Course on:

Cryostats for accelerator superconducting devices

Exercises

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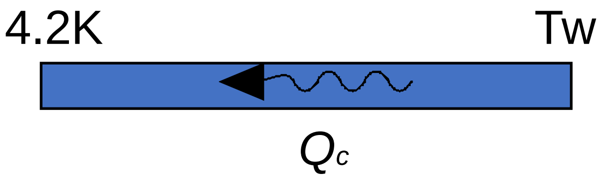
Thermal conductivity integrals (conductance) for some materials [W/m]

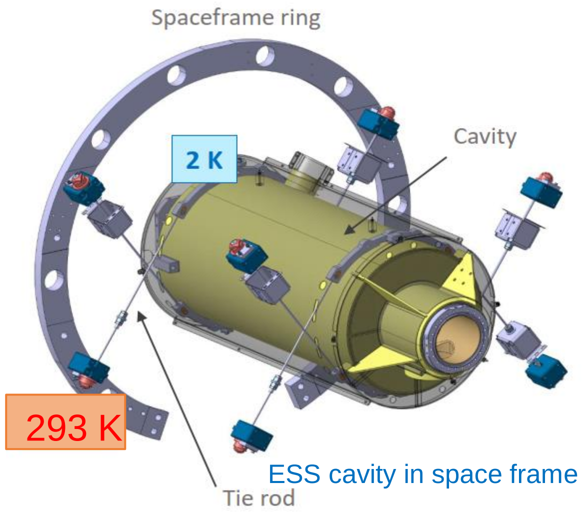
$$Qc = \frac{A}{L} \cdot \int_{4.2K}^{Tw} k(T)dT$$



Highest T *(Lowest T =4.2 K)*	20 K	80 K	290 K
OFHC Copper	11000	60’600	152’000
DHP Copper	395	5’890	46’100
Aluminium 1100	2740	23’300	72’100
Aluminium 2024	160	2’420	22’900
Titanium (Ti-6Al-4V)	170	1’193	1’363
Stainless steel	16,3	349	3’060
Typical Glass-fiber/Epoxy Composite G-10	2	18	153

1. Conduction in supports

- A. Calculate conduction heat loads for the tie rods suspension system:
  - Rod diameter = 6 mm (A = 28 mm^2)
  - Rod length (L) = 500 mm
  - Materials: st.steel (304L) or Ti grade 5 (Ti-6Al-4V)
- B. Calculate conduction heat loads with an intermediate heat intercept at 80 K:
  - Calculate optimal position along the length for Ti grade 5 (Ti-6Al-4V)
  - Calculate heat loads



- A. Conduction (integrals from tables or calculation from Cryostat Toolbox):

$$\dot{Q}_{2K} = \frac{A}{L}\int_{2\,K}^{293\,K} k_{st.\,st.}(T)dT = 28/500 \times 2.97 = 166\text{ mW}$$

$$\dot{Q}_{2K} = \frac{A}{L}\int_{2\,K}^{293\,K} k_{Ti}(T)dT = 28/500 \times 1.36 = 76\text{ mW}$$

- B. Calculate optimal position means <u>minimizing total plug power</u>:
  - $C_{80\,K}$: 16 W/W (30% Carnot)
  - $C_{2\,K}$: 990 W/W (15% Carnot)

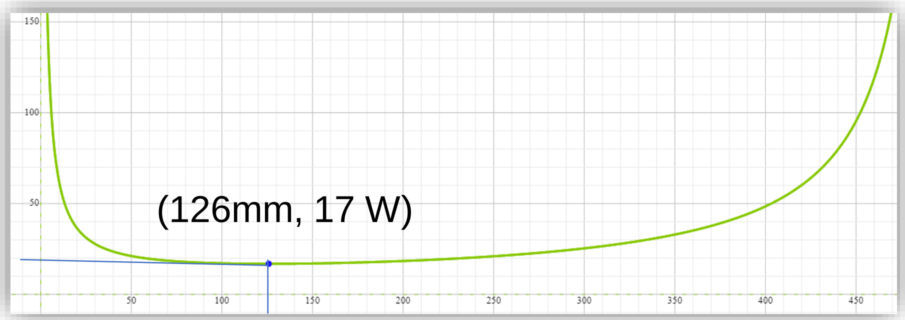
$$W_{tot} = W_{2K} + W_{80K} = C_{2\,K}\left(\frac{A}{(L-x)}\int_{2\,K}^{80\,K} k_{Ti}(T)dT\right) + C_{80\,K}\left(\frac{A}{x}\int_{80\,K}^{293\,K} k_{Ti}(T)dT\right) = \frac{990\cdot 28}{(500-x)}1.19 + \frac{16\cdot 28}{x}0.17$$

$$\frac{d}{dx}\left(\frac{32'986}{(500-x)} + \frac{76}{x}\right) = \frac{32'986}{(500-x)^2} - \frac{76}{x^2} = 0$$

$$\xrightarrow{\text{yields}} \quad x = 126\text{ mm};\ W_{tot} = 17\text{ W}$$

$$\dot{Q}_{80K} = 264\text{ mW}$$

$$\dot{Q}_{2K} = 13\text{ mW}$$

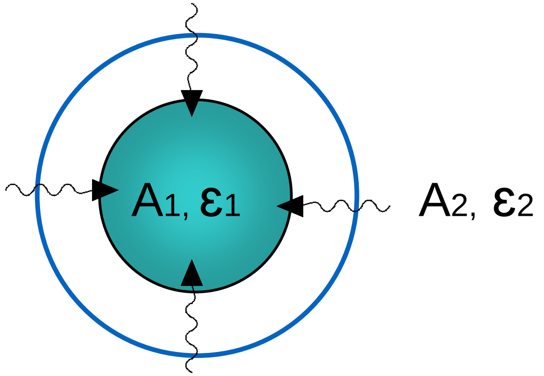


2. Calculate thermal radiation

- Thermal radiation HL for a 1-m cryostat unit length
- Vacuum vessel diameter: 1m (A_2= π x 1= 3.14 m^2)
- Cold mass diameter: 0.5 m (A_1= π x 0.5 = 1.57 m^2)
- T_1 cold mass: 2 K
- T_2 vac.vessel: 293 K
- ε_1 = 0.12 (st.steel, mec.polished, 2 K)
- ε_2 = 0.2 (low carbon.steel, mec.polished, 293 K)

→ $q_{1\text{-}2}$ = 63.5 W (41.6 W/m^2)

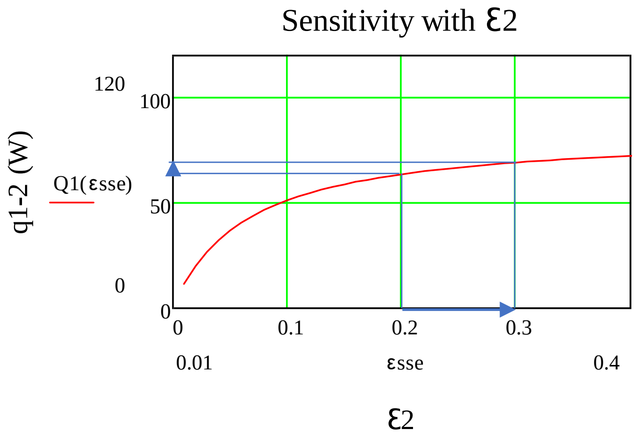
$$q_{1\_2} = \frac{\sigma A_1 (T_1{}^4 - T_2{}^4)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

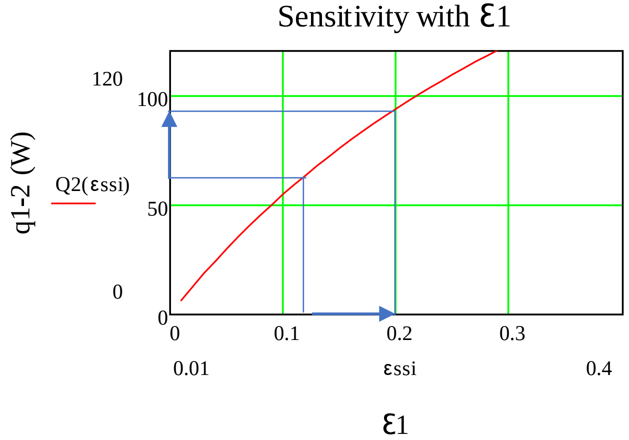


Sensitivity wrt ε uncertainties:

- ε1 = 0.18 (+50%), ε2 = 0.20 → $q_{1\text{-}2}$ = 87 W (+37%)
- ε1 = 0.12, ε2 = 0.30 (+50%) → $q_{1\text{-}2}$ = 69 W (+9%)
- ε1 = 0.18 (+50%), ε2 = 0.30 (+50%) → $q_{1\text{-}2}$ = 98 W (+54%)
- Note: assuming black bodies ($\varepsilon_1 = \varepsilon_2 = 1$) → $q_{1\text{-}2}$ = 656 W ! (x10!)

→ Knowing ε_1 is 4 times more important than ε_2

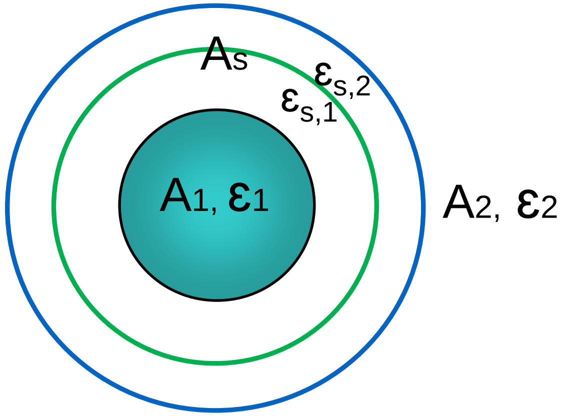




3. Thermal radiation with a floating shield

- Floating Al shield

$$q1-2=\frac{\sigma(T1^4-T2^4)}{\underbrace{\frac{1-\varepsilon 1}{\varepsilon 1 A1}+\frac{1}{A1F1s}+\frac{1-\varepsilon s,1}{\varepsilon s,1As}}_{\text{A1 to S gap}}+\underbrace{\frac{1-\varepsilon s,2}{\varepsilon s,2As}+\frac{1}{AsFs2}+\frac{1-\varepsilon 2}{\varepsilon 2A2}}_{\text{S to A2 gap}}}$$



$$q1-2=\frac{\sigma(T1^4-T2^4)}{\frac{1-\varepsilon 1}{\varepsilon 1\ A1}+\frac{1}{A1}+\frac{1-\varepsilon s}{\varepsilon s\ As}+\frac{1-\varepsilon s}{\varepsilon s\ As}+\frac{1}{As}+\frac{1-\varepsilon 2}{\varepsilon 2\ A2}}$$

Floating Al shield:

- Thermal shield diameter: 0.75 m (A_s= π x 0.75 = 2.35 m²)
- F_{1s}= 1; F_{2s}= 1
- T_s = 80 K (first guess)
- $\varepsilon_{s,1} = \varepsilon_{s,2}$ = 0.1 (Aluminum, mec.polished, 80 K)

→ $q_{1\text{-}2} = q_{1\text{-}s} = q_{s\text{-}2}$ = 28.5 W

- Calculate Ts by *trial and error* to obtain power balance $q_{1\text{-}s} = q_{s\text{-}2}$

→ T_s = 260 K → increase $\varepsilon_{s,1} = \varepsilon_{s,2}$ (0.15?) and recalculate

→ $q_{1\text{-}2} = q_{1\text{-}s} = q_{s\text{-}2}$ = 35.4 W (to be compared to 63.5 W without shield)

→ $q_{1\text{-}s}$ = 35.4 W (22.5 W/m²), $q_{s\text{-}2}$ = 35.4 W (15 W/m²)

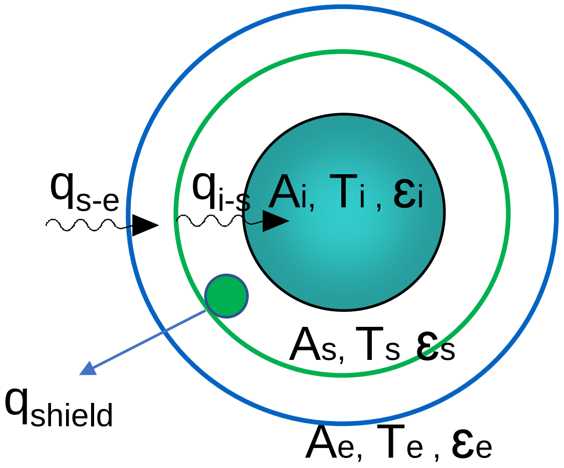
→ 1 floating shield reduces to almost ½ the radiation to the low T (close to flat plates approximation for this geometry)

4. Thermal radiation with an actively cooled shield

- Actively cooled intermediate shield
  - T_s = 80 K (now an input)
  - ε_s = 0.1 (Aluminum, mec.polished, 80 K)
  - Radiation between vessel and shield:

→ $q_{s\text{-}e}$ = 76 W (32 W/m²)

$$q_{s\_e} = \frac{\sigma As(T_s{}^4 - T_e{}^4)}{\frac{1}{\varepsilon_s} + \frac{A_s}{A_e}\left(\frac{1}{\varepsilon_e} - 1\right)}$$



- Radiation between shield (80 K) and helium vessel (2 K):
- ε_i = 0.12 (st.steel, mec.polished, 2 K)

→ $q_{i\text{-}s}$ = 0.26 W (0.16 W/m²) (x 136 times less than with floating shield !)

$$q_{i\_s} = \frac{\sigma Ai(T_i{}^4 - T_s{}^4)}{\frac{1}{\varepsilon_i} + \frac{A_i}{A_s}\left(\frac{1}{\varepsilon_s} - 1\right)}$$

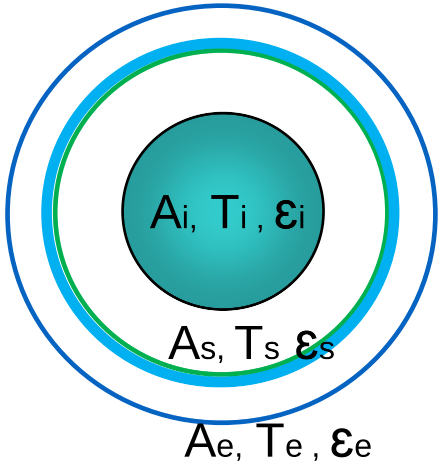
- Radiation heat extraction from thermal shield (not to be forgotten!):

→ $q_{shield} = q_{s\text{-}e} - q_{i\text{-}s}$ = 75.74 W

→ Actively cooled shield at T~ 80 K drastically reduces radiation to the lowest T and "deviates" heat load to the shield *(but do not forget to "pay" for its cooling!)*

5. Thermal radiation with a floating shield with MLI

- MLI with N reflectors, floating shields
- Radiation between vac.vessel and shield with MLI (based on flat plat approximation):



$$q_{s-e} = \frac{\sigma A_{av}(T_s^{\,4} - T_e^{\,4})}{(N+1)(\frac{2}{\varepsilon_{av}} - 1)}$$

- N = 30
- T_s = 80 K (first guess)
- ε_e = 0.2 (low carbon.steel, mec.polished, 293 K)
- ε_s = 0.08 (Aluminium reflector, electrolytical deposition, 80 K)
- ε_{av} = ½ (ε_s+ ε_e) (average emissivity)
- A_{av} = ½ (A_s + A_e) (average area)

→ q_{s-e} = 2.7 W (0.14 W/m²) (with T_s = 80 K)

- Radiation between shield (80 K) and helium vessel (formula between enclosed cylinders):

→ q_{i-s} = 0.23 W (0.012 W/m²) (with T_s = 80 K)

- Calculate T_s by *trial and error* (and tune ε_s) to obtain power balance q_{s-e} = q_{i-s} (floating shield condition)

→ T_s = 143 K, → ε_s= 0.1

$q_{s-e} = q_{i-s}$

→ q_{s-e} = 2.6 W (1.65 W/m²)

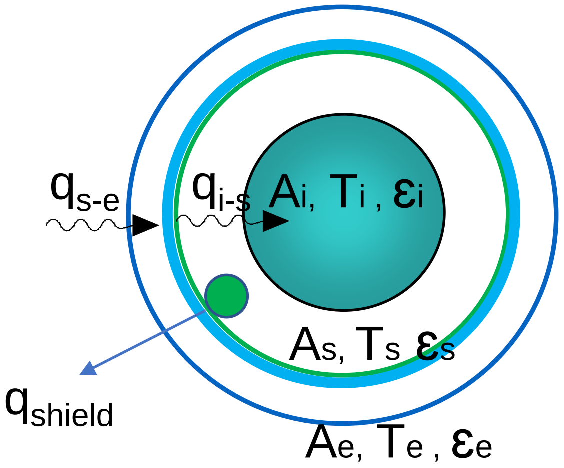
→ q_{i-s} = 2.6 W (0.96 W/m²)

→ MLI (30 layers) reduce the HL to the thermal shield (and cold surface) by a factor x 15 wrt simple shield

6. Thermal radiation with an actively cooled shield with MLI

- MLI with N reflectors, actively cooled shield
- Radiation between shield and vac.vessel (flat plat approximation):

$$q_{s-e} = \frac{\sigma\, Aav\,(Ts^4 - Te^4)}{(N+1)(\frac{2}{\varepsilon_{av}} - 1)}$$



- N = 30
- T_s = 80 K (now an input)
- ε_s = 0.08 (Aluminium reflector, electrolytical deposition, 80 K)
- ε_{av} = ½ (ε_s+ ε_e) (average emissivity)
- A_{av} = ½ (A_s + A_e) (average area)

→ q_{s-e} = 2.78 W (1.0 W/m^2)

- Radiation between shield (80 K) and helium vessel (2 K) (formula between enclosed cylinders):

→ q_{i-s} = 0.23 W (0.14 W/m^2) (x10 times less than with floating shield !)

- Radiation heat extraction from thermal shield (not to be forgotten!):

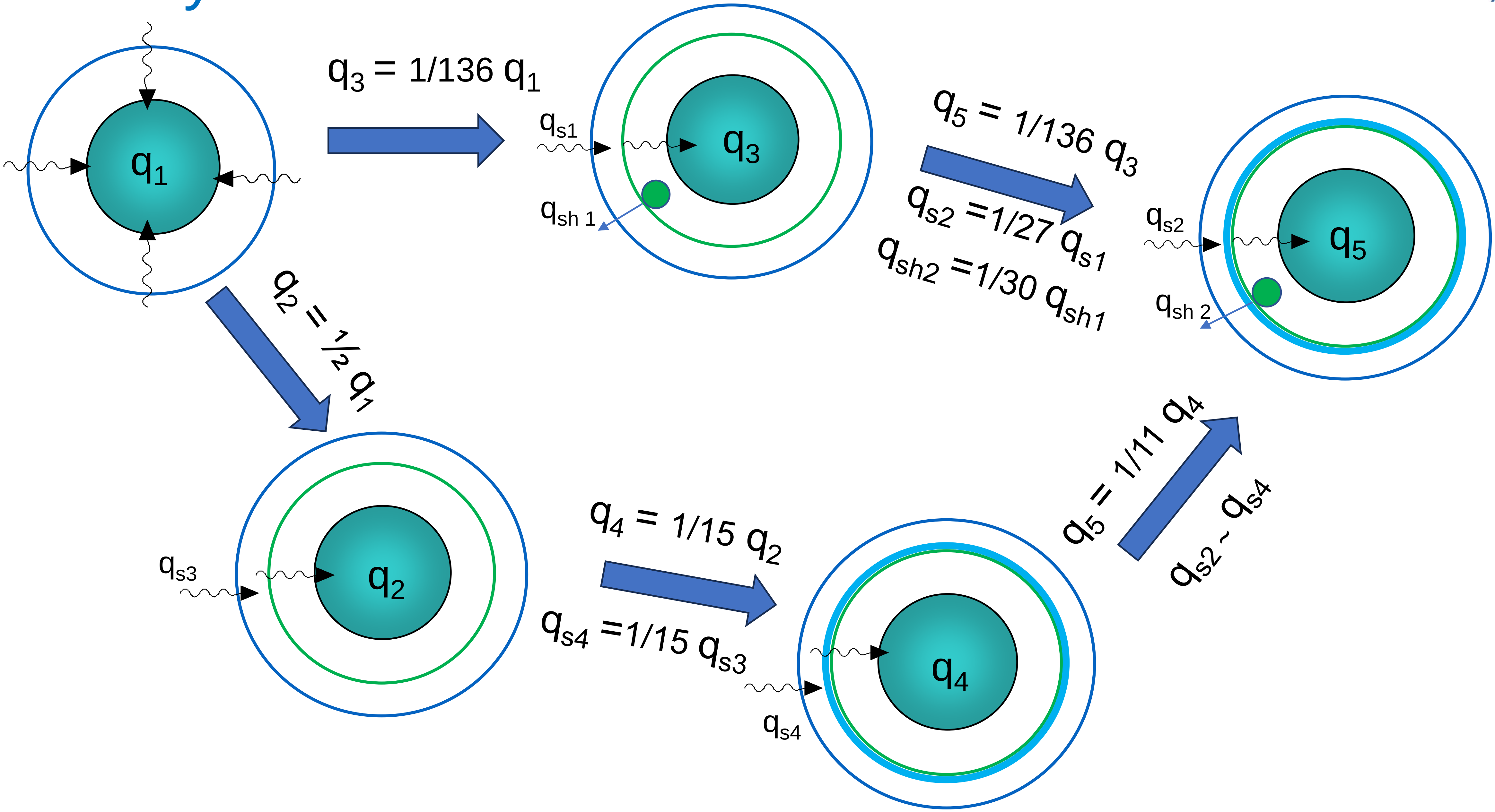
→ q_{shield} = q_{s-e} - q_{i-s} = 2.55 W

$$q_{i-s} = \frac{\sigma As(T_s{}^4 - T_i{}^4)}{\frac{1}{\varepsilon_s} + \frac{A_s}{A_i}\left(\frac{1}{\varepsilon_i} - 1\right)}$$

→ Actively cooled shield at ~80 K drastically reduces Heat Load to the lowest T
→ MLI reduces the Heat Loads to the shield

An actively cooled shield at ~80 K with MLI is standard practice in all helium cryostats!

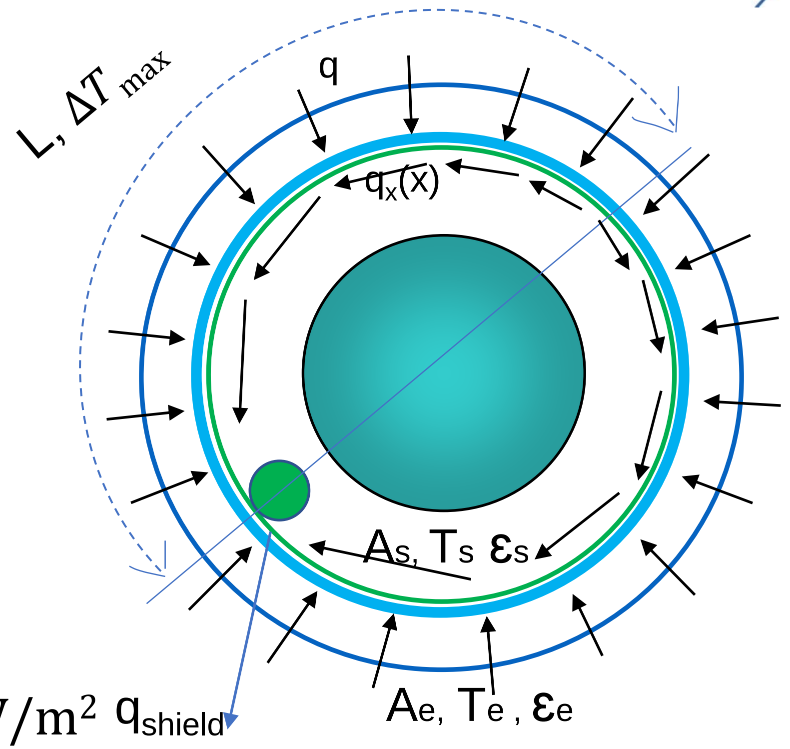
Summary



q1
q3 = 1/136 q1
qs1
q3
qsh 1
q5 = 1/136 q3
qs2 =1/27 qs1
qsh2 =1/30 qsh1
qs2
q5
qsh 2
q2 = ½ q1
qs3
q2
q4 = 1/15 q2
qs4 =1/15 qs3
q4
qs4
q5 = 1/11 q4
qs2 ~ qs4

7. calculate a thermal shield thickness in an LHC-type cryostat

$$t = \frac{q \cdot L^2}{2 \cdot k \cdot \Delta T_{\max}}$$



- Symmetry: point opposite of cooling line is an isolated tip
- Diameter thermal shield: 0.75 m → A_s= π x 0.75 x 1 = 2.35 m²
- Take ½ shield: $L = \frac{1}{2}\frac{A_s}{1\ m}$ = ½ 2.35 m = 1.17 m
- Uniform heat deposition (previous exercise): $q = \frac{q_{shield}}{A_s} = \frac{2.55\ W}{2.35\ m^2} = 1.08\ \mathrm{W/m^2}$
- Let's set the maximum $\Delta T_{\max} = 5\ \mathrm{K}$
- Take k aluminium (Al6061) at ~80 K: 85 W/(K m)

→t = 1.7 mm

Remark:

- If thermal shield in Cu alloy (e.g. CuNi70-70)→ k= 37 W/(K m) → t = 0.74 mm …too thin sheets for mech. stability