Benchmarking Proton Bremsstrahlung for Dark Sector Production

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with Adam Ritz, 2108.05900 and to appear
Proton Bremsstrahlung

- Primary production channel in the forward direction for intermediate mass range 0.5 - 1.5 GeV at the proton beam facilities
  [deNiverville, Pospelov, Ritz ’16]
- Important regime near vectors ($\rho, \omega, ...$), and scalar ($f_0, ...$) meson resonances

- Equivalent Particle Approx. or F.W.W.: a well-known approach in QED for relativistic collinear scattering & radiation
  [Kim, Tsai ’73; Gribov & Lipatov ’72; Baier, Fadin & Khoze ’73; Altarelli & Parisi ’77; Bjorken, Essig et al ’09 ...]
Elastic Scattering via Pomeron Exchange

- Donnachie & Landshoff (DL) model [DL ’82, 84, 11, 13]

- Single pomeron exchange fits data well for low $t$

\[
\frac{d\sigma^{\text{el}}}{dt} \simeq \frac{1}{4\pi} (Y_{p} F_{1}(t))^{4} |G(s, t)|^{2}
\]

- Pomeron Trajectory contributes a power $s^{\alpha(t)-1}$ to the scattering amplitude

\[
G_{p}(s, t) = (s\alpha_{p}')(\alpha_{p}(t))^{-1}, \quad \Gamma\mu(t) = Y_{p} F_{1}(t)\gamma\mu
\]

\[
\alpha_{p}(t) \approx 1.08 + 0.25t
\]
Radiation in Quasi-Elastic scattering

• Modeling forward $pp$ scattering with Pomeron

Observe the large cancellation between ISR & FSR in quasi-elastic scattering
Soft Photon Production

• Contrast the radiation in quasi-elastic processes involving the Pomeron with the soft photon approximation (SPA) for SM photon production

• Soft singularity for $k \to 0$

$$d\sigma|_{\text{SPA}} \sim \frac{1}{k} \left( \frac{p'^\mu}{p \cdot k} - \frac{p'^\mu}{p' \cdot k} \right)^2 d\sigma_{\text{el}}$$

[Low ‘58, Burnett-Kroll ‘68, Bell-VanRoyen ‘69]
Radiation in Diffractive Processes

• Non-Diffractive processes constitute up to 60% of $\sigma_{tot}$

• The dominant contribution comes from ISR in non-single diffractive scattering.
Production via ISR

• “Equivalent proton” or Quasi-Real Approx.

Collinear radiation with low $p_T$
intermediate $p'$ near on-shell

$$\frac{p_T^2}{4z(1-z)^2p_p^2} \ll 1$$

[Khoze, Fadin] [Altarelli, Parisi]

• The cross section divides into a splitting probability and the cross section of the subprocess involving the $p'$ with the reduced energy:

$$d\sigma^{pp}\rightarrow Df(s) \approx dP_{p\rightarrow p'}D \times \sigma_{pp}^{NSD}(s')$$
Production via ISR

- The dark vector is radiated with timelike momentum, thus can mix with hadronic resonances, while the intermediate proton is slightly off-shell, so the vertex is a transition form-factor

- **Time-like** nucleon form factor:
  - Mixing with meson resonances

[Faessler et al ‘09]
[Adamuscin et al ‘16]
Revisiting Proton Bremsstrahlung

Analyze the “equivalent proton” or quasi-real approximation (QRA) to calculate the splitting function for ISR; Convolute with $\sigma_{NSD}$ (not total)

Red band from varying transition form-factor scale $\Lambda \sim m_p$ from 1 to 2 GeV
Revisiting Proton Bremsstrahlung

Analyze the “equivalent proton” or quasi-real approximation (QRA) to calculate the splitting function for ISR; Convolute with $\sigma_{NSD}$ (not total)

Hadronic generalization with an “equivalent pomeron” provides an excellent approximation to (suppressed) quasi-elastic ISR+FSR radiation

Dark Vector Radiation, $\sqrt{s} = 14$ [TeV]

$\pi^0 \rightarrow \gamma V$

$\eta \rightarrow \gamma V$

$\theta_V^{CM} < 1$ mrad

Drell–Yan

Quasi–Real Approx. (NSD)

Pomeron–Exchange (Elastic)

Modified W.W. Approx.
An “equivalent vector” approximation is used to estimate the ISR splitting function; convoluted with $\sigma_{tot}$.

Hadronic generalization with an “equivalent pomeron” provides an excellent approximation to (suppressed) quasi-elastic ISR+FSR radiation.

Analyze the “equivalent proton” or quasi-real approximation (QRA) to calculate the splitting function for ISR; convolute with $\sigma_{NSD}$ (not total).
Impact on Sensitivity

[SF & Ritz '21]
Benchmarks - $\rho$ production

• Compare these mechanisms for inclusive rho production with data

\[ p+p \rightarrow \rho^0 + X, \quad \sqrt{s} = 27.5 \text{ [GeV]} \]

\[ d\sigma / dx_F \text{ [mb]} \]

• Consider regime with $p_L \ll p_L^{\text{max}}$ (small $x_F$) and $p_T < \text{GeV}$

Inclusive $p+p \rightarrow \rho^0 + X$, $E_{\text{beam}} = 400$ [GeV]

Preliminary

[SF & Ritz, to appear]
Other Benchmarks

• Compare these mechanisms for inclusive photon production with data from LHCf and for dimuon production with data from COMPASS

[COMPASS Collaboration ’13] [LHCf experiment ’17]

• What is the kinematic regime in which coherent ISR transit to partonic-level radiation? Insights form future measurements of PDFs at small $x$. 
Back-up Slides
Overview

• Sub-GeV WIMP requires new annihilation channel through light mediators as part of a dark sector
  [Boehm & Silk et al.]
  [Pospelov, Ritz, Voloshin '07]

• Visible decays of dark photon

\[ m_{A'} < 2m_\chi \]
Hadronic Production

$\pi^{\pm,0}$, $\eta$, $K^{\pm}$ Decay
Proton Bremmstrahlung
Drell-Yan

FASER is taking data now
Modeling forward pp scattering

• Regge Theory: [Regge; Chew, Frautschi ‘61]
  - Trajectory $\alpha(t)$ contributes a power $s^{\alpha(t)-1}$ to the scattering amplitude

$\alpha(t) \approx \frac{1}{2} + 0.9t$

$\sim s^{-\frac{1}{2}}$ for $\rho, \omega, f_2, a_2$ trajectories
Modeling forward pp scattering

- Pomeron Trajectory: [Gribov ‘62]

  - Reggeons are not enough!
  Need to include exchange of another object with trajectory

\[ \alpha_P(t) \approx 1.08 + 0.25t \]
Elastic Scattering via Pomeron Exchange

• Donnachie & Landshoff (DL) model  [D&L ’82, ’84, ’11, ’13]

Effective Propagator & Vertex for Soft Pomeron Exchange:

\[
\frac{d\sigma^{el}}{dt} \sim \frac{1}{4\pi} \left[ Y_P F_1(t) \right]^4 |G(s, t)|^2
\]

Diffractive minimum: analogue to Fraunhofer diffraction: \(|t| \sim p^2 \theta^2\)
Quasi-Real (Fermion pole) Approximation

- Ultrarealistic fermion & radiation is highly collinear

\[ k'_{\mu} = (zp_p + \frac{p^2_T + m^2_D}{2zp_p}, p_T, zp_p) \quad p_T, m_p (m_D) \ll E_p (E_k) \]

- Intermediate-fermion being near on-shell

\[ A(p, p_j) \rightarrow A(p - k, p_j) \frac{i(p - k + m_p)}{(p - k)^2 - m^2_p} \]

- With the cost of being non-covariant:

\[ E_{p'} = \sqrt{(\vec{p} - \vec{k})^2 + m^2_p} \]
Quasi-Real (Fermion pole) Approximation

• Replace the $p'$ propagator by the polarization sum for an on-shell proton

$$\mathcal{M}_{r}^{p \rightarrow D f}(p, k, p_j) \approx \sum_{r'} \mathcal{M}_{r'}^{p \rightarrow f}(p', p_j) \left( \frac{V_{r'r}^D}{2k \cdot p - m_D^2} \right)$$

• Collinear emission does not change the proton helicity

Only the helicity conserving transitions contribute:

$$\frac{1}{2} \sum_{r(, \lambda)} |\mathcal{M}_{r(, \lambda)}^{p \rightarrow D f}(p, k, p_j)|^2 = g_D^2 \left( \frac{Z}{H} \right)^2 \mathcal{I}_D |\mathcal{M}_{p'p \rightarrow f}|^2$$

**Vertex Functions**

$$V_{r'r, \lambda}^V = g_V \bar{u}'(p') \gamma^\lambda(k) u(p)$$

$$V_{r'r}^S = g_S \bar{u}'(p') u(p)$$

![Diagram showing collinear emission](image)
Radiation in Non-Single Diffractive Processes

- The dominant contribution comes from ISR in non-single diffractive scattering.

❖ **Time-like** nucleon form factor:  
  - Mixing with meson resonances  
  [Faessler et al ’09]  
  [Adamuscin et al ’16]

❖ **Transition (Off-Shell)** form factor:  
  - Accounts for the suppression when the intermediate $p'$ goes far off-shell  
  
\[ F_{pp\gamma D}(p'^2) = \frac{\Lambda_p^4}{\Lambda_p^4 + (p'^2 - m_p^2)^2} \]  
  [Feuster & Mosel ’98]
Hadronical Generalization of the FWW Approx.

- Assumptions:
  - Highly collinear radiation with small virtuality of pomeron momentum

\[ t_{\text{min}} = -q_{\text{min}}^2 \approx -H^2/(2z(1-z)p_p)^2 \]

- Cloud of effective flux of pomeron

\[ \chi_p = \int_{t_{\text{min}}}^{t_{\text{max}}} dt \, (t - t_{\text{min}}) |A_{\text{el}}(s, t)|^2 \]

Amplitude \( \sim (s^{-\alpha' t})^2 \)

compare with \( \frac{1}{t^2} \) for photon

\[ \left( \frac{d\sigma_{pp \rightarrow ppD}^{\text{el}}}{dz dp_T^2} \right)_{\text{WW}} \approx \frac{\alpha_D}{16\pi^2} \frac{z(1-z)}{H^2} \times (A_D^{22}|_{t=t_{\text{min}}}) \chi_p \]

Elastic Amplitude
Modified WW Approximation

- Modified version of both fermion-pole and photon-pole approaches [Blümlein & Brunner ’13]

- The **splitting function** is convoluted with $\sigma_{\text{tot}}(s')$

\[
 w_V(z, p_T^2) = \frac{\alpha_e}{2\pi} |F_{1V,(m_V^2)}|^2 \frac{1}{H} \left[ \frac{1+(1-z)^2}{z} - 2z(1-z) \left( \frac{2m_p^2 + m_V^2}{H} - z^2 \frac{2m_p^4}{H^2} \right) 
+ 2z(1-z)(1+(1-z)^2) \frac{m_p^2 m_V^2}{H^2} + 2z(1-z)^2 \frac{m_V^4}{H^2} \right]
\]
Benchmarks - $\rho$ production

- Contrast these mechanisms for inclusive rho production with data from NA27

![Graph 1: $p+p \rightarrow \rho^0 + X$, $\sqrt{s} = 27.5$ [GeV]]

![Graph 2: Inclusive $p+p \rightarrow \rho^0 + X$, $E_{beam} = 400$ [GeV], $\theta = 10$ mrad]

[SF & Ritz, to appear]