

Introduction

Rationale: In cryogenic fuel tanks, sufficient vapour pressure must be maintained to enable fuel flow. Effect of sloshing on heat transfer between liquid and vapour phases is generally observed to result in a pressure drop in stratified tanks – highly dependent on fill level & excitation.

Aims: Model and extract heat transfer correlations for planar stable and chaotic sloshing in spherical cryogenic tanks, using ANSYS FLUENT.

Experimental Data (Moran et al, 1994)

Liquid hydrogen sloshing tests conducted at NASA's Lewis Research Centre:

- Spherical tank ($R = 0.75\text{m}$, $V = 1.75\text{m}^3$).
- Lateral sloshing only.
- Data (sufficient for model comparison) provided for two tests (see below).

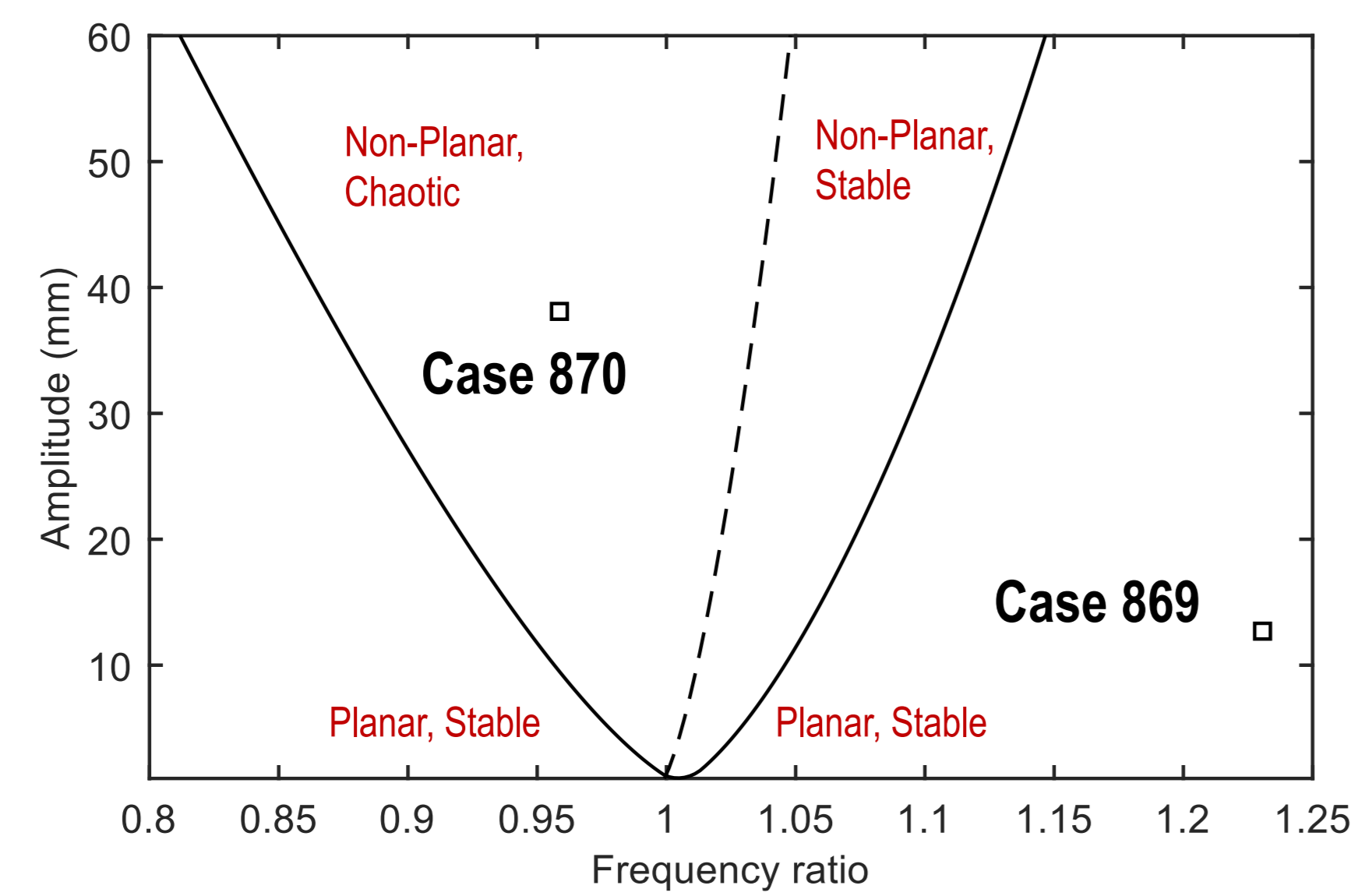


Figure 1: Theoretical lateral sloshing stability map for spherical tank at 67% fill [1], assuming inviscid fluid. Frequency ratio = f/f_1 .

Overview of sloshing experiments [1]

Case #	Fill	Sloshing "mode"	Sloshing frequency	Sloshing amplitude	Re_s
869	64%	1 st mode planar (stable)	0.95 Hz	$\pm 0.0127\text{ m}$ Linearly increased over 30s	5.9×10^6
870	67%	Chaotic	0.74 Hz	$\pm 0.0381\text{ m}$ Linearly increased over 10s	6.6×10^4

Methods

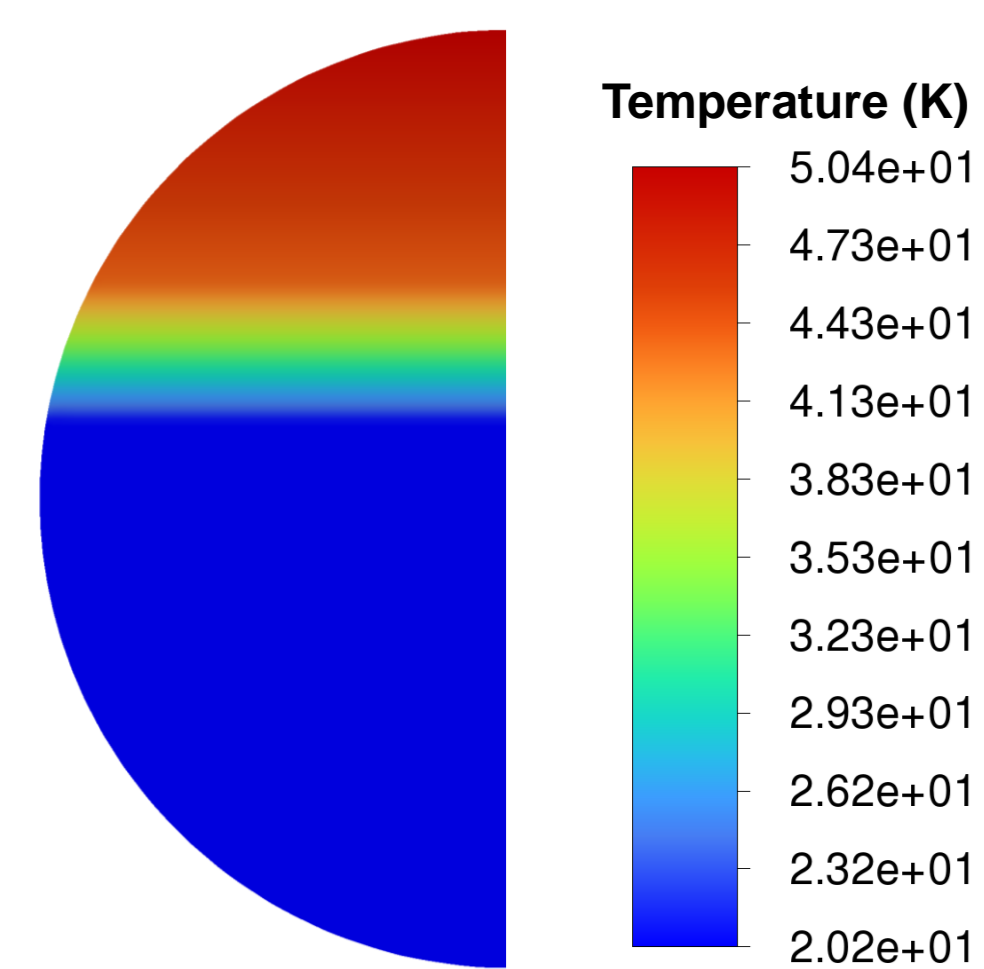


Figure 2: Initial liquid and vapour temperature for Case 870. This is assumed to be identical for Case 869.

Model Overview	Solution Methods
<ul style="list-style-type: none"> RANS turbulence modelling Lee Model for interfacial mass transfer Structured mesh (~890K cells) Adiabatic wall conditions Liquid treated as incompressible fluid Vapour treated as ideal gas 	<ul style="list-style-type: none"> PISO for velocity-pressure coupling Second order upwind for all convective terms Convergence criteria: e-10 for energy, e-4 for all others. Fixed step size of 5e-4 s

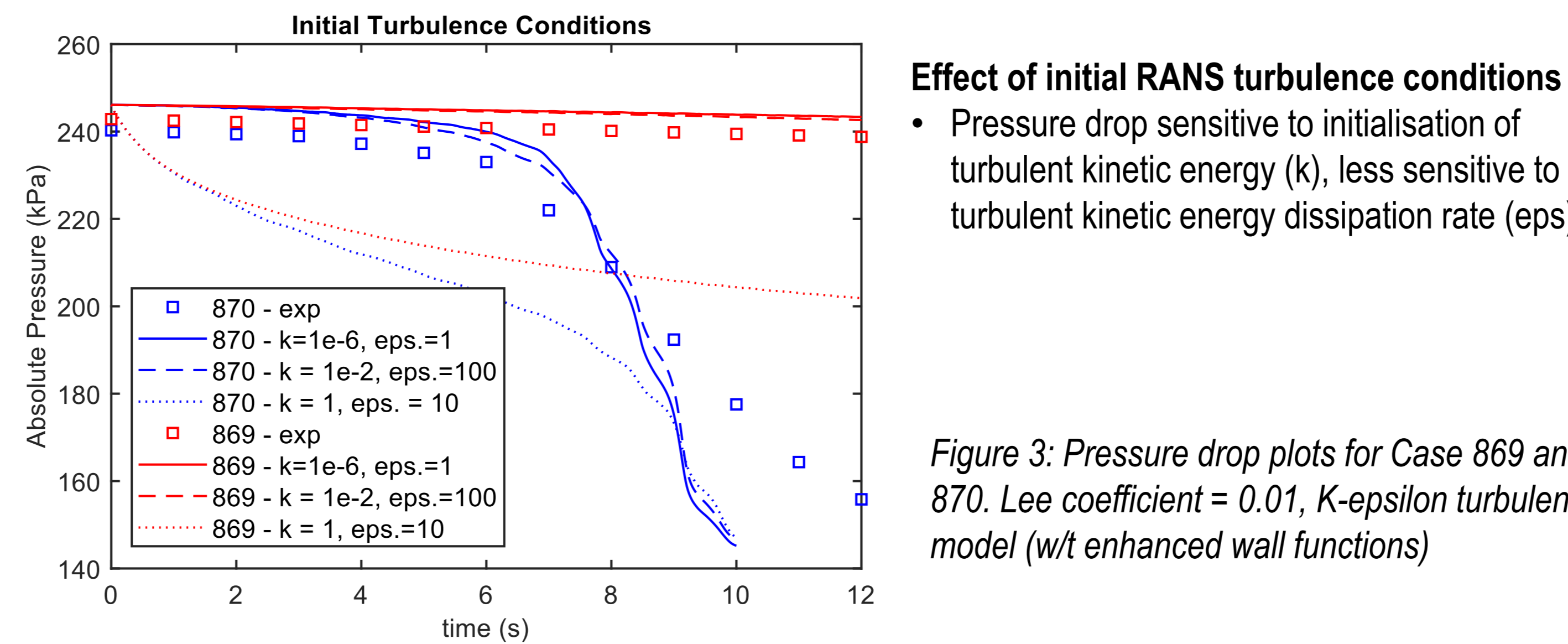
Initial Conditions

Initial gauge pressure of 143 kPaG. Initial liquid temperature approximated as semi-infinite solid:

$$T_l(x) = T_{\text{sat,p1}} - (T_{\text{sat,p1}} - T_{l,i}) \operatorname{erf}\left(\frac{x}{\alpha_1 t_{\text{ramp}}}\right)^{-0.5}$$

$$T_{\text{sat,p1}} = 23.6\text{ K}, T_{l,i} = 20.223\text{ K}, \alpha_1 = 1.247 \cdot 10^{-7}, t_{\text{ramp}} = 28\text{ s}$$

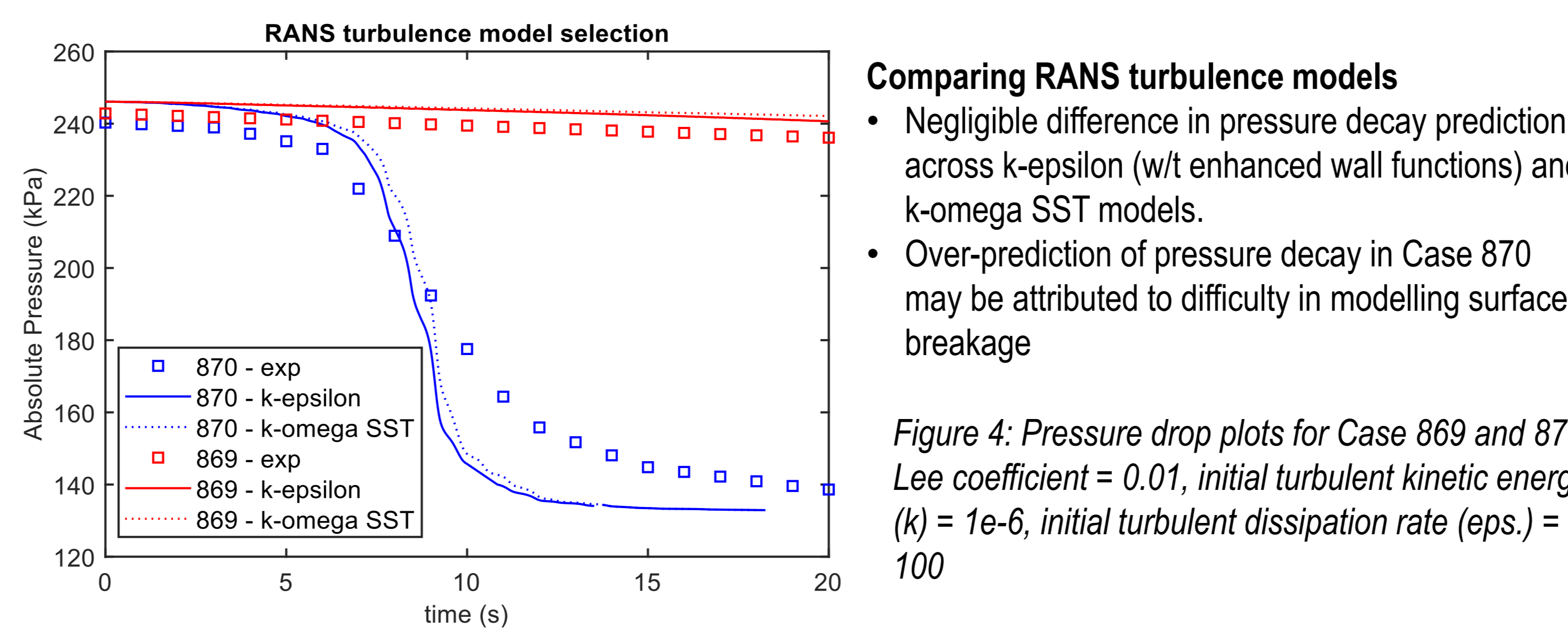
Model Comparison with Exp. Data



Effect of initial RANS turbulence conditions

- Pressure drop sensitive to initialisation of turbulent kinetic energy (k), less sensitive to turbulent kinetic energy dissipation rate (ϵ).

Figure 3: Pressure drop plots for Case 869 and 870. Lee coefficient = 0.01, K-epsilon turbulence model (w/ enhanced wall functions)



Comparing RANS turbulence models

- Negligible difference in pressure decay prediction across k-epsilon (w/ enhanced wall functions) and k-omega SST models.
- Over-prediction of pressure decay in Case 870 may be attributed to difficulty in modelling surface breakage

Figure 4: Pressure drop plots for Case 869 and 870. Lee coefficient = 0.01, initial turbulent kinetic energy (k) = $1e-6$, initial turbulent dissipation rate (ϵ) = 100

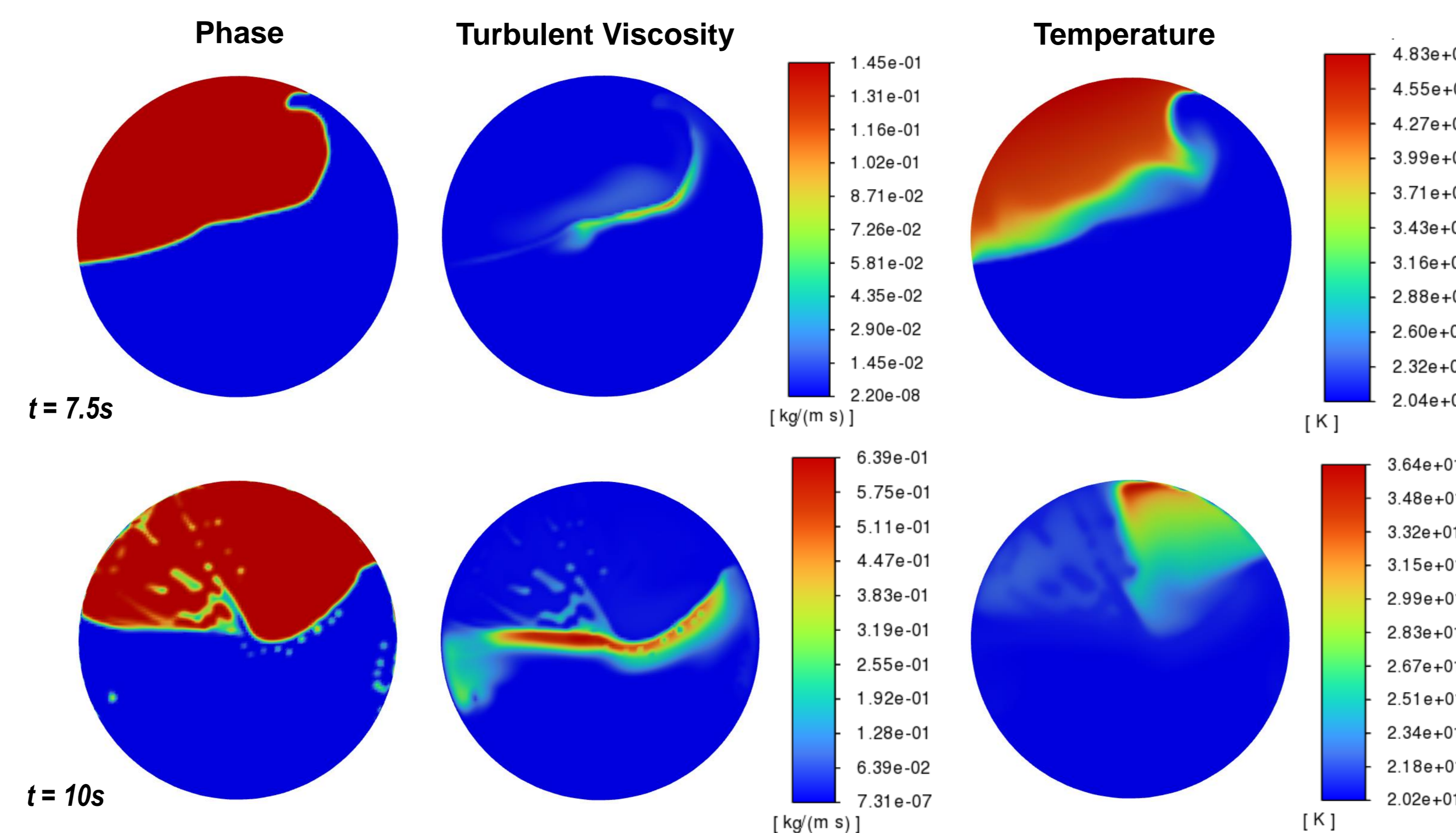


Figure 5: Case 870, k-epsilon, Lee coefficient = 0.01

Key Observations

- Negligible difference for Lee coefficient variation between 0.01 to 0.001. Over-estimation of condensation ≥ 0.1 .
- Model is sensitive to initialisation of turbulence kinetic energy and dissipation.
- For chaotic sloshing (Case 870), rate of pressure drop is over-estimated between 8 – 10 s, during overturning and splashing of liquid.
- Model appears able to capture "final" pressure in Case 870 and rate of pressure decay in Case 869.

[1] Moran M. E., M.N.B., Kudlac M. T., Habersbusch M. S., Saturnino G. A. *Experimental results of hydrogen sloshing a 62 cubic foot (1750 liter) tank. in 30th Joint Propulsion Conference and Exhibit.* 1994.

[2] Ludwig, C., M.E. Dreyer, and E.J. Hopfinger, *Pressure variations in a cryogenic liquid storage tank subjected to periodic excitations.* International Journal of Heat and Mass Transfer, 2013. **66**: p. 223-234.

Extracting Heat and Mass Transfer

Vapour-Interface & Interface-Liquid Heat Transfer

Nusselt number defined as: $Nu = \frac{q dx}{k dT_{t=0}}$

Ludwig et al. (2013) [2] derived the following empirical correlation for interface-liquid heat transfer based on 9 data points in the literature:

$$Nu_{int-liq} = \left(\frac{Re_s}{Re_{s,c}}\right)^{0.69} Pr^{1/3} \text{ Where: } Re_{s,c} = 4 \cdot 10^3 \pm 20\%, \quad Re_s = \frac{2\pi f b^2}{\nu}$$

f_1 = 1st mode nat. frequency, b = wave amplitude, ν = kin. viscosity, ϵ_1 = sloshing eigenvalue
 $f_{1,870} = 0.772\text{ Hz}$, $f_{1,869} = 0.785\text{ Hz}$ [1], $b_{870} = 0.45\text{ m}$, $b_{869} = 0.045\text{ m}$ [2]

Based on the assumptions:

- Initial liquid temperature profile can be approximated using an semi-infinite solid analogy.
- Vapour-interface heat transfer is negligible ($\dot{Q}_{int-liq} = \dot{m}_{cond} h_{fg}$).
- Vapour can be treated as an ideal gas.

Estimated Nusselt Numbers [2]

Case #	Nu_{Ludwig} [2]		$Nu_{model,avg,5-10s}$		$Nu_{model,max}$	
	Vap – int	Int – liq	Vap – int	Int – liq	Vap – int	Int – liq
869	-	7.1 – 8.1	236.4	12.1	329.4	17.1
870	-	157.2 – 180.5	4,209.1	183.9	7,993.2	343.4

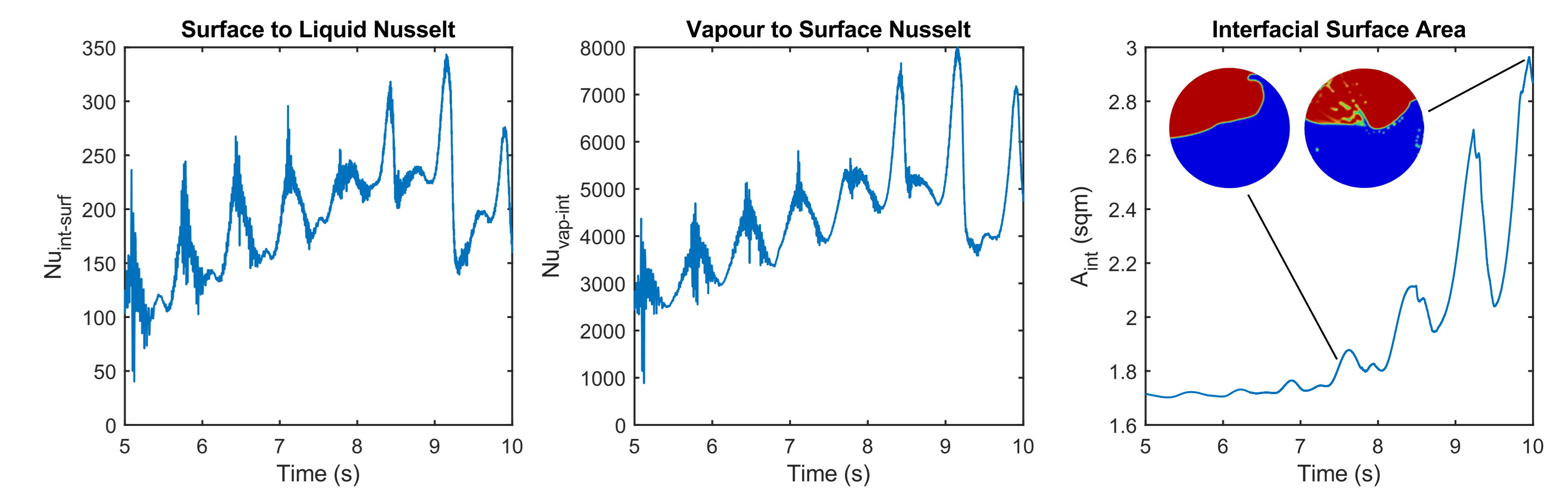


Figure 6: Nusselt number for vapour-interface (left) interface-liquid (centre) heat transfer, and interfacial surface area (right) for Case 870.

Key Observations

- Extracted Nu generally higher than predicted by empirical correlation and appears highly time-dependent, with fluctuations corresponding to sloshing half-periods.
- Increase in interfacial surface area (due to splashing) predicted to be large contributor to pressure drop.
- Vapour-interface heat transfer and sensible heat transfer predicted to be non-negligible contributor to overall pressure decay.

Conclusion

- VOF method appears able to capture pressure decay during "stable planar" sloshing – however, pressure decay over-predicted in "chaotic" sloshing case.
- Vapour-interface and interface-liquid heat transfer oscillations highly temporal.
- Extracted Nusselt number for interface-liquid heat transfer generally higher than that predicted by existing correlations.

Future Modelling Work

- Further investigation of lateral sloshing at different amplitudes and excitation frequencies.
- Investigating contribution of tank wall thermal mass and initial temperature.

Acknowledgements: The author acknowledges financial support from the Australian Govt and the Woodside Monash Energy Partnership in the completion of this work.