

# Thermodynamic analysis of a high cooling capacity dilution refrigerator under the critical velocity limitation of the dilute phase

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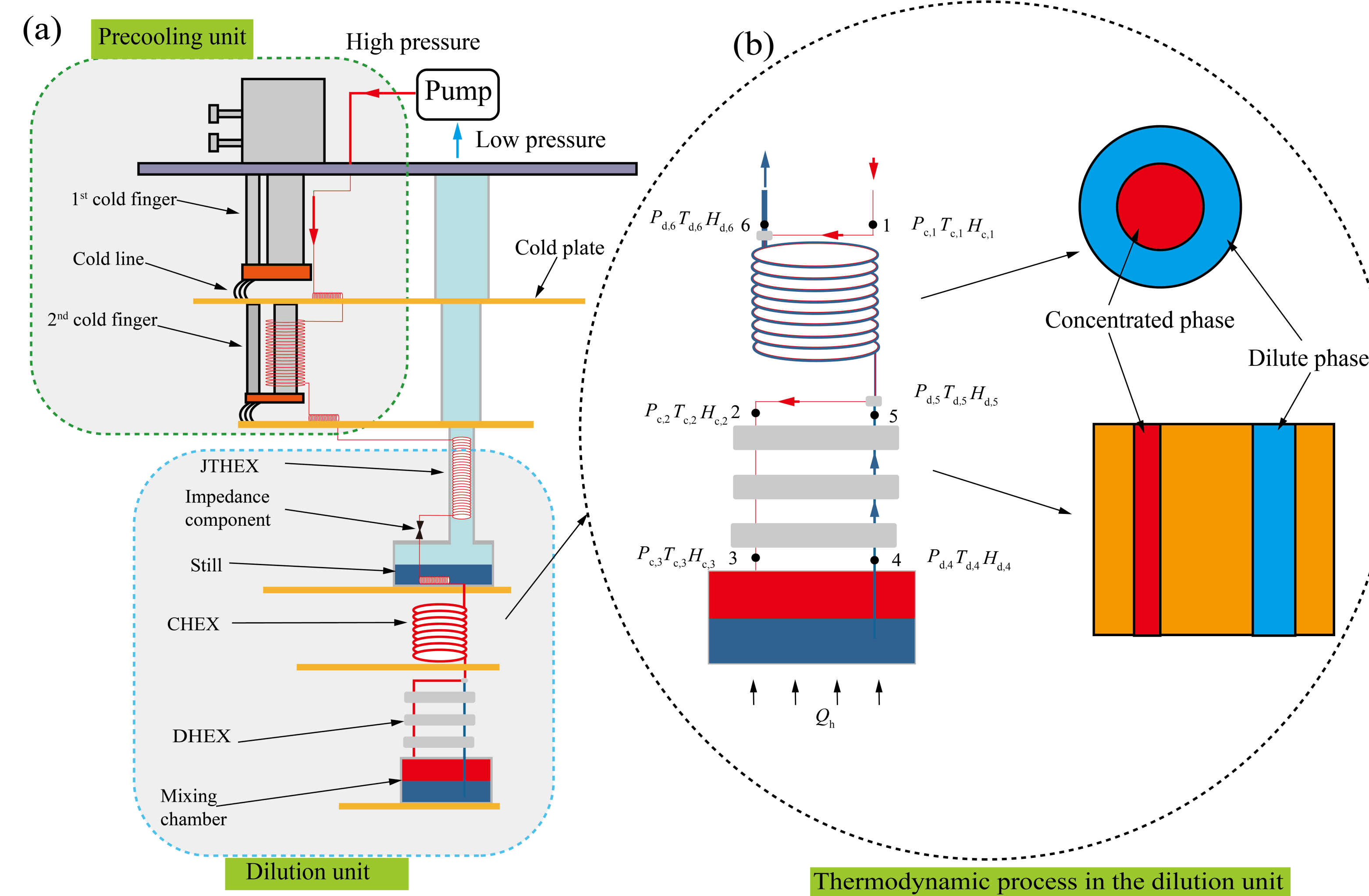
## Introduction

- ❖ The rapid development of superconducting quantum computing technology has stimulated the demand for cryogen-free dilution refrigerators with high cooling capacity (>1000 μW@100 mK, >10 μW@10 mK).
- ❖ The method of increasing the mass flow rate to obtain a high cooling capacity is limited by the critical velocity of the dilute phase.
- ❖ Increasing the flow rate to achieve high cooling capacity while increasing viscous heating significantly degrades refrigeration performance at temperatures ranging from 10 mK to 100 mK.

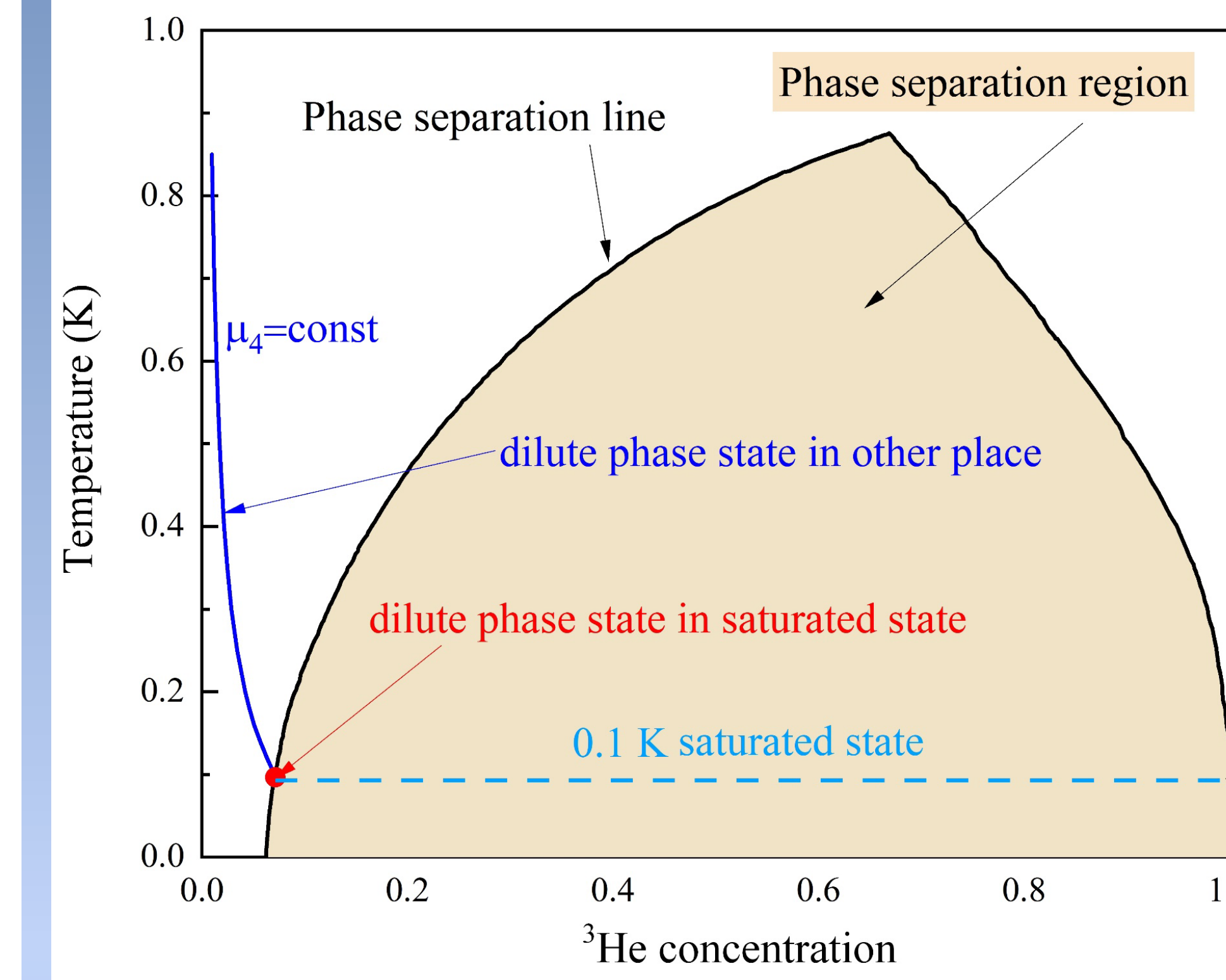
### Highlights

- Equal chemical potential of superfluid <sup>4</sup>He.
- Real physical properties of <sup>3</sup>He and <sup>3</sup>He-<sup>4</sup>He mixtures.
- Comprehensive analysis of viscous heating under the critical velocity.

## Schematic diagram of dilution refrigerator



## Critical velocity and viscous heating



Tube diameter (mm)	Critical velocity (m/s)	Molar flow rate (μmol/s)
0.5	0.0350	184
1	0.0210	440
2	0.0122	1024
4	0.0070	2339
5	0.0058	3040
6	0.0050	3762
10	0.0033	6806
20	0.0018	15062

$v_{d,3} = [5 \times 10^{-6} \times \ln(\frac{d}{15 \times 10^{-6}})] / d$

Below the critical velocity, there is no friction between <sup>3</sup>He and <sup>4</sup>He atoms, which is the mechanical vacuum model. Under the assumption of this model, <sup>4</sup>He in the dilute phase satisfies the assumption of equal chemical potential.

❖ The enthalpy of the dilute phase outside the phase interface is considered to be distributed along the equal chemical potential line.

Viscous heating

$Z = \frac{128l}{\pi(2R_1)^4}$   
 $\Delta P = (\eta\dot{V})Z$   
 $Q_{vi} = \Delta P \cdot \dot{V}$

Normal flow

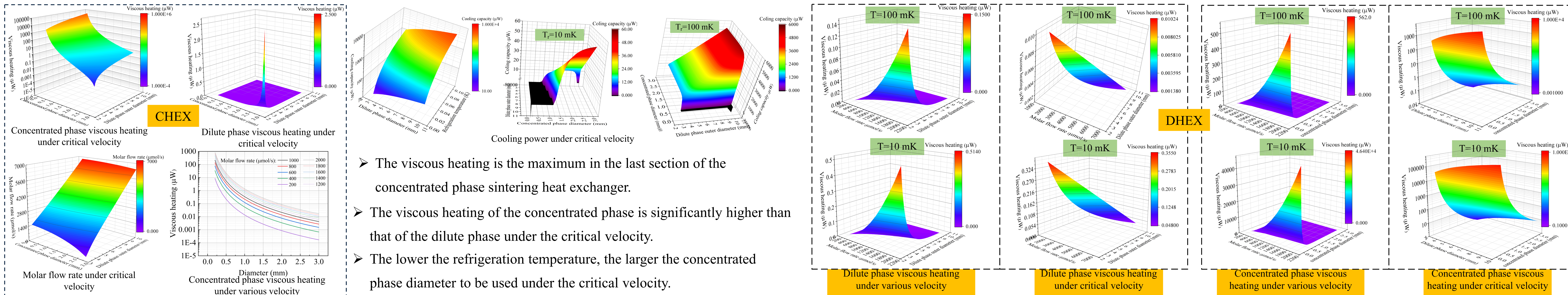
Annular flow

$$Z = 8 \frac{l}{\pi \left[ R_2^4 - R_1^4 - \frac{(R_2^2 - R_1^2)^2}{\ln R_2 / R_1} \right]}$$

$$\Delta P = (\eta\dot{V})Z$$

$$Q_{vi} = \Delta P \cdot \dot{V}$$

## Result



- The viscous heating is the maximum in the last section of the concentrated phase sintering heat exchanger.
- The viscous heating of the concentrated phase is significantly higher than that of the dilute phase under the critical velocity.
- The lower the refrigeration temperature, the larger the concentrated phase diameter to be used under the critical velocity.

## Conclusions

- ◆ The cooling capacity under a critical velocity of 100 mK and 10 mK refrigeration temperatures has been studied. And the tube diameters at 0-50 μW@10 mK, 0-5000 μW@100 mK have been discussed.
- ◆ With the increasing tube diameter, the critical velocity decreases, and the molar flow rate increases.
- ◆ Under critical velocity, there is a minimum diameter limit for cooling capacity. When the refrigeration temperature is 10 mK, the concentrated phase diameter needs to be greater than 1.5 mm. When the refrigeration temperature is 100 mK, the concentrated phase diameter needs to be greater than 0.5 mm.