Transversity 2011
3rd international workshop on
Transverse Polarization Phenomena in Hard Scattering
Veli Lošinj (Croatia), Aug. 29th - Sept. 2nd 2011

Unpolarized and Polarized Fragmentation Functions

(only for light quarks in vacuum)

for a review see also
Parton fragmentation in the vacuum and in the medium
Mini-workshop ECT*, 25-28 Feb. 2008
arXiv:0804.2021 [hep-ph]

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Outline

- Unpol. 1-hadron Fragm. Functions (1h FF)

status of "collinear" parametrizations
what do we know about 1h "TMD" FF ?

- Pol. 1h FF: the Collins function

- Models of 1h FF

- 2h FF (or Dihadron Fragm. Functions – DiFF)

BELLE (+BaBar?) data and parametrizations (next 2 talks)
BELLE+HERMES (+COMPASS) data and extraction of h_1 (Braun)
extraction of e and h_L with DiFF at JLab (Avakian)

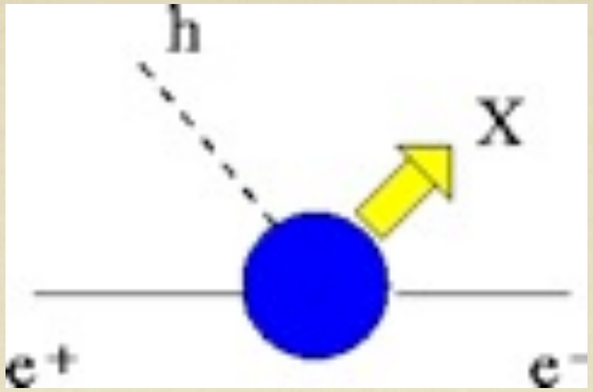
- Conclusions and Outlooks

unpolarized 1h FF

1h FF main source of data

$e^+e^- \to hX$

$h = \pi^{\pm}, K^{\pm}, K^0{}_S, p, \bar{p}, \Lambda, \bar{\Lambda}$



Energy range

- $\sqrt{s}$=12-36 GeV at DESY (ARGO, JADE, CELLO, TASSO)
- $\sqrt{s}$=29 at SLAC (HRS, MARK II, TPC)
- $\sqrt{s}$=58 at KEK (TOPAZ)

} 80's

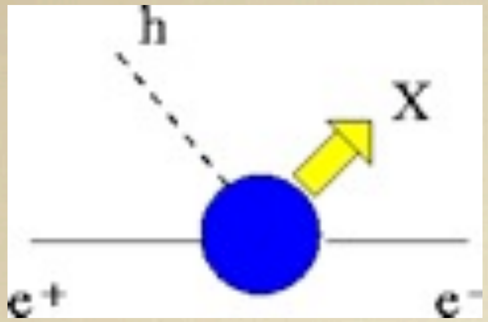
- $\sqrt{s}$=91.2 (Z0) at LEP-1 (ALEPH, DELPHI, OPAL)
- “ “ “ at SLAC (SLD)
- $\sqrt{s}$=133-209 at LEP-2 (DELPHI, L3, OPAL)

} '95-'06

- $\sqrt{s}$=10.58 (Υ_{4S}) at B-factories (BaBar, BELLE, CLEO)

- $5\times10^{-3} \leq z \leq 0.8$

$e^+e^- \to hX$



$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma^h}{dz} \equiv F^h(z,Q^2)$$
$$= \sum_{i=q,\bar q} D_i^h(z,Q^2) \quad \text{at LO}$$
$$= \sum_{i=q,\bar q,g} C_i(z,Q^2)\otimes D_i^h(z,Q^2) \quad \text{beyond}$$

- direct connection (at LO) to parton-to-hadron FF
- C_i known up to NNLO in $\overline{\rm MS}$ (Mitov & Moch (2006))
- flavor analysis ~ {u,d,s} + c + b

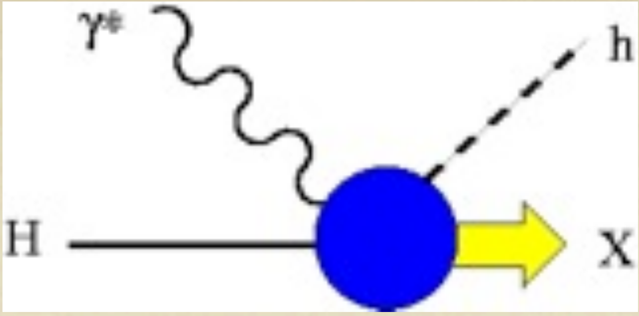
  except OPAL (full separation)

but

* D_g^h less constrained
* access only to $D_q^h + D_{\bar q}^h = D_q^{h/\bar h}$ (at LO)
* virtuality fixed by c.m. energy $Q=\sqrt{s}/2$

$e^{\pm}p \to e^{\pm}hX$

$h = \pi^{\pm}, K^{\pm}, h^{\pm}, \Lambda, \bar{\Lambda}$



Energy range

- $1 \leq Q \leq 200$ GeV at HERA (H1, ZEUS, HERMES)
- $1 \leq Q \leq 5$ at CERN (COMPASS)
- $1 \leq Q \leq 10$ also at NOMAD with ν_μ probes

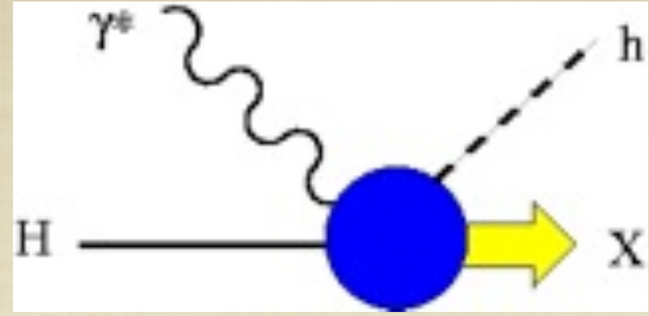
- $0.1 \leq z < 1$

– larger phase space in $\{z, Q^2\}$ than in e^+e^-

– separate D_q^h from $D_{\bar{q}}^h$ (at least for $x_B \geq 0.1$)

$e^{\pm}p \rightarrow e^{\pm}hX$

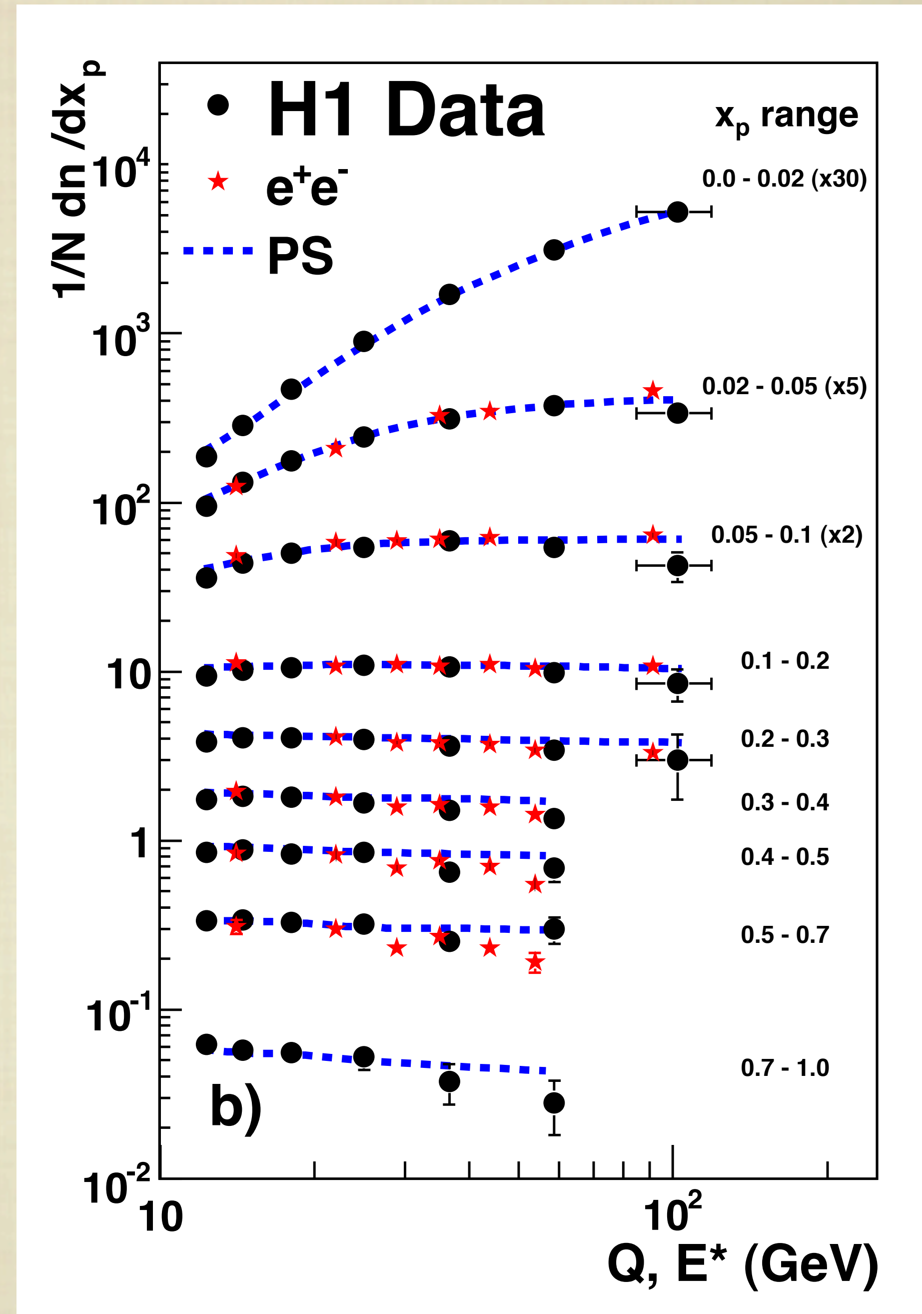
$h = \pi^{\pm}, K^{\pm}, h^{\pm}, \Lambda, \bar{\Lambda}$



- SIDIS in Breit frame
  $x_p = P_h / Q/2$
  $h^{\pm}$ scaled mom. distr.
  $1/N \, dn/dx_p$

- compare with e^+e^-
  at $E^* \equiv Q = \sqrt{s}/2$

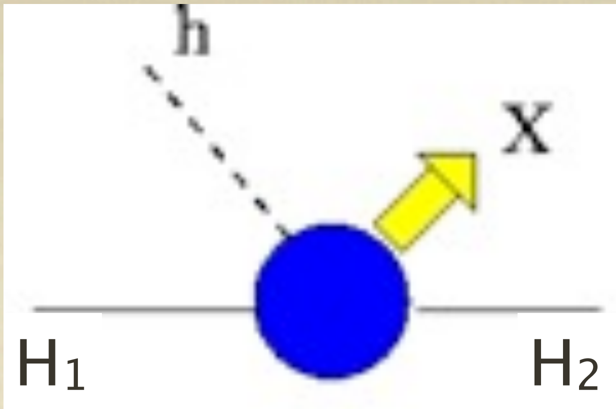
➤ universality test



H1 Coll., P.L. **B654** (2007) 148

$\overset{(-)}{p p} \rightarrow hX$

$h = \pi^{\pm,0}, K^{\pm}, K_s^0, p, \bar{p}, \Lambda, \bar{\Lambda}$



<u>Energy range</u>

- mid η, $1 \leq P_\perp(\pi^0) \leq 20$ GeV — at RHIC (PHENIX) — pp $\sqrt{s}=200$
- large η>0, $1 \leq P_\perp(\pi^0 - \pi^\pm, K^\pm) \leq 10$ — at RHIC (STAR – BRAHMS) — pp $\sqrt{s}=200$
- mid η, $1 \leq P_\perp(K_s^0, p, \bar{p}, \Lambda, \bar{\Lambda}) \leq 10$ — at RHIC (STAR) — pp $\sqrt{s}=200$
- $80 \leq M_{jj} \leq 600$, $1 \leq P_\perp(h^\pm) \leq 20$ — at CDF — $p\bar{p}$

– constrain D_g^h, especially at $x_B \ll 1$

– probe FF at large z (complementary to e^+e^-)

– $1/N\, dn/dx_p$ test universality with e^+e^- and SIDIS

status of parametrizations

before 2007

- AKK Albino, Kniehl, Kramer, 2005
- BKK Binnewies, Kniehl, Kramer, 1995
- BFG Bourhis, Fontannaz, Guillet, 1998
- BFGW Bourhis, Fontannaz, Guillet, Werlen, 2001
- CGRW Chiappetta, Greco, Guillet, Rolli, Werlen, 1994
- GRV Glück, Reya, Vogt, 1993
- KKP Kniehl, Kramer, Potter, 2000
- Kr Kretzer, 2000

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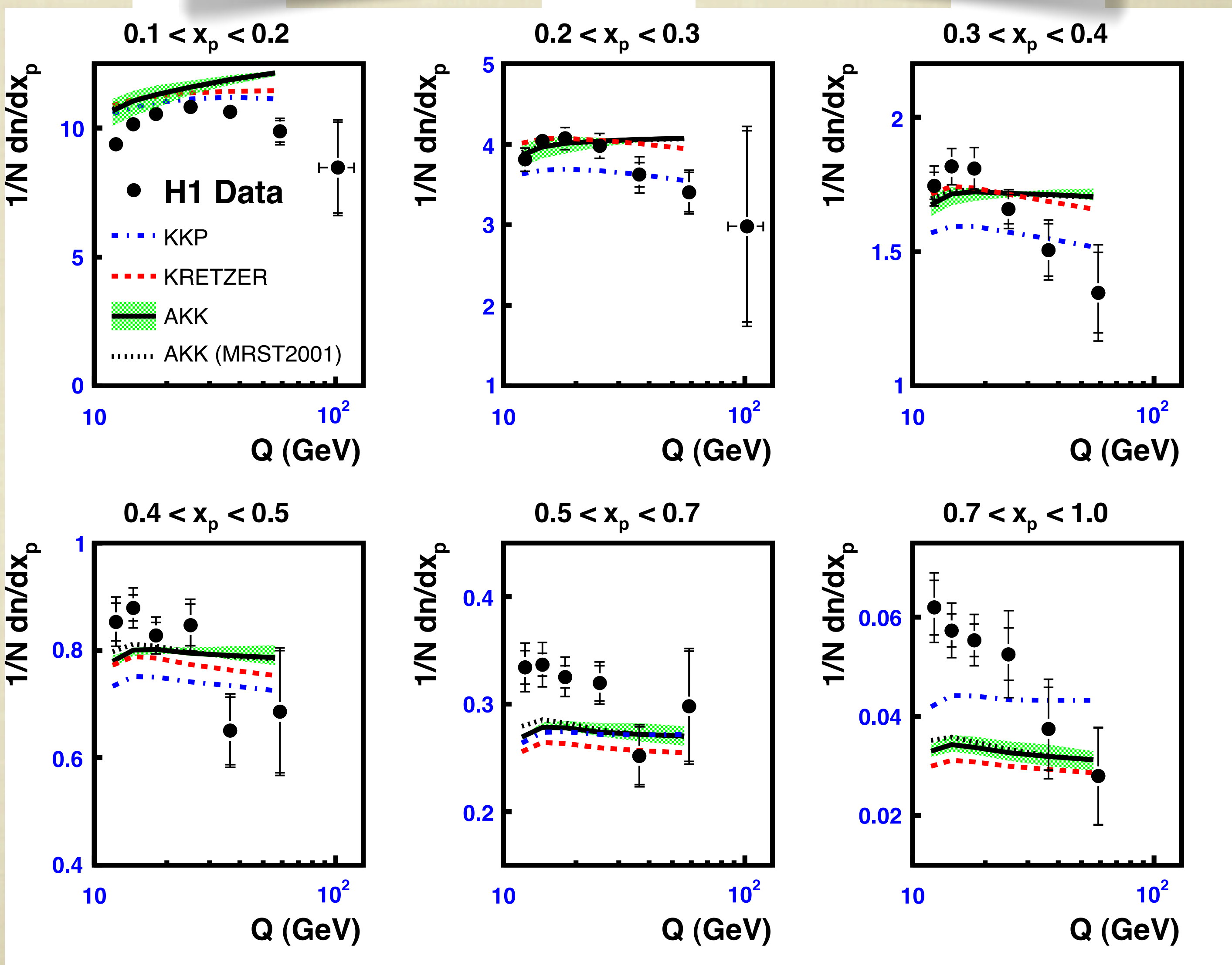
fail to reproduce scaling violations of recent H1 data



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status of parametrizations

<u>after 2007</u>

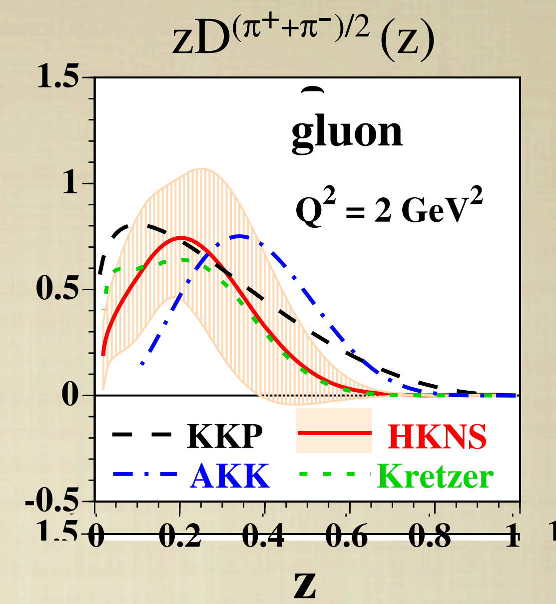
- AKK08 Albino, Kniehl, Kramer, 2008
- DSS De Florian, Sassot, Stratmann, 2007
- HKNS Hirai, Kumano, Nagai, Sudoh, 2007

main ingredients

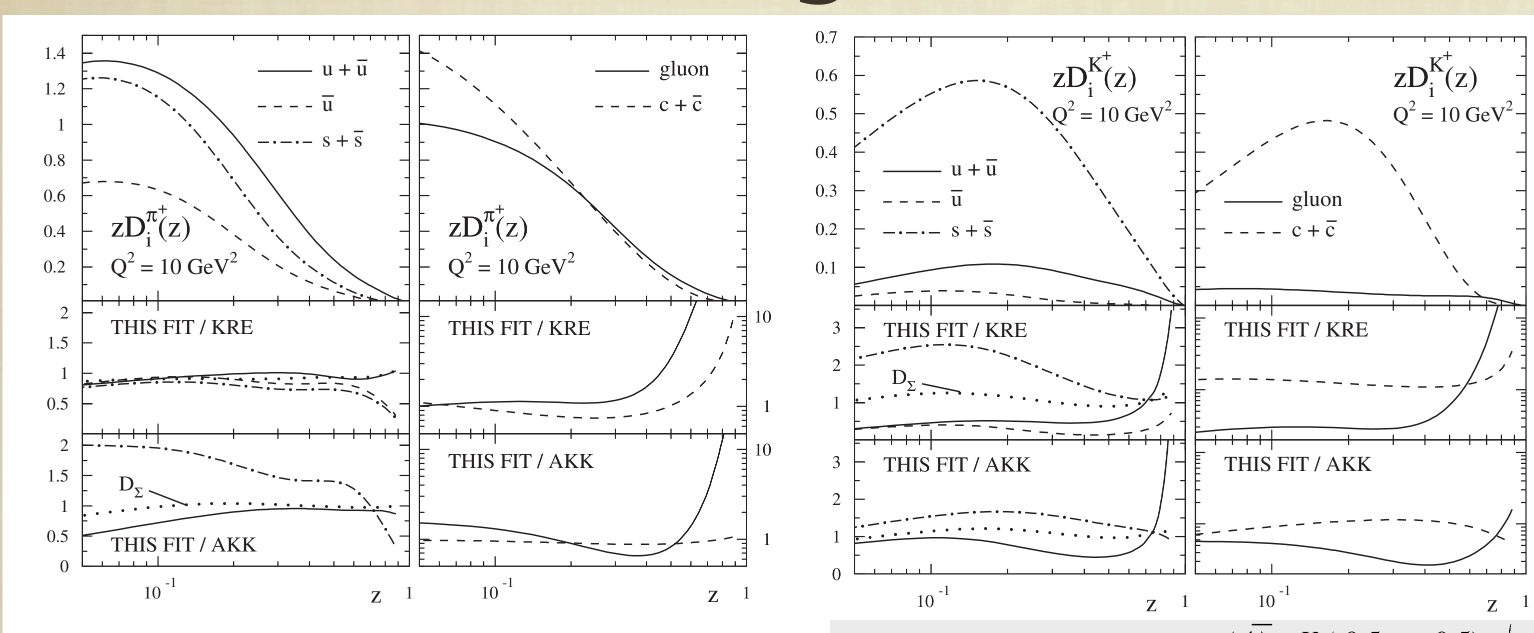
DSS	AKK08	HKNS
e^+e^- SIDIS pp	e^+e^- pp $p\bar{p}$	e^+e^-
$\pi^\pm$, $K^\pm$, p, $\bar{p}$, $h^\pm$ (, Λ)	$\pi^\pm$, $K^\pm$, K_s^0, p, $\bar{p}$, Λ, $\bar{\Lambda}$	$\pi^\pm$, π^0, $K^\pm$, $K^0+\bar{K}^0$, n, $p+\bar{p}$
$0.05-0.1 \leq z$ $1 \leq Q^2 \leq 10^5$ GeV^2	$0.05 \leq z$ $2 \leq Q^2 \leq 4\times10^4$ GeV^2	$0.01 \leq z$ $1 \leq Q^2 \leq 10^8$ GeV^2
NLO DGLAP in Mellin space $D(z,Q_0)=Nz^a(1-z)^b[1-c(1-z)^d]$ N fixed by $\sum_h \int dz\, zD_i^h(z,Q^2)=1$	NLO DGLAP in Mellin space + resum $\log^n(1-z)/1-z$ at NLL $D(z,Q_0)$ and N fixed as DSS	NLO DGLAP direct integration $D(z,Q_0)=Nz^a(1-z)^b$ N fixed as DSS
SU(2) symmetric unfavoured $d+\bar{d} \propto u+\bar{u}$	SU(2) symmetric favoured (π) and unfavoured build $D_i^{h_++h_-}$, $D_i^{h_+-h_-}$	SU(2) symmetric favoured and unfavoured s = unfavoured
Lagrange multipliers	no error analysis	Hessian errors
	$m_h\neq0$ effect $\rightarrow$ $z\neq x_p$ resum log's at NLL also in C_i	

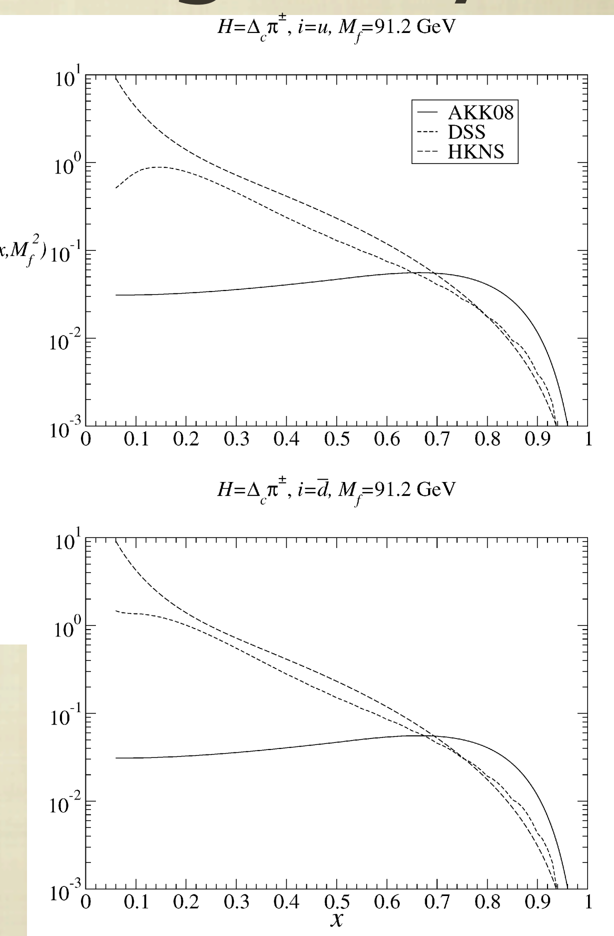
main differences

- HKNS: no constrain on D_g^h from pp data, reliable at LHC ?

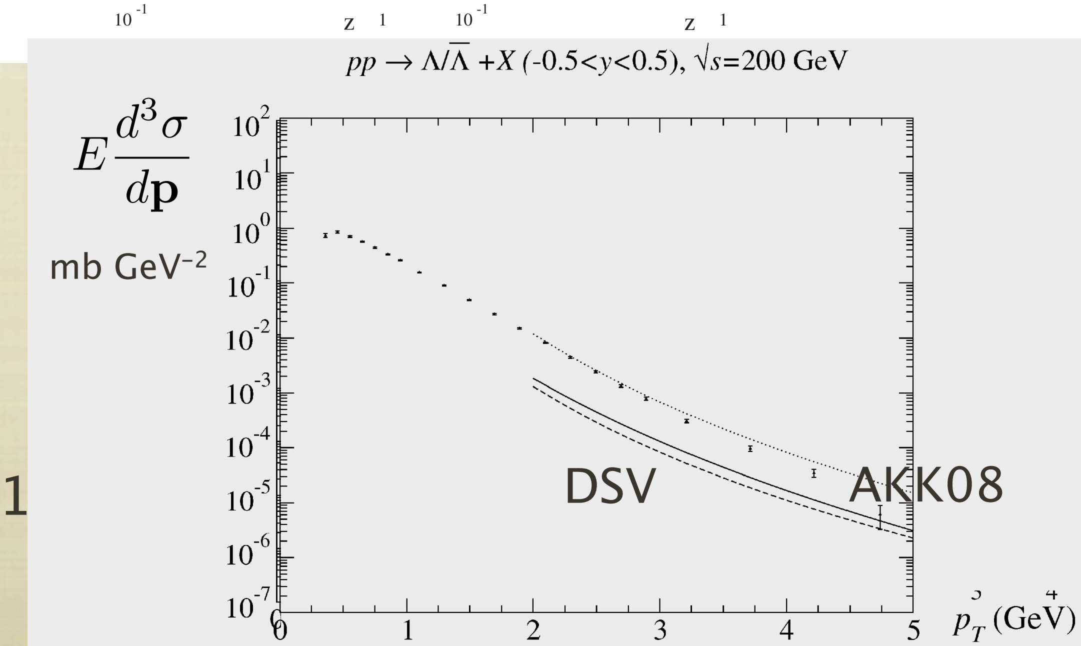


- AKK08-DSS discrepancies at large z and charge asym.



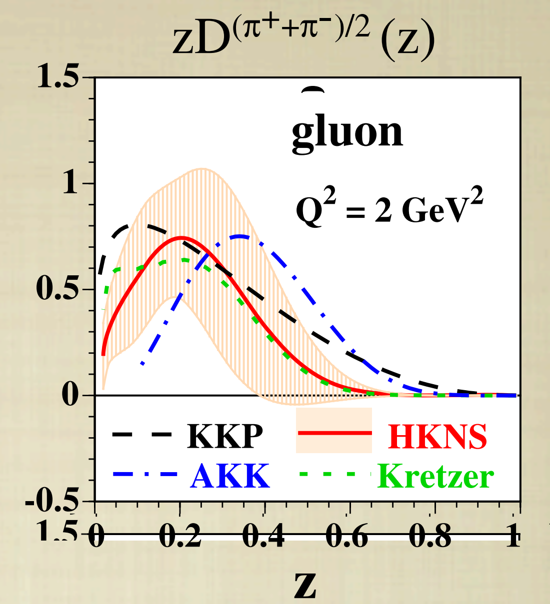


- the puzzle of STAR $\Lambda,\bar{\Lambda}$ data



STAR Coll., P.R.C 75 (07) 064901

main differences



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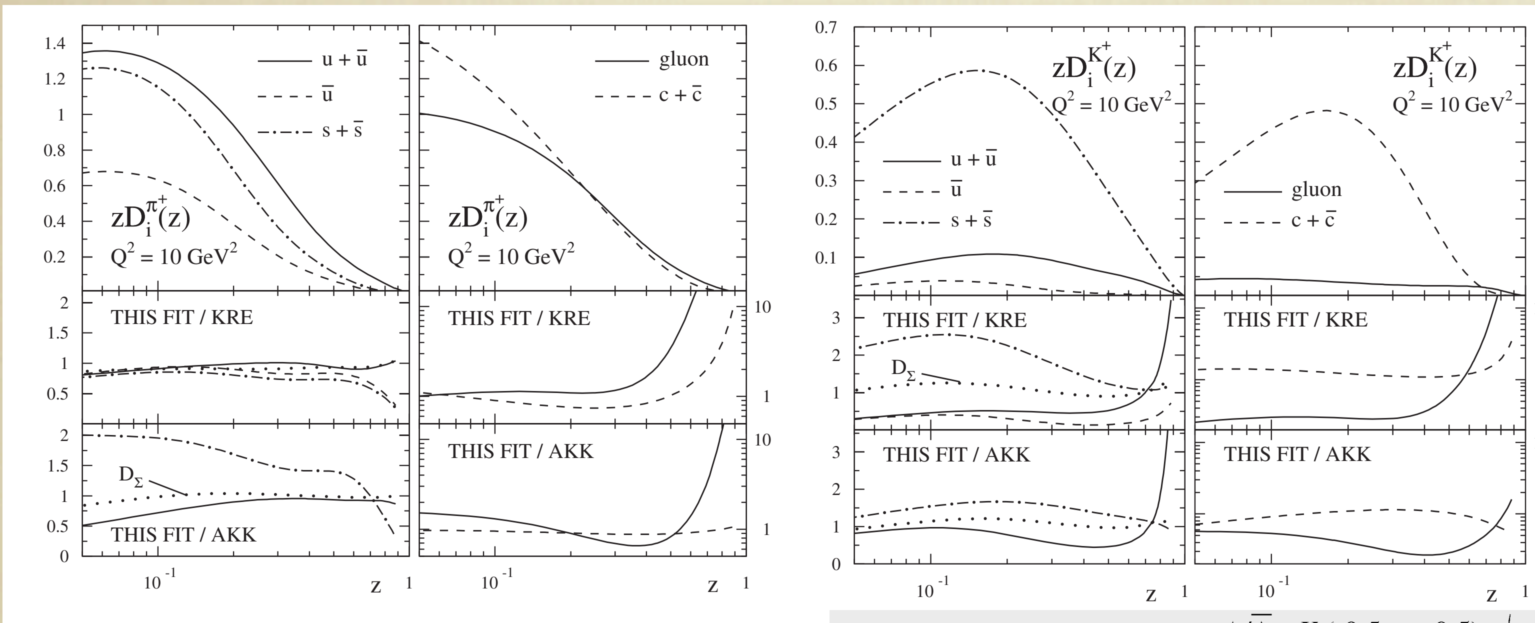

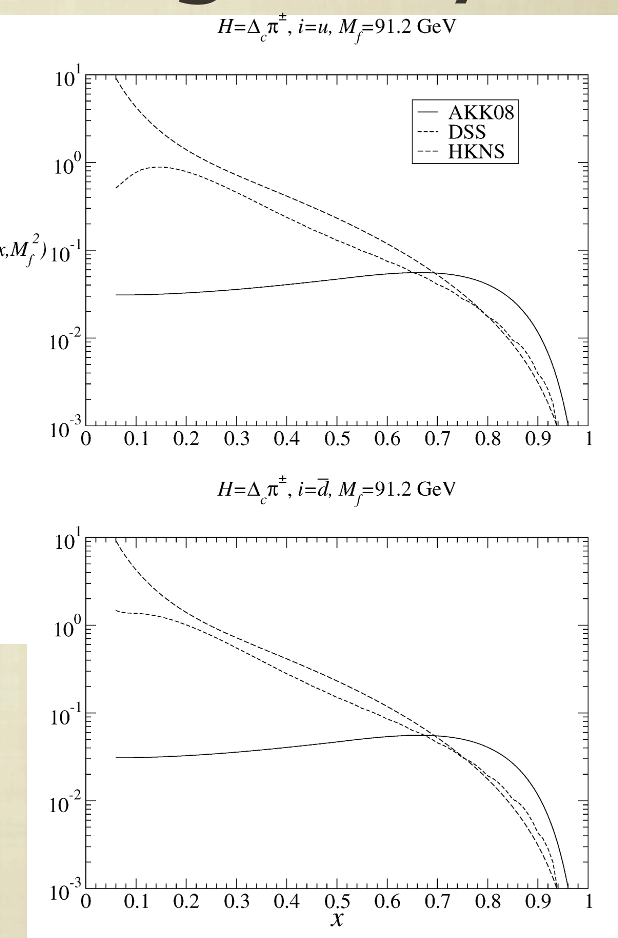

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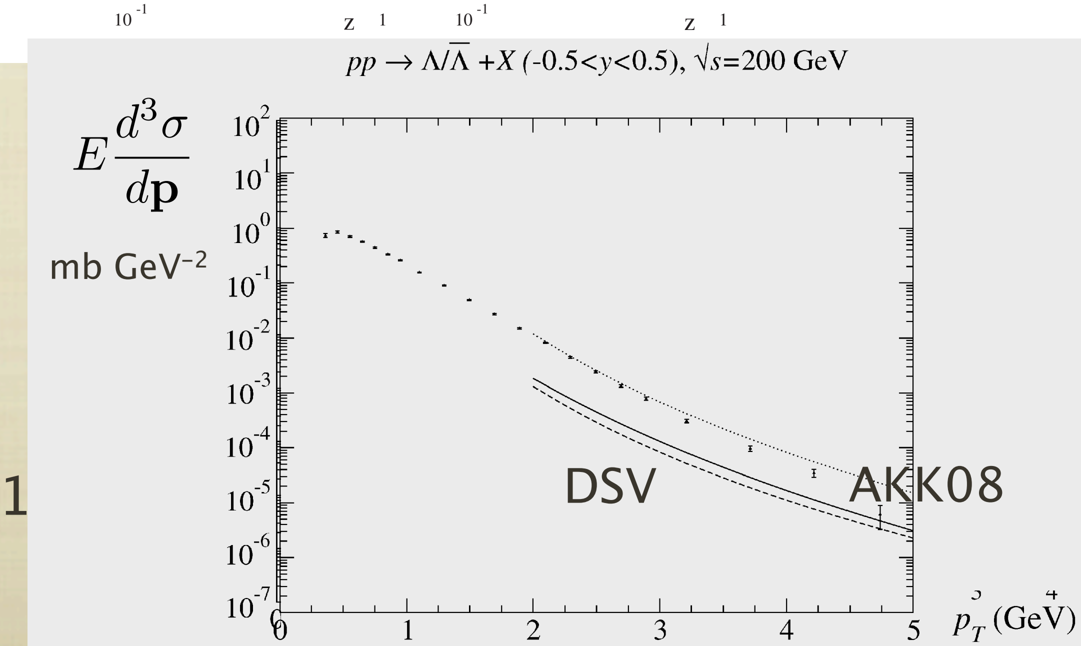
STAR Coll., P.R.C **75** (07) 064901



future of parametrizations

- towards NNLO analysis

Almasy, Moch, Vogt, arXiv:1107.2263 [hep-ph]
Albino et al., arXiv:1108.3948 [hep-ph]

$$\frac{dD_i^h(z,Q^2)}{d\ln Q^2} = \sum_{i=q,\bar{q},g} P_{ji}(z,Q^2) \otimes D_j^h(z,Q^2)$$

non-singlet $\mathcal{O}(\alpha_s^3)$

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– $\sigma^{K^\pm} - 2\sigma^{K_s^0} = [C_u - C_d] \otimes D_{u-d}^{K^\pm}$ at any order for SU(2) sym.

NS $K^\pm$ FF directly from data with NNLO C_i
but data put not enough constrains yet

Albino, Christova, P.R.D**81** (10) 094031

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– determine "non-perturbative" error from FF

➤ need a common interface like LHAPDF
at present only http://www.pv.infn.it/~radici/FFdatabase

what about 1h TMD FF ?

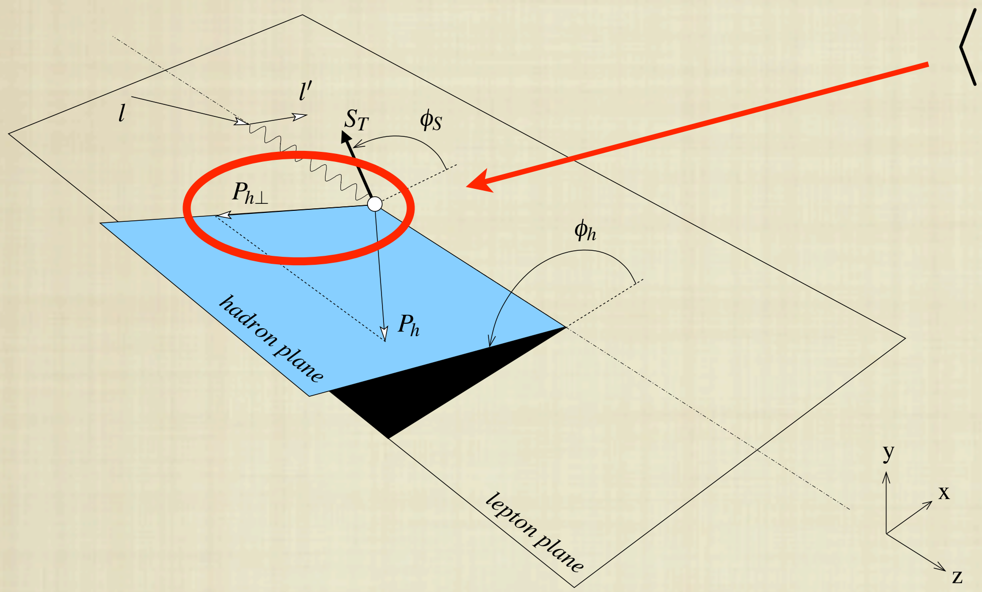
Gaussian ansatz for SIDIS $d\sigma_{UU}$

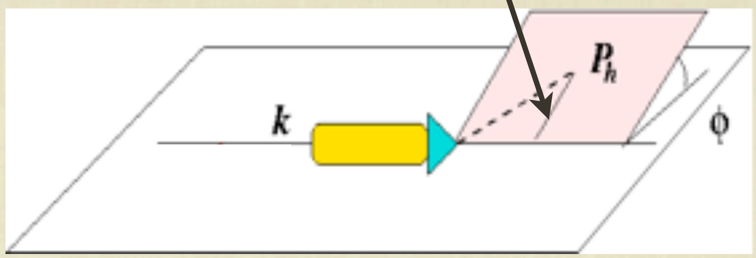
$$f_1^q(x, \mathbf{p}_T) = f_1^q(x) \frac{\exp\left[-\mathbf{p}_T^2/\langle \mathbf{p}_T^2 \rangle\right]}{\pi \langle \mathbf{p}_T^2 \rangle}$$

$$D_1^q(z, \mathbf{K}_T) = D_1^q(z) \frac{\exp\left[-\mathbf{K}_T^2/\langle \mathbf{K}_T^2 \rangle\right]}{\pi \langle \mathbf{K}_T^2 \rangle}$$

$$\longrightarrow \quad \frac{d\sigma_{UU}(P_{h\perp})}{dz d\mathbf{P}_{h\perp}^2} = \frac{d\sigma_{UU}(0)}{dz d\mathbf{P}_{h\perp}^2} \exp\left[-\mathbf{P}_{h\perp}^2/\langle \mathbf{P}_{h\perp}^2 \rangle\right]$$

$$\langle \mathbf{P}_{h\perp}^2 \rangle = z^2 \langle \mathbf{p}_T^2 \rangle + \langle \mathbf{K}_T^2 \rangle \qquad \mathbf{K}_T = -z \mathbf{k}_T$$





$\langle \mathbf{p}_T^2 \rangle = 0.25$, $\langle \mathbf{K}_T^2 \rangle = 0.20$ GeV^2 by fitting Cahn effect in EMC data ('83)

(Anselmino et al., P.R.D**71** (05) 074006)

$\langle \mathbf{p}_T^2 \rangle = 0.33$, $\langle \mathbf{K}_T^2 \rangle = 0.16$ GeV^2 by reproducing HERMES $\langle P_{h\perp} \rangle$ data ('98–'00)

(Collins et al., P.R.D**73** (06) 014021)

used in many phenomenological studies, <u>but</u>...

$\langle \mathbf{p}_T^2 \rangle = 0.25 \;,\; \langle \mathbf{K}_T^2 \rangle = 0.20 \; \mathrm{GeV}^2$ $A_{UU}^{\cos\phi}$ in EMC not only from Cahn effect

$\langle \mathbf{p}_T^2 \rangle = 0.33 \;,\; \langle \mathbf{K}_T^2 \rangle = 0.16 \; \mathrm{GeV}^2$ HERMES data not corrected for acceptance effects

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Since 2007, new data (including cosφ and cos2φ) from JLab, HERMES, COMPASS

combined analysis of SIDIS and (old+new) DY data

(Schweitzer, Teckentrup, Metz, P.R.D**81** (10) 094019)

➤ new parameters $\langle \mathbf{p}_T^2 \rangle = 0.38 \pm 0.06\ ,\ \langle \mathbf{K}_T^2 \rangle = 0.16 \pm 0.01\ \mathrm{GeV}^2$

➤ various tests of Gaussian ansatz

➤ p_T and K_T broadening with s ↗ $\langle \mathbf{p}_T^2(s) \rangle = 0.3 + C_h\, s$

$$C_p = 7 \times 10^{-4}$$
$$C_\pi = 2.1 \times 10^{-3}$$

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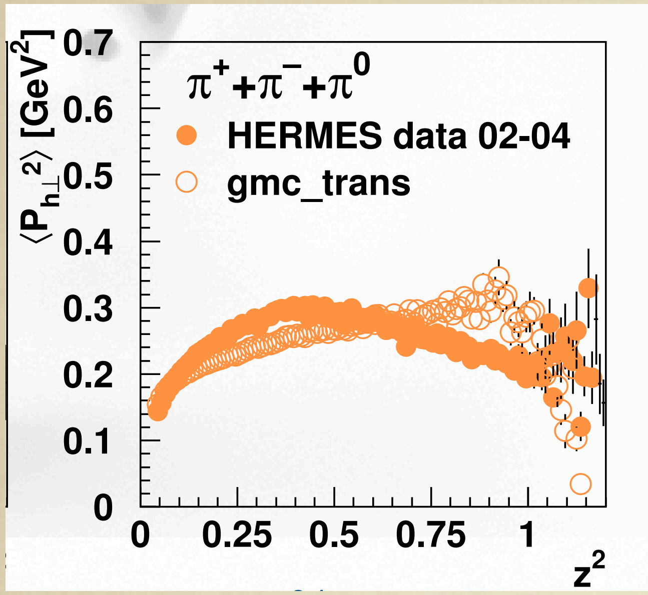
$$\langle \mathbf{p}_T^2(s) \rangle = 0.3 + C_h\, s$$

$$\begin{aligned} C_p &= 7 \times 10^{-4} \\ C_\pi &= 2.1 \times 10^{-3} \end{aligned}$$

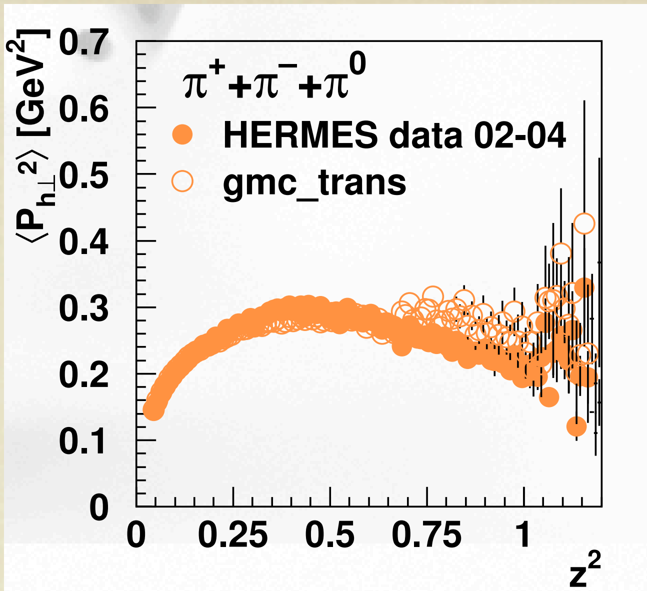
BUT...

➤ gmc_trans MC (Schnell, ECT* '07)

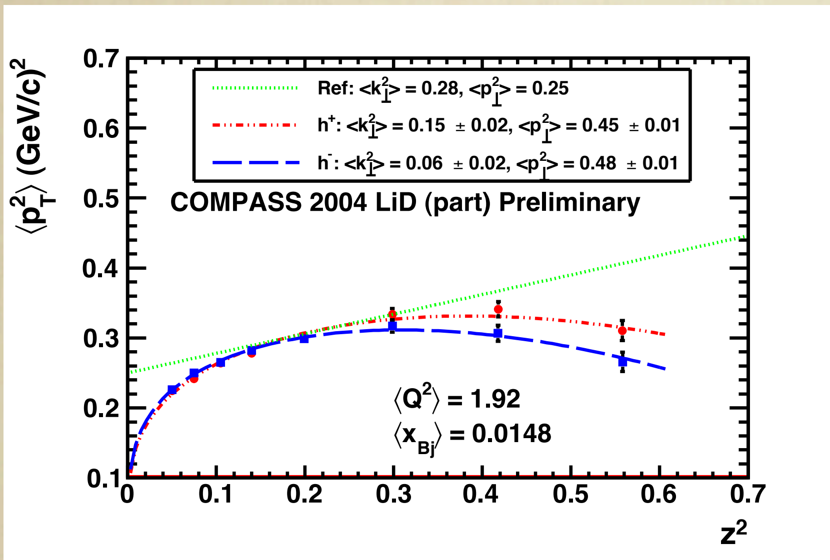
$\langle \mathbf{P}_{h\perp}^2 \rangle = z^2 \langle \mathbf{p}_T^2 \rangle + \langle \mathbf{K}_T^2 \rangle$



$\langle \mathbf{P}_{h\perp}^2 \rangle = z^2 \langle \mathbf{p}_T^2 \rangle + \langle \mathbf{K}_T^2(z) \rangle$



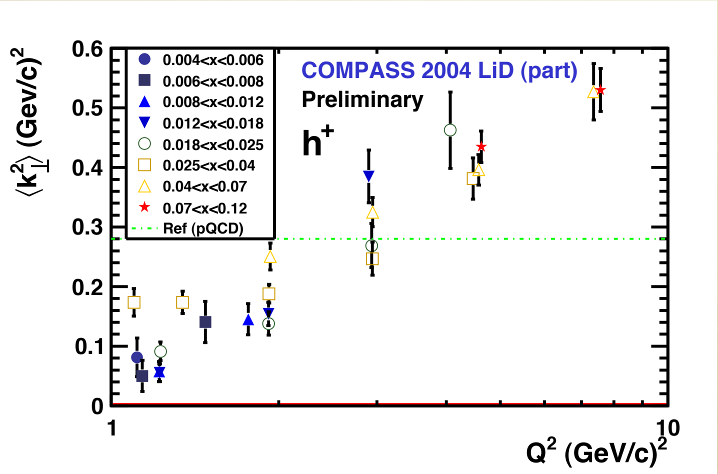
$$\langle \mathbf{p}_T^2 \rangle = 0.14 \;\; \mathrm{GeV}^2$$
$$\langle \mathbf{K}_T^2 \rangle = 0.42\, z^{0.54} (1-z)^{0.37} \;\; \mathrm{GeV}^2$$

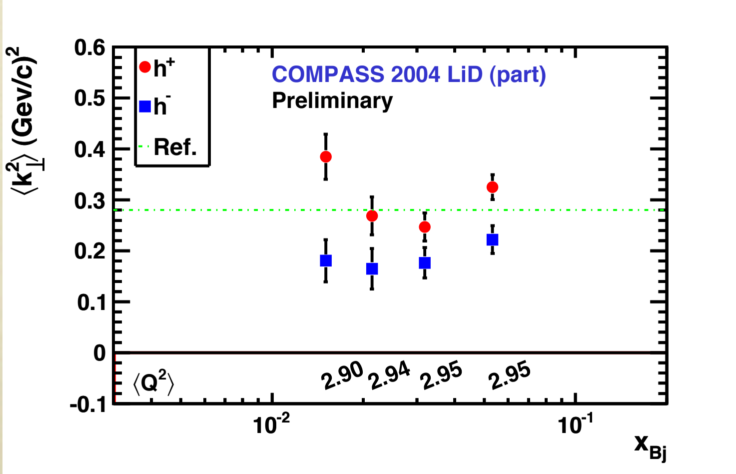


➤ similarly COMPASS (Rajotte, arXiv:1008.5125)

$$\langle \mathbf{p}_T^2 \rangle_{h+} = 0.15 \;\; \mathrm{GeV}^2$$
$$\langle \mathbf{K}_T^2 \rangle_{h+} = 0.45\, z^{0.5} (1-z)^{1.5} \;\; \mathrm{GeV}^2$$
$$\langle \mathbf{p}_T^2 \rangle_{h-} = 0.06 \;\; \mathrm{GeV}^2$$
$$\langle \mathbf{K}_T^2 \rangle_{h-} = 0.48\, z^{0.5} (1-z)^{1.5} \;\; \mathrm{GeV}^2$$

fit with constant $\langle p_T^2 \rangle, \langle K_T^2 \rangle$





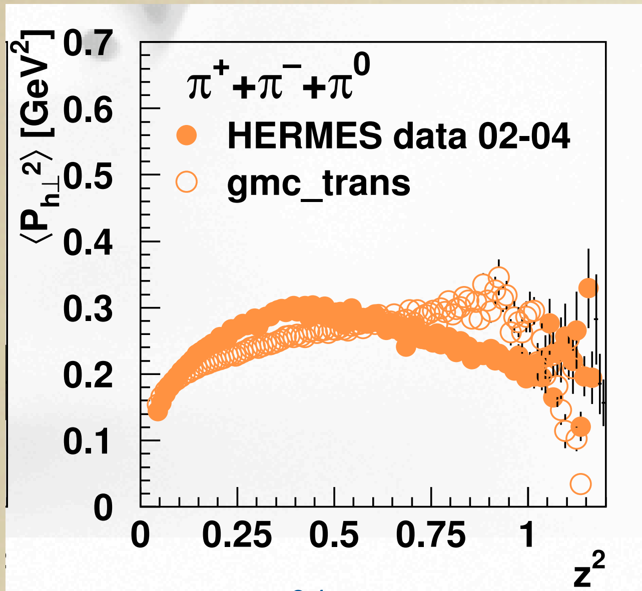
also

$\langle p_T^2(x,Q^2) \rangle_{h+} \neq \langle p_T^2(x,Q^2) \rangle_{h-}$

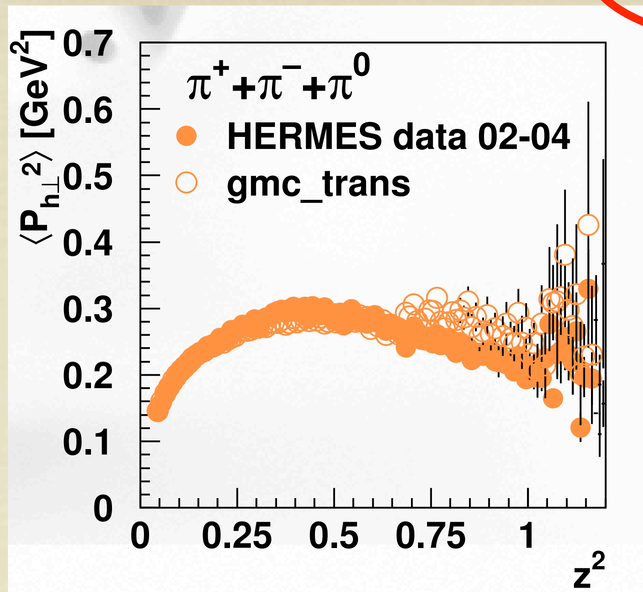
Moreover,...

➤ gmc_trans MC (Schnell, ECT* '07)

$\langle \mathbf{P}_{h\perp}^2 \rangle = z^2 \langle \mathbf{p}_T^2 \rangle + \langle \mathbf{K}_T^2 \rangle$



$\langle \mathbf{P}_{h\perp}^2 \rangle = z^2 \langle \mathbf{p}_T^2 \rangle + \langle \mathbf{K}_T^2(z) \rangle$



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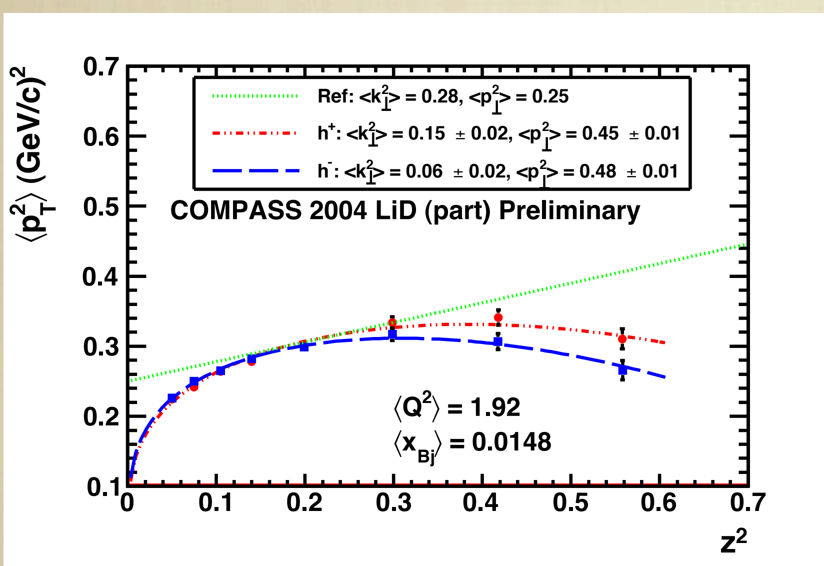
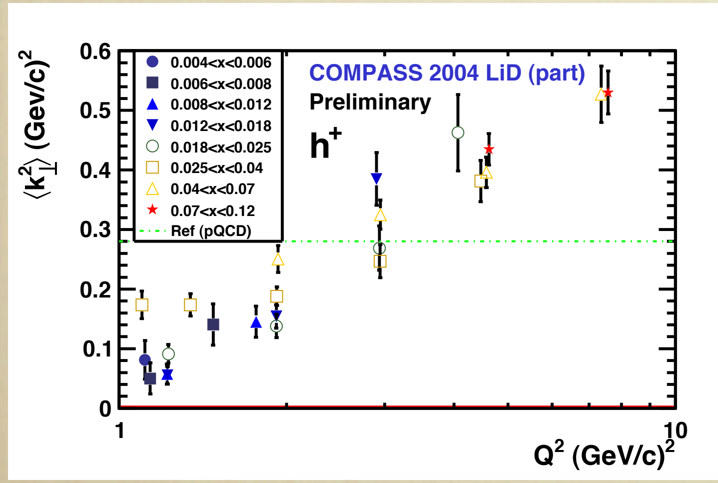
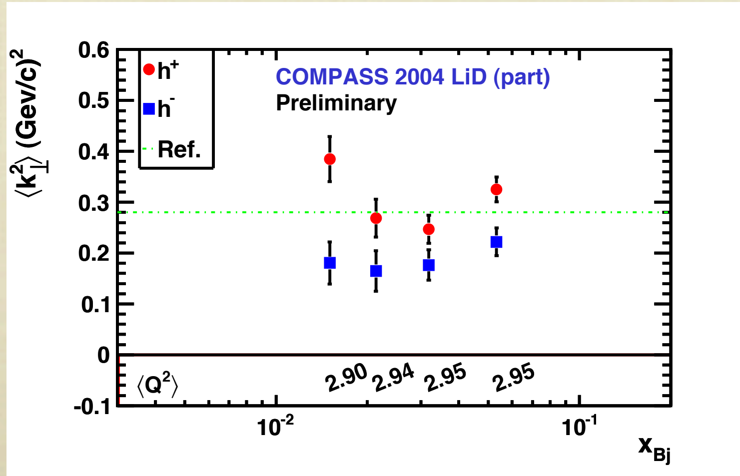


➤ similarly COMPASS (Rajotte, arXiv:1008.5125)

$$
\begin{aligned}
\langle \mathbf{p}_T^2 \rangle_{h+} &= 0.15 \;\; \mathrm{GeV}^2 \\
\langle \mathbf{K}_T^2 \rangle_{h+} &= 0.45 \, z^{0.5} \, (1-z)^{1.5} \;\; \mathrm{GeV}^2 \\
\langle \mathbf{p}_T^2 \rangle_{h-} &= 0.06 \;\; \mathrm{GeV}^2 \\
\langle \mathbf{K}_T^2 \rangle_{h-} &= 0.48 \, z^{0.5} \, (1-z)^{1.5} \;\; \mathrm{GeV}^2
\end{aligned}
$$

fit with constant $\langle p_T^2 \rangle, \langle K_T^2 \rangle$





also

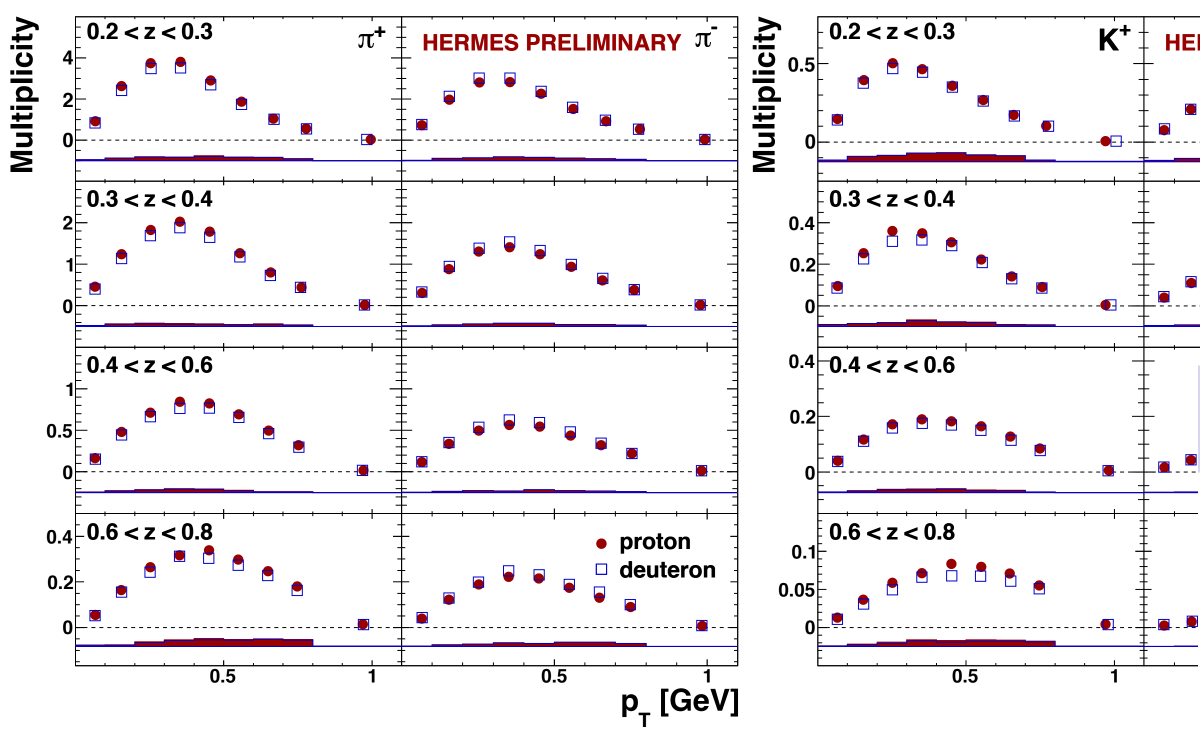
$\langle p_T^2(x,Q^2) \rangle_{h+} \neq \langle p_T^2(x,Q^2) \rangle_{h-}$

Moreover,...

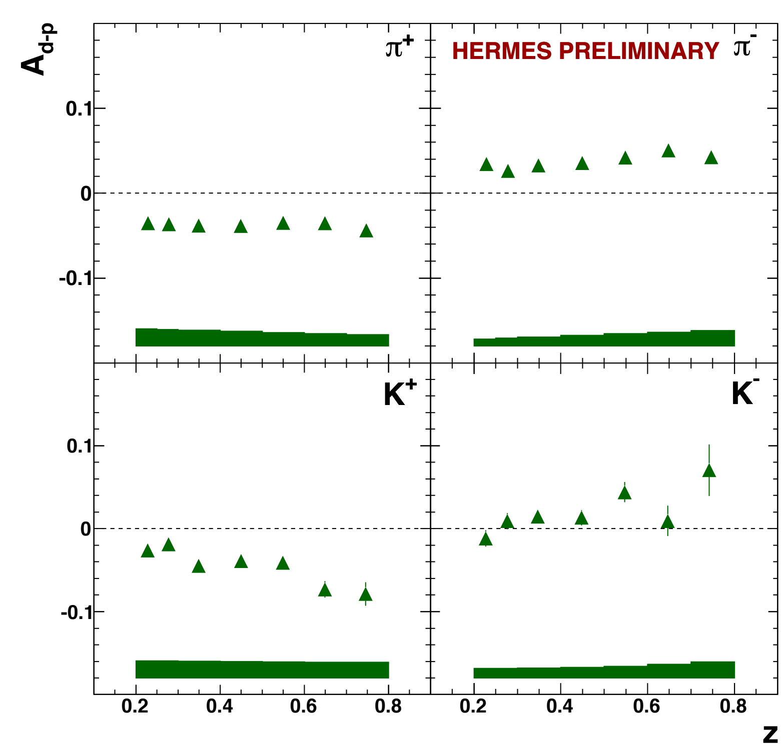
➤ HERMES multiplicity (Joosten, DIS 2011)

Results: Projections vs zp_T

- Disentanglement of z and p_T
- Access to the transverse intrinsic quark p_T and fragmentation k_T.



Proton-deuteron multiplicity asymmetry



definition:

$$A^h_{d-p} \equiv \frac{\mathcal{M}^h_d - \mathcal{M}^h_p}{\mathcal{M}^h_d + \mathcal{M}^h_p}$$

- Reflects different valence quark content
- Improved precision by cancellations in the systematic uncertainty

..., moreover,...

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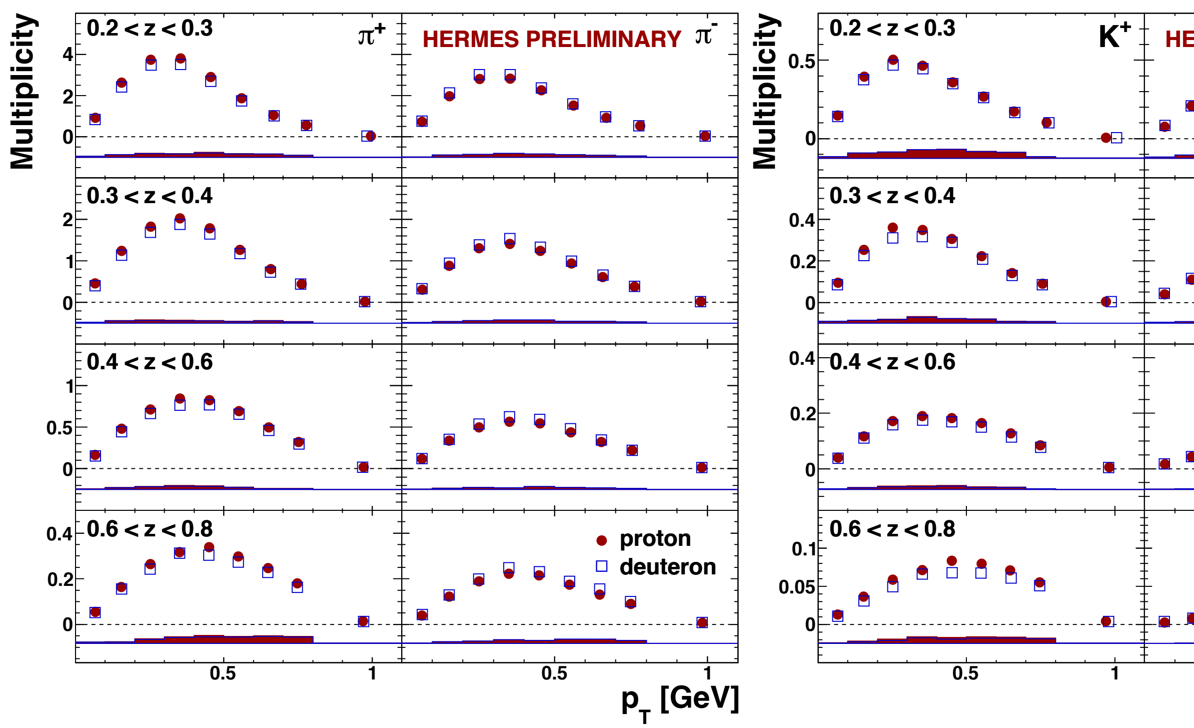
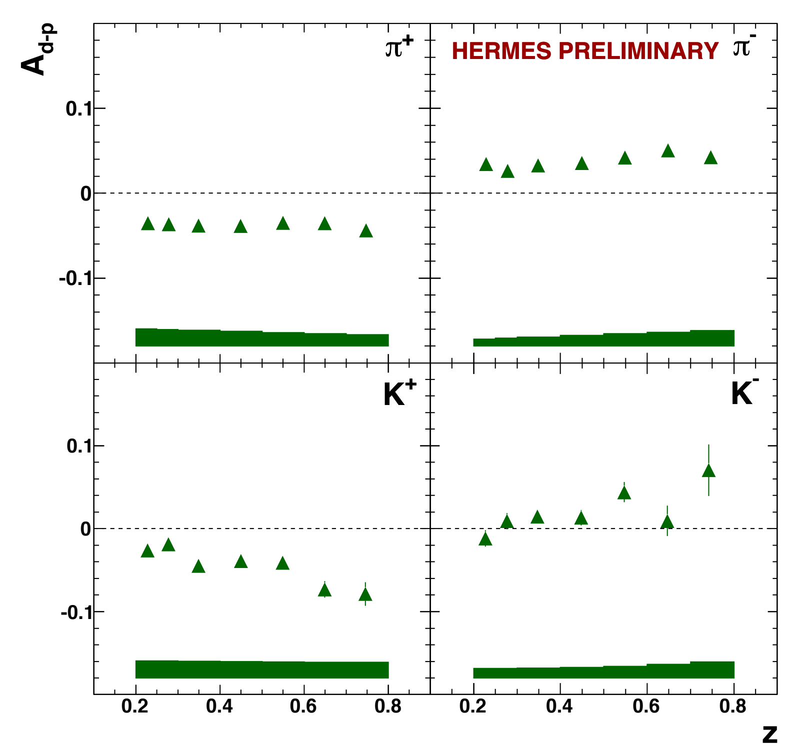


..., moreover,...

Proton-deuteron multiplicity asymmetry



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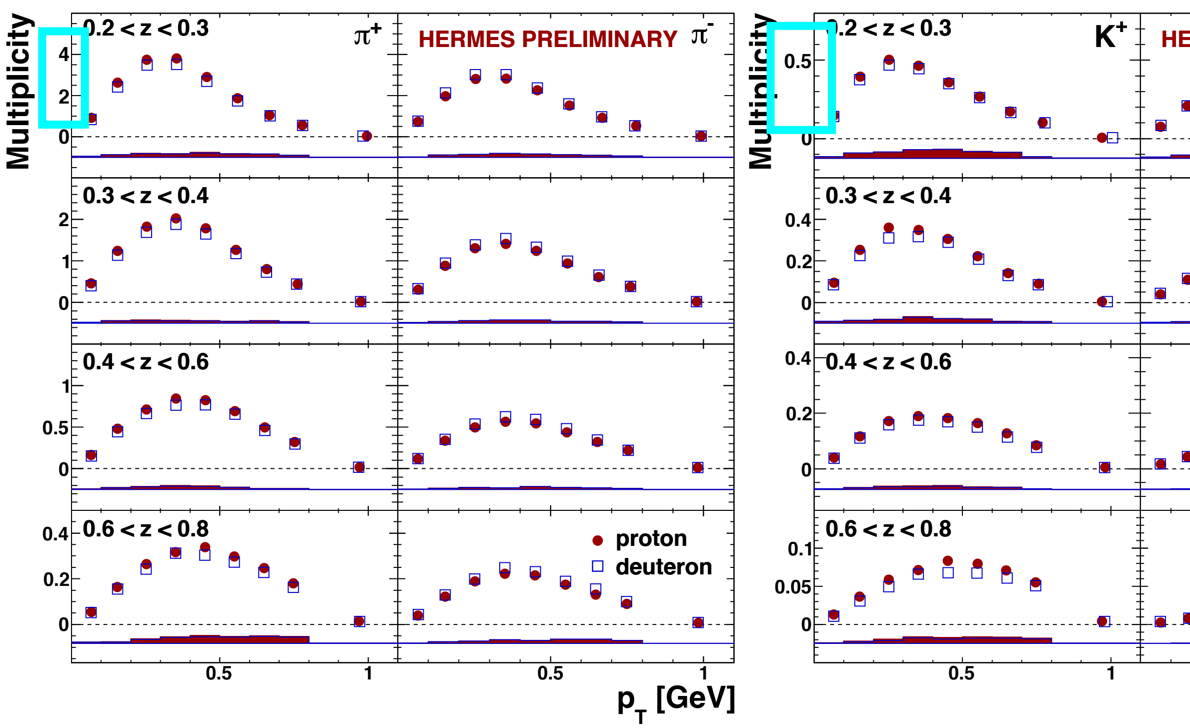
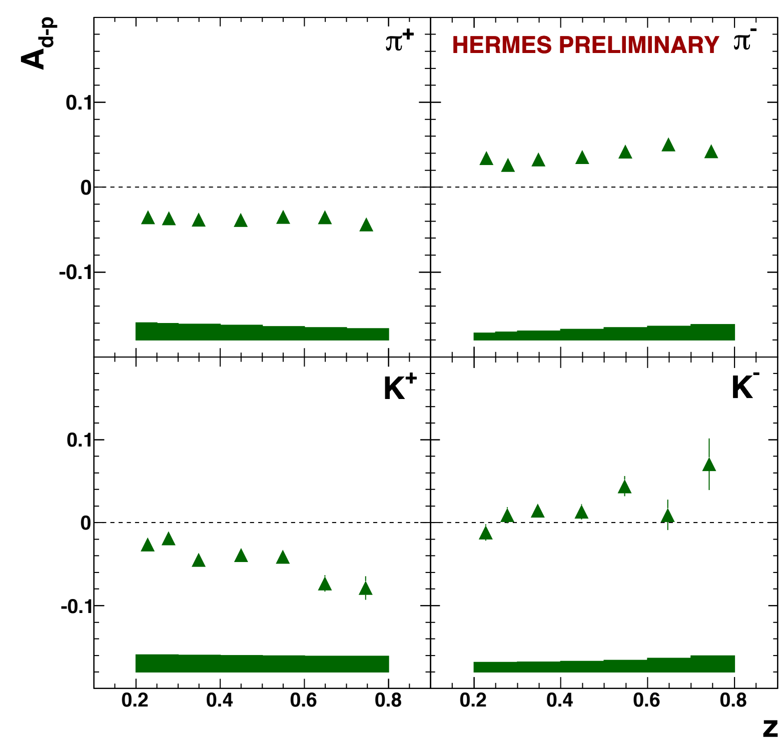


evidence for flavor dependence

Proton-deuteron multiplicity asymmetry



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- Improved precision by cancellations in the systematic uncertainty

..., moreover,...

1h TMD FF evolution

(Rogers & Aybat, P.R.D**83** (11) 114042)

in config. space

$$D_i^h(z,\mathbf{b}_T;Q,\zeta) = \quad A \quad \times \quad B \quad \times \quad C$$

1h TMD FF evolution

(Rogers & Aybat, P.R.D**83** (11) 114042)

in config. space

$$D_i^h(z,\mathbf{b}_T;Q,\zeta) = \quad A \quad \times \quad B \quad \times \quad C$$

$$\updownarrow$$

$$\sum_j C_{ij} \otimes D_j^h(z)$$

at $b_T \ll 1/\Lambda_{QCD}$

1h TMD FF evolution

(Rogers & Aybat, P.R.D**83** (11) 114042)

in config. space

$D_i^h(z,\mathbf{b}_T;Q,\zeta)$ = A × B × C

A ↕ $\sum_j C_{ij} \otimes D_j^h(z)$ at $b_T \ll 1/\Lambda_{QCD}$

B ↕ ~ exp[anom. dim.]

1h TMD FF evolution

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C ↔ $e^{-g(Q)\, \mathbf{b}_T^2}$

nonperturb.
g universal
scale dep.
at low K_T

1h TMD FF evolution

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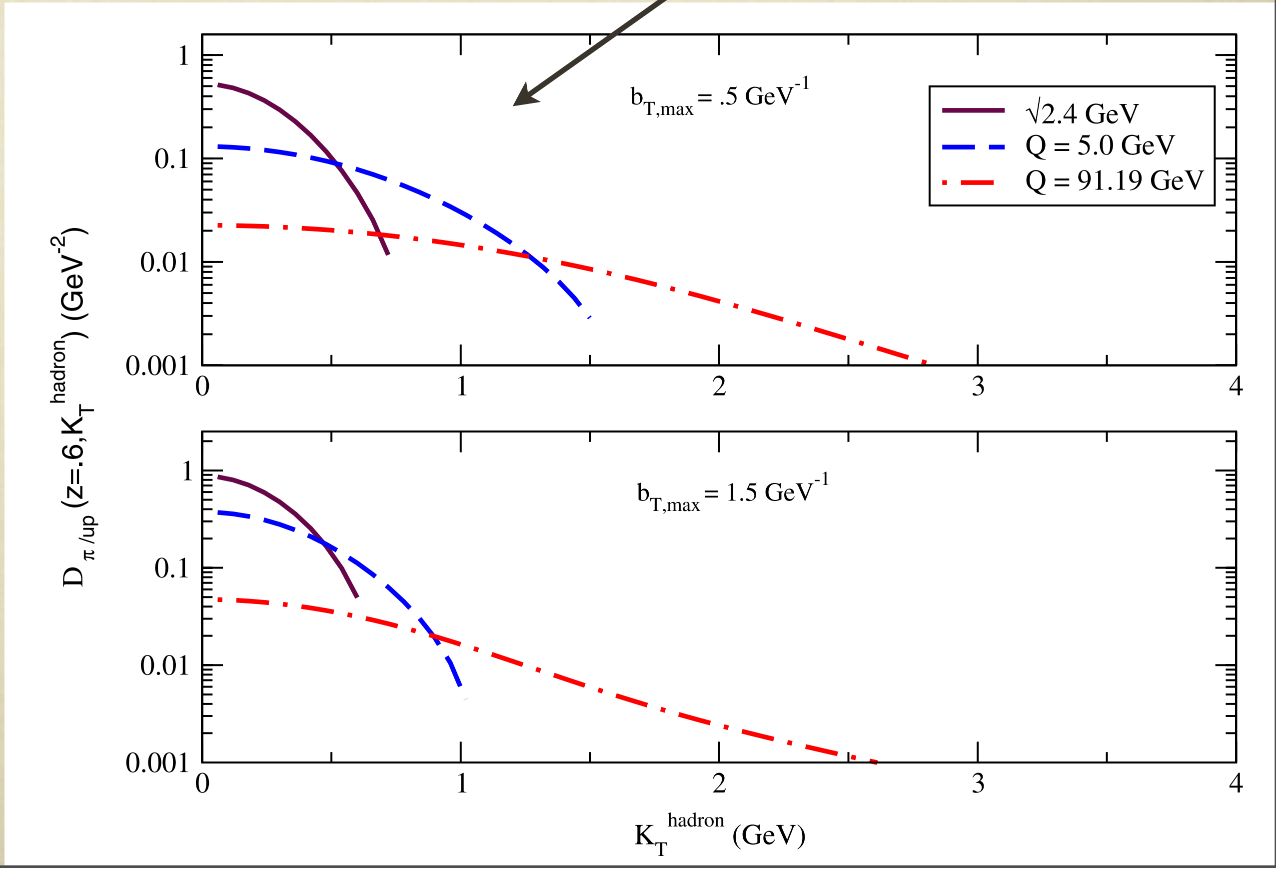
scale dep. at low K_T

$o(\alpha_s^0)$ ≈ $D_i^h(z)$ $\exp[-g(Q)\,\mathbf{b}_T^2]$

Gaussian ← TMD FF → BLNY fit ⇒ fix g(Q)

$Q_0^2=2.4$ $Q^2=M_Z^2$

$u \to \pi^+$



1h TMD FF evolution

(Rogers & Aybat, P.R.D**83** (11) 114042)

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$D_i^h(z,\mathbf{b}_T;Q,\zeta) = A \times B \times C$

A ↔ $\sum_j C_{ij} \otimes D_j^h(z)$ at $b_T \ll 1/\Lambda_{QCD}$

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nonperturb.
g universal
scale dep.
at low K_T

$o(\alpha_s^0) \approx D_i^h(z) \quad \exp[-g(Q)\,\mathbf{b}_T^2]$

Gaussian ← TMD FF → BLNY fit ⇒ fix g(Q)

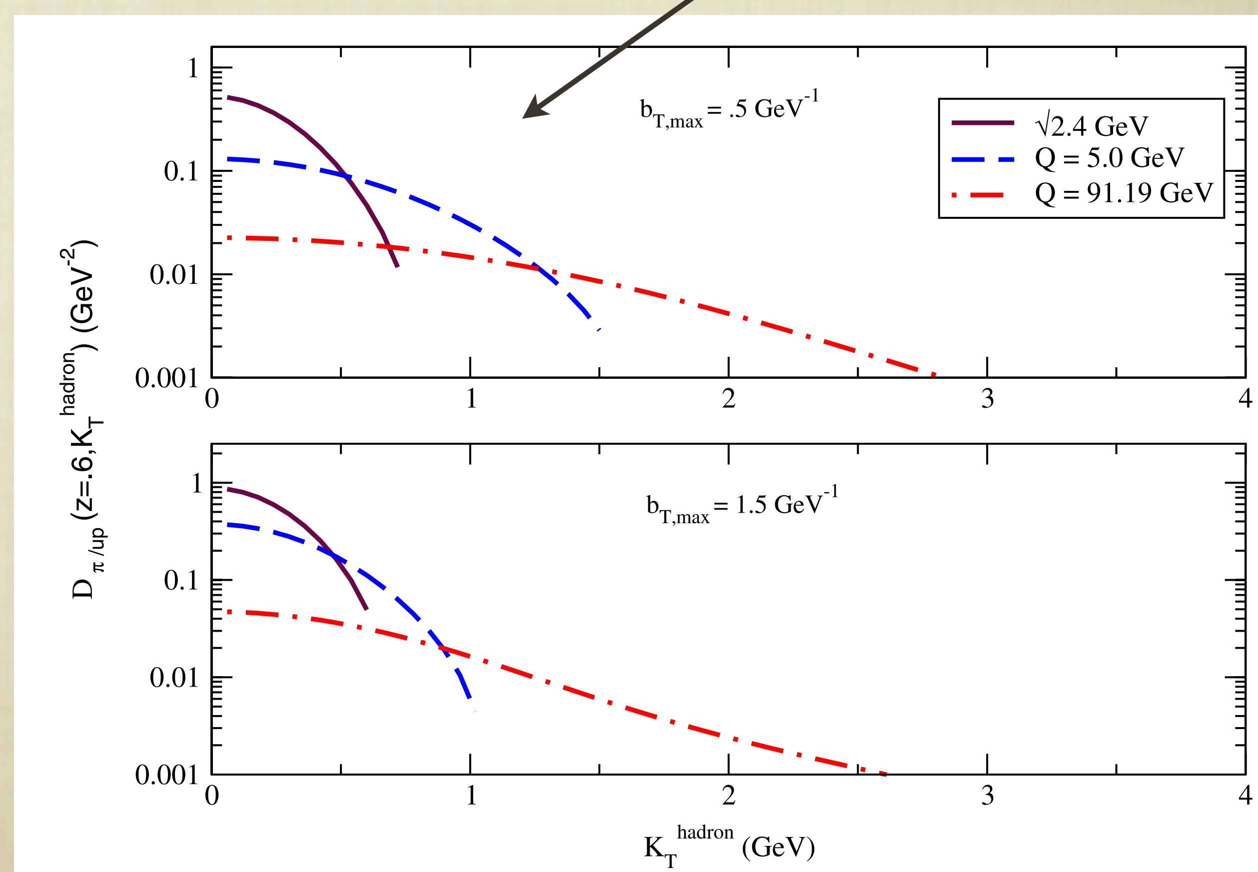
$Q_0^2=2.4$ $Q^2=M_Z^2$

strong evolution effects

$<k_T^2>^{1/2}(M_Z^2)$

b_{Tmax} GeV^{-1}	Gauss GeV	TMD FF GeV
0.5	1.74	2.15
1.5	1.06	1.85

$u \to \pi^+$

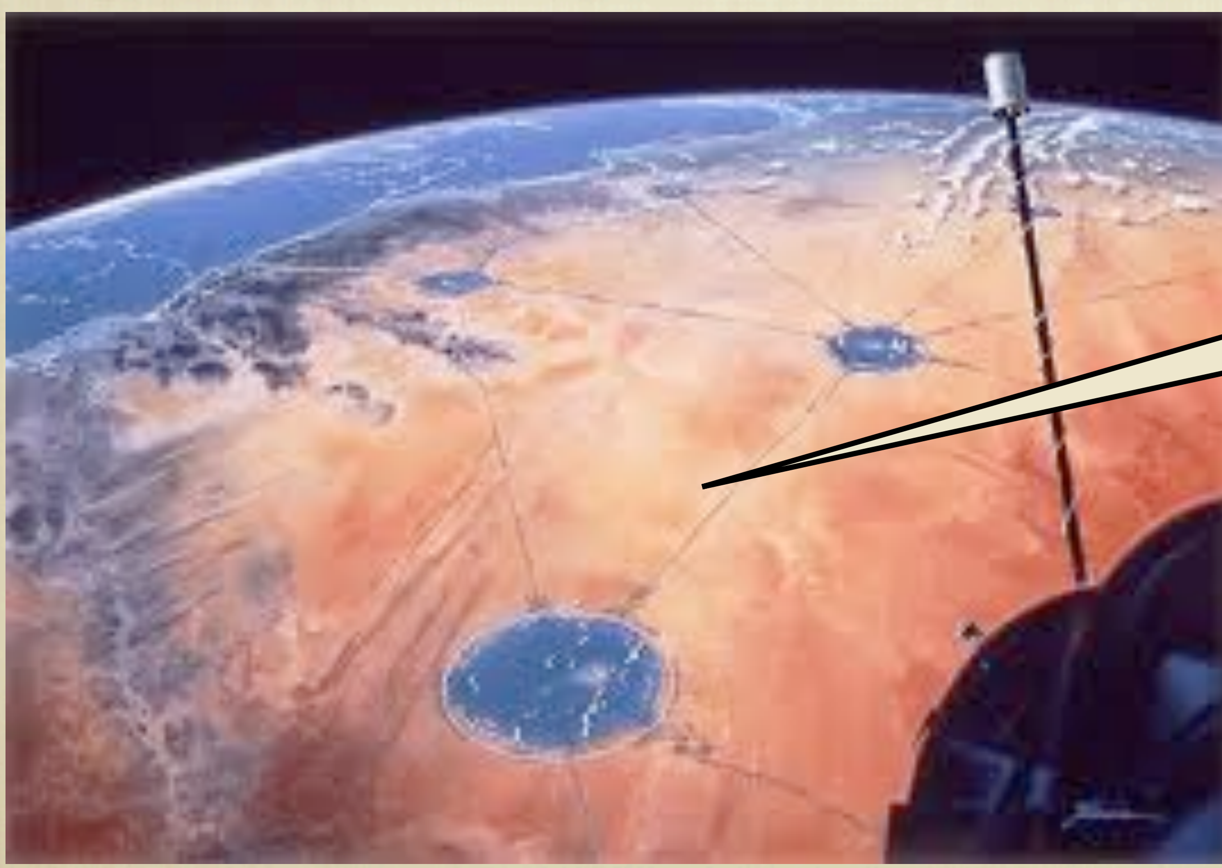


gaussian ansatz: too narrow point of view ?

It’s amazing...

gaussian ansatz: too narrow point of view ?

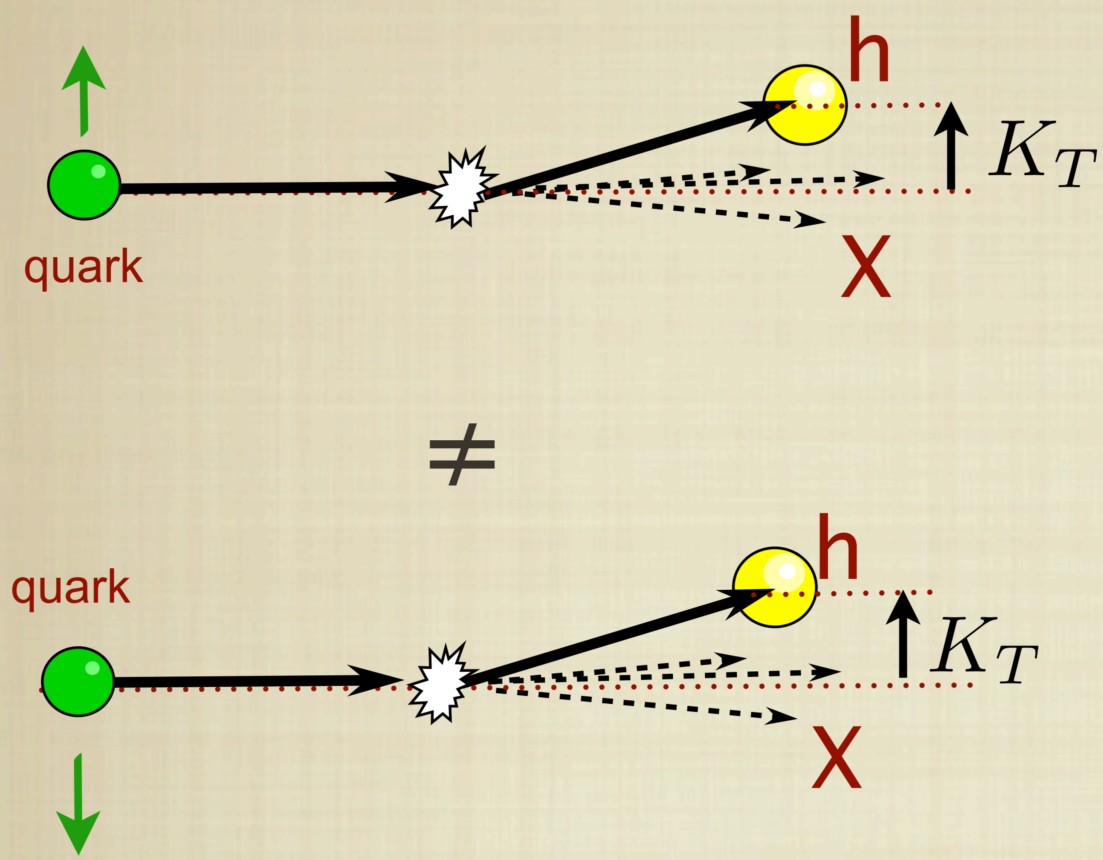
It's amazing...



... how the earth
looks so **FLAT** ...

polarized 1h TMD FF

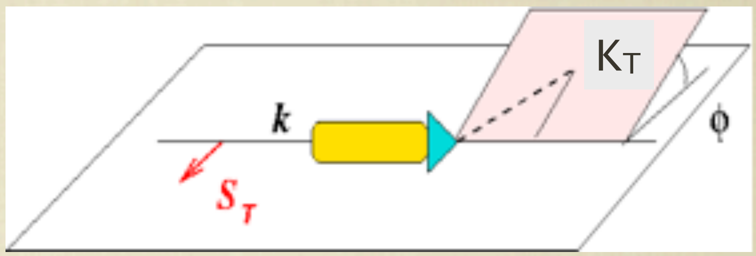
the Collins function



density of h produced by $q^\uparrow$

$$D_{h/q^\uparrow}(z, \mathbf{K}_T) = D_1^q(z, \mathbf{K}_T^2) + H_1^{\perp\, q}(z, \mathbf{K}_T^2)\, \frac{|\mathbf{K}_T|}{zM_h}\, \mathbf{S}_\perp^q \cdot \underbrace{(\hat{\mathbf{k}}_q \times \hat{\mathbf{K}}_T)}_{\sim\, \sin\phi}$$

transv. motion of h = spin analyzer of fragm. q



positivity bound

$$|H_1^{\perp\, q}(z, \mathbf{K}_T^2)|\, \frac{|\mathbf{K}_T|}{zM_h} \leq D_1^q(z, \mathbf{K}_T^2)$$

Schäfer-Teryaev sum rule

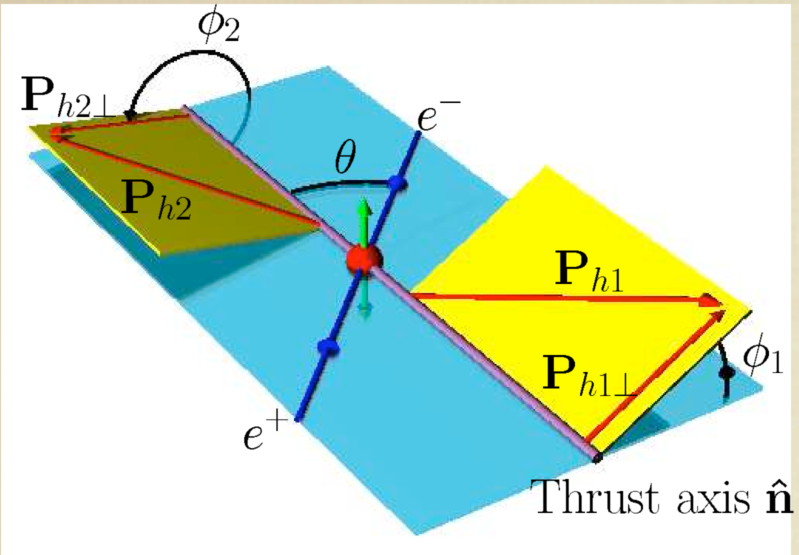
$$\sum_{h, S_h} \int_0^1 dz\, zM_h\, H_1^{\perp\, (1) q}(z) = 0$$

Schäfer & Teryaev, P.R.D**61** (00) 077903
Meissner, Metz, Pitonyak, P.L.**B690** (10) 296

$$H_1^{\perp\, (n) q}(z) = \int d\mathbf{K}_T \frac{1}{2} \left(\frac{\mathbf{K}_T^2}{z^2 M_h^2} \right)^n H_1^{\perp\, q}(z, \mathbf{K}_T^2)$$

extraction of Collins function

$e^+e^- \to \pi^+\pi^- X$



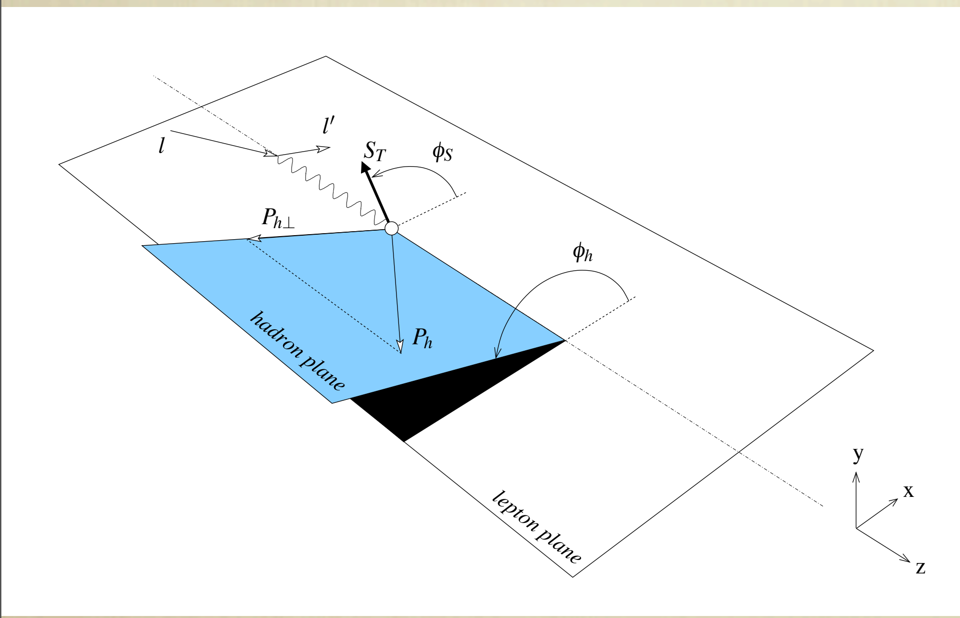
$$A^{\cos(\phi_1+\phi_2)}(\cos\theta, z, \bar{z}) = \frac{\sin^2\theta}{1+\cos^2\theta} \, \frac{\sum_q e_q^2 \, H_{1,q\to h_1}^{\perp(1/2)}(z) \, H_{1,\bar{q}\to h_2}^{\perp(1/2)}(\bar{z})}{\sum_q e_q^2 \, D_{1,q\to h_1}(z) \, D_{1,\bar{q}\to h_2}(\bar{z})}$$

old data: Abe et al. (Belle), P.R.L.**96** (06) 232002

new data: Seidl et al. (Belle), P.R.D**78** (08) 032011

"thrust axis" method, or Collins-Soper frame
also "cos(2ϕ_0)" method, or Gottfried-Jackson frame

in combination with SIDIS



$$A_{UT}^{\sin(\phi_h+\phi_S)} \propto - \frac{\sum_q e_q^2 \left[h_1^q \otimes H_{1,q\to h}^{\perp} \right](x, z, P_{h\perp}^2)}{\sum_q e_q^2 \, f_1^q(x) \;\; D_{1,q\to\pi}(z)}$$

old data: Airapetian et al. (HERMES), P.R.L.**94** (05) 012002
Ageev et al. (COMPASS), N.P.**B765** (07) 31

new data: Diefenthaler et al. (HERMES), arXiv:0706.2242
Alekseev et al. (COMPASS), P.L.**B673** (09) 127

extraction of Collins function

$e^+e^- \to \pi^+\pi^- X$

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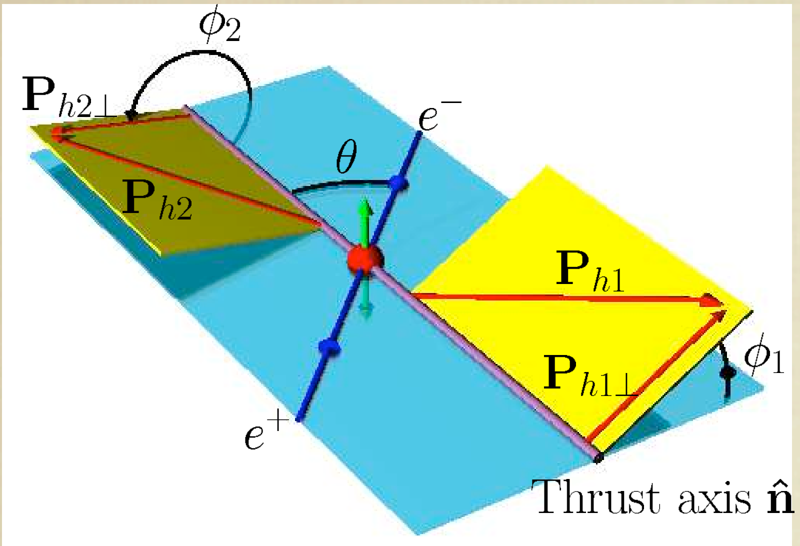

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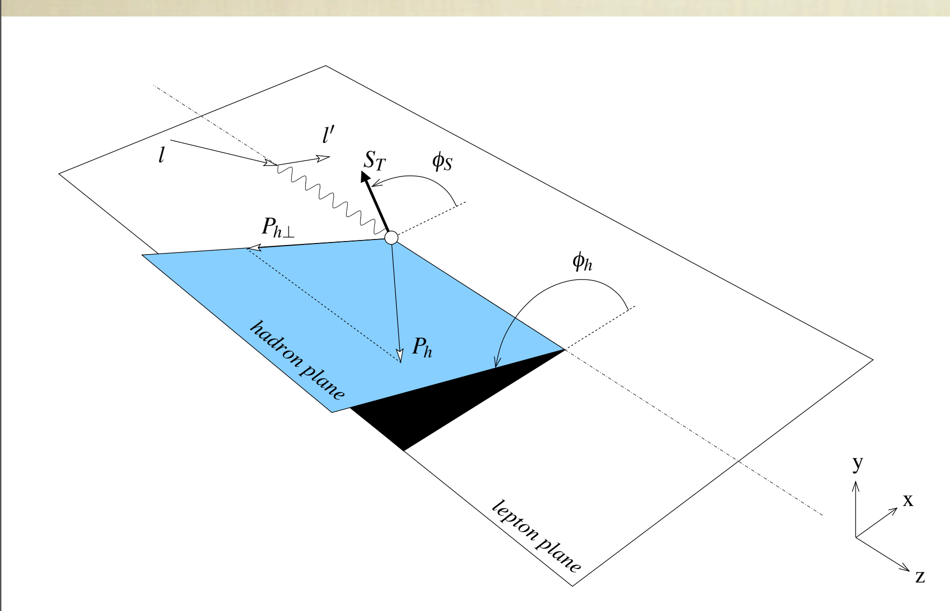
"thrust axis" method, or Collins-Soper frame
also "cos(2ϕ₀)" method, or Gottfried-Jackson frame

need
- factoriz. th. ☑
- universality ☑
- evolution ✘

in combination with SIDIS

$$A_{UT}^{\sin(\phi_h+\phi_S)} \propto -\frac{\sum_q e_q^2 \left[h_1^q \otimes H_{1,q\to h}^{\perp} \right](x,z,P_{h\perp}^2)}{\sum_q e_q^2\, f_1^q(x)\; D_{1,q\to\pi}(z)}$$



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extraction of Collins function

$$H_{1\,q\to h_1}^{\perp(1/2)}\,H_{1\,\bar{q}\to h_2}^{\perp(1/2)}$$

$$\Updownarrow$$

$$h_1^q \otimes H_{1\,q\to h}^{\perp}$$

Gaussian ansatz:

$$\begin{aligned} D_1^q(z,\mathbf{K}_T) &= D_1^q(z)\,\frac{\exp\left[-\mathbf{K}_T^2/\langle\mathbf{K}_T^2\rangle\right]}{\pi\langle\mathbf{K}_T^2\rangle} \\ H_1^{\perp\,q}(z,\mathbf{K}_T) &= F_{N^q,\gamma,\delta}(z)\,D_1^q(z)\,\frac{\exp\left[-\mathbf{K}_T^2/\langle\mathbf{K}_T^2\rangle_C\right]}{\pi\langle\mathbf{K}_T^2\rangle_C} \end{aligned}$$

extraction of Collins function

$$H_{1\,q\to h_1}^{\perp(1/2)}\, H_{1\,\bar{q}\to h_2}^{\perp(1/2)}$$

↕

$$h_1^q \otimes H_{1\,q\to h}^{\perp}$$

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$$H_1^{\perp\,q}(z,\mathbf{K}_T) = F_{N^q,\gamma,\delta}(z)\,D_1^q(z)\,\frac{\exp\left[-\mathbf{K}_T^2/\langle\mathbf{K}_T^2\rangle_C\right]}{\pi\langle\mathbf{K}_T^2\rangle_C}$$

2 channels: favoured , unfavoured

5 params: N^{fav}, N^{unfav}, γ, δ, M_H

extraction of Collins function

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- factoriz. th. ☑
- universality ☑
- evolution ✘ $\longleftrightarrow$ LO DGLAP for $D_1(z)$ and $H_1^{\perp(n)}(z) \sim D_1(z)$

extraction of Collins function

$$H^{\perp(1/2)}_{1\,q\to h_1}\; H^{\perp(1/2)}_{1\,\bar q\to h_2}$$

$$\updownarrow$$

$$h_1^q \otimes H^{\perp}_{1\,q\to h}$$

Gaussian ansatz:

$$D_1^q(z,\mathbf{K}_T) = D_1^q(z)\,\frac{\exp\left[-\mathbf{K}_T^2/\langle\mathbf{K}_T^2\rangle\right]}{\pi\langle\mathbf{K}_T^2\rangle}$$

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2 channels: favoured , unfavoured

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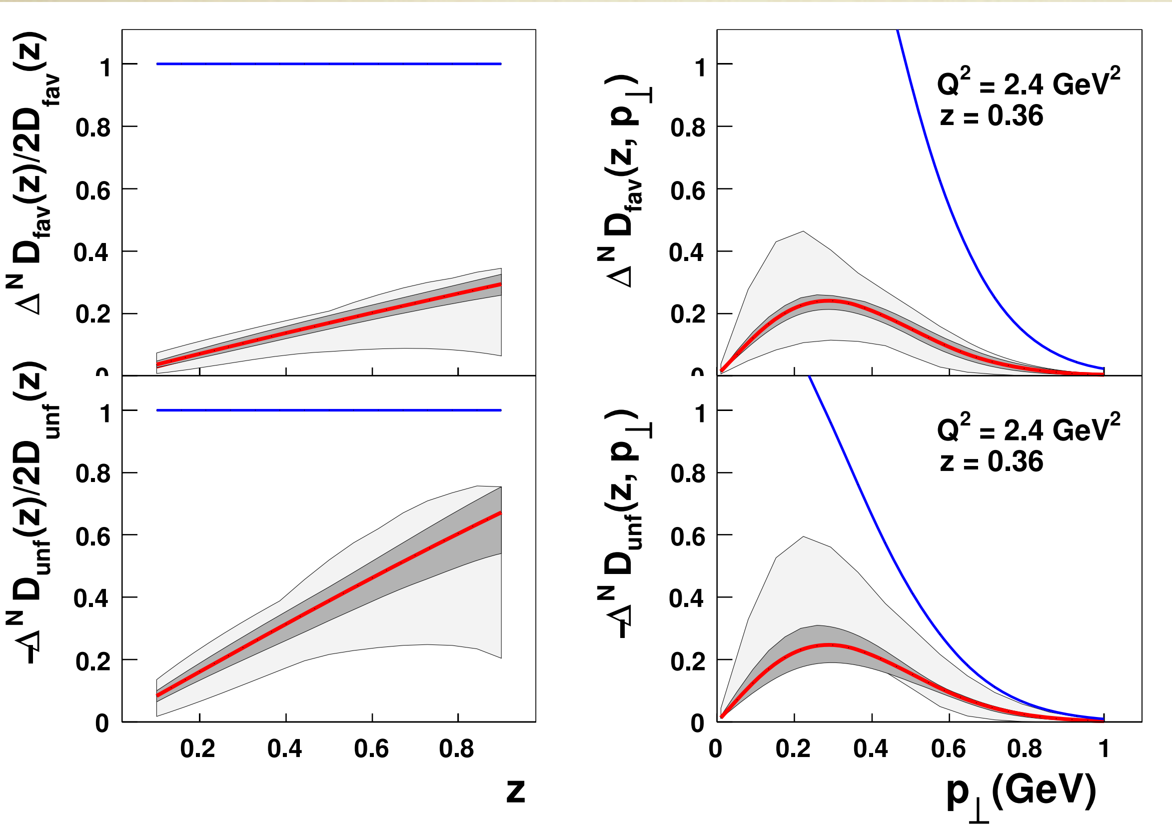
- factoriz. th. ☑
- universality ☑
- evolution ✘ ⟷ LO DGLAP for $D_1(z)$ and $H_1^{\perp(n)}(z) \sim D_1(z)$

$2\, H_1^{\perp(1/2)}\; /\; D_1$



old data ▭
Anselmino et al., P.R.D75 (07) 054032

new data ▭
Anselmino et al., N.P.**B191**(Pr.Sup.) (09) 98

positivity bound —

error band
$\Delta\chi^2 \approx 17$

see also Vogelsang & Yuan, P.R.D**72** (05) 054028
Efremov, Goeke, Schweitzer, P.R.D**73** (06) 094025

extraction of Collins function

$$H^{\perp(1/2)}_{1\,q\to h_1}\, H^{\perp(1/2)}_{1\,\bar{q}\to h_2}$$

$\updownarrow$

$$h_1^q \otimes H^{\perp}_{1\,q\to h}$$

Gaussian ansatz:

$$D_1^q(z,\mathbf{K}_T) \;=\; D_1^q(z)\,\frac{\exp\left[-\mathbf{K}_T^2/\langle\mathbf{K}_T^2\rangle\right]}{\pi\langle\mathbf{K}_T^2\rangle}$$

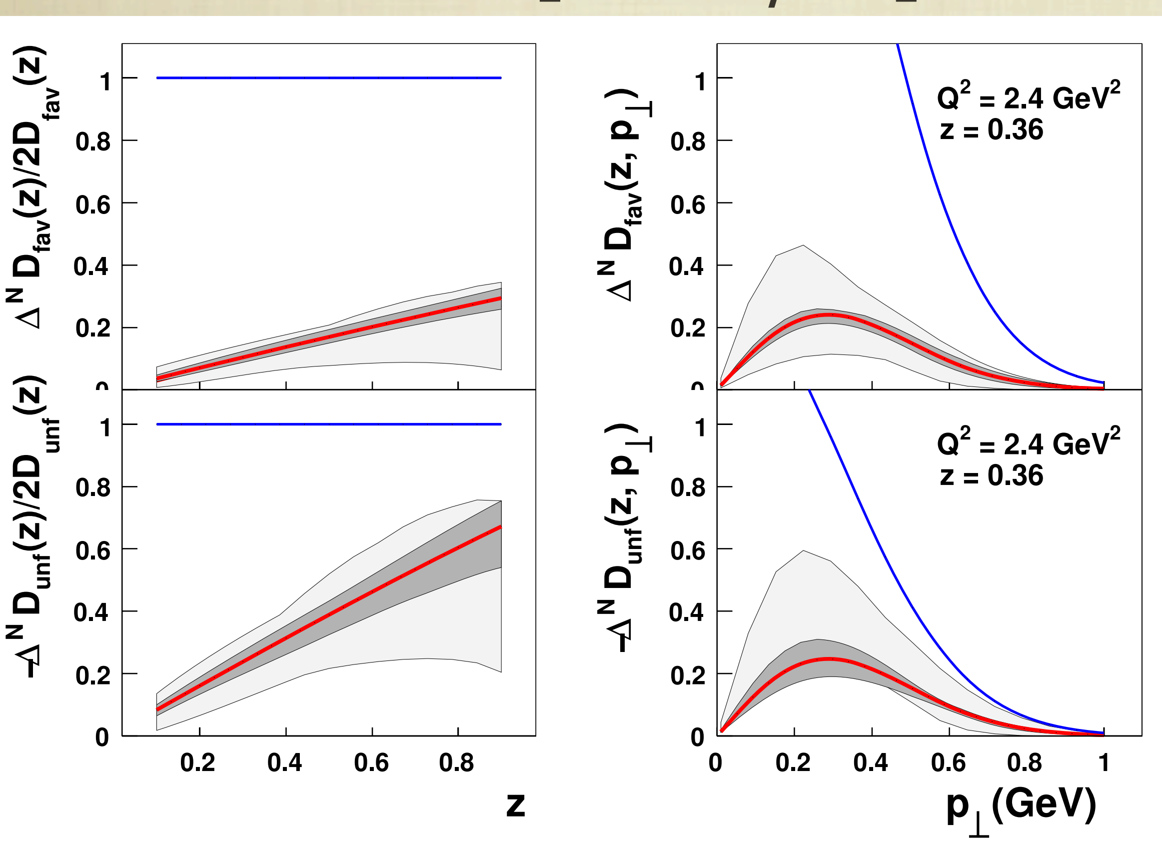
$$H_1^{\perp\, q}(z,\mathbf{K}_T) \;=\; F_{N^q,\gamma,\delta}(z)\, D_1^q(z)\,\frac{\exp\left[-\mathbf{K}_T^2/\langle\mathbf{K}_T^2\rangle_C\right]}{\pi\langle\mathbf{K}_T^2\rangle_C}$$

- factoriz. th. ☑
- universality ☑
- evolution ✘ ⟷ LO DGLAP for $D_1(z)$ and $H_1^{\perp(n)}(z) \sim D_1(z)$

2 channels: favoured , unfavoured

5 params: N^{fav}, N^{unfav}, γ, δ, M_H

$2\, H_1^{\perp(1/2)} \,/\, D_1$



➤ unfav. ≈ – fav.

old data
Anselmino et al., P.R.D75 (07) 054032

new data
Anselmino et al., N.P.**B191**(Pr.Sup.) (09) 98

error band
$\Delta\chi^2 \approx 17$

positivity bound

see also Vogelsang & Yuan, P.R.D**72** (05) 054028
Efremov, Goeke, Schweitzer, P.R.D**73** (06) 094025

But...

- access only to $H_1^{\perp(n)}(z) \Rightarrow$ K_T dep. unconstrained

  $<K_T^2>_C \neq <K_T^2>$ but flavor-/ z-/ Q^2-independent

- SIDIS kin.: $x \leq 0.3$, $0.2 \leq z \leq 0.7$, $Q^2 = 2.5$ (need EIC)

- only fav./unfav. flavors (u & d)

- LO DGLAP evolution of $H_1^{\perp(n)}(z) \sim D_1(z)$
  but the chiral-odd kernel of $H_1^{\perp(1)} \sim h_1$ (Kang, P.R.D**83** (11) 036006)

- full TMD evolution missing [$Q_{Belle}^2 \sim 100 \leftrightarrow Q_{SIDIS}^2 \sim 2.5$]

  $H_1^{\perp(1)}$ kernel: diag. piece ($\sim h_1$) + off-diag. piece (small ?)

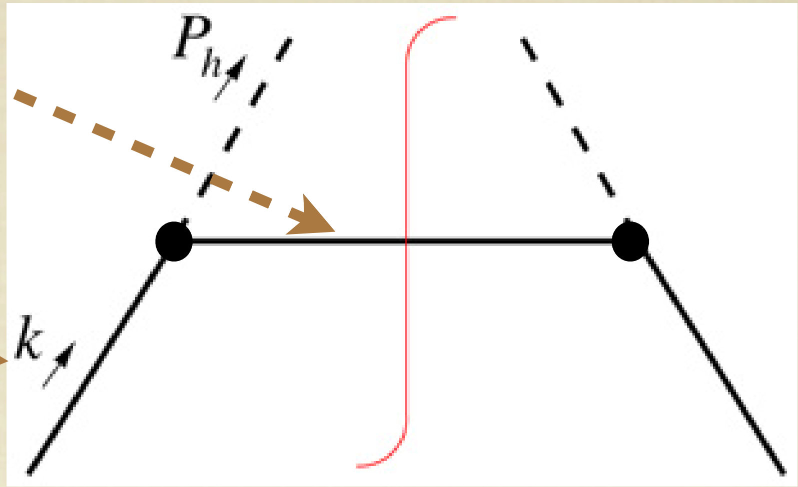
  $D_{1T}^{\perp(1)}$ kernel: diag. piece ($\sim D_1$) + off-diag. piece

(Kang, P.R.D**83** (11) 036006; see also Meissner, Metz, P.R.L.**102** (09) 172003; Yuan, Zhou, P.R.L.**103** (09) 052001)
Boer et al., P.R.L.**105** (10) 202001; Gamberg et al., P.R.D**83** (11) 071503(R)

models of 1h (TMD) FF

1^{st} category: the spectator approximation

on-shell spectator



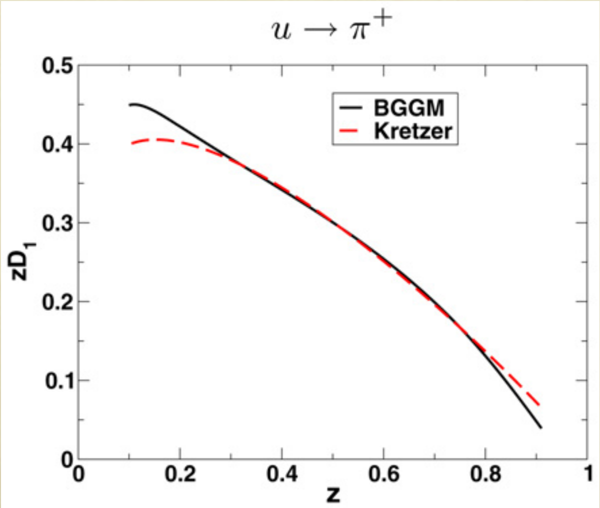
- ➤ δ funct. ⇒ q-q correlator analytic
- ➤ off-shell $k^2(z)$ analytic
- ✖ only favoured channel

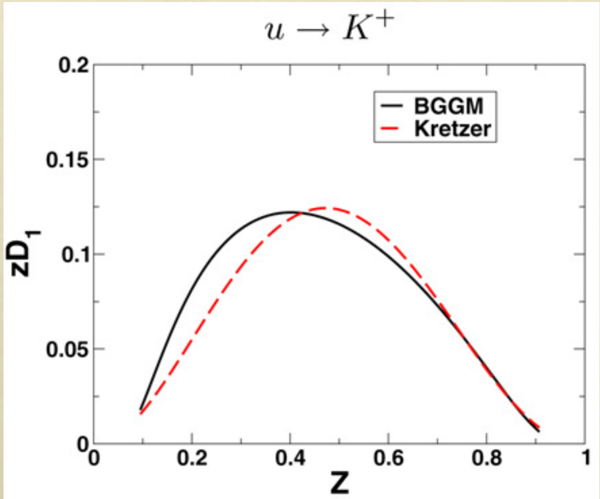
- qπ vertex: PS $g_{\pi q}\gamma_5\tau_i$ (Jakob et al., N.P.**A626** (97); Bacchetta et al., P.L.**B506** (01), **B659** (08); Gamberg et al., P.R.D**68** (03); Amrath et al., P.R.D**71** (05))

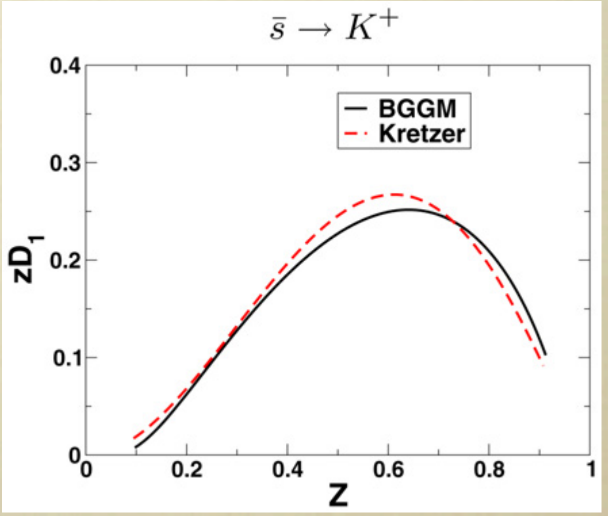
  PV $g_{\pi q}\gamma_5 \not{P}_h$ (Bacchetta et al., P.R.D**65** (02), P.L.**B574** (03); Amrath et al., P.R.D**71** (05))

$g_{\pi q}(z,k^2) \sim \exp[-k^2/\Lambda^2(z)]$ (Gamberg et al., P.R.D**68** (03); Bacchetta et al., P.L.**B659** (08))

fit $D_1{}^q$ to Kretzer
@ Q^2=0.4 GeV^2



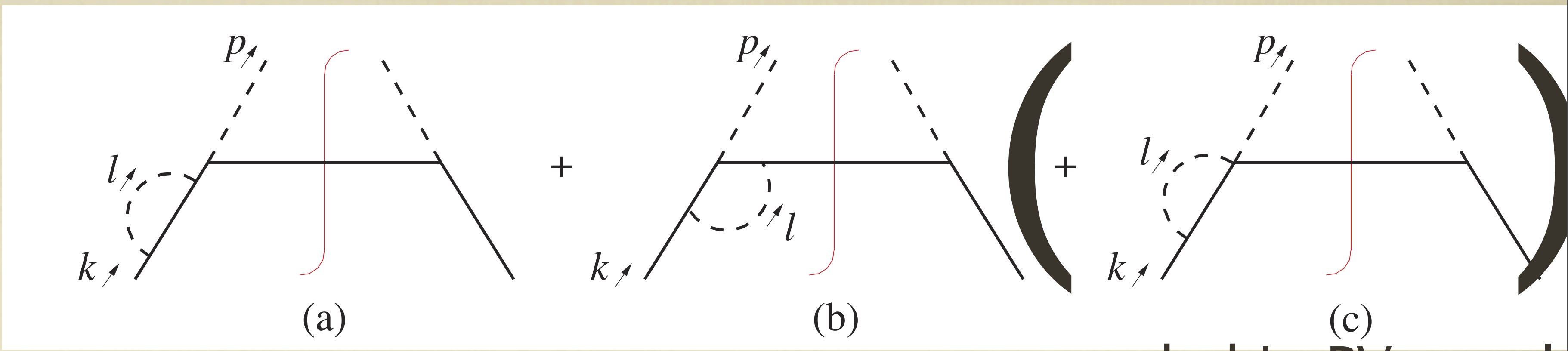




the spectator approximation: the Collins funct.

interference from:

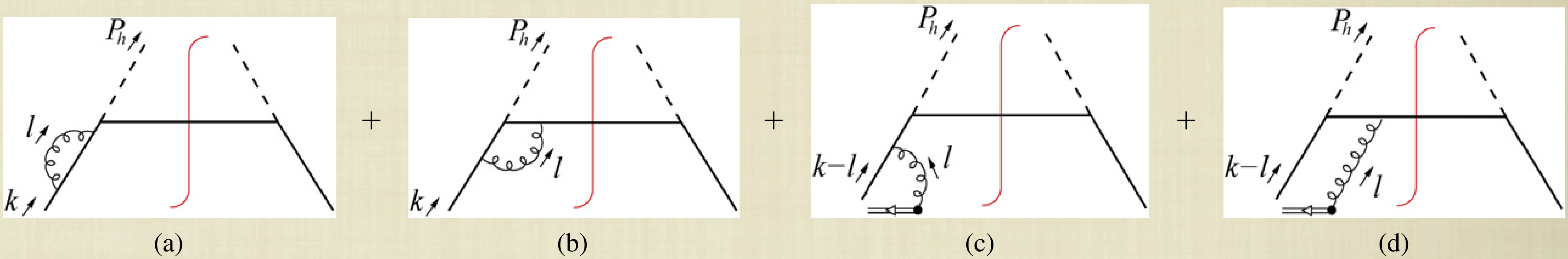
π loops



needed in PV coupl.

and / or

g loops



the spectator approximation: the Collins funct.

interference from:

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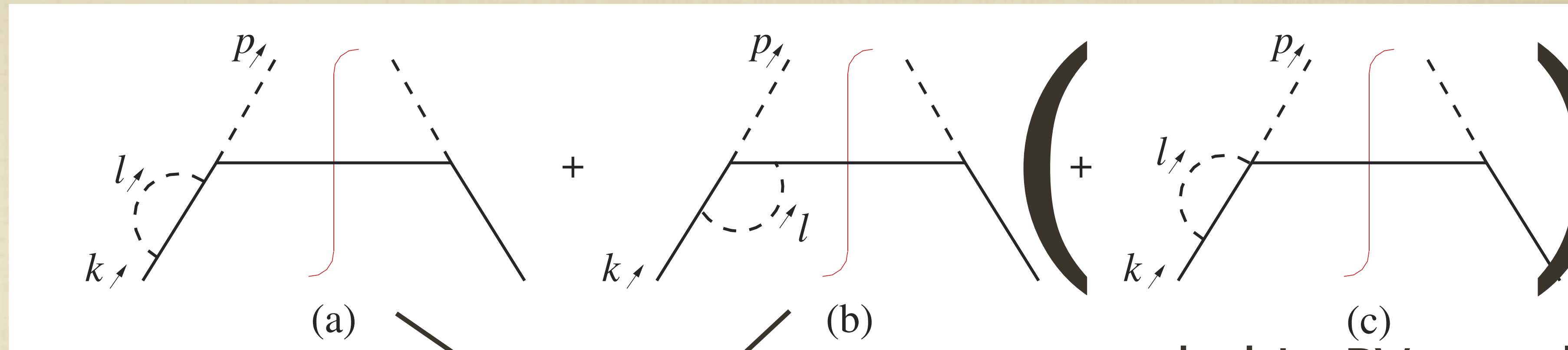
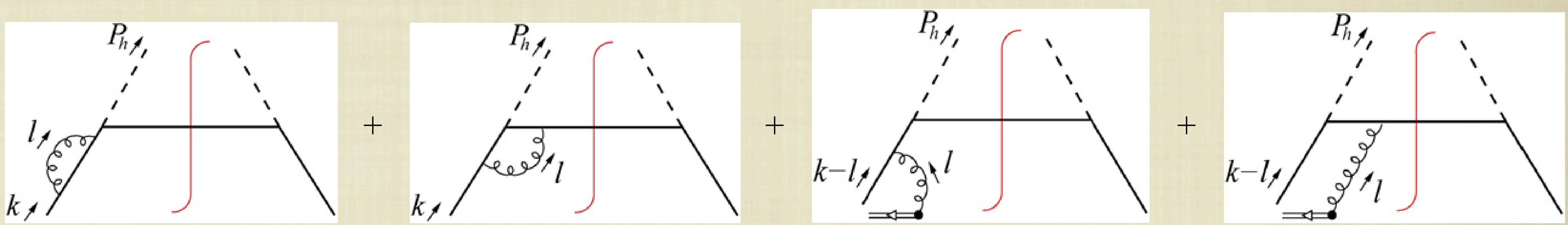


needed in PV coupl.

large cancellations ⇒ c) dominant

and / or

g loops



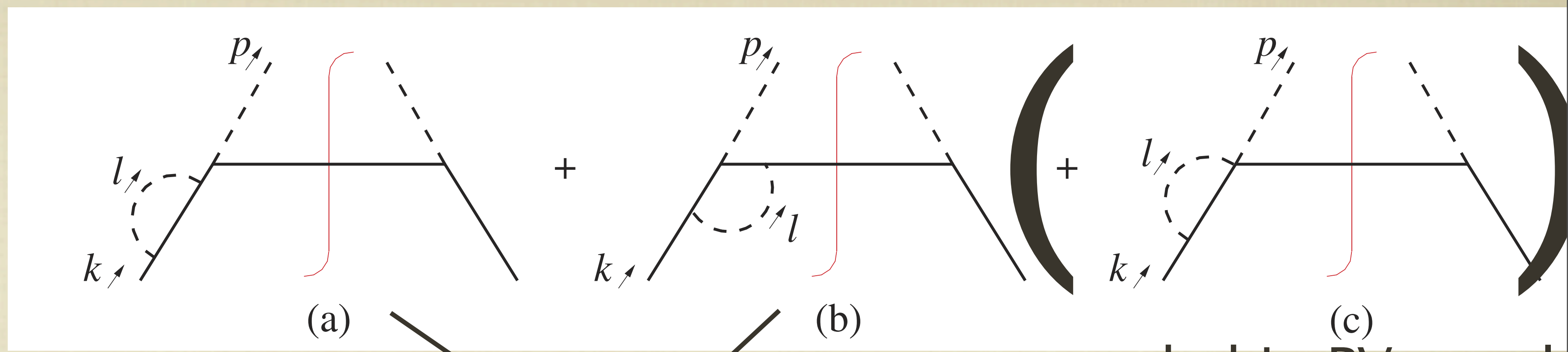
(a) (b) (c) (d)

large cancellations = 0 dominant

the spectator approximation: the Collins funct.

interference from:

π loops

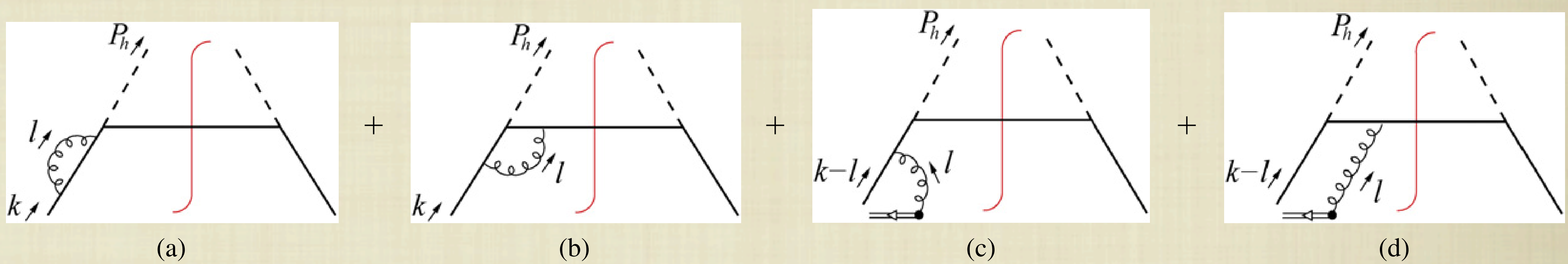


needed in PV coupl.

and / or

large cancellations ⇒ c) dominant

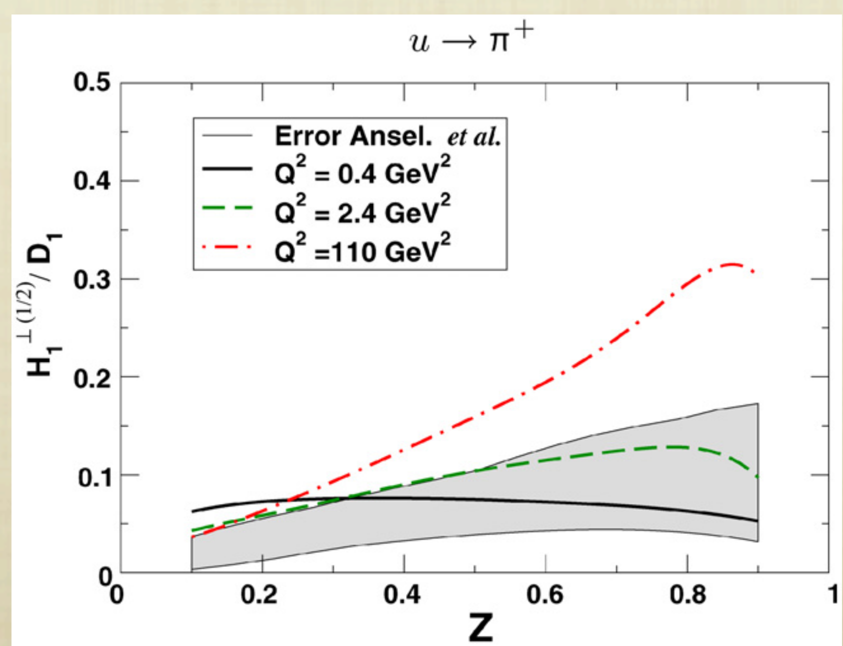
g loops



large cancellations = 0 dominant

PS coupling
g loops
params fitted to D_1

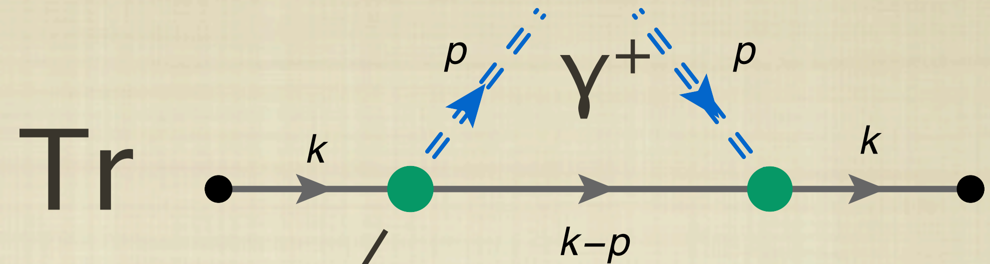
Bacchetta et al., P.L.**B659** (08)



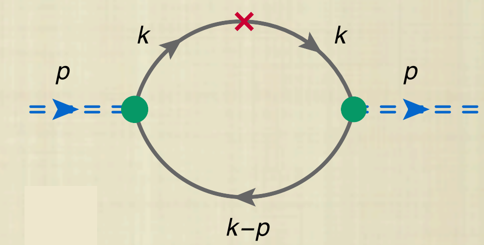
2^{nd} category: the NJL–jet model

(Ito et al., P.R.D**80** (09) 074008)

- elementary fragm. $d_q^h(z)$ from



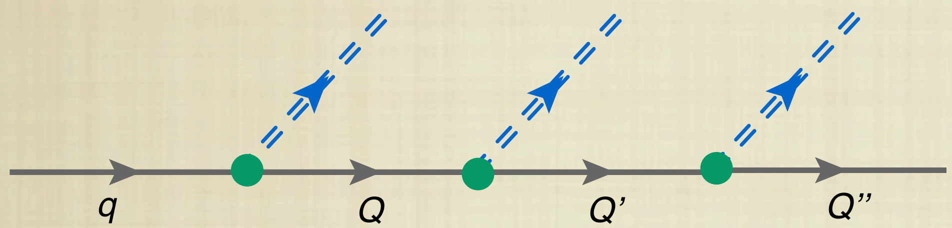
$$\frac{1}{g^2_{qh}} = -\frac{\partial}{\partial k^2}$$



$$\Big|_{k^2=M_h^2}$$

- multiplicative ansatz

$$D_q^h(z) = d_q^h(z) + \sum_Q [d_q^Q \otimes D_Q^h](z)$$

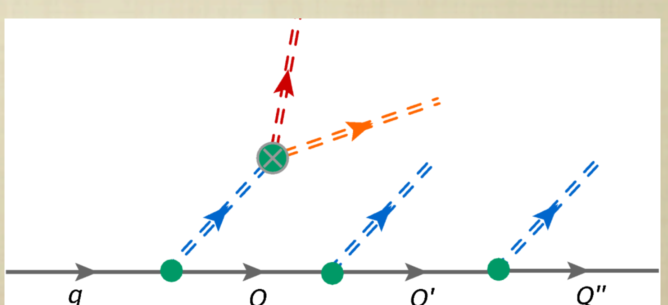


- mom. sum rule satisfied in Bjorken limit (#h's → ∞)

- probabilistic interpretation → Monte Carlo (sample based on d_q^h)

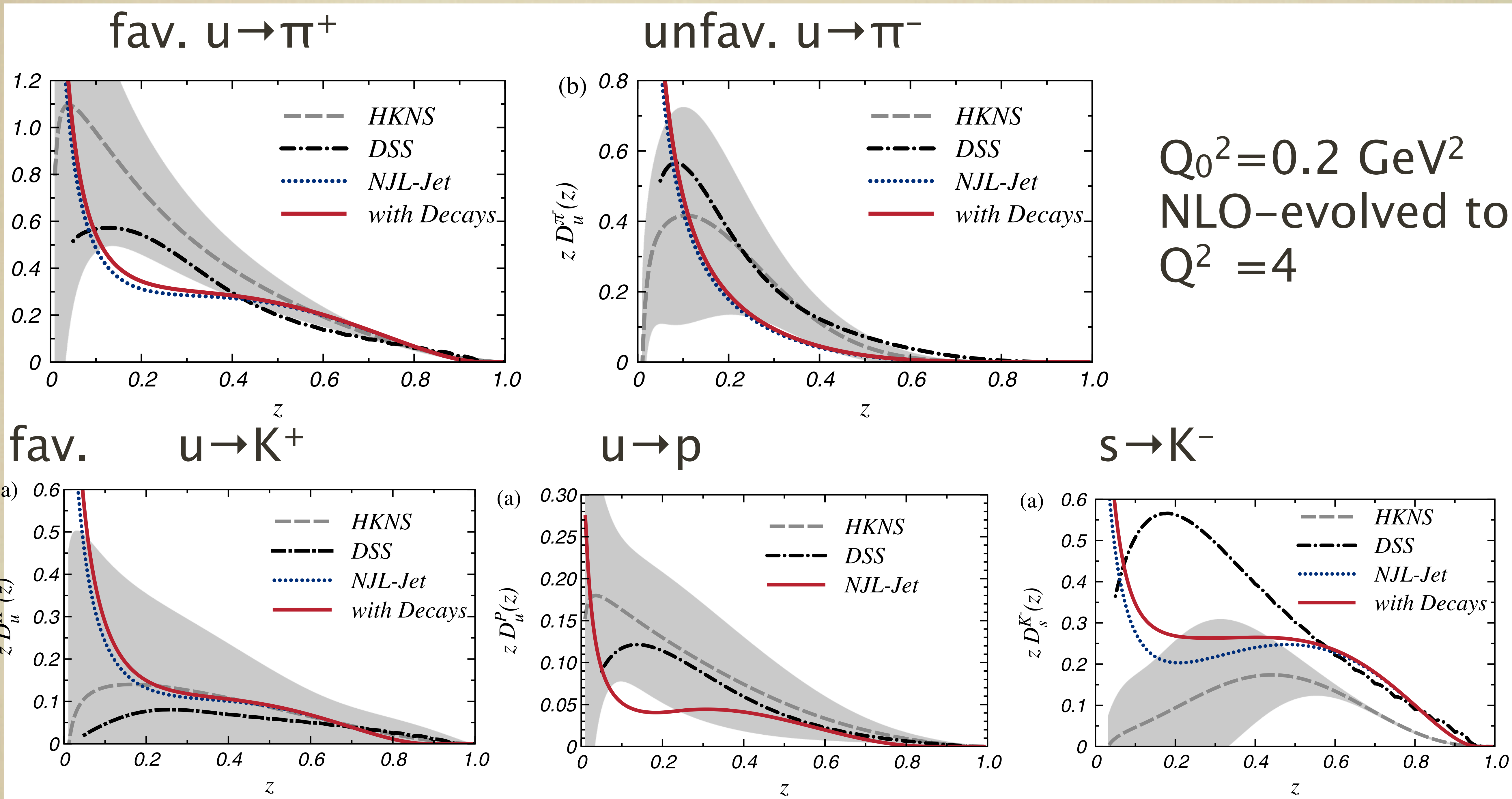
$$D_q^h(z)\,\Delta z = 1/N \;\sum_N \; N_q^h(z,z+\Delta z)$$ MC for N→∞ → ansatz

- h = π, K, ρ, K*, φ, p, n → spect. diquark model
  only scalar



results for the MC ~ NJL-jet model

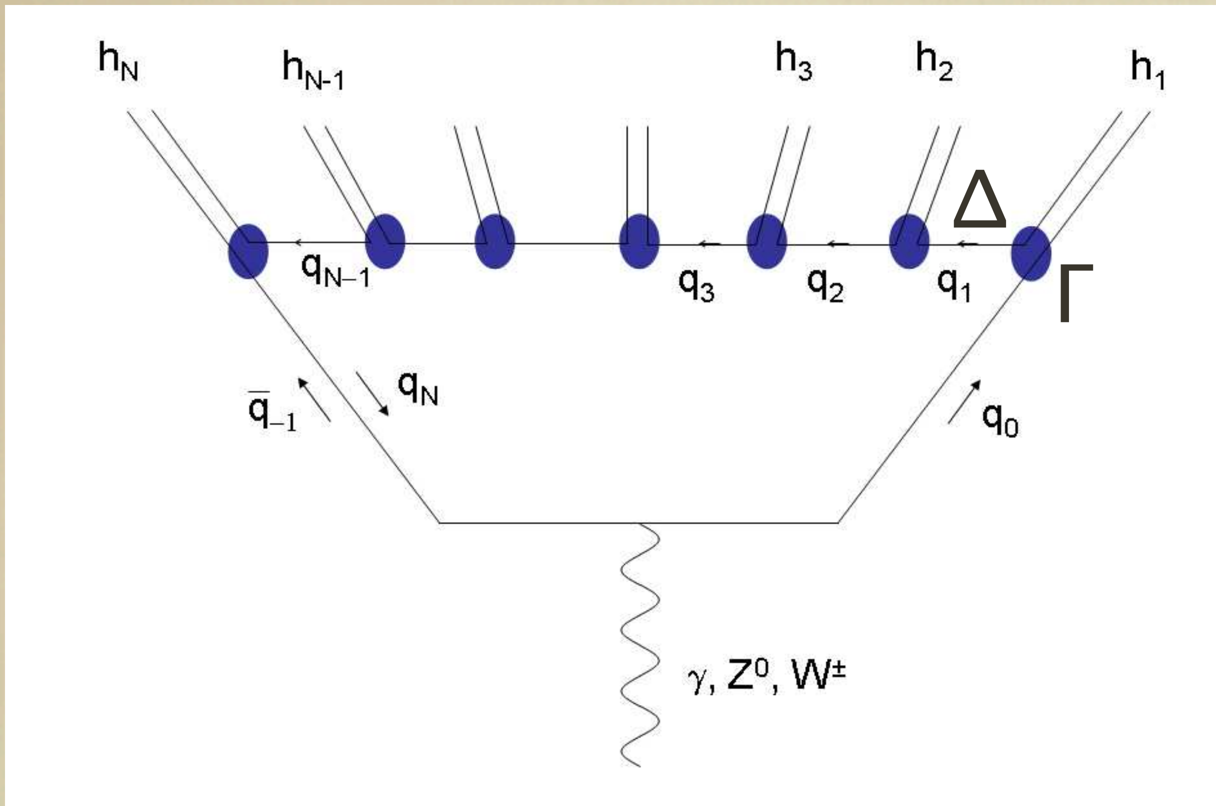
(Matevosyan et al., P.R.D**83** (11) 114010)



- ✖ $zD(z,Q_0^2)\to$const for $z\to 0$ (mult.→∞), larger effect at Q^2=4
- ✖ LB regular. scheme ⇒ $z_{min}(h) \le z \le z_{max}(h)$

3rd category: recursive model with spin

(Artru, arXiv:1001.1061)



$e^+e^- \to q_0\bar{q}_{-1} \to h_1 + h_2 + \ldots h_N$

$p_{q0} = p_{h1} + p_{q1}$

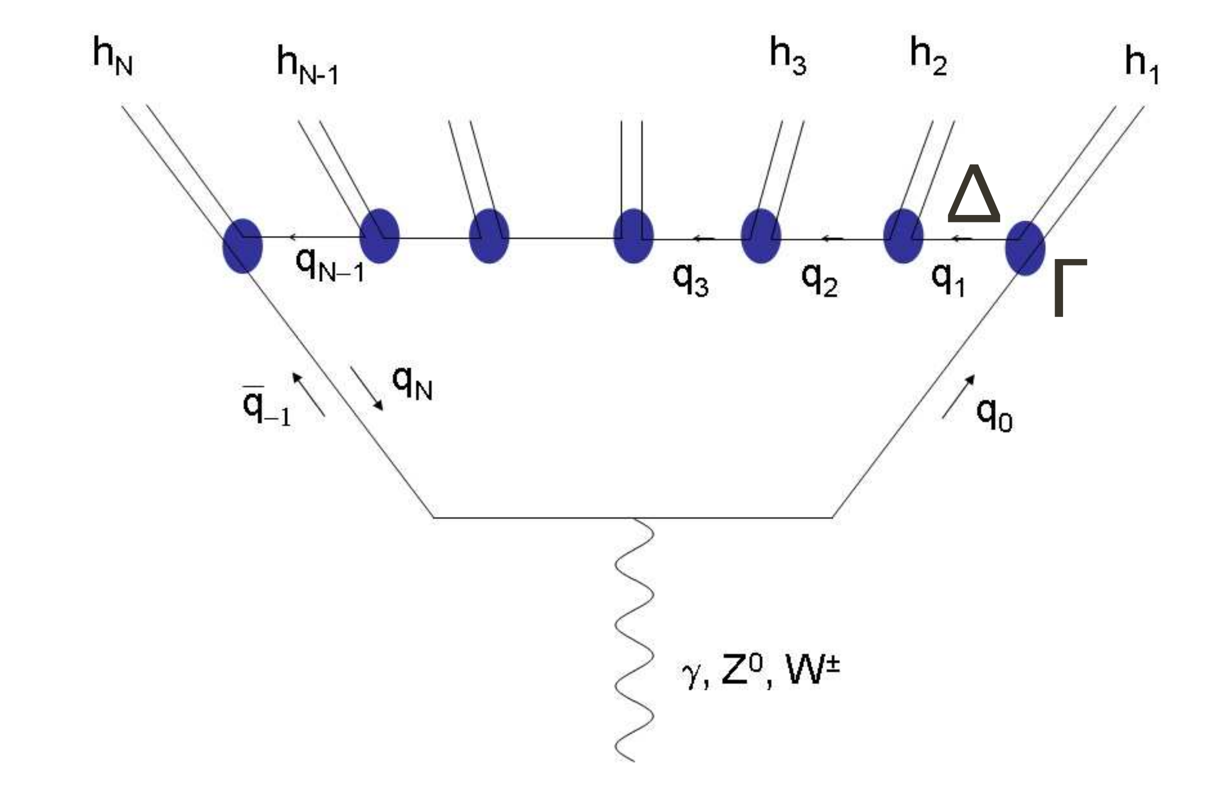
$p_{q1} = p_{h2} + p_{q2}$

......

$$\mathcal{M} = \bar{v}(-1)\ \Gamma(N)\ \Delta(N-1) \ldots\ \Delta(1)\ \Gamma(1)\ u(0)$$

3rd category: recursive model with spin

(Artru, arXiv:1001.1061)



$e^+e^- \to q_0\bar{q}_{-1} \to h_1+h_2+\ldots h_N$

$p_{q0}=p_{h1}+p_{q1}$

$p_{q1}=p_{h2}+p_{q2}$

......

$$\mathcal{M}=\bar{v}(-1)\ \Gamma(N)\ \Delta(N-1)\ldots\ \Delta(1)\ \Gamma(1)\ u(0)$$

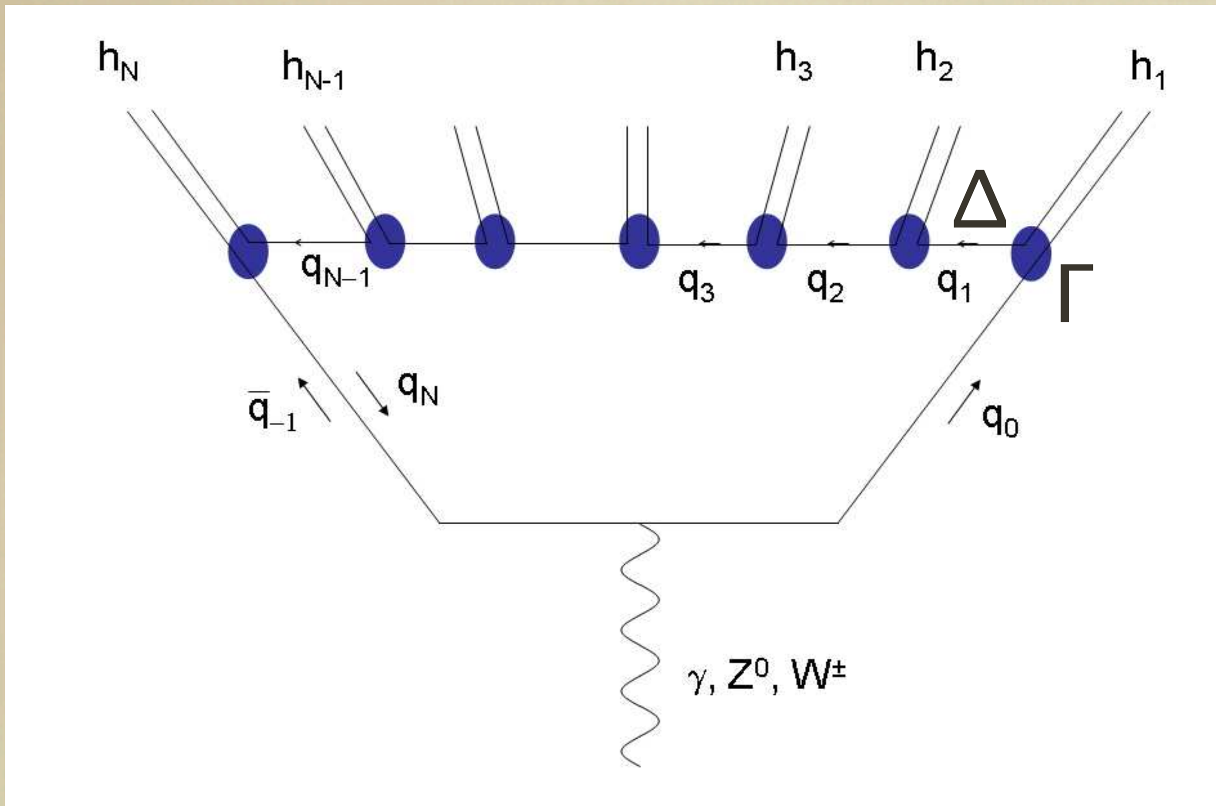
Simplifications :

1- Γ=const.

2- $\Delta(p_q) \approx \exp[-b\mathbf{p}_{qT}^2/2]\ [\ \mu(\mathbf{p}_{qT}^2)+i\ \boldsymbol{\sigma}\cdot\check{z}\times\mathbf{p}_{qT}\ ]$ with b some parameter

3rd category: recursive model with spin

(Artru, arXiv:1001.1061)



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$$\mathcal{M}\mathcal{M}^\dagger \approx \exp[-b\mathbf{p}_{h1T}^2\ldots.-b\mathbf{p}_{hNT}^2]\ \mathrm{Tr}\ \{M_1..M_N\ (1+\mathbf{S}_0\cdot\boldsymbol{\sigma})\ M_N^\dagger..M_1^\dagger\}$$

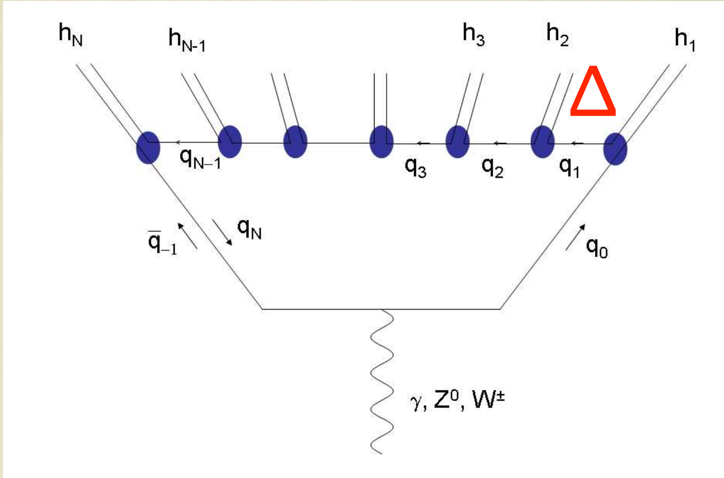
$$M_i=[\mu(\mathbf{p}_{hiT}^2)+i\ \boldsymbol{\sigma}\cdot\check{z}\times\mathbf{p}_{hiT}]\ \sigma_z$$

recursive model with spin: Collins and jet handedness

(Artru, arXiv:1001.1061)

N=1: the Collins effect

$$\mathcal{M}\mathcal{M}^\dagger \approx \exp[-b\mathbf{p}_{h1T}^2]\ \mathrm{Tr}\ \{M_1\ (1+\mathbf{S}_0\cdot\boldsymbol{\sigma})\ M_1^\dagger\}$$
$$= \exp[-b\mathbf{p}_{h1T}^2]\ [\sigma^0(\mathbf{p}_{h1T}^2)+\mathrm{Im}(\mu)\ \mathbf{S}_0\cdot\check{\mathbf{z}}\times\mathbf{p}_{h1T}]$$



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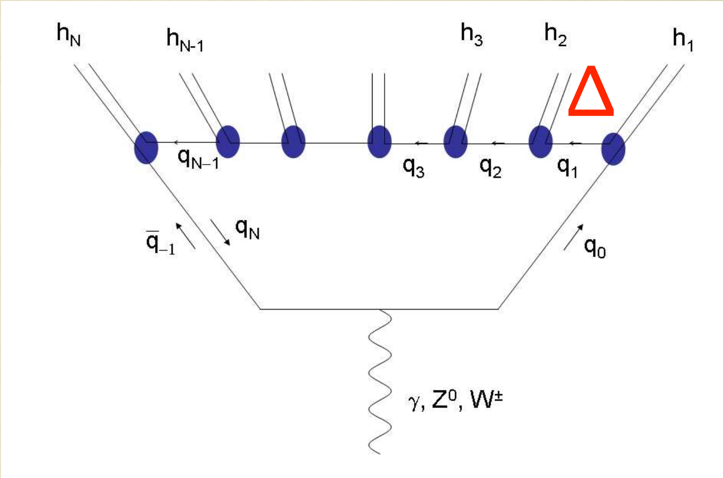
$\mathcal{M}\mathcal{M}^\dagger \approx \exp[-b\mathbf{p}_{h1T}{}^2]\ \mathrm{Tr}\,\{M_1\,(1+\mathbf{S}_0\cdot\boldsymbol{\sigma})\,M_1{}^\dagger\}$

$= \exp[-b\mathbf{p}_{h1T}{}^2]\ [\sigma^0(\mathbf{p}_{h1T}{}^2)+\mathrm{Im}(\mu)\ \mathbf{S}_0\cdot\check{\mathbf{z}}\times\mathbf{p}_{h1T}]$



N=2: iterated Collins effect + jet handedness

$\mathcal{M}\mathcal{M}^\dagger \approx \exp[-b\mathbf{p}_{h1T}{}^2-b\mathbf{p}_{h2T}{}^2]\ \mathrm{Tr}\,\{M_1M_2\,(1+\mathbf{S}_0\cdot\boldsymbol{\sigma})\,M_2{}^\dagger M_1{}^\dagger\}$

$= \ldots + A(\mathbf{p}_{h2T}{}^2)\ \mathrm{Im}(\mu)\ \mathbf{S}\cdot\check{\mathbf{z}}\times\mathbf{p}_{h1T} + A'(\mathbf{p}_{h1T}{}^2)\ \mathrm{Im}(\mu)\ \mathbf{S}\cdot\check{\mathbf{z}}\times\mathbf{p}_{h2T}$

$-\ 2\ \mathrm{Im}(\mu^2)\ S_z\ \check{\mathbf{z}}\cdot\mathbf{p}_{h1T}\times\mathbf{p}_{h2T}$

recursive model with spin: Collins and jet handedness

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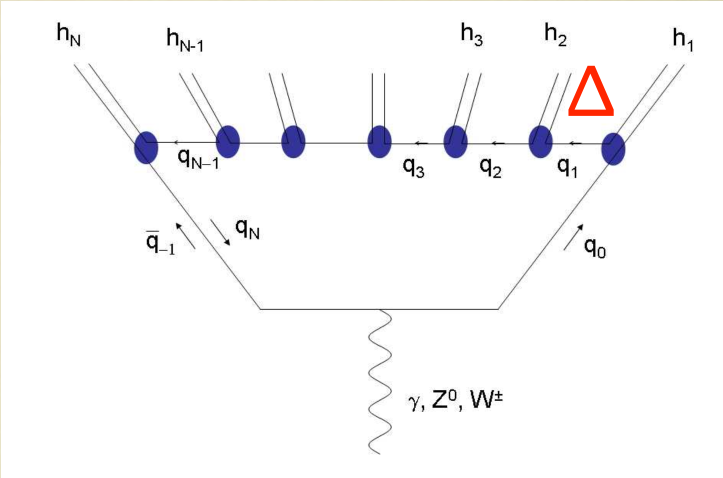
$$= \exp[-b\mathbf{p}_{h1T}{}^2]\ [\sigma^0(\mathbf{p}_{h1T}{}^2)+\mathrm{Im}(\mu)\ \mathbf{S}_0\cdot\check{\mathbf{z}}\times\mathbf{p}_{h1T}]$$



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why ?

recursive model with spin: Collins and jet handedness

(Artru, arXiv:1001.1061)

- define $R_N = M_1..M_N (1+\mathbf{S}_0\cdot\boldsymbol{\sigma}) M_N^\dagger..M_1^\dagger$

  recursive property $R_N = M_N R_{N-1} M_N^\dagger$

recursive model with spin: Collins and jet handedness

(Artru, arXiv:1001.1061)

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  recursive property $R_N = M_N R_{N-1} M_N^\dagger$

- implies $\mathbf{S}_N = 1/\mathrm{Tr}\{R_N\} [\mathrm{Im}(\mu)\ \check{\mathbf{z}}\times\mathbf{p}_{qNT} + \mathcal{R}(\check{\mathbf{z}};\mu,\mathbf{p}_{qT}^2)\ \mathbf{S}_{N-1}]$
  ➤ $\mathrm{Im}(\mu)\neq 0 \Rightarrow \mathbf{S}_{NT}\neq 0$ even if $\mathbf{S}_{N-1}=0$

  helicity $S_{N-1z} \leftrightarrow$ transversity S_{NT}

recursive model with spin: Collins and jet handedness

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➤ jet handedness = 1) $S_{0z} \rightarrow \mathbf{S}_{1T} \,||\, \mathbf{p}_{h1T} \neq 0$

2) Collins effect $\check{\mathbf{z}}\cdot\mathbf{p}_{h2T}\times\mathbf{S}_{1T} \Rightarrow \check{\mathbf{z}}\cdot\mathbf{p}_{h2T}\times\mathbf{p}_{h1T}$

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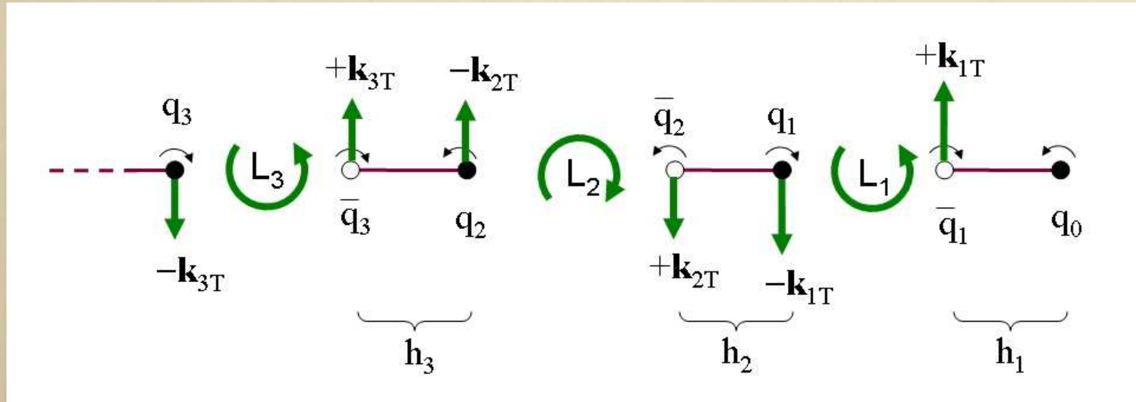
- implies $S_{Nz}=D_{LL}(|\mu^2|)\,S_{N-1z}$; $\mathbf{S}_{NT} = D_{TT}(|\mu^2|)\,\mathbf{S}_{N-1T}$ $\quad 2|D_{TT}| \leq 1+D_{LL}$
- $D_{TT}<0 \;\Rightarrow$ alternate Collins effects on h_1, h_2.. as in Lund $^3P_0$ model

unfav. ~ – fav.



recursive model with spin: Collins and jet handedness

(Artru, arXiv:1001.1061)

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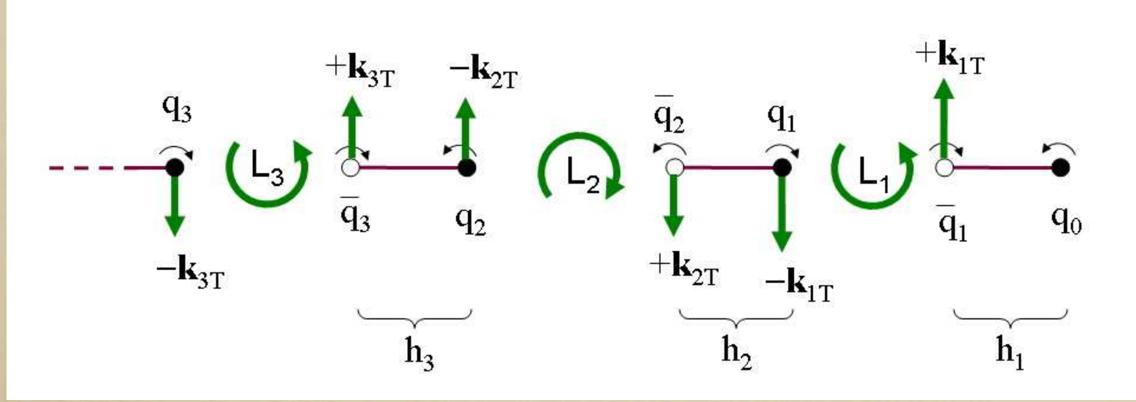
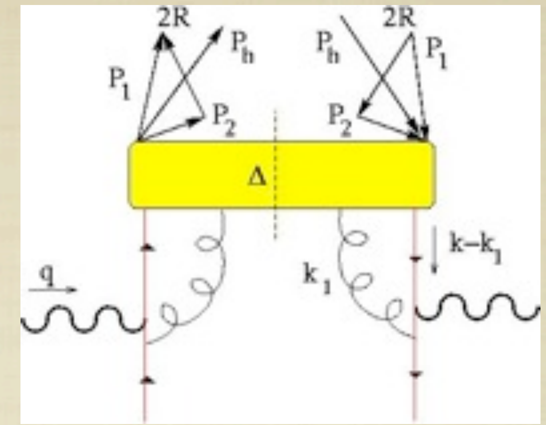


BUT $A_{UT}^{Coll}(K^-) \sim 0$ at HERMES
different trend at COMPASS
large $A_{UU}^{\cos 2\phi}(K-)$ at HERMES

2h FF (DiFF)

Di-hadron Fragm. Functions (DiFF)

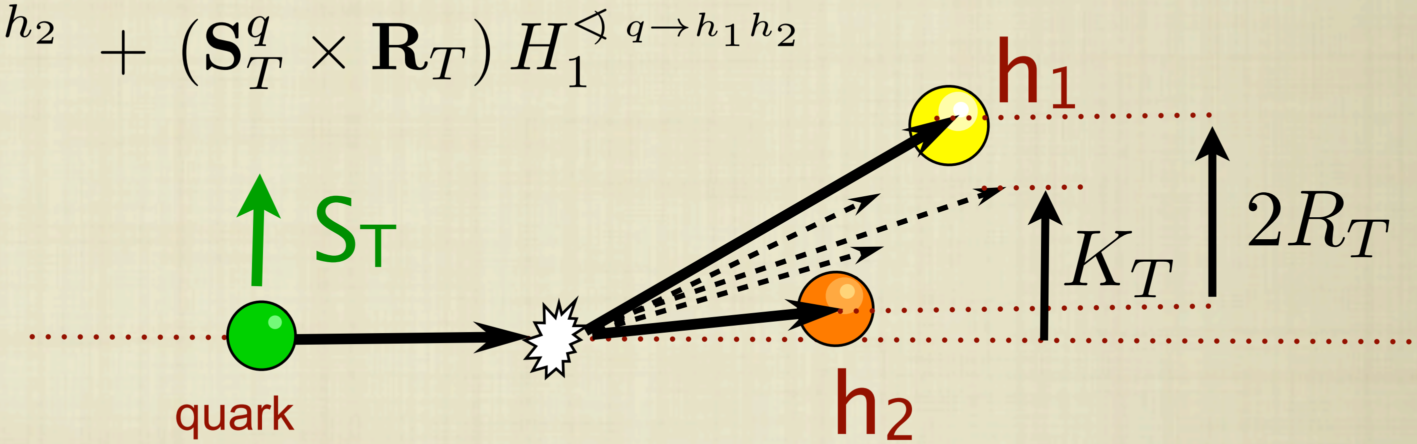


from q-q correlator Δ(z_1,z_2,$\mathbf{K}_T$,$\mathbf{R}_T$)
project out (at leading twist):

$$\mathrm{Tr}\left[\Delta\,\gamma^-\right] \quad\rightarrow\quad D_1^{q\rightarrow h_1 h_2}(z_1, z_2, K_T^2, R_T^2, \mathbf{K}_T\cdot\mathbf{R}_T)$$

$$\mathrm{Tr}\left[\Delta\,\gamma^-\gamma_5\right] \quad\rightarrow\quad (\mathbf{R}_T\times\mathbf{K}_T)\,G_1^{\perp\,q\rightarrow h_1 h_2}$$

$$\mathrm{Tr}\left[\Delta\,i\sigma^{i-}\gamma_5\right] \quad\longrightarrow\quad (\mathbf{S}_T^q\times\mathbf{K}_T)\,H_1^{\perp\,q\rightarrow h_1 h_2} + (\mathbf{S}_T^q\times\mathbf{R}_T)\,H_1^{\sphericalangle\,q\rightarrow h_1 h_2}$$



First suggested in Konishi et al., P.L.**B78** (78)

Polarized DiFF in Collins et al., N.P.**B420** (94); Jaffe et al., P.R.L.**80** (98); Artru & Collins, Z.Ph.C**69** (96)

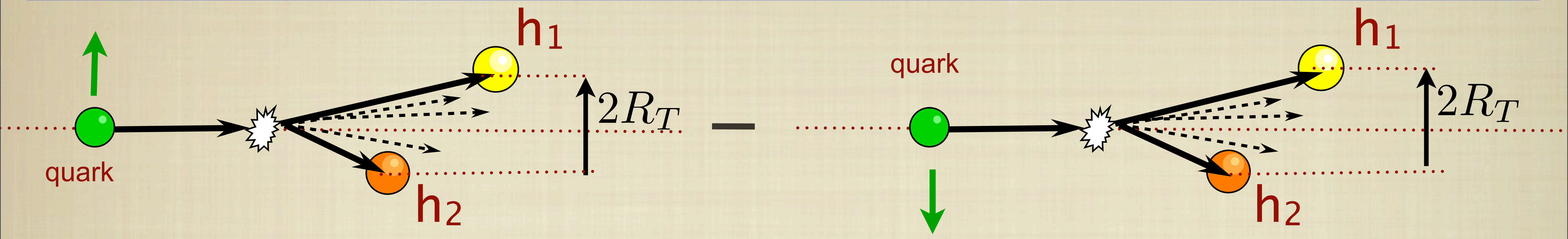
Jet handedness in Efremov et al.,P.L.**B284** (92); Stratmann & Vogelsang, P.L.**B295** (92); Boer et al.,P.R.D**67** (03)

full analysis at twist 2 Bianconi et al., P.R.D**62** (00); at twist 3 Bacchetta & Radici, P.R.D**69** (04)

LO evolution eqs. Ceccopieri et al., P.L.**B650** (07)

chiral-odd $H_1^{\sphericalangle\, q \to h_1 h_2}$ survives $\int d\mathbf{K}_T$ ($H_1^{\perp\, q\to h}$ doesn't)

(memo: h_1,h_2 must be distinguishable!)



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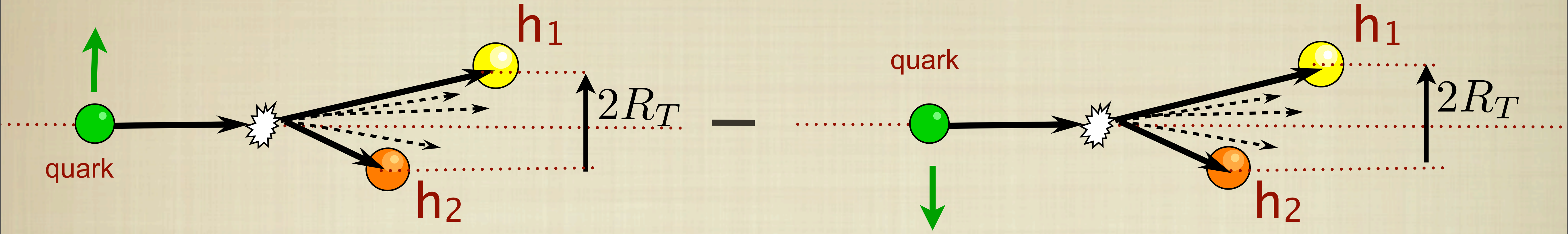
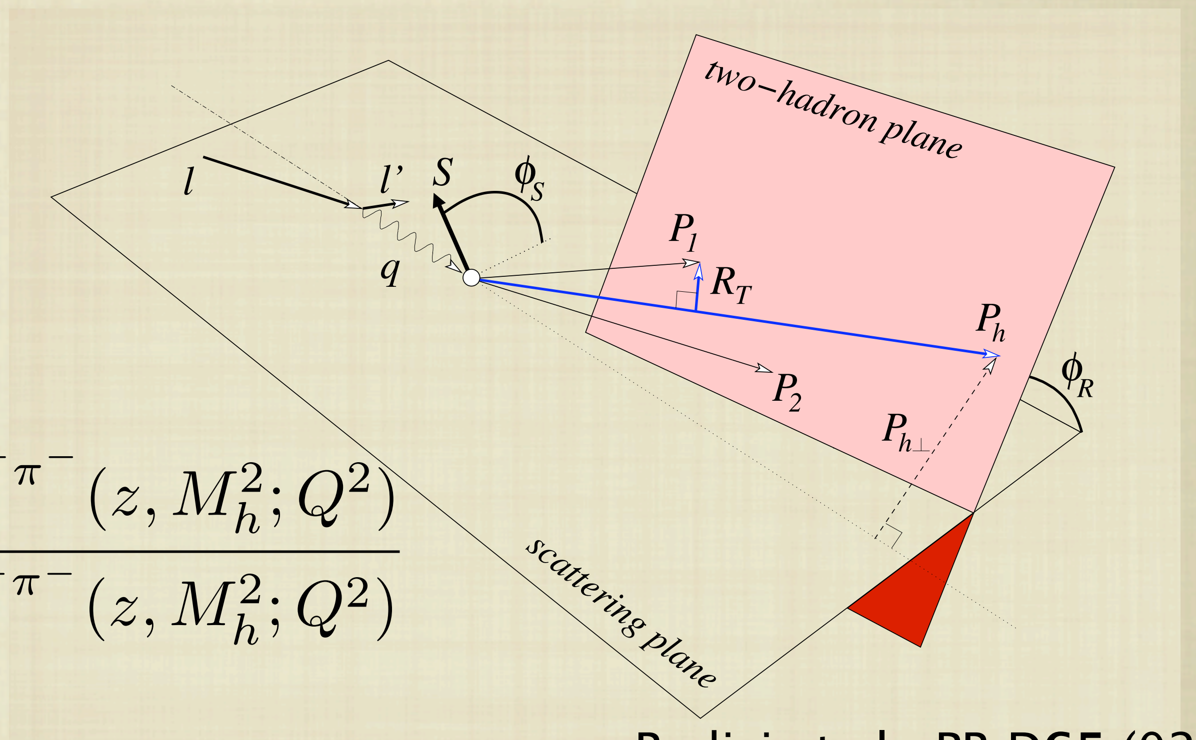


partner of transversity



$$A_{UT}^{\sin(\phi_R+\phi_S)\sin\theta}(x,z,M_h^2;Q^2) =$$

$$-\,C_y\,\frac{|\boldsymbol{R}|}{M_h}\,\frac{\sum_q e_q^2\, h_1^q(x,Q^2)\;H_1^{\sphericalangle\, q\to\pi^+\pi^-}(z,M_h^2;Q^2)}{\sum_q e_q^2\, f_1^q(x,Q^2)\;D_1^{\sphericalangle\, q\to\pi^+\pi^-}(z,M_h^2;Q^2)}$$

Radici et al., PR D**65** (02)
Bacchetta & Radici, PR D**67** (03)

- coll. fact. → simple product (no ⊗)
- DGLAP (LO) evolution
- universality
- cleaner $e^+e^-\to(\pi^+\pi^-)(\pi^+\pi^-)X$ (expect less background)

2007: preliminary SIDIS data on D from COMPASS: $A_{UT} \sim 0$

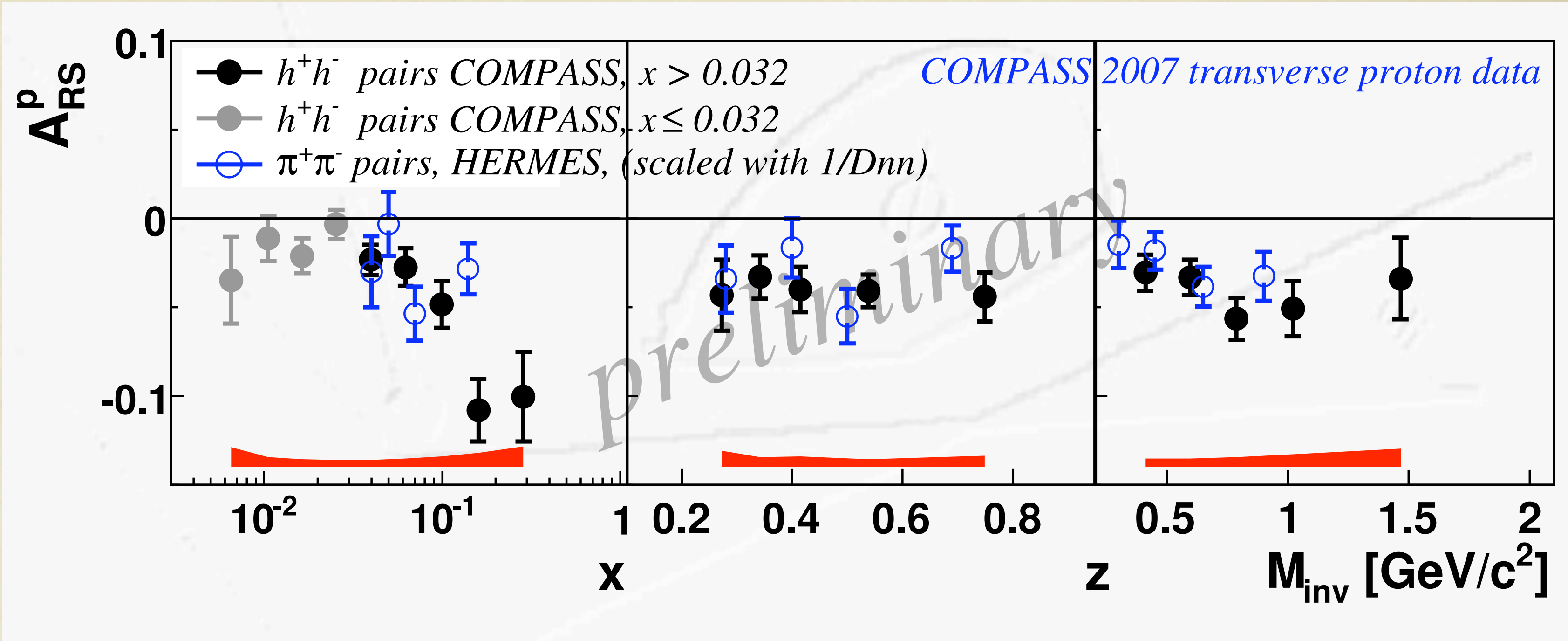
Martin, hep-ex/0702002

2008: first SIDIS data on $p^\uparrow$ from HERMES

Airapetian et al. (HERMES), JHEP 06 (08)

2009: preliminary data on $p^\uparrow$ from COMPASS

Wollny (COMPASS), DIS 2009, arXiv:0907.0961



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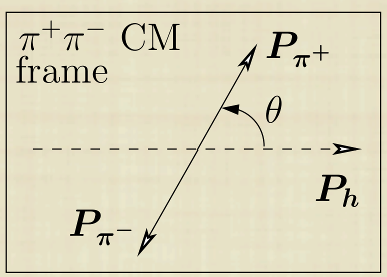
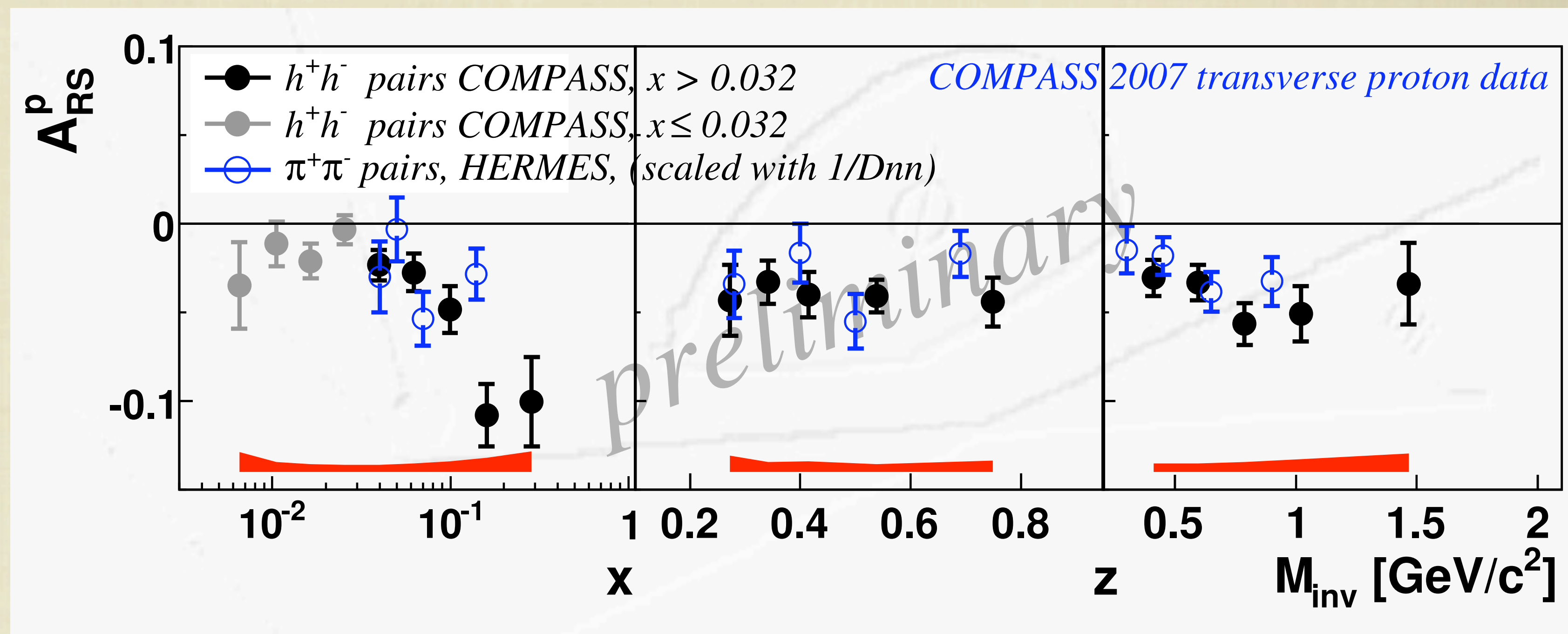
observable is

$$A_{UT}^{\sin(\phi_R+\phi_S)\sin\theta}$$





partial wave expansion

Bacchetta & Radici, P.R.D**67** (03)

$$H_1^{\sphericalangle} = H_{1,\,sp}^{\sphericalangle}(z, M_h^2) + \cos\theta\, H_{1,\,pp}^{\sphericalangle}(z, M_h^2)$$

access to interference $(\pi^+\pi^-)_s \leftrightarrow (\pi^+\pi^-)_p$

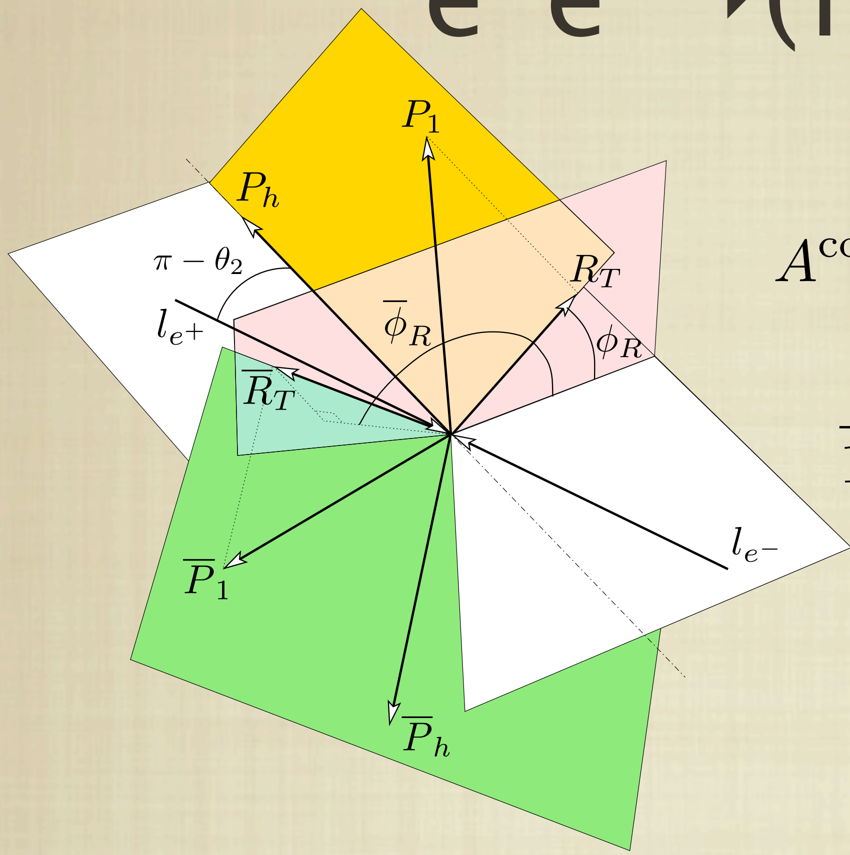
model prediction Bacchetta & Radici, P.R.D**74** (06)

model analysis Bacchetta et al., P.R.D**79** (09); She et al., P.R.D**77** (08)

2011: the BELLE data for a_{12R}

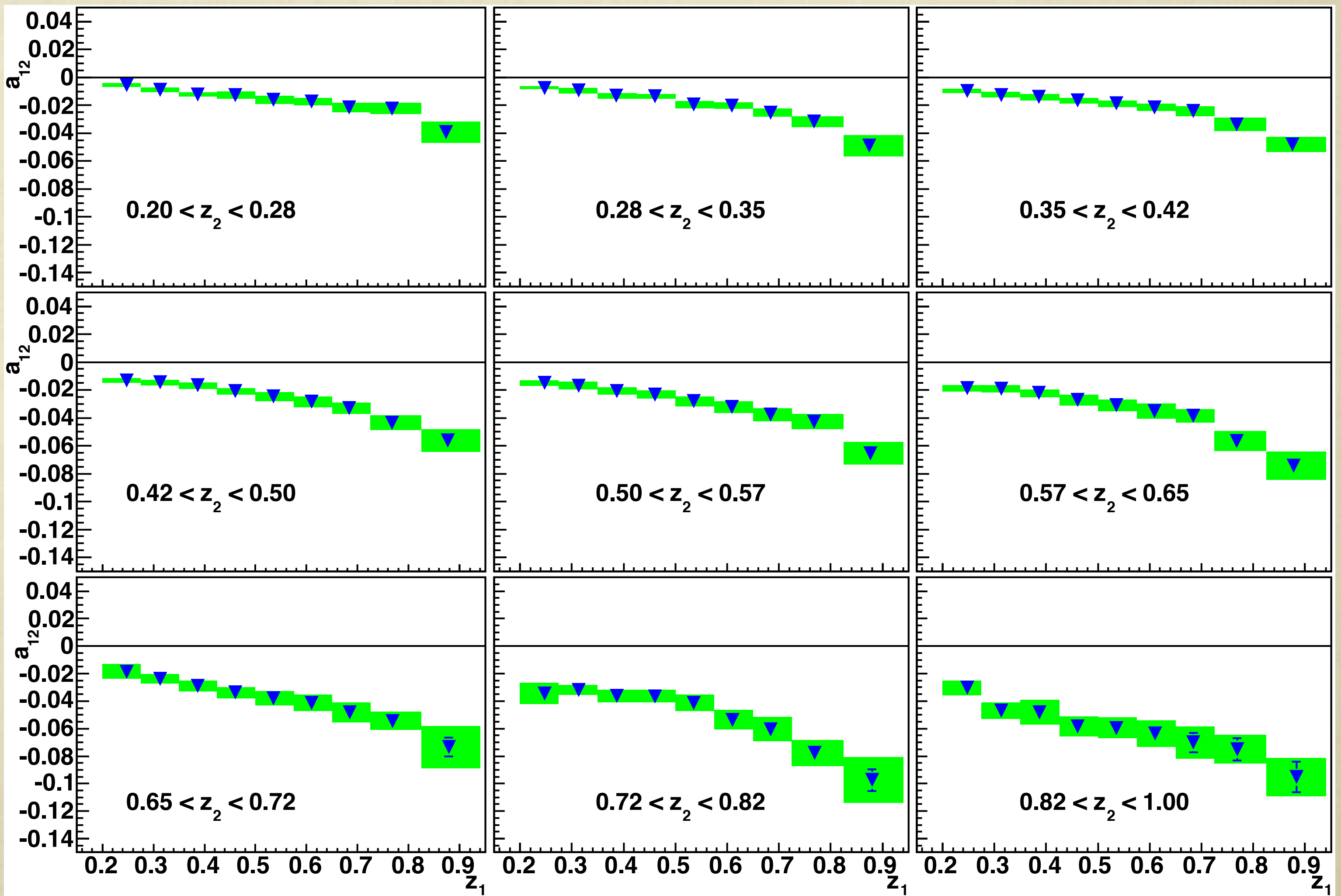
$e^+e^- \to (\pi^+\pi^-)(\pi^+\pi^-)X$

Artru & Collins, Z.Ph.C**69** (96)
Boer et al., P.R.D**67** (03)



$$A^{\cos(\phi_R+\overline{\phi}_R)}(\cos\theta_2, z, M_h^2, \overline{z}, \overline{M}_h^2) \equiv a_{12R} \propto$$

$$\frac{\sin^2\theta_2}{1+\cos^2\theta_2} \frac{|\mathbf{R}_T|}{M_h} \frac{|\overline{\mathbf{R}}_T|}{\overline{M}_h} \frac{\sum_q e_q^2 H^{\sphericalangle}_{1,q\to\pi^+\pi^-}(z, M_h^2)\, H^{\sphericalangle}_{1,\bar{q}\to\pi^+\pi^-}(\overline{z}, \overline{M}_h^2)}{\sum_q e_q^2 D_{1,q\to\pi^+\pi^-}(z, M_h^2)\, D_{1,\bar{q}\to\pi^+\pi^-}(\overline{z}, \overline{M}_h^2)}$$



Vossen et al. (BELLE),
arXiv:1104.2425 [hep-ex]

parametrizing DiFF: fitting BELLE data

$$a_{12R} = \frac{\langle \sin^2\theta_2\rangle}{\langle 1+\cos^2\theta_2\rangle}\,\frac{\langle \sin\theta\rangle |\boldsymbol{R}|}{M_h}\,\frac{\langle \sin\overline{\theta}\rangle |\overline{\boldsymbol{R}}|}{\overline{M}_h}\,\frac{\sum_q e_q^2\, H^{\sphericalangle}_{1,q\to\pi^+\pi^-}(z,M_h^2)\, H^{\sphericalangle}_{1,\bar{q}\to\pi^+\pi^-}(\overline{z},\overline{M}_h^2)}{\sum_q e_q^2\, D_{1,q\to\pi^+\pi^-}(z,M_h^2)\, D_{1,\bar{q}\to\pi^+\pi^-}(\overline{z},\overline{M}_h^2)}$$

$$|\boldsymbol{R}| = \frac{M_h}{2}\sqrt{1-\frac{4m_\pi^2}{M_h^2}}$$

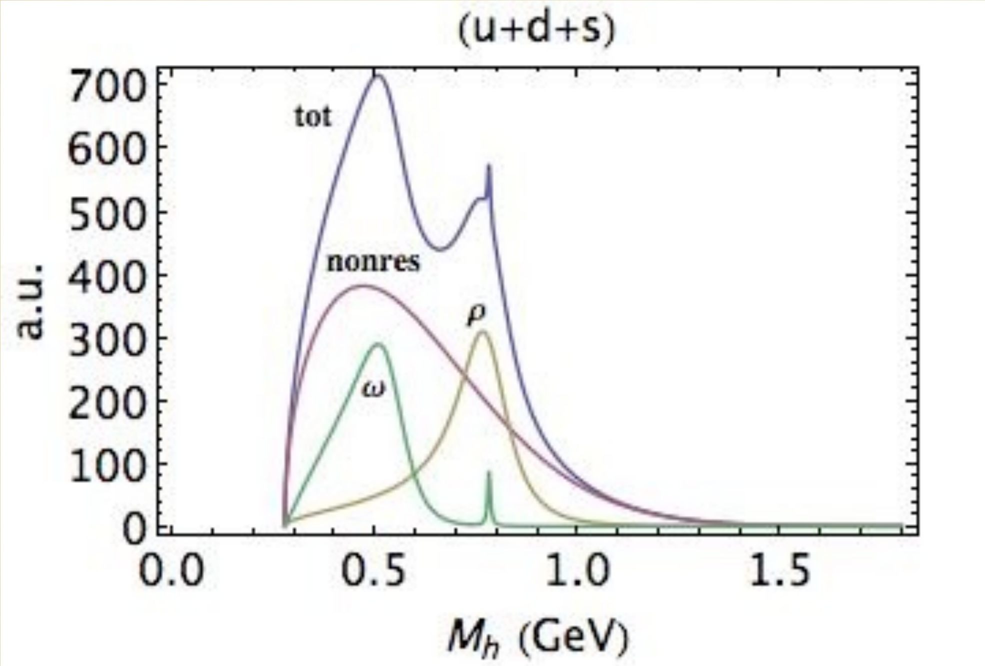
strategy

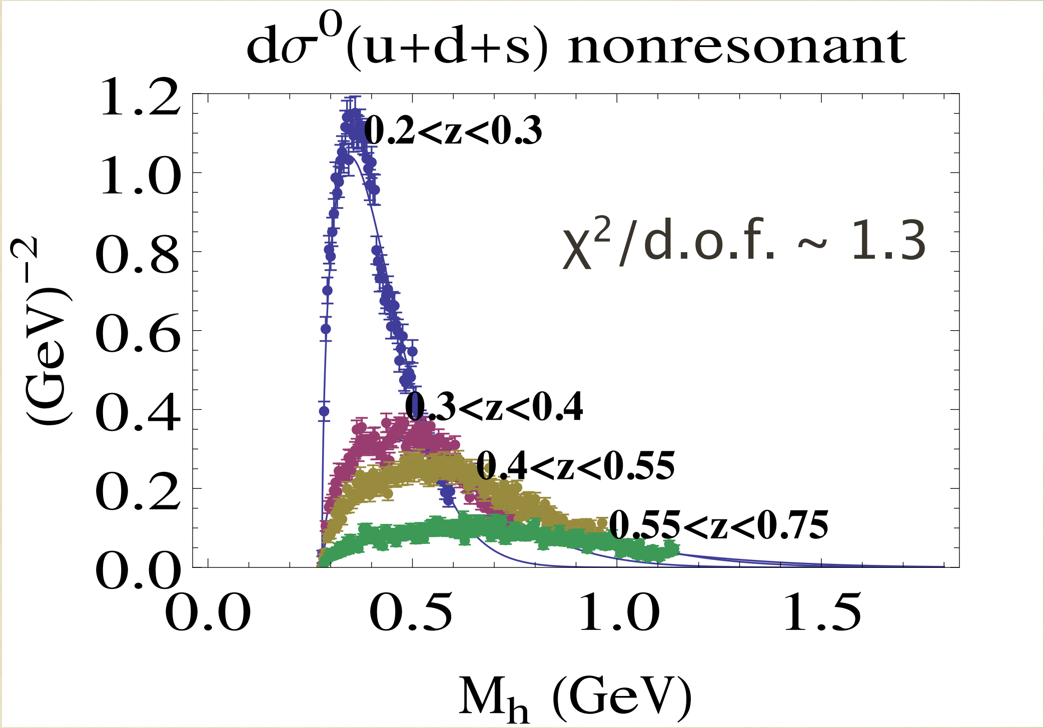
1. fit the denominator using the unpolarized cross section generated by PYTHIA MC adapted to BELLE

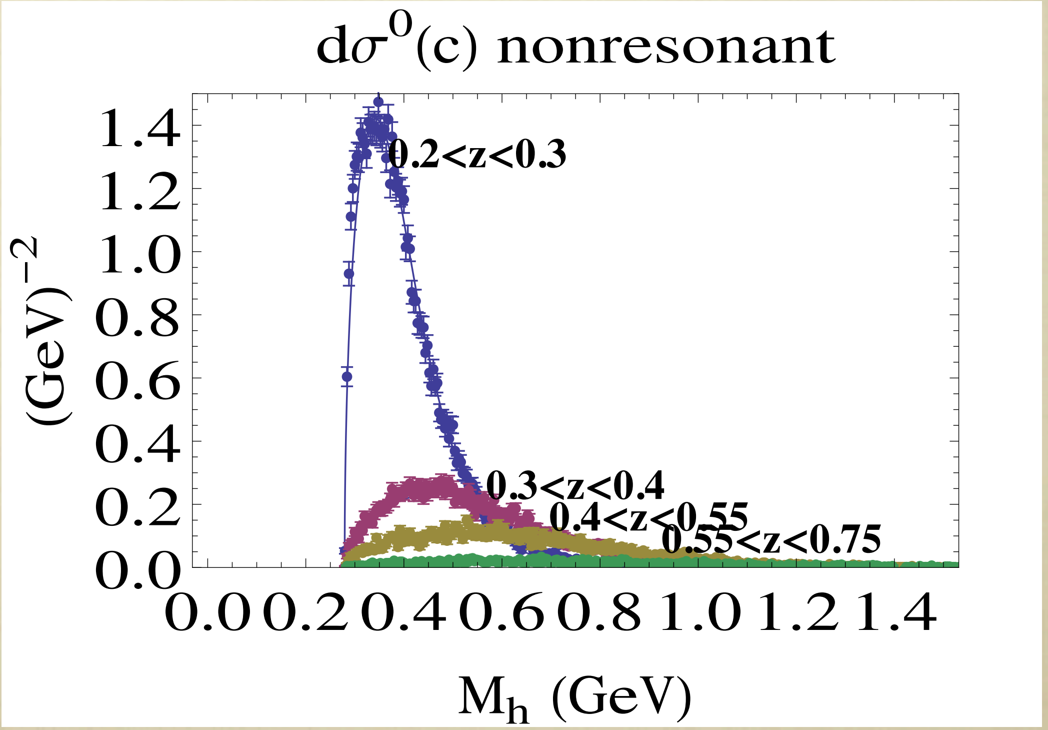
2. fit the asymmetry a_{12R} multiplied by denominator (≈ [statistical error]$^{-1}$)
→ get the numerator, bin by bin

1. fitting the BELLE (MC) $d\sigma^0 \to D_1^{q\to\pi^+\pi^-}$

1. flavor decomposition: {uds} – charm
2. resonant (ρ,ω; only {uds}) and nonresonant contributions







Courtoy et al., arXiv:1012.0054 [hep-ph]

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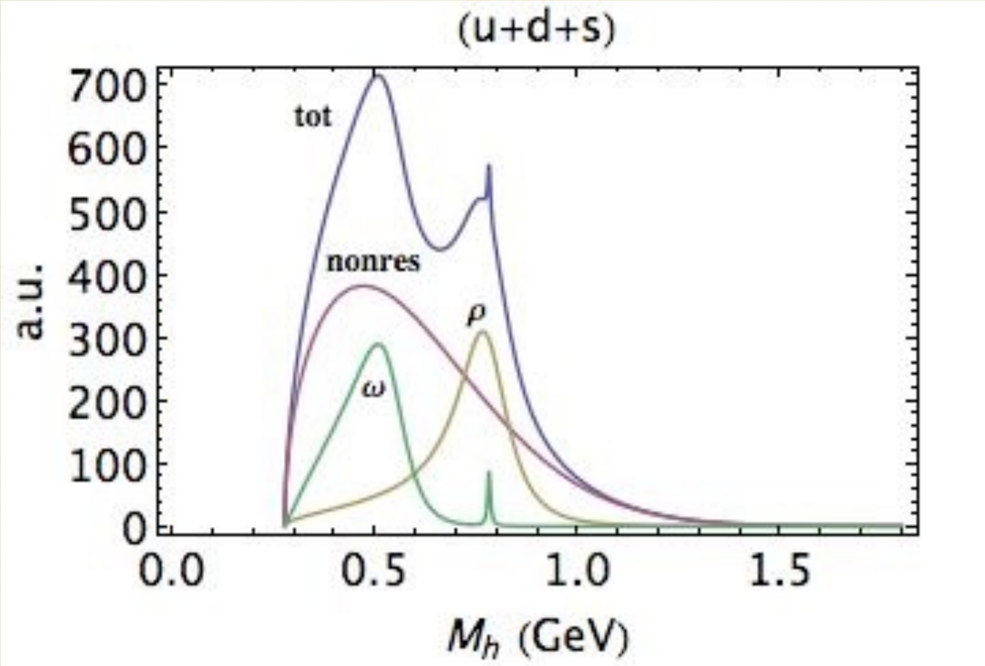
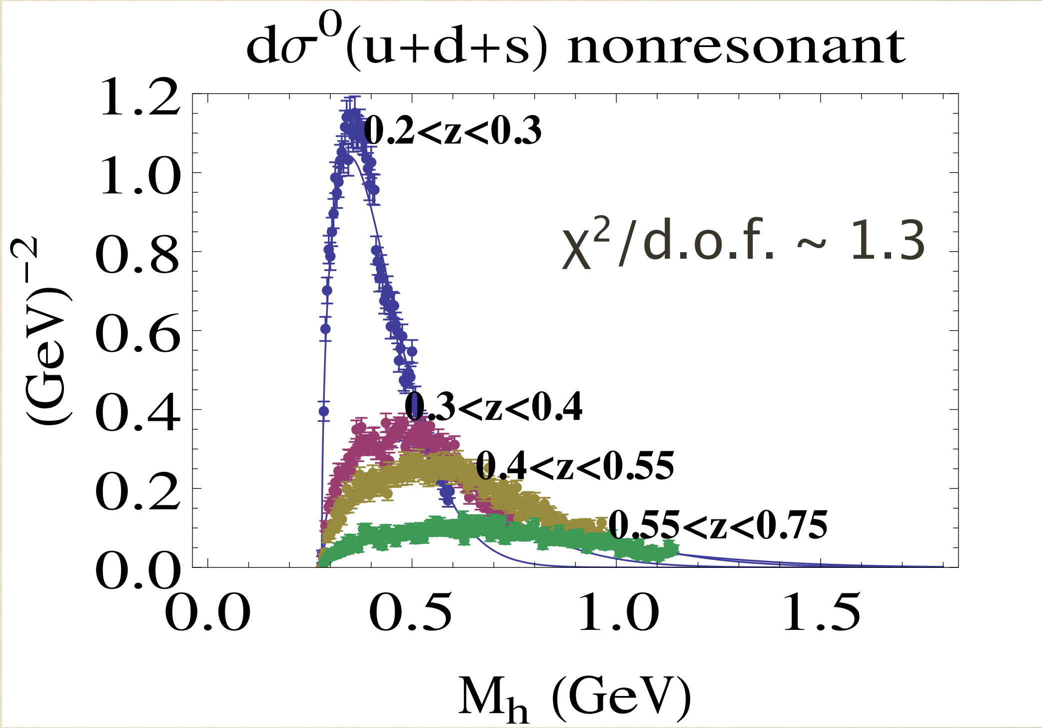
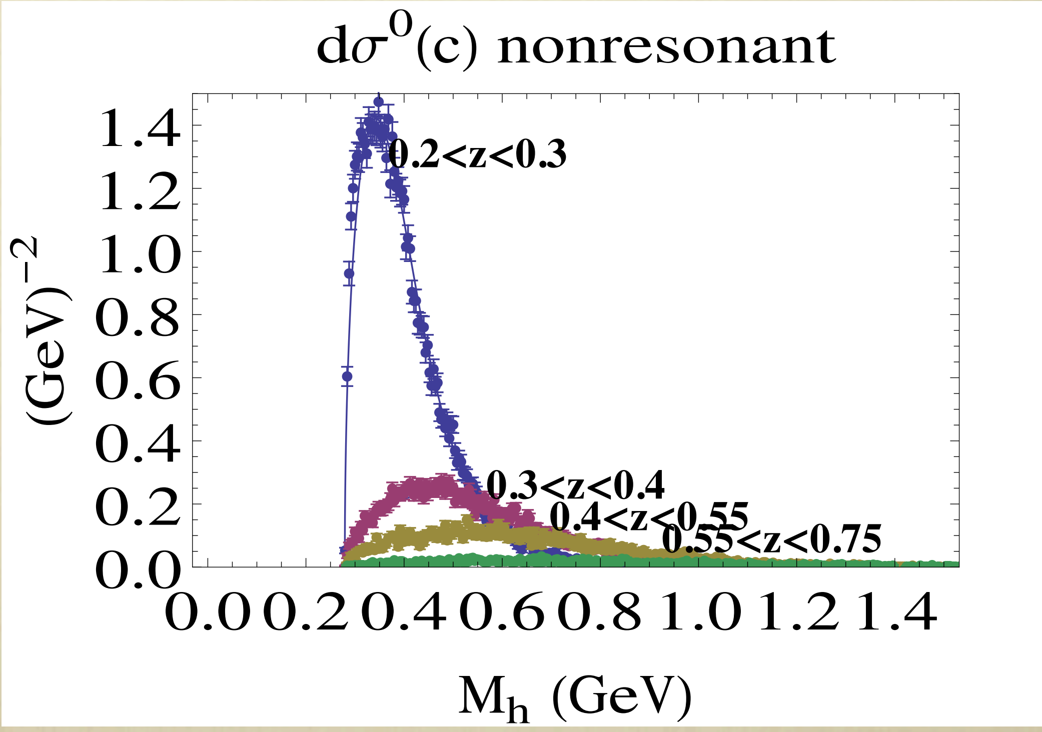




- big effect from charm
- no factorization of (z,M_h) depend.

work in progress for $d\sigma^0 \times a_{12R}$ ...

... but ...



Courtoy et al., arXiv:1012.0054 [hep-ph]

1^{st} extraction of transversity in coll. framework

Bacchetta et al., P.R.L. **107** (11)

$$A_{UT}^{\mathrm{SIDIS}}(x,z,M_h^2;Q^2) = -C_y \, \frac{|\boldsymbol{R}|}{M_h} \, \frac{\sum_q e_q^2 \, h_1^q(x,Q^2) \;\, H_1^{\sphericalangle\, q\to\pi^+\pi^-}(z,M_h^2;Q^2)}{\sum_q e_q^2 \, f_1^q(x,Q^2) \;\, D_1^{\sphericalangle\, q\to\pi^+\pi^-}(z,M_h^2;Q^2)}$$

$$\int dz \int dM_h^2$$

$$A_{UT}^{\mathrm{SIDIS}}(x,Q^2) = -C_y \, \frac{\sum_q e_q^2 \, h_1^q(x,Q^2) \;\, n_q^{\perp}(Q^2)}{\sum_q e_q^2 \, f_1^q(x,Q^2) \;\, n_q(Q^2)}$$

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assume charge/isospin symmetry

$$D_1^u = D_1^d = D_1^{\bar u} = D_1^{\bar d}$$

$$D_1^s = D_1^{\bar s} = N_s D_1^u \qquad D_1^c = D_1^{\bar c}$$

$$H_1^{\sphericalangle u} = -H_1^{\sphericalangle d} = -H_1^{\sphericalangle \bar u} = H_1^{\sphericalangle \bar d}$$

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$$x h_1^{u_v}(x,Q^2) - \frac{1}{4} x h_1^{d_v}(x,Q^2)$$

$$= -\frac{A_{UT}^{\text{SIDIS}}(x,Q^2)}{C_y} \frac{n_u(Q^2)}{n_u^{\uparrow}(Q^2)} \sum_{q=u,d,s} \frac{e_q^2 N_q}{e_u^2} x f_1^{q+\overline{q}}(x,Q^2)$$

HERMES (+ COMPASS) MSTW08LO

1st extraction of transversity in coll. framework

Bacchetta et al., P.R.L. **107** (11)

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goal $\longrightarrow$ $x h_1^{u_v}(x,Q^2) - \frac{1}{4} x h_1^{d_v}(x,Q^2)$

BELLE

$$= -\frac{A_{UT}^{\rm SIDIS}(x,Q^2)}{C_y} \, \frac{n_u(Q^2)}{n_u^{\uparrow}(Q^2)} \sum_{q=u,d,s} \frac{e_q^2 N_q}{e_u^2} x f_1^{q+\overline{q}}(x,Q^2)$$

HERMES (+ COMPASS)

MSTW08LO

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1. start from $D_1^{q=u,s,c}(z,M_h; Q_0^2=1)$, $H_1^{\sphericalangle u}(z,M_h; Q_0^2=1)$ Bacchetta & Radici, P.R.D74 (06)
   resonant + nonresonant channel inspired by spect. model
2. evolve at LO with HOPPET (updating with chiral-odd kernel)
3. fit $d\sigma^0$ from PYTHIA (adatped to BELLE) and $d\sigma^0 \times a_{12R}$ bin by bin
4. integrate D_1^q and $H_1^{\sphericalangle u}$ in HERMES range $0.5 \leq M_h \leq 1$, $0.2 \leq z \leq 0.7$
5. get $n_u^{\uparrow}(Q^2)/n_u(Q^2)$: $Q^2=2.5$ GeV^2 $n_u^{\uparrow}/n_u = -0.251 \pm 0.006_{ex} \pm 0.023_{th}$
   $[n_u^{\uparrow}/n_u\,(2.5)]$ / $[n_u^{\uparrow}/n_u\,(100)]$ ~ 92%(±8%)

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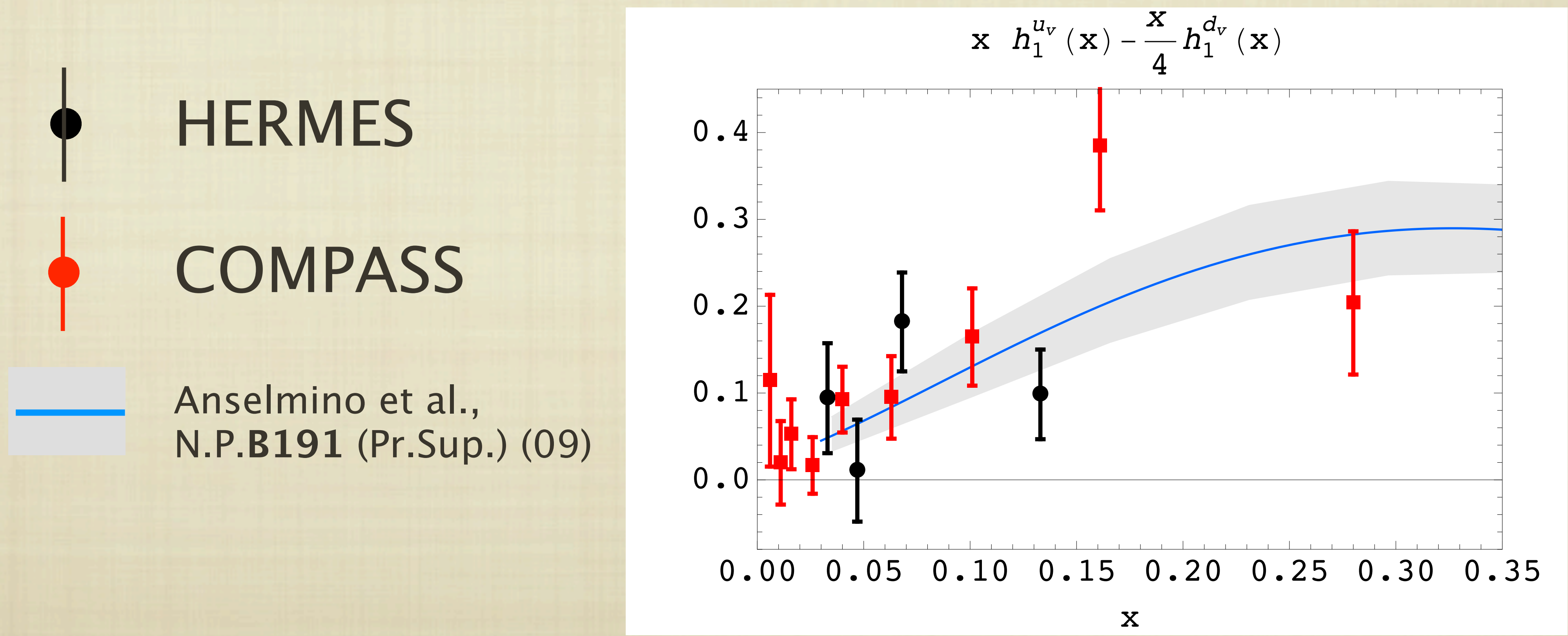

1^{st} extraction of transversity in coll. framework

Bacchetta et al., P.R.L. **107** (11)

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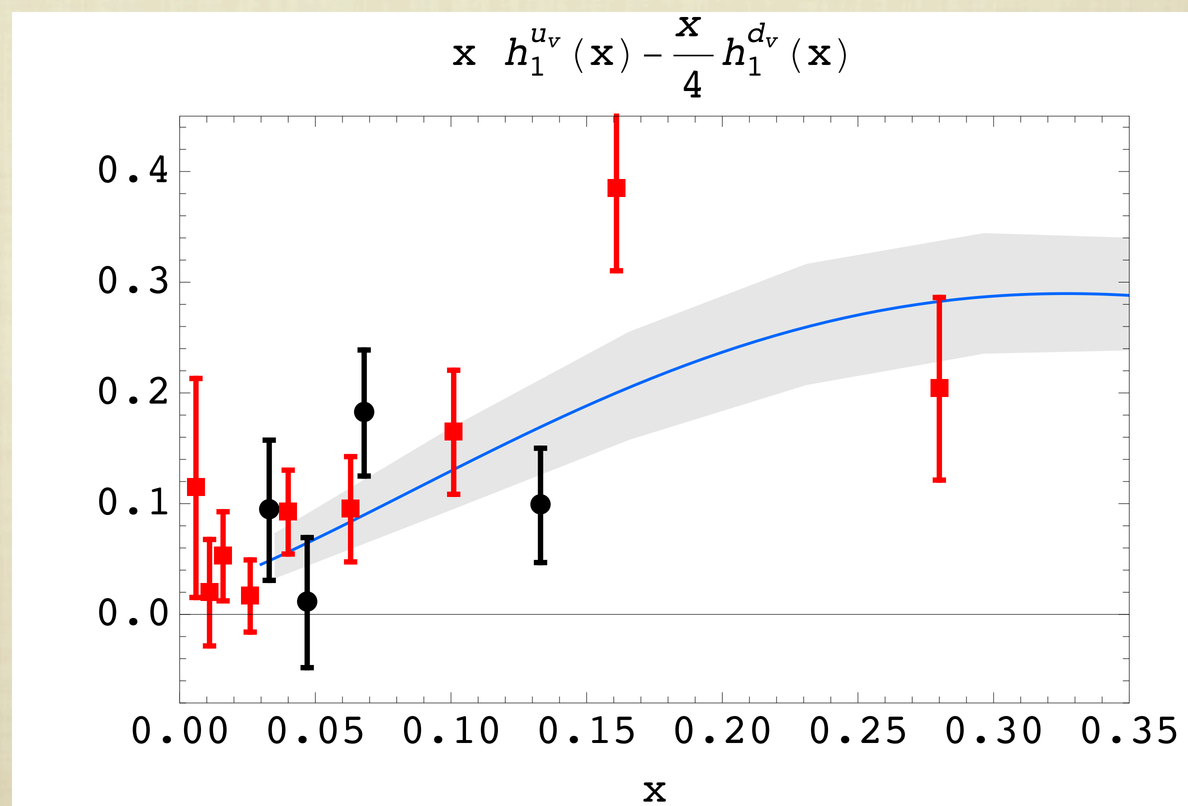
$\Delta\chi^2=1$ { HERMES

COMPASS

$\Delta\chi^2\sim17$ { Anselmino et al., N.P.**B191** (Pr.Sup.) (09)



several “BUT..”
work in progress
stay tuned..