Numerical Study on Transient Heat Transfer in Forced Flow of Superfluid Helium

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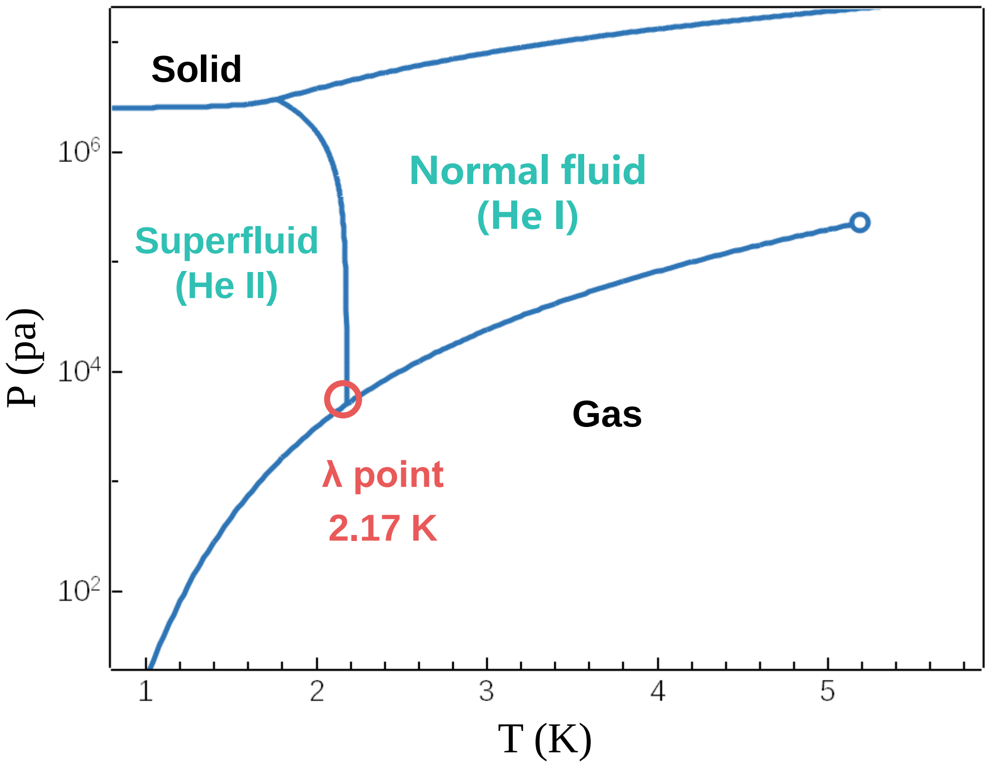
Date: 2024.07.23

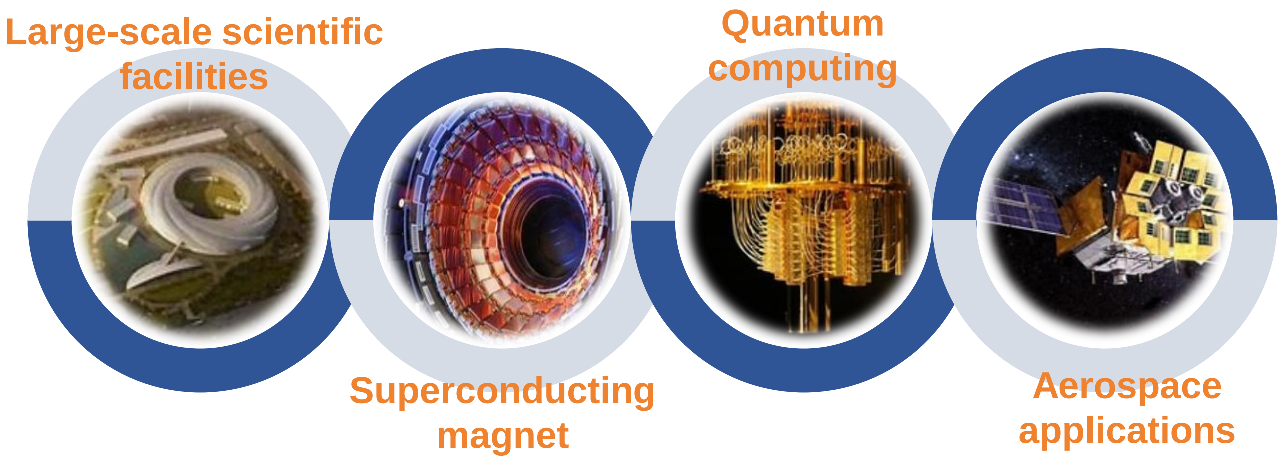
Superfluid Helium

Helium Phase Diagram



- Under saturated vapor pressure, liquid helium undergoes **Bose-Einstein condensation** and becomes superfluid helium below the **lambda transition temperature**.

- Superfluid helium is widely used in heat transfer and cooling of advanced devices and large-scale projects.



- Landau proposed the two-fluid model in 1941, which considers superfluid helium as a mixture **of superfluid and normal components**.

	Density	Entropy	Viscosity
Superfluid component	ρ_s	0	0
normal component	ρ_n	s	η_n

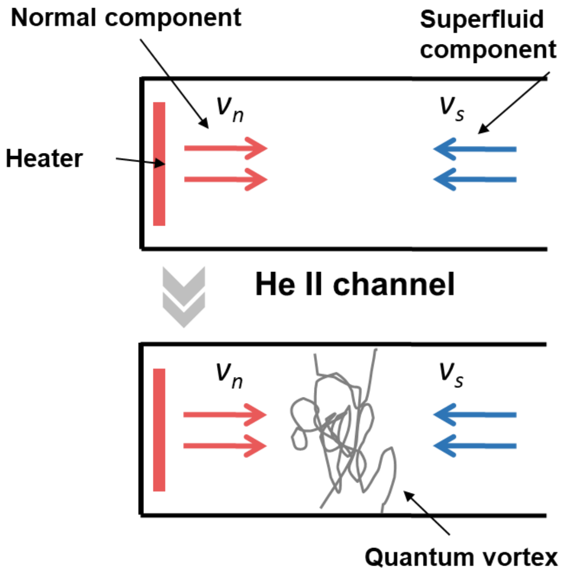
Heat Transfer in Superfluid Helium

- **Thermal Counterflow:** The viscous normal component carries all the heat and flows away from the heat source; the inviscid superfluid component moves in the opposite direction to satisfy the conservation of mass:

$$q = \rho s T \mathbf{v}_n$$

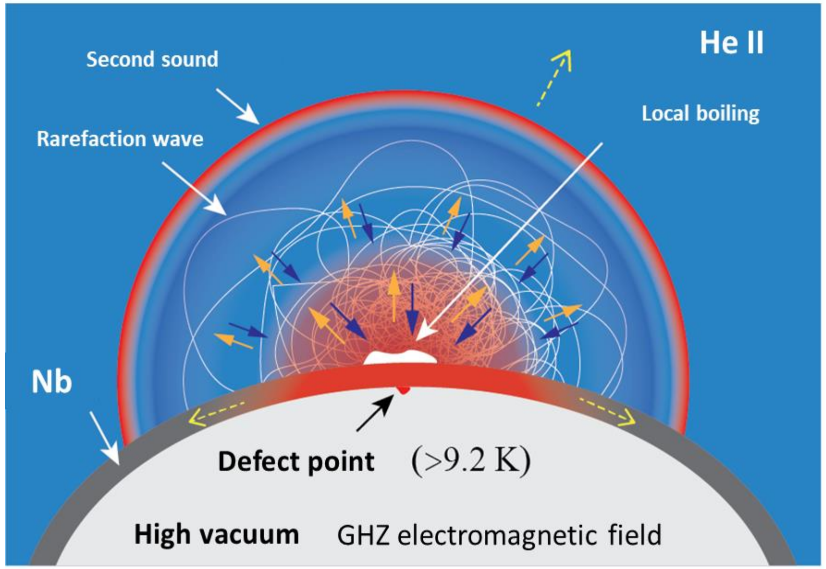
- The **superconductor quench** is a typical transient heat transfer process involving superfluid helium;
- However, when **the velocity difference between the normal and superfluid components** exceeds a certain threshold, **quantum turbulence** occurs in superfluid helium, which makes it difficult for conventional fluid dynamics models to describe its heat and mass transfer characteristics accurately.

Thermal Counterflow



NHMFL's Experiment*

Transient quench



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* Mastracci, Brian , and W. Guo . "An exploration of thermal counterflow in He II using particle tracking velocimetry." Physical Review Fluids 3.6(2018).

NS Equation and Vinen Equation

- The whole superfluid helium satisfies the following continuity equation:

$$\frac{\partial \rho}{\partial t}+\nabla\cdot\left(\rho_n \mathbf{v}_n+\rho_s \mathbf{v}_s\right)=0$$

- The normal component follows the momentum equation as follows:

$$\frac{\partial \rho_n \mathbf{v}_n}{\partial t}+\nabla\cdot\left(\rho_n \mathbf{v}_n \mathbf{v}_n\right)=-\frac{\rho_n}{\rho}\nabla p-\rho_s s\nabla T+\nabla\cdot\left(\mu_n \nabla \mathbf{v}_n\right)+F_{ns}$$

- The superfluid component is:

$$\frac{\partial \rho_s \mathbf{v}_s}{\partial t}+\nabla\cdot\left(\rho_s \mathbf{v}_s \mathbf{v}_s\right)=-\frac{\rho_s}{\rho}\nabla p+\rho_s s\nabla T-F_{ns}$$

- The whole superfluid helium satisfies the following continuity equation:

$$\frac{\partial \rho s}{\partial t}+\nabla\cdot\left(\rho s \mathbf{v}_n\right)=\frac{F_{ns}\square\mathbf{v}_{ns}}{T}$$

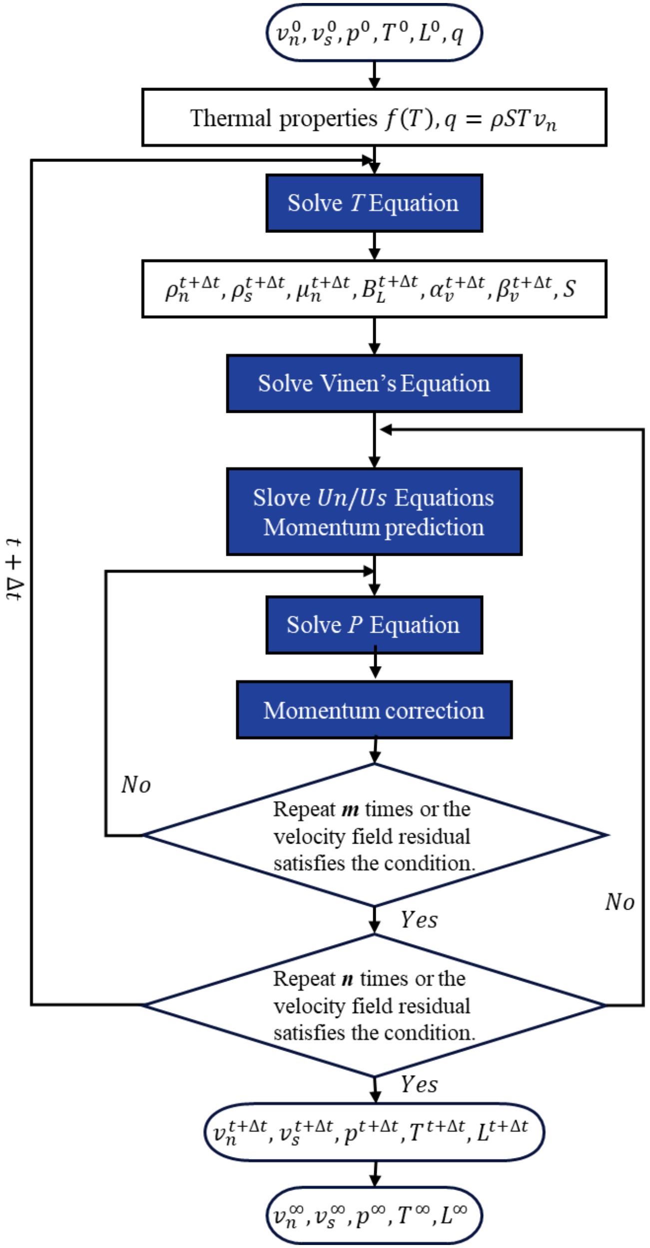
- **Mutual Friction：**

$$F_{ns}=\frac{\kappa}{3}\frac{\rho_s\rho_n}{\rho}B_L L v_{ns}$$

- The **quantum effect** in superfluid helium is introduced by the Vinen equation:

$$\frac{\partial L}{\partial t}+\nabla\cdot\left(\mathbf{v}_L L\right)=\alpha_V\,|v_{ns}|\,L^{3/2}-\beta_V L^2+\gamma_V\,|v_{ns}|^{5/2}$$

SuperHeliumFoam



$v_n^0, v_s^0, p^0, T^0, L^0, q$

Thermal properties $f(T), q = \rho S T v_n$

Solve T Equation

$\rho_n^{t+\Delta t}, \rho_s^{t+\Delta t}, \mu_n^{t+\Delta t}, B_L^{t+\Delta t}, \alpha_v^{t+\Delta t}, \beta_v^{t+\Delta t}, S$

Solve Vinen's Equation

Slove Un/Us Equations
Momentum prediction

Solve P Equation

Momentum correction

Repeat ***m*** times or the velocity field residual satisfies the condition.

No

Yes

Repeat ***n*** times or the velocity field residual satisfies the condition.

No

Yes

$t+\Delta t$

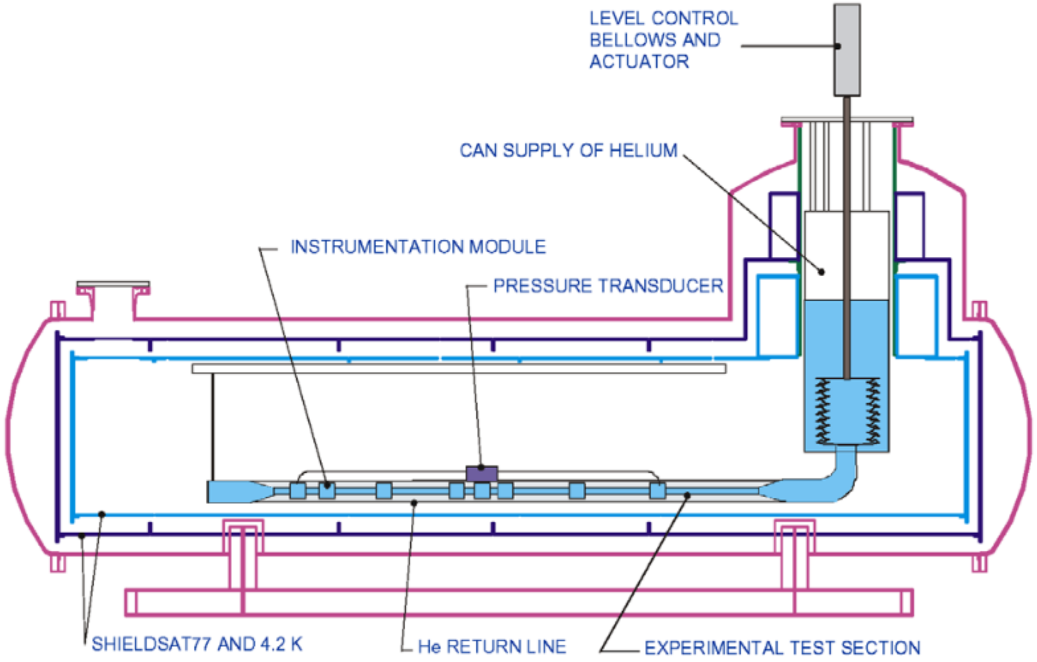
$v_n^{t+\Delta t}, v_s^{t+\Delta t}, p^{t+\Delta t}, T^{t+\Delta t}, L^{t+\Delta t}$

$v_n^{\infty}, v_s^{\infty}, p^{\infty}, T^{\infty}, L^{\infty}$

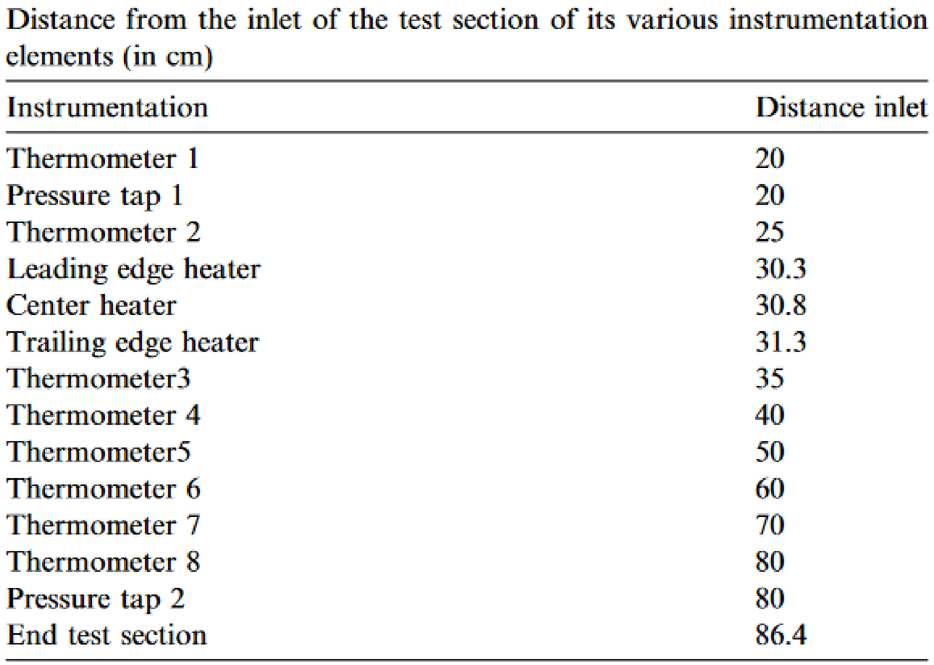
- **The solver is developed based on the Pimple algorithm.**
- **Due to the high experimental flow velocity, which results in a large Reynolds number, we have introduced the k-epsilon turbulence model into the momentum equation of the normal fluid; the superfluid component is left untreated.**
- **The thermophysical properties used in the calculations are provided by Hepak, and the parameters involving Vinen's equation are recommended by Kondaurova et al.**
- **The use of OpenFOAM endows it with the potential for three-dimensional computations while also taking into account quantum effects.**

Comparison with Experiment

- This simulation reproduces the experimental results of Fuzier et.al , whose experimental setup is shown as follows:

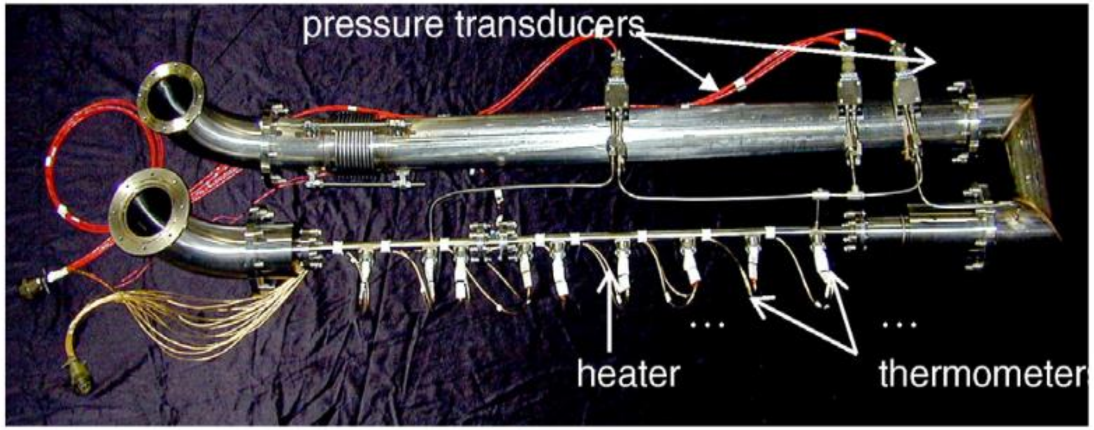


Fuzier’s Experiment Setup*



Distance from the inlet of the test section of its various instrumentation elements (in cm)

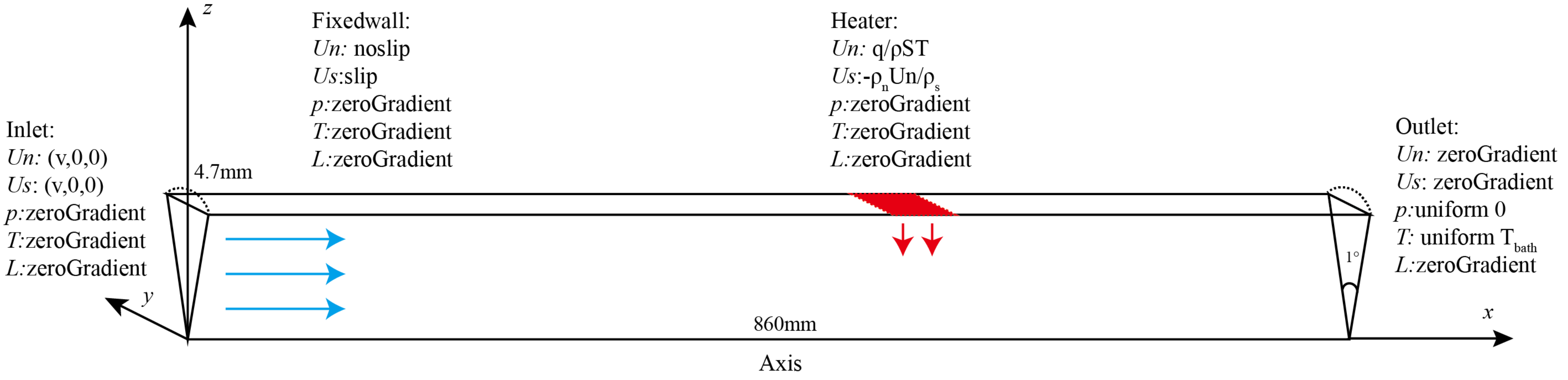
Instrumentation	Distance inlet
Thermometer 1	20
Pressure tap 1	20
Thermometer 2	25
Leading edge heater	30.3
Center heater	30.8
Trailing edge heater	31.3
Thermometer3	35
Thermometer 4	40
Thermometer5	50
Thermometer 6	60
Thermometer 7	70
Thermometer 8	80
Pressure tap 2	80
End test section	86.4



*Fuzier, S. , and S. W. V. Sciver . "Experimental measurements and modeling of transient heat transfer in forced flow of He II at high velocities." Cryogenics 48.3-4(2008):130-137.

Geometric Model

- The computational domain is set up as shown in the figure below：



Inlet:
Un: (v,0,0)
Us: (v,0,0)
p:zeroGradient
T:zeroGradient
L:zeroGradient

Fixedwall:
Un: noslip
Us:slip
p:zeroGradient
T:zeroGradient
L:zeroGradient

Heater:
Un: q/ρST
Us:$-\rho_n Un/\rho_s$
p:zeroGradient
T:zeroGradient
L:zeroGradient

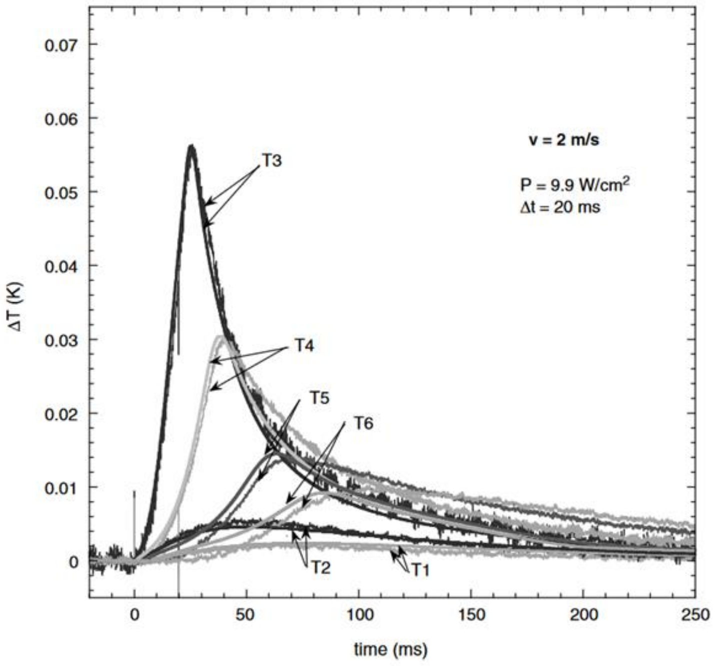
Outlet:
Un: zeroGradient
Us: zeroGradient
p:uniform 0
T: uniform T_{bath}
L:zeroGradient

- To simplify the computational load, a two-dimensional axisymmetric mesh is used for the calculations;
- Consistent with the experiment, the calculated flow channel has a length of 860 mm and a radius of 5.0 mm;
- The boundary conditions for the main physical quantities are also indicated on the diagram.

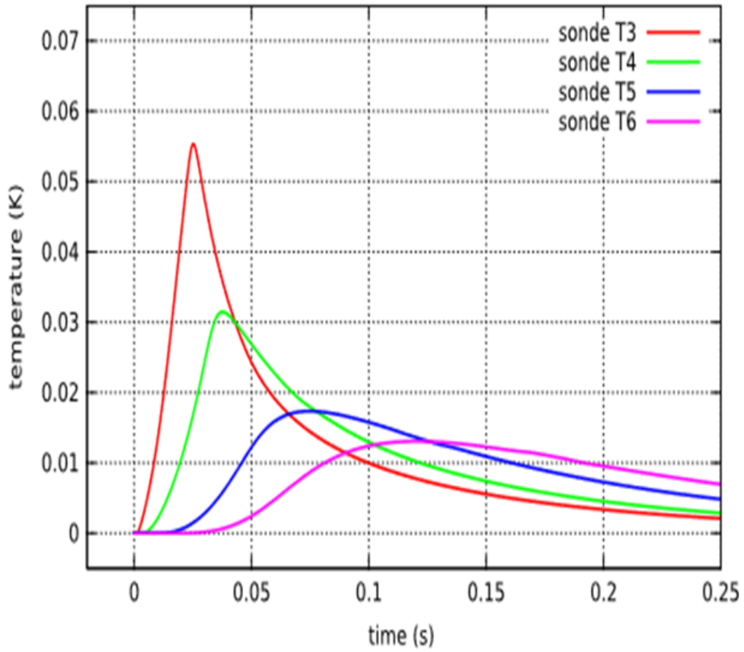
Comparison with Experiment and Computation

$$v = \mathbf{2\ m/s};\ P = 9.9\ \mathrm{W/cm^2};\ \Delta t = 20\ \mathrm{ms};\ T_{bath} = 1.7\ \mathrm{K}$$

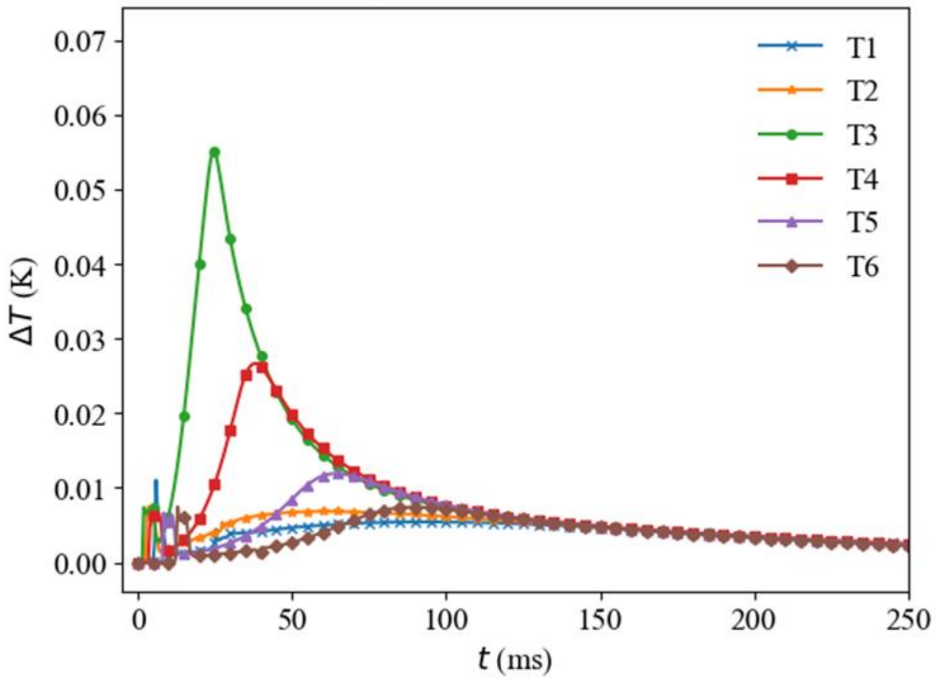
Fuzier’ Experiment



Soulaine’ Computation*



Our Computation



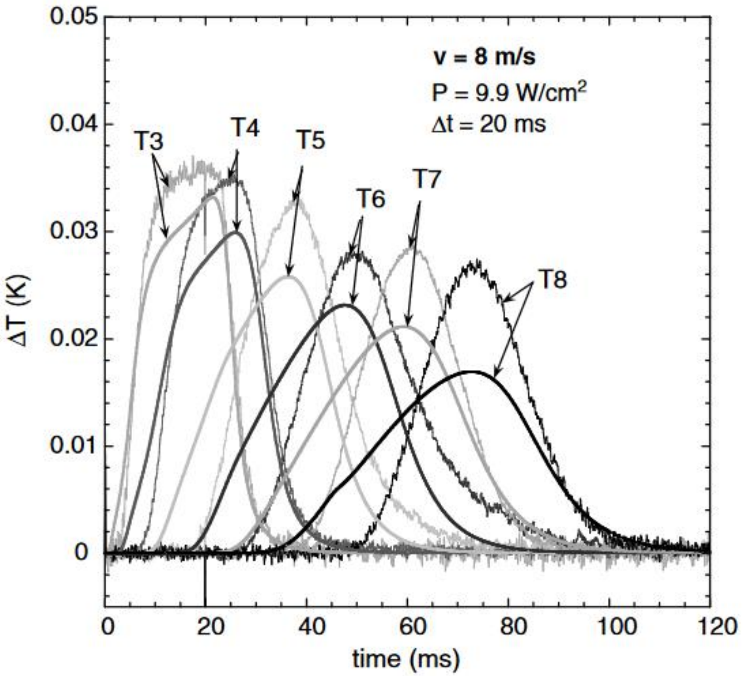
- **The calculated temperature variations at each measurement point over time are in good agreement with previous experiments and numerical simulations.**

*Soulaine, Cyprien , et al. "A PISO-like algorithm to simulate superfluid helium flow with the two-fluid model." Computer Physics Communications 187(2015):20-28.

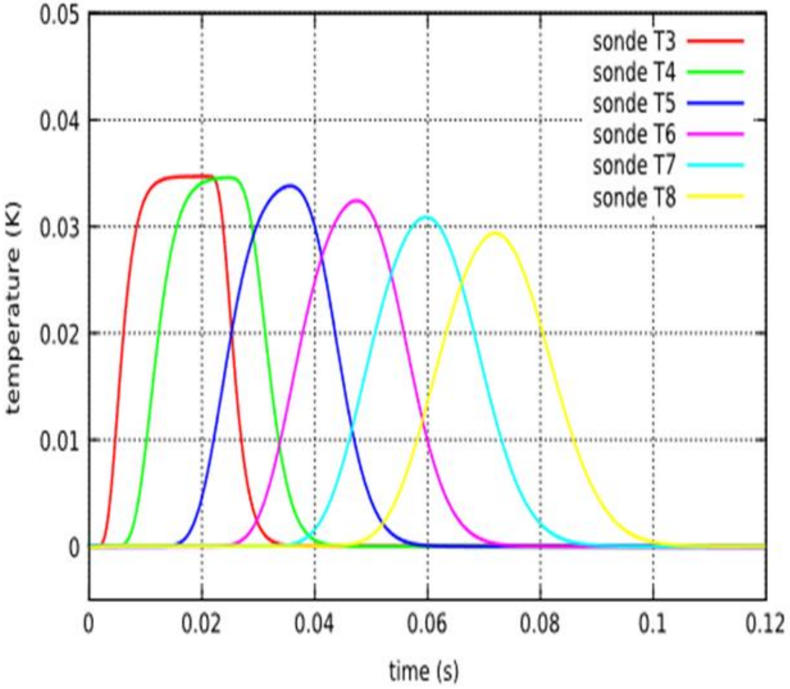
Comparison with Experiment and Computation

$$v = 8\ \mathrm{m/s};\ P = 9.9\ \mathrm{W/cm^2};\ \Delta t = 20\ \mathrm{ms};\ T_{bath} = 1.7\ \mathrm{K}$$

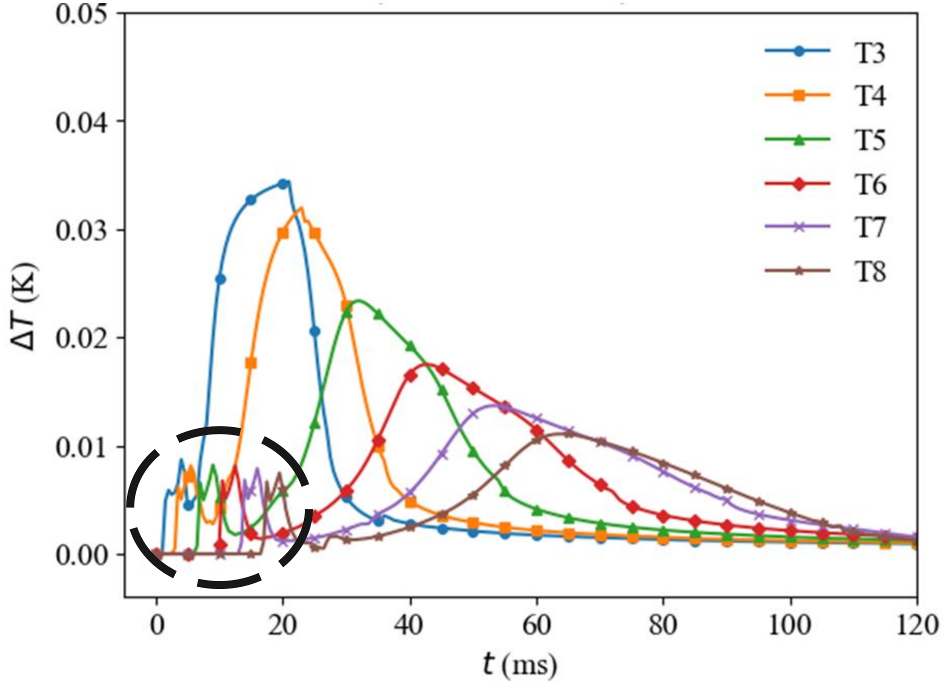
Fuzier' Experiment



Soulaine' Computation



Our Computation



- **Before the heat transfer, another type of temperature fluctuation was observed, which was calculated to be the propagation of second sound:**

$$u_T = \frac{(0.08-0.07)\mathrm{m}}{(0.0195-0.016)\mathrm{s}} = 28.57\mathrm{m/s} \quad \approx \quad u_c = \sqrt{\frac{Ts^2\rho_s}{c\rho_n}} \approx \sqrt{\frac{Ts\rho_s}{3\rho_n}} \approx 28.21\mathrm{m/s}$$

RESULTS

Distribution of Variables Along the Flow Direction in 2m/s

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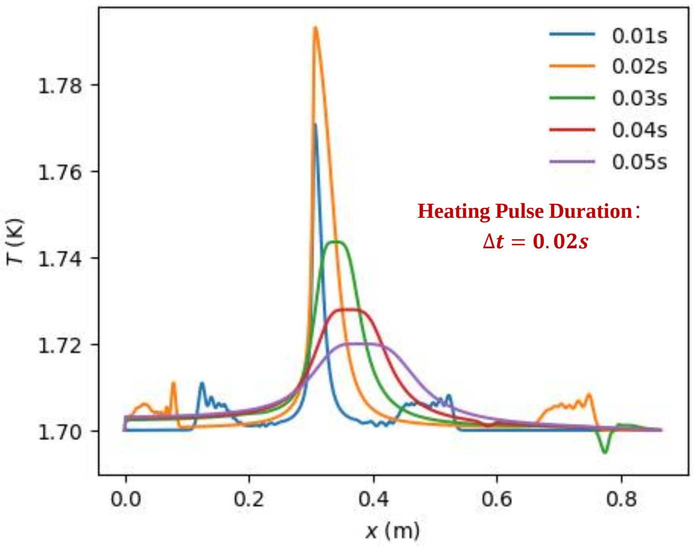
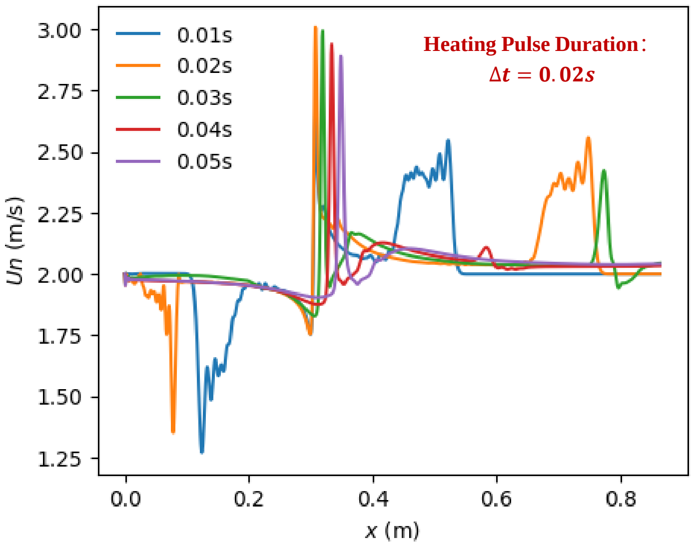
Temperature Distribution



Heating Pulse Duration：
$\Delta t = 0.02s$

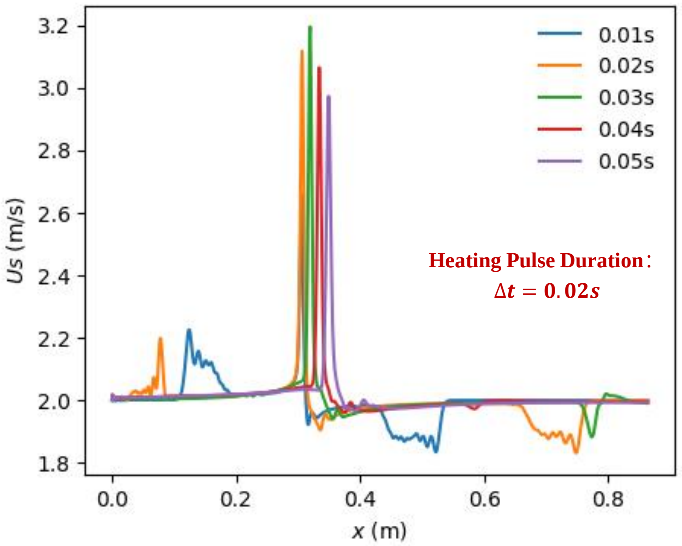
- At the moment the heater is turned off, the temperature reaches its maximum value, and after the shutdown, **the peak moves downstream.**
- For the **normal components**, due to thermal counterflow, the velocity upstream of the heater is suppressed, with the maximum reduction being **12.25%,** while the velocity downstream is enhanced, with the highest increase reaching **50%. The opposite is true for the superfluid components.**
- The total velocity exhibits only a **single peak**, with no distinction between upstream and downstream
- After the heater is deactivated, the peak velocity, along with the temperature, **moves downstream**.

Normal Component Velocity Distribution



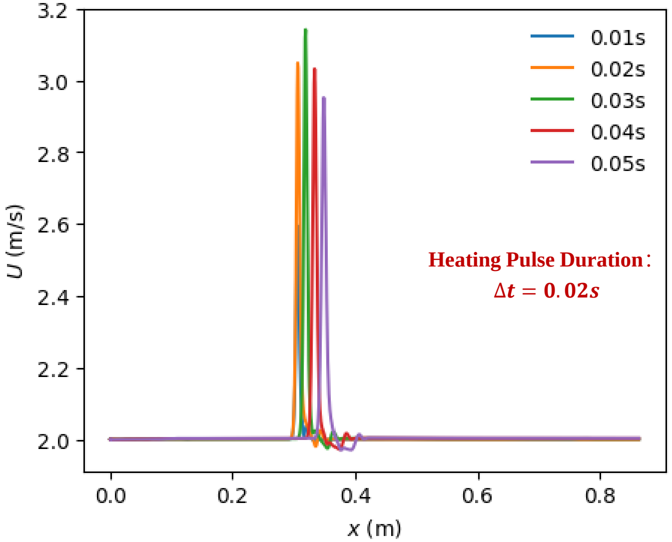
Heating Pulse Duration：
$\Delta t = 0.02s$

Superfluid Component Velocity Distribution



Heating Pulse Duration：
$\Delta t = 0.02s$

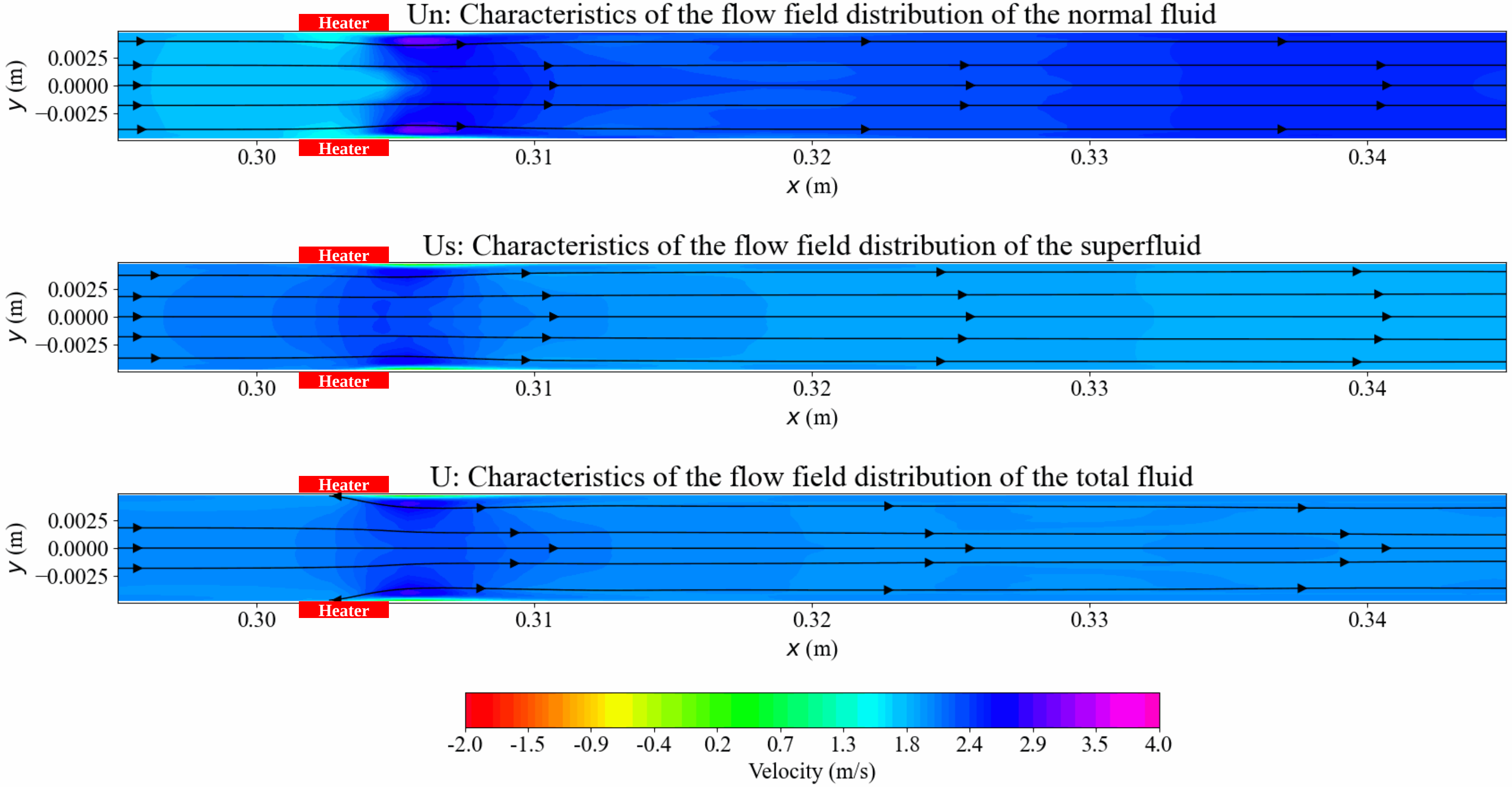
Total Velocity Distribution



Heating Pulse Duration：
$\Delta t = 0.02s$

The 2D Velocity Profile in 2m/s

Time: 0.005s (Heater on)



- After the heater is activated, the velocity near the heater **increases sharply**, forming a distinct **velocity boundary layer**;
- The flow field contracts near the heater, forming a flow structure similar to a “nozzle”;
- **This structure is due to the heat input, which imparts a radial component to the velocity field;**
- After the heater is turned off, the structure moves downstream with the main flow, and the upstream flow field returns to normal.

RESULTS

Distribution of Variables Along the Flow Direction in 8m/s

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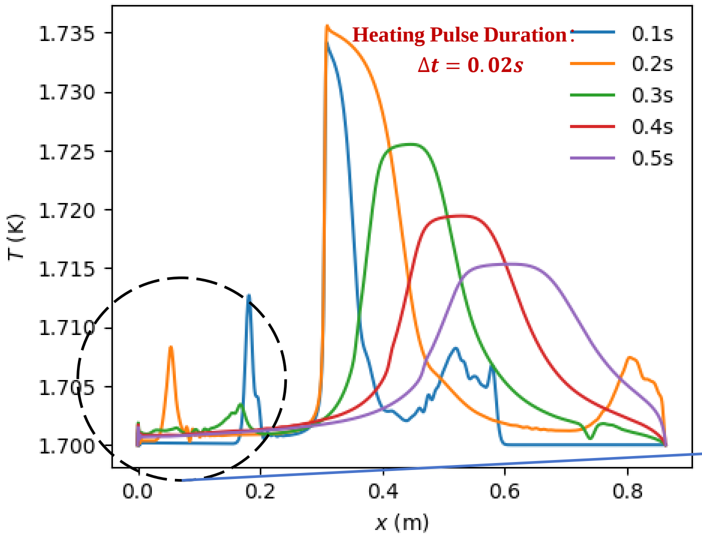
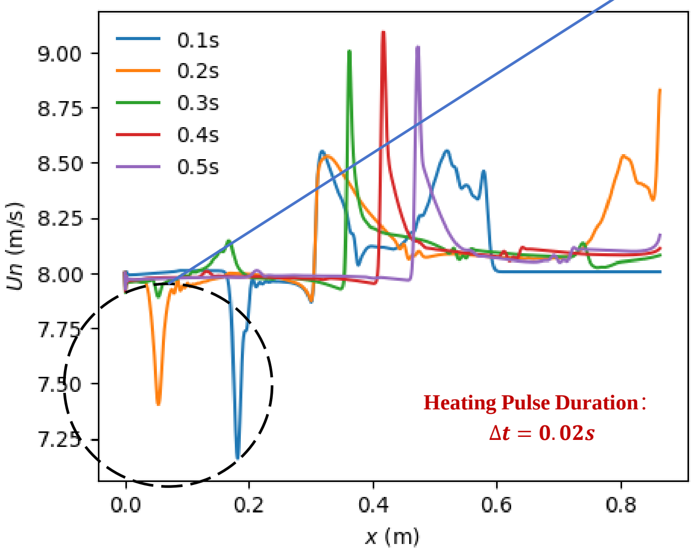
Temperature Distribution

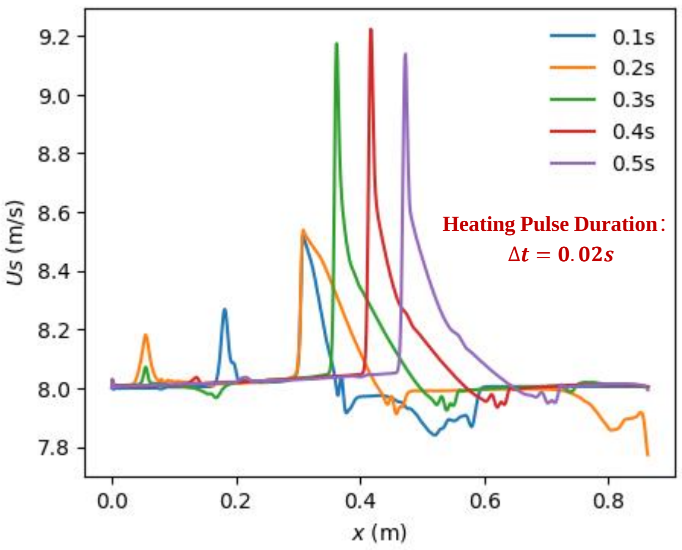


- As the incoming flow velocity increases, the fundamental phenomena remain the same as those observed at lower flow velocities.
- However, a higher incoming flow velocity **suppresses the thermal counterflow**. At this time, for the normal components, the maximum reduction upstream is **1.63%**, while the maximum increase downstream is **15%.**
- Furthermore, at higher flow velocities, the extreme peak of velocity occurs after the heater is turned off.
- **The transmission of the second sound also causes fluctuations in the velocity field**, which propagate more quickly than the effects induced by the heater. Moreover, the impact it has on the normal and superfluid components, as well as upstream and downstream, is the opposite.

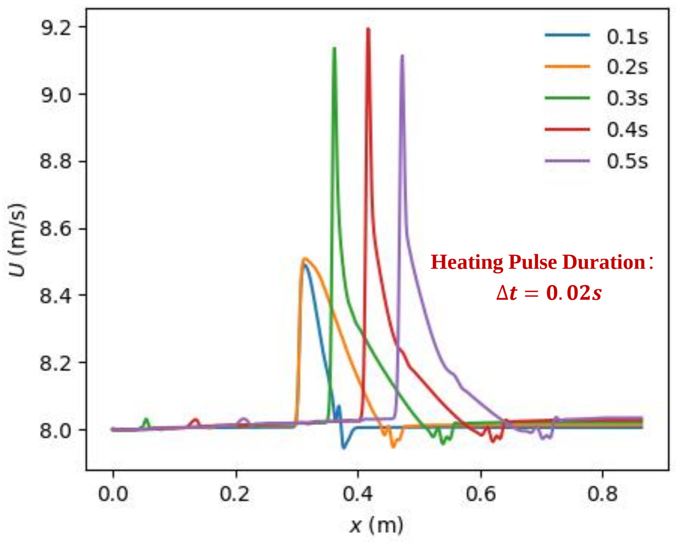
Normal Component Velocity Distribution



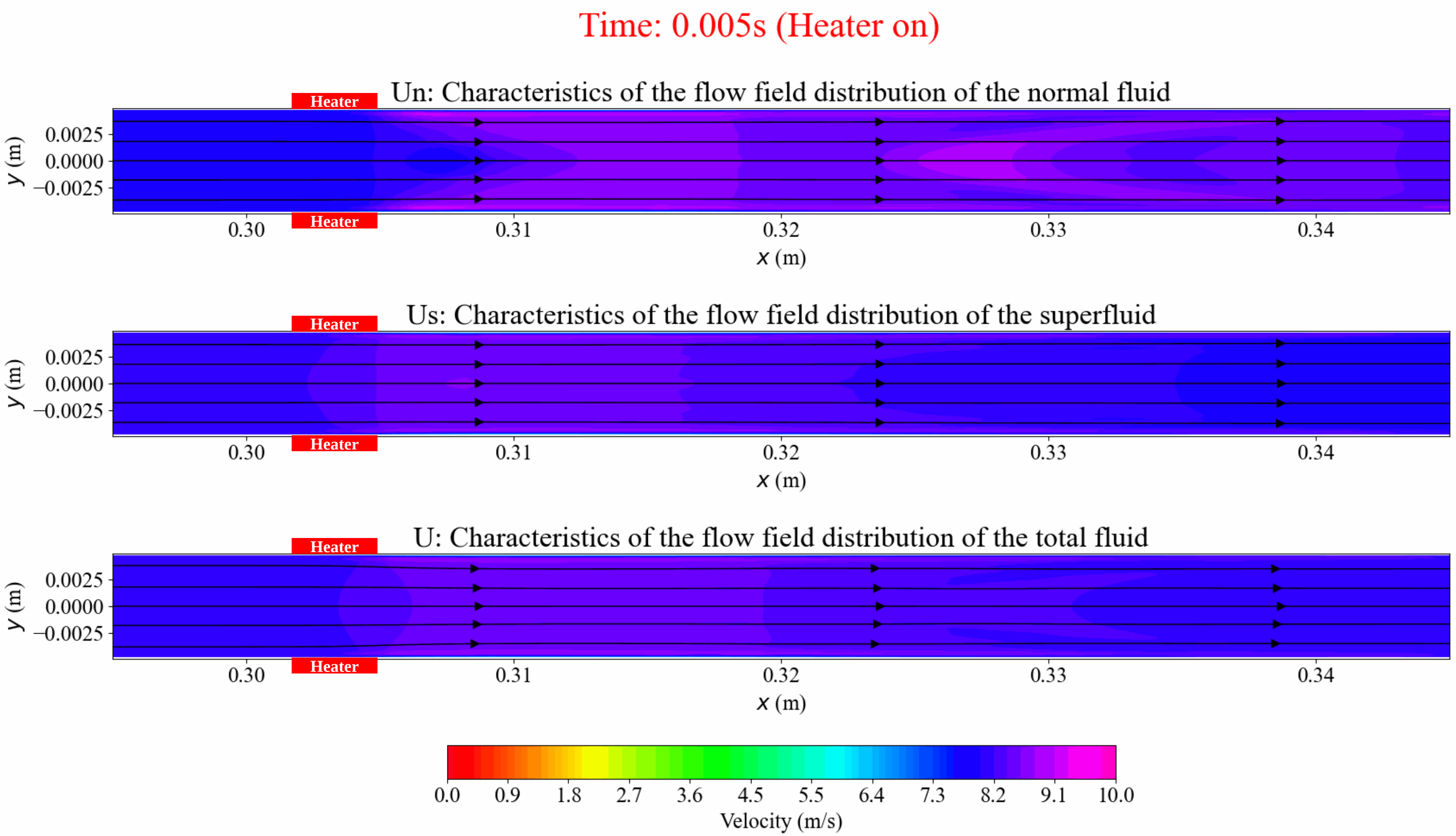
Superfluid Component Velocity Distribution



Total Velocity Distribution



The 2D Velocity Profile in 8m/s



- At high velocity, the velocity **boundary layer still exists**;
- the flow field still contracts radially, **but the degree of contraction is not as pronounced as it is at lower velocity.**
- After the heater is turned off, the velocity at which the nozzle structure moves downstream is also **faster** compared to the situation with lower incoming flow velocities.

CONCLUSION

- We have combined the **Navier-Stokes equations** with the **Vinen equations** to develop an OpenFOAM solver that takes into account **quantum effects** in the flow of superfluid helium.

- The model has been validated by the experiments of Fuzier et al., and it can effectively reflect the energy transfer and flow information in the superfluid helium flow field, as well as capture the **propagation of the second sound**.

- In the forced flow heat transfer, we **observed the suppressive effect of thermal counterflow on the upstream normal fluid flow field and the enhancing effect on the downstream**, while the superfluid experiences the opposite. This effect diminishes with an increase in the incoming flow velocity.

- In the 2D flow field analysis, we found the existence of **the boundary layer** caused by the heater and the **“nozzle” phenomenon**; after the heater is turned off, the “nozzle” structure will detach from the heater and move downstream.

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浙江大學
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CRYOBOAT
A group attracted to cold

Thank you!

Any Question?