On the speed of sound in dense two colour QCD

A Quick Update

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ITPQFTM+ 2023
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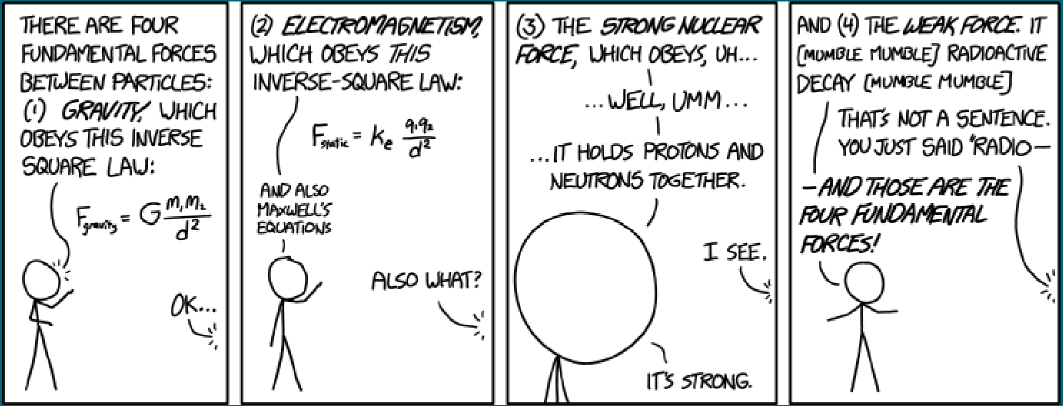
Ollscoil Mhá Nuad
Ollscoil na hÉireann Má Nuad



"Of these four forces, there's one we don't really understand." "Is it the weak force or the strong–" "It's gravity."

1

[1] https://xkcd.com/1489/

Euclidean QCD Lagrangian given by

$$\mathscr{L}_{QCD} = \frac{1}{4}\mathrm{Tr}\, G_{\mu\nu}G_{\mu\nu} + \bar{\psi}\left[\gamma_\mu\left(\partial_\mu + ig_s A_\mu\right) + m\right]\psi \quad (1)$$

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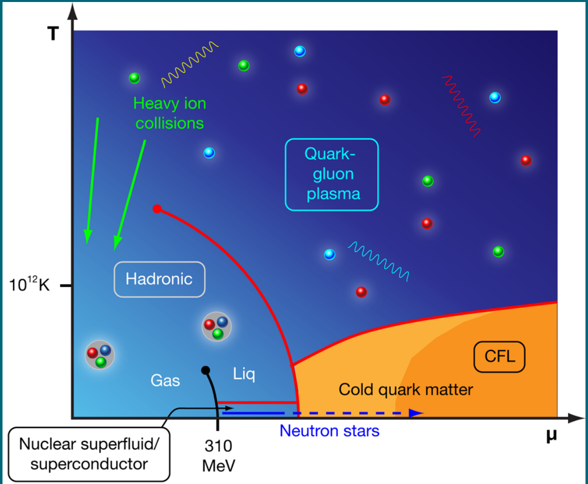
- For $g_s \ll 1$ can use perturbation theory (very high temperature/very short distances)
- For $g_s \sim 1$ or non-perturbative phenomena must use non-perturbative approach



A sketch of the QCD Phase Diagram

- Want to solve the path integral

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[\Phi] \mathcal{O}[\Phi] e^{-S[\Phi]} \tag{2}$$

- Space-time itself is discretised
- 4-Dimensional Lattice with periodic boundary conditions.
- Fermions lie on vertices, gauge bosons on the links

Gluon fields:

$$A_\mu(x) \in \mathfrak{su}(N) \tag{3}$$
$$U_\mu(n) \in SU(N) \tag{4}$$

Gauge Action:

$$S_g[A] = -\frac{1}{2g^2}\int d^4x G^a_{\mu\nu} G^a_{\mu\nu} \tag{5}$$
$$S_g[U] = -\frac{a^4}{2g^2}\sum_{n\in\Lambda} \mathrm{Tr}\, G_{\mu\nu}(n) G^{\mu\nu}(n) \tag{6}$$

This discretisation is not unique

Fermion Action:

$$S_F\left[\psi, \bar{\psi}, A\right] = \sum_{f=1}^{N_f} \int d^4x \bar{\psi}^{(f)}(x) \left(\gamma_\mu \left(\partial_\mu + iA_\mu(x) \right) + m^{(f)} \right) \psi^{(f)}(x) \tag{7}$$

$$S_F[\psi, \bar{\psi}, U] = a^4 \sum_{n\in\Lambda} \sum_{f=1}^{N_f} \bar{\psi}^{(f)}(n) \left(\sum_{\mu=1}^{4} \gamma_\mu \frac{U_\mu(n)\psi_\mu^{(f)}(n+\hat{\mu}) - U_\mu^\dagger(n-\hat{\mu})\psi^{(f)}(n-\hat{\mu})}{2a} + m\psi^{(f)}(n) \right) \tag{8}$$

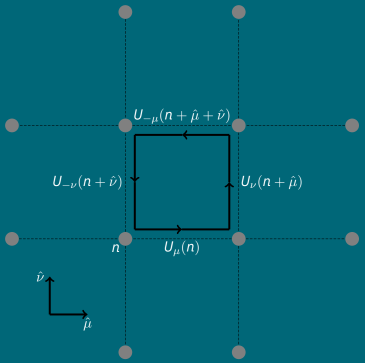


Figure: A Plaquette.

- Bosonic observables represented by closed loop on the lattice (Wilson loop)
- Closed loop in temporal extent (Polyakov loop) is order parameter for deconfinement
- Simplest closed loop is a 1×1 square called a *plaquette*

$$U_{\mu\nu}(n) = U_{\mu}(n)U_{\nu}(n+\hat{\mu})U_{-\mu}(n+\hat{\mu}+\hat{\nu})U_{-\nu}(n+\hat{\nu}) = U_{\mu}(n)U_{\nu}(n+\hat{\mu})U^{\dagger}_{\mu}(n+\hat{\nu})U^{\dagger}_{\nu}(n) \quad (9)$$

- Gauge configurations produced with probability weight

$$e^{-S[U]} = \det M[U] e^{-S_g[U]} \tag{10}$$

using Hybrid Monte Carlo (HMC). $M[U]$ fermion matrix.
  - Metropolis algorithm is inefficient. Updates one site per step.
  - updating all sites naïvely gives very large changes to action
  - HMC involves a global update in a fictitious time τ defined by *Hamiltonian* instead.

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Generating Configurations

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Fun Fact!

For an exact calculation, $\delta H = 0$ due to energy conservation so all fields would be accepted.

- Observables on the lattice are dimensionless, must set the scale to give a physical meaning.
- Choose an observable and use it to fix scale.
- We use string tension $\sigma = (440\,\text{MeV})^2$ and compare using static quark potential.

Ensemble	β	κ	a (fm)	$\frac{m_\pi}{m_\rho}$	m_π (MeV)
V. Coarse	1.7	0.178	0.233	$0.779(4)$	$688(11)$
Coarse	1.9	0.1680	$0.178(6)$	$0.805(9)$	$717(25)$
Light	1.7	0.1810	$0.189(4)$	$0.61(5)$	$638(33)$
Fine	2.1	0.1577	$0.138(6)$	$0.810(7)$	$637(28)$
Light-Fine					

Table: Table of lattice parameters, lattice spacings and pion masses

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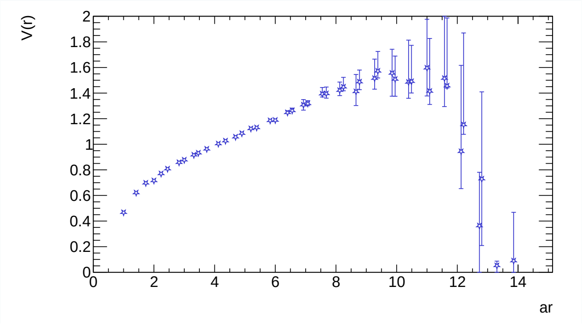

Figure: Static Quark Potential for $\beta = 2.0$, $\kappa = 0.1642$ on a $16^3 \times 32$ lattice.

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Table: Table of lattice parameters, lattice spacings and pion masses

1. Produce gauge configs for chose of β and κ.
2. Calculate pseudoscalar/vector correlators and extract mass ratio.
3. Evaluate Wilson Loops/Lines numerically
4. Fit Wilson lines to form
$$W(r,\tau) = Ae^{-V(r)\tau} \tag{11}$$
with fit parameters A and $V(r)$
5. Fit $V(r)$ to the Cornell Form in lattice units
$$aV(r) = aV_0 + \frac{\alpha}{\left(\frac{r}{a}\right)} + a^2\sigma\frac{r}{a} \tag{12}$$

Momentum Space Massless Free Fermion Propagator

$$\left.\tilde{D}(p)^{-1}\right|_{m=0} = \frac{i \sum_{\mu=1}^{4} \gamma_\mu p_\mu}{p^2} \tag{13}$$

$$\left.\tilde{D}(p)^{-1}\right|_{m=0} = \frac{i a^{-1} \sum_{\mu=1}^{4} \gamma_\mu \sin\left(p_\mu a\right)}{a^{-2} \sum_{\mu=1}^{4} \sin^2(p_\mu a)} \tag{14}$$

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  This is the infamous fermion doubling problem. Techniques for removing these doublers include
  - Wilson Fermions (increase mass of doublers so they decouple from the theory)
  - (Rooted) Staggered Fermions (mixes Dirac and space-time indices)
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- Want translational invariance, hermiticity and locality; you get an equal number of left and right handed fermions. No-Go Theorem![2]
- This work uses Wilson Fermions, which explicitly break chiral symmetry.

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Wilson Fermions

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Wilson Fermions

- Want to distinguish between the real $p_\mu = 0$ pole and the $p_\mu = \frac{\pi}{a}$ poles
- Add term to momentum space Dirac operator that is
  1. 0 if $p_\mu = 0$
  2. Non-zero if $p_\mu = \frac{\pi}{a}$

- Naïve Dirac operator given by

$$\tilde{D}(p) = m + \frac{i}{a}\sum_{\mu=1}^{4}\gamma_\mu \sin(p_\mu a) \tag{15}$$

- Instead consider

$$\tilde{D}(p) = m + \frac{i}{a}\sum_{\mu=1}^{4}\gamma_\mu \sin(p_\mu a) + \frac{1}{a}\sum_{\mu=1}^{4}\left(1 - \cos\left(p_\mu a\right)\right) \tag{16}$$

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- For $p_\mu = \frac{\pi}{a}$, we get an additional "mass" of $\frac{2(d-1)}{a}$
- In $a \to 0$ limit these extra masses are infinite so decouple
- Chiral symmetry is *explicitly* broken by the Wilson term
- Get order $\mathcal{O}(a)$ discretisation errors

- But we can do better!!!
- Let

$$Q_{\mu\nu} = U_{\mu,\nu}(n) + U_{\mu,-\nu}(n) + U_{-\mu,-\nu}(n) + U_{-\mu,\nu}(n) \tag{17}$$

Wilson Fermions

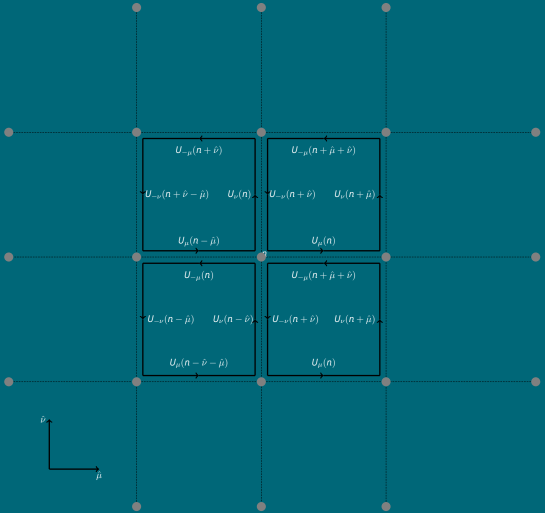

Figure: Clover action

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- Then the *Symanzik Improved* action of the form

$$S_I = S_{\text{Wilson}} + c_{\text{sw}} a^5 \sum_{n\in\Lambda} \sum_{\mu<\nu} \bar{\psi}(n) \frac{1}{2} \sigma_{\mu\nu} \left(\frac{i}{8a^2} \left(Q_{\mu\nu}(n) - Q_{\nu\mu}(n) \right) \right) \psi(n) \tag{18}$$

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- This is a work in progress

Justifying my Existence

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  so can use HMC
- Theory is *qualitatively similar* showing QCD properties such as deconfinement and chiral symmetry breaking.

QC_2D

DISCLAIMER!!!!

- We are **NOT** claiming that the strong force actually follows $SU(2)$ instead of $SU(3)$.
  We are merely using $SU(2)$ to probe an otherwise inaccessible régime using lattice techniques, much like running 2-d instead of 3-d simulations.
- The individual quarks are still fermionic.

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Things to keep in mind

- Lattice coupling $\beta = \frac{2N_c}{g^2}$ will have smaller value than seen in real QCD.

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Things to keep in mind

- Lattice coupling $\beta = \frac{2N_c}{g^2}$ will have smaller value than seen in real QCD.
- Max 10 flavours to have asymptotic freedom (vs up-to 16 in three-colour QCD)
- Baryons are now quark-quark pairs (diquarks), so follow Bose-Einstein Statistics[3]
- BEC phase appears at high density
- Now get 5 massless Goldstone bosons in the chiral limit.

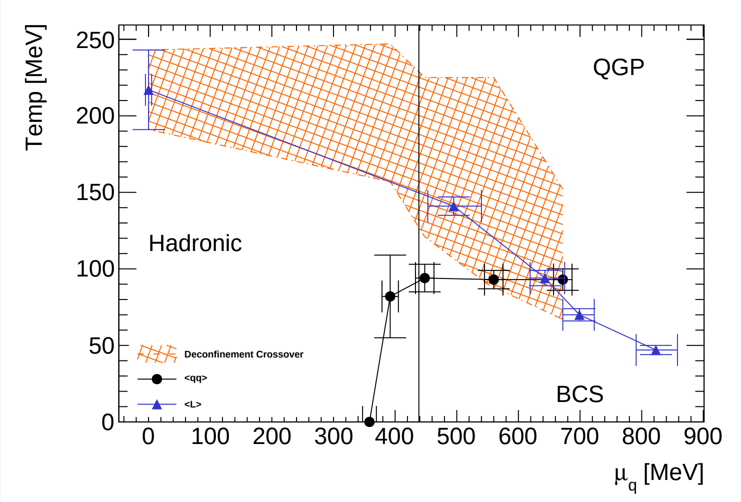
[3] This makes people *really* uncomfortable

- These results were first presented at XQCD 2022 and XV QCHS Conference[4]
- Using a spatial extent $N_s = 24$
- Conducted a temperature scan on coarse lattice at $a\mu = 0.400$
- Temperature is given by

$$T = \frac{1}{a_\tau N_\tau} \tag{19}$$

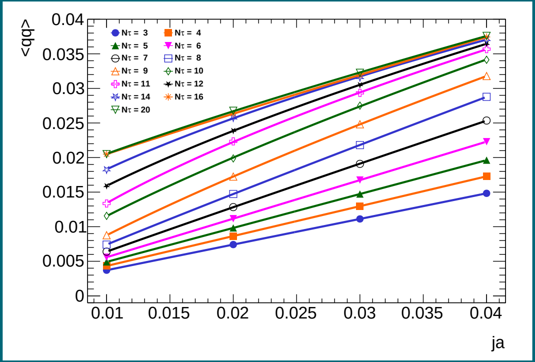
- Varying the number of sites along the time direction allows us to complete a temperature scan

[4] Lawlor et al. 2022.



Phase diagram of QC_2D for $\frac{m_\pi}{m_\rho} = 0.80(1)$.

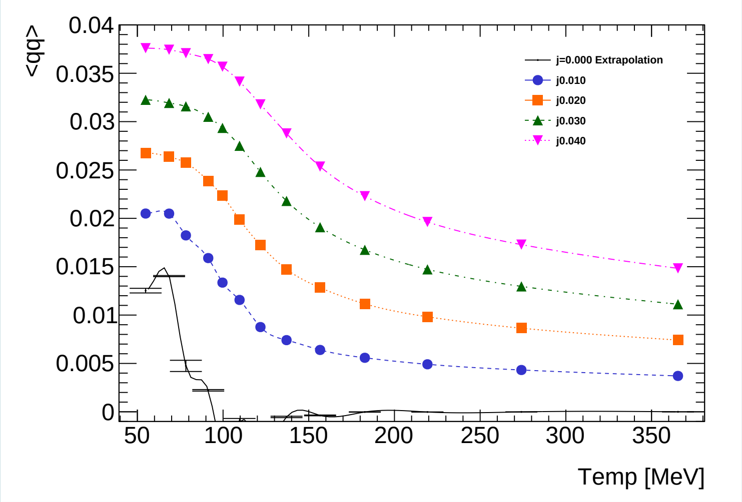
- Molecular dynamics requires inverting a large sparse matrix via conjugate gradient. [a]
- At non-zero baryon density, fermion matrix acquires non-zero density of very small eigenvalues, slowing down the computation
- Diquark source j lifts these eigenvalues, with "physical" results recovered by extrapolation of j to zero.



Diquark condensate vs Diquark Source

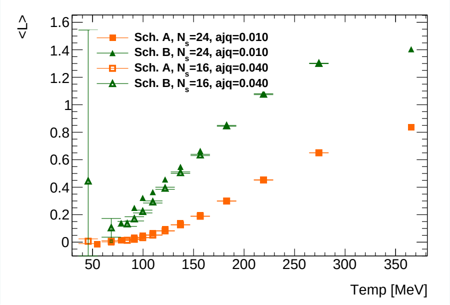
[a] i.e. make supercomputer go brrr

Superfluid Transition



Diquark condensate vs Temperature

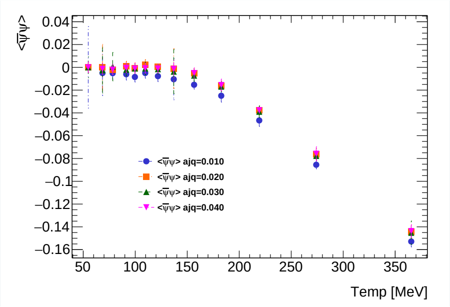
- The superfluid phase transition occurs around $T \sim 100$ MeV.
- This indicates that the superfluid phase transition is indeed distinct from the deconfinement crossover.



Renormalised Polyakov loop with two renormalisation schemes. These results are also compatible with earlier results on smaller volumes.

A: $L_R(N_\tau = 4, \mu = 0) = 0.5$

B: $L_R(N_\tau = 4, \mu = 0) = 1.0$



Unrenormalised but subtracted chiral condensate

The change in behaviour from constant to decreasing at $T \sim 150$ MeV suggests that the crossover coincides with the deconfinement crossover, not the superfluid transition.

Neutron Star (NS)

- Equation of State (EoS) of dense nuclear matter an unanswered question
- NS form natural labs for exploring dense nuclear matter
- Measurements of NS tidal deformabilities using Gravitational Wave (GW) indicates EoS soft at nuclear densities.
- Pulsar observations indicate stiff EoS at $M > 2M_{\odot}$. Non-monotonic C_s?

	CEFT	Dense NM	Pert. QM	CFTs
c_s^2	$\ll 1$	$[0.8, 1]$	$\lesssim 1/3$	1/3

Table: C_s^2 predictions in four different limits.[a]

[a] Annala et al. 2023.

Table: Radius constraints for neutron stars for $\simeq 1.4 M_\odot$ and $\simeq 2.1 M_\odot$ NSs. taken from Minamikawa et al. 2023

	radius [km]	mass [$M_\odot$]
GW170817 (primary)	$11.9^{+1.4}_{-1.4}$	$1.46^{+0.12}_{-0.10}$
GW170817 (second)	$11.9^{+1.4}_{-1.4}$	$1.27^{+0.09}_{-0.09}$
J0030+0451 (NICER)	$13.02^{+1.24}_{-1.06}$	$1.44^{+0.15}_{-0.14}$
J0030+0451 (NICER)	$12.71^{+1.14}_{-1.19}$	$1.34^{+0.15}_{-0.16}$
PSR J0740+6620 (NICER)	$12.35^{+0.75}_{-0.75}$	$2.08^{+0.07}_{-0.07}$
PSR J0740+6620 (NICER)	$12.39^{+1.30}_{-0.98}$	$2.08^{+0.07}_{-0.07}$

- Recent results (Kojo and Suenaga 2022) indicate QC_2D is applicable NS studies.
- First principles calculation of C_s^2 using QC_2D
- Coarse lattice parameters. $12^3 \times 24 \Rightarrow T \approx 50$ MeV
- Work in progress. No diquark extrapolation

Pressure

$$C_s^2 = \frac{dP}{d\varepsilon} \tag{20}$$

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- Two different approaches for pressure

Integral Method

$$P(\mu) = \int_0^\mu n_q(\mu')d\mu' \tag{23}$$

Integral Method

$$P(\mu) = \int_0^{\mu} n_q(\mu')d\mu' \qquad (23)$$

- $n_q(\mu)$ has form of cubic polynomial
- Fit $n_q(\mu)$, extract coefficients and integrate analytically.
- Currently use polynomial interpolation, cubic spline on the cards.

Integral Method

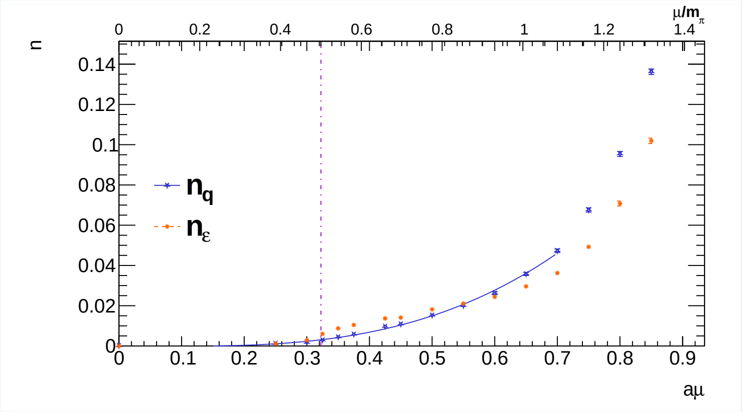

Figure: Quark and Energy Number Densities

Derivative Method

Conformal Anomaly

$$T_{\mu\mu} = \varepsilon - 3p \tag{23}$$

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- Calculate gluonic and fermionic components separately.

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$$T^{q}_{\mu\mu} = -a\frac{\partial\kappa}{\partial a}\kappa^{-1}\left(4N_f N_c - \langle\bar{\psi}\psi\rangle\right) \tag{25}$$

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- Needs fewer configurations
- Karsch Coefficients need different lattice spacings.

$T_{\mu\mu}$

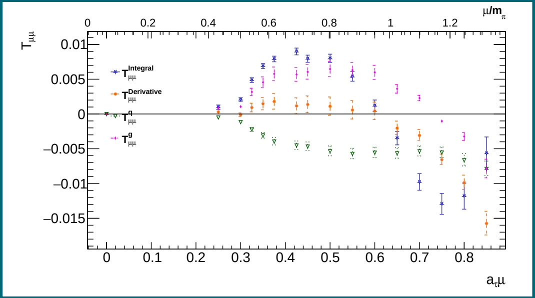

Figure: Unrenormalised Conformal Anomaly evaluated using the derivative and integral method, with the gluonic and fermionic components shown for the derivative method.

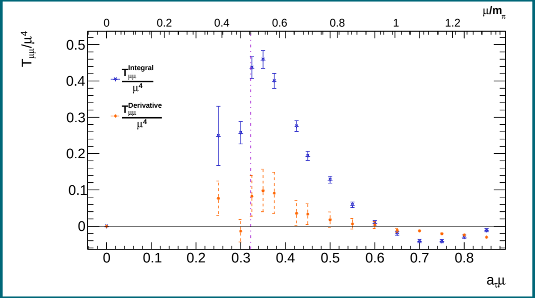

Figure: Renormalised Conformal Anomaly

c_s^2

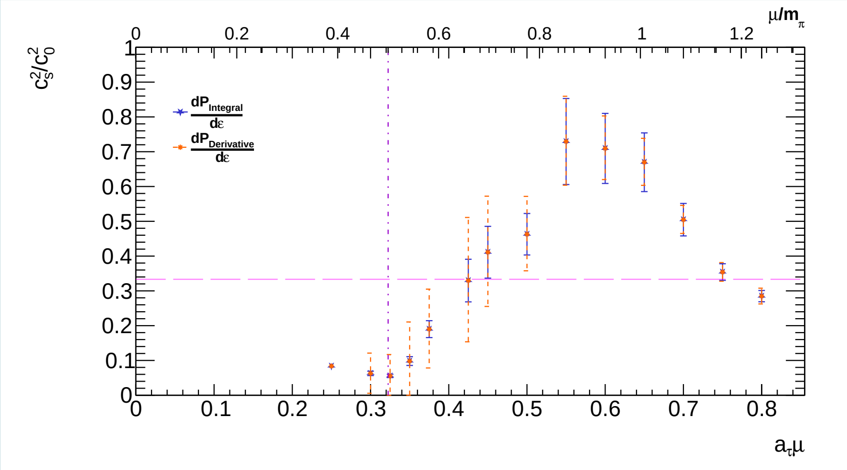

Figure: Speed of sound squared calculated using derivative and integral methods. Calculated using a $12^3 \times 24$ coarse lattice. Results are consistent with Iida and Itou 2022

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- Get the code to actually run correctly on GPUs
- EuroHPC benchmark access just awarded (LUMI-C/Vera).

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QC_2D	Two Colour QCD
QCD	Quantum Chromodynamics
HMC	Hybrid Monte Carlo
RHIC	Relativistic Heavy Ion Collisions
EoS	Equation of State
NS	Neutron Star
GW	Gravitational Wave