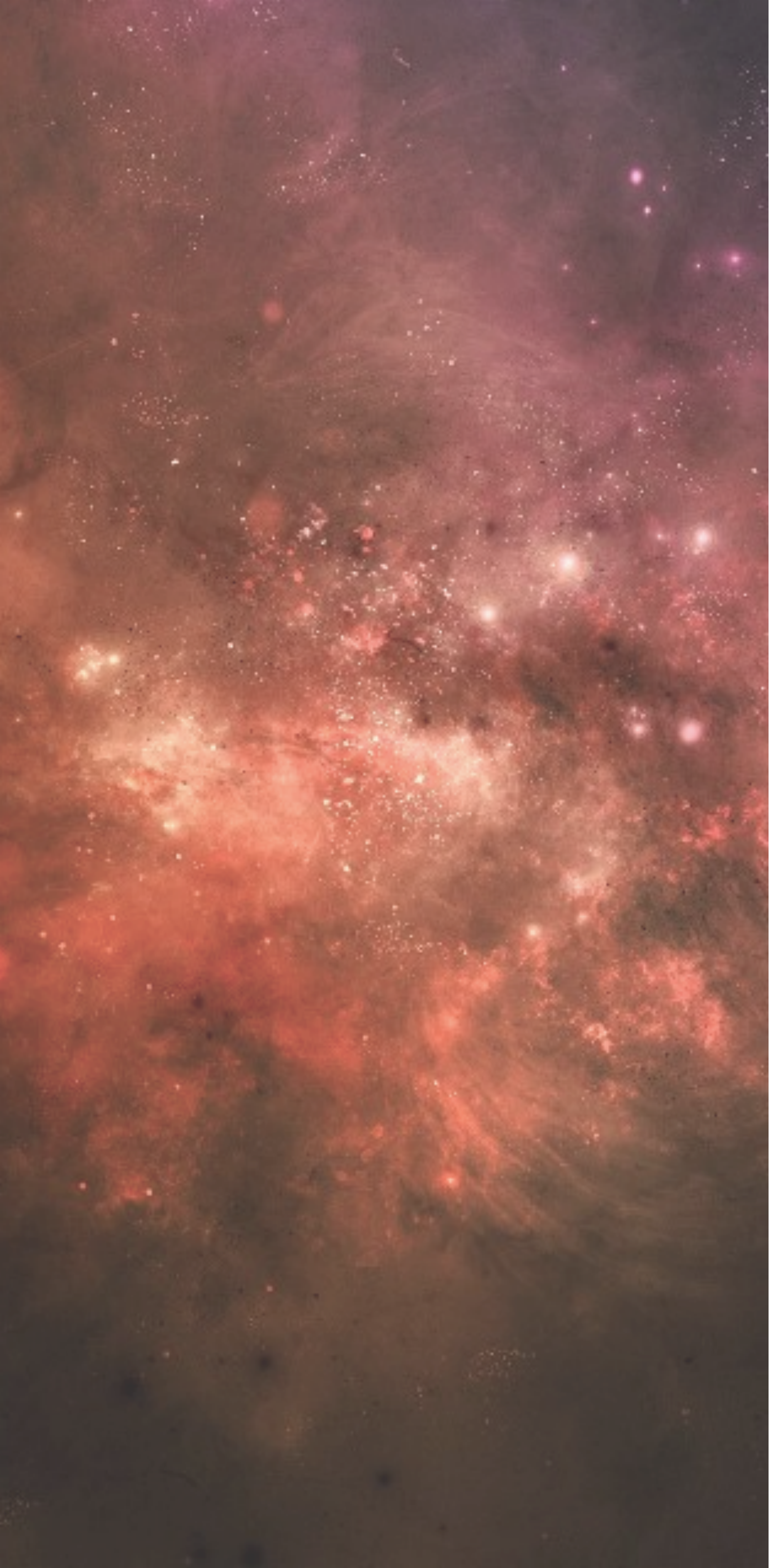
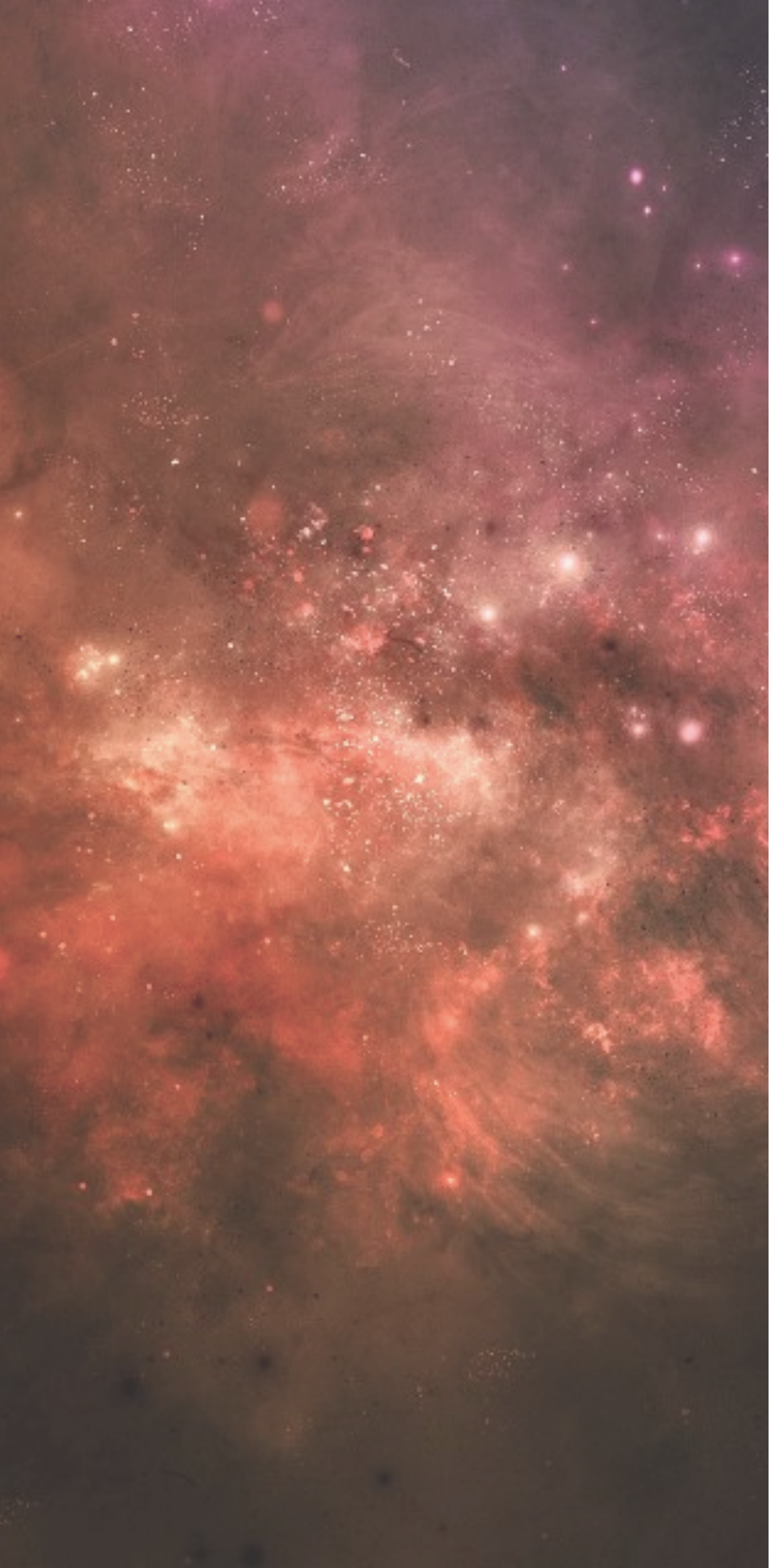


Fast and Differentiable Big Bang Nucleosynthesis

Hongwan Liu

with Cara Giovanetti, Mariangela Lisanti,
Siddharth Mishra-Sharma and Joshua Ruderman
240X.XXXXX, 240X.XXXXX

ALPS Workshop 5 April 2024



BBN: You’re Doing It Wrong!

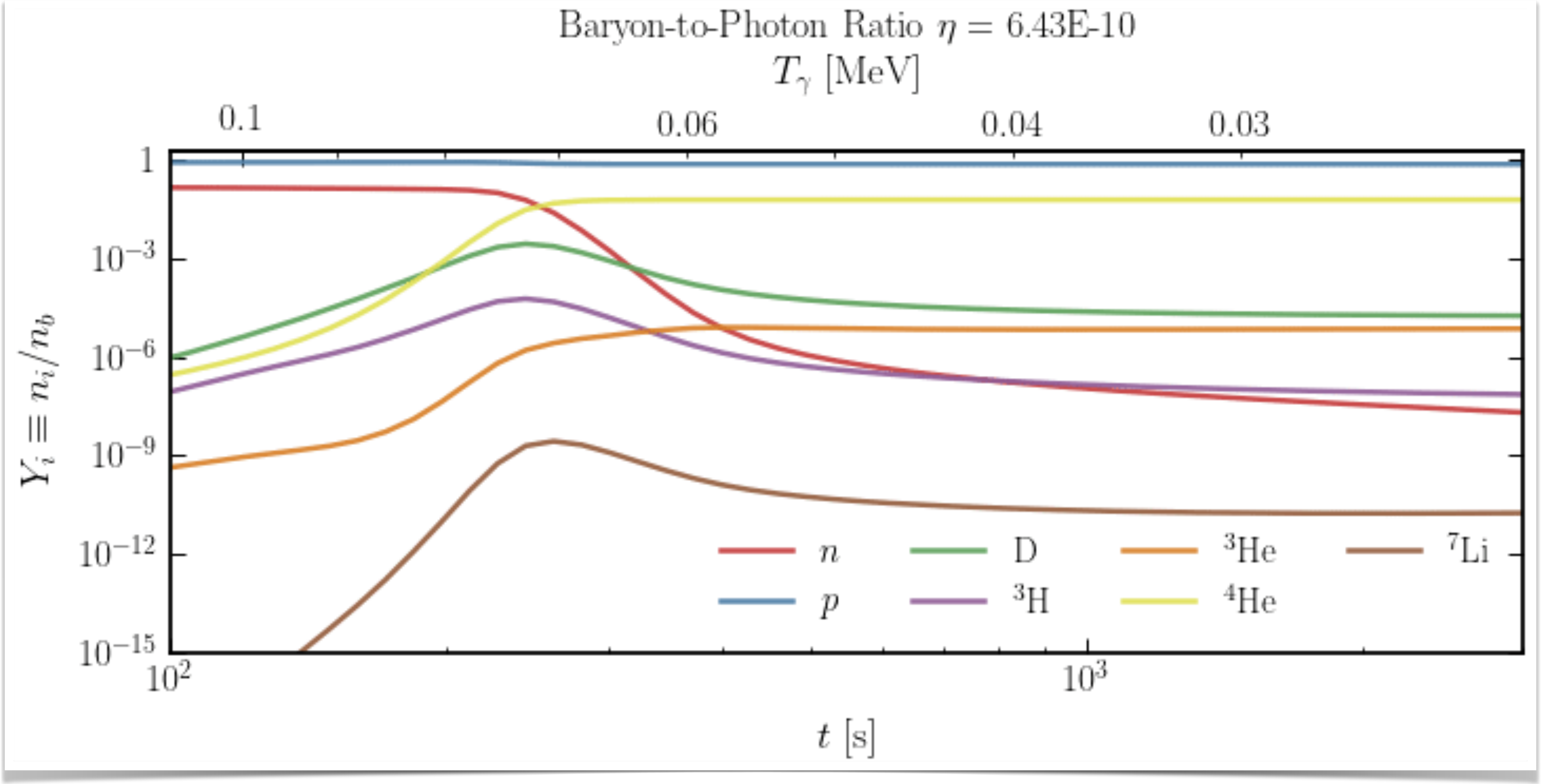
credit: Yoni Kahn, Risa Wechsler

Hongwan Liu

with Cara Giovanetti, Mariangela Lisanti,
Siddharth Mishra-Sharma and Joshua Ruderman
240X.XXXXX, 240X.XXXXX

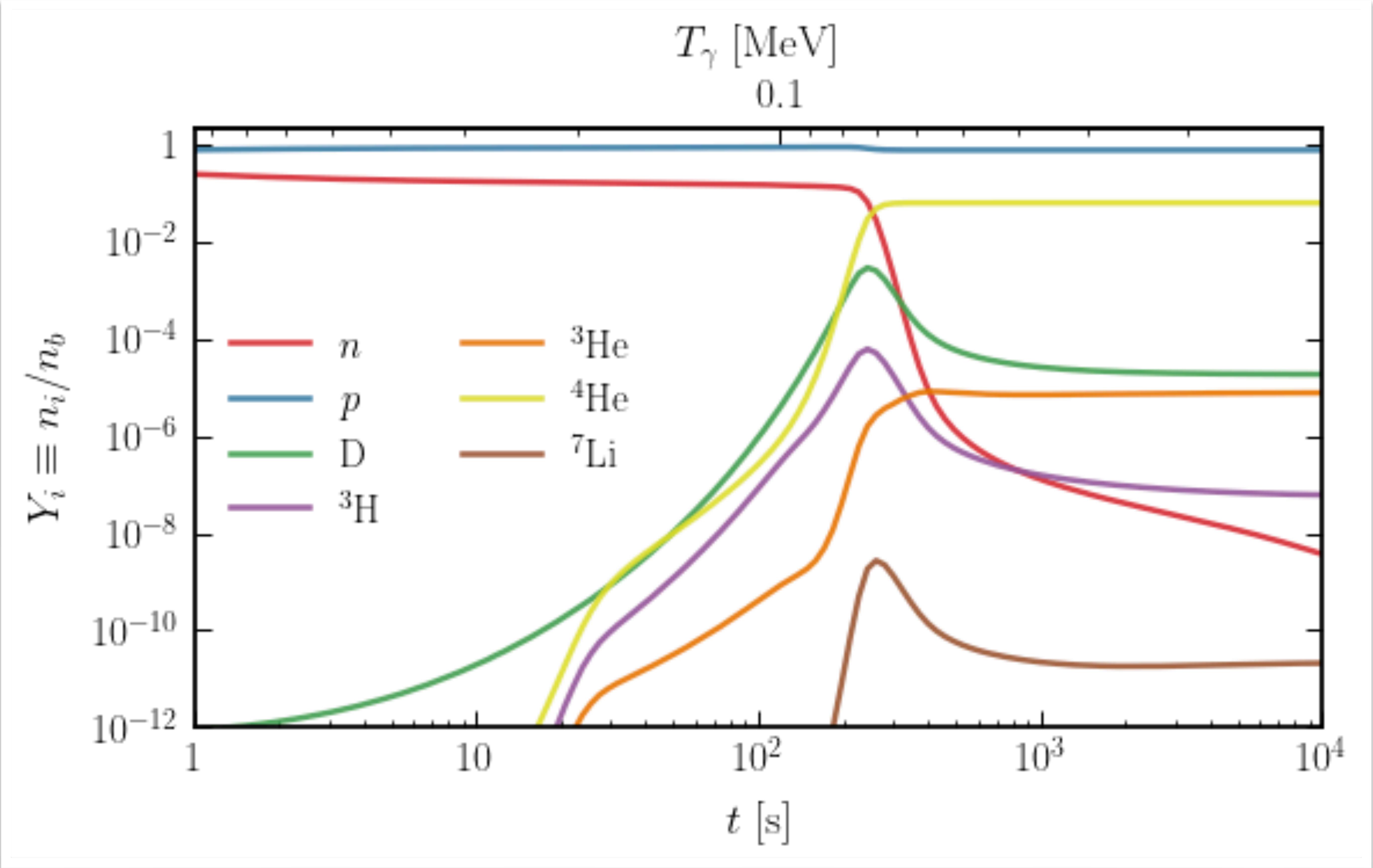
ALPS Workshop 5 April 2024

Punchline



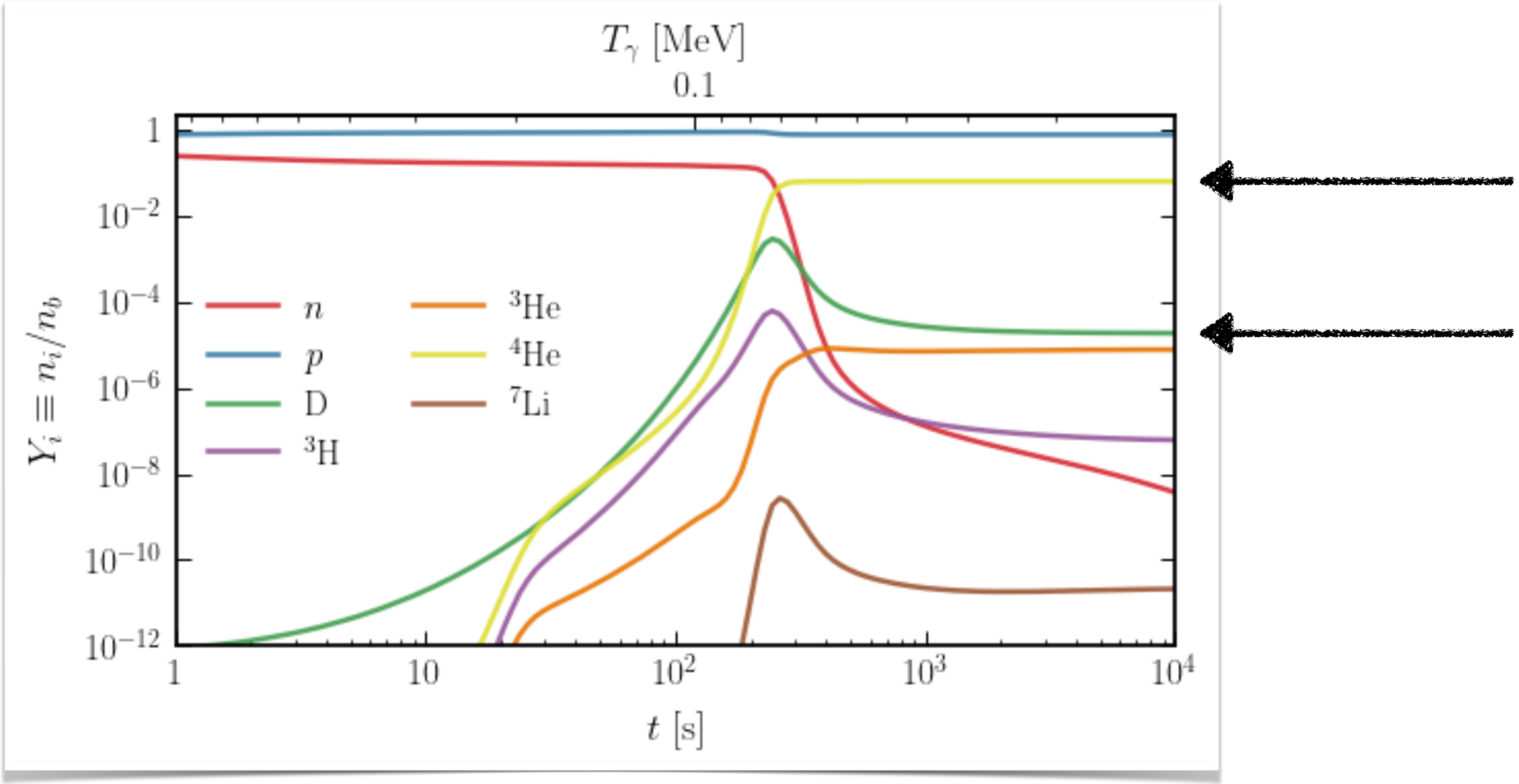
Fast, accurate BBN calculation that can elevate BBN analyses to the same level as CMB analyses, where it should be!

BBN: Formation of Light Elements



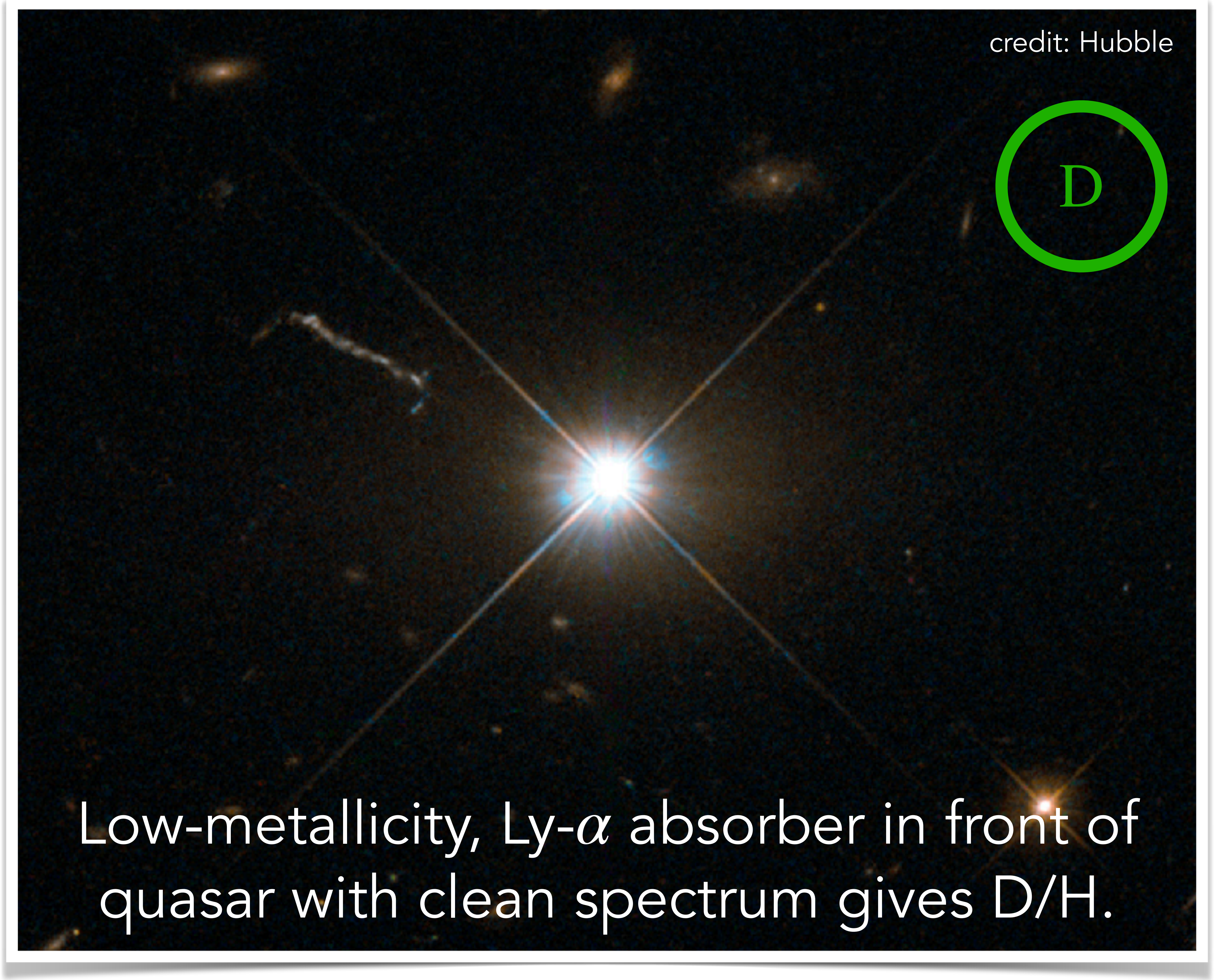
Nuclear reactions active in the hot, early Universe.
Gradually shut off as the Universe cools, freezes out to **primordial abundance**.

Abundance Sensitive to MeV Scale Physics

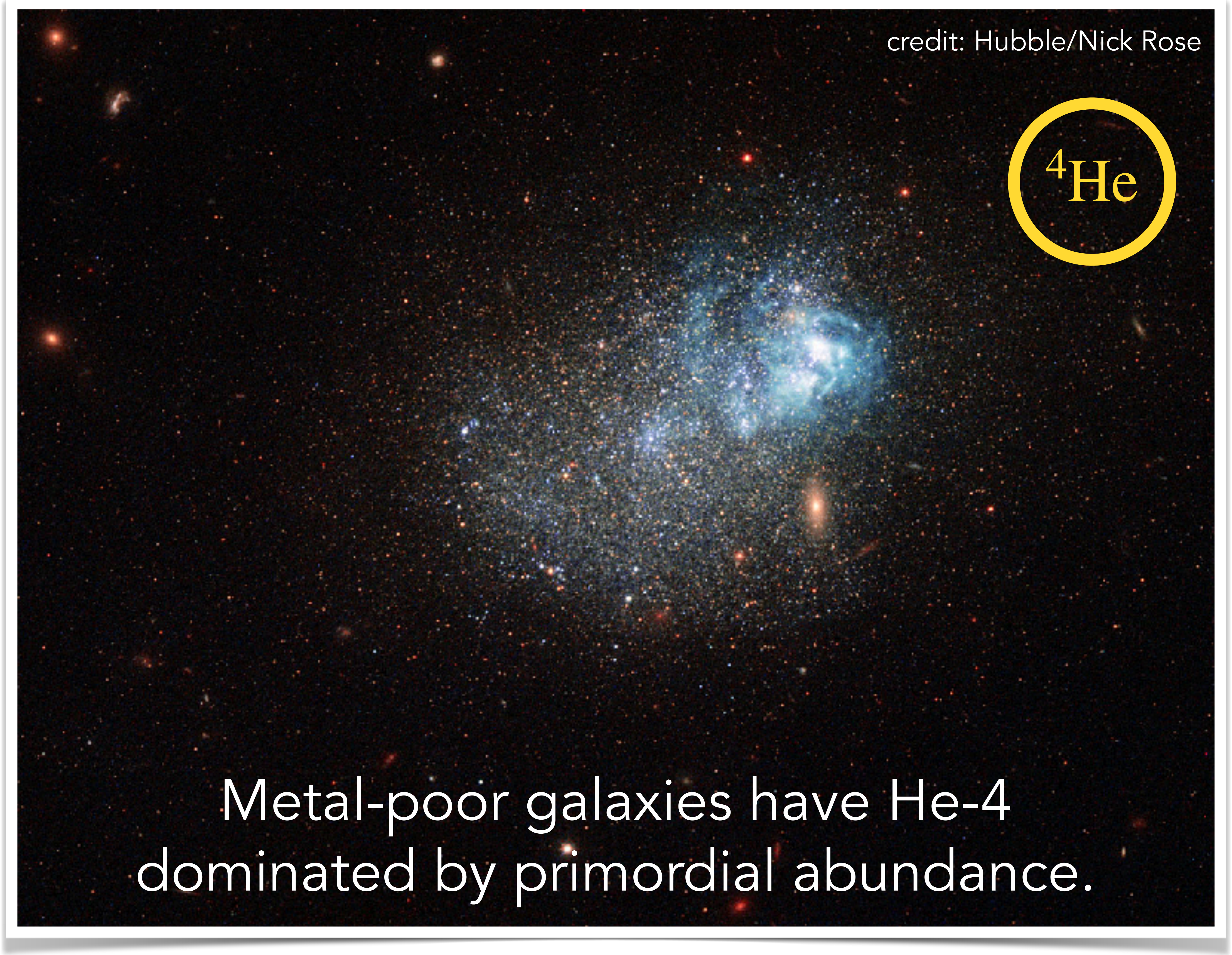


Focus on **deuterium** ($\mathrm{D/H} \equiv n_\mathrm{D}/n_p$) and **helium-4** ($Y_p \equiv \rho_{^4\mathrm{He}}/\rho_b$).

Observed Primordial Abundance


credit: Hubble

Low-metallicity, Ly-α absorber in front of quasar with clean spectrum gives D/H.

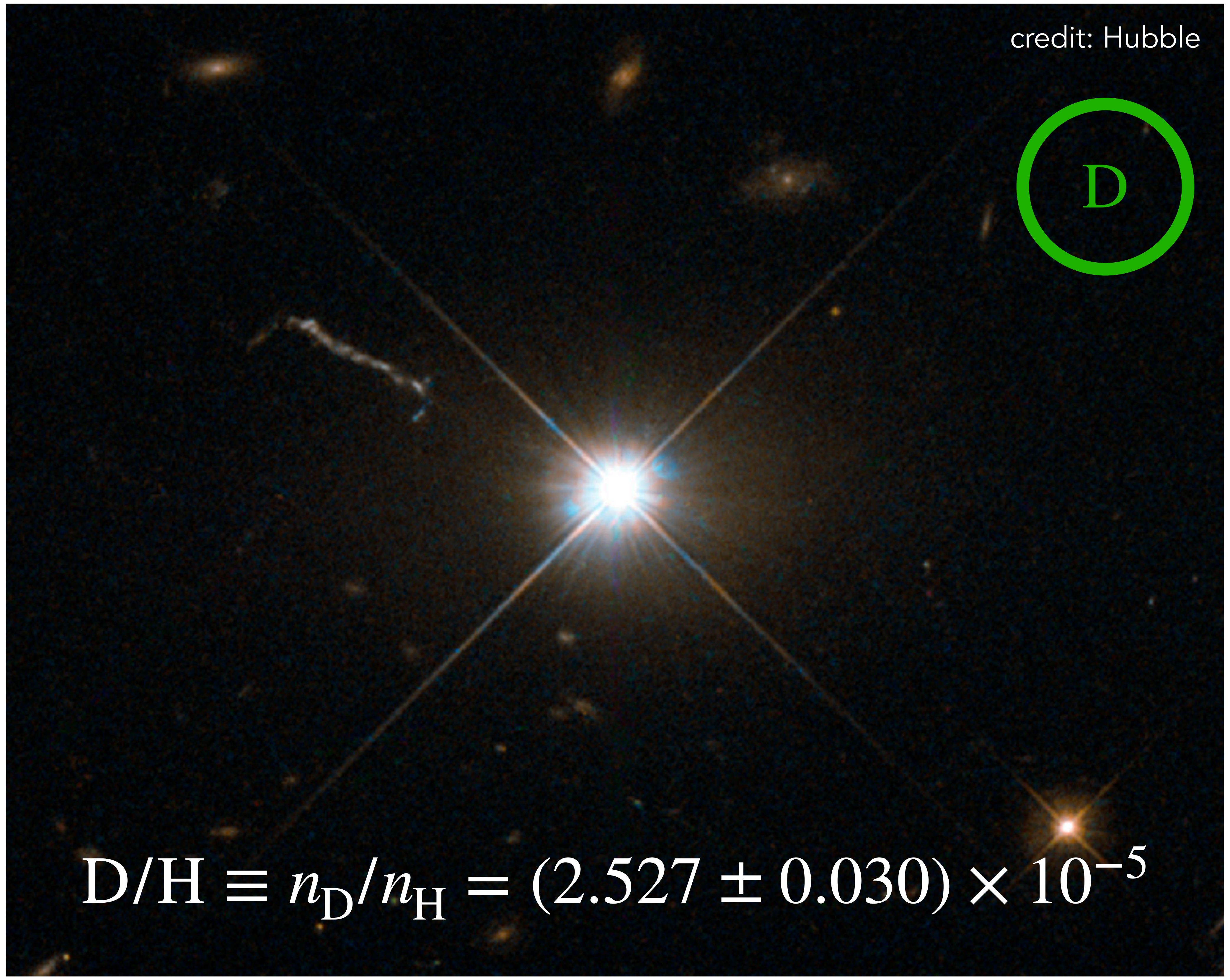

credit: Hubble/Nick Rose

Metal-poor galaxies have He-4 dominated by primordial abundance.

To test cosmology with BBN, we need to measure **primordial abundance**.
Achieved for **deuterium** and **helium-4** (lithium-7 is murky).

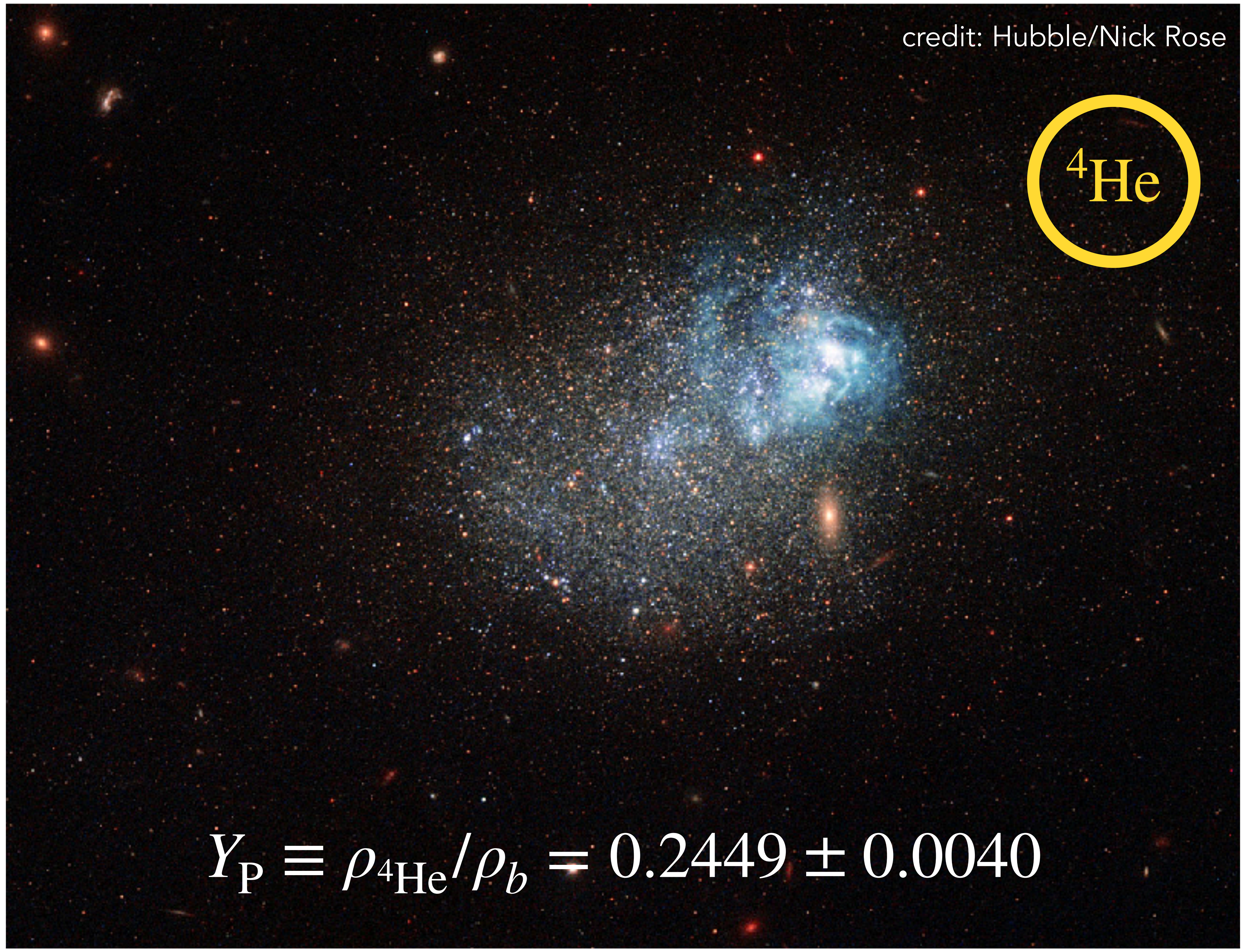
Aver, Olive & Skillman 1503.08146; Cooke, Pettini & Steidel 1710.11129

1% Experimental Precision



credit: Hubble

$$\mathrm{D/H} \equiv n_\mathrm{D}/n_\mathrm{H} = (2.527 \pm 0.030) \times 10^{-5}$$

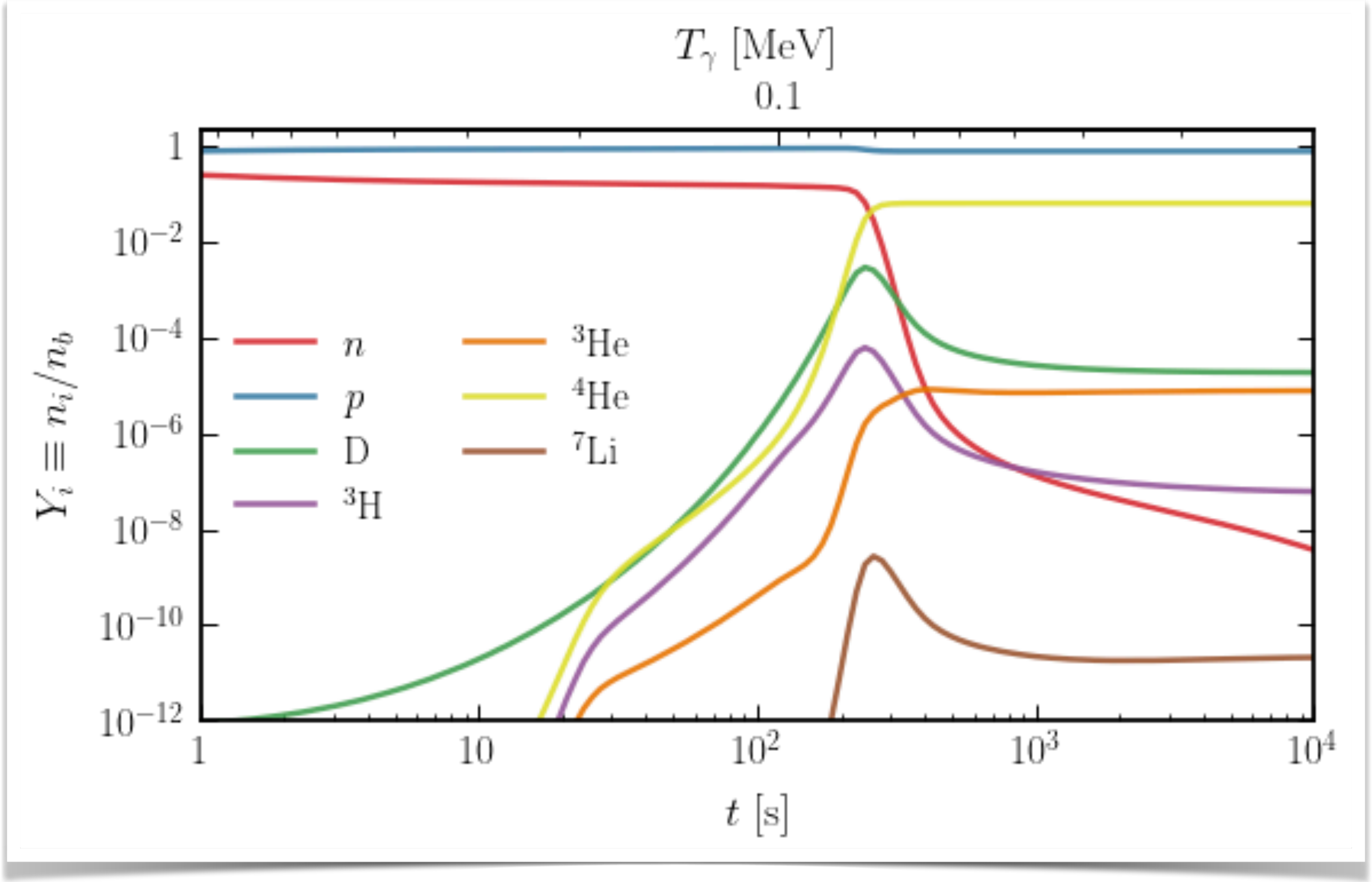


credit: Hubble/Nick Rose

$$Y_\mathrm{P} \equiv \rho_{^4\mathrm{He}}/\rho_b = 0.2449 \pm 0.0040$$

Can be a powerful test of cosmology, if theory uncertainties are under control!

How do we predict elemental abundances?



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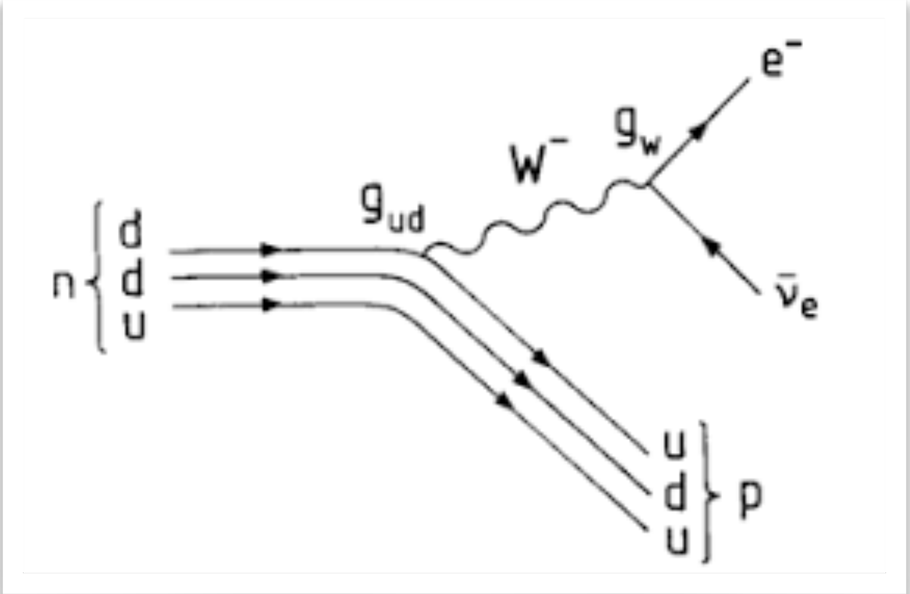
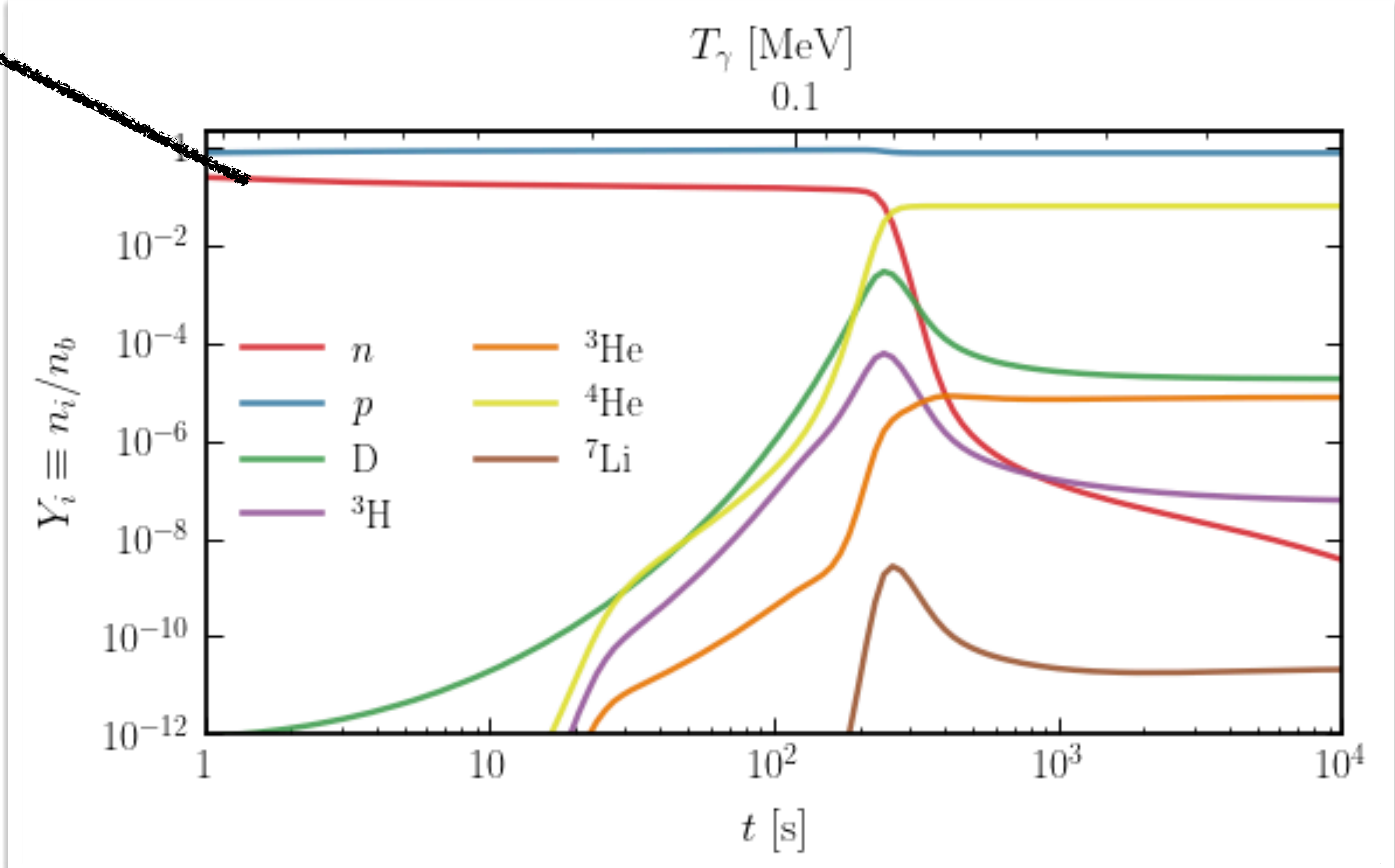


Electroweak physics governing $n \leftrightarrow p$



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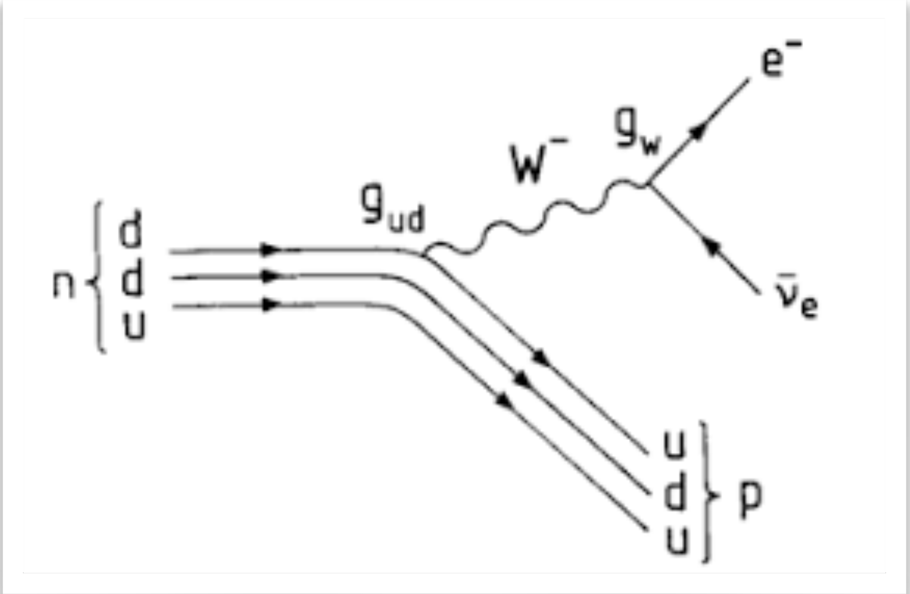

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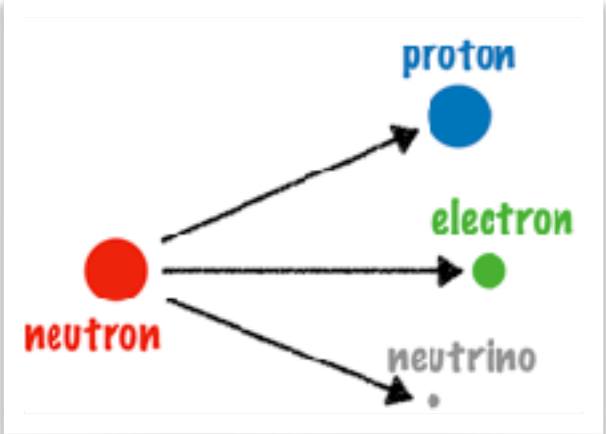
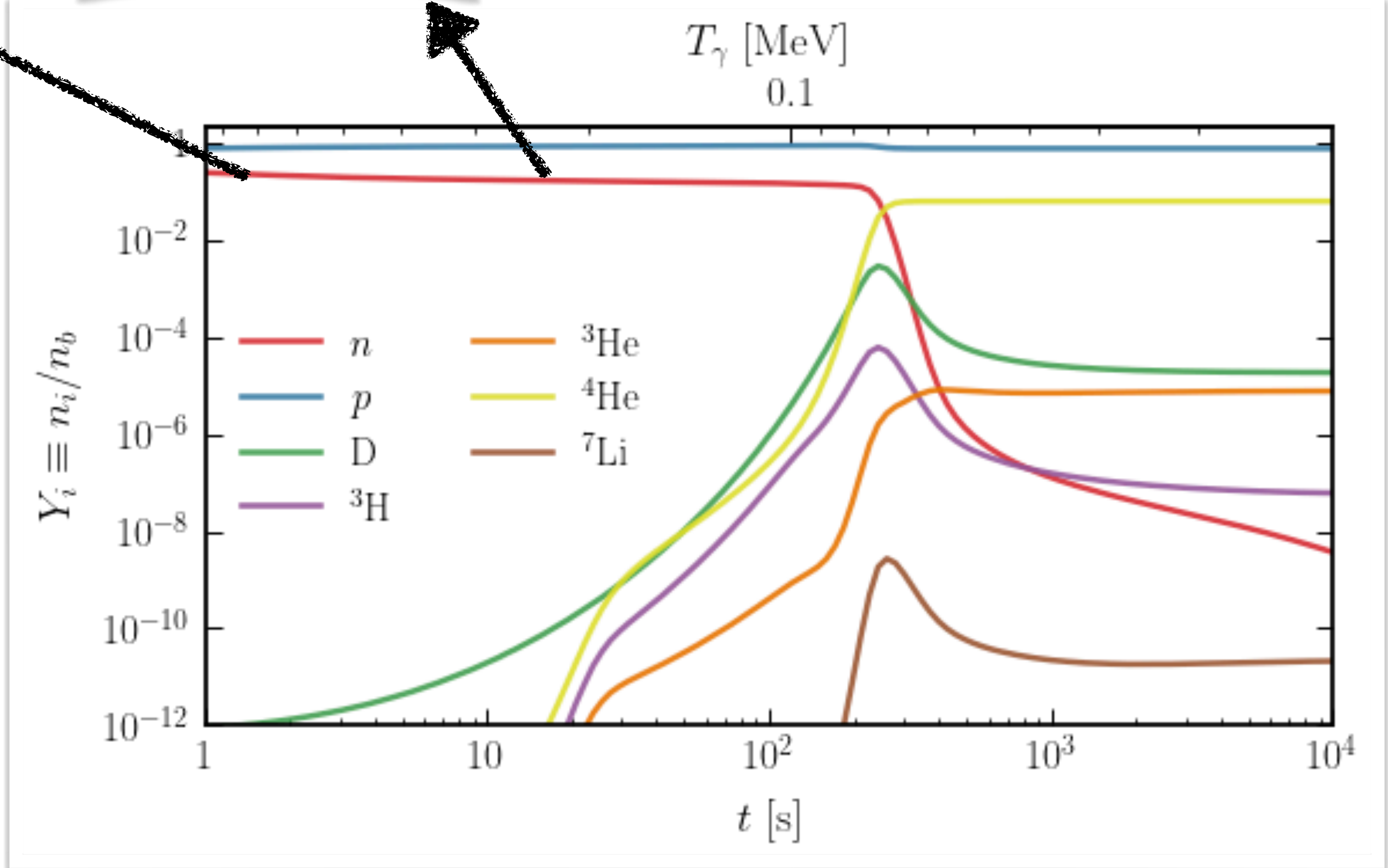


Neutron decay lifetime τ_n



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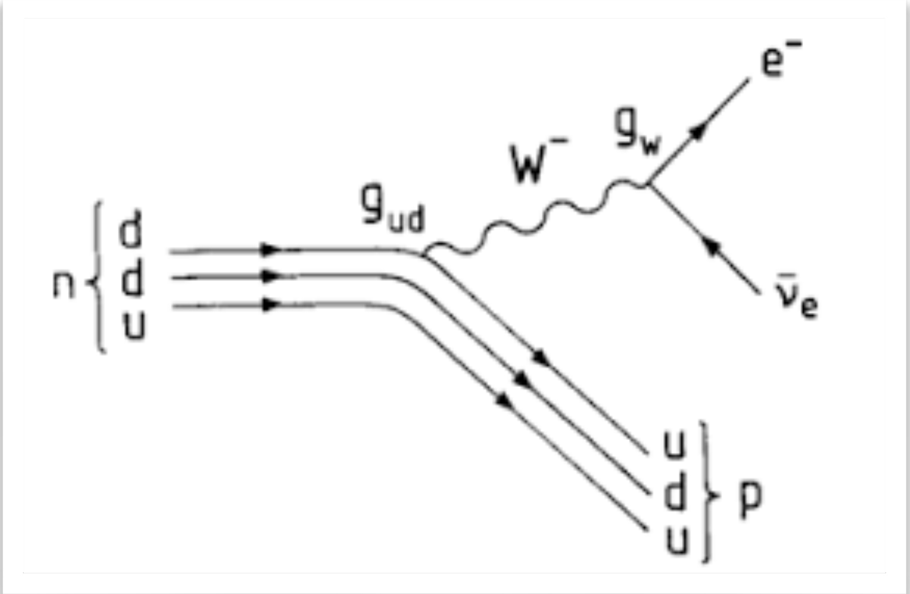

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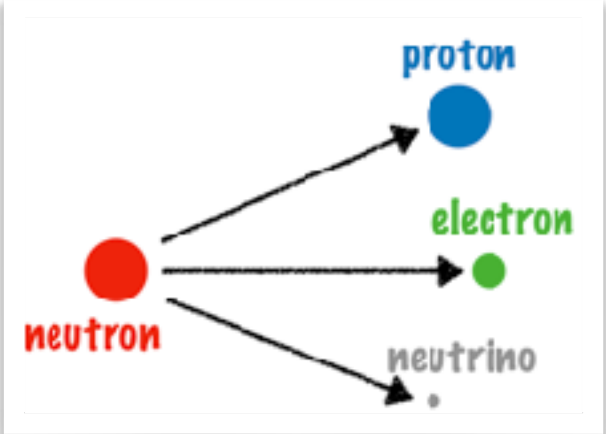

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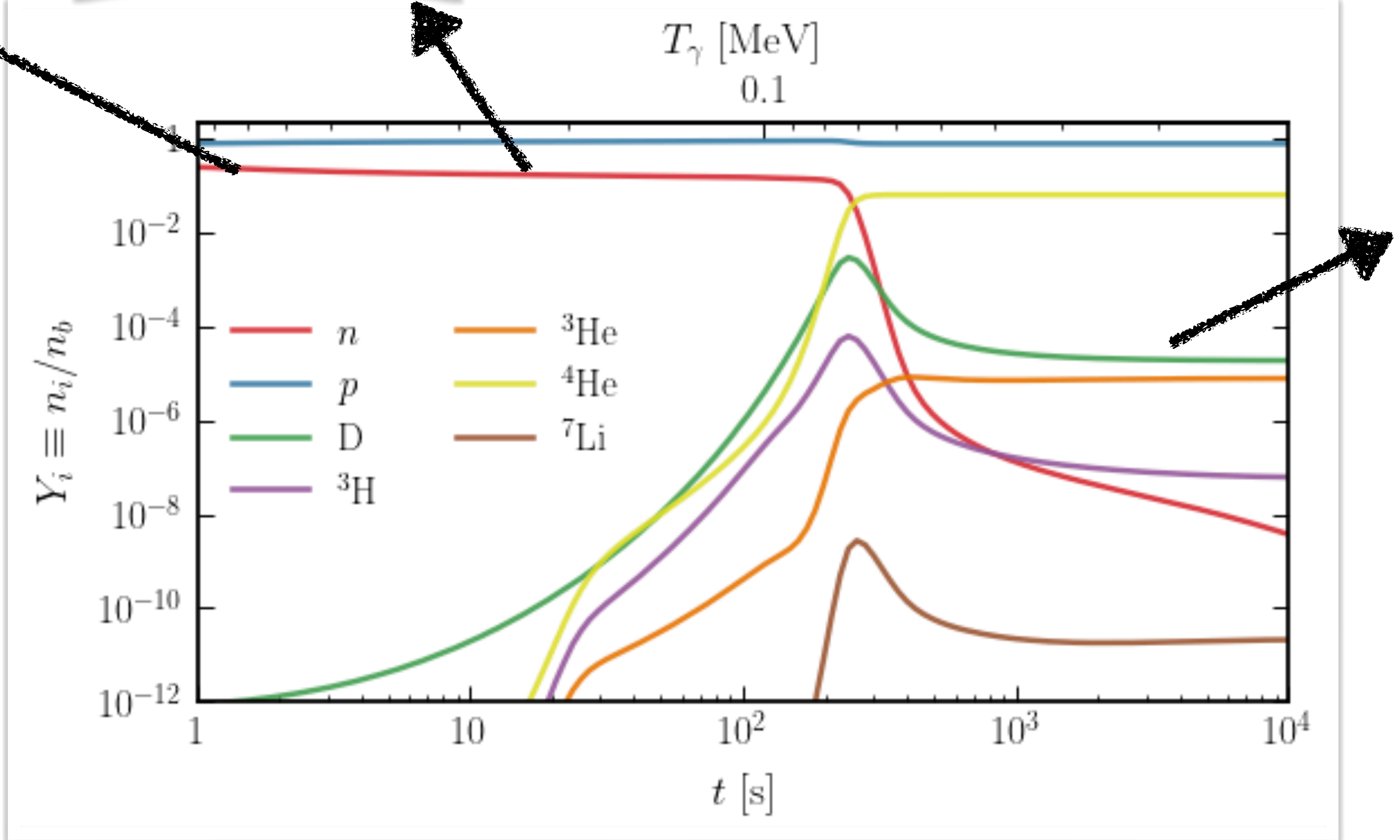
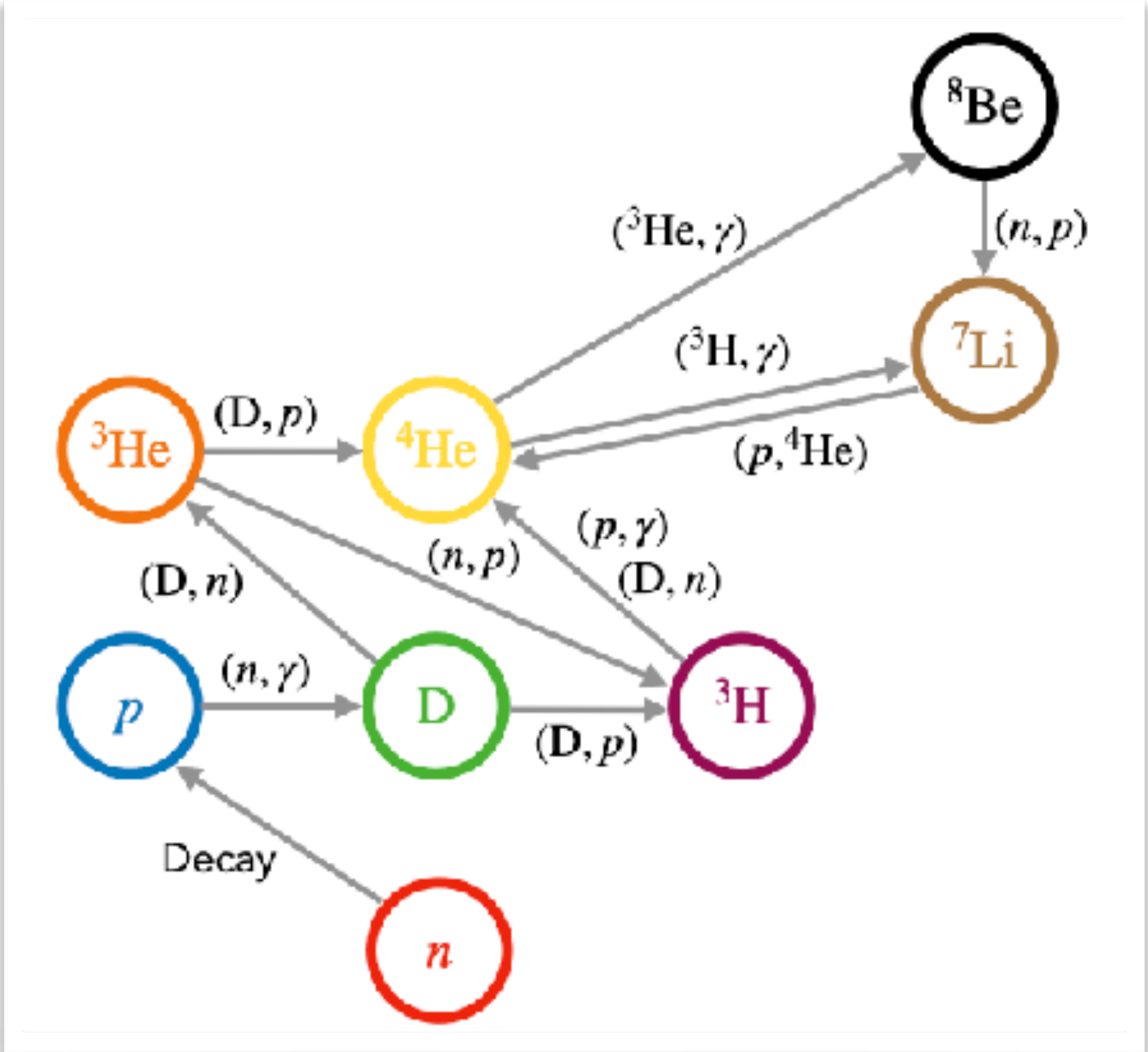
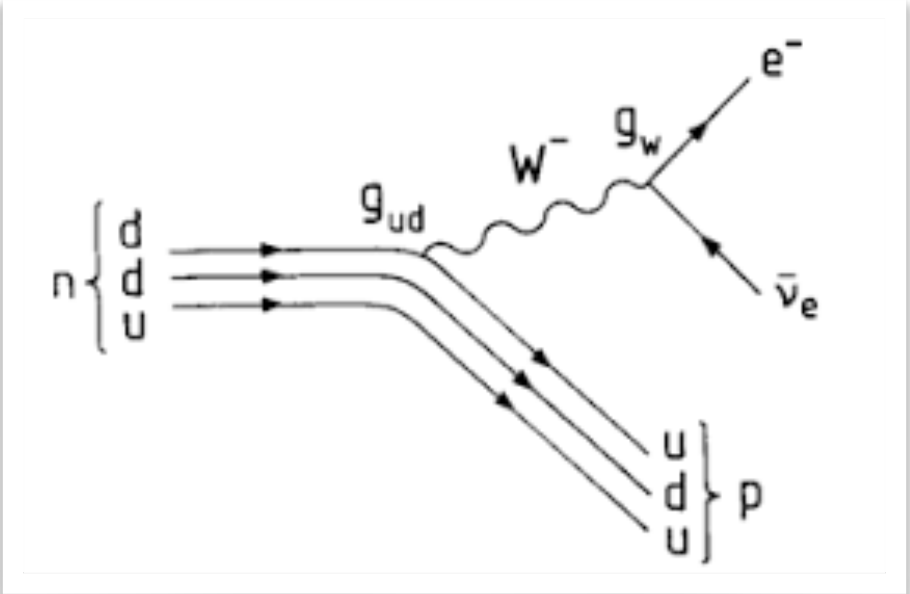




Nuclear reaction rates

How do we predict elemental abundances?



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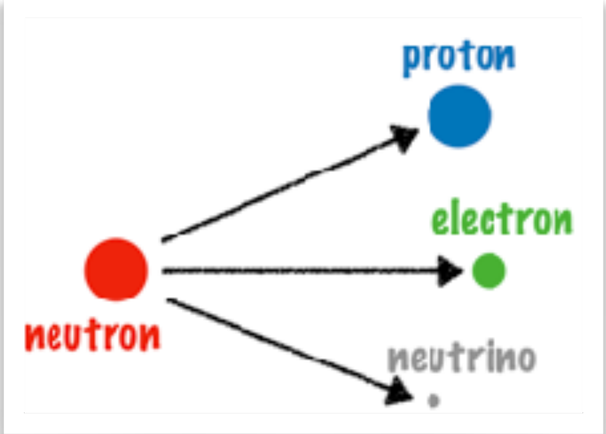
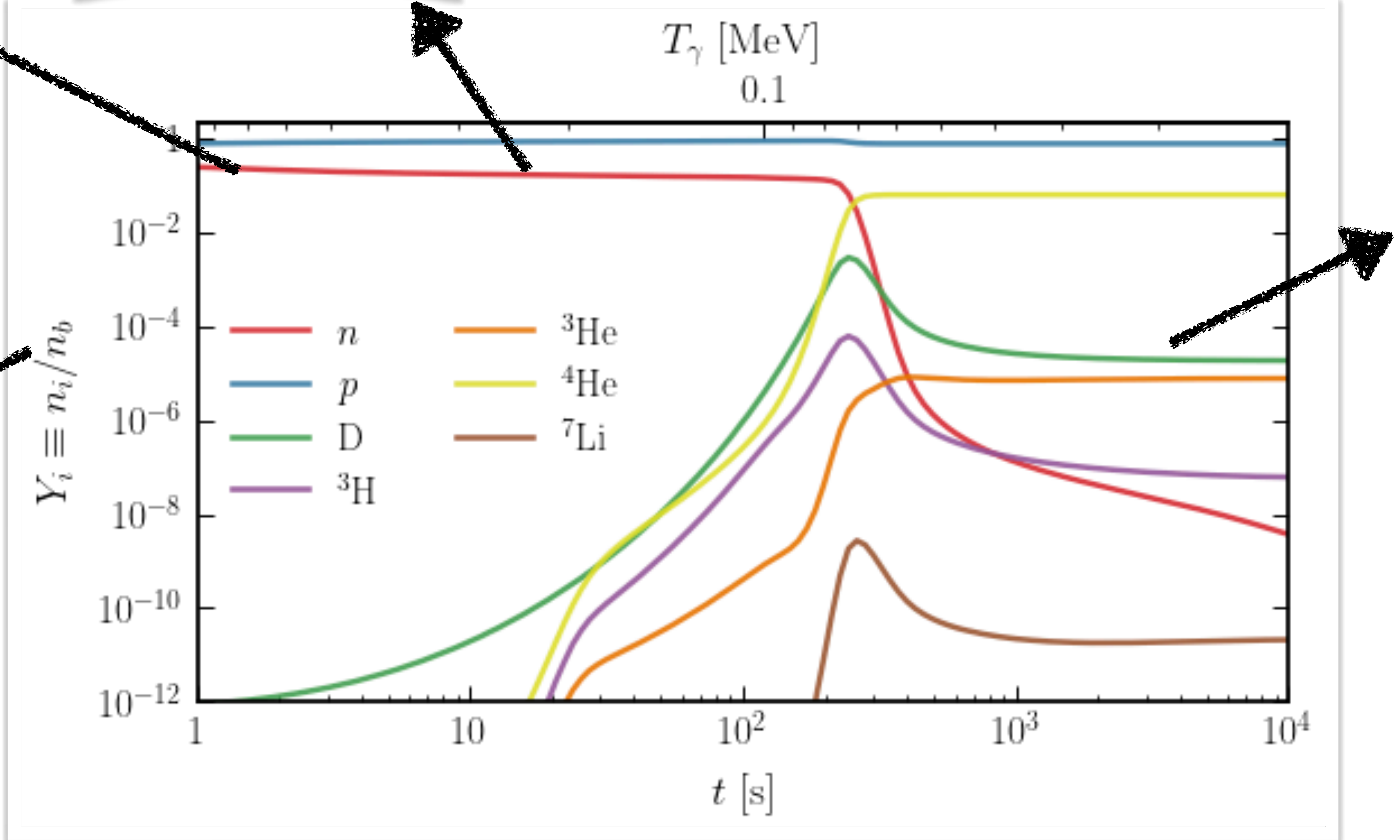
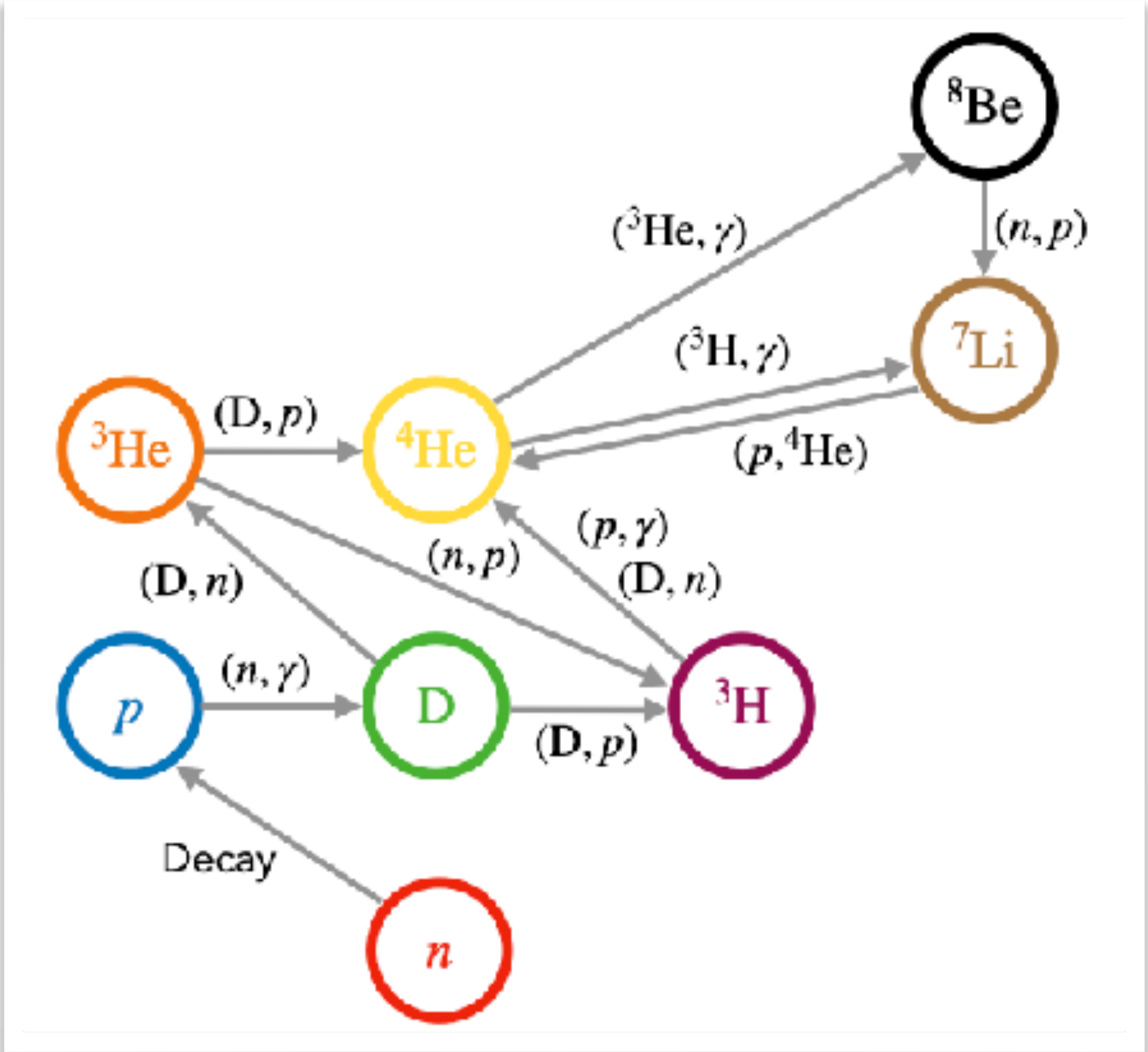


Neutron decay lifetime τ_n



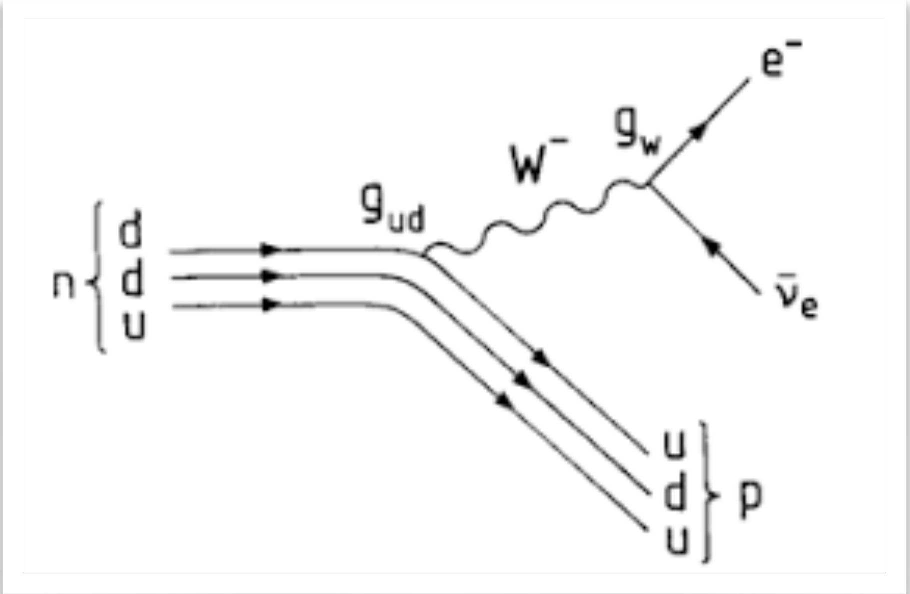


Nuclear reaction rates

Baryon abundance ω_b / photon-to-baryon ratio η

Solve differential equations of the form: $\dot{Y}_\text{D} = Y_p Y_n \Gamma_{np\to\text{D}\gamma}(T_\gamma, \eta) - Y_\text{D}\Gamma_{\text{D}\gamma\to np}(T_\gamma, \eta) + \cdots$

Test of Cosmology: Baryon-to-Photon Ratio



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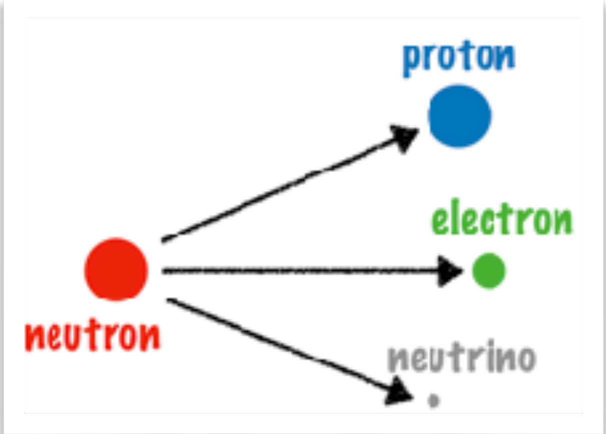
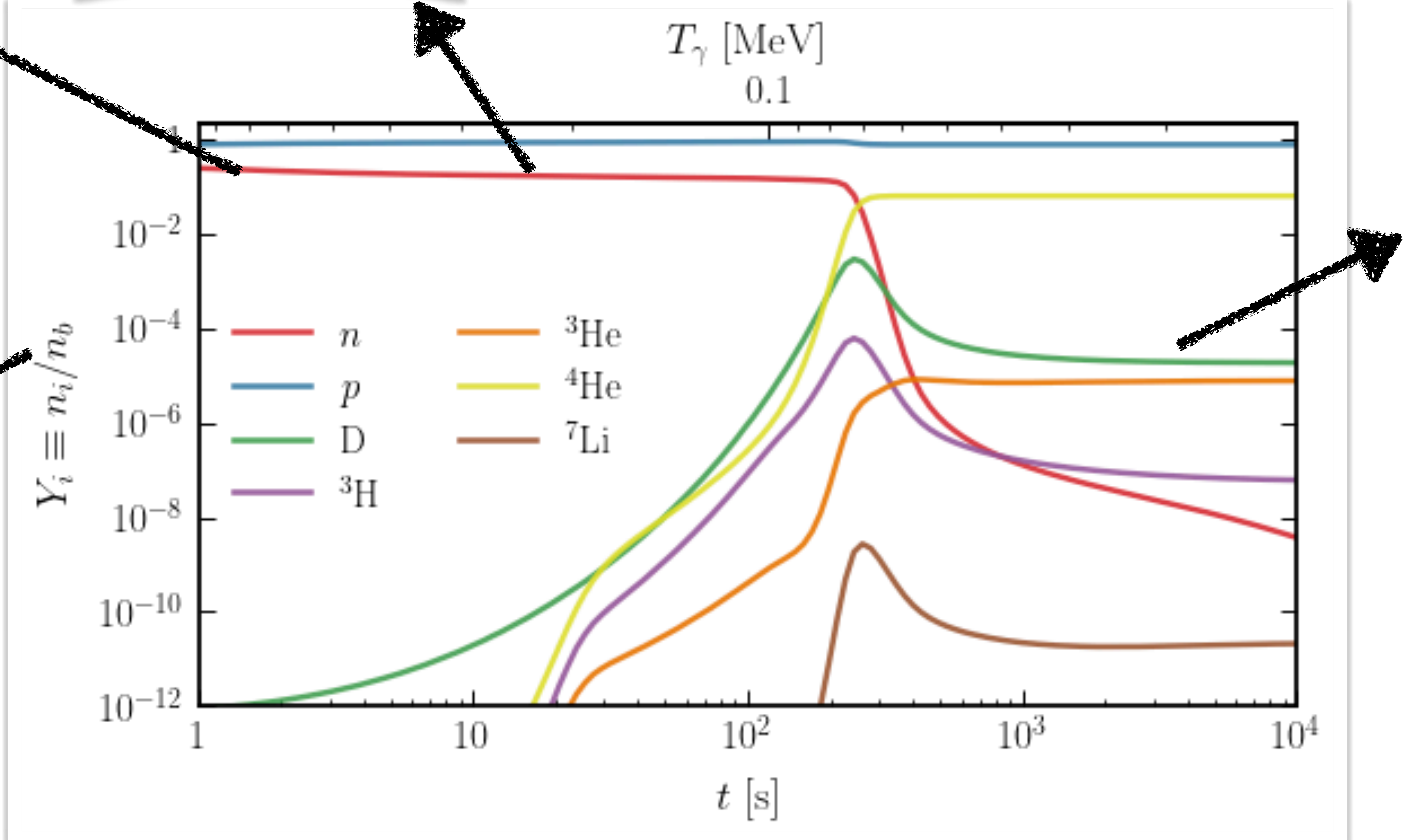
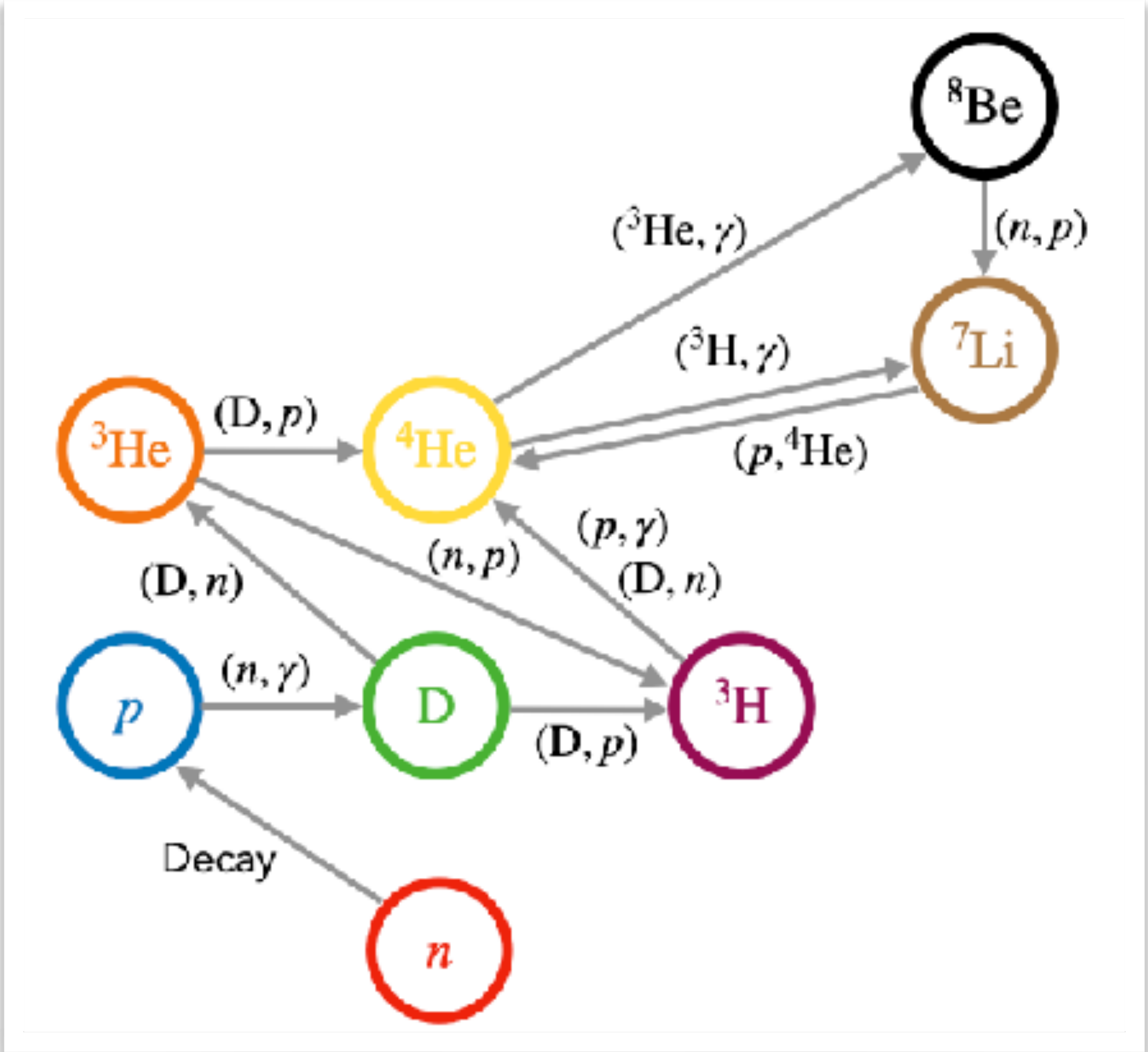


Neutron decay lifetime τ_n





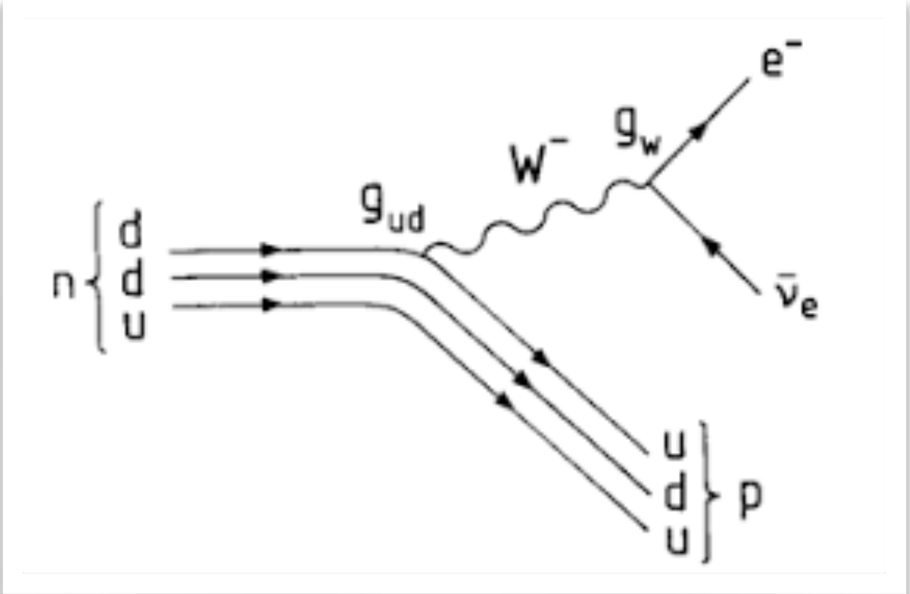
Nuclear reaction rates

Baryon abundance ω_b / photon-to-baryon ratio η

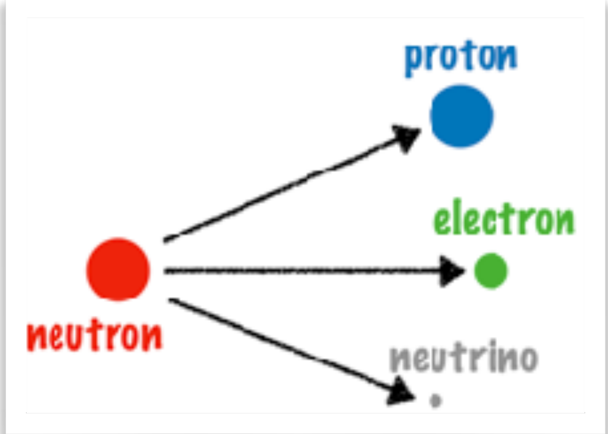
BBN is capable of determining η or ω_b at **very early times**.

Comparable uncertainty to **Planck CMB** if we assume ΛCDM cosmology.

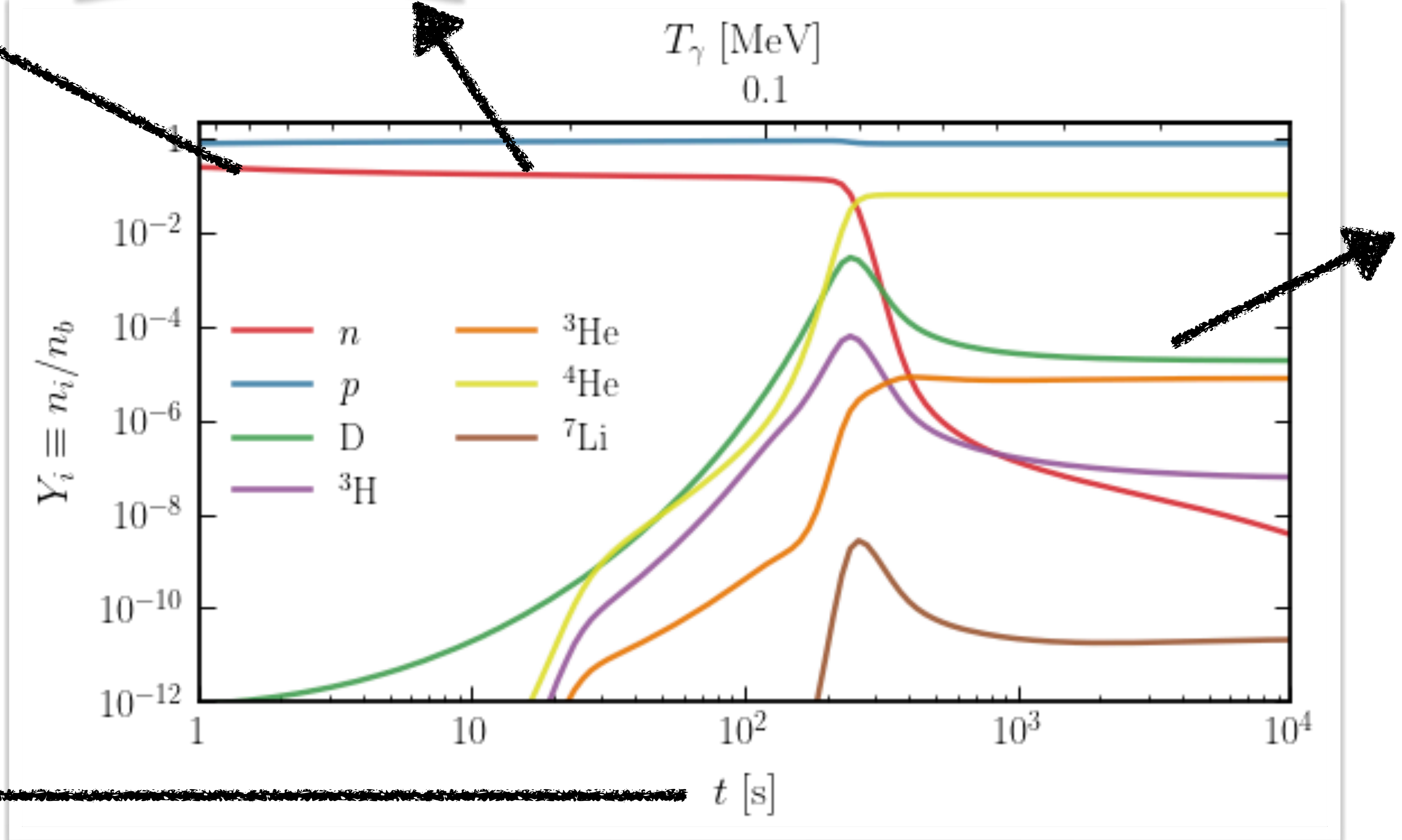
Test of Beyond-ΛCDM Cosmology: $\Delta N_{\rm eff}$

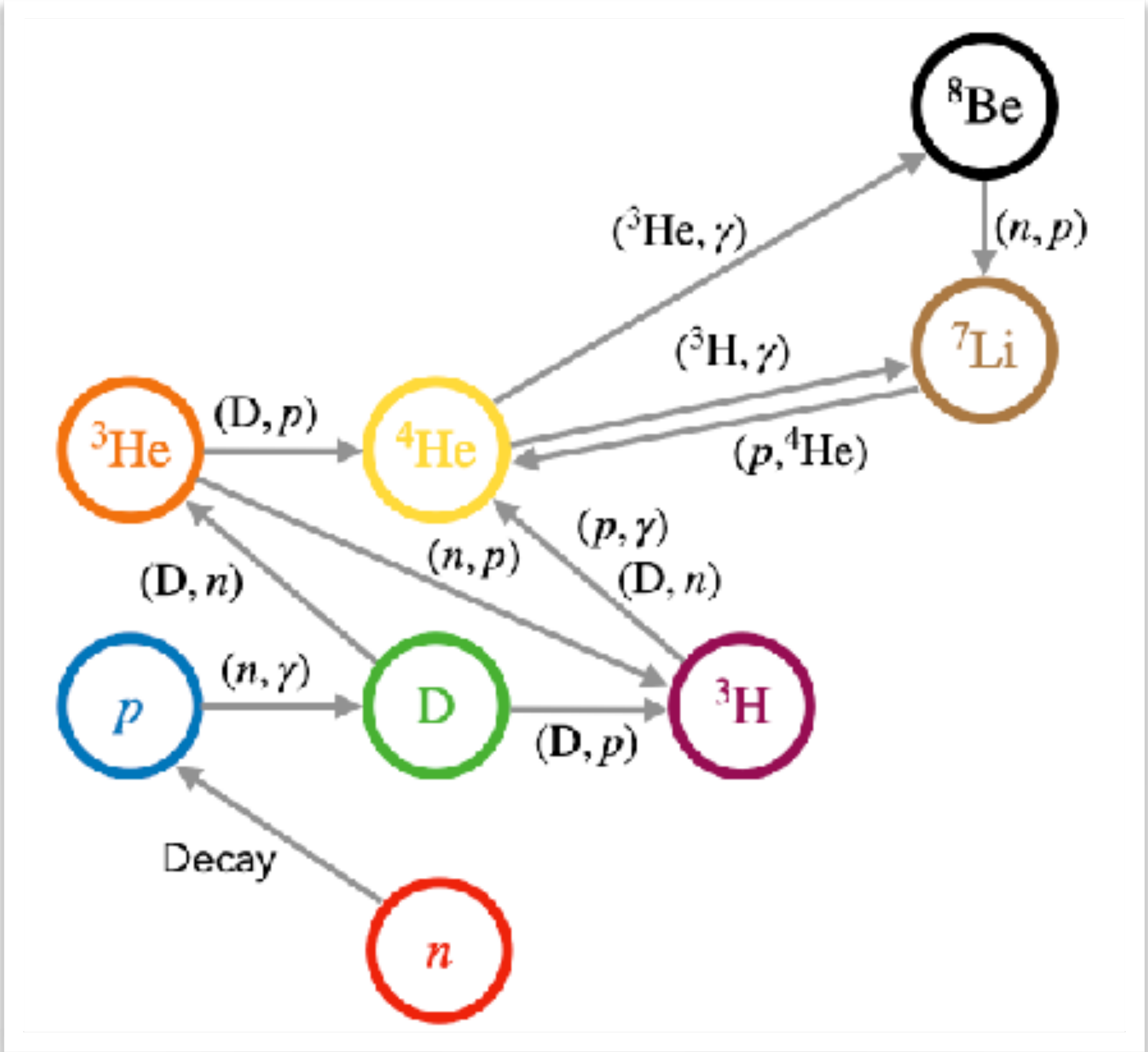


Electroweak physics governing $n \leftrightarrow p$



Neutron decay lifetime τ_n





Nuclear reaction rates

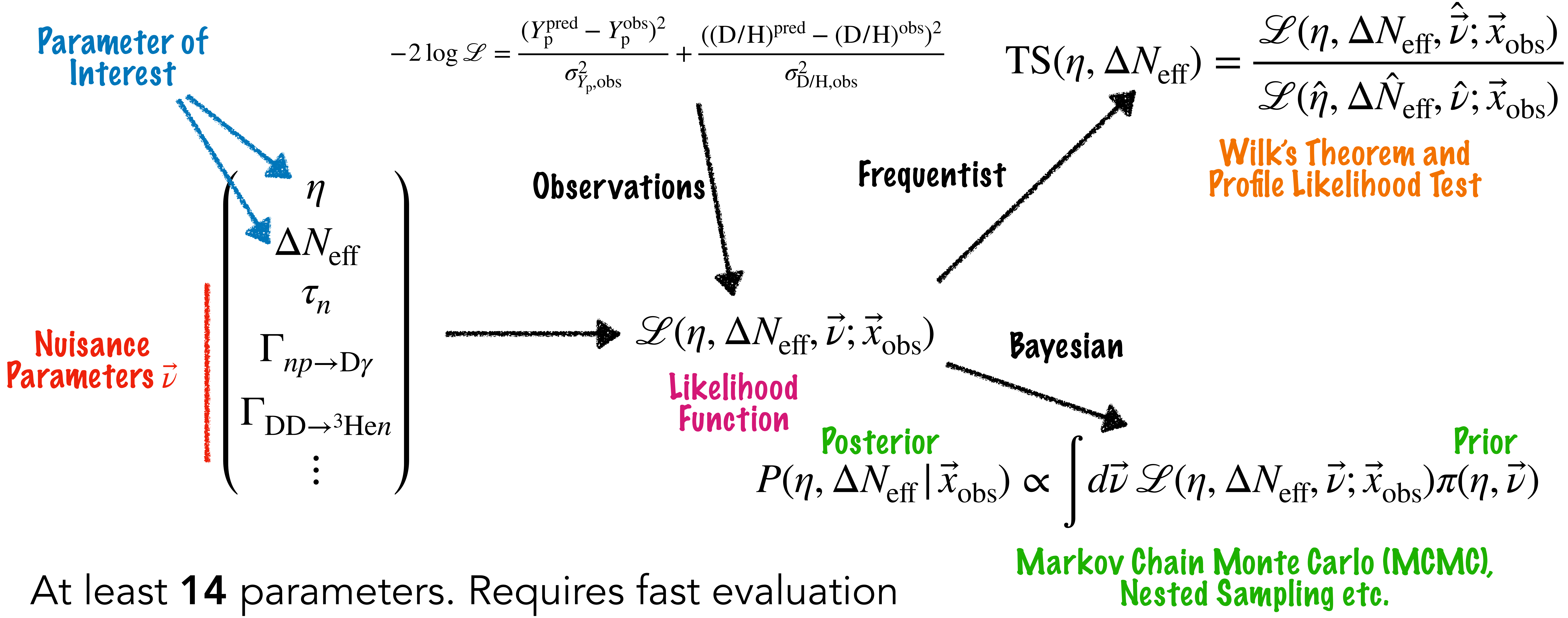
Baryon abundance ω_b / photon-to-baryon ratio η

Relativistic Degrees of Freedom $\Delta N_{\rm eff}$

Extra energy density alters the **relation between T_γ and t.**

BBN can test important scenarios in ways that CMB $\Delta N_{\rm eff}$ cannot.

Principled Parameter Estimation



Parameter of Interest

Nuisance Parameters $\vec{\nu}$

$$\begin{pmatrix} \eta \\ \Delta N_{\mathrm{eff}} \\ \tau_n \\ \Gamma_{np\to \mathrm{D}\gamma} \\ \Gamma_{\mathrm{DD}\to {}^3\mathrm{He}n} \\ \vdots \end{pmatrix}$$

$$-2\log\mathscr{L} = \frac{(Y_{\mathrm{p}}^{\mathrm{pred}} - Y_{\mathrm{p}}^{\mathrm{obs}})^2}{\sigma^2_{Y_{\mathrm{p}},\mathrm{obs}}} + \frac{((\mathrm{D/H})^{\mathrm{pred}} - (\mathrm{D/H})^{\mathrm{obs}})^2}{\sigma^2_{\mathrm{D/H},\mathrm{obs}}}$$

Observations

$$\mathscr{L}(\eta, \Delta N_{\mathrm{eff}}, \vec{\nu}; \vec{x}_{\mathrm{obs}})$$

Likelihood Function

Frequentist

$$\mathrm{TS}(\eta, \Delta N_{\mathrm{eff}}) = \frac{\mathscr{L}(\eta, \Delta N_{\mathrm{eff}}, \hat{\hat{\vec{\nu}}}; \vec{x}_{\mathrm{obs}})}{\mathscr{L}(\hat{\eta}, \Delta \hat{N}_{\mathrm{eff}}, \hat{\nu}; \vec{x}_{\mathrm{obs}})}$$

Wilk's Theorem and Profile Likelihood Test

Bayesian

Posterior

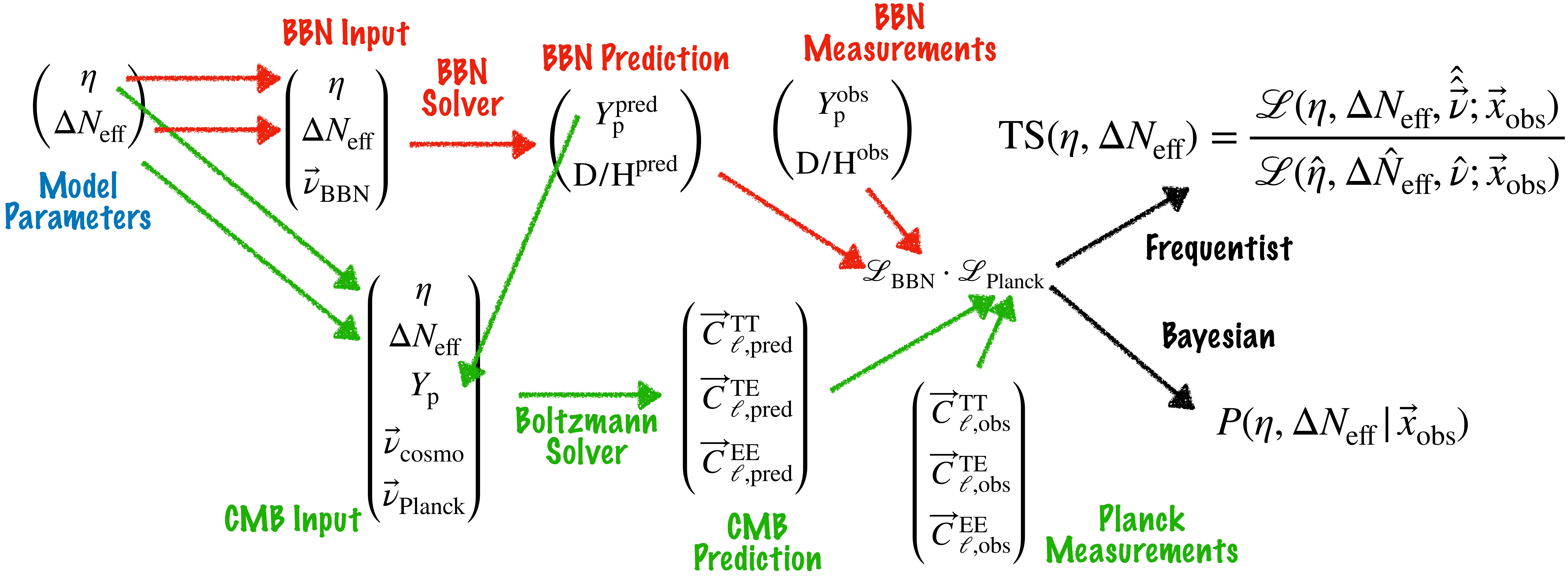
Prior

$$P(\eta, \Delta N_{\mathrm{eff}} \,|\, \vec{x}_{\mathrm{obs}}) \propto \int d\vec{\nu}\, \mathscr{L}(\eta, \Delta N_{\mathrm{eff}}, \vec{\nu}; \vec{x}_{\mathrm{obs}})\pi(\eta, \vec{\nu})$$

Markov Chain Monte Carlo (MCMC), Nested Sampling etc.

At least **14** parameters. Requires fast evaluation to explore 14-D space: **~1 second per run**.

BBN+CMB Combination



Model Parameters

$\begin{pmatrix} \eta \\ \Delta N_{\rm eff} \end{pmatrix}$

BBN Input

$\begin{pmatrix} \eta \\ \Delta N_{\rm eff} \\ \vec{\nu}_{\rm BBN} \end{pmatrix}$

BBN Solver

BBN Prediction

$\begin{pmatrix} Y_{\rm p}^{\rm pred} \\ {\rm D/H}^{\rm pred} \end{pmatrix}$

BBN Measurements

$\begin{pmatrix} Y_{\rm p}^{\rm obs} \\ {\rm D/H}^{\rm obs} \end{pmatrix}$

CMB Input

$\begin{pmatrix} \eta \\ \Delta N_{\rm eff} \\ Y_{\rm p} \\ \vec{\nu}_{\rm cosmo} \\ \vec{\nu}_{\rm Planck} \end{pmatrix}$

Boltzmann Solver

CMB Prediction

$\begin{pmatrix} \vec{C}_{\ell,\rm pred}^{\rm TT} \\ \vec{C}_{\ell,\rm pred}^{\rm TE} \\ \vec{C}_{\ell,\rm pred}^{\rm EE} \end{pmatrix}$

Planck Measurements

$\begin{pmatrix} \vec{C}_{\ell,\rm obs}^{\rm TT} \\ \vec{C}_{\ell,\rm obs}^{\rm TE} \\ \vec{C}_{\ell,\rm obs}^{\rm EE} \end{pmatrix}$

$\mathscr{L}_{\rm BBN} \cdot \mathscr{L}_{\rm Planck}$

Frequentist

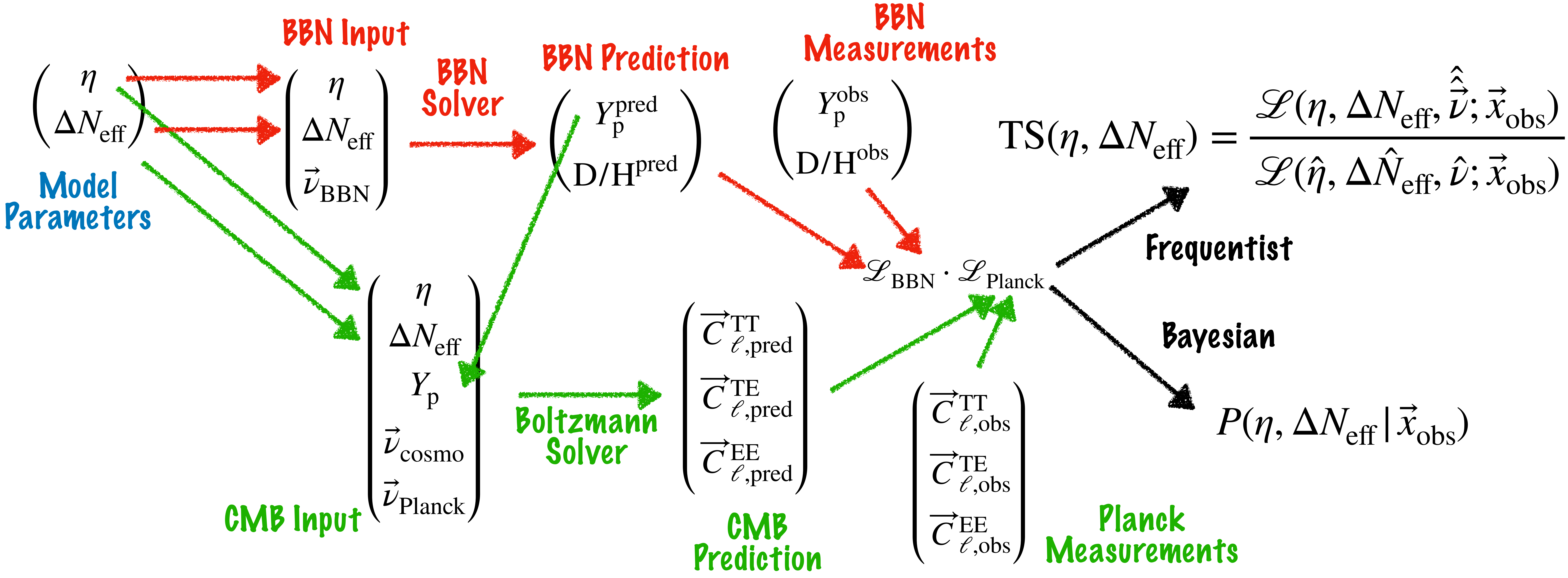
$$\mathrm{TS}(\eta, \Delta N_{\rm eff}) = \frac{\mathscr{L}(\eta, \Delta N_{\rm eff}, \hat{\hat{\vec{\nu}}}; \vec{x}_{\rm obs})}{\mathscr{L}(\hat{\eta}, \Delta \hat{N}_{\rm eff}, \hat{\nu}; \vec{x}_{\rm obs})}$$

Bayesian

$$P(\eta, \Delta N_{\rm eff} \,|\, \vec{x}_{\rm obs})$$

Comparable power for η and $\Delta N_{\rm eff}$ (for inert, relativistic species).
Combination is important!

BBN+CMB Combination



Model Parameters

$$\begin{pmatrix} \eta \\ \Delta N_\text{eff} \end{pmatrix}$$

BBN Input

$$\begin{pmatrix} \eta \\ \Delta N_\text{eff} \\ \vec{\nu}_\text{BBN} \end{pmatrix}$$

BBN Solver

BBN Prediction

$$\begin{pmatrix} Y_\text{p}^\text{pred} \\ \text{D/H}^\text{pred} \end{pmatrix}$$

BBN Measurements

$$\begin{pmatrix} Y_\text{p}^\text{obs} \\ \text{D/H}^\text{obs} \end{pmatrix}$$

CMB Input

$$\begin{pmatrix} \eta \\ \Delta N_\text{eff} \\ Y_\text{p} \\ \vec{\nu}_\text{cosmo} \\ \vec{\nu}_\text{Planck} \end{pmatrix}$$

Boltzmann Solver

CMB Prediction

$$\begin{pmatrix} \vec{C}_{\ell,\text{pred}}^\text{TT} \\ \vec{C}_{\ell,\text{pred}}^\text{TE} \\ \vec{C}_{\ell,\text{pred}}^\text{EE} \end{pmatrix}$$

Planck Measurements

$$\begin{pmatrix} \vec{C}_{\ell,\text{obs}}^\text{TT} \\ \vec{C}_{\ell,\text{obs}}^\text{TE} \\ \vec{C}_{\ell,\text{obs}}^\text{EE} \end{pmatrix}$$

$$\mathscr{L}_\text{BBN} \cdot \mathscr{L}_\text{Planck}$$

Frequentist

$$\text{TS}(\eta, \Delta N_\text{eff}) = \frac{\mathscr{L}(\eta, \Delta N_\text{eff}, \hat{\hat{\vec{\nu}}}; \vec{x}_\text{obs})}{\mathscr{L}(\hat{\eta}, \Delta \hat{N}_\text{eff}, \hat{\nu}; \vec{x}_\text{obs})}$$

Bayesian

$$P(\eta, \Delta N_\text{eff} \,|\, \vec{x}_\text{obs})$$

BBN code should ideally be **much faster than CLASS/CAMB+Planck Likelihood**.

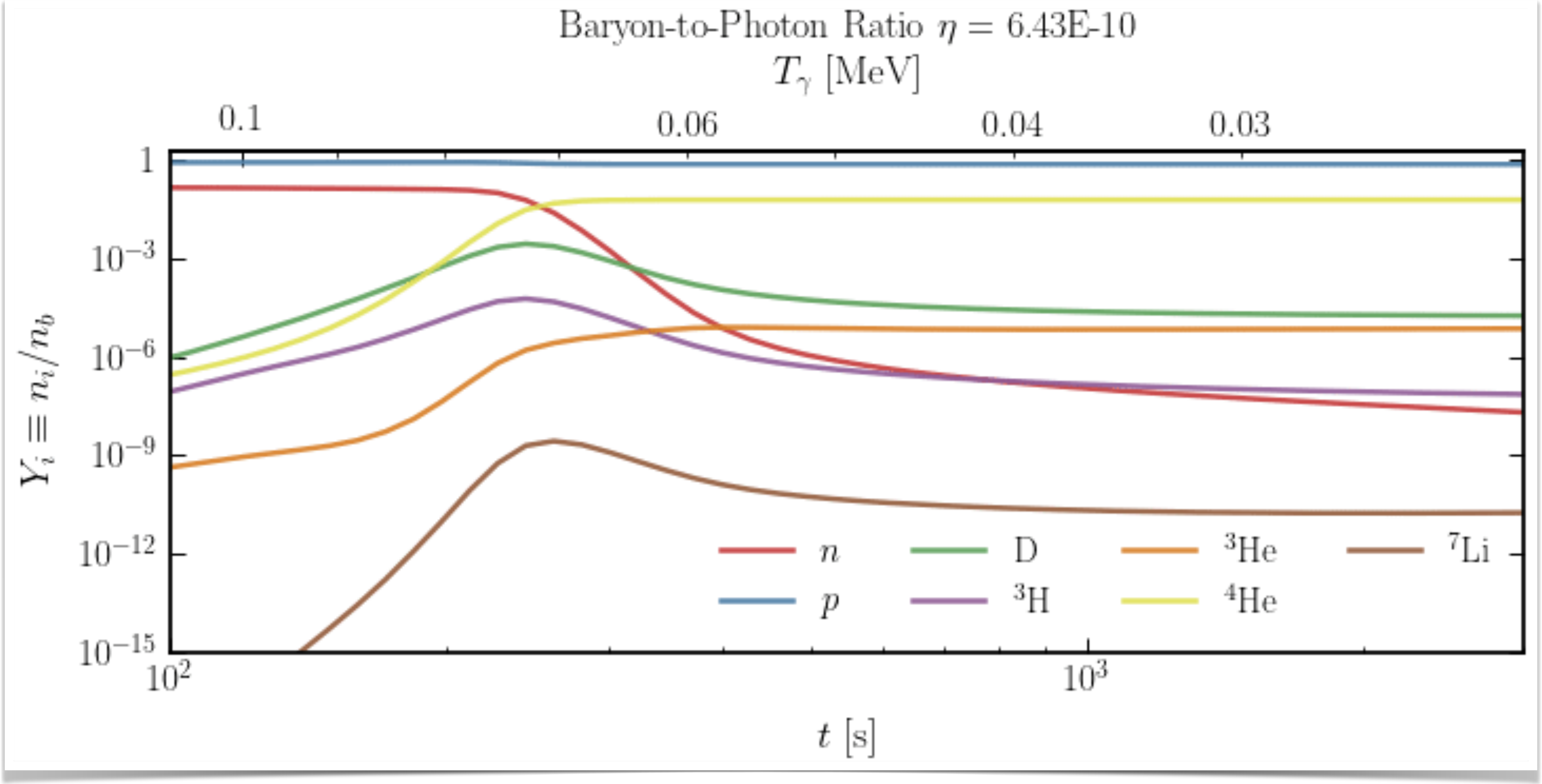
Existing Codes Can't Do This

Name	Language	Time Per Solve	Comments
AlterBBN	C	< 1s	Incomplete implementation of weak rates, old nuclear rates. No BSM neutrino decoupling.
PRIMAT	Mathematica	O(10 min)	Extremely accurate, but very slow. No BSM neutrino decoupling.
PArthENoPE	Fortran	< 1s	Fast, but requires significant modification for parameter estimation. No BSM neutrino decoupling.
PRyMordial	Python	O(10 s)	Accurate. Full parameter estimation possible, but slow. Written with new physics in mind.

All current BBN codes have to **make compromises** when it comes to parameter estimation, **very significant ones** in combination with CMB.

LINX: Light Isotope Nucleosynthesis with JAX

50 evaluations:
12 seconds
after compilation
(about 50 seconds)



Python-based code that is **compiled**, **easily vectorized** and **differentiable**.
Makes for very fast code, but lots of "sharp bits" to avoid.

AlterBBN: Arbey+ 1806.11095, PRIMAT: Pitrou+ 1801.08023, PArthENoPE: Gariazzo+ 2103.05027, PRyMordial: Burns+ 2307.07061

Existing, Open Source Codes

Name	Language	Time Per Solve	Comments
AlterBBN	C	< 1s	Incomplete implementation of weak rates, old nuclear rates. No BSM neutrino decoupling.
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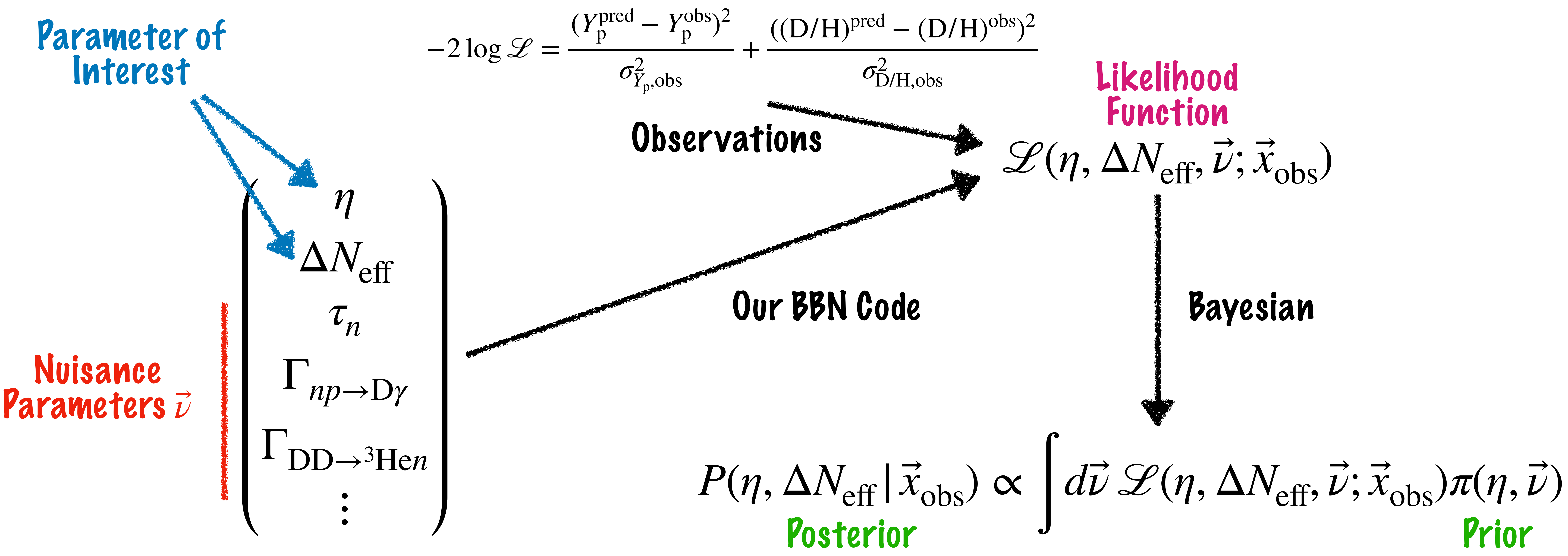
PArthENoPE	Fortran	< 1s	Fast, but requires significant modification for parameter estimation. No BSM neutrino decoupling.
PRyMordial	Python	O(10 s)	Accurate. Full parameter estimation possible, but slow. Written with new physics in mind.
LINX	**Python+JAX**	**O(0.1s)**	**Inspired by PRyMordial: equally accurate. Fast enough for MCMC methods.**

BBN-Only Procedure



Parameter of Interest

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$$\begin{pmatrix} \eta \\ \Delta N_{\rm eff} \\ \tau_n \\ \Gamma_{np\to {\rm D}\gamma} \\ \Gamma_{{\rm DD}\to {}^3{\rm He}n} \\ \vdots \end{pmatrix}$$

$$-2\log\mathscr{L} = \frac{(Y_{\rm p}^{\rm pred} - Y_{\rm p}^{\rm obs})^2}{\sigma^2_{Y_{\rm p},{\rm obs}}} + \frac{(({\rm D/H})^{\rm pred} - ({\rm D/H})^{\rm obs})^2}{\sigma^2_{\rm D/H,obs}}$$

Observations

Our BBN Code

Likelihood Function

$$\mathscr{L}(\eta, \Delta N_{\rm eff}, \vec{\nu}; \vec{x}_{\rm obs})$$

Bayesian

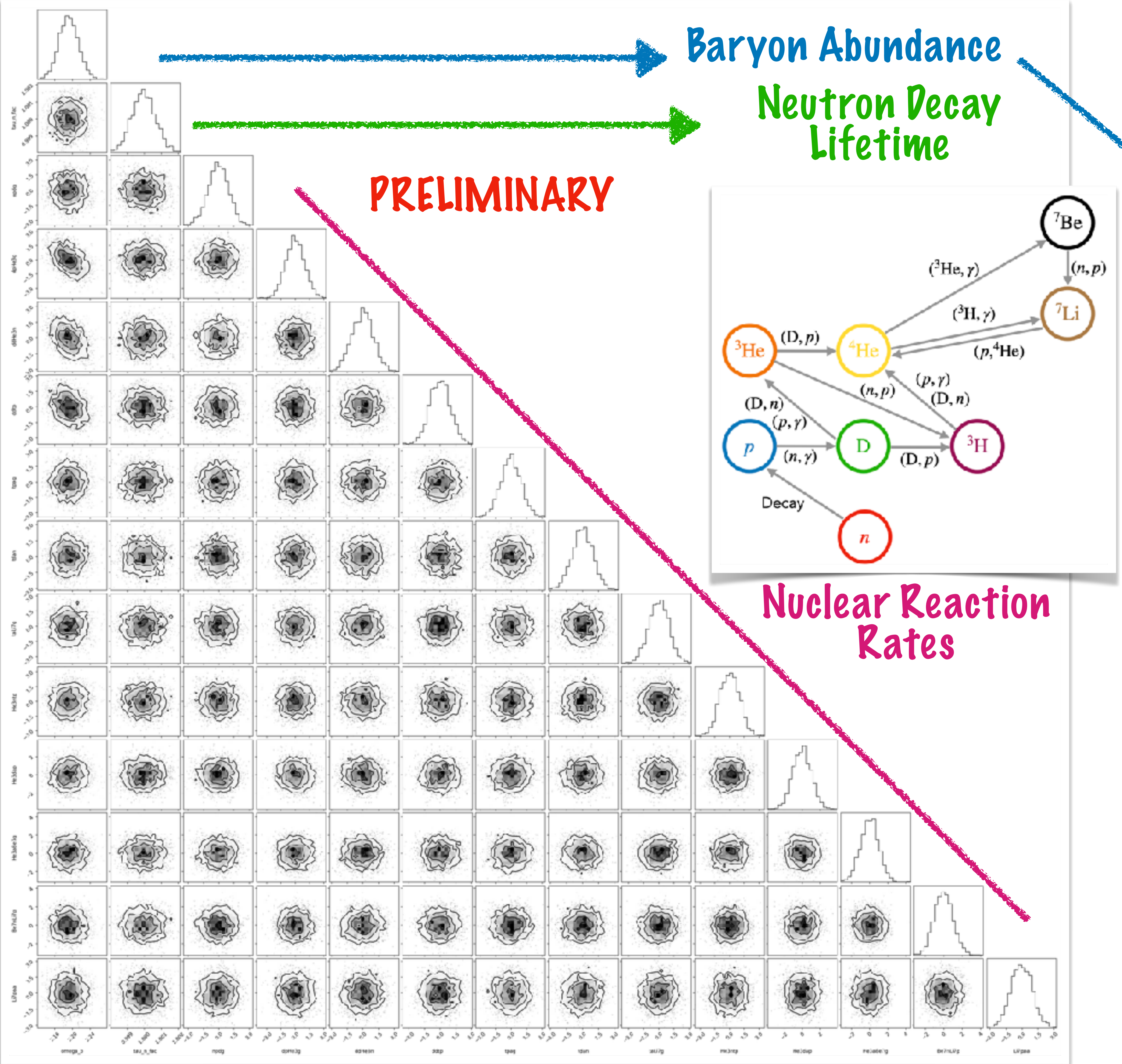
$$P(\eta, \Delta N_{\rm eff} \,|\, \vec{x}_{\rm obs}) \propto \int d\vec{\nu}\, \mathscr{L}(\eta, \Delta N_{\rm eff}, \vec{\nu}; \vec{x}_{\rm obs})\pi(\eta, \vec{\nu})$$

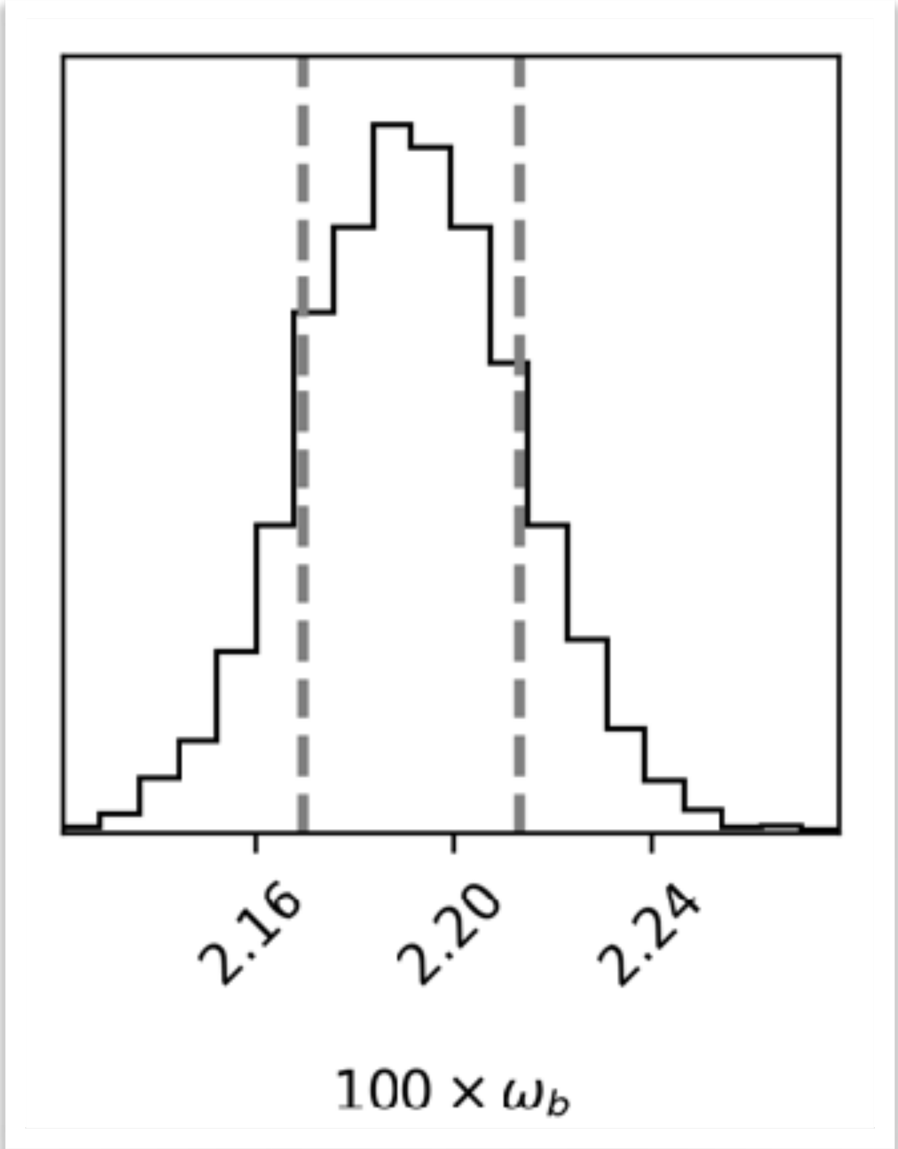
Posterior

Prior

Posterior estimation using **nested sampling**. Rapid evaluation using our code.

BBN-Only, Baryon Abundance





PRELIMINARY
(PRIMAT Rates)

$$100\Omega_b h^2 = 2.192 \pm 0.022$$

Takes **3 CPU-hours**. Estimated to be **100x faster** than PRyMordial (**10^4x** faster than PRIMAT).

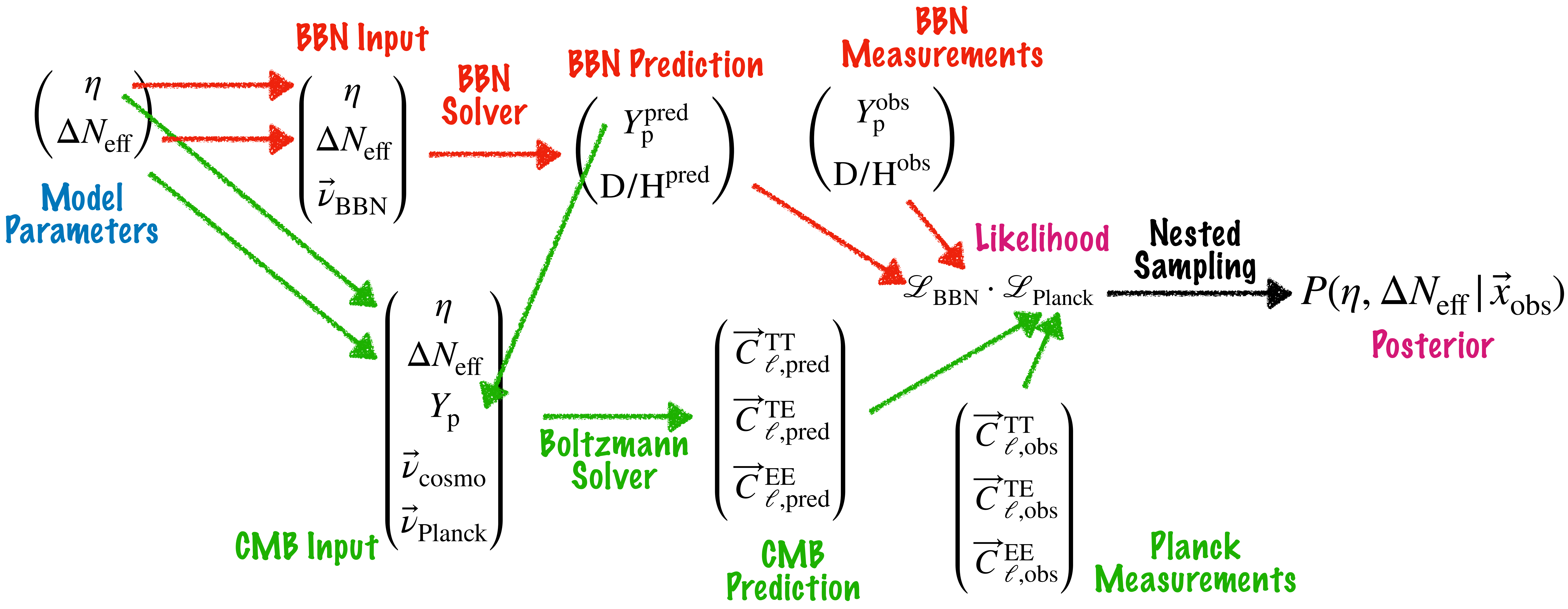
Schöneberg 2401.15054, Yeh+ 2207.13133, Planck Collab. 1807.06209

BBN Only, Baryon Abundance

Analysis	$100\Omega_b h^2$
LINX, PRIMAT Rates	2.192 ± 0.022 (Preliminary)
PRyMordial, PRIMAT Rates (Schöneberg)	2.195 ± 0.021
LINX, NACREII Rates	2.234 ± 0.058 (Preliminary)
PRyMordial, NACREII Rates (Schöneberg)	2.231 ± 0.055
Yeh+	2.209 ± 0.043
CMB, LCDM + Y_p	2.236 ± 0.020

Good agreement with previous analyses with full marginalization over rates.
Some scatter depending on choice of rates/codes/observations.

BBN+CMB Procedure



$\begin{pmatrix} \eta \\ \Delta N_{\rm eff} \end{pmatrix}$

Model Parameters

BBN Input

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BBN Prediction

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Boltzmann Solver

CMB Prediction

$\begin{pmatrix} \vec{C}_{\ell,{\rm pred}}^{\rm TT} \\ \vec{C}_{\ell,{\rm pred}}^{\rm TE} \\ \vec{C}_{\ell,{\rm pred}}^{\rm EE} \end{pmatrix}$

Planck Measurements

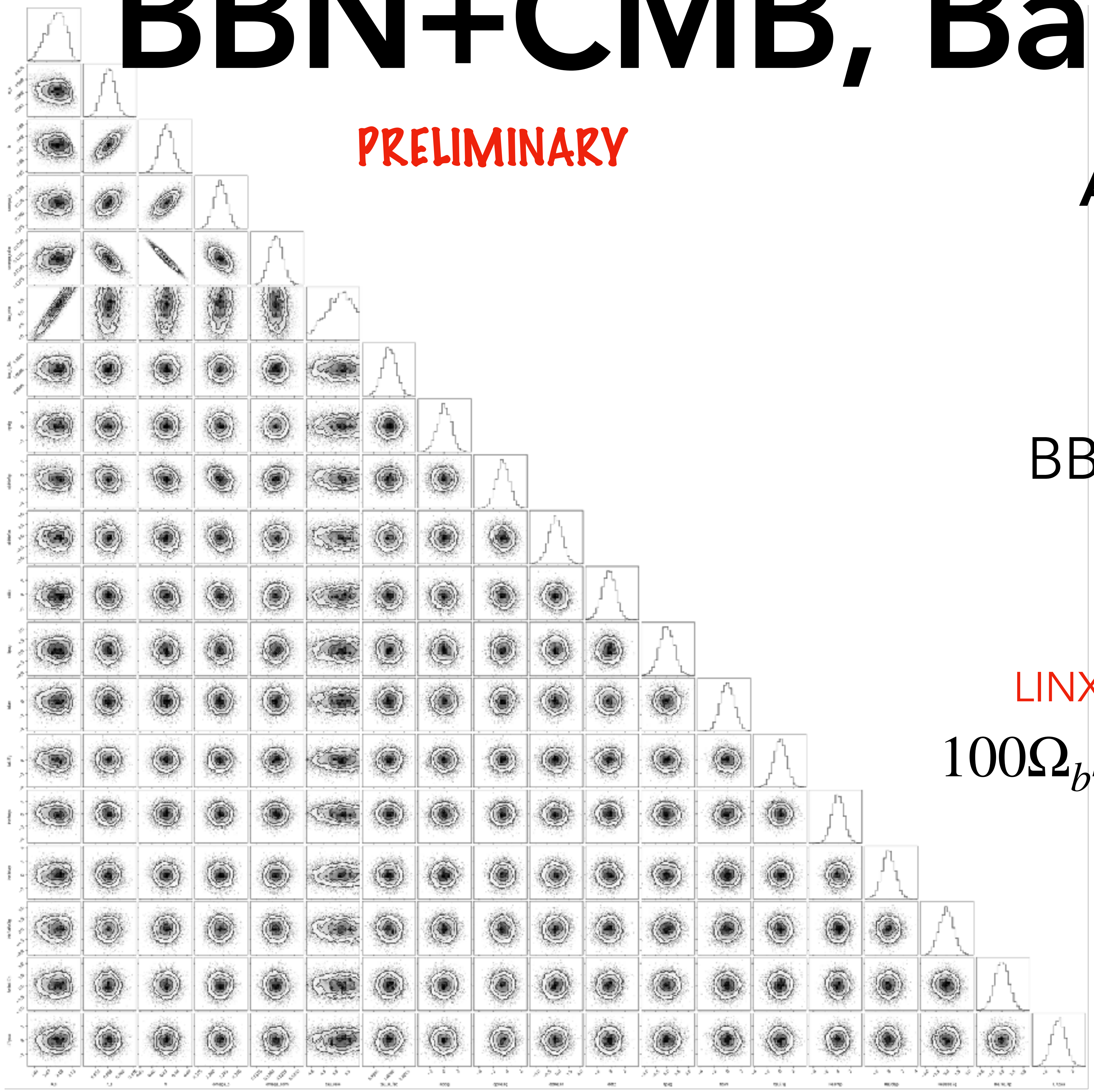
$\begin{pmatrix} \vec{C}_{\ell,{\rm obs}}^{\rm TT} \\ \vec{C}_{\ell,{\rm obs}}^{\rm TE} \\ \vec{C}_{\ell,{\rm obs}}^{\rm EE} \end{pmatrix}$

Likelihood

$\mathscr{L}_{\rm BBN} \cdot \mathscr{L}_{\rm Planck} \xrightarrow{\text{Nested Sampling}} P(\eta, \Delta N_{\rm eff} \,|\, \vec{x}_{\rm obs})$

Posterior

BBN+CMB, Baryon Abundance



PRELIMINARY

At least 19 parameters to vary. Many more if we include Planck nuisance parameters.

BBN analysis approaching **the same rigor as CMB analyses**: important for 1% precision that we already have.

LINX BBN Only (PRIMAT)

$$100\Omega_b h^2 = 2.192 \pm 0.022$$

Planck CMB Only

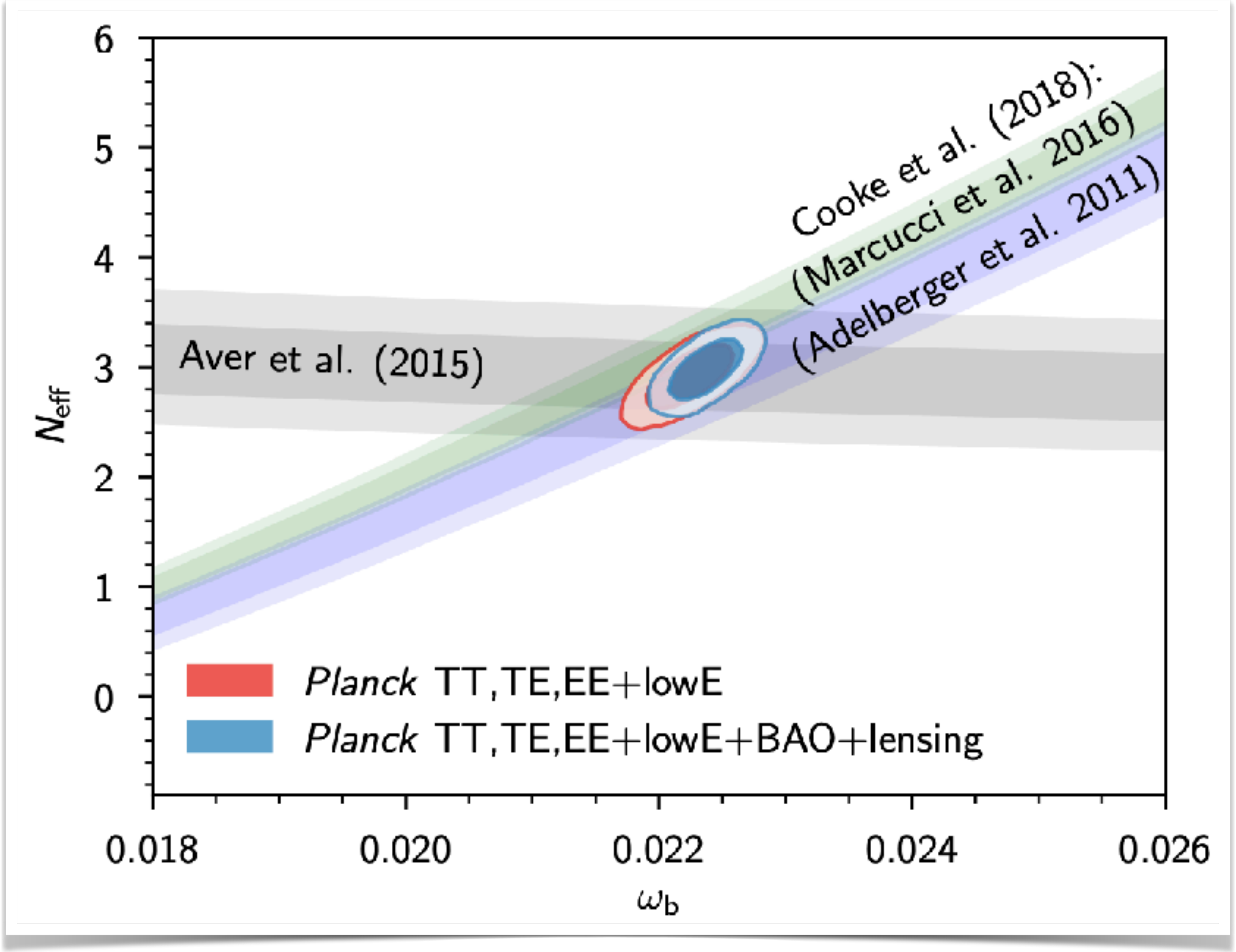
$$100\Omega_b h^2 = 2.236 \pm 0.015$$

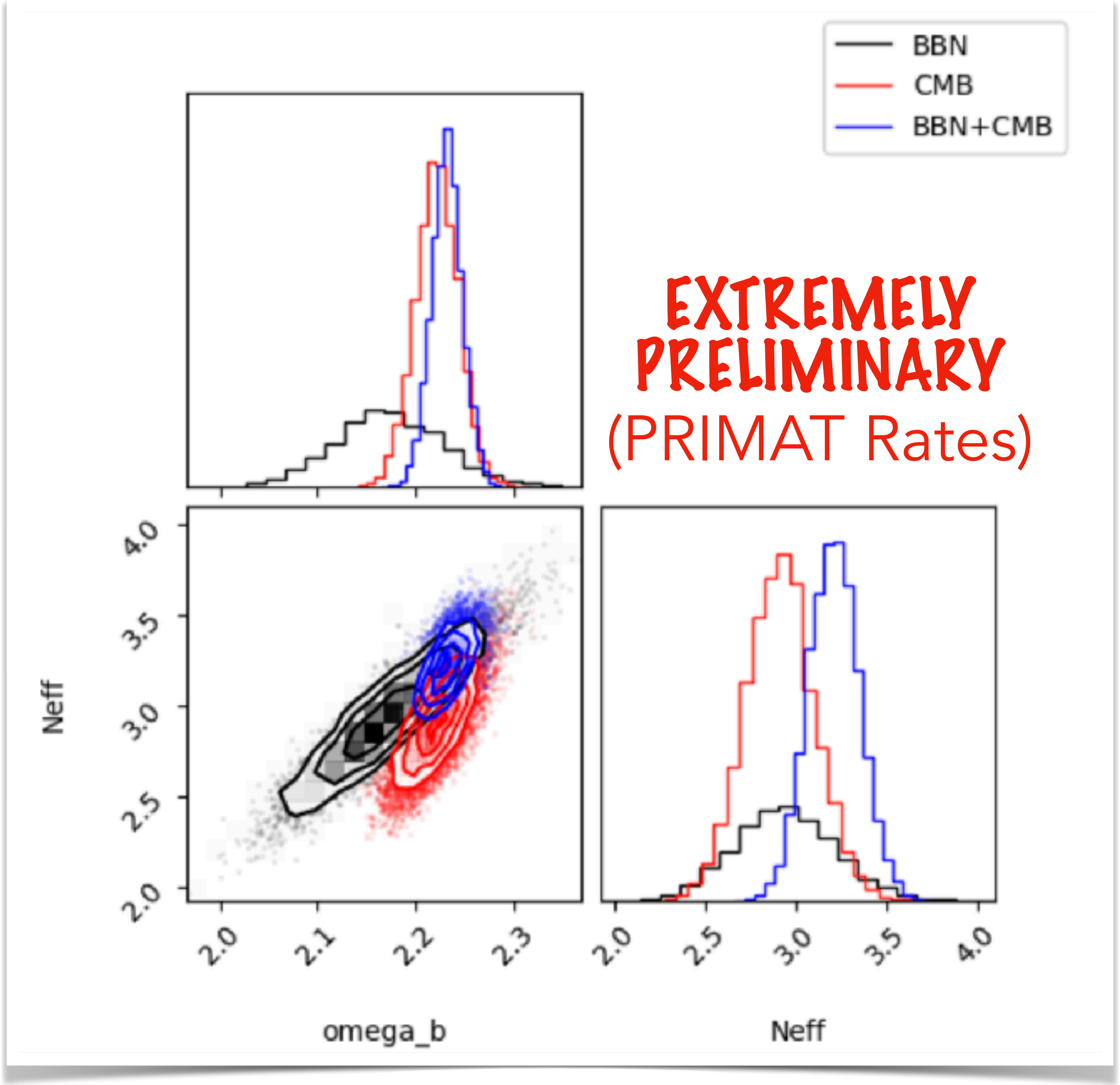
LINX BBN (PRIMAT)+Planck CMB

$$100\Omega_b h^2 = 2.219 \pm 0.012$$

Note that using PRIMAT rates, there is a tension between CMB and BBN.

BBN+CMB, η and $\Delta N_{\rm eff}$



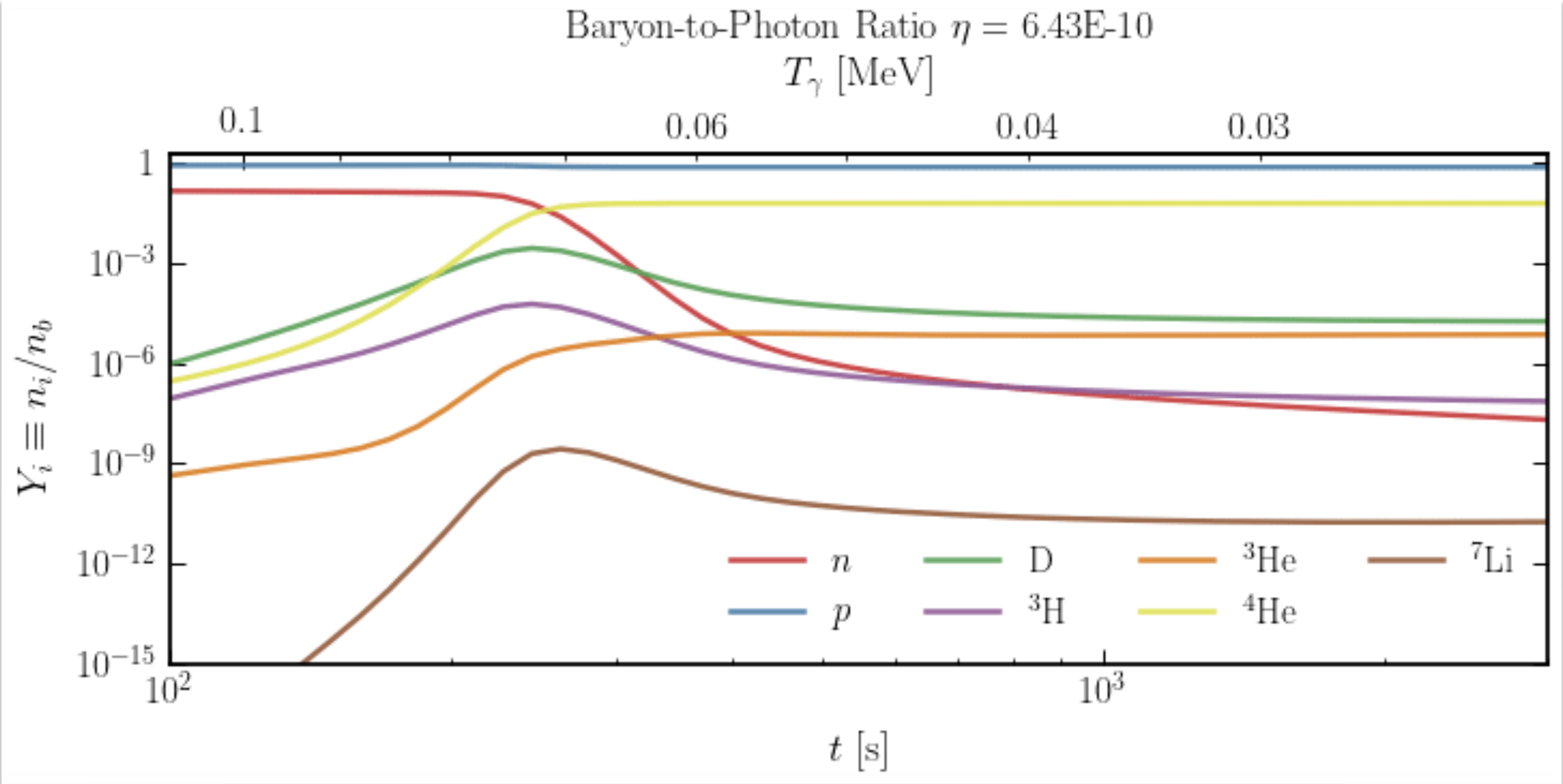


EXTREMELY PRELIMINARY
(PRIMAT Rates)

Constraining power about equal currently: important to combine properly!
We can do so, **including over all Planck nuisance parameters**.

JAX: High-Performance Array Computing

50 evaluations:
12 seconds
after compilation
(about 50 seconds)

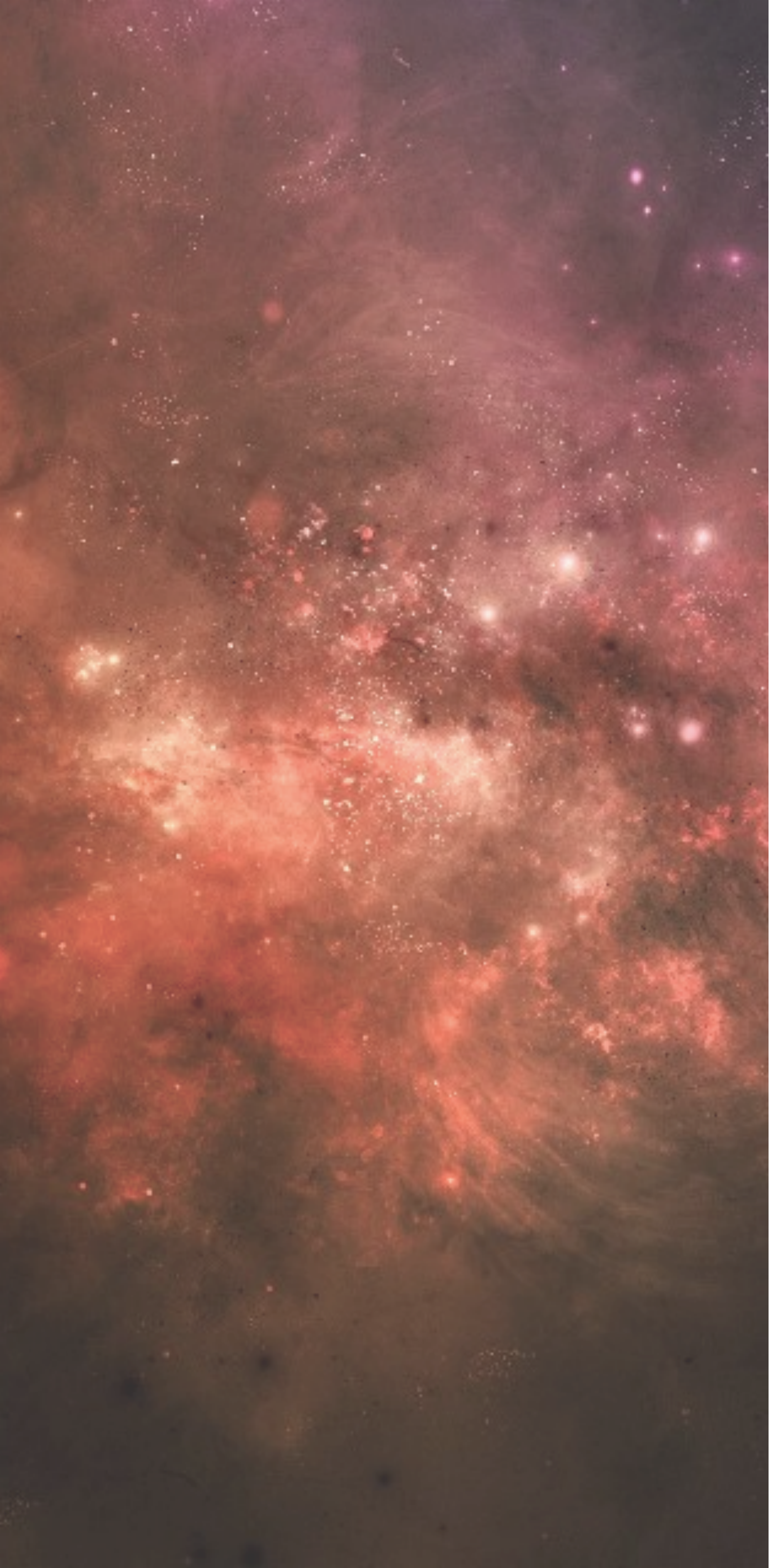


Python-based code that is **compiled**, **easily vectorized** and **differentiable**.
Makes for very fast code, but lots of "sharp bits" to avoid.

Differentiable Code

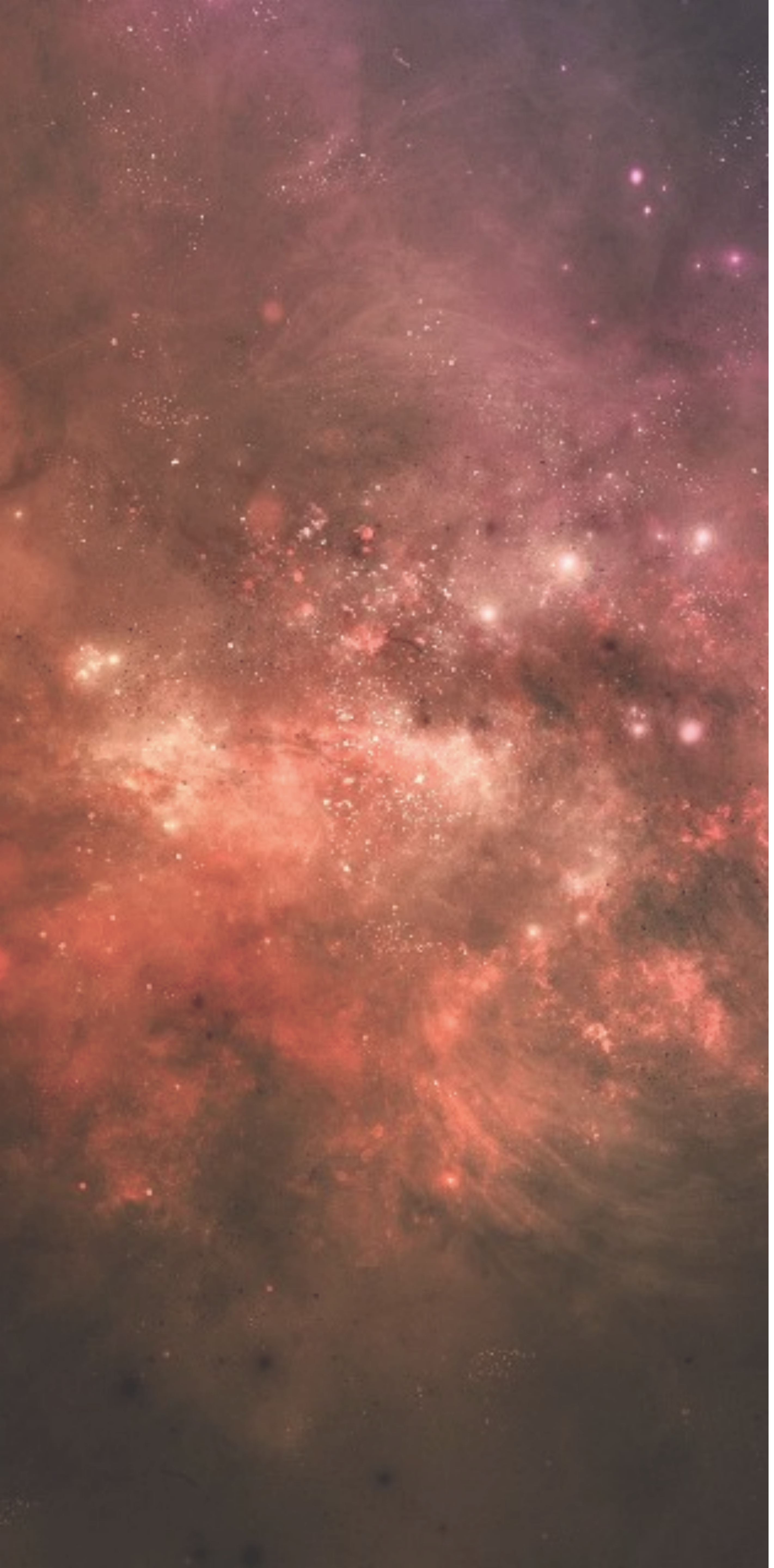
$$[\eta, \Delta N_{\mathrm{eff}}, \vec{\nu}] \longrightarrow \text{JAX} \longrightarrow \mathscr{L}(\eta, \Delta N_{\mathrm{eff}}, \vec{\nu}; \vec{x}_{\mathrm{obs}}), \frac{\partial \mathscr{L}}{\partial \eta}, \frac{\partial \mathscr{L}}{\partial \Delta N_{\mathrm{eff}}}, \frac{\partial \mathscr{L}}{\partial \vec{\nu}}$$

JAX can automatically compute the derivative of any function, enabling the use of **significantly faster MCMC techniques**. This is the future of parameter estimation!

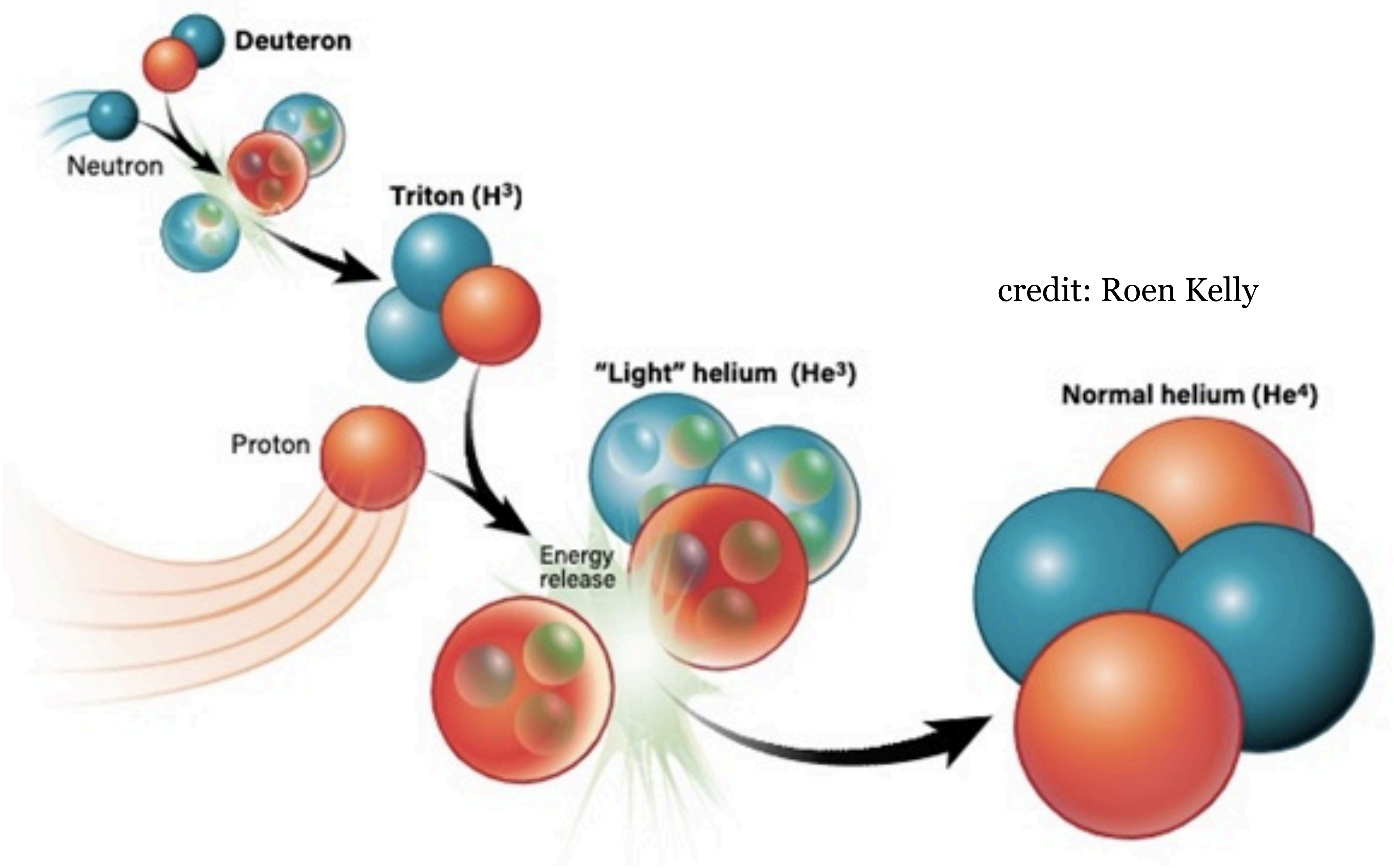


LINX enables **fast parameter estimation** for BBN, at the **same level of rigor as CMB**.

Looking forward: new physics, **model-independent constraints.**

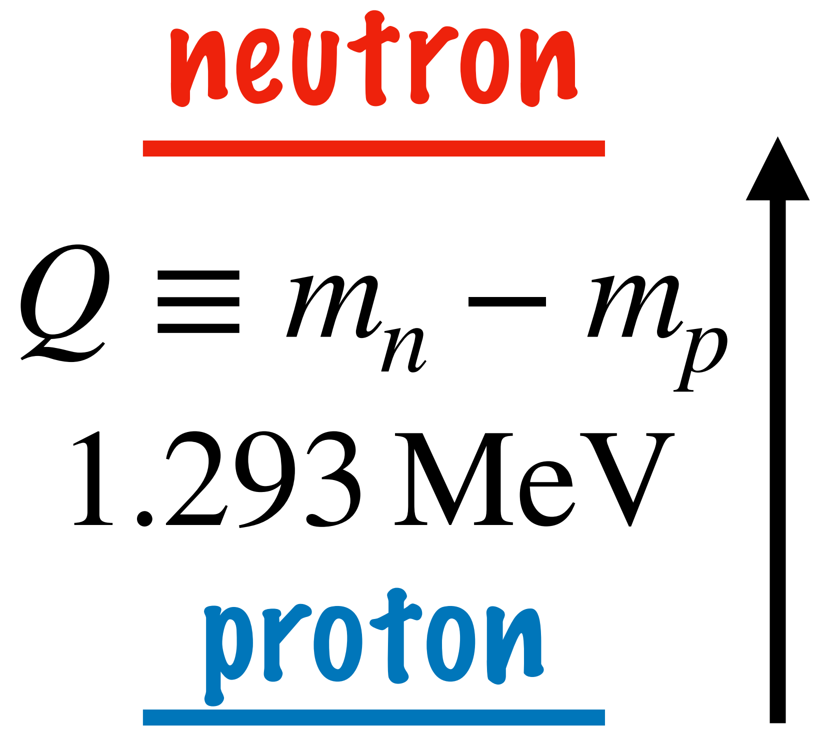


Backup Slides



credit: Roen Kelly

$T \gg Q$: Free Protons and Neutrons



neutron

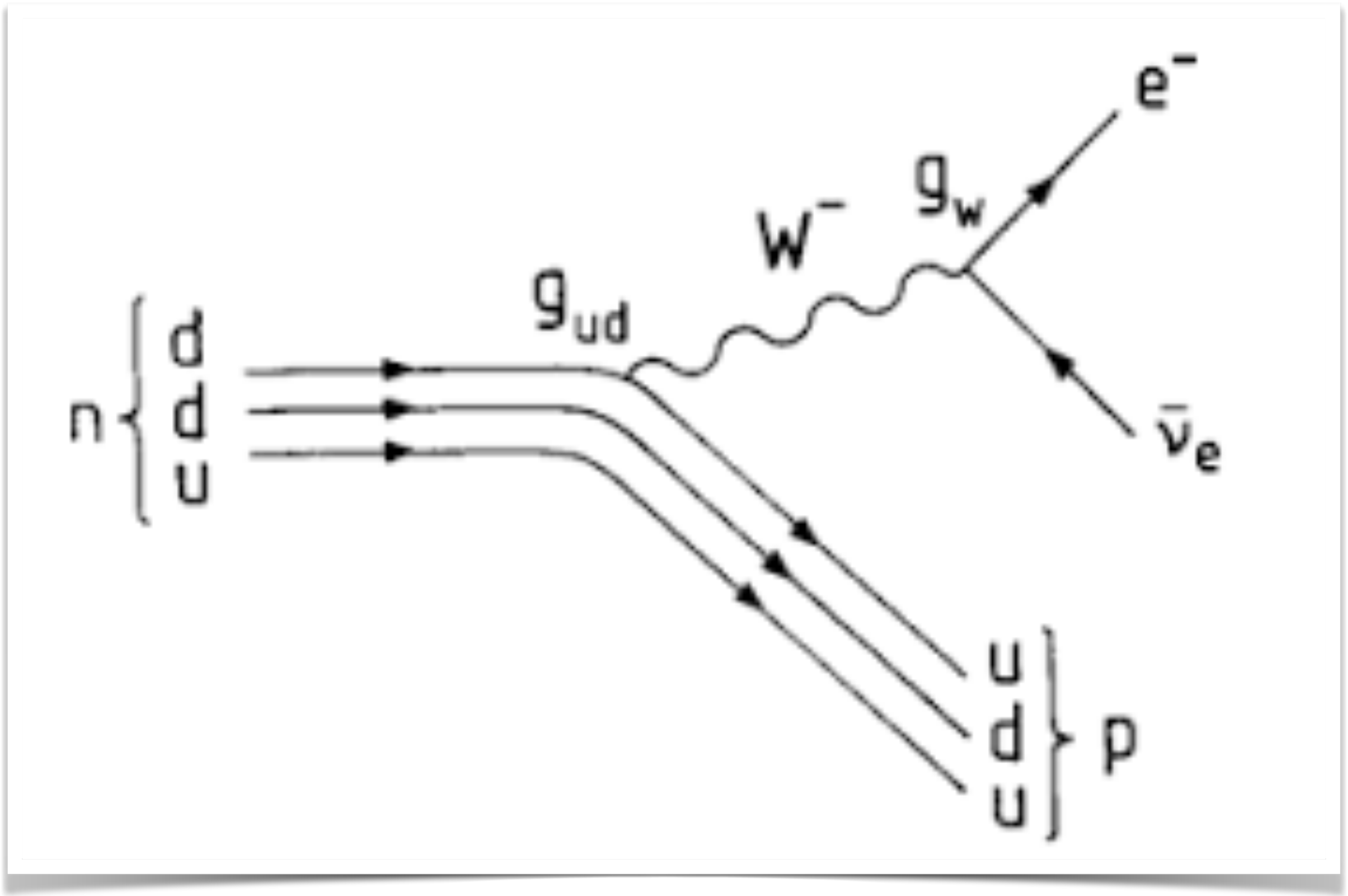
$Q \equiv m_n - m_p$

$1.293\,\mathrm{MeV}$

proton

$$n + e^+ \leftrightarrow p + \bar{\nu}_e$$

$$n + \nu_e \leftrightarrow p + e^-$$



Interconversions of n and p rapid through **weak interactions**.

n and p in **chemical equilibrium**, $n_n \approx n_p$.

$T \sim 2\,\mathrm{MeV}, t \sim 1\mathrm{s}$

$T \sim Q$: $n \to p$ Easy, $p \to n$ Hard

neutron

$Q \equiv m_n - m_p$

1.293 MeV

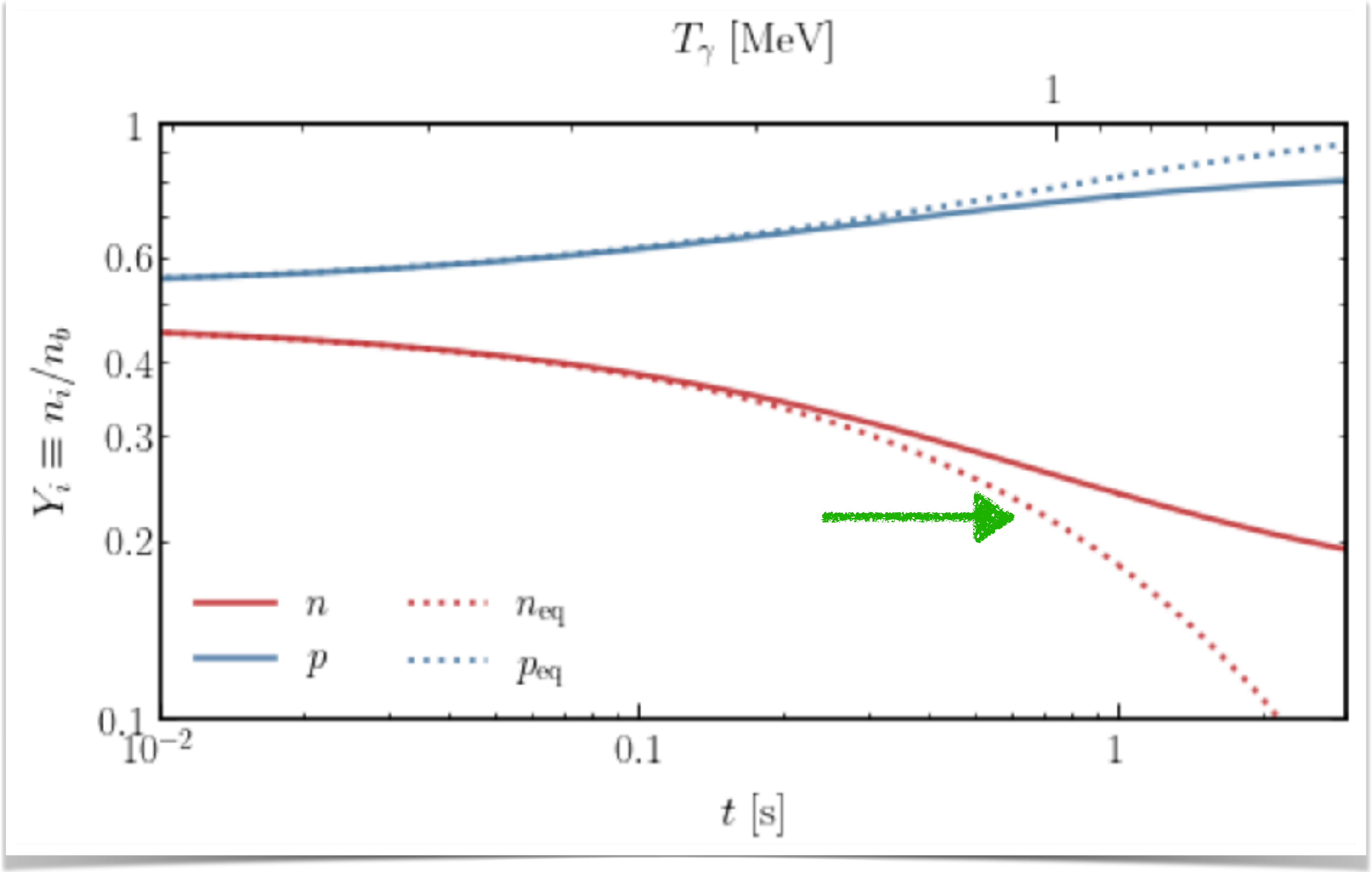
proton

EASY

$$n + e^+ \leftrightarrow p + \bar{\nu}_e$$

$$n + \nu_e \leftrightarrow p + e^-$$

HARD



Hard to find p with enough energy to go back to n.

As expected from equilibrium, $n_n/n_p \sim \exp(-Q/T)$.

n Freezeout

neutron

$Q \equiv m_n - m_p$

1.293 MeV

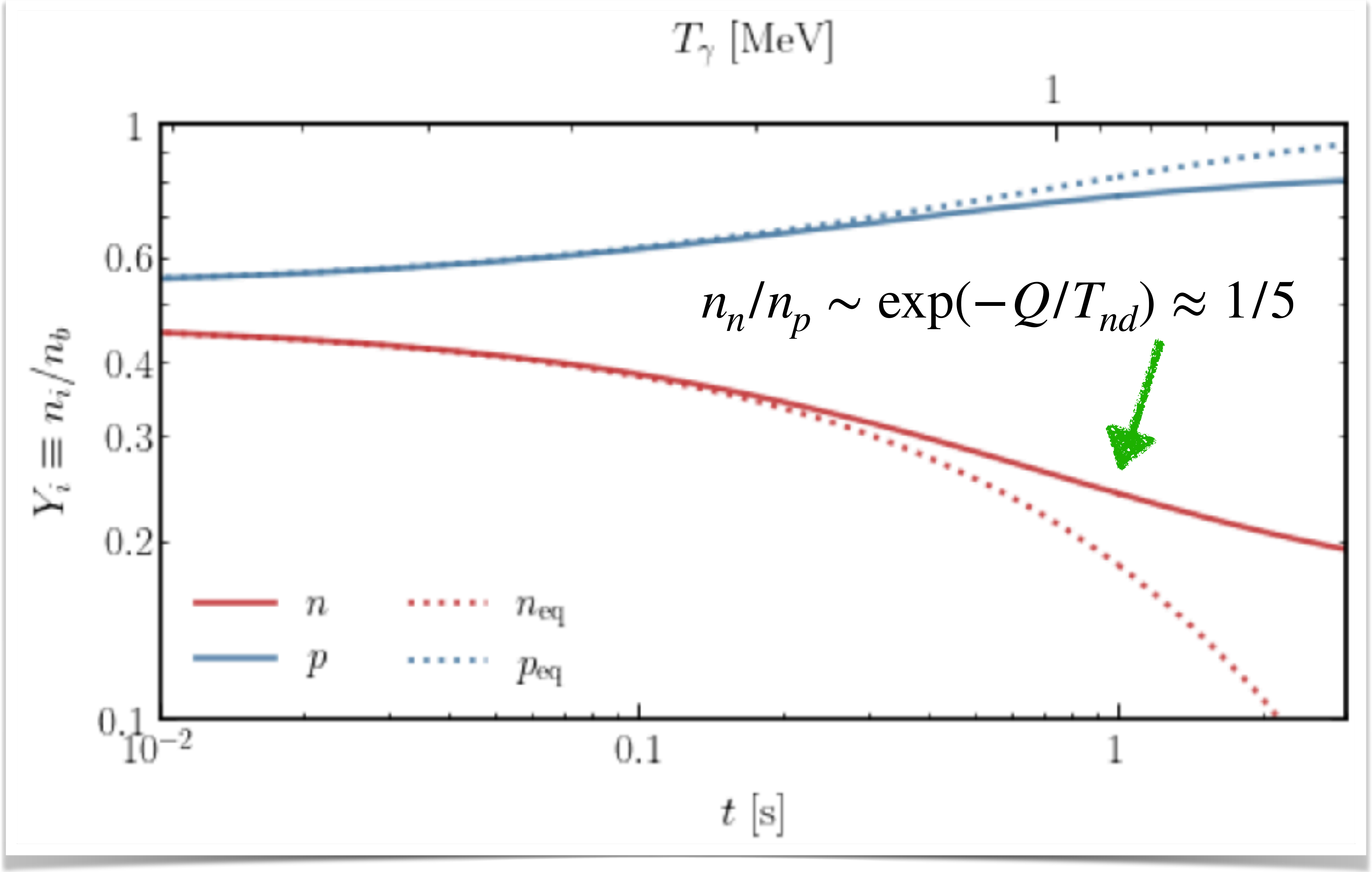
proton

HARD

$$n + e^+ \leftrightarrow p + \bar{\nu}_e$$

$$n + \nu_e \leftrightarrow p + e^-$$

HARD

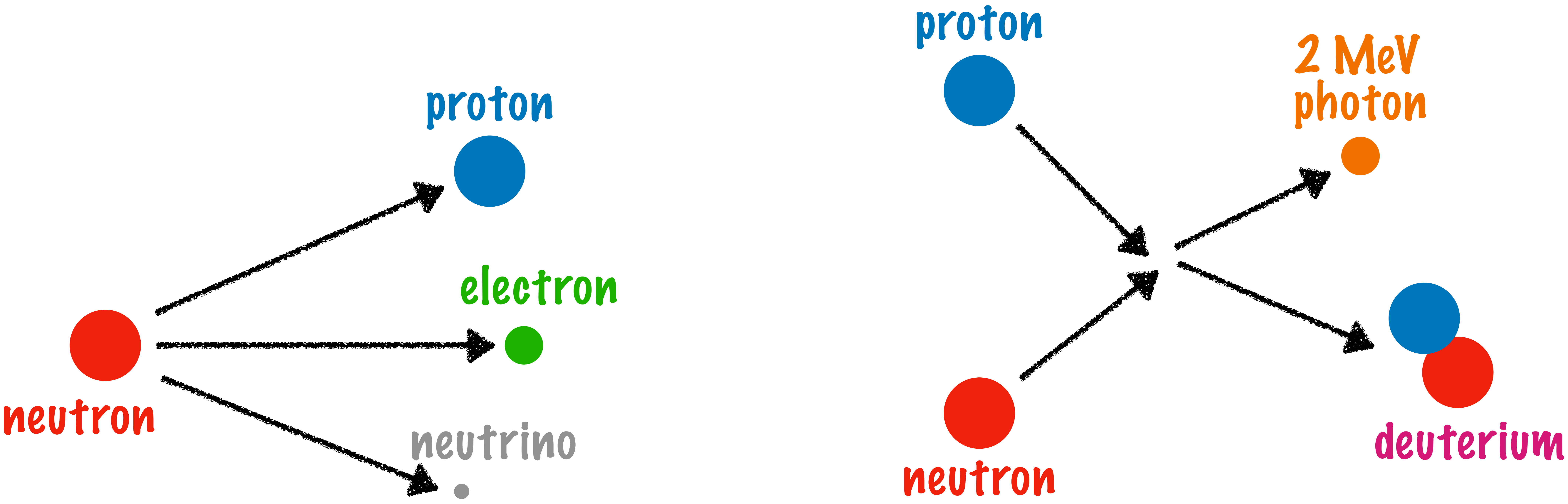


$n_n/n_p \sim \exp(-Q/T_{nd}) \approx 1/5$

Universe cools to $T \equiv T_{nd} \approx 0.84$ MeV.

Interconversion now **too slow** to change n abundance over cosmic time.

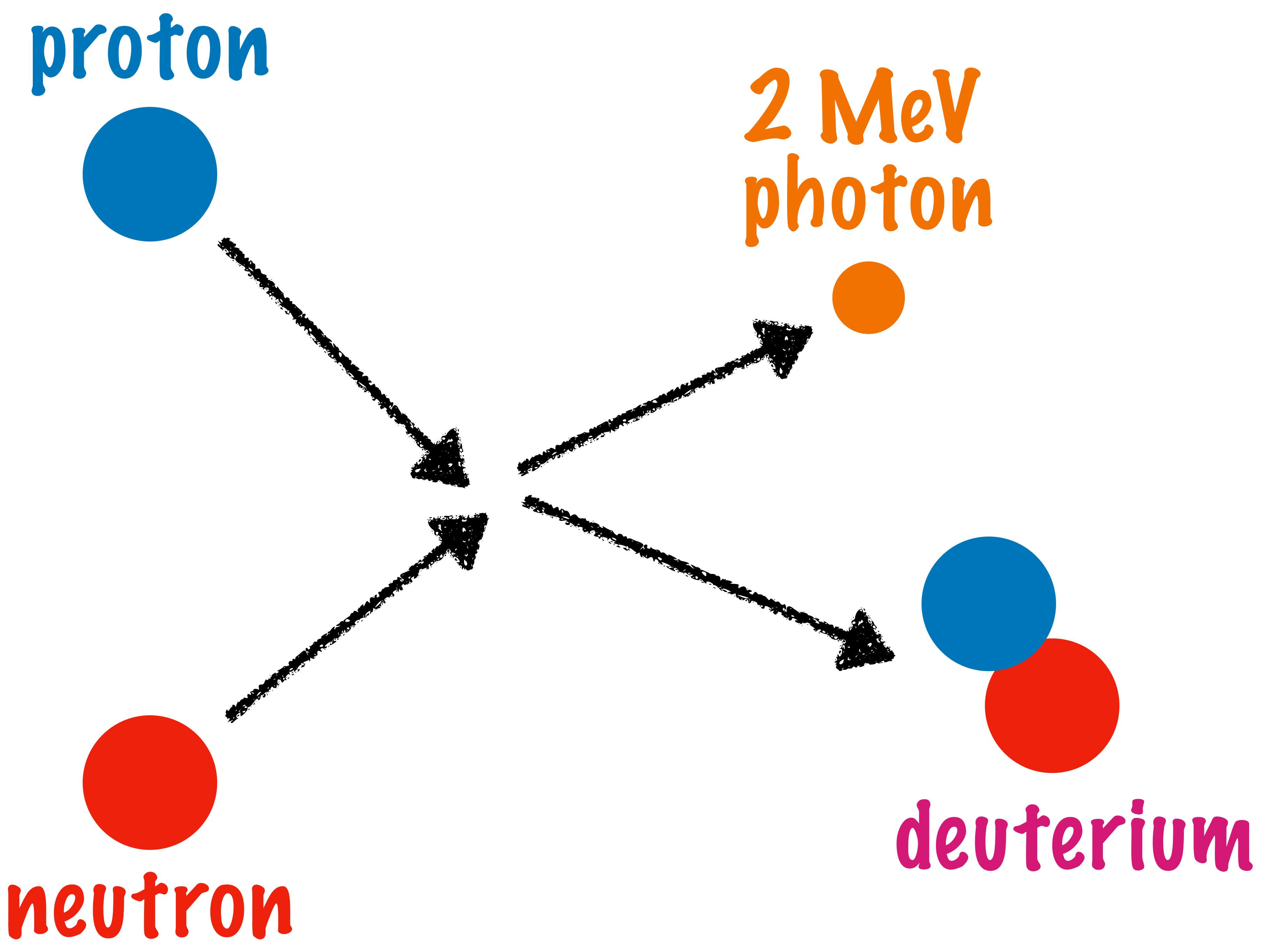
Decay or Form a Bound State



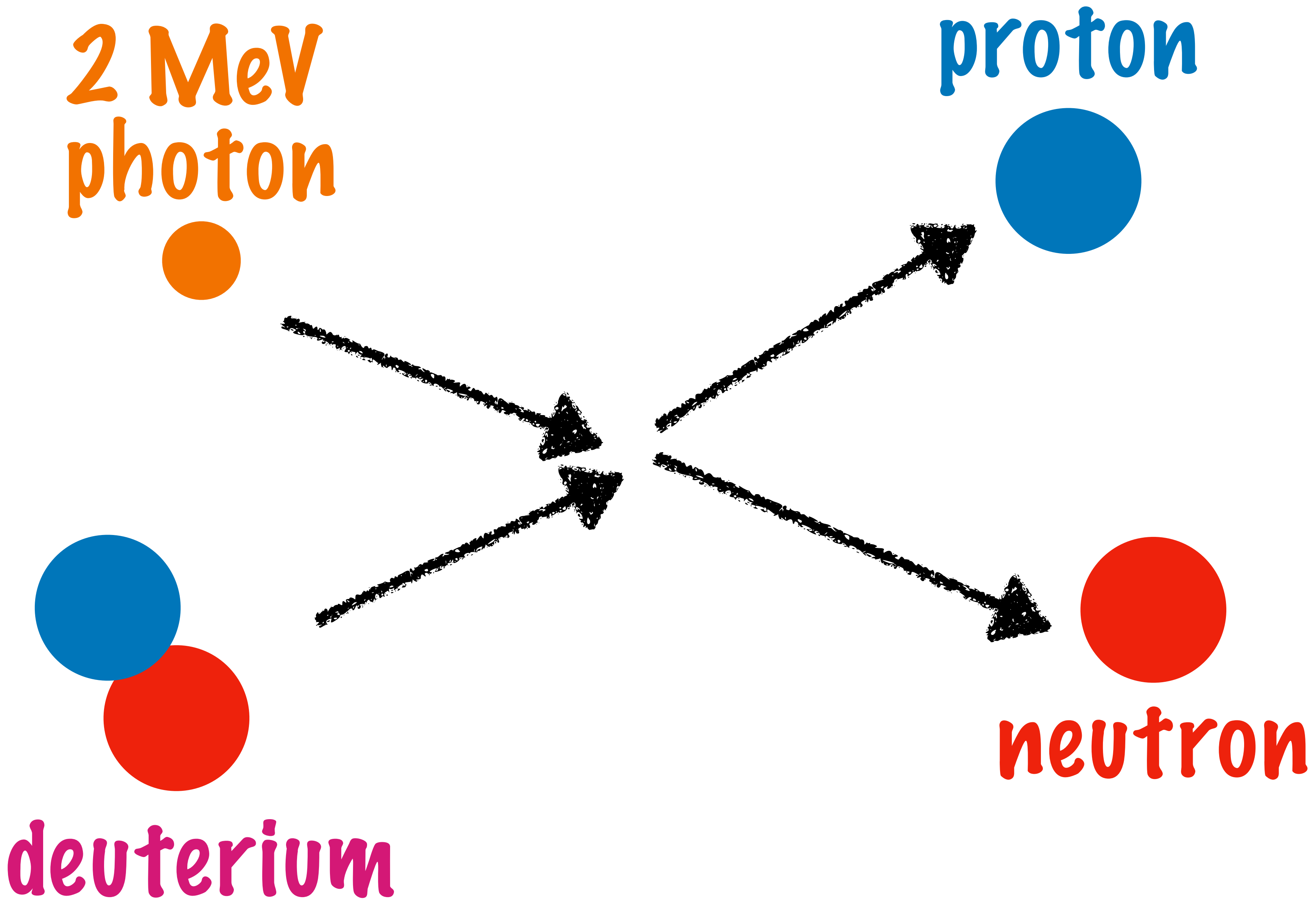
Free neutrons are **unstable**, $\tau_n = 879.4 \pm 0.6$ seconds*, but many **stable bound states**, starting with **deuterium**.

*with interesting discrepancies between inclusive/non-inclusive measurements.

Lots of 2 MeV Photons



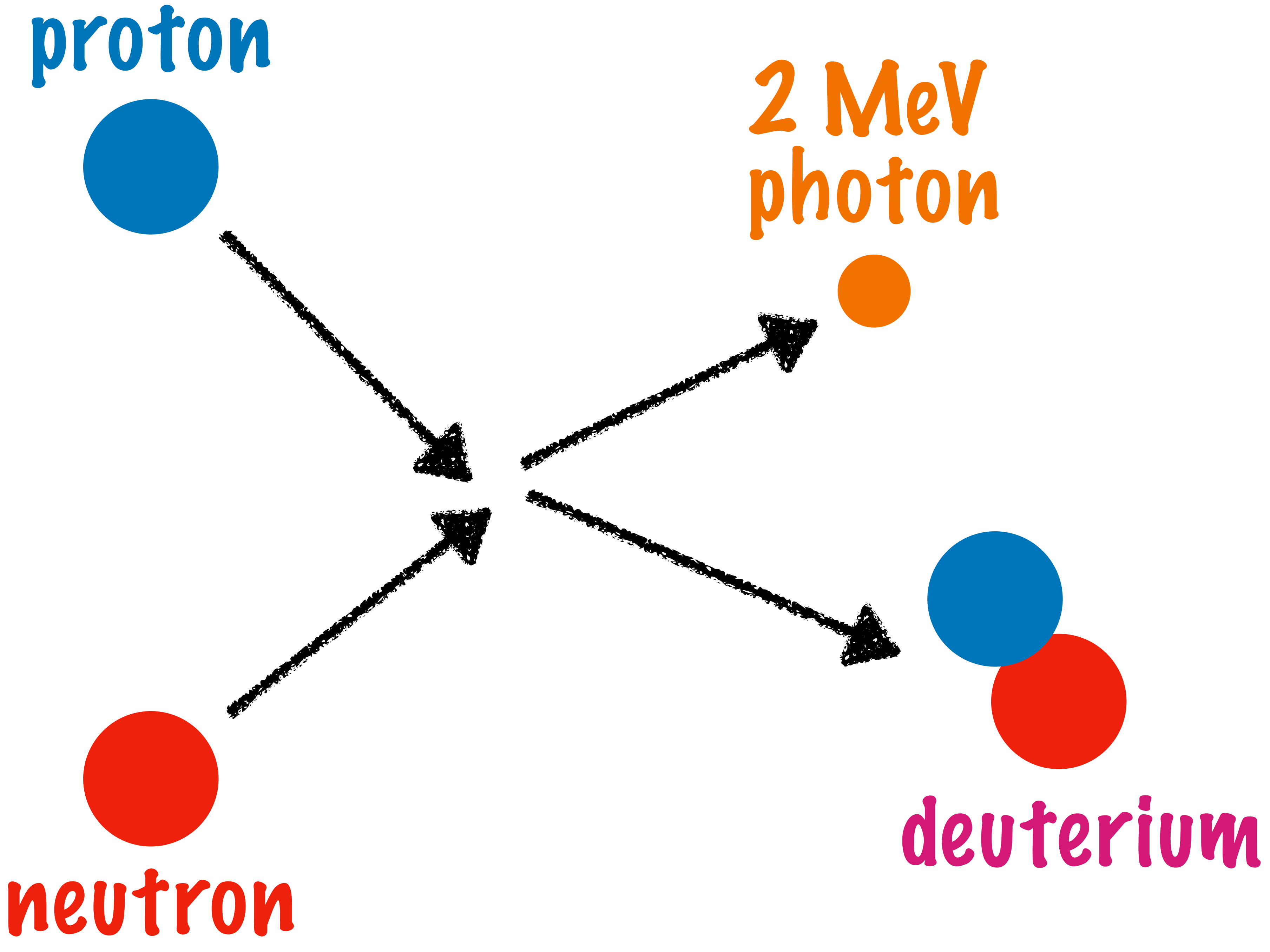
$p + n \rightarrow D + \gamma$



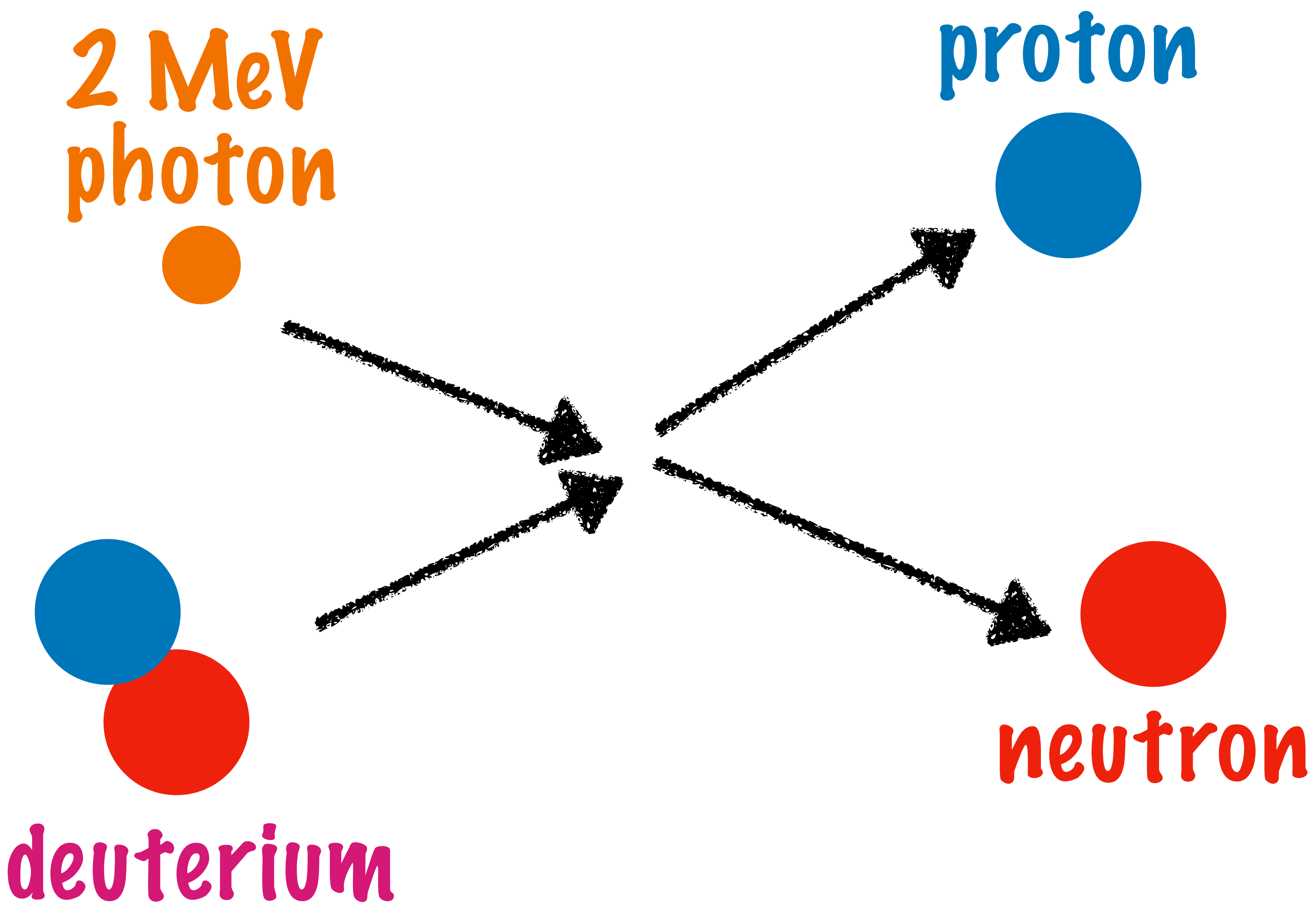
$D + \gamma \rightarrow p + n$

Even when $T < B_D \approx 2$ MeV the binding energy of D,
there are still 10^{10} photons for every baryon.

n Freezeout at 1MeV, D Forms at 0.1 MeV



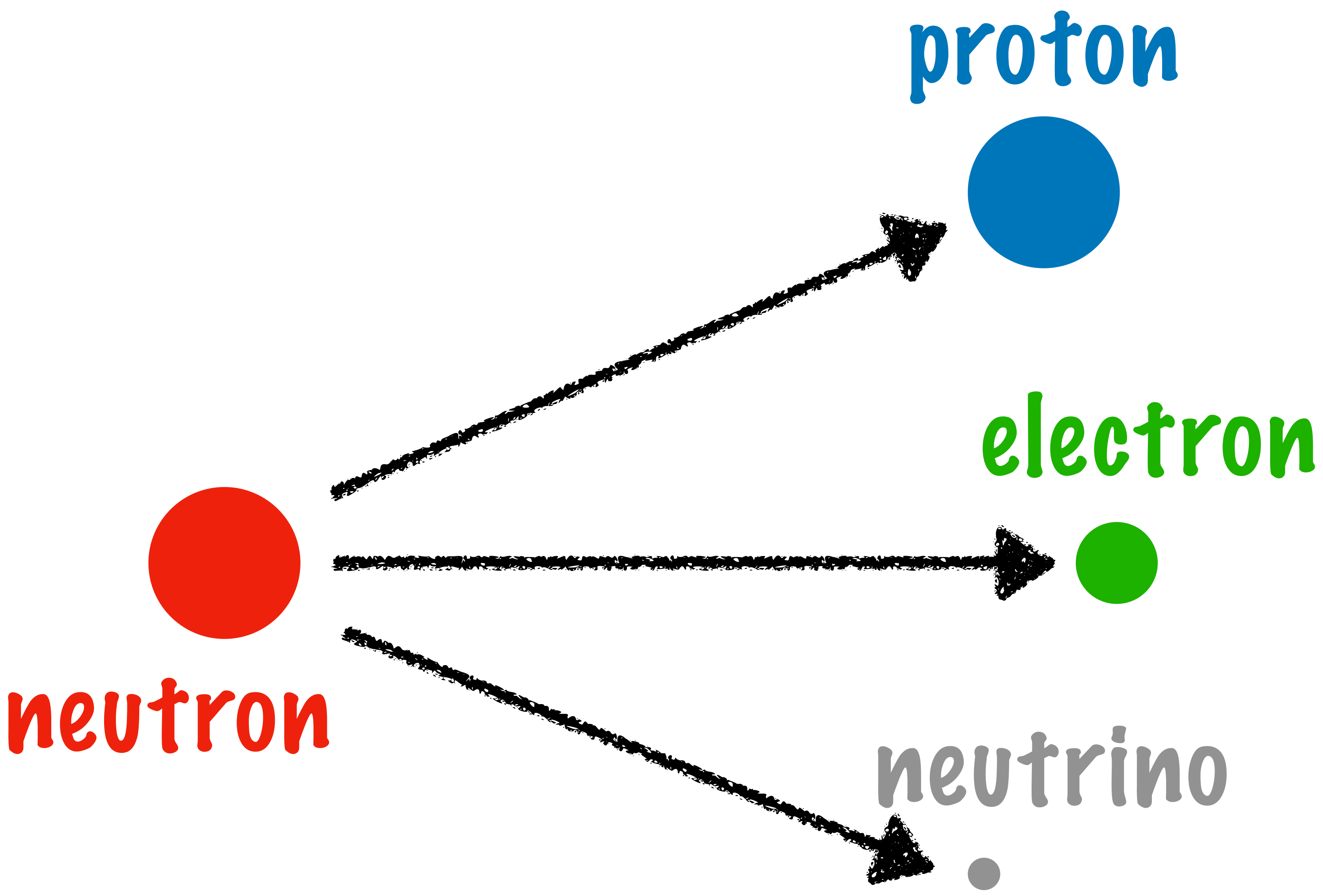
$p + n \rightarrow D + \gamma$



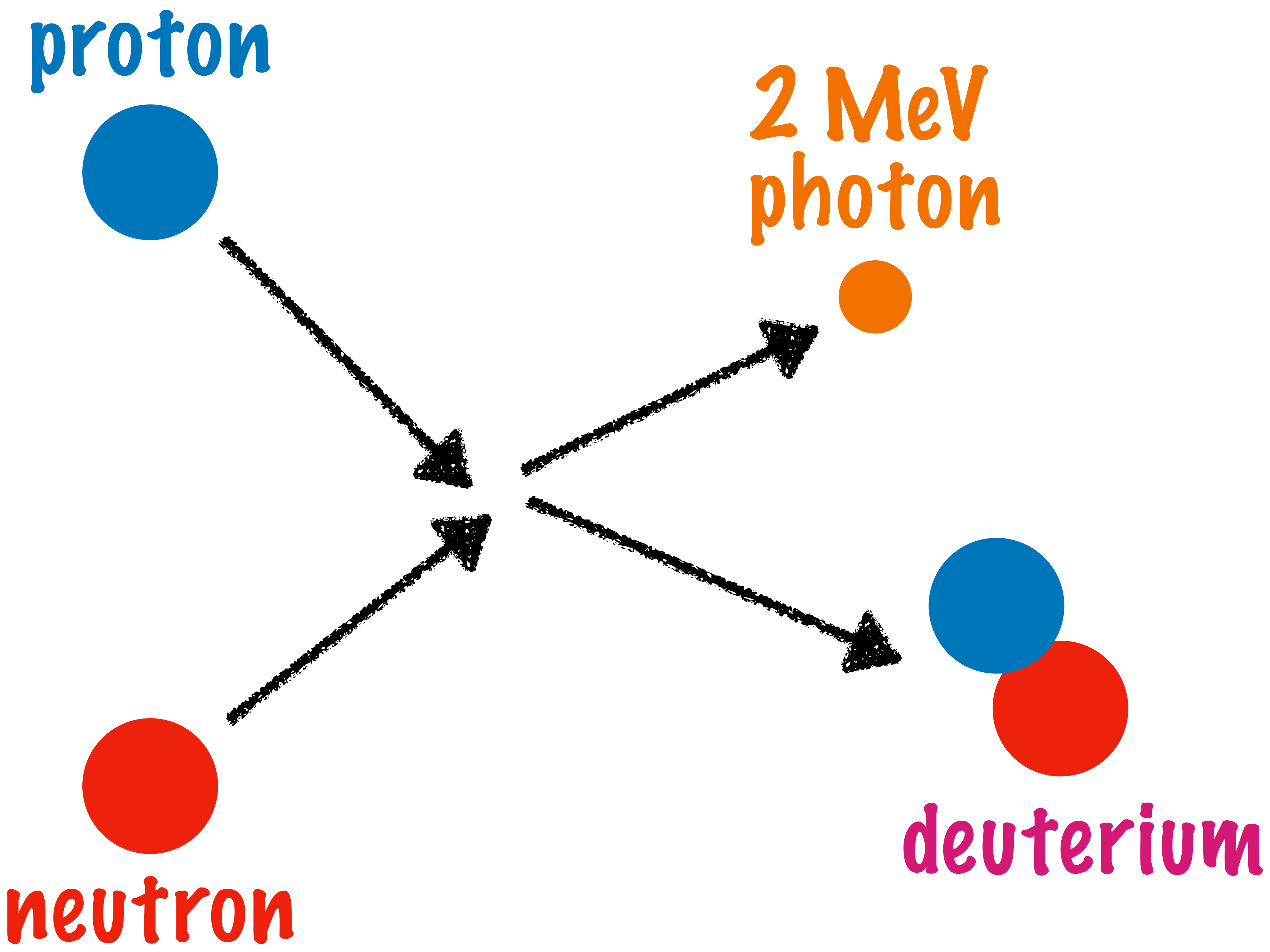
$D + \gamma \rightarrow p + n$

D can only form efficiently once $10^{10} e^{-B_D/T} \sim 1$, or $T \sim 0.1$ MeV.
This is known as the **deuterium bottleneck**.

n Decays Between 1 MeV and 0.1 MeV



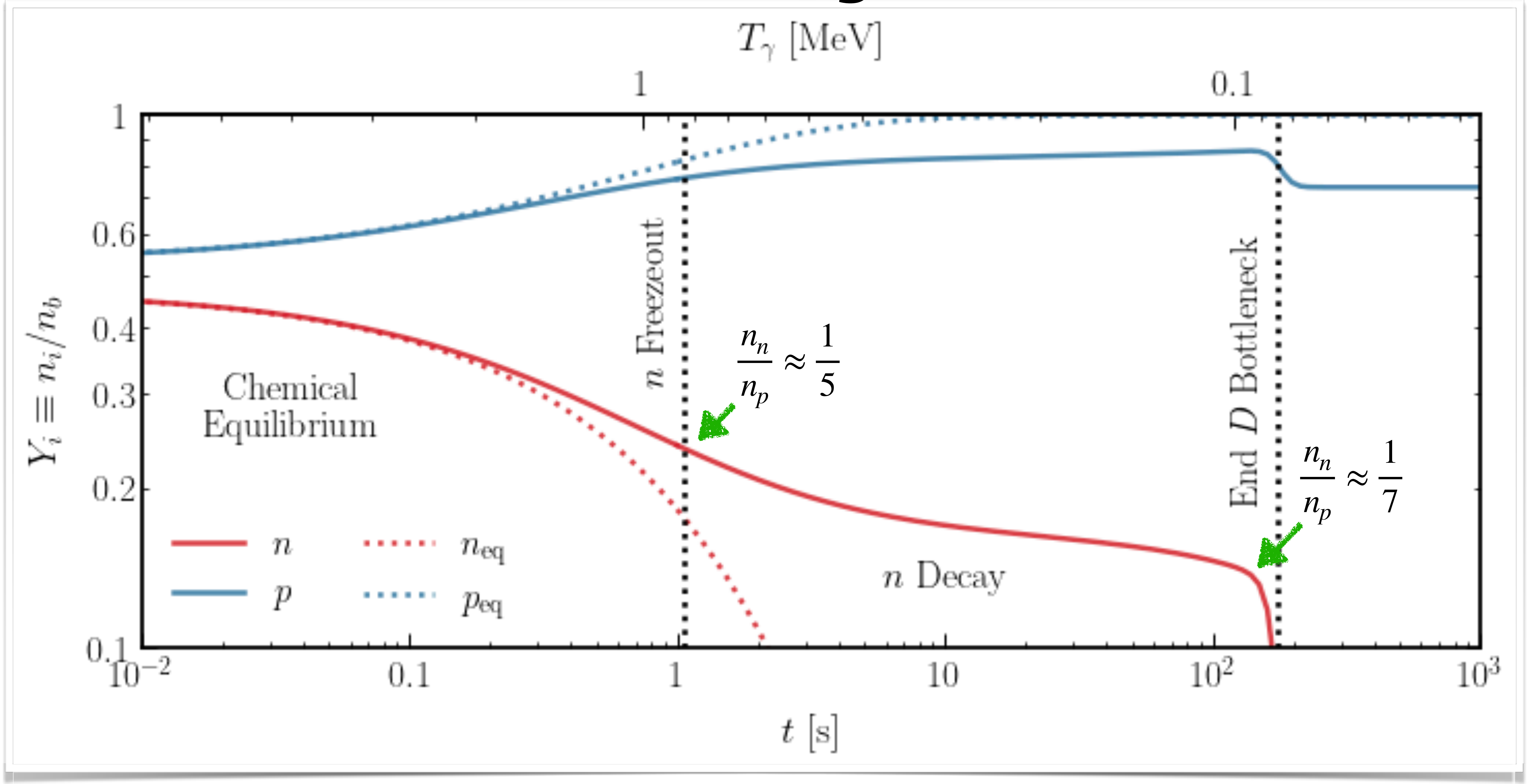
$1\,\mathrm{MeV} \gtrsim T \gtrsim 0.1\,\mathrm{MeV}$



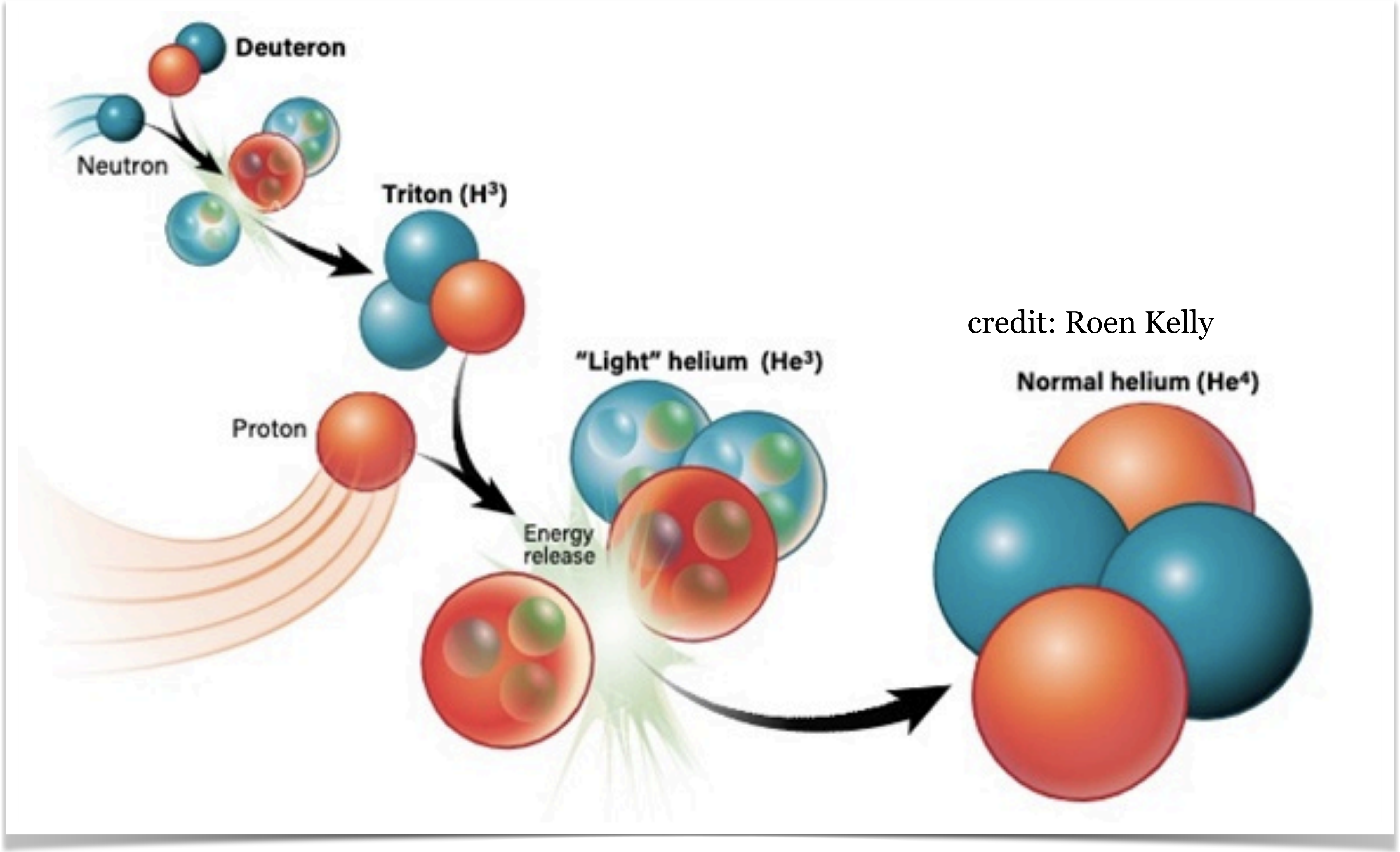
$T \lesssim 0.1\,\mathrm{MeV}$

Neutrons primarily are free, and decay for ~ 100 s after freezeout, before finally getting locked away in stable nuclei.

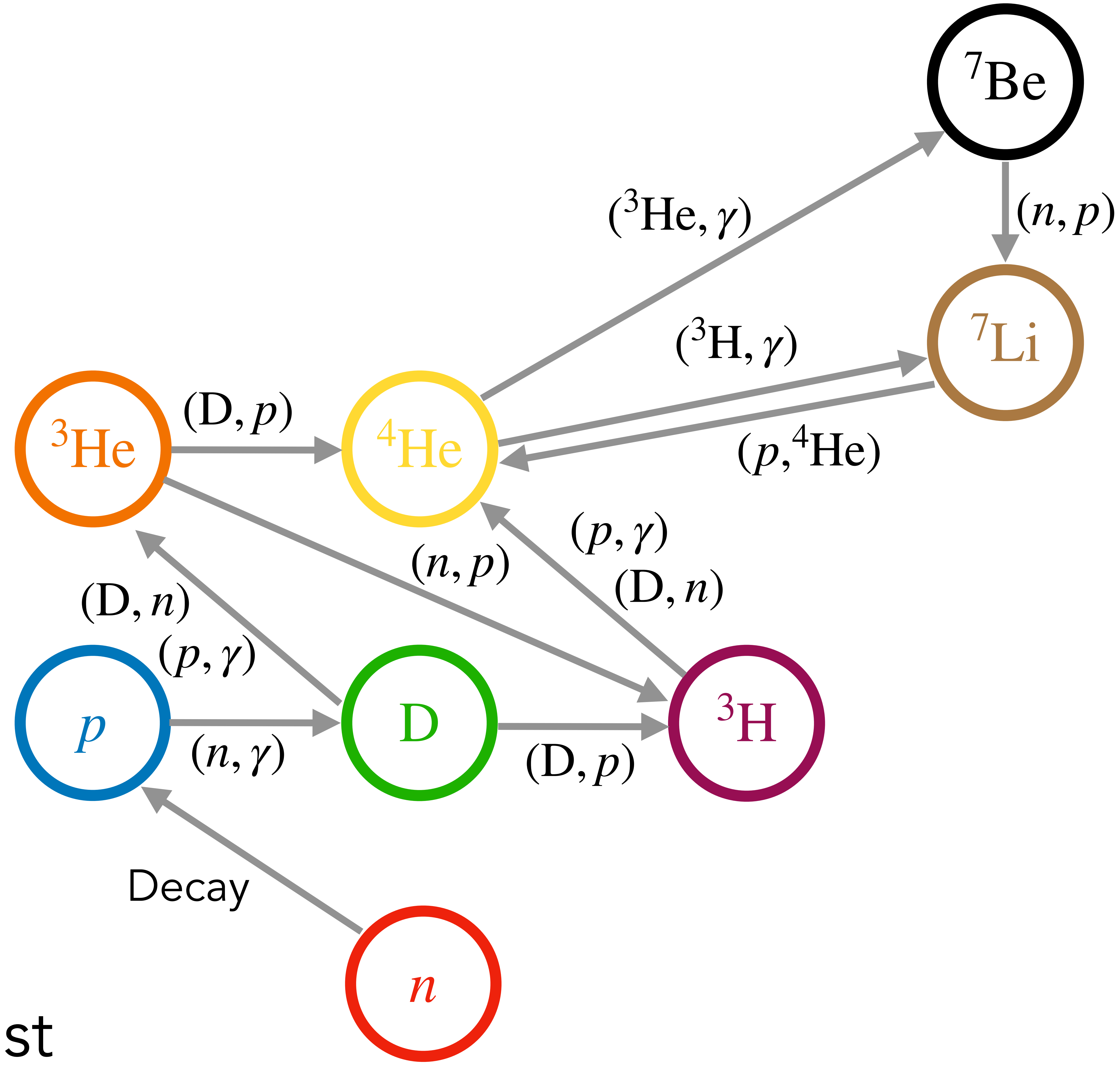
Neutrons Summary



Nucleosynthesis

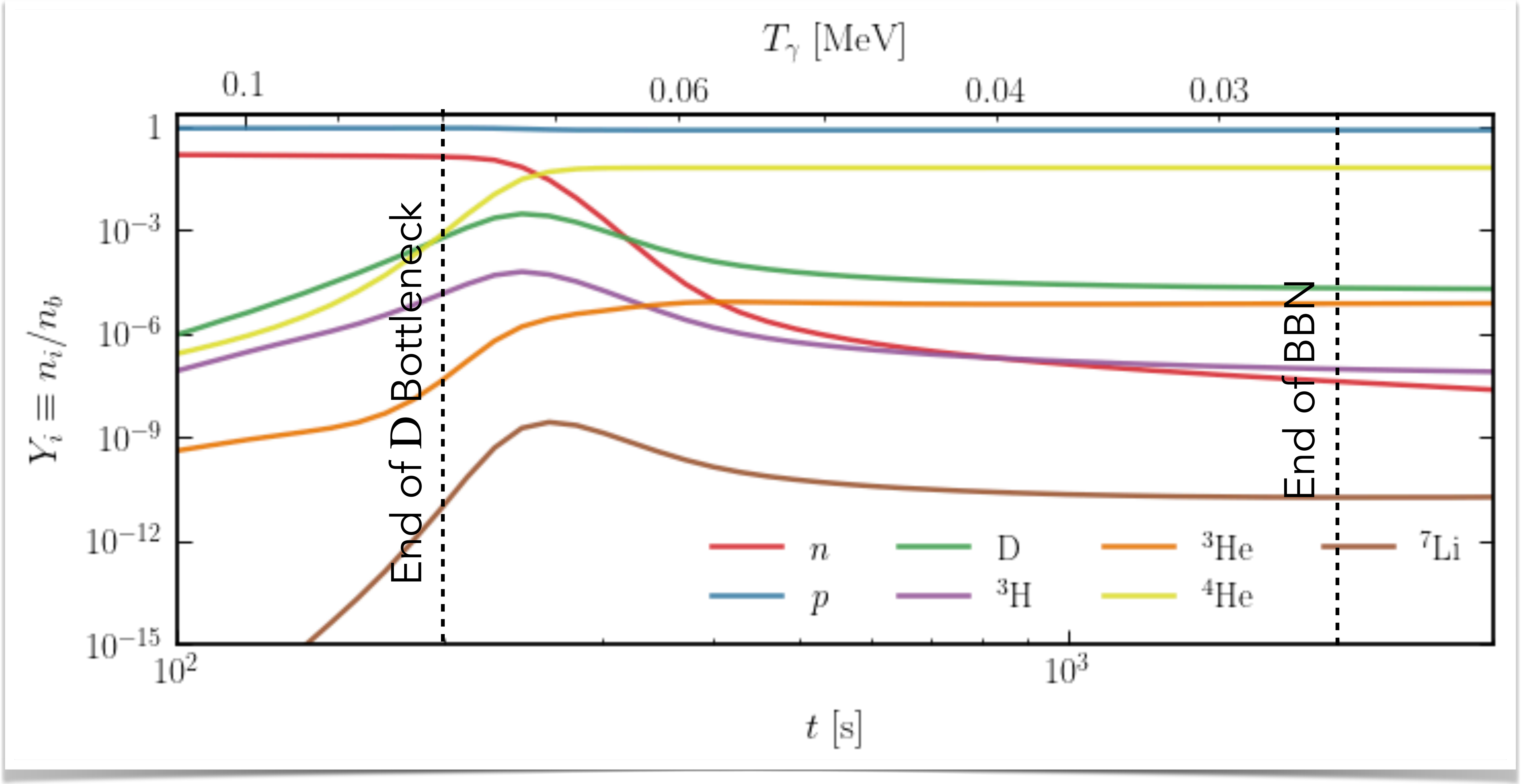


credit: Roen Kelly

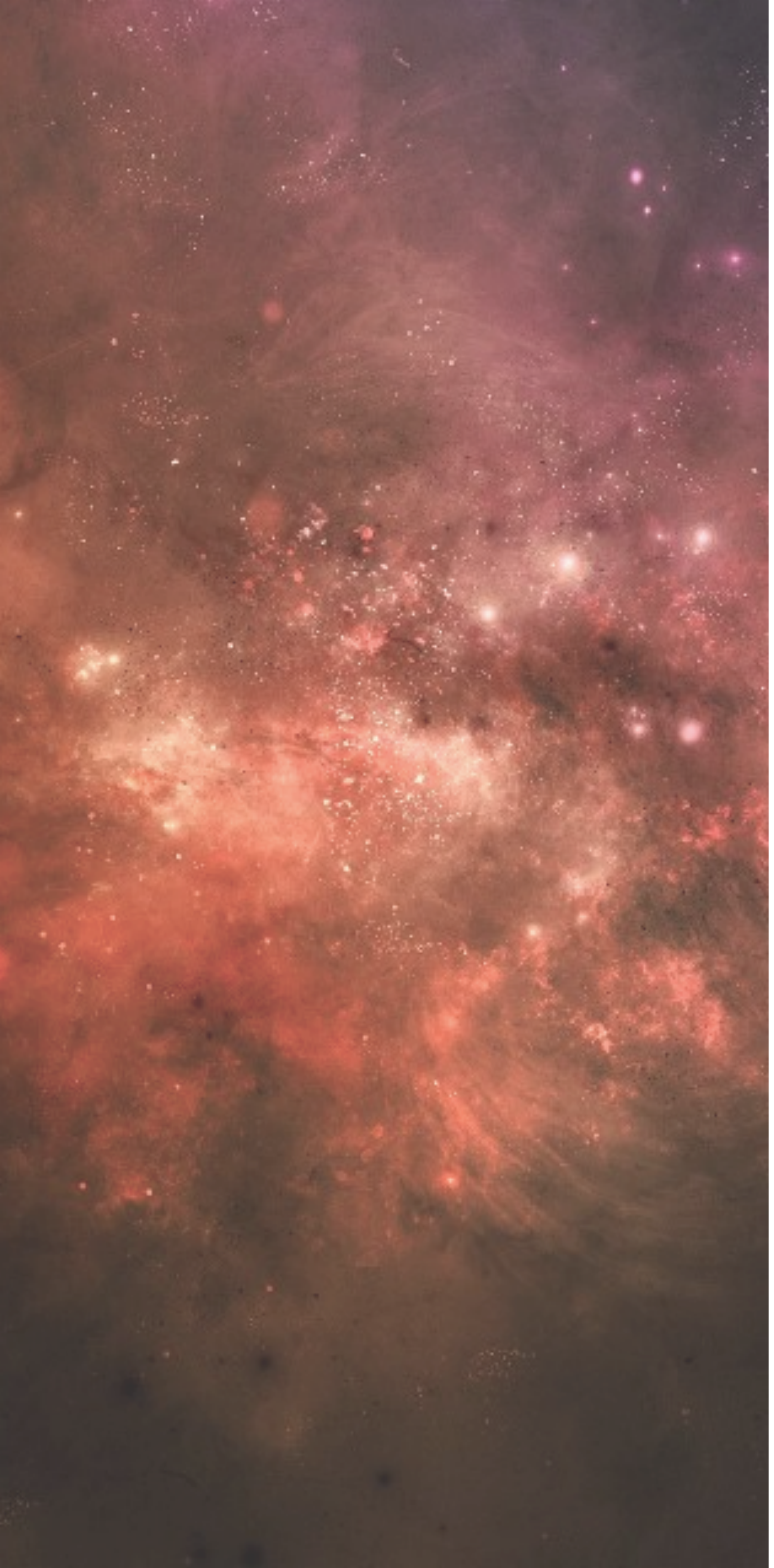


Reaction network converts neutrons into almost all **Helium-4**, the most stable light nucleus.

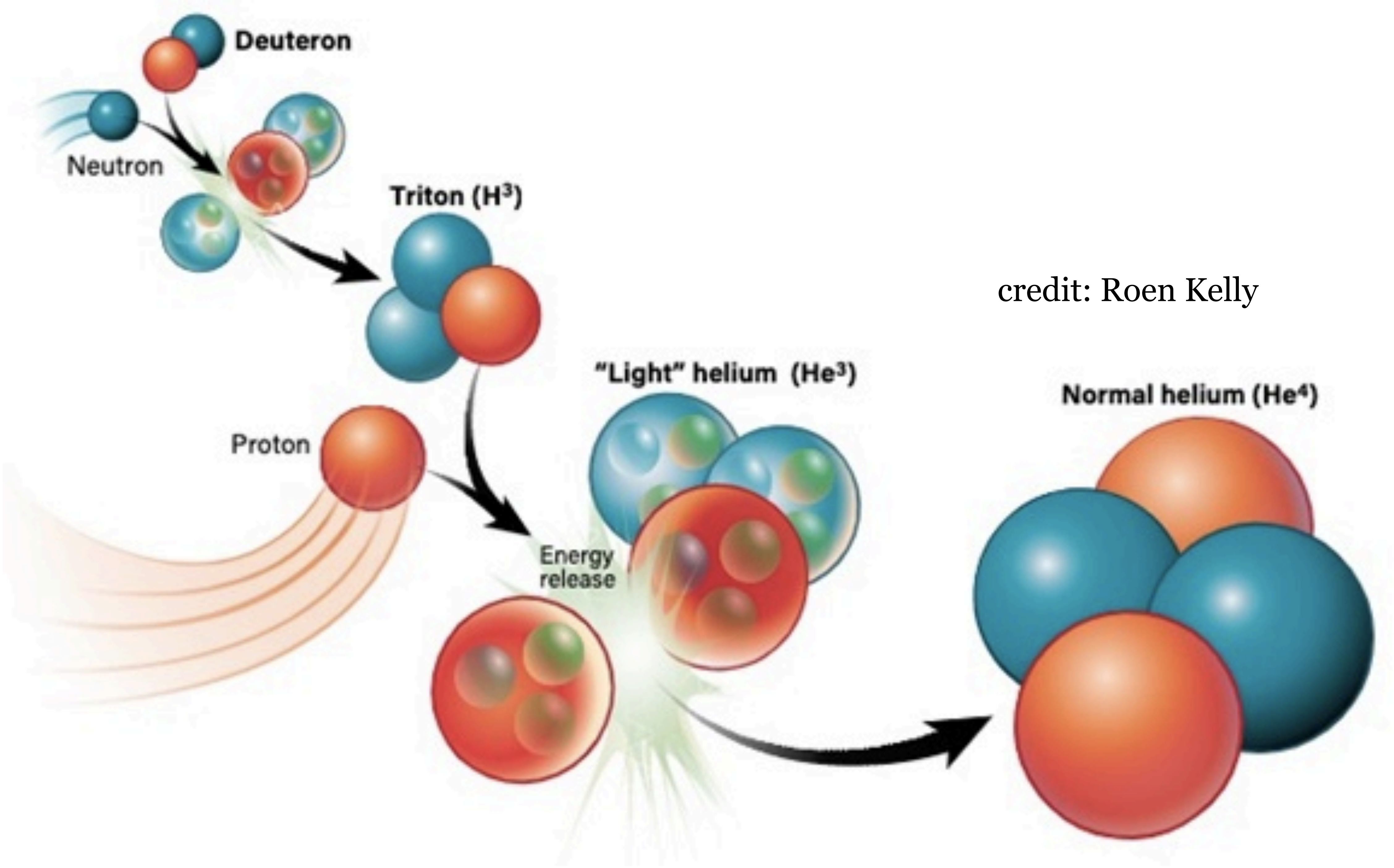
Nucleosynthesis



Efficient conversion from n to mostly $^4\mathrm{He}$, with $n_{^4\mathrm{He}}/n_b \approx (n_n/2)/n_b \approx 1/16$.
Nuclear reactions freeze out and **primordial abundance** is reached.

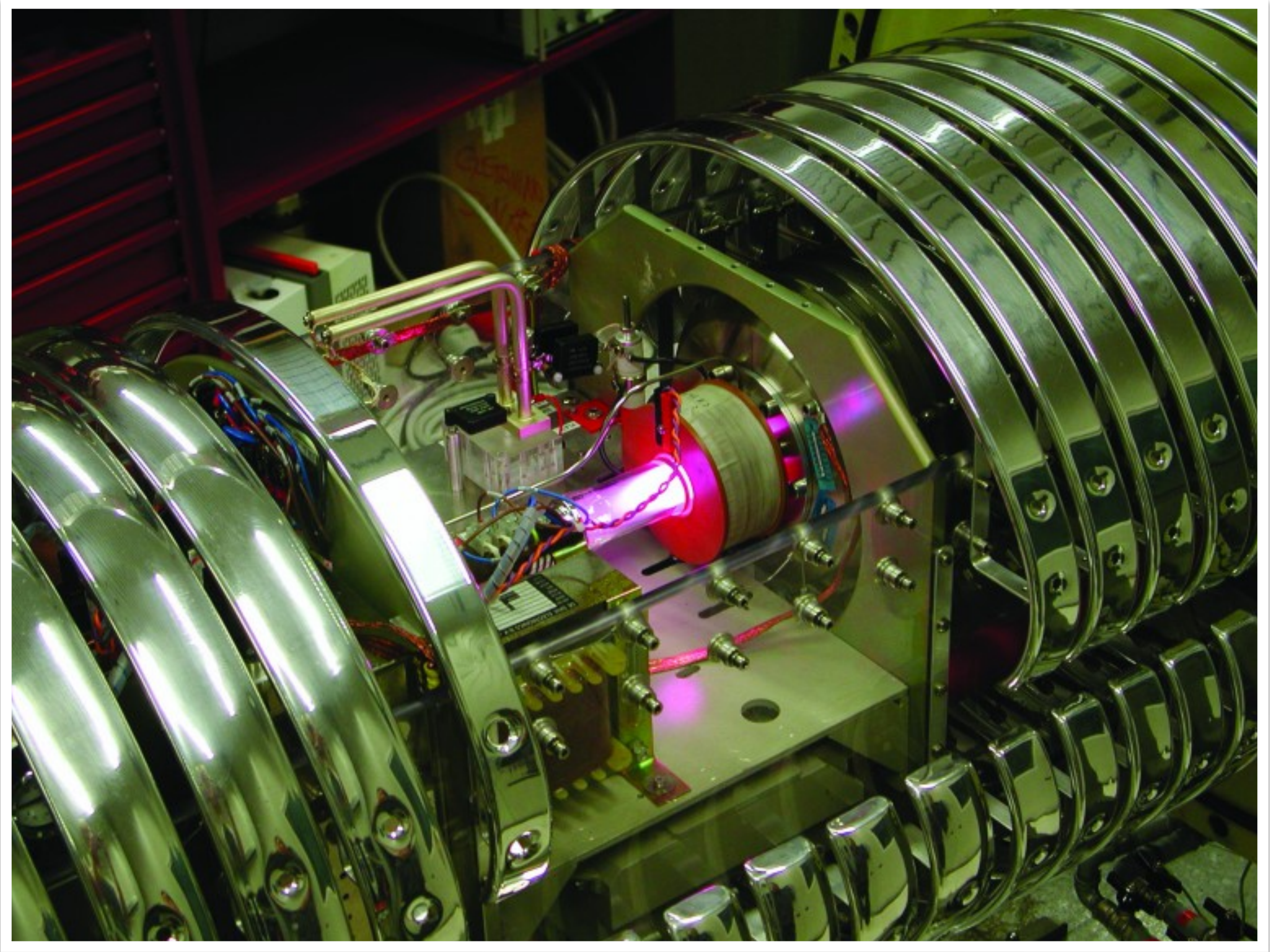


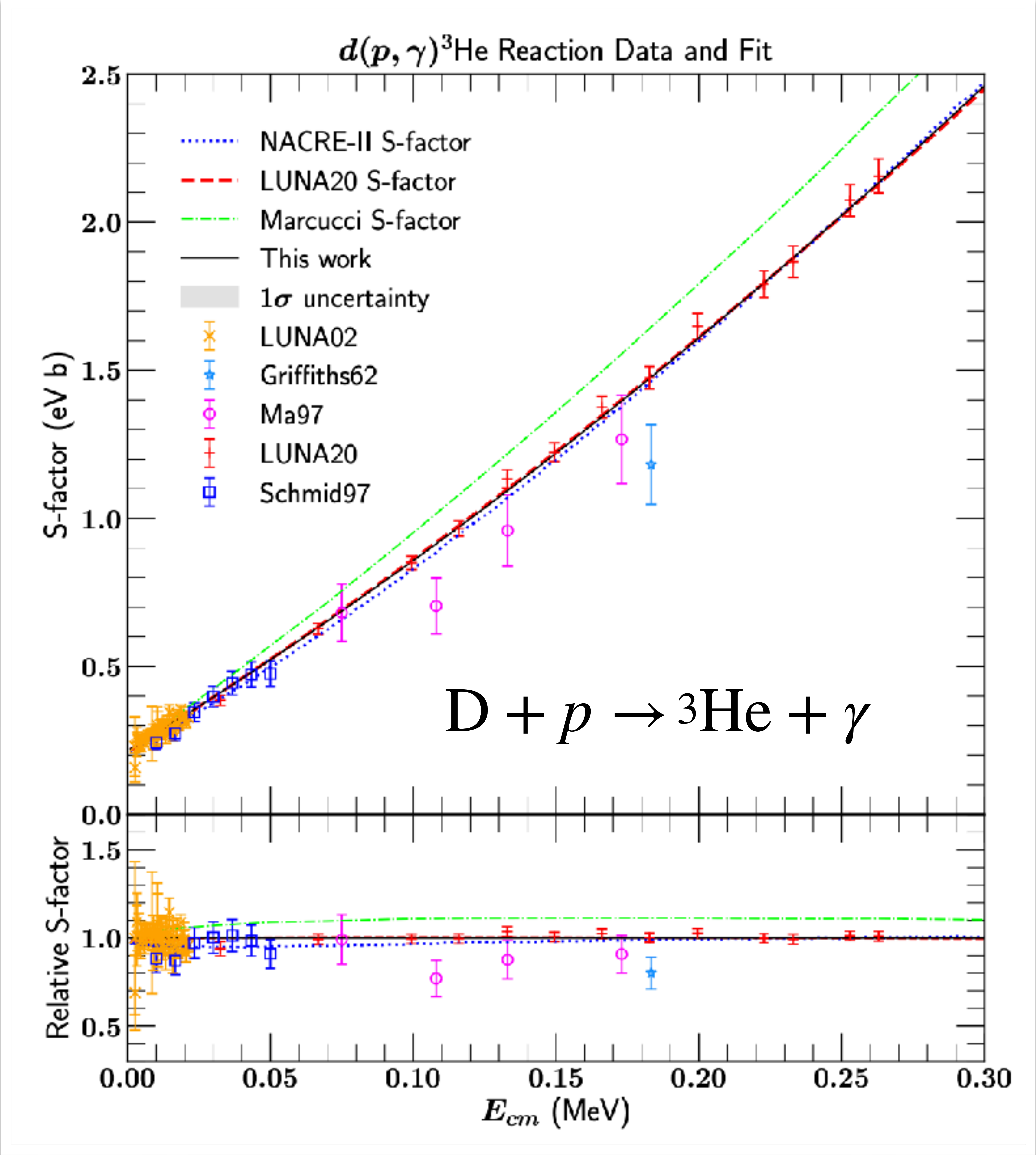
BBN Measurements and Theory Prediction



credit: Roen Kelly

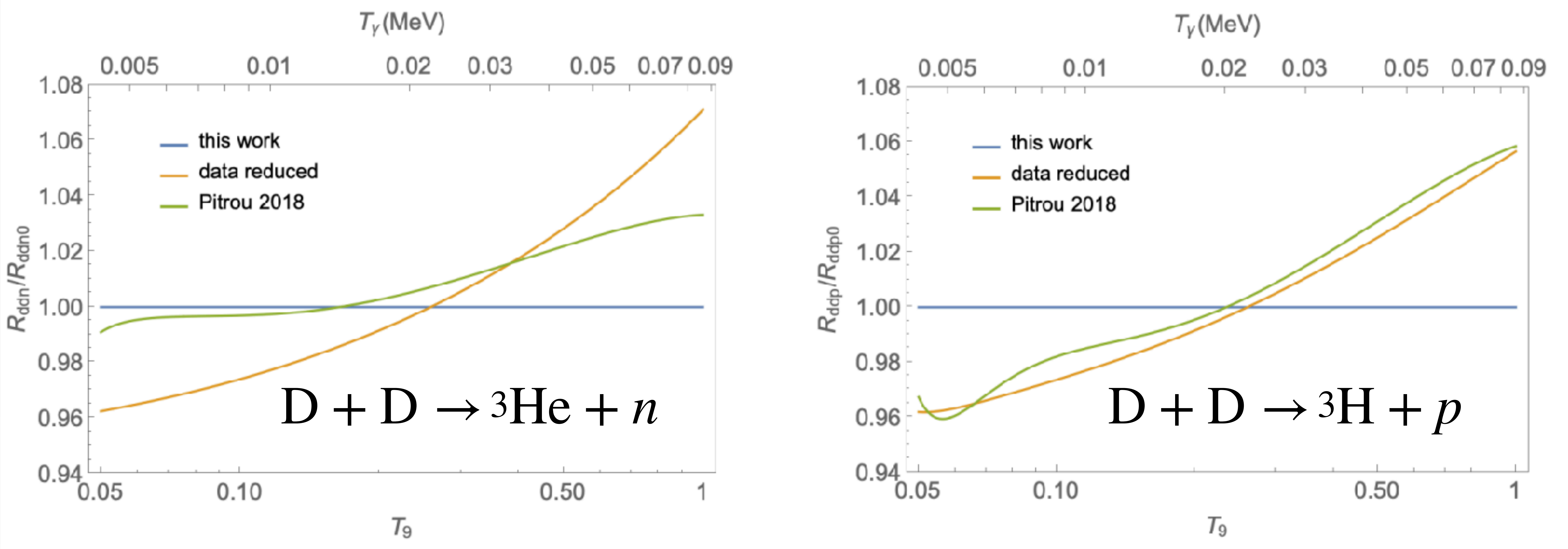
LUNA





$$\mathrm{D} + p \rightarrow {}^3\mathrm{He} + \gamma$$

Difference in Rates



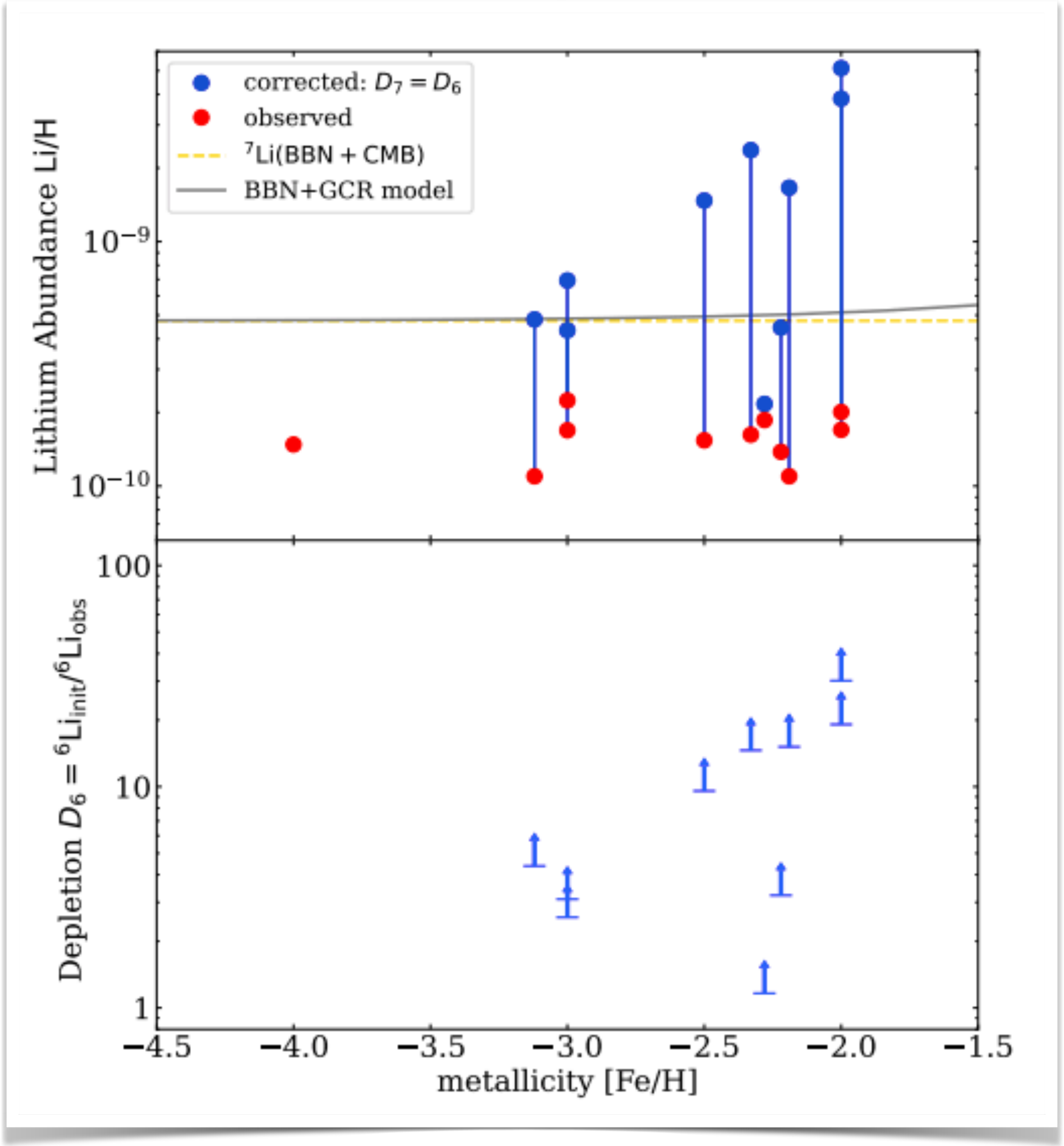
$\mathrm{D} + \mathrm{D} \to {}^3\mathrm{He} + n$

$\mathrm{D} + \mathrm{D} \to {}^3\mathrm{H} + p$

Figure 10. Ratios between the ddn (left) and ddp(right) rate obtained adopting different datasets and statistical approach, see text, and their benchmark values obtained in this work, R_{ddn0} and R_{ddp0}, versus temperature in 10^9 K.

PRIMAT vs. other rates affected by dataset choice, fitting for overall normalization of experimental data.

The Lithium Problem?



The hope is to measure Li-7 primordial abundance in **low metallicity stars**, and that **no destruction of Li-7 takes place**.

Measurements consistently **lower** than BBN prediction.

But what if they are destroyed in the star? Can check with **Li-6**.

Li-6 produced via **cosmic rays**: can predict abundance, check with measurements. If no significant destruction, then we would similarly expect no destruction for Li-7 (more stable).

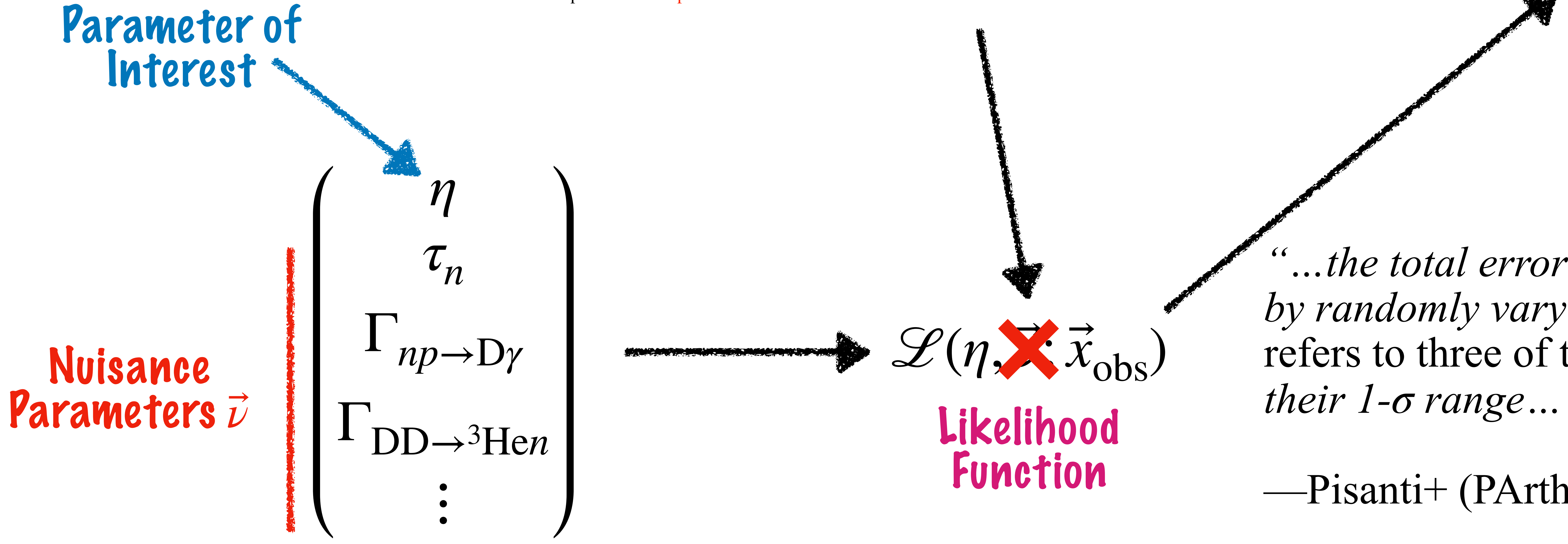
*We find Li-6 abundance **much lower** than expected from cosmic ray production, indicating **significant destruction** of Li-6 in these systems.*

We therefore can't be sure that Li-7 abundance has not been reduced from its primordial value in these systems.

e.g. Pisanti+ 2011.11537, Yeh+ 2011.13874

“Theory Uncertainty”

$$-2\log\mathscr{L} = \frac{(Y_\mathrm{p}^\mathrm{pred}-Y_\mathrm{p}^\mathrm{obs})^2}{\sigma^2_{Y_\mathrm{p},\mathrm{obs}}+\sigma^2_{Y_\mathrm{p},th}} + \frac{((\mathrm{D/H})^\mathrm{pred}-(\mathrm{D/H})^\mathrm{obs})^2}{\sigma^2_{\mathrm{D/H},\mathrm{obs}}+\sigma^2_{Y_\mathrm{D/H},th}}$$

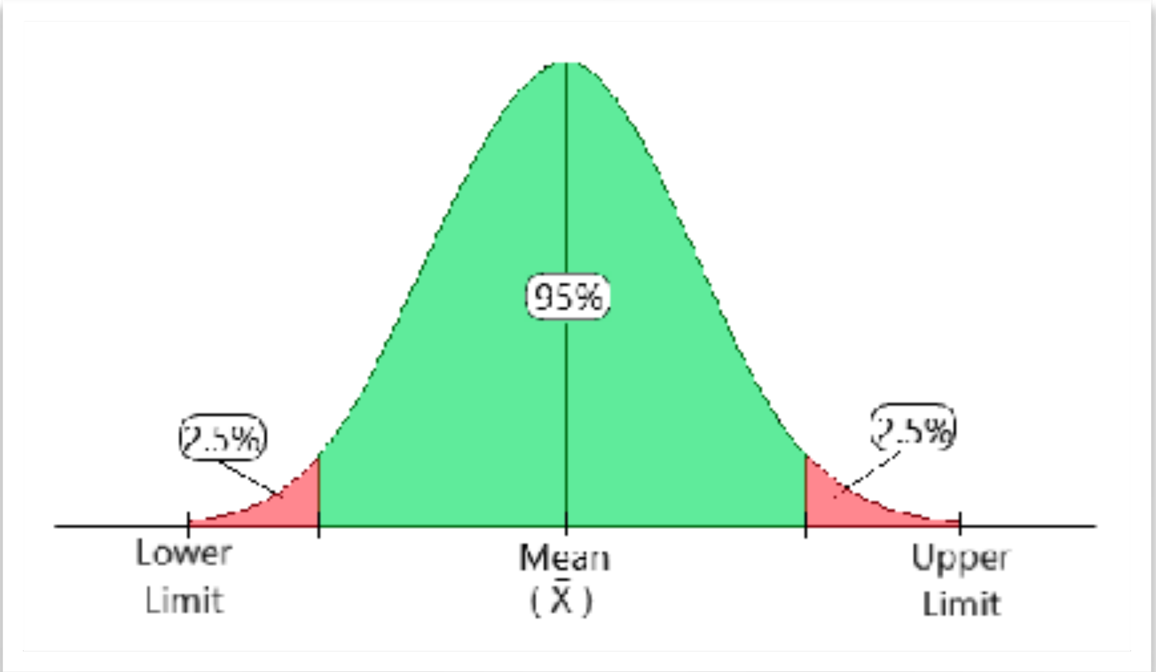


Parameter of Interest

Nuisance Parameters $\vec{\nu}$

$$\begin{pmatrix}\eta\\ \tau_n\\ \Gamma_{np\to\mathrm{D}\gamma}\\ \Gamma_{\mathrm{DD}\to{}^3\mathrm{He}n}\\ \vdots\end{pmatrix} \longrightarrow \mathscr{L}(\eta, \cancel{\vec{\nu}}; \vec{x}_\mathrm{obs})$$

Likelihood Function



Max. Likelihood Confidence Interval

Method (1): uncertainty in predicted abundances estimated by **varying over $\vec{\nu}$.**

Added in quadrature into $\mathscr{L}$.

“…the total error on Deuterium prediction is obtained by randomly varying all rates [possibly ‘all’ here only refers to three of the nuclear rates] *simultaneously in their 1-σ range…”*

—Pisanti+ (PArthENoPE) 2011.11537

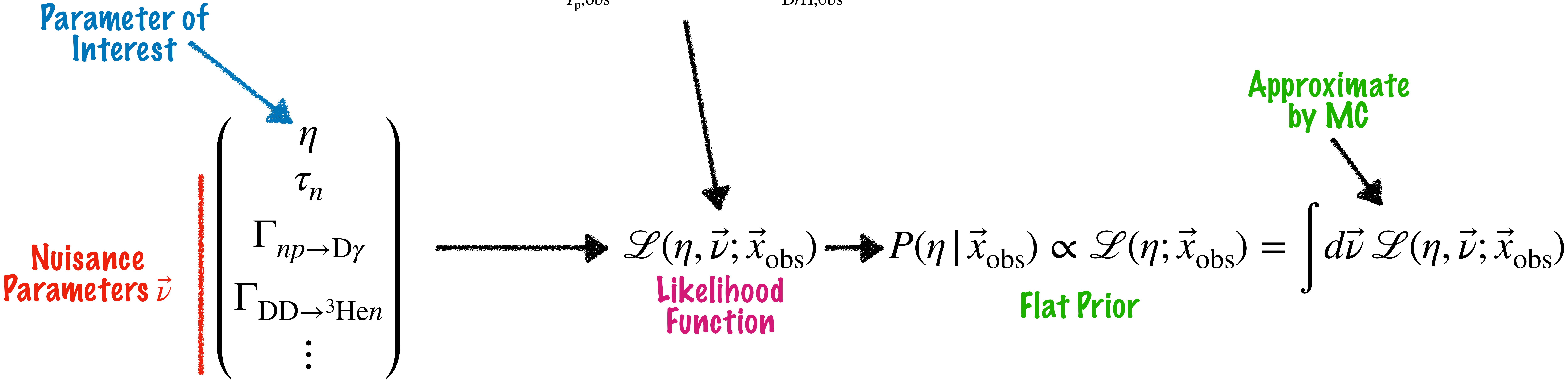
“For each grid point, i.e., each value of η…, we run the BBN code 10000 times allowing the key nuclear rates to vary… From these we infer BBN likelihood functions for the entire grid.”

—Yeh+ 2011.13874

Pitrou+ (PRIMAT) 2011.11320

Limited Monte Carlo

$$-2\log\mathscr{L} = \frac{(Y_{\mathrm{p}}^{\mathrm{pred}} - Y_{\mathrm{p}}^{\mathrm{obs}})^2}{\sigma^2_{Y_{\mathrm{p}},\mathrm{obs}}} + \frac{((\mathrm{D/H})^{\mathrm{pred}} - (\mathrm{D/H})^{\mathrm{obs}})^2}{\sigma^2_{\mathrm{D/H,obs}}}$$



$$\begin{pmatrix} \eta \\ \tau_n \\ \Gamma_{np\to\mathrm{D}\gamma} \\ \Gamma_{\mathrm{DD}\to{}^3\mathrm{He}n} \\ \vdots \end{pmatrix} \longrightarrow \mathscr{L}(\eta,\vec{\nu};\vec{x}_{\mathrm{obs}}) \longrightarrow P(\eta\,|\,\vec{x}_{\mathrm{obs}}) \propto \mathscr{L}(\eta;\vec{x}_{\mathrm{obs}}) = \int d\vec{\nu}\,\mathscr{L}(\eta,\vec{\nu};\vec{x}_{\mathrm{obs}})$$

Method (2): limited Monte Carlo with ~20,000 points, **per value of η**.

This is ~2 grid points per side of a 14-D cube.

Simplifications for Joint Analysis

1. **Planck 1807.06209**
   (5/5) Full Planck CMB likelihood. Y_p is fed into CMB prediction.
   (1/5) BBN likelihood uses constant theory uncertainty for $\mathrm{D/H}$ and Y_p.

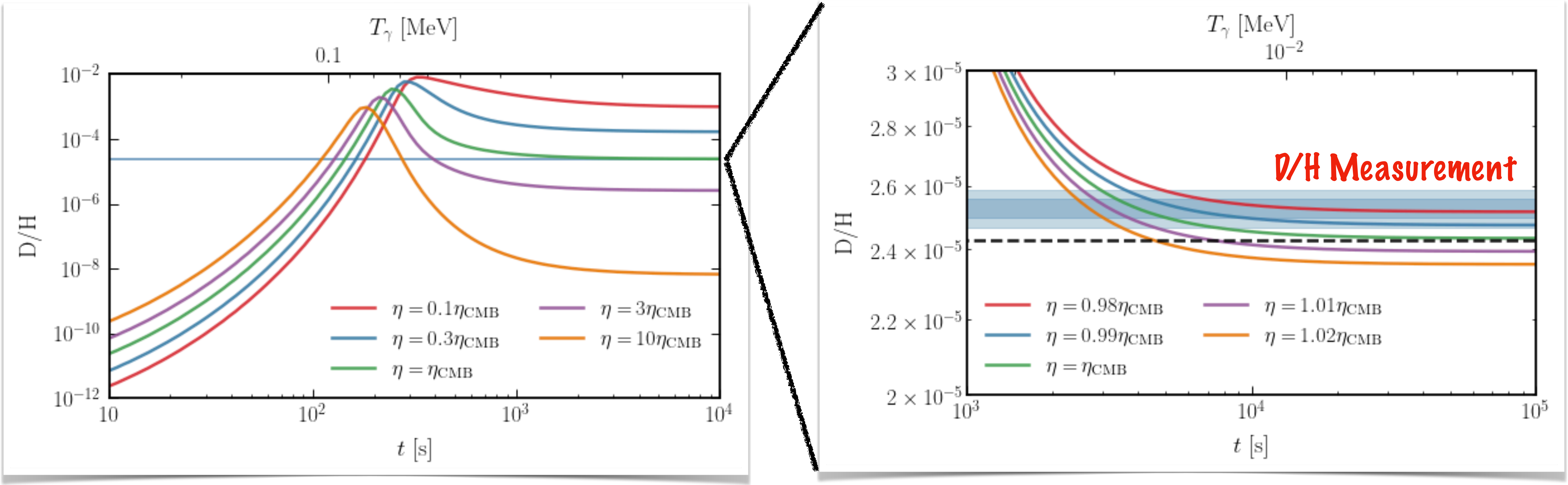
2. **Pitrou+ (PRIMAT) 2011.11320**
   (2/5) η from CMB posterior used as prior. Y_p from BBN has no impact on CMB.
   (4/5) Full BBN likelihood, estimated with 20,000 MC samples.

3. **Pisanti+ (PArthENoPE) 2011.11537**
   (2/5) η from CMB posterior included as theory uncertainty in CMB likelihood. Y_p from BBN has no impact on CMB.
   (3/5) BBN likelihood includes theory uncertainty at each parameter point, by scanning 3 rates over 1σ range.

4. **Yeh+ 2011.13874, 2207.13133**
   (3/5) η and Y_p from CMB posterior included as theory uncertainty in CMB likelihood. But Y_p and η should not be treated independently!
   (4/5) BBN likelihood includes theory uncertainty at each parameter point, by drawing 10,000 MC samples.

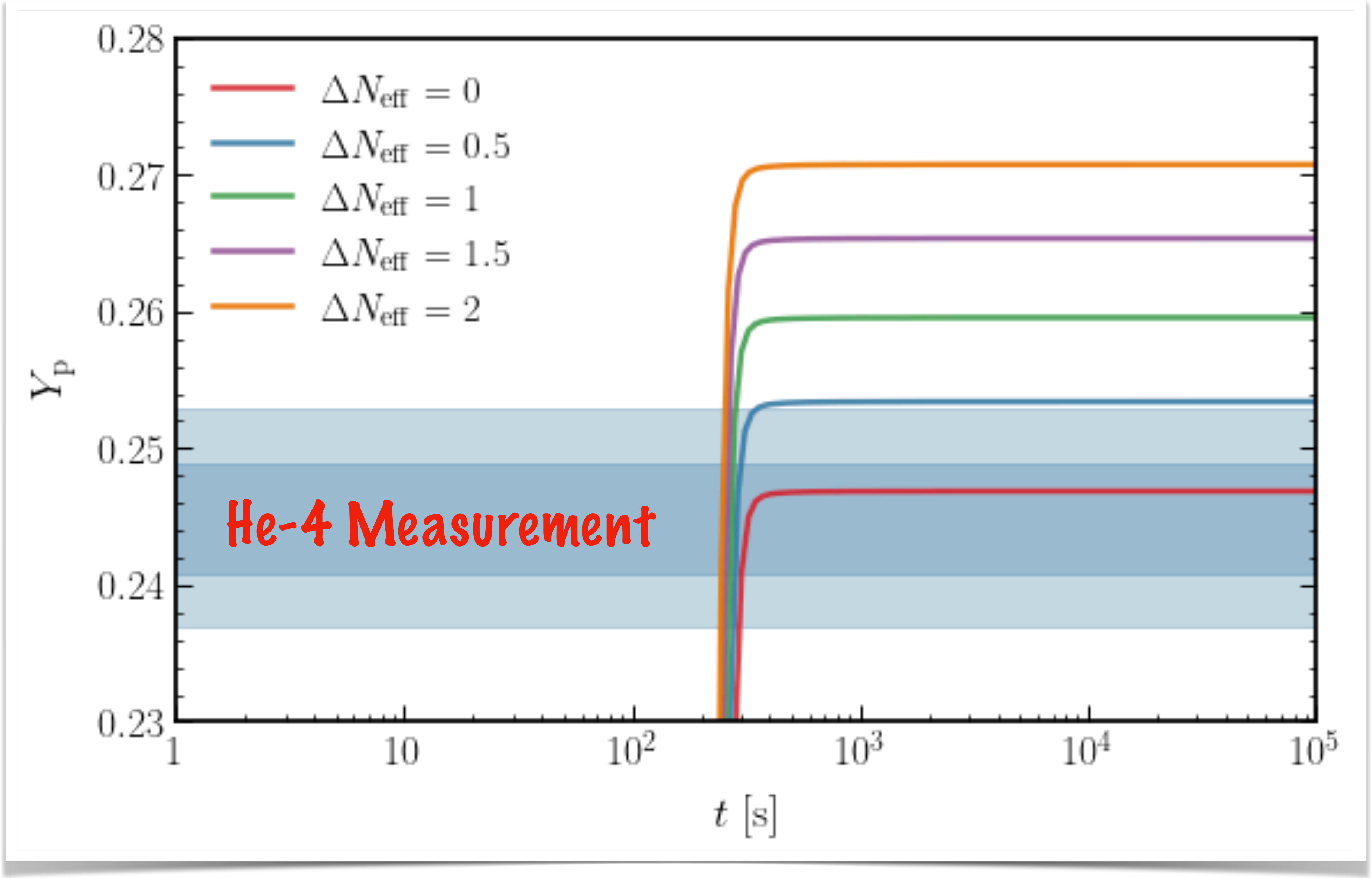
Mukhanov astro-ph/0303073; Cooke, Pettini & Steidel 1710.11129

D/H **Sensitivity to** η / $\Omega_b h^2$



Deuterium abundance set by **freezeout** of deuterium-destroying processes, with rates that depend sensitively on η.

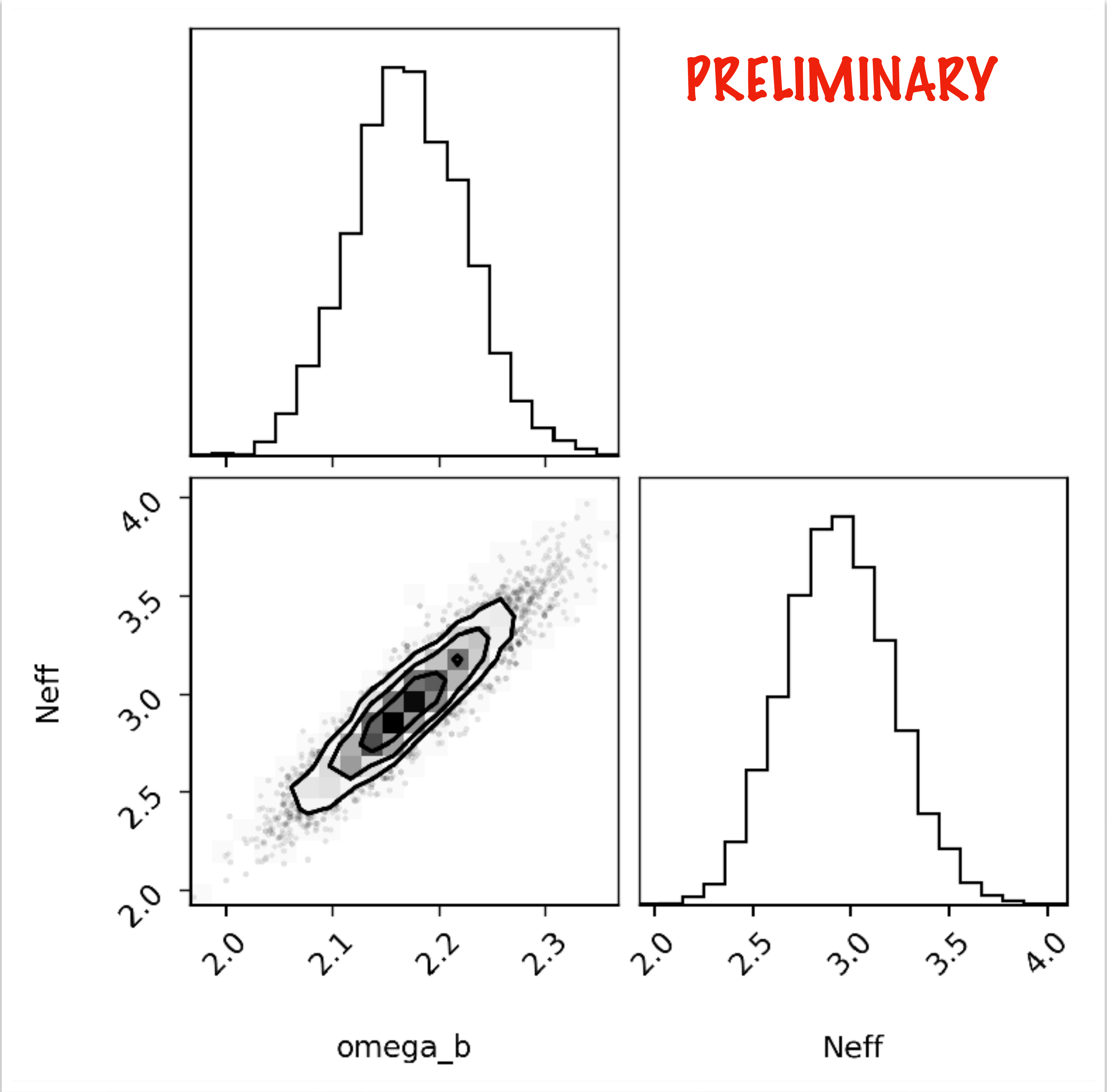
Y_p Sensitivity to ΔN_eff



Helium abundance set by **neutron abundance before deuterium bottleneck**.

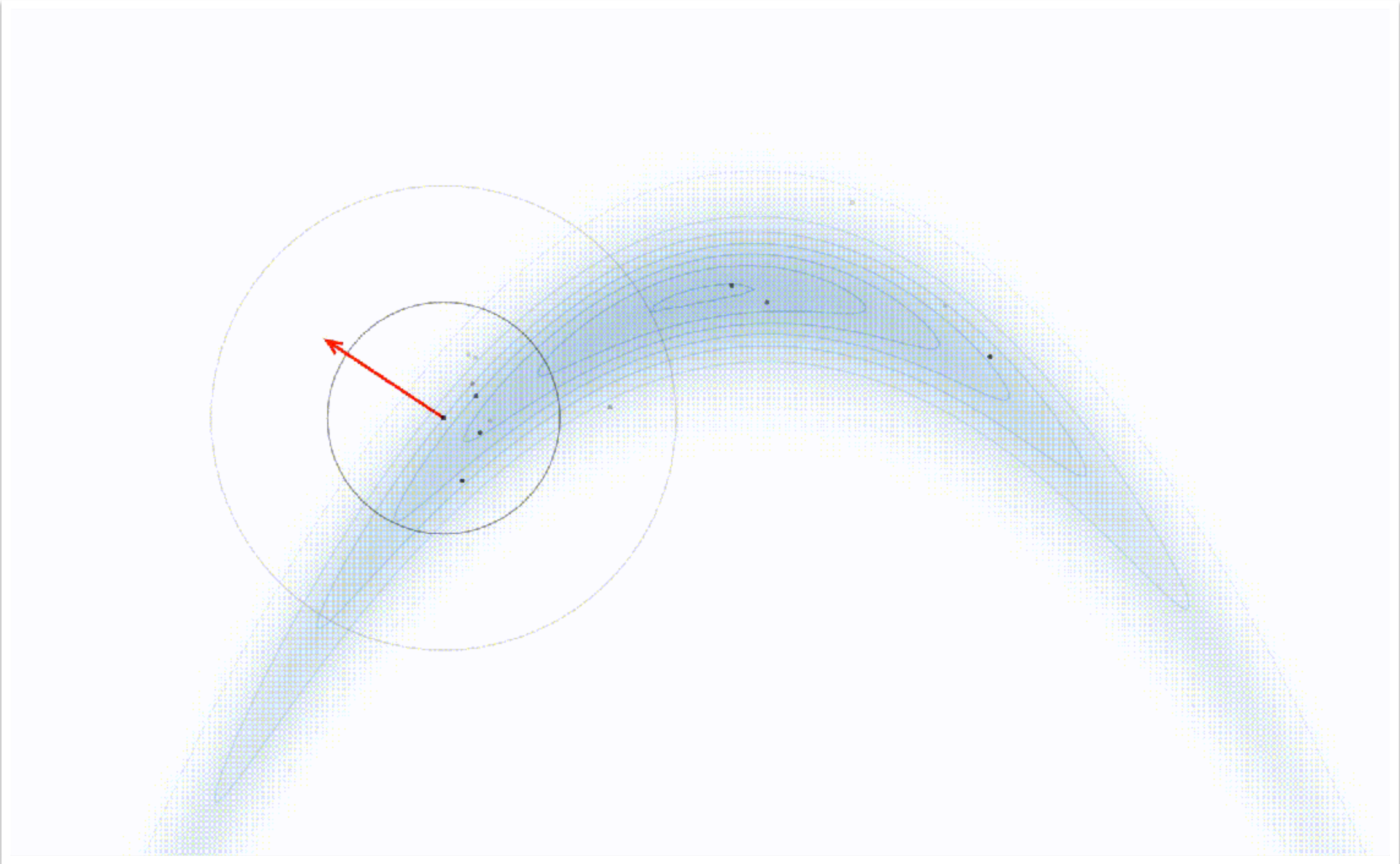
Higher ΔN_eff means faster expansion, **earlier neutron freezeout**.

BBN Only, η and $\Delta N_{\rm eff}$



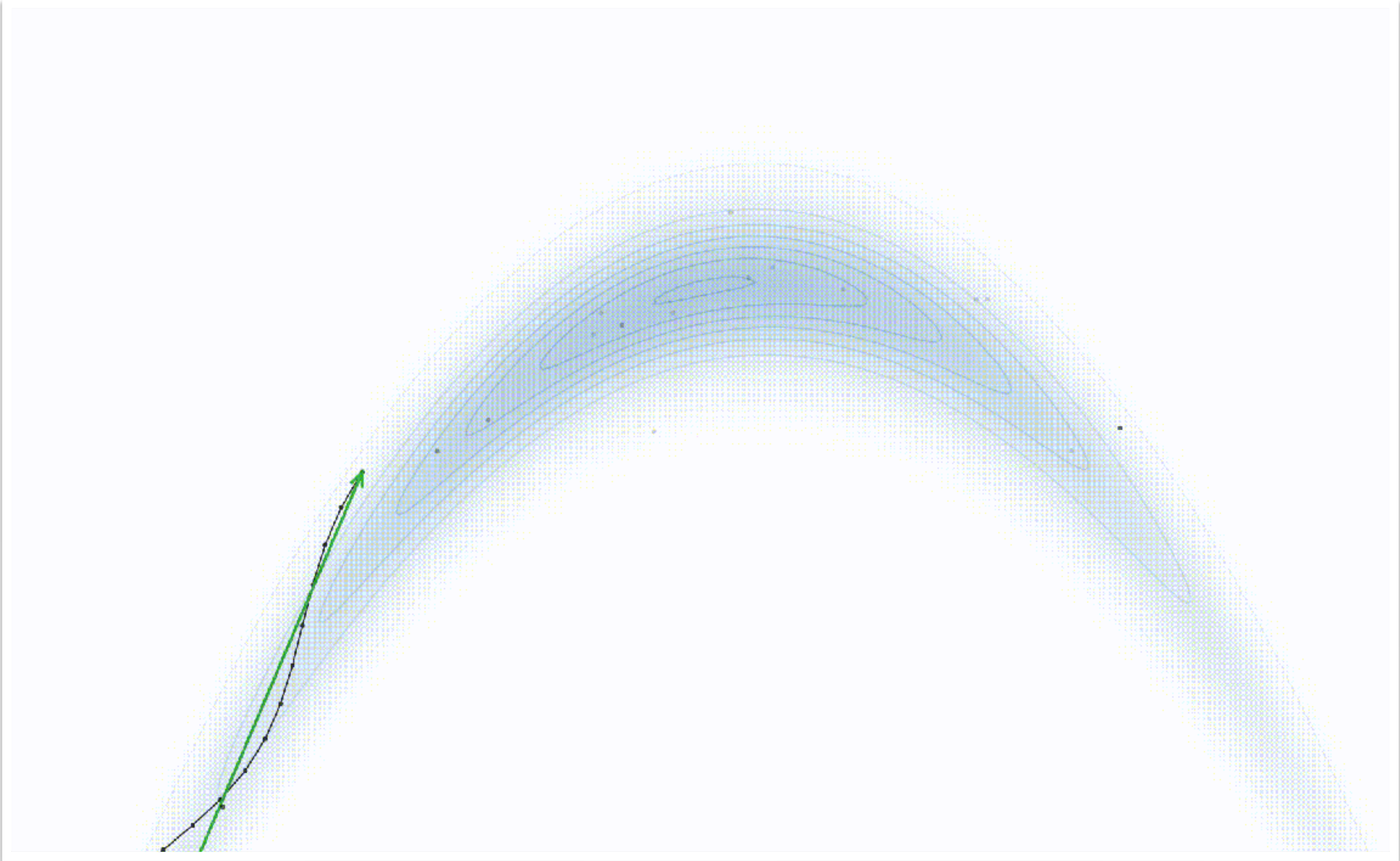
Analysis	$100\Omega_b h^2$	$N_{\rm eff}$
LINX, PRIMAT Rates	2.172 ± 0.054 (Preliminary)	2.94 ± 0.28 (Preliminary)
PRyMordial, PRIMAT Rates (Schöneberg)	2.172 ± 0.055	2.92 ± 0.28
LINX, NACREII Rates	2.217 ± 0.076 (Preliminary)	2.96 ± 0.28 (Preliminary)
PRyMordial, NACREII Rates (Schöneberg)	2.212 ± 0.072	2.93 ± 0.27
Yeh+	2.189 ± 0.059	2.89 ± 0.23
CMB, LCDM + $N_{\rm eff}$ + Y_p	2.227 ± 0.022	2.80 ± 0.29

Markov-Chain Monte Carlo (MCMC)



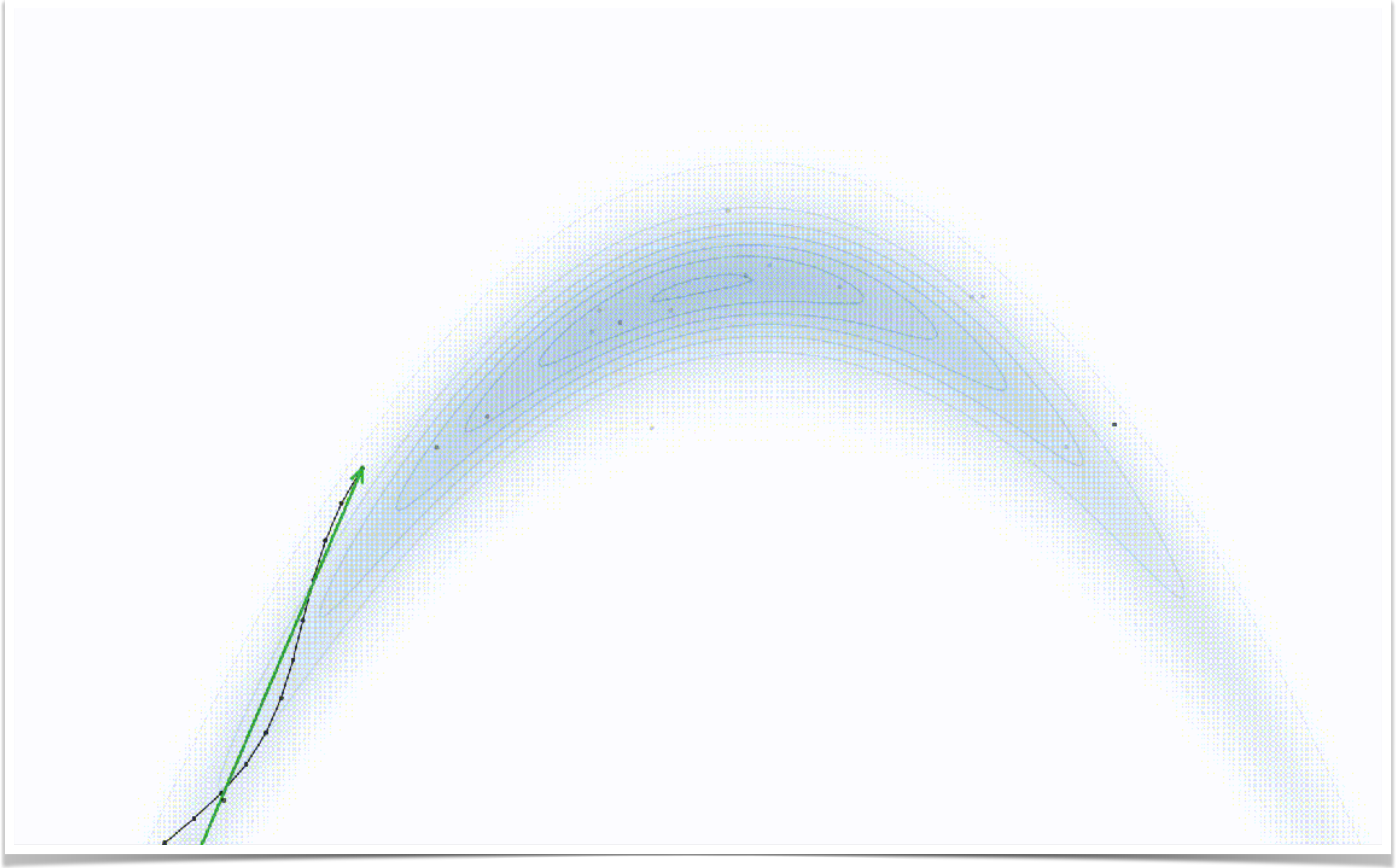
Explore posterior by **random walk**, with acceptance probability of each step being equal to the **ratio of the posteriors** of before and after. **Converges to the posterior**.

Hamiltonian Monte Carlo (HMC)



We can do better than random walk. Points should stay **near the typical set**. Treat the posterior as **potential energy** to enable this!

Hamiltonian Monte Carlo (HMC)



To realize this: we need **derivatives of the likelihood** at every point.