Topical Meeting on Bc decays

Bc->J/Ψ M decays in the iPQCD formalism

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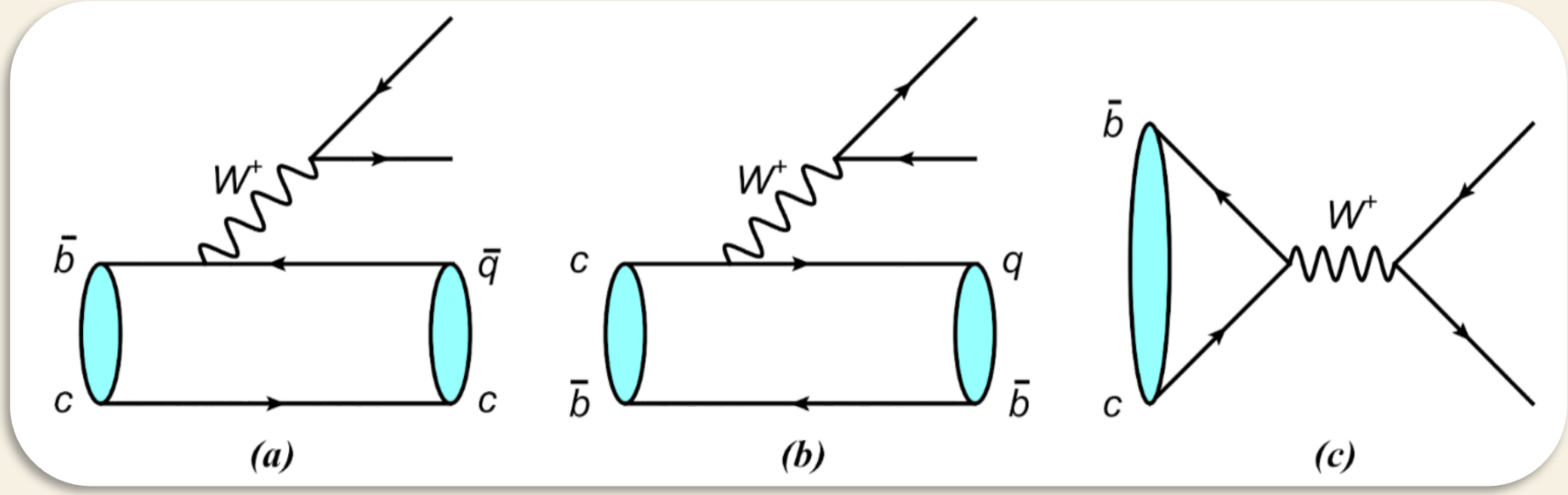
INTRODUCTION and MOTIVATION

- A Bc meson is different from the typical heavy-light B meson and heavy-heavy quarkonia. The Bc-meson decays contain rich heavy quark dynamics in perturbative and non-perturbative regimes.



- From the experimental side,
  - The first discovery of Bc meson at Tevatron in 1998 proclaimed the beginning of its experimental studies.
  - And, the running of LHC since 2009 promoted the measurements on Bc-meson decays to further understand the heavy flavor physics.
  - The LHC experiments have measured many multi-body nonleptonic Bc-meson decays through relative ratios of the branching fractions between the related Bc decays.
  - Some interesting results have been reported. Typically,

See Pereima's talk

☑ Bc->J/\psi K

$$R_{K/\pi}^{\rm Exp} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi K^+)}{\mathcal{B}(B_c^+ \to J/\psi \pi^+)} = 0.069 \pm 0.020 \ ,$$

LHCb arXiv:1306.6723

$$R_{K/\pi}^{\prime\,\rm Exp} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi K^+)}{\mathcal{B}(B_c^+ \to J/\psi \pi^+)} = 0.079 \pm 0.008 \ .$$

LHCb arXiv:1607.06823

☑ Bc->J/\psi π+π-π+ Exhibits an *a*1 peak

$$R^{\text{Exp}}_{3\pi/\pi} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi\pi^+\pi^-\pi^+)}{\mathcal{B}(B_c^+ \to J/\psi\pi^+)} = 2.41 \pm 0.45 \,,$$ LHCb arXiv:1204.0079

$$R'^{\,\text{Exp}}_{3\pi/\pi} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi\pi^+\pi^-\pi^+)}{\mathcal{B}(B_c^+ \to J/\psi\pi^+)} = 2.55 \pm 0.87 \,,$$ CMS arXiv:1410.5729

$$R^{\text{Avg}}_{3\pi/\pi} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi\pi^+\pi^-\pi^+)}{\mathcal{B}(B_c^+ \to J/\psi\pi^+)} = 2.45 \pm 0.40 \,,$$ HFLAV arXiv:2206.07501

☑ Bc->J/\psi K+K-π+

$$R^{\text{Exp}}_{2K\pi/\pi} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi K^+K^-\pi^+)}{\mathcal{B}(B_c^+ \to J/\psi\pi^+)} = 0.53 \pm 0.11 \,,$$ LHCb arXiv:1309.0587

☑ Bc->J/\psi K+K-π+ and Bc->J/\psi π+π-π+

$$R^{\text{Exp}}_{2K\pi/3\pi} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi K^+K^-\pi^+)}{\mathcal{B}(B_c^+ \to J/\psi\pi^+\pi^-\pi^+)} = 0.185 \pm 0.014 \,.$$ LHCb arXiv:2111.03001

Mode	[32]	[33]	[34]	[35]	[36]	[37]	[38]	[39]	[40]	[41]	[21]	[42]	[29]	[43]	[44]	Data
$R_{K/\pi}$	0.077	0.076	0.052	0.074	0.049	0.082	0.076	0.079	0.088	–	0.075	0.079	0.082	$0.076^{+0.015}_{-0.015}$	$0.075^{+0.005}_{-0.005}$	$0.079^{+0.008}_{-0.008}$
$R_{K^*/\rho}$	0.054	0.054	0.054	0.057	0.038	0.063	0.057	0.058	0.050	–	0.053	–	0.057	0.059	0.056	–
$R_{\rho/\pi}$	3.01	3.22	2.85	2.85	19.31	2.62	2.88	3.16	–	5.29	2.77	–	3.31	3.52	5.65	–
$R_{K^*/\pi}$	0.16	0.17	0.15	0.16	0.73	0.16	0.16	0.18	–	0.26	0.15	0.16	0.19	0.21	0.32	–
$R_{a_1/\pi}$	–	–	4.0	–	–	–	–	–	–	–	5.5	–	–	–	–	–

- From the theoretical side,
  - Partial decay modes such as Bc->J/\psi P, J/\psi V and J/\psi a1 have been investigated mainly based on naïve factorization in many different models, but the predictions of the individual branching fractions vary in a wide range.
  - Most of the relative ratios between the branching fractions of these mentioned Bc-meson decays and Bc->J/\psi \pi are basically consistent with each other within errors, despite their individual decay rates with large discrepancies.

- For the Bc->J/\psi P decays for example,

$$R_{K/\pi} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi K^+)}{\mathcal{B}(B_c^+ \to J/\psi \pi^+)} \simeq \frac{|V_{us}|^2}{|V_{ud}|^2} \cdot \frac{f_K^2}{f_\pi^2} \sim 0.081 \,,$$

- However, for the decays with suppressed or vanished factorizable emission amplitudes while with enhanced nonfactorizable ones such as Bc->J/\psi S, Bc->J/\psi T, and so on, we should go beyond naïve factorization to explore the rich but complicated dynamics within the factorization framework based on QCD.

- The applications of conventional PQCD approach to the Bc-meson decays have been reviewed shortly in 2014.

Xiao and Liu, arXiv:1401.0151

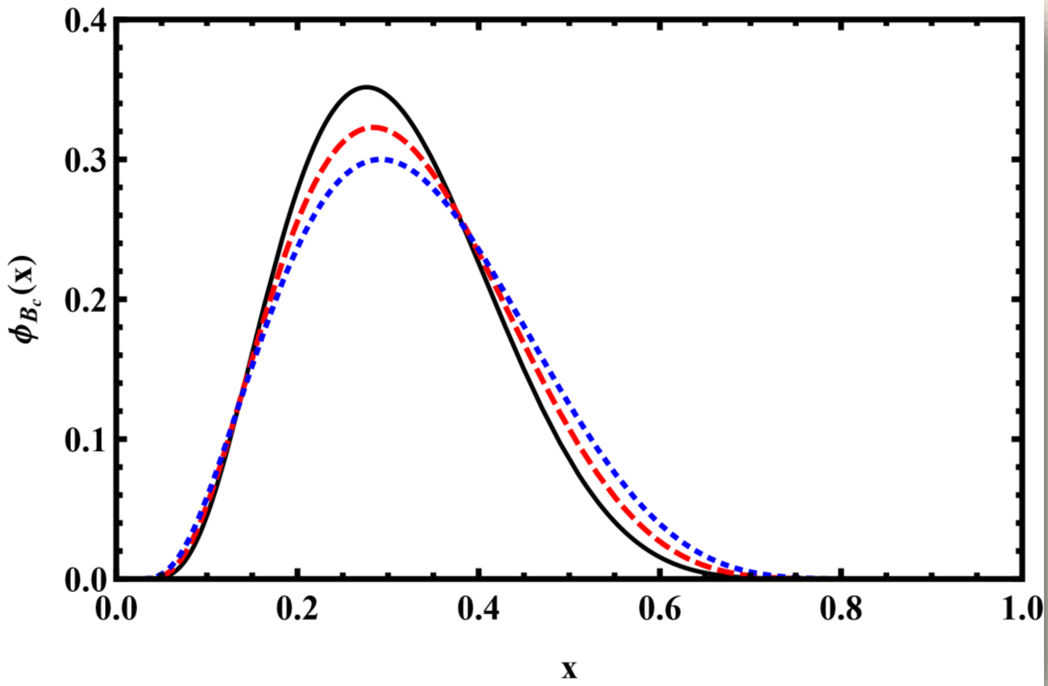
- But, the finite charm quark mass makes the Bc meson more complicated than the typical B mesons. Two important ingredients are demanded to further improve the PQCD formalism:

☑ appropriate Bc-meson distribution amplitude to highly suppress the strong phase in the Bc to J/\psi transition form factor;

Liu, Li, and Xiao, arXiv:1801.06145

$$\phi_{B_c}(x,b) = \frac{f_{B_c}}{2\sqrt{2N_c}} N_{B_c} x(1-x) \exp\left[-\frac{(1-x)m_c^2 + x m_b^2}{8\beta_{B_c}^2 x(1-x)}\right] \exp\left[-2\beta_{B_c}^2 x(1-x) b^2\right],$$

Figure 2 Behavior of Bc-meson distribution amplitude on the shape parameter: black-solid, red-dashed and blue-dotted lines correspond to β_{Bc} = 0.9, 1.0, 1.1, respectively.



☑ new Sudakov factor for Bc-meson decays derived by resumming the term ln(mb/mc), in addition to the ordinary one ln(mb/Λ_{QCD}).

Liu, Li, and Xiao, arXiv:2006.12786

Liu, arXiv:2305.00713

$$S_c(Q,b) = \frac{a_1}{2\beta_1}\left\{\hat{Q}\ln\hat{Q} - \hat{c}\ln\hat{c} - (\hat{Q}-\hat{c})(1+\ln\hat{b})\right\} + \frac{a_2}{4\beta_1^2}\left\{-\ln\frac{\hat{Q}}{\hat{c}} + \frac{\hat{Q}-\hat{c}}{\hat{b}}\right\} + \frac{a_1}{4\beta_1}\ln\frac{\hat{Q}}{\hat{c}}\ln\frac{e^{2\gamma_E-1}}{2}$$

$$\hat{Q} \equiv \ln[xP^+/\Lambda],\ \hat{c} \equiv \ln[m_c/(\sqrt{2}\Lambda)],\ \text{and}\ \hat{b} \equiv \ln[1/(b\Lambda)],$$

Notice that, when the replacement \hat{c} -> \hat{b} is adopted, then the formula will recover the Sudakov factor for B-meson decays with one-loop running coupling at next-to-leading-logarithm accuracy.

- Hence, the improved PQCD (iPQCD) formalism is more self-consistent at leading order and is also ready for the decays B and Bc to charmonia plus light mesons.

Perturbative Calculations

◆ The weak effective Hamiltonian could be written as,

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} \left\{ V_{cb}^* V_{uq} [C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu)] \right\} + {\rm h.c.}\ ,$$

with local four-quark operators

$$O_1 = \bar{q}_\alpha \gamma_\mu (1-\gamma_5) u_\beta \, \bar{c}_\beta \gamma_\mu (1-\gamma_5) b_\alpha\ , \qquad O_2 = \bar{q}_\alpha \gamma_\mu (1-\gamma_5) u_\alpha \, \bar{c}_\beta \gamma_\mu (1-\gamma_5) b_\beta\ ;$$

◆ The standard procedures in the PQCD approach are utilized in the Bc->J/\psi M decays

$$A(B_c^+ \to J/\psi M^+) \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \cdot {\rm Tr}\left[C(t) \Phi_{B_c}(x_1, b_1) \Phi_M(x_2, b_2) \Phi_{J/\psi}(x_3, b_3) H(x_i, b_i, t) e^{-S(t)} \right]\ ,$$

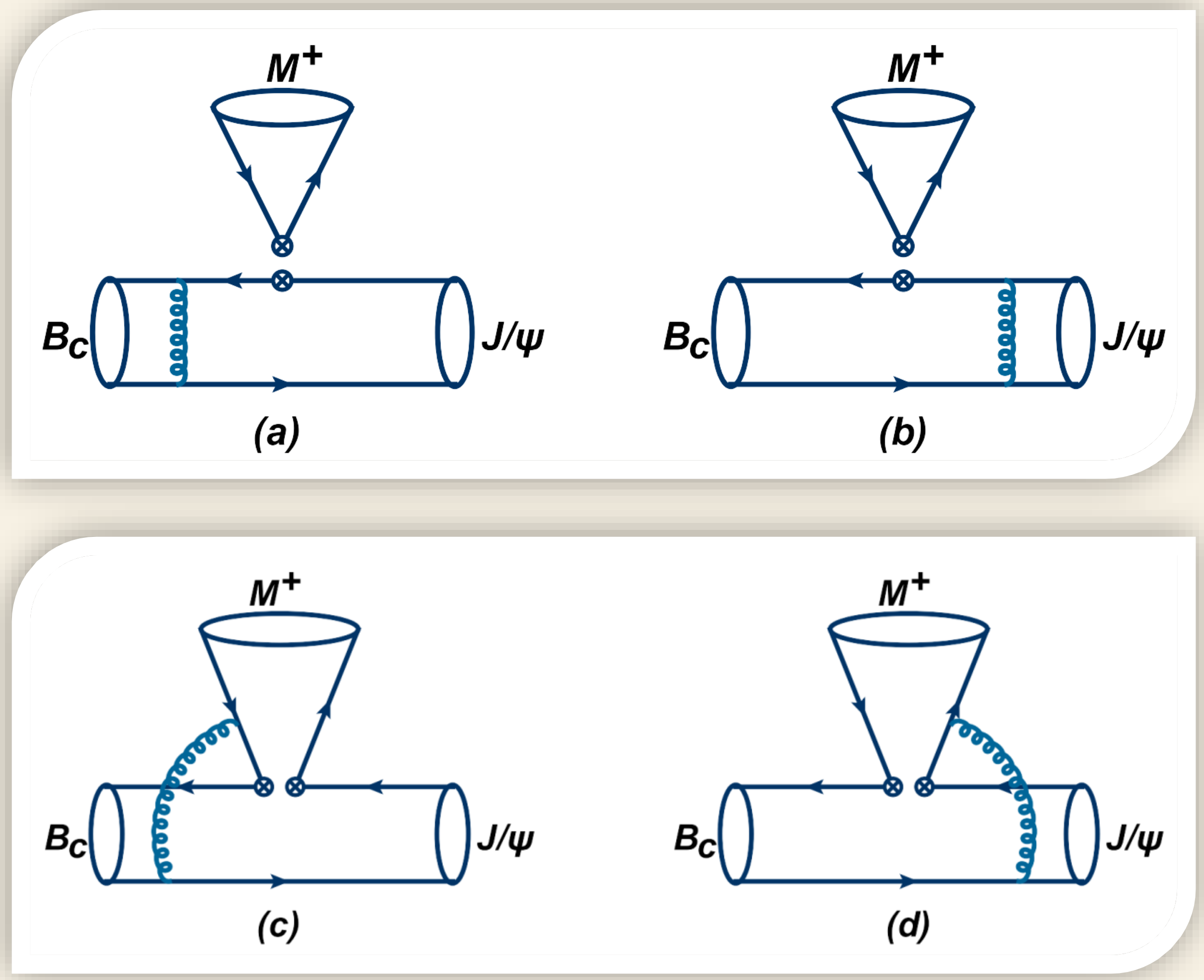


Figure 3 Feynman diagrams for Bc-> J/\psi M at leading order in the iPQCD formalism: (a) and (b) are the factorizable emission diagrams, and (c) and (d) are the nonfactorizable emission ones.

The factorization formulas for the decays Bc->J/\psi M could be found in the Appendix B of arXiv:2305.00713 explicitly.

Results and Discussions

A. Bc->J/\psi (P, V, a1)

- Branching fractions for Bc-> J/\psi P,

$$\mathcal{B}(B_c^+ \to J/\psi\pi^+) = 1.17^{+0.31}_{-0.23}(\beta_{B_c})^{+0.08}_{-0.08}(f_M)^{+0.00}_{-0.00}(a_\pi) \times 10^{-3}\,, \quad 1.17^{+0.32}_{-0.24}$$

$$\mathcal{B}(B_c^+ \to J/\psi K^+) = 8.68^{+2.32}_{-1.73}(\beta_{B_c})^{+0.64}_{-0.62}(f_M)^{+0.66}_{-0.63}(a_K) \times 10^{-5}\,, \quad 8.68^{+2.50}_{-1.94}$$

➢ They are consistent generally with those already available in the literature within errors and particularly with those in the QCDF approach. Furthermore, they agree well with those in CCQM and LCSR presented very recently.

$B_c^+ \to J/\psi\pi^+$	$B_c^+ \to J/\psi K^+$
0.101 ± 0.02 %	0.008 ± 0.002 %

$$\mathcal{B}(B_c^+ \to J/\psi\pi^+) = (0.136^{+0.002}_{-0.002})\%,$$

$$\mathcal{B}(B_c^+ \to J/\psi K^+) = (0.010^{+0.000}_{-0.000})\%,$$

Issadykov and Ivanov, arXiv:1804.00472

Cheng et al, arXiv:2107.08405

◆ Relative ratio,

$$R^{\text{Theo}}_{K/\pi} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi K^+)}{\mathcal{B}(B_c^+ \to J/\psi \pi^+)} = 0.074^{+0.006}_{-0.005} ,$$

- Within errors, it is in good consistency with the latest measurement, and also with the naïve expectation and the very recent predictions in CCQM and LCSR.
- Dramatical cancelation occurs in the nonfactorizable emission diagrams under isospin limit.
- Slight deviation for $R_{K/\pi}$ between the iPQCD prediction and the naïve expectation is attributed to the SU(3)-flavor symmetry breaking effects appearing in the nonfactorizable emission amplitudes proportional to kaon DA at twist-2.
- As a byproduct, the above naïve expectation for the relative ratios between the branching fractions of Bc to other charmonia plus a kaon and a pion could also be anticipated.

◆ Branching fractions for Bc-> J/\psi V,

$$\mathcal{B}(B_c^+ \to J/\psi\rho^+) = 3.69^{+1.02}_{-0.76}(\beta_{B_c})^{+0.28}_{-0.27}(f_M)^{+0.01}_{-0.00}(a_\rho) \times 10^{-3}\,, \quad 3.69^{+1.06}_{-0.81}$$

$$\mathcal{B}(B_c^+ \to J/\psi K^{*+}) = 2.23^{+0.62}_{-0.46}(\beta_{B_c})^{+0.20}_{-0.18}(f_M)^{+0.02}_{-0.01}(a_{K^*}) \times 10^{-4}\,, \quad 2.23^{+0.65}_{-0.49}$$

➢ Evident discrepancies appear in lots of predictions with different approaches and models. The iPQCD predictions agree well with those in CCQM.

$$\begin{aligned} \mathrm{Br}_{LC}(B_c \to J/\psi\rho) &= 0.38\%, \\ \mathrm{Br}_{QM}(B_c \to J/\psi\rho) &= 0.44\%, \\ \mathrm{Br}_{SR}(B_c \to J/\psi\rho) &= 0.48\%. \end{aligned}$$

Likhoded and Luchinsky , arXiv:0910.3089

$B_c^+ \to J/\psi\,\rho^+$	$B_c^+ \to J/\psi\,K^{*+}$
0.334 ± 0.067 %	0.019 ± 0.004 %

Issadykov and Ivanov, arXiv:1804.00472

◆ Relative ratio,

$$R^{\mathrm{Theo}}_{K^*/\rho} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi K^{*+})}{\mathcal{B}(B_c^+ \to J/\psi\rho^+)} = 0.060^{+0.002}_{-0.002}\,,$$

➢ It roughly agrees with the value 0.066 anticipated by naïve factorization and is indeed close to the similar one in Bc->J/\psi P.

$$R^{\rm Theo}_{\rho/\pi} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi\rho^+)}{\mathcal{B}(B_c^+ \to J/\psi\pi^+)} = 3.15^{+0.09}_{-0.10} , \qquad R^{\rm Theo}_{K^*/\pi} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi K^{*+})}{\mathcal{B}(B_c^+ \to J/\psi\pi^+)} = 0.19^{+0.01}_{-0.01} .$$

- They agree basically with those available in the literature within errors and are expected to be tested in the future measurements.

◆ Branching fraction for Bc-> J/\psi a1,

$$\mathcal{B}(B_c^+ \to J/\psi a_1^+) = 5.90^{+1.63}_{-1.24}(\beta_{B_c})^{+0.66}_{-0.64}(f_M)^{+0.00}_{-0.00}(a_{a_1}) \times 10^{-3} , \; 5.90^{+1.76}_{-1.40}$$

- The similar QCD behavior with ρ meson and a bit larger decay constant fa1 lead to this larger branching fraction.

- By assuming the validity of narrow-width approximation and with the equal branching ratios of a1+->π+π-π+ and a1+->π+π0π0 around 50%,

arXiv:0708.0050

$$\mathcal{B}(B_c^+ \to J/\psi\pi^+\pi^-\pi^+)_{\rm iPQCD} \equiv \mathcal{B}(B_c^+ \to J/\psi a_1^+) \cdot \mathcal{B}(a_1^+ \to \pi^+\pi^-\pi^+) = 2.95^{+0.88}_{-0.70} \times 10^{-3} ,$$

- It is surprisingly well consistent with the predictions in the literature within errors.

$$\begin{aligned} \mathrm{Br}_{LC}\,(B_c \to J/\psi + 3\pi) &= 0.52\%, \\ \mathrm{Br}_{QM}\,(B_c \to J/\psi + 3\pi) &= 0.64\%, \\ \mathrm{Br}_{SR}\,(B_c \to J/\psi + 3\pi) &= 0.77\%. \end{aligned}$$

Likhoded and Luchinsky, arXiv:0910.3089

$$Br(B_c^+ \to J/\psi + \pi^+\pi^-\pi^+) \simeq 0.3\%.$$

Rakitin and Koshkarev, arXiv:0911.3287

$B_c \to J/\psi + (3\pi)_{\rm ch}$	0.39 %	0.45 %	0.14 %

Luchinsky, arXiv:1208.1398

- Relative ratio,

$$R^{\rm Theo}_{3\pi/\pi} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi\pi^+\pi^-\pi^+)_{\rm iPQCD}}{\mathcal{B}(B_c^+ \to J/\psi\pi^+)_{\rm iPQCD}} = 2.52^{+0.05}_{-0.10}\,,$$

- It is well consistent with the measurements by LHCb and CMS, and the averaged value by HFLAV within errors.

- Additionally,

$$\mathcal{B}(a_1^+ \to K^+K^-\pi^+)_{\rm iPQCD} \equiv R^{\rm Exp}_{2K\pi/\pi} \cdot \frac{\mathcal{B}(B_c^+ \to J/\psi\pi^+)}{\mathcal{B}(B_c^+ \to J/\psi a_1^+)} \approx (10.5^{+2.0}_{-1.9})\%\,,$$

- The detection on this value would help understand the nature of a1.

$$\mathcal{B}(B_c^+ \to J/\psi K^+ K^- \pi^+)_{\mathrm{iPQCD}} \equiv R^{\mathrm{Exp}}_{2K\pi/3\pi} \cdot \mathcal{B}(B_c^+ \to J/\psi \pi^+ \pi^- \pi^+)_{\mathrm{iPQCD}} = 5.46^{+1.68}_{-1.40} \times 10^{-4} \, .$$

- It is roughly consistent with the predictions given in different form factors within a bit large errors.

0.081% and 0.03% Luchinsky, arXiv:1307.0953

$$R^{\mathrm{Theo}}_{a_1/\pi} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi a_1^+)}{\mathcal{B}(B_c^+ \to J/\psi \pi^+)} = 5.04^{+0.10}_{-0.15} \, ,$$

$$R^{\mathrm{Theo}}_{a_1/\rho} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi a_1^+)}{\mathcal{B}(B_c^+ \to J/\psi \rho^+)} = 1.60^{+0.10}_{-0.11} \, ,$$

- Tests on these two ratios could help further understand the QCD behavior of a1.
- But, without individual decay rates, the F.E.A. dominated decays cannot help reveal the dynamics and constrain the shape parameter β_{Bc}.
- The explorations on the modes governed by large nonfactorizable decay amplitudes are of great necessity.

B. Bc->J/\psi (b1, S)

- Branching fraction for Bc-> J/\psi b1,

$$\mathcal{B}(B_c^+ \to J/\psi b_1^+) = 7.93^{+2.43}_{-1.83}(\beta_{B_c})^{+0.93}_{-0.89}(f_M)^{+3.09}_{-2.59}(a_{b_1}) \times 10^{-4} \, . \; 7.93^{+4.04}_{-3.29}$$

  - The anti-symmetric QCD behavior of b1-meson DA at twist-2 and the nearly vanished decay constant fb1+ result in a smaller branching fraction of Bc->J/\psi b1 than that of Bc->J/\psi a1.

- Relative ratio,

$$R^{\rm Theo}_{b_1/\pi} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi b_1^+)}{\mathcal{B}(B_c^+ \to J/\psi \pi^+)} = 0.68^{+0.26}_{-0.22} \, ,$$

  - Its experimental measurements could provide useful hints to test the reliability of the adopted iPQCD formalism and phenomenologically constrain β_{Bc}.

$$R^{\text{Theo}}_{b_1/\rho} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi b_1^+)}{\mathcal{B}(B_c^+ \to J/\psi \rho^+)} = 0.21^{+0.09}_{-0.07}\,.$$

- ➢ Tests on this ratio could provide information on the QCD behavior of b1 meson.

- ◆ Different from Bc->J/\psi b1 with angular decomposition, the Bc-> J/\psi S decays have only longitudinal contributions due to conservation of the angular momentum.

- ◆ The factorizable emission amplitudes are strongly suppressed due to the highly small vector decay constants, while the asymmetric QCD behavior of twist-2 DAs leads to large nonfactorizable emission contributions.

- ◆ The light scalars are considered as two-quark structure mesons. Two possible scenarios:
  - ➢ In S1, a0 and κ are ground states, then a0(1450) and K0*(1430) are the first excited states;
  - ➢ In S2, a0(1450) and K0*(1430) are ground states, then a0 and κ are the four-quark states.

- Branching fractions for Bc-> J/\psi a0 and J/\psi κ,

$$\mathcal{B}(B_c^+ \to J/\psi a_0^+) = 5.98^{+1.77}_{-1.39}(\beta_{B_c})^{+0.80}_{-0.78}(f_M)^{+1.31}_{-1.22}(B_i) \times 10^{-4}\,, \quad 5.98^{+2.34}_{-2.01}$$

$$\mathcal{B}(B_c^+ \to J/\psi \kappa^+) = 1.31^{+0.50}_{-0.36}(\beta_{B_c})^{+0.18}_{-0.18}(f_M)^{+0.41}_{-0.35}(B_i) \times 10^{-5}\,, \quad 1.31^{+0.67}_{-0.53}$$

with relative ratios,

$$R^{\rm Theo}_{a_0/\pi} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi a_0^+)}{\mathcal{B}(B_c^+ \to J/\psi \pi^+)} = 0.51^{+0.13}_{-0.12}\,, \qquad R^{\rm Theo}_{\kappa/\pi} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi \kappa^+)}{\mathcal{B}(B_c^+ \to J/\psi \pi^+)} = 0.011^{+0.005}_{-0.004}\,,$$

  - The future measurements on these results could help identify the q\bar q components in a0 and κ.

- Branching fractions for Bc-> J/\psi a0(1450) and J/\psi K0*(1430),

$$\mathcal{B}(B_c^+ \to J/\psi a_0'^+) = \begin{cases} 6.39^{+2.09}_{-1.56}(\beta_{B_c})^{+1.52}_{-1.37}(f_M)^{+2.19}_{-1.89}(B_i) \times 10^{-4} \\ 2.20^{+0.71}_{-0.54}(\beta_{B_c})^{+0.54}_{-0.48}(f_M)^{+1.37}_{-1.02}(B_i) \times 10^{-4} \end{cases}, \quad \begin{matrix} 6.39^{+3.39}_{-2.81} \\ 2.20^{+1.63}_{-1.25} \end{matrix}$$

$$\mathcal{B}(B_c^+ \to J/\psi K_0^{*+}) = \begin{cases} 3.22^{+0.81}_{-0.66}(\beta_{B_c})^{+0.74}_{-0.64}(f_M)^{+0.37}_{-0.35}(B_i) \times 10^{-5} \\ 2.23^{+0.87}_{-0.70}(\beta_{B_c})^{+0.55}_{-0.51}(f_M)^{+0.65}_{-0.45}(B_i) \times 10^{-5} \end{cases}, \quad \begin{matrix} 3.22^{+1.16}_{-0.98} \\ 2.23^{+1.22}_{-0.98} \end{matrix}$$

with relative ratios,

$$R^{\rm Theo}_{a_0'/\pi} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi a_0'^{\,+})}{\mathcal{B}(B_c^+ \to J/\psi \pi^+)} = \begin{cases} 0.55^{+0.19}_{-0.17} \\ 0.19^{+0.12}_{-0.09} \end{cases}, \quad R^{\rm Theo}_{K_0^*/\pi} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi K_0^{*+})}{\mathcal{B}(B_c^+ \to J/\psi \pi^+)} = \begin{cases} 0.028^{+0.003}_{-0.003} \\ 0.019^{+0.006}_{-0.005} \end{cases}$$

- The experimental measurements on these values could test the reliability of the adopted iPQCD formalism, and further help constrain the shape parameter β_{Bc}.

$$R^{\rm Theo}_{\kappa/a_0} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi \kappa^+)}{\mathcal{B}(B_c^+ \to J/\psi a_0^+)} = 0.022^{+0.002}_{-0.002}, \quad R^{\rm Theo}_{K_0^*/a_0'} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi K_0^{*+})}{\mathcal{B}(B_c^+ \to J/\psi a_0'^{\,+})} = \begin{cases} 0.050^{+0.014}_{-0.008} \\ 0.101^{+0.050}_{-0.022} \end{cases}$$

- The future examinations on these ratios could help understand the QCD dynamics of these light scalars, and further identify the preferred scenario.
- The theoretical errors are remarkably large due to much less constraints experimentally. Nevertheless, the Bc->J/\psi S decays can still provide good chances to understand the involved dynamics because of the limit of naïve factorization in such kinds of decays.

C. Bc->J/\psi T

- The tensors cannot be produced via V, A, S, and P currents. Therefore, the factorizable contributions with T-emission in Bc->J/\psi T are forbidden intuitively.

- The branching fractions of Bc->J/\psi T are contributed purely by the nonfactorizable emission amplitudes. It means that Bc->J/\psi T must be explored theoretically going beyond naïve factorization.

- Branching fractions for Bc-> J/\psi T,

$$\mathcal{B}(B_c^+ \to J/\psi a_2^+) = 1.39^{+0.45}_{-0.33}(\beta_{B_c})^{+0.19}_{-0.18}(f_M) \times 10^{-4} \,, \quad 1.39^{+0.49}_{-0.38}$$

$$\mathcal{B}(B_c^+ \to J/\psi K_2^{*+}) = 9.05^{+2.91}_{-2.22}(\beta_{B_c})^{+1.03}_{-0.98}(f_M) \times 10^{-6} \,, \quad 9.05^{+3.09}_{-2.43}$$

- By assuming the equal branching ratios of a2+->π+π-π+ and a2+->π+π0π0 and the validity of narrow-width approximation, with $\mathcal{B}$(a2->3 π) =(70.1\pm 2.7)%, an interesting branching fraction associated with relative ratio could be obtained as,

$$\mathcal{B}(B_c^+ \to J/\psi a_2^+(\to \pi^+\pi^-\pi^+)) \equiv \mathcal{B}(B_c^+ \to J/\psi a_2^+) \cdot \mathcal{B}(a_2^+ \to \pi^+\pi^-\pi^+) = 0.49^{+0.17}_{-0.14} \times 10^{-4} ,$$

$$R^{a_2}_{3\pi/\pi} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi a_2^+(\to \pi^+\pi^-\pi^+))}{\mathcal{B}(B_c^+ \to J/\psi \pi^+)} = 0.042^{+0.002}_{-0.004} .$$

- These two values await future examinations to support the iPQCD formalism in dealing with these types of Bc-meson decays.

◆ Relative ratios,

$$R^{\rm Theo}_{a_2/\pi} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi a_2^+)}{\mathcal{B}(B_c^+ \to J/\psi \pi^+)} = 0.12^{+0.01}_{-0.01} , \qquad R^{\rm Theo}_{K_2^*/\pi} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi K_2^{*+})}{\mathcal{B}(B_c^+ \to J/\psi \pi^+)} = 0.008^{+0.000}_{-0.001} ,$$

- These ratios can help explore the Bc->J/\psi T decays at LHC, even CEPC in the future.

◆ One more interesting relative ratio,

$$R^{\text{Theo}}_{K_2^*/a_2} \equiv \frac{\mathcal{B}(B_c^+ \to J/\psi K_2^{*+})}{\mathcal{B}(B_c^+ \to J/\psi a_2^+)} = 0.065^{+0.002}_{-0.002}\,.$$

- It could be cleanly written as (f_{K2^*}/f_{a2})^2*|V_{us}/V_{ud}|^2~0.066 due to SU(3)-flavor symmetry in the T-meson leading-twist distribution amplitudes.
- It agrees perfectly with naïve expectation $R_{K^*/\rho}$ while is slightly larger than that in iPQCD, because of SU(3) symmetry breaking effects in nF.E.A destructively interfering with F.E.A in the Bc->J/\psi K* mode.

Summary and Outlook

1. The relative ratios $R_{K/\pi}$, $R_{3\pi/\pi}$ and so on are highly consistent with data and most predictions, though no available individual decay rates experimentally. The iPQCD predictions await future tests at relevant experiments.

2. The branching fractions and their relative ratios of Bc->J/\psi S, J/\psi T and J/\psi $b1$, governed by nF.E.A, are predicted in iPQCD for the first time and will be confronted with future tests. The precise measurements could help constrain β_{Bc} to further understand (non)perturbative dynamics.

3. The model-independent ratio $R_{K2^*/a2}$ is obtained in iPQCD and is expected to shed light on the information of β_{Bc} promisingly.

4. Suggest our experimental colleagues to focus on the Bc-meson decays predominated by nF.E.A. that could help examine the reliability of iPQCD.

Outlook

- To precisely determine β_{Bc} , one-loop QCD corrections to the Bc-meson wave function, correlated with productions and decays of the Bc meson is urgently demanded;
- To precisely study Bc decays to charm mesons, a new Sudakov factor is demanded due to the charm quark mass effects;
- To extend the iPQCD formalism to baryonic Bc-meson decays, new Sudakov factors for baryon are important to make the framework self-consistent;
- ……

Thanks for your attention!

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Analytic Calculations

$$F_e(S) = -8\pi C_F m_{B_c}^4 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_c}(x_1,b_1)(r_3^2-1)$$
$$\times\left\{\left[r_3(r_b+2x_3-2)\phi^t_{J/\psi}(x_3)-(2r_b+x_3-1)\phi^L_{J/\psi}(x_3)\right]h_a(x_1,x_3,b_1,b_3)E_f(t_a)\right.$$
$$\left.+\left[r_3^2(x_1-1)-r_c\right]\phi^L_{J/\psi}(x_3)h_b(x_1,x_3,b_1,b_3)E_f(t_b)\right\},$$

$$M_e(S) = \frac{32}{\sqrt{6}}\pi C_F m_{B_c}^4 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_c}(x_1,b_1)\phi_S(x_2)(r_3^2-1)$$
$$\times\left\{\left[(r_3^2-1)(x_1+x_2-1)\phi^L_{J/\psi}(x_3)+r_3(x_3-x_1)\phi^t_{J/\psi}(x_3)\right]E_f(t_c)\right.$$
$$\times h_c(x_1,x_2,x_3,b_1,b_2)+\left[(2x_1-(x_2+x_3)+r_3^2(x_2-x_3))\phi^L_{J/\psi}(x_3)\right.$$
$$\left.\left.+r_3(x_3-x_1)\phi^t_{J/\psi}(x_3)\right]h_d(x_1,x_2,x_3,b_1,b_2)E_f(t_d)\right\}.$$

$$F_e^L(V) = 8\pi C_F m_{B_c}^4 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_c}(x_1, b_1)\sqrt{1-r_3^2}$$
$$\times\Big\{ \left[r_3(r_b + 2x_3 - 2)\phi^t_{J/\psi}(x_3) - (2r_b + x_3 - 1)\phi^L_{J/\psi}(x_3) \right] h_a(x_1, x_3, b_1, b_3)$$
$$\times E_f(t_a) + \left[r_3^2(x_1 - 1) - r_c \right] \phi^L_{J/\psi}(x_3) h_b(x_1, x_2, b_1, b_2) E_f(t_b) \Big\} ,$$

$$F_e^N(V) = 8\pi C_F m_{B_c}^4 r_2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_c}(x_1, b_1)$$
$$\times\Big\{ \Big[(r_3^2(r_b + 4x_3 - 2) + r_b - 2)\phi^T_{J/\psi}(x_3) - r_3((4r_b + x_3(1 + r_3^2) - 2))$$
$$\times \phi^v_{J/\psi}(x_3) \Big] h_a(x_1, x_3, b_1, b_3) E_f(t_a) - r_3 \left[r_3^2 + 2r_c - 2x_1 + 1 \right] \phi^v_{J/\psi}(x_3)$$
$$\times h_b(x_1, x_2, b_1, b_2) E_f(t_b) \Big\} ,$$

$$F_e^T(V) = 16\pi C_F m_{B_c}^4 r_2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_c}(x_1, b_1)$$
$$\times\Big\{ \left[(r_b - 2)\phi^T_{J/\psi}(x_3) + r_3 x_3 \phi^v_{J/\psi}(x_3) \right] h_a(x_1, x_3, b_1, b_3) E_f(t_a)$$
$$- r_3 \phi^v_{J/\psi}(x_3) h_b(x_1, x_2, b_1, b_2) E_f(t_b) \Big\} ,$$

$$M_e^L(V) = \frac{32}{\sqrt{6}}\pi C_F m_{B_c}^4 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_c}(x_1,b_1)\phi_V(x_2)\sqrt{1-r_3^2}$$
$$\times\Big\{\Big[(r_3^2-1)(x_1+x_2-1)\phi_{J/\psi}^L(x_3)+r_3(x_3-x_1)\phi_{J/\psi}^t(x_3)\Big]E_f(t_c)$$
$$\times h_c(x_1,x_2,x_3,b_1,b_2)+\Big[(2x_1-(x_2+x_3)+r_3^2(x_2-x_3))\phi_{J/\psi}^L(x_3)$$
$$+r_3(x_3-x_1)\phi_{J/\psi}^t(x_3)\Big]h_d(x_1,x_2,x_3,b_1,b_2)E_f(t_d)\Big\},$$

$$M_e^N(V) = -\frac{32}{\sqrt{6}}\pi C_F m_{B_c}^4 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_c}(x_1,b_1) r_2$$
$$\times\Big\{\big[(r_3^2(x_1-x_2-2x_3+1)+x_1+x_2-1)\phi_V^v(x_2)+(1-r_3^2)^2(x_1+x_2-1)$$
$$\times\phi_V^a(x_2)\big]\phi_{J/\psi}^T(x_3)E_f(t_c)h_c(x_1,x_2,x_3,b_1,b_2)+\big[((r_3^2(x_1+x_2-2x_3)+x_1$$
$$-x_2)\phi_V^v(x_2)+(1-r_3^2)^2(x_1-x_2)\phi_V^a(x_2))\phi_{J/\psi}^T(x_3)+2r_3((x_3-x_2)r_3^2$$
$$+x_2+x_3-2x_1)\phi_V^v(x_2)\phi_{J/\psi}^v(x_3)\big]h_d(x_1,x_2,x_3,b_1,b_2)E_f(t_d)\Big\},$$

$$M_e^T(V) = -\frac{64}{\sqrt{6}}\pi C_F m_{B_c}^4 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_c}(x_1, b_1) r_2$$
$$\times\Big\{ \left[(r_3^2(x_1 - x_2 - 2x_3 + 1) + x_1 + x_2 - 1)\phi_V^a(x_2) + (x_1 + x_2 - 1)\right.$$
$$\times\phi_V^v(x_2)\left.\right] \phi_{J/\psi}^T(x_3) E_f(t_c) h_c(x_1, x_2, x_3, b_1, b_2) + \left[((x_1 - x_2)\phi_V^v(x_2)\right.$$
$$+(x_1(1 + r_3^2)^2 + x_2(r_3^2 - 1) - 2r_3^2 x_3)\phi_V^a(x_2))\phi_{J/\psi}^T(x_3) + 2r_3(x_3(r_3^2 + 1)$$
$$\left. -x_2(r_3^2 - 1) - 2x_1)\phi_V^v(x_2)\phi_{J/\psi}^v(x_3)\right] h_d(x_1, x_2, x_3, b_1, b_2) E_f(t_d)\Big\} .$$

arXiv:0705.2797

- f_M includes f_{Bc} = (0.489 \pm 0.005) GeV and $f_{J/\psi}$ = (0.405 \pm 0.014) GeV. arXiv:0412335

$$
\begin{aligned}
S_c(Q,b) = \frac{a_1}{2\beta_1}\Big\{&\hat{Q}\ln\hat{Q} - \hat{c}\ln\hat{c} - (\hat{Q}-\hat{c})(1+\ln\hat{b})\Big\} \\
&+\frac{a_2}{4\beta_1^2}\Big\{-\ln\frac{\hat{Q}}{\hat{c}} + \frac{\hat{Q}-\hat{c}}{\hat{b}}\Big\} + \frac{a_1}{4\beta_1}\ln\frac{\hat{Q}}{\hat{c}}\ln\frac{e^{2\gamma_E-1}}{2} \\
&+\frac{a_1\beta_2}{4\beta_1^3}\Big\{\ln\frac{\hat{Q}}{\hat{c}} + \frac{1}{2}(\ln^2 2\hat{Q} - \ln^2 2\hat{c}) - \frac{1}{\hat{b}}(1+\ln 2\hat{b})(\hat{Q}-\hat{c})\Big\} \\
&-\frac{a_2\beta_2}{4\beta_1^4}\Big\{\frac{1}{4\hat{b}^2}(1+2\ln 2\hat{b})(\hat{Q}-\hat{c}) + \frac{3}{4}(\frac{1}{\hat{Q}} - \frac{1}{\hat{c}}) + \frac{1}{2}(\frac{\ln 2\hat{Q}}{\hat{Q}} - \frac{\ln 2\hat{c}}{\hat{c}})\Big\} \\
&+\frac{a_2\beta_2^2}{16\beta_1^6}\Big\{(\frac{2}{27} + \frac{2}{9}\ln 2\hat{b} + \frac{1}{3}\ln^2 2\hat{b})\frac{\hat{Q}-\hat{c}}{\hat{b}} + \frac{19}{108}(\frac{1}{\hat{Q}^2} - \frac{1}{\hat{c}^2}) + \frac{5}{18}(\frac{\ln 2\hat{Q}}{\hat{Q}^2} - \frac{\ln 2\hat{c}}{\hat{c}^2}) \\
&+\frac{1}{6}(\frac{\ln^2 2\hat{Q}}{\hat{Q}^2} - \frac{\ln^2 2\hat{c}}{\hat{c}^2})\Big\} + \frac{a_1\beta_2}{8\beta_1^3}\Big\{\frac{1}{\hat{Q}} - \frac{1}{\hat{c}} + \frac{\ln 2\hat{Q}}{\hat{Q}} - \frac{\ln 2\hat{c}}{\hat{c}}\Big\}\ln\frac{e^{2\gamma_E-1}}{2}\ ,
\end{aligned}
$$

TABLE II. Decay constants(in GeV) and Gegenbauer moments for light vectors.

f_ρ	f_ρ^T	$a_{1\rho}^{\parallel}$	$a_{2\rho}^{\parallel}$	$a_{1\rho}^{\perp}$	$a_{2\rho}^{\perp}$
0.107 ± 0.006	0.105 ± 0.021	–	0.15 ± 0.07	–	0.14 ± 0.06
f_{K^*}	$f_{K^*}^T$	$a_{1K^*}^{\parallel}$	$a_{2K^*}^{\parallel}$	$a_{1K^*}^{\perp}$	$a_{2K^*}^{\perp}$
0.118 ± 0.005	0.077 ± 0.014	0.03 ± 0.02	0.11 ± 0.09	0.04 ± 0.03	0.10 ± 0.08

TABLE III. Scalar decay constant $\bar{f}_S$ (in GeV) and Gegenbauer moments $B_{1,3}$ for light scalars

Scalars	$\bar{f}_S$	B_1	B_3
a_0	0.365 ± 0.020	-0.93 ± 0.10	0.14 ± 0.08
κ	0.340 ± 0.020	-0.92 ± 0.11	0.15 ± 0.09
$a_0(1450)$	-0.280 ± 0.030	0.89 ± 0.20	-1.38 ± 0.18
	0.460 ± 0.050	-0.58 ± 0.12	-0.49 ± 0.15
$K_0^*(1430)$	-0.300 ± 0.030	0.58 ± 0.07	-1.20 ± 0.08
	0.445 ± 0.050	-0.57 ± 0.13	-0.42 ± 0.22

TABLE IV. Decay constants (in GeV) and Gegenbauer moments for light axial-vectors.

f_{a_1}	$a^{\parallel}_{0a_1}$	$a^{\parallel}_{1a_1}$	$a^{\parallel}_{2a_1}$	$a^{\perp}_{0a_1}$	$a^{\perp}_{1a_1}$	$a^{\perp}_{2a_1}$
0.238 ± 0.010	1	–	-0.02 ± 0.02	0	-1.04 ± 0.34	–
f_{b_1}	$a^{\parallel}_{0b_1}$	$a^{\parallel}_{1b_1}$	$a^{\parallel}_{2b_1}$	$a^{\perp}_{0b1}$	$a^{\perp}_{1b_1}$	$a^{\perp}_{2b_1}$
0.180 ± 0.008	0.0028 ± 0.0026	-1.95 ± 0.35	–	1	–	0.03 ± 0.19
$f_{K_{1A}}$	$a^{\parallel}_{0K_{1A}}$	$a^{\parallel}_{1K_{1A}}$	$a^{\parallel}_{2K_{1A}}$	$a^{\perp}_{0K_{1A}}$	$a^{\perp}_{1K_{1A}}$	$a^{\perp}_{2K_{1A}}$
0.250 ± 0.013	1	0.00 ± 0.26	-0.05 ± 0.03	0.08 ± 0.09	-1.08 ± 0.48	0.02 ± 0.20
$f_{K_{1B}}$	$a^{\parallel}_{0K_{1B}}$	$a^{\parallel}_{1K_{1B}}$	$a^{\parallel}_{2K_{1B}}$	$a^{\perp}_{0K_{1B}}$	$a^{\perp}_{1K_{1B}}$	$a^{\perp}_{2K_{1B}}$
0.190 ± 0.010	0.14 ± 0.15	-1.95 ± 0.45	0.02 ± 0.10	1	0.17 ± 0.22	-0.02 ± 0.22

TABLE V. Decay constants(in GeV) for light tensors.

f_{a_2}	$f^T_{a_2}$	$f_{K_2^*}$	$f^T_{K_2^*}$
0.107 ± 0.006	0.105 ± 0.021	0.118 ± 0.005	0.077 ± 0.014

- The Bc->J/\psi M decay amplitude is decomposed into

$$A^{(\sigma)}(B_c \to J/\psi M^+) = V_{cb}^* V_{uq}(F_e^{(\sigma)} \cdot f_M + M_e^{(\sigma)}).$$

- For the decays Bc->J/\psi P and J/\psi S,

$$\begin{aligned}\mathcal{B}(B_c^+ \to J/\psi M^+) &\equiv \tau_{B_c} \cdot \Gamma(B_c^+ \to J/\psi M^+) \\ &= \tau_{B_c} \cdot \frac{G_F^2 m_{B_c}^3}{32\pi} \cdot \Phi(r_2, r_3) \cdot |A(B_c^+ \to J/\psi M^+)|^2 ,\end{aligned}$$

- For the decays Bc->J/\psi V, J/\psi A and J/\psi T,

$$\begin{aligned}\mathcal{B}(B_c^+ \to J/\psi M^+) &\equiv \tau_{B_c} \cdot \Gamma(B_c^+ \to J/\psi M^+) \\ &= \tau_{B_c} \cdot \frac{G_F^2 |\mathbf{P_c}|}{16\pi m_{B_c}^2} \sum_{\sigma=L,N,T} A^{\sigma\,\dagger}(B_c^+ \to J/\psi M^+) A^{\sigma}(B_c^+ \to J/\psi M^+)\end{aligned}$$

- Longitudinal polarization fractions for Bc-> J/\psi V,

$$f_L(B_c^+ \to J/\psi\rho^+) = (89.1^{+0.1}_{-0.1})\% \,, \qquad f_L(B_c^+ \to J/\psi K^{*+}) = (85.6^{+0.2}_{-0.2})\% \,,$$

$$f_L(B_c^+ \to J/\psi a_1^+) = (74.8^{+0.0}_{-0.3})\% \,,$$

$$f_L(B_c^+ \to J/\psi b_1^+) = (98.9^{+0.0}_{-0.0})\% \,.$$

- Longitudinal polarization fractions for Bc-> J/\psi T,

$$f_L(B_c^+ \to J/\psi a_2^+) = (96.1^{+0.1}_{-0.1})\% \,, \qquad f_L(B_c^+ \to J/\psi K_2^{*+}) = (95.3^{+0.0}_{-0.2})\% \,,$$

- Main inputs and CKM matrix elements

$$\Lambda_{\overline{\rm MS}}^{(f=4)} = 0.250\,, \quad m_W = 80.41\,, \quad m_{B_c} = 6.275\,, \quad m_{J/\psi} = 3.097\,,$$

$$\tau_{B_c} = 0.507\,, \quad m_b = 4.8\,, \quad m_c = 1.5\,. \quad f_{Bc} = 0.489, \quad f_{J/\psi} = 0.405$$

$$V_{\rm CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

with A=0.790 and λ = 0.22650

- Decays governed by F.E.A. first, then those sensitive to nF.E.A.