TRANSVERSE ANGULAR MOMENTUM: NEW RESULTS

Elliot Leader

Imperial College London

Two topics:

1) **Brief comment on comparison of longitudinal and transverse sum rules**

2) **New relation between transverse angular momentum and GPDs.**

Derivation of a sum rule

Two steps:

Derivation of a sum rule

Two steps:

1) Derive expression for

$$\langle \text{Nucleon}; P, S \,|\, \boldsymbol{J} \,|\text{Nucleon}; P, S \,\rangle$$

Derivation of a sum rule

Two steps:

1) Derive expression for

$$\langle \text{Nucleon}; P, S \,|\, \boldsymbol{J} \,|\text{Nucleon}; P, S \,\rangle$$

2) Express $|\text{Nucleon}; P, S\,\rangle$ as a Fock expansion in terms of the constituents of the nucleon.

The super-quick approach to (1)

We know what a ROTATION does to a state, so we know matrix elements of R.

But, e.g.

$$R_z(\beta) = e^{-i\beta \boldsymbol{J}_z}$$

The super-quick approach to (1)

We know what a ROTATION does to a state, so we know matrix elements of R.

But, e.g.

$$R_z(\beta) = e^{-i\beta \boldsymbol{J}_z}$$

so that we get the matrix element of J_z using

$$\boldsymbol{J}_z = i \frac{d}{d\beta} R_z(\beta) \Big|_{\beta=0}$$

The traditional approach to (1)

Typically the angular momentum density involves the energy-momentum tensor density $t^{\mu\nu}(x)$ in the form e.g.

$$\boldsymbol{J}_z = \boldsymbol{J}^3 = \int dV\,[x t^{02}(x) - y t^{01}(x)]$$

The traditional approach to (1)

Typically the angular momentum density involves the energy-momentum tensor density $t^{\mu\nu}(x)$ in the form e.g.

$$\boldsymbol{J}_z = \boldsymbol{J}^3 = \int dV \, [x t^{02}(x) - y t^{01}(x)]$$

The factors x, y cause trouble. End up with things like

$$\int dV x \langle P, S | t^{02}(0) | P, S \rangle$$

The matrix element is independent of x so we are faced with $\int dV x = \infty$? or $= 0$? Totally ambiguous!

The problem is an old one: In ordinary QM plane wave states give infinities.

The problem is an old one: In ordinary QM plane wave states give infinities.

The solution is an old one: Build a wave packet, a superposition of physical plane wave states...... but.... it is a looooooooong, complicated calculation.

Both approaches give same result

$$\langle \text{Nucleon}; P, S \,|\, \boldsymbol{J} \,|\text{Nucleon}; P, S \,\rangle = \frac{1}{2}\boldsymbol{s} + \text{delta function}$$

where $\boldsymbol{s}$ is the REST FRAME SPIN VECTOR.

Both approaches give same result

$$\langle \text{Nucleon}; P, S \,|\, \boldsymbol{J} \,|\text{Nucleon}; P, S \,\rangle = \frac{1}{2}\boldsymbol{s} + \text{delta function}$$

where $\boldsymbol{s}$ is the REST FRAME SPIN VECTOR.

KEY POINT: This result is INDEPENDENT OF WHETHER $\boldsymbol{s}$ IS LONGITUDINAL OR TRANSVERSE.

Comparison of longitudinal and transverse sum rules

First ever use of the transverse sum rule

First moment of u and d transversity from Anselmino et al arXiv:0812.4366 ....assumes sea quark transversity zero

$$J_{Tr} = \frac{1}{2} = 0.16^{+0.07}_{-0.14} + L_{Tr}$$

Compare with

$$J_z = \frac{1}{2} = 0.42 \pm 0.19 + L_z \quad \text{for } \Delta G > 0$$

or

$$J_z = -0.21 \pm 0.46 + L_z \quad \text{for changing sign } \Delta G$$

New relation between transverse angular momentum and GPDs

Although painful, the traditional approach is fruitful, because it connects matrix elements of $\boldsymbol{J}$ with matrix elements of the energy momentum tensor $t^{\mu\nu}$.

The most general form of the matrix elements of $t^{\mu\nu}$, say for quarks, is (similar for gluons)

$$\begin{aligned}\langle P', S' \,|\, t_q^{\mu\nu}(0) \,|\, P, S \rangle \;&=\; [\bar{u}'\gamma^{\mu}u\,\bar{P}^{\nu} + (\mu \leftrightarrow \nu)]\mathbb{D}_q(\Delta^2)/2 \\ &\quad - \left[\frac{i\Delta_{\rho}}{2M}\,\bar{u}'\sigma^{\mu\rho}u\,\bar{P}^{\nu} + (\mu \leftrightarrow \nu)\right][\mathbb{D}_q(\Delta^2)/2 - \mathbb{S}_q(\Delta^2)] \\ &+\frac{\bar{u}'u}{2M}\left[\frac{1}{2}[\mathbb{G}_q(\Delta^2) - \mathbb{H}_q(\Delta^2)](\Delta^{\mu}\Delta^{\nu} - \Delta^2 g^{\mu\nu}) + M^2\mathbb{R}_q(\Delta^2)g^{\mu\nu}\right]\end{aligned}$$

where

$$u \equiv u(P,S) \qquad u' \equiv u(P',S') \qquad \Delta = P' - P.$$

Comparing with the definition of GPDs one finds

$$\int_{-1}^{1} dx x H_q(x,0,0) = \mathbb{D}_q = \text{momentum fraction carried by quarks}$$

Comparing with the definition of GPDs one finds

$$\int_{-1}^{1} dx x H_q(x,0,0) = \mathbb{D}_q = \text{momentum fraction carried by quarks}$$

Therefore

$$\int_{-1}^{1} dx x [H_q(x,0,0) + H_G(x,0,0)] = \mathbb{D}_q + \mathbb{D}_G = 1$$

Also, comparing with GPDs,

$$\int_{-1}^{1} dx x E_q(x,0,0) = (2\,\mathbb{S}_q - \mathbb{D}_q)$$

From these

$$\int_{-1}^{1} dx x H_q(x,0,0) + \int_{-1}^{1} dx x E_q(x,0,0) = 2\,\mathbb{S}_q.$$

From these

$$\int_{-1}^{1} dx x H_q(x, 0, 0) + \int_{-1}^{1} dx x E_q(x, 0, 0) = 2\,\mathbb{S}_q.$$

Longitudinal polarized nucleon

BLT showed that the expectation value

$$\langle\langle\, J_z(\text{quark})\,\rangle\rangle = \mathbb{S}_q$$

From these

$$\int_{-1}^{1} dx x H_q(x,0,0) + \int_{-1}^{1} dx x E_q(x,0,0) = 2\,\mathbb{S}_q.$$

Longitudinal polarized nucleon

BLT showed that the expectation value

$$\langle\langle\, J_z(\text{quark})\,\rangle\rangle = \mathbb{S}_q$$

so that

$$\int_{-1}^{1} dx x [H_q(x,0,0) + E_q(x,0,0)] = 2\,\langle\langle\, J_z(\text{quark})\rangle\rangle$$

which is the relation first derived by Ji.

How to test these results?

Ji likes to define

$$L_z(\text{quark}) = \frac{1}{2}\int_{-1}^{1} dx x[H_q(x,0,0)+E_q(x,0,0)]-\frac{1}{2}\Delta\Sigma_{\bar{M}S}$$

How to test these results?

Ji likes to define

$$L_z(\text{quark}) = \frac{1}{2}\int_{-1}^{1} dx x[H_q(x,0,0)+E_q(x,0,0)]-\frac{1}{2}\Delta\Sigma_{\bar{M}S}$$

Compare with model calculation? Or with Lattice calculation?

How to test these results?

Ji likes to define

$$L_z(\text{quark}) = \frac{1}{2}\int_{-1}^{1} dx x[H_q(x,0,0)+E_q(x,0,0)]-\frac{1}{2}\Delta\Sigma_{\bar{M}S}$$

Compare with model calculation? Or with Lattice calculation?

Or compare

$$\frac{1}{2}\int_{-1}^{1} dx x[H_q(x,0,0) + E_q(x,0,0)]$$

directly with calculation of $\langle\langle\, J_z(\text{quark})\rangle\rangle$.

How to test these results?

Ji likes to define

$$L_z(\text{quark}) = \frac{1}{2}\int_{-1}^{1} dx x [H_q(x,0,0) + E_q(x,0,0)] - \frac{1}{2}\Delta\Sigma_{\bar{M}S}$$

Compare with model calculation? Or with Lattice calculation?

Or compare

$$\frac{1}{2}\int_{-1}^{1} dx x [H_q(x,0,0) + E_q(x,0,0)]$$

directly with calculation of $\langle\langle J_z(\text{quark})\rangle\rangle$.

Or test

$$\int_{-1}^{1} dx x [E_q(x,0,0) + E_G(x,0,0)] = 0$$

Transversely polarized nucleon

From BLT

$$\langle\langle\, J_x(\text{quark}\,\rangle\rangle \;=\; \frac{1}{2M}\,[(M - P_0)\,\mathbb{D}_q + 2P_0\,\mathbb{S}_q]$$
$$\;=\; \frac{1}{2M}\,[(P_0\,(2\,\mathbb{S}_q - \mathbb{D}_q) + M\,\mathbb{D}_q]$$

Transversely polarized nucleon

From BLT

$$\langle\langle\, J_x(\text{quark}\,\rangle\rangle \;=\; \frac{1}{2M}\,[(M - P_0)\,\mathbb{D}_q + 2P_0\,\mathbb{S}_q]$$
$$\;=\; \frac{1}{2M}\,[(P_0\,(2\,\mathbb{S}_q - \mathbb{D}_q) + M\,\mathbb{D}_q]$$

Thus, new result:

$$\langle\langle\, J_x(\text{quark}\,\rangle\rangle = \frac{1}{2M}\left[P_0 \int_{-1}^{1} dx x E_q(x,0,0) + M \int_{-1}^{1} dx x H_q(x,0,0)\right]$$

$$\langle\langle\, J_x(\text{quark}\,\rangle\rangle = \frac{1}{2M}\left[P_0 \int_{-1}^{1} dx x E_q(x,0,0) + M \int_{-1}^{1} dx x H_q(x,0,0)\right]$$

Dependence on P_0 ? Expected or unexpected?

$$\langle\langle\, J_x(\text{quark}\,)\rangle\rangle = \frac{1}{2M}\left[P_0 \int_{-1}^{1} dx x E_q(x,0,0) + M \int_{-1}^{1} dx x H_q(x,0,0)\right]$$

Dependence on P_0 ? Expected or unexpected?

Expected: supported by a purely classical picture where the orbital angular momentum is generated by the quark rotating about the CM of the nucleon.

$$\langle\langle\, J_x(\text{quark}\,)\rangle\rangle = \frac{1}{2M}\left[P_0 \int_{-1}^{1} dx x E_q(x,0,0) + M \int_{-1}^{1} dx x H_q(x,0,0)\right]$$

Dependence on P_0 ? Expected or unexpected?

Expected: supported by a purely classical picture where the orbital angular momentum is generated by the quark rotating about the CM of the nucleon.

Of course

$$\langle\langle\, J_x(\text{quark}\,)\rangle\rangle + \langle\langle\, J_x(\text{gluon}\,)\rangle\rangle$$

is independent of P_0 since

$$\int_{-1}^{1} dx x [E_q(x,0,0) + E_G(x,0,0)] = 0$$

How to test these results?

Calculate $\langle\langle\, J_x(\text{quark}\,\rangle\rangle$ on the Lattice ?

How to test these results?

Calculate $\langle\langle\, J_x(\text{quark}\,\rangle\rangle$ on the Lattice ?

Define

$$\langle\langle\, L_x(\text{quark})\,\rangle\rangle = \langle\langle\, J_x(\text{quark})\,\rangle\rangle - \frac{1}{2}\int dx\, \Delta_T q(x)$$

and calculate $\langle\langle\, L_x(\text{quark})\,\rangle\rangle$ in model or on Lattice?

Summary

1) Have given new relations between GPD structure functions and Transverse angular momentum of quarks and gluons.

2) Would be very interesting to try to test these.