P-Wave Charmonia Production in Exclusive B_c-Meson Decays

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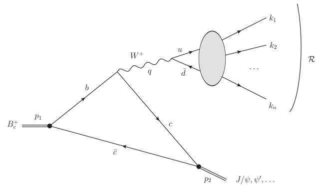
Introduction

- $B_c^+ = (\bar{b}c)$ combines both charmonia and bottomonia properties
- Can be used to
  - study QCD in confinment and assymtotoc freedom regimes
  - check existing models for heavy quarkonia description
- $B_c \to (\bar{c}c) + \mathcal{R}$ decays can be studied using the factorization model
- Good results for vector charmonia:
  - $B_c \to \psi^{(')} + e\nu, \pi, \rho, 3\pi, \ldots$
- What about P-wave states
  - $B_c \to \chi_{cJ} + e\nu, \pi, \rho, 3\pi, \ldots$

Content

Factorization

Diagram and amplitude factorise:



$$\mathcal{M}\left(B_c \to (\bar{c}c) + \mathcal{R}\right) = H^\mu \epsilon_\mu^{(\mathcal{R})}$$

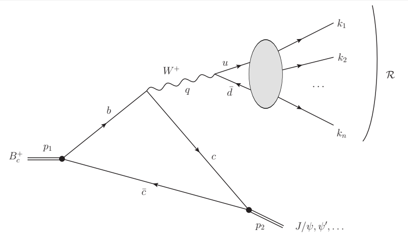
where

- H^μ is the $B_c \to (\bar{c}c)W$ transition vertex
  - Potential model, QCD sum rules, etc
- $\epsilon_\mu^{(\mathcal{R})}$ is effective $W \to \mathcal{R}$ polarization vector
  - ChPt, phonomenologr, resonance approximation, etc

The differential width is

$$\frac{d\Gamma}{dq^2} \sim \frac{d\Gamma_T}{dq^2}\rho_T\left(q^2\right) + \frac{d\Gamma_L}{dq^2}\rho_L\left(q^2\right)$$

$B_c \to \chi_{cJ} + \mathcal{R}$



$$\mathcal{M}\left(B_c \to \chi_{cJ} + \mathcal{R}\right) = H^{\mu} \epsilon_{\mu}^{(\mathcal{R})}$$

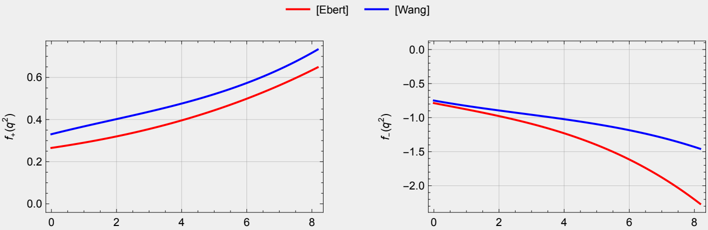
Form-factors of the $B_c \to \chi_{cJ} W$ transitions were considered in

- D. Ebert, R. N. Faustov, V. O. Galkin, PRD 82 (2010) 034019
- Zhi-hui Wang et al, J. Phys. G, 39 (2012) 015009
- E. Hernandez et al, PRD 74 (2006) 074008
- etc.

I will use these form factors, compare published branching fractions and consider some other decay modes

χ_{c0}, Form Factors

$$H_\mu = f_+\left(q^2\right)\left(p_1 + p_2\right)_\mu + f_-\left(q^2\right)\left(p_1 - p_2\right)_\mu$$



$f_-(q^2)$ gives no contribution to most of the decays
$f_+(q^2)$ are proportional to each other

$B_c \to \chi_{c0}\pi$

[Ebert]:

Paper	:	$\mathrm{Br} = 0.021\%$
this	:	$\mathrm{Br} = 0.022\%$
ratio	:	1.034

[Wang]:

Paper	:	$\mathrm{Br} = 0.031 \pm 0.004\%$
this	:	$\mathrm{Br} = 0.035\%$
ratio	:	1.127 ± 0.145

Good agreement

$B_c \to \chi_{c0} e\nu$

		[Ebert]
Paper	:	$\mathrm{Br} = 0.087\%$
this	:	$\mathrm{Br} = 0.089\%$
ratio	:	1.021

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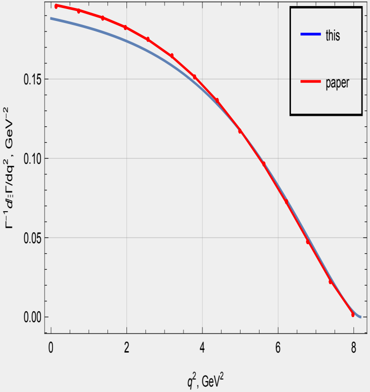

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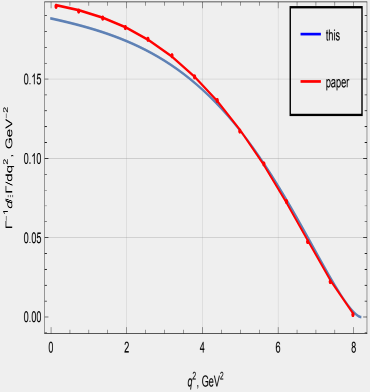

[Wang]		
Paper	:	$\mathrm{Br} = 0.13 \pm 0.03\%$
this	:	$\mathrm{Br} = 0.138\%$
ratio	:	1.058 ± 0.244

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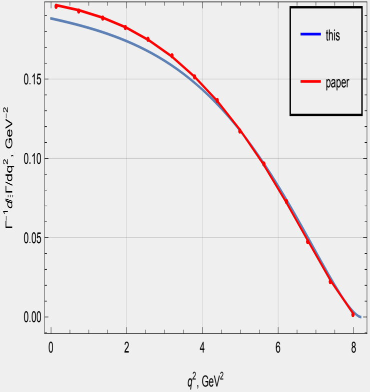

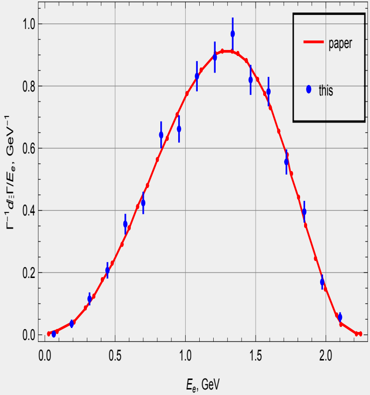

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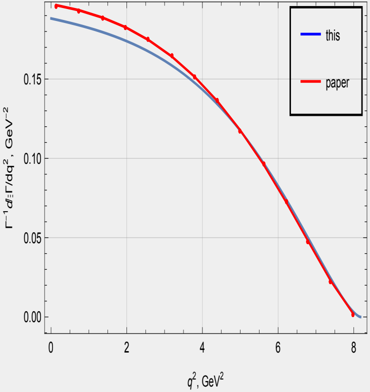

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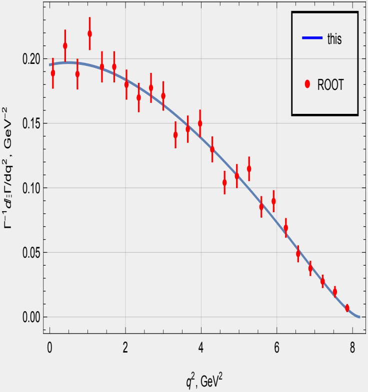
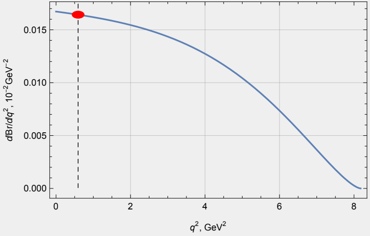


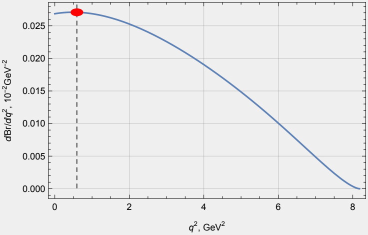
$B_c \to \chi_{c0}\rho$

$$\mathrm{Br}\left[B_c \to \chi_{c0}\rho\right] = 6\pi^2 f_\rho^2 \left. \frac{d\mathrm{Br}\left(B_c \to \chi_{c0} + e\nu\right)}{dq^2} \right|_{q^2 = m_\rho^2}$$

Ebert



Wang

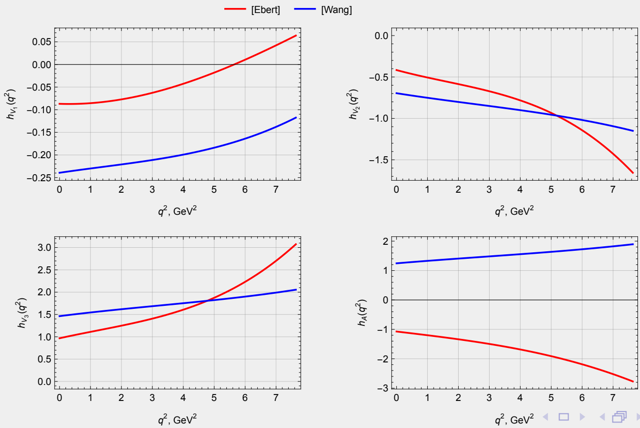


Paper	:	$\mathrm{Br} = 0.058\%$
this	:	$\mathrm{Br} = 0.055\%$
ratio	:	0.943

Paper	:	$\mathrm{Br} = 0.076 \pm 0.009\%$
this	:	$\mathrm{Br} = 0.09\%$
ratio	:	1.186 ± 0.14

χ_{c1}, Form Factors

$$H_\mu = \frac{2ih_A(q^2)}{M_1+M_2} e_{\mu\nu\rho\sigma}\epsilon^\nu p_1^\rho p_2^\sigma + (M_1+M_2)h_{V_1}(q^2)\epsilon_\mu + \left[h_{V_2}(q^2)p_{1\mu} + h_{V_3}(q^2)p_{2\mu}\right]\frac{\epsilon q}{M_1}$$



$B_c \to \chi_{c1}\pi$

[Ebert]:

Paper	:	$\mathrm{Br} = 0.02\%$
this	:	$\mathrm{Br} = 0.001\%$
ratio	:	0.032

[Wang]:

Paper	:	$\mathrm{Br} = 0.002 \pm 0.\%$
this	:	$\mathrm{Br} = 0.003\%$
ratio	:	1.339 ± 0.128

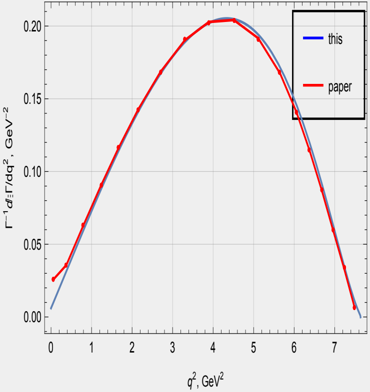
$B_c \to \chi_{c1} e \nu$

		[Ebert]
Paper	:	$\mathrm{Br} = 0.082\%$
this	:	$\mathrm{Br} = 0.082\%$
ratio	:	1.002

$B_c \to \chi_{c1} e\nu$

[Ebert]

Paper	:	$\mathrm{Br} = 0.082\%$
this	:	$\mathrm{Br} = 0.082\%$
ratio	:	1.002



$B_c \to \chi_{c1} e\nu$

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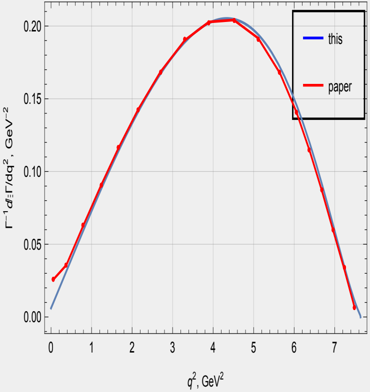
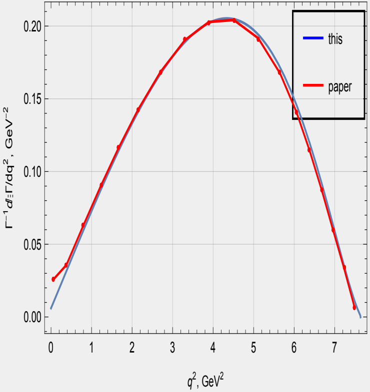
Paper	:	$\mathrm{Br} = 0.082\%$
this	:	$\mathrm{Br} = 0.082\%$
ratio	:	1.002

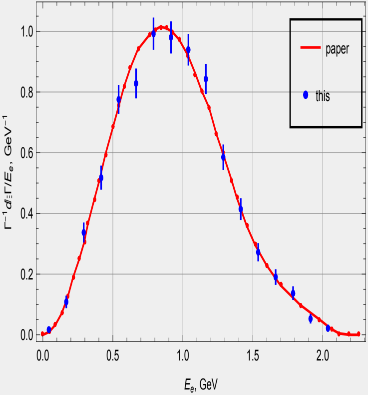


[Wang]		
Paper	:	$\mathrm{Br} = 0.11 \pm 0.03\%$
this	:	$\mathrm{Br} = 0.115\%$
ratio	:	1.042 ± 0.284

$B_c \to \chi_{c1} e\nu$

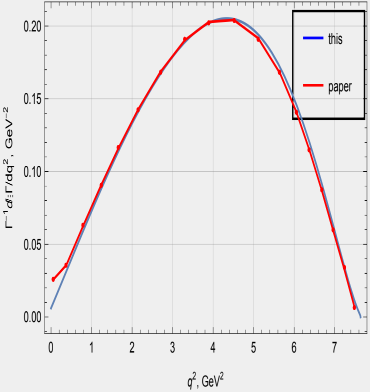
	[Ebert]			[Wang]
Paper	:	Br $= 0.082\%$	Paper	:	Br $= 0.11 \pm 0.03\%$
this	:	Br $= 0.082\%$	this	:	Br $= 0.115\%$
ratio	:	1.002	ratio	:	1.042 ± 0.284





$B_c \to \chi_{c1} e\nu$

[Ebert]		
Paper	:	Br = 0.082%
this	:	Br = 0.082%
ratio	:	1.002



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Paper	:	Br = 0.11 ± 0.03%
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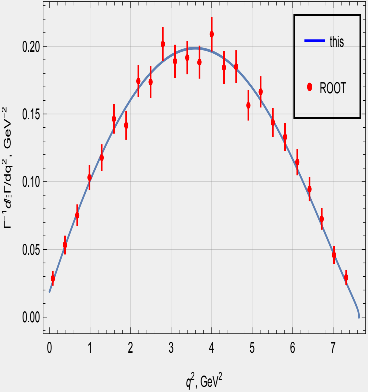
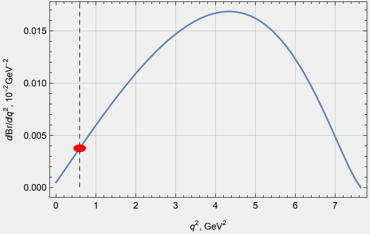


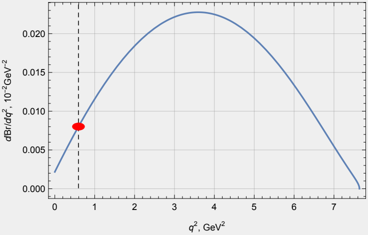
$B_c \to \chi_{c1}\rho$

$$\mathrm{Br}\left[B_c \to \chi_{c1}\rho\right] = 6\pi^2 f_\rho^2 \left.\frac{d\mathrm{Br}\left(B_c \to \chi_{c1} + e\nu\right)}{dq^2}\right|_{q^2=m_\rho^2}$$

Ebert



Wang

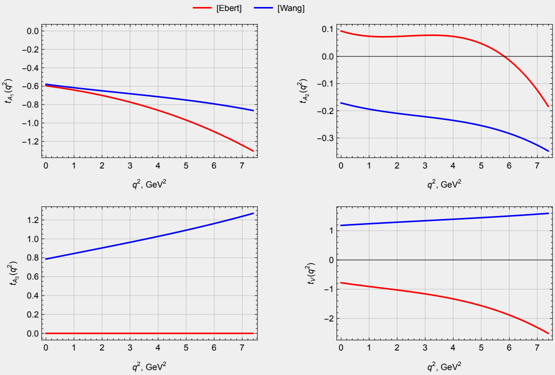


Paper	:	$\mathrm{Br} = 0.015\%$
this	:	$\mathrm{Br} = 0.013\%$
ratio	:	0.845

Paper	:	$\mathrm{Br} = 0.023 \pm 0.002\%$
this	:	$\mathrm{Br} = 0.027\%$
ratio	:	1.171 ± 0.102

χ_{c2}, Form Factors

$$H_\mu = \frac{2 i t_V(q^2)}{M_1 + M_2} \epsilon^{\mu\nu\rho\sigma} \epsilon^*_{\nu\alpha} \frac{p_1^\alpha}{M_1} p_1 p_1 + (M_1 + M_2) t_{A_1}(q^2) \epsilon^{*\mu\alpha} \frac{p_{1\alpha}}{M_1} +$$
$$[t_{A_2}(q^2) p_1^\mu + t_{A_3}(q^2) p_2^\mu] \epsilon^*_{\alpha\beta} \frac{p_1^\alpha p_1^\beta}{M_1^2},$$



$B_c \to \chi_{c2}\pi$

[Ebert]:

Paper	:	$\mathrm{Br} = 0.038\%$
this	:	$\mathrm{Br} = 0.041\%$
ratio	:	1.078

[Wang]:

Paper	:	$\mathrm{Br} = 0.021 \pm 0.005\%$
this	:	$\mathrm{Br} = 0.024\%$
ratio	:	1.136 ± 0.27

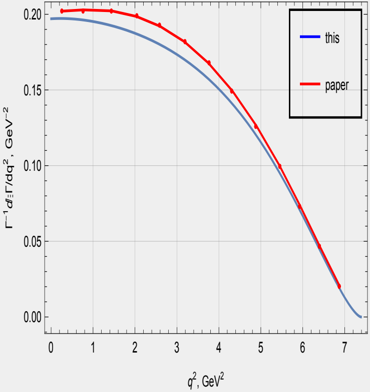
$B_c \to \chi_{c2} e\nu$

		[Ebert]
Paper	:	$\mathrm{Br} = 0.16\%$
this	:	$\mathrm{Br} = 0.16\%$
ratio	:	1.003

$B_c \to \chi_{c2} e\nu$

[Ebert]

Paper	:	$\mathrm{Br} = 0.16\%$
this	:	$\mathrm{Br} = 0.16\%$
ratio	:	1.003



$B_c \to \chi_{c2} e\nu$

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Paper	:	$\mathrm{Br} = 0.16\%$
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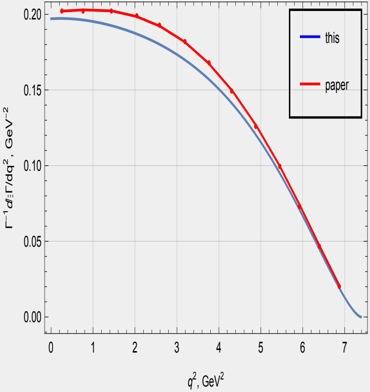
ratio	:	1.003



[Wang]		
Paper	:	$\mathrm{Br} = 0.1 \pm 0.03\%$
this	:	$\mathrm{Br} = 0.097\%$
ratio	:	0.971 ± 0.291

$B_c \to \chi_{c2} e \nu$

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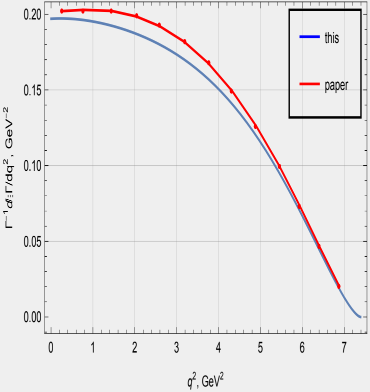
Paper	:	$\mathrm{Br} = 0.16\%$
this	:	$\mathrm{Br} = 0.16\%$
ratio	:	1.003



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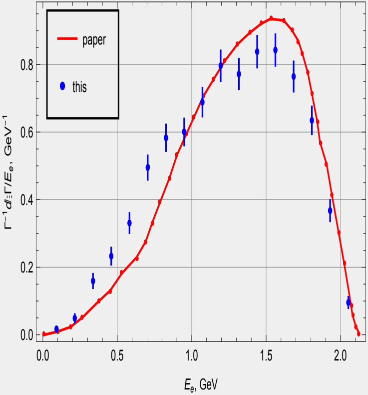
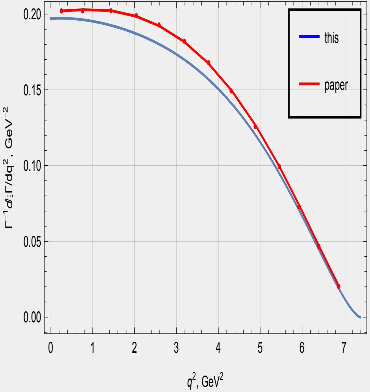
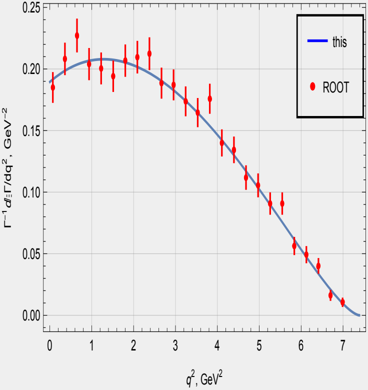


$B_c \to \chi_{c2} e\nu$

	[Ebert]			[Wang]
Paper	:	$\mathrm{Br} = 0.16\%$	Paper	:	$\mathrm{Br} = 0.1 \pm 0.03\%$
this	:	$\mathrm{Br} = 0.16\%$	this	:	$\mathrm{Br} = 0.097\%$
ratio	:	1.003	ratio	:	0.971 ± 0.291

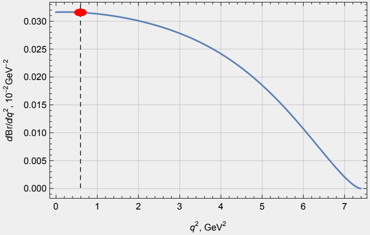




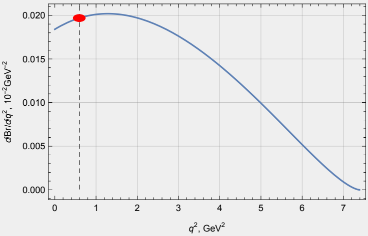
$B_c \to \chi_{c2}\rho$

$$\mathrm{Br}\left[B_c \to \chi_{c2}\rho\right] = 6\pi^2 f_\rho^2 \left.\frac{d\mathrm{Br}\left(B_c \to \chi_{c2} + e\nu\right)}{dq^2}\right|_{q^2=m_\rho^2}$$

Ebert



Wang



Paper	:	$\mathrm{Br} = 0.11\%$
this	:	$\mathrm{Br} = 0.105\%$
ratio	:	0.955

Paper	:	$\mathrm{Br} = 0.056 \pm 0.011\%$
this	:	$\mathrm{Br} = 0.066\%$
ratio	:	1.171 ± 0.23

Results

Original branchng fractions:

	χ_{c0} [Ebert]	χ_{c0} [Wang]	χ_{c1} [Ebert]	χ_{c1} [Wang]	χ_{c2} [Ebert]	χ_{c2} [Wang]
$e\nu$	0.087	0.13 ± 0.03	0.082	0.11 ± 0.03	0.16	0.1 ± 0.03
π	0.021	0.031 ± 0.004	0.02	0.0021 ± 0.0002	0.038	0.021 ± 0.005
ρ	0.058	0.076 ± 0.009	0.015	0.023 ± 0.002	0.11	0.056 ± 0.011

Predicitons for branching fractions of the considered decays

	χ_{c0} [Ebert]	χ_{c0} [Wang]	χ_{c1} [Ebert]	χ_{c1} [Wang]	χ_{c2} [Ebert]	χ_{c2} [Wang]
$e\nu$	0.0889	0.138	0.0822	0.115	0.16	0.0971
π	0.0217	0.0349	0.000644	0.00281	0.041	0.0238
ρ	0.0547	0.0901	0.0127	0.0269	0.105	0.0656

Ratios (this/paper)

	χ_{c0} [Ebert]	χ_{c0} [Wang]	χ_{c1} [Ebert]	χ_{c1} [Wang]	χ_{c2} [Ebert]	χ_{c2} [Wang]
$e\nu$	1.02	1.06 ± 0.244	1.	1.04 ± 0.284	1.	0.971 ± 0.291
π	1.03	1.13 ± 0.145	0.0322	1.34 ± 0.128	1.08	1.14 ± 0.27
ρ	0.943	1.19 ± 0.14	0.845	1.17 ± 0.102	0.955	1.17 ± 0.23

$W \to \mathcal{R}$

$W \to \mathcal{R}$ amplitude can be calculated using

- ChPt
- resonance approximation
- Phenomenology
- etc

Spectral functions are defined as

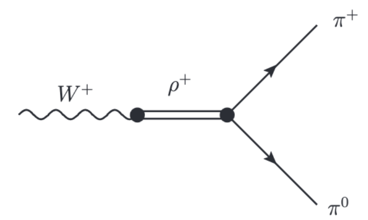
$$\int \frac{d\Phi(q \to k_1 \dots k_n)}{2\pi} \epsilon_\mu \epsilon^*_\nu = (q_\mu q_\nu - q^2 g_{\mu\nu})\rho_T(q^2) + q_\mu q_\nu \rho_L(q^2)$$

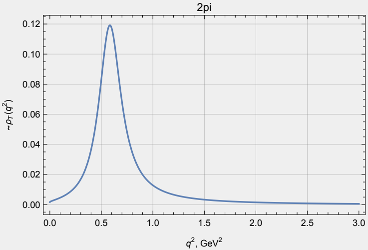
Checked comaring with

- $e^+e^- \to \mathcal{R}$
- $\tau \to \nu_\tau \mathcal{R}$
- etc

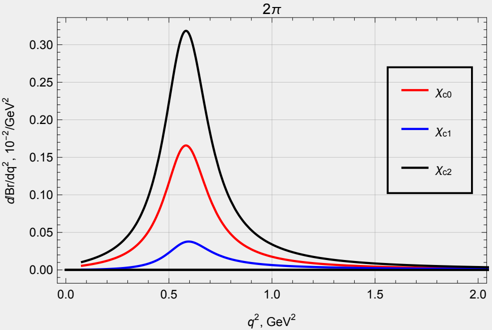
$W \to 2\pi$

A.V. Berezhnoy, A.K. Likhoded, A.V. Luchinsky,
1104.0808 [hep-ph]





$B_c \to \chi_{cJ} + 2\pi$

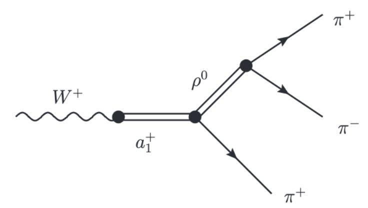


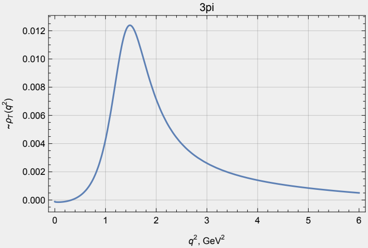
$B_c \to \chi_{cJ} + 2\pi$ decay branching fractions (in 10^{-2})

	[Ebert]	[Wang]
χ_{c0}	0.057	0.094
χ_{c1}	0.015	0.031
χ_{c2}	0.11	0.068

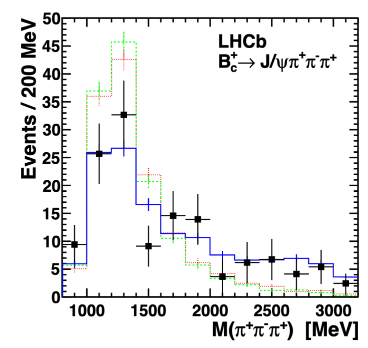
$W \to 3\pi$

A.V. Berezhnoy, A.K. Likhoded, A.V. Luchinsky, 1104.0808 [hep-ph]

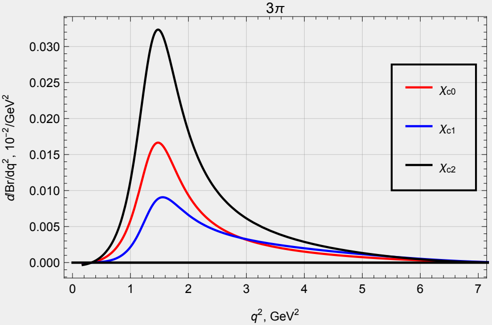




Agrees with CERN arXiv 1204.0079 [hep-ex]



$B_c \to \chi_{cJ} + 3\pi$

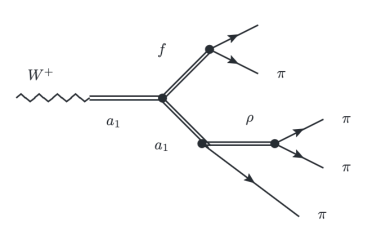


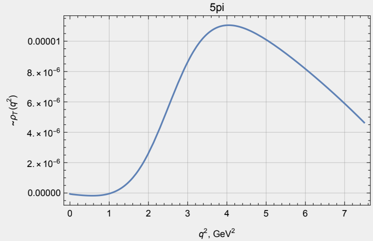
$B_c \to \chi_{cJ} + 3\pi$ decay branching fractions (in 10^{-2})

	[Ebert]	[Wang]
χ_{c0}	0.021	0.034
χ_{c1}	0.016	0.025
χ_{c2}	0.041	0.026

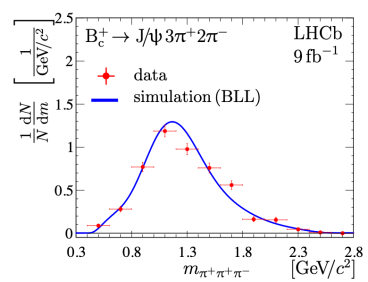
$W \to 5\pi$

A.V. Luchinsky,
PRD 86 (2012) 074024

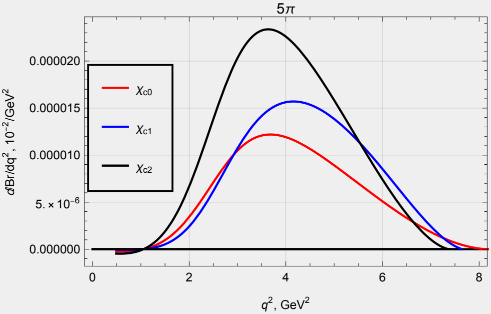




Agrees with CERN [2208.08660 [hep-ex]]



$B_c \to \chi_{cJ} + 5\pi$



$B_c \to \chi_{cJ} + 5\pi$ decay branching fractions (in 10^{-6})

	[Ebert]	[Wang]
χ_{c0}	0.38	0.56
χ_{c1}	0.49	0.63
χ_{c2}	0.67	0.39

EvtGen

All described dacays modes are added to BC_VHAD model

- Typical decay file:

```
Decay B_c+
  1.000 chi_c0 pi+ pi+ pi+ pi- pi- BC_VHAD 1;
Enddecay
```

- All usual for model out states: $e\nu$, 2π, 3π, 5π, ...
- Particles' order is determined automatically

rrF: read root File

Useful tool that helps you to debug EvtGen models

- Analyzing ROOT file produced by **simpleEvtGenRO**
- Extract any histograms without writing any code
- Lots of output variables (m, m^2, p_T, p_T^2, etc)
- Simple cuts
- Different output format (ROOT, csv)

Example: $\tau \to e\nu_e\nu_\tau$

```
./simpleEvtGenRO.exe tau- ../src/tau_enu.dec 100000
./rrF.exe -i evtOutput.root -o out.root -s -v m2_23 -v e_1
```

Available at GitHub

- `https://github.com/alexey-luchinsky/rrF.git`

Also with python interface

Conclusion

- Considering $B_c \to \chi_{cJ} + \mathcal{R}$ decays
- Factorization model, spectral function approach
- Form-factor sets from
  - D. Ebert, R. N. Faustov, V. O. Galkin, PRD 82 (2010) 034019
  - Zhi-hui Wang et al, J. Phys. G, 39 (2012) 015009
- Resonable agreement with papers' results, with one exception

  Ebert : $B_c \to \chi_{c1}\pi$
- New results for $R = 2\pi, 3\pi, 5\pi$
- New decays added to EvtGen model (BC_VHAD)

Thank you for your attention