec{B}\right)\right]^3 |P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

Ji decomposition

- Δq from polarized DIS
- $J_q \equiv \frac{1}{2}\Delta q + L_q = \frac{1}{2}\int_0^1 dx \, [H_q(x,0,0) + E_q(x,0,0)]$ from DVCS
- J_g in principle from gluon-GPDs; in practice $J_g = \frac{1}{2} - J_q$
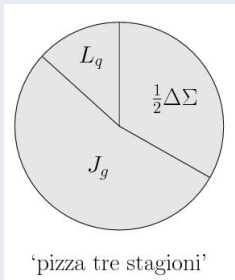- spin 1 $\longrightarrow$ Goldstein, Liuti, ..

$$\vec{J}_\gamma = \int d^3r \, \vec{r} \times \left(\vec{E} \times \vec{B} \right) = \int d^3r \, \vec{r} \times \left[\vec{E} \times \left(\vec{\nabla} \times \vec{A} \right) \right]$$

$$= \int d^3r \left[E^j \left(\vec{r} \times \vec{\nabla} \right) A^j - \vec{r} \times (\vec{E} \cdot \vec{\nabla}) \vec{A} \right]$$

$$= \int d^3r \left[E^j \left(\vec{r} \times \vec{\nabla} \right) A^j + \left(\vec{r} \times \vec{A} \right) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A} \right]$$

- replace 2nd term (eq. of motion $\vec{\nabla} \cdot \vec{E} = ej^0 = e\psi^\dagger\psi$), yielding

$$\vec{J}_\gamma = \int d^3r \left[\psi^\dagger \vec{r} \times e\vec{A}\psi + E^j \left(\vec{x} \times \vec{\nabla} \right) A^j + \vec{E} \times \vec{A} \right]$$

- $\psi^\dagger \vec{r} \times e\vec{A}\psi$ cancels similar term in electron OAM $\psi^\dagger \vec{r} \times (\vec{p} - e\vec{A})\psi$
$\hookrightarrow$ decomposing $\vec{J}_\gamma$ into spin and orbital also shuffles angular momentum from photons to electrons!

Ji decomposition

'pizza tre stagioni'
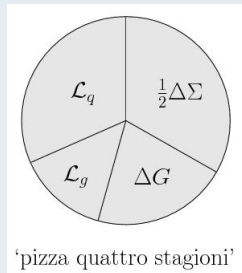
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- $i\vec{D} = i\vec{\partial} - g\vec{A}$

Jaffe decomposition

'pizza quattro stagioni'

light-cone framework & gauge $A^+ = 0$

$$\frac{1}{2} = \sum_q \frac{1}{2}\Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}_q = \int d^3r \langle P,S| \, \bar{q}(\vec{r})\gamma^+\Big(\vec{r} \times i\vec{\partial}\Big)^z q(\vec{r})|P,S\rangle$$
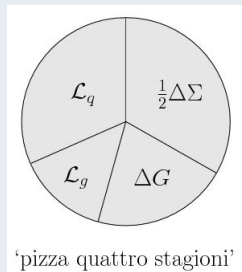
$$\Delta G = \varepsilon^{+-ij}\int d^3r \,\langle P,S| \,\mathrm{Tr}F^{+i}A^j \,|P,S\rangle$$

$$\mathcal{L}_g = 2\int d^3r \langle P,S|\mathrm{Tr}F^{+j}\Big(\vec{x} \times i\vec{\partial}\Big)^z A^j|P,S\rangle$$

Jaffe decomposition

- Δq from polarized DIS
- ΔG from $\Delta g(x)$
  ($\overrightarrow{p}\overleftarrow{p}$ & $\frac{d}{d\ln Q^2}\Delta q(x)$)
- ΔG gauge invariant! Nonlocal for $A^+ \neq 0$
- no exp./lattice access to $\mathcal{L}_q$, $\mathcal{L}_g$
- only $\mathcal{L} \equiv \mathcal{L}_g + \sum_q \mathcal{L}_q$, by subtraction
  $\mathcal{L} = \frac{1}{2} - \Delta G - \sum_q \frac{1}{2}\Delta q$

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$$\mathcal{L}_q = \int d^3 r \langle P,S| \,\bar{q}(\vec{r})\gamma^+\Big(\vec{r}\times i\vec{\partial}\Big)^z q(\vec{r})|P,S\rangle$$

$$\Delta G = \varepsilon^{+-ij}\int d^3 r \,\langle P,S|\,\mathrm{Tr}F^{+i}A^j\,|P,S\rangle$$

$$\mathcal{L}_g = 2\int d^3 r \langle P,S|\mathrm{Tr}F^{+j}\Big(\vec{x}\times i\vec{\partial}\Big)^z A^j|P,S\rangle$$

- L_q matrix element of

$$q^\dagger \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A} \right) \right]^z q = \bar{q}\gamma^0 \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A} \right) \right]^z q$$

- $\mathcal{L}_q^z$ matrix element of ($\gamma^+ = \gamma^0 + \gamma^z$)

$$\left. \bar{q}\gamma^+ \left[\vec{r} \times i\vec{\partial} \right]^z q \right|_{A^+=0}$$

- (for $\vec{p} = 0$) matrix element of $\bar{q}\gamma^z \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A} \right) \right]^z q$ vanishes (parity!)

↪ L_q identical to matrix element of $\bar{q}\gamma^+ \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A} \right) \right]^z q$ (nucleon at rest)

↪ even in light-cone gauge, L_q^z and $\mathcal{L}_q^z$ still differ by matrix element of $\left. q^\dagger \left(\vec{r} \times g\vec{A} \right)^z q \right|_{A^+=0} = q^\dagger \left(r^x g A^y - r^y g A^x \right) q \big|_{A^+=0}$

scalar diquark model

- 'mother functions' $\psi_s^S(x, \mathbf{k}_\perp)$
- $\hookrightarrow \mathcal{L}_q$ from $|\psi_s^S(x, \mathbf{k}_\perp)|^2$
- GPDs from overlap integrals of $\psi^\dagger \psi$
- $\hookrightarrow L_q$ from Ji
- $L_q = \mathcal{L}_q$.
  No surprise since
  $L_q - \mathcal{L}_q \sim \langle q^\dagger \vec{r} \times \vec{A} q \rangle$ and no
  $\vec{A}$ in scalar diquark model
- $L_q(x) \neq \mathcal{L}_q(x)$
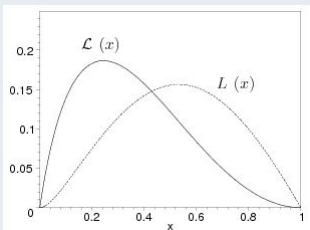
M.B. + Hikmat BC,
PRD **79**, 071501 (2009)

QED for dressed e^- in QED

- 'mother functions' $\psi_{sh}^S(x, \mathbf{k}_\perp)$
- $\hookrightarrow \mathcal{L}_q$ from $|\psi_{sh}^S(x, \mathbf{k}_\perp)|^2$
- GPDs from overlap integrals of $\psi^\dagger \psi$
- $\hookrightarrow L_q$ from Ji
- $\mathcal{L}_e = L_e + \frac{\alpha}{4\pi} \neq L_e$
- Wakamatsu-pizza?
- Goldstein-pizza?

Bakker Leader Trueman

Bakker Leader Trueman

$\perp$ pol. nucleon at $p \to \infty$

$$J_q^x = L_q^x + S_q^x = \int d^3r \left[y T_q^{0z}(\vec{r}) - z T_q^{0y}(\vec{r}) \right]$$

BIG   small

$\hookrightarrow p$ pol. in $+\hat{x}$ direction has CoM shifted by $\frac{1}{2M_N}$ in $+\hat{y}$ direction!
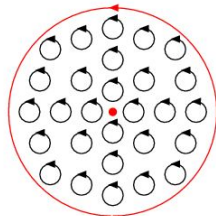
- for wave packet centered around origin:
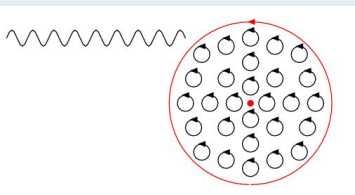 $J^x \sim \frac{1}{2M_N} P_N \longrightarrow \infty$
$\hookrightarrow$ subtract that infinity
- quark flavor q has CoM shifted by $\frac{1}{2M_N} \int dx\, x E(x_q, 0, 0)$
- $J_q^x \sim \frac{1}{2M_N} \int dx\, x E(x_q, 0, 0) P_N \longrightarrow \infty$

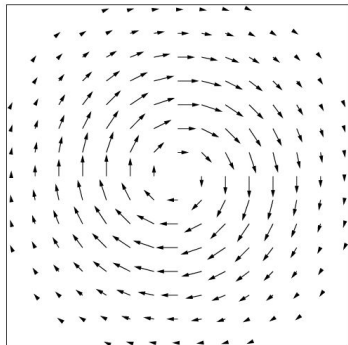## q with polarization $\odot$
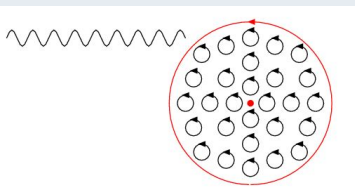

↪ counterclockwise current from lower component

↪ q distribution shifted to top

unpolarized target

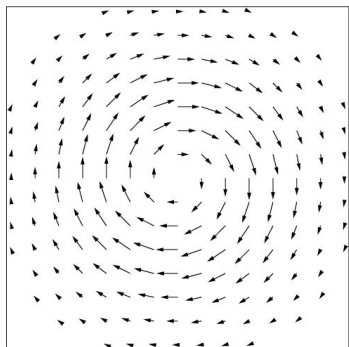- all q polns. equally likely

q with polarization $\odot$

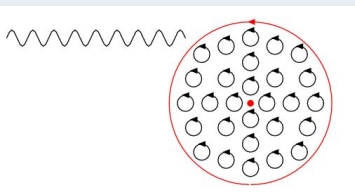$\hookrightarrow$ counterclockwise current from lower component

$\hookrightarrow$ q distribution shifted to top

unpolarized target

- q with pol. $\uparrow$ shifted to left

q with polarization $\odot$

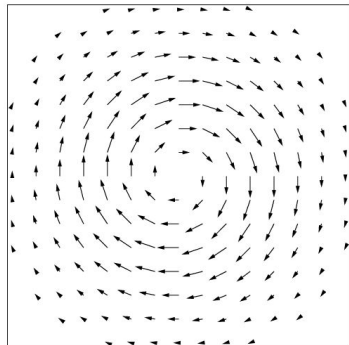$\hookrightarrow$ counterclockwise current from lower component

$\hookrightarrow$ q distribution shifted to top

unpolarized target

- q with pol. $\downarrow$ shifted to right

unpolarized target

- q with pol. → shifted to top

q with polarization ⊙
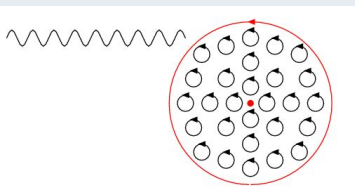

↪ counterclockwise current from lower component

↪ q distribution shifted to top

q with polarization $\odot$

$\hookrightarrow$ counterclockwise current from lower component

$\hookrightarrow$ q distribution shifted to top

unpolarized target

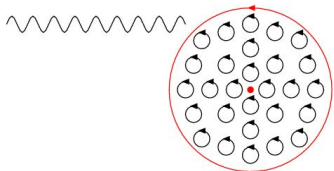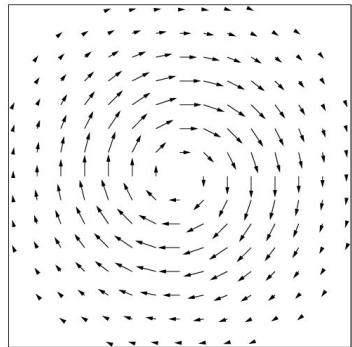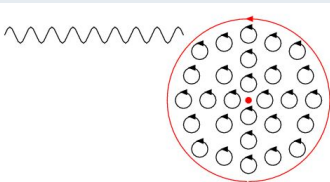- q with pol. $\leftarrow$ shifted to bottom

q with polarization $\odot$

lattice calculations (QCDSF)
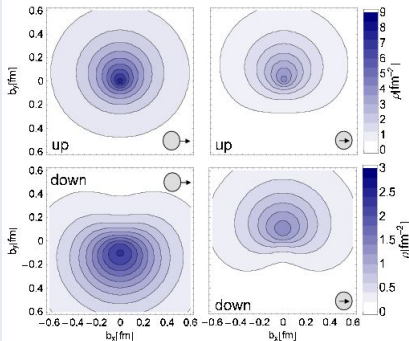
unpolarized target
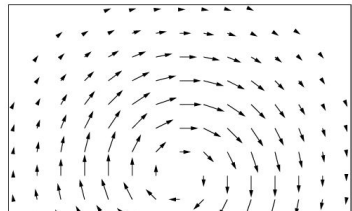
- transversity distribution in unpol. target described by chirally odd GPD $\bar{E}_T$
- $\bar{E}_T > 0$ for u & d (QCDSF)
- connection $h_1^\perp(x, \mathbf{k}_\perp) \leftrightarrow \bar{E}_T$ similar to $f_{1T}^\perp(x, \mathbf{k}_\perp) \leftrightarrow E$.
- $\hookrightarrow$ $h_1^\perp(x, \mathbf{k}_\perp) < 0$ for u/p, d/p, u/π, $\bar{d}/\pi$, ..(MB+BH, 2008)
- different valence quarks add coherently $|h_1^\perp| > |f_1^\perp|$ (MB+BH; Musch)

higher twist in polarized DIS

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$
- $g_1 = \frac{1}{2} \sum_q e_q^2 g_1^q$ with $g_1^q = q^\uparrow(x) + \bar{q}^\uparrow(x) - q^\downarrow(x) - \bar{q}^\downarrow(x)$
- g_2 involves quark-gluon correlations
- $\hookrightarrow$ no parton interpret. as difference between number densities for g_2
- for $\perp$ pol. target, g_1 & g_2 contribute equally

$$\sigma_{LT} \propto g_T \equiv g_1 + g_2$$

- $\hookrightarrow$ 'clean' separation between g_2 and $\frac{1}{Q^2}$ corrections to g_1
- What can one learn from g_2?
- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx \, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P, S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P, S \right\rangle$$

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P, S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P, S \right\rangle$$

$$\sqrt{2} G^{+y} = G^{0y} + G^{zy} = -E^y + B^x$$

$\hookrightarrow$ write $d_2 = \frac{1}{4}\left(\chi_E + 2\chi_M\right)$ with

$$2M^2 \vec{S} \chi_E = \left\langle P, S \left| \vec{j}_a \times \vec{E}_a \right| P, S \right\rangle$$

$$2M^2 \vec{S} \chi_B = \left\langle P, S \left| j_a^0 \times \vec{B}_a \right| P, S \right\rangle$$

called color-electric and magnetic
'polarizabilities'

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P, S \left| \bar{q}(0)gG^{+y}(0)\gamma^+ q(0) \right| P, S \right\rangle$$
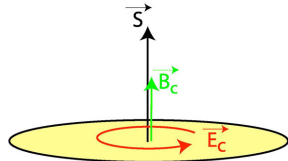
$$\sqrt{2}G^{+y} = G^{0y} + G^{zy} = -E^y + B^x$$

↪ write $d_2 = \frac{1}{4}\left(\chi_E + 2\chi_M\right)$ with

$$2M^2\vec{S}\chi_E = \left\langle P, S \left| \vec{j}_a \times \vec{E}_a \right| P, S \right\rangle$$

$$2M^2\vec{S}\chi_B = \left\langle P, S \left| j_a^0 \times \vec{B}_a \right| P, S \right\rangle$$

called color-electric and magnetic
'polarizabilities'

names misleading:

these are **not** the response of the nucleon due to an applied color field!

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P, S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P, S \right\rangle$$

$$\sqrt{2}G^{+y} = G^{0y} + G^{zy} = -E^y + B^x$$

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P,S \right\rangle$$
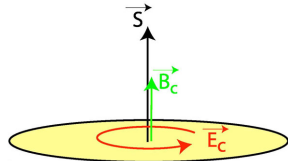
$$\sqrt{2} G^{+y} = G^{0y} + G^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B} \right)^y \text{ for } \vec{v} = (0,0,-1)$$

$\hookrightarrow$ $d_2 \leftrightarrow$ average **color Lorentz force** acting on quark moving with $v = c$ in $-\hat{z}$ direction in the instant after being struck by γ^*

$$\langle F^y \rangle = -2M^2 d_2 = -\frac{M}{P^{+2}S^x} \left\langle P,S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P,S \right\rangle$$

cf. Qiu-Sterman matrix element $\langle k_\perp^y \rangle \equiv \int_0^1 dx \int d^2 k_\perp \, k_\perp^2 f_{1T}^\perp(x, k_\perp^2)$

$$\langle k_\perp^y \rangle = -\frac{1}{2p^+} \left\langle P,S \left| \bar{q}(0) \int_0^\infty dx^- g G^{+y}(x^-) \gamma^+ q(0) \right| P,S \right\rangle$$

semi-classical interpretation: average $k_\perp$ in SIDIS obtained by correlating the quark density with the transverse impulse acquired from (color) Lorentz force acting on struck quark along its trajectory to (light-cone) infinity

matrix element defining d_2 $\leftrightarrow$ 1st integration point in QS-integral

color Lorentz force

$d_2 \leftrightarrow$ average **color Lorentz force** acting on quark moving with $v = c$ in $-\hat{z}$ direction in the instant after being struck by γ^*

$$\langle F^y \rangle = -2M^2 d_2 = -\frac{M}{P^{+2}S^x} \langle P, S \left| \bar{q}(0)gG^{+y}(0)\gamma^+ q(0) \right| P, S \rangle$$

sign of d_2

- $\kappa_q/p \longrightarrow$ sign of deformation
- $\hookrightarrow$ direction of average force
- $\hookrightarrow$ $d_2^u > 0$, $d_2^d < 0$
- cf. $f_{1T}^{\perp u} < 0$, $f_{1T}^{\perp u} < 0$

magnitude of d_2

- $\langle F^y \rangle = -2M^2 d_2 = -10\frac{GeV}{fm}$
- expect partial cancellation of forces in SSA
- $\hookrightarrow |\langle F^y \rangle| \ll \sigma \approx 1\frac{GeV}{fm}$
- $\hookrightarrow d_2 = \mathcal{O}(0.01)$

lattice (Göckeler et al., 2005)

$d_2^u \approx 0.010$, $d_2^d \approx -0.0056$
please: up to date lattice calcs.

color Lorentz force

$e_2 \leftrightarrow$ average **color Lorentz force** (in $\hat{y}$-direction) acting on quark (with transversity $\hat{x}$) moving with $v = c$ in $-\hat{z}$ direction in the instant after being struck by γ^*

$$\langle F^y \rangle = M^2 e_2 \equiv M^2 \int_0^1 dx\, \bar{e}_2(x) = \frac{M}{4P^{+2}} \sum_{i=1,2} \langle P \left| \bar{q}(0) g G^{+i}(0) \sigma^{+i} q(0) \right| P \rangle$$

chirally even

- GPD $E_q \Rightarrow \mathbf{b}_\perp$ deformation of unpol. q distr. in $\perp$ pol. target
- $\hookrightarrow f_{1T}^\perp$
- $\hookrightarrow d_2 \equiv \int dx\, x^2 \bar{g}_2$ force

chirally odd

- GPD $\bar{E}_T \Rightarrow \mathbf{b}_\perp$ deformation of quarks with transversity in unpol. target
- $\hookrightarrow h_1^\perp$
- $\hookrightarrow e_2 \equiv \int dx\, x^2 \bar{e}_2$ force

lattice (Göckeler et al., 2005)

$d_2^u \approx 0.010$, $d_2^d \approx -0.0056$
please: up to date lattice calcs.

lattice

B.Musch ($\rightarrow$ H.Avakian)
subtraction of m_q term?

- Deeply Virtual Compton Scattering (DVCS) $\longrightarrow$ GPDs
↪ impact parameter dependent PDFs $q(x, \mathbf{b}_\perp)$
- $E^q(x, 0, -\boldsymbol{\Delta}_\perp^2) \leftrightarrow \kappa_{q/p}$ (contribution from quark flavor q to anomalous magnetic moment)
- $E^q(x, 0, -\boldsymbol{\Delta}_\perp^2) \longrightarrow \perp$ deformation of PDFs for $\perp$ polarized target
- $\perp$ deformation $\leftrightarrow$ (sign of) SSA (Sivers; Boer-Mulders)
- parton interpretation for Ji-relation
- $L_q \neq \mathcal{L}_q$
- higher-twist ($\int dx\, x^2 \bar{g}_2(x)$, $\int dx\, x^2 \bar{e}(x)$) $\leftrightarrow$ $\perp$ force in DIS
- $\perp$ deformation $\leftrightarrow$ (sign of) quark-gluon correlations ($\int dx\, x^2 \bar{g}_2(x)$, $\int dx\, x^2 \bar{e}(x)$)