Positivity at the LHC

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LHC EFT WG
CERN
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$\mathcal{L}$?

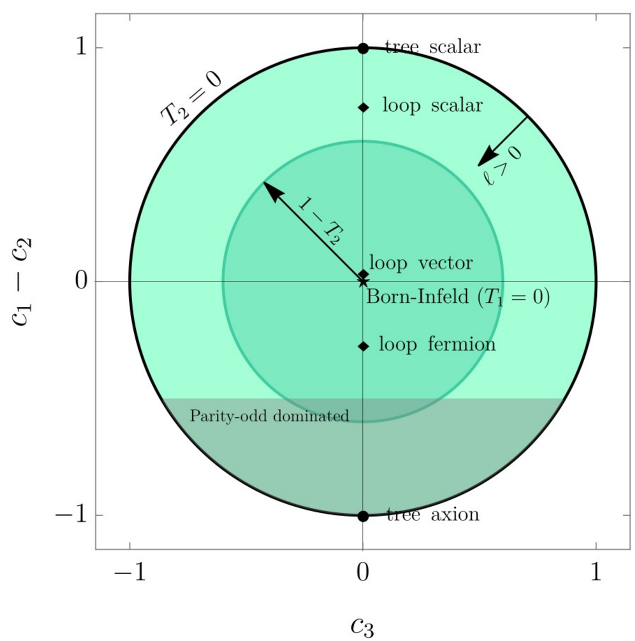
$$\mathcal{L} = -(FF) + c_1(FF)^2 + c_2(F\widetilde{F})^2 + c_3(FF)(F\widetilde{F}) + \dots$$

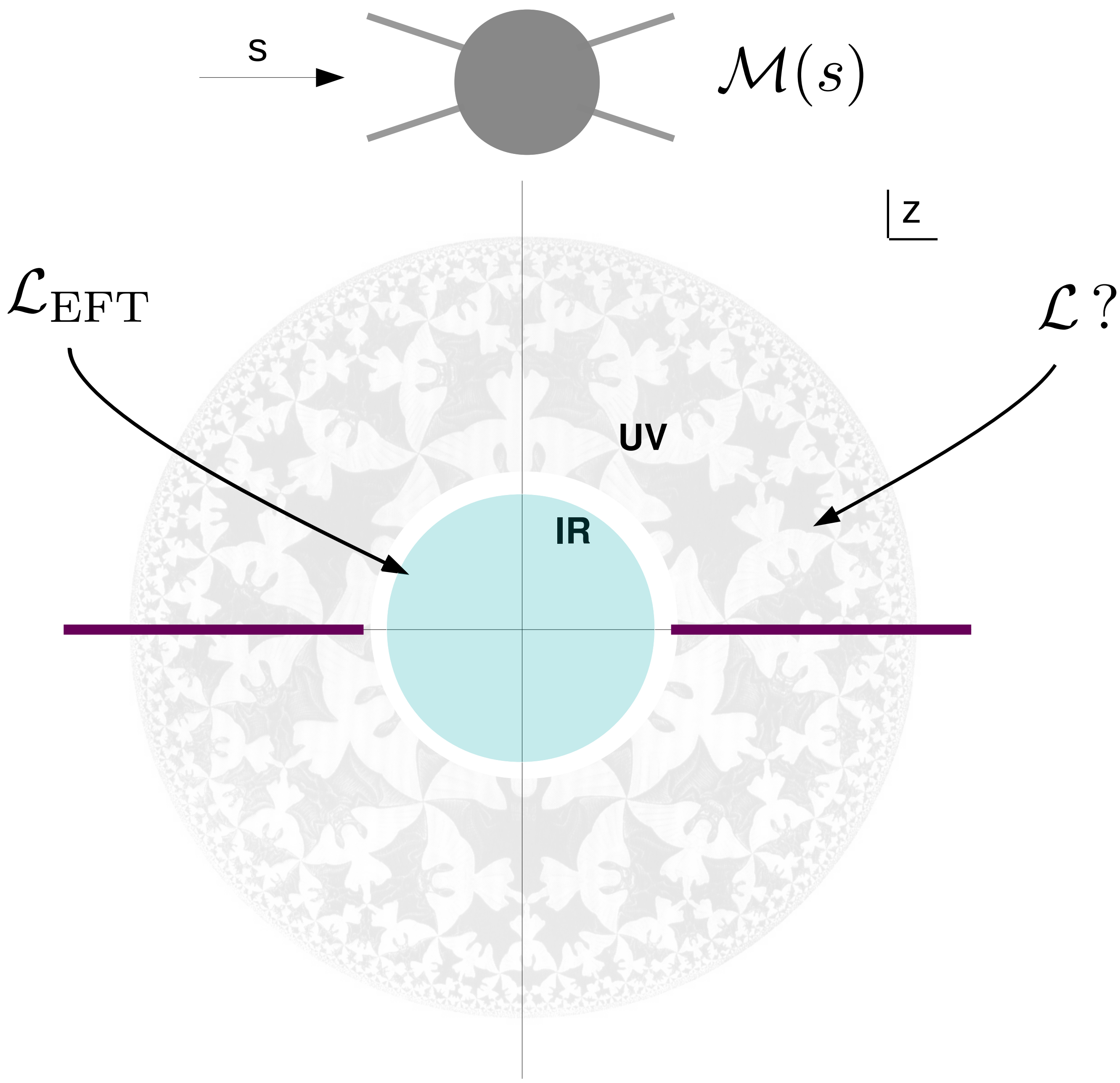
$$(FF) \equiv \frac{1}{4}F_{\mu\nu}F_{\mu\nu} \qquad (F\widetilde{F}) = \frac{1}{4}F_{\mu\nu}\widetilde{F}_{\mu\nu}$$

Consistent UV completion only if $c_1 > 0\,, \quad c_2 > 0\,, \quad c_1 c_2 - c_3^2 > 0$

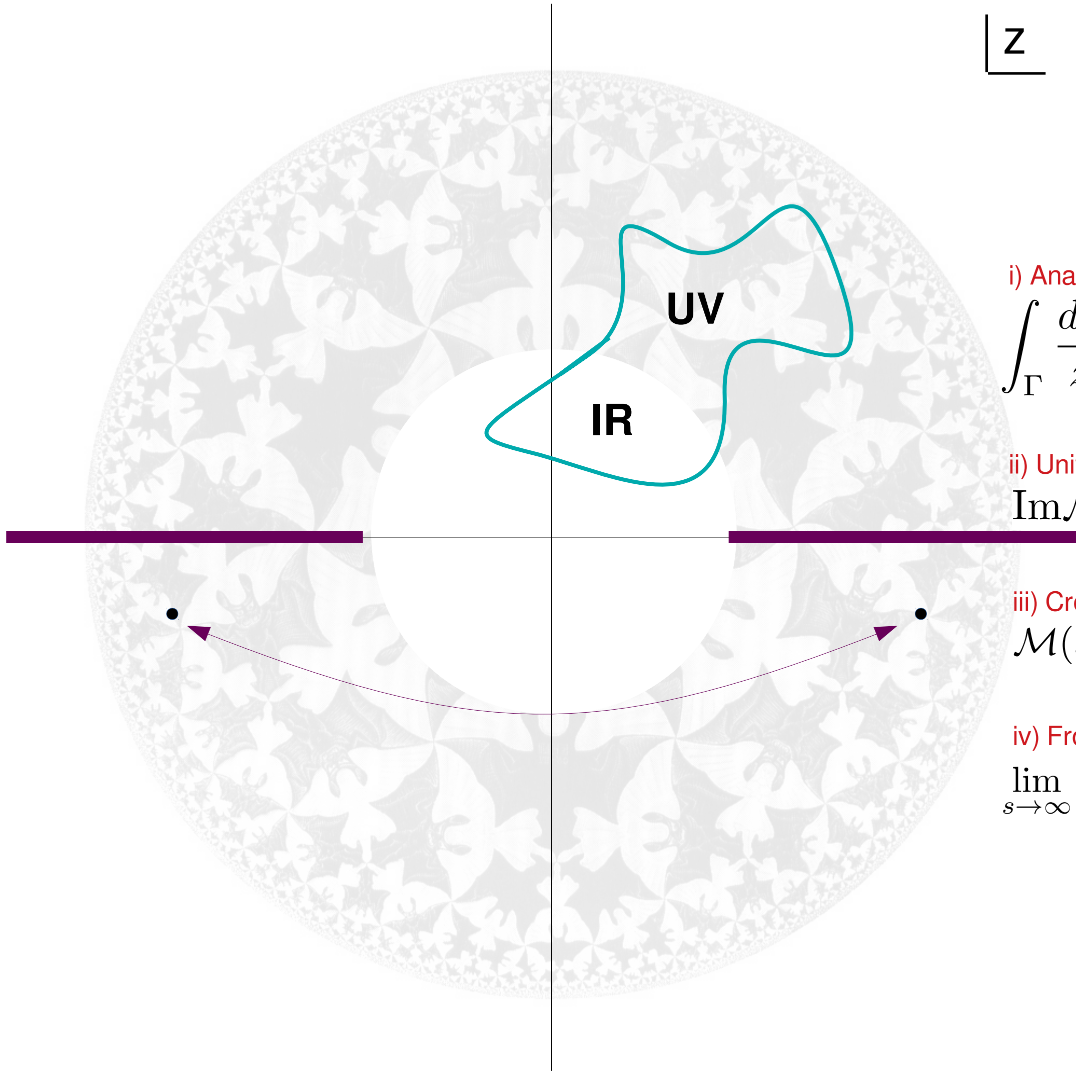
“Positivity” is a map between the IR and the UV.

$$c_1 > 0\,, \quad c_2 > 0\,, \quad c_1 c_2 - c_3^2 > 0$$





s
$\mathcal{M}(s)$
z
$\mathcal{L}_{\rm EFT}$
$\mathcal{L}\,?$
UV
IR



i) Analyticity

$$\int_\Gamma \frac{dz}{z}\mathcal{M}(z) \;=\; 0$$

ii) Unitarity

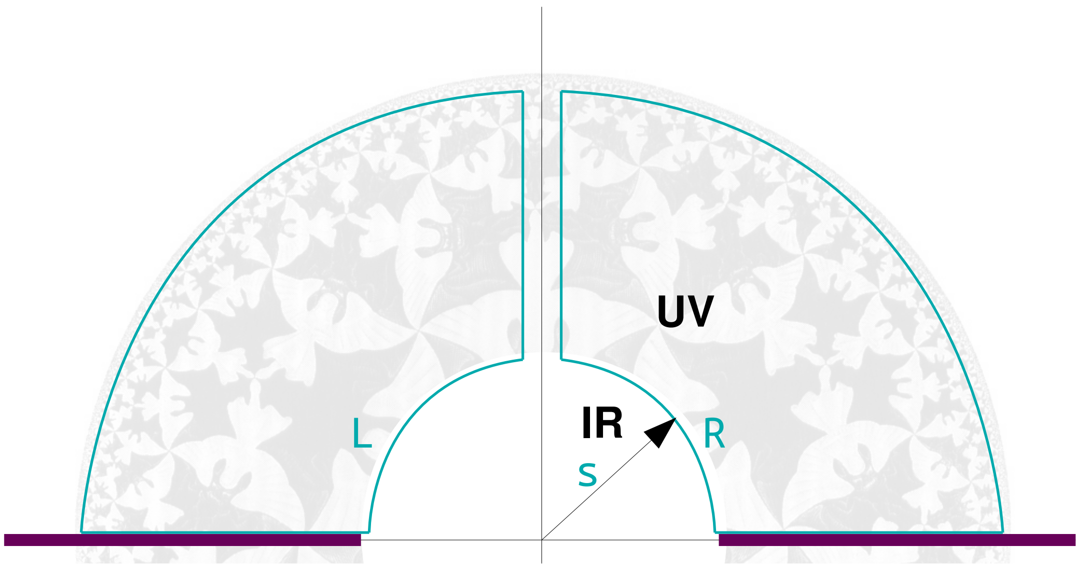
$$\mathrm{Im}\mathcal{M}(s+i\epsilon) \geq 0$$

iii) Crossing

$$\mathcal{M}(s) = \mathcal{M}^*(-s^*)$$

iv) Froissart bound

$$\lim_{s\to\infty} \mathcal{M}(s)/s^2 = 0$$



$$a_n^R = \int_R \frac{dz}{i\pi}\frac{\mathcal{M}(z)}{z^{3+n}} = \int_s^\infty \frac{dz}{i\pi}\frac{\mathcal{M}(z)}{z^{3+n}} + \int_{i\infty}^{is} \frac{dz}{i\pi}\frac{\mathcal{M}(z)}{z^{3+n}}$$

n even:

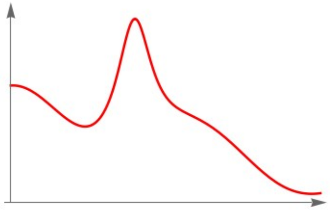
$$a_n^R + a_n^L = \frac{2}{\pi}\int_s^\infty \frac{dz}{z}\frac{\mathrm{Im}\mathcal{M}(z)}{z^{2+n}}$$

$$a_n^R - a_n^L = \frac{2}{i\pi}\int_s^\infty \frac{dz}{z}\frac{\mathrm{Re}\mathcal{M}(z)}{z^{2+n}} + 2I_n$$

Bellazzini, Elias-Miro, Rattazzi, MR, Riva '20

$$a_n^R + a_n^L = \frac{2}{\pi}\int_s^\infty \frac{dz}{z}\frac{\mathrm{Im}\mathcal{M}(z)}{z^{2+n}}$$

$\mathrm{Im}\mathcal{M}(z) \geq 0$



$\{a_n\}$

IR-calculable

$$\int_s^\infty \frac{dz}{z}\frac{1}{z^{2+n}}\bullet$$

$$m_n = \int_0^1 d\mu(x)x^n\,, \quad d\mu(x) \geq 0$$

When is an EFT UV-completable? → **When a bunch of numbers can be identified with moments of a positive distribution?**

When can a bunch of numbers be identified with moments of a positive distribution?

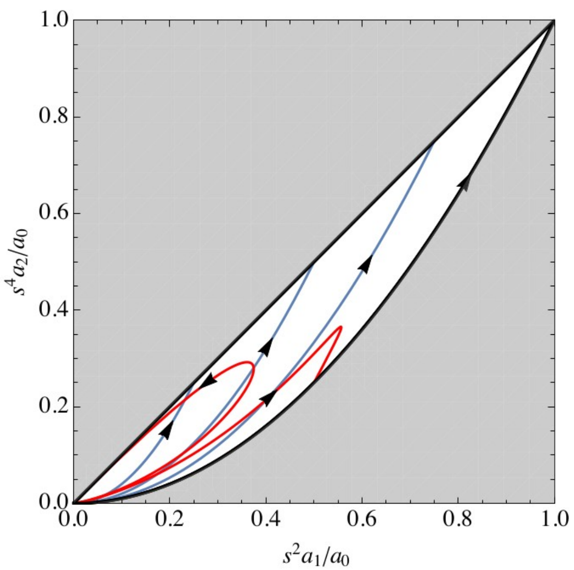
$\{a_0,\, a_1\,, \dots\}$ moments of a positive distribution in [0,1] **iff**

$$\begin{pmatrix} a_0 & a_1 & \dots & a_n \\ a_1 & a_2 & & \\ \dots & & \ddots & \vdots \\ a_n & & \dots & a_{2n} \end{pmatrix} \succ 0$$

$$\begin{pmatrix} a_0 - a_1 & a_1 - a_2 & \dots & a_n - a_{n+1} \\ a_1 - a_2 & a_2 - a_3 & & \\ \dots & & \ddots & \vdots \\ a_n - a_{n+1} & & \dots & a_{2n} - a_{2n+1} \end{pmatrix} \succ 0$$

2d

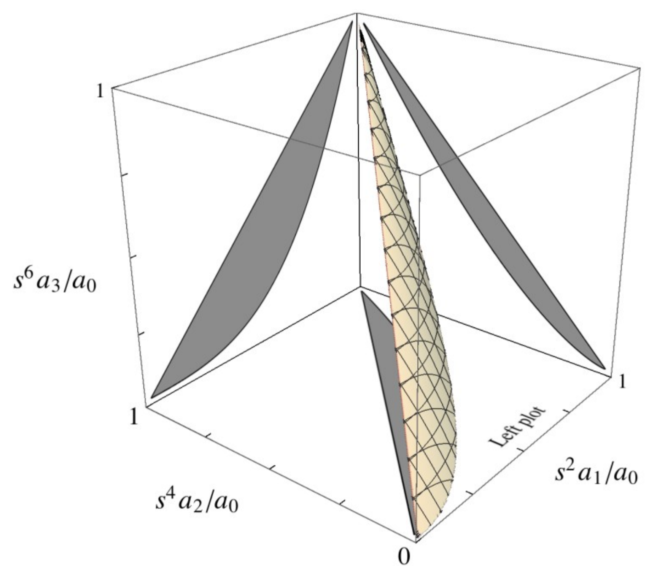
$$\begin{pmatrix} a_0 & a_1 \\ a_1 & a_2 \end{pmatrix} \succ 0\,, \quad a_1 > 0\,, \quad a_0 > \hat{s}^2 a_1\,, \quad a_1 > \hat{s}^2 a_2$$



3d

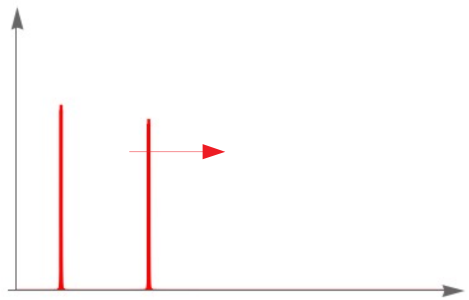
$$\begin{pmatrix} a_0 & a_1 \\ a_1 & a_2 \end{pmatrix} \succ 0\,, \qquad \begin{pmatrix} a_0 - a_1\hat{s}^2 & a_1 - a_2\hat{s}^2 \\ a_1 - a_2\hat{s}^2 & a_2 - a_3\hat{s}^2 \end{pmatrix} \succ 0\,,$$

$$\begin{pmatrix} a_1 & a_2 \\ a_2 & a_3 \end{pmatrix} \succ 0\,, \qquad a_1 > \hat{s}^2 a_2\,,$$



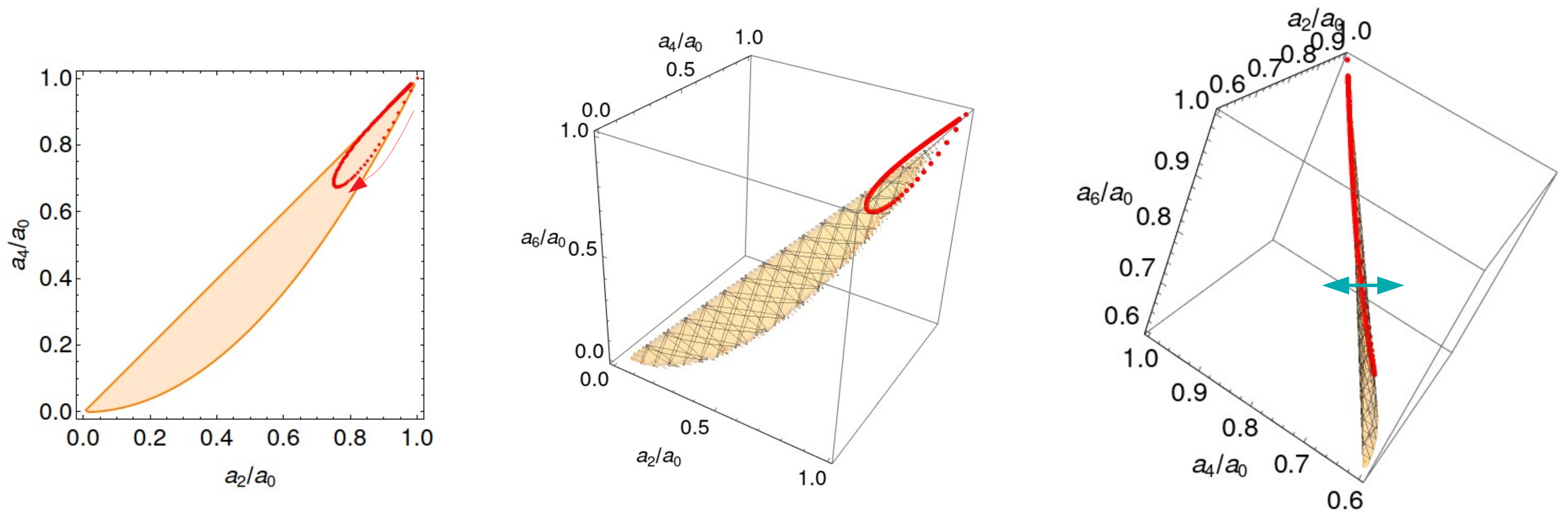
Convexity → **Understanding of the boundary = Understanding of the entire space**

Boundary of n-dimensional space given by (n-1)-particles



Arcs from two narrow resonances

Space of arcs shrinks
faster than exponentially with n!

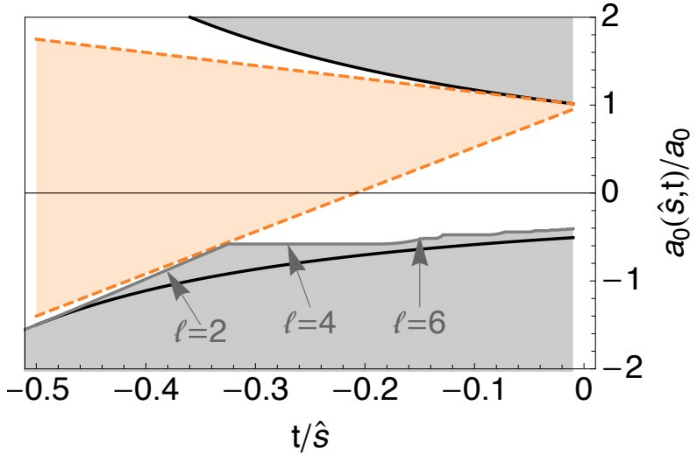


Finite t

Arcs at finite t get a t- and l- dependent kernel

$$a_n(s,t) \;=\; \frac{2}{\pi}\int_s^\infty \frac{dz}{z}\sum_\ell \frac{\mathrm{Im} f_\ell(z)}{z^{2+n}} I_{n,\ell}(t,z)$$

$$\min_{\ell,z} I_{n,\ell}(t,z) \leq \frac{a_n(s,t)}{a_n(s,0)} \leq \frac{s+t/2}{(s(s+t))^{n+2}}$$



Upper bound on the arc is t-dependent but lower bound is t- and l- dependent.

$$a_n(s,t) \;=\; c_2 - tc_3 + \ldots$$

Upper bound comes from t=0,

but lower bound from finite t,
at the intersection of l=2 and l=4 partial waves

Full unitarity? $2\mathrm{Im} f_\ell(s) \geq |f_\ell(s)|^2$

Pert. IR theory: $\mathcal{M}(s) = c_2 s^2 + s^4(c_4 + \beta_4 \log(-is)) + s^6(c_6 + \beta_6 \log(-is)) + \ldots$ MR ‘22

$$\frac{\text{relevance of Unitarity bounds}}{\text{relevance of Convexity bounds}} \sim \frac{\beta_4}{c_4}\frac{c_2}{c_4 s^2} \sim \frac{\text{loop expansion}}{\text{derivative expansion}}$$

Bounds from full unit. become more relevant in theories and regimes where the loop expansion is more relevant than the derivative expansion

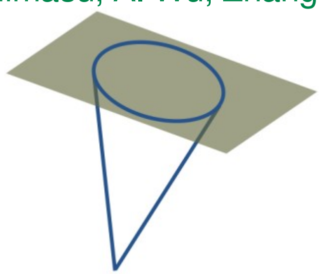
Generic ansatz: $\mathcal{M}(s) = \sum_{a,b,c} \rho^a(s)\rho^b(t)\rho^c(u)$ Paulos, Penedones, Toledo, v Rees, Vieira ‘17

Impose unit. Numerically & maximise e.g. amplitude at a point

Chen, Mimasu, A. Wu, Zhang, Zhou ‘23

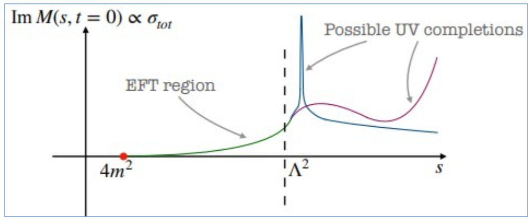
SDP:

Impose unit. and (tree-) crossing on partial waves and dispersively relate then with IR obs.



Generic ansatz + perturbativity: Elias-Miró, Guerreri, Gumus ‘22

$\mathcal{M}(s) = \sum_{a,b,c} \rho^a(s)\rho^b(t)\rho^c(u)$ +



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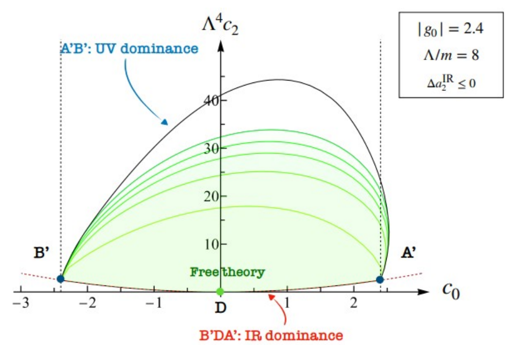


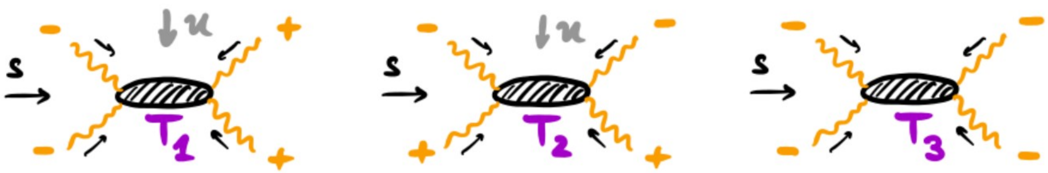
Multichannel EFTs

Remmen, Rodd '19
Li, Xu, Yang, Zhang, Zhou '21
Haring, Hebbar, Karateev, Meineri, Penedones '22
Durieux, Remmen, MR, Rodd 'WIP

Example, a U(1) vector:

$$\mathcal{L} = -(FF) + c_1 (FF)^2 + c_2 (F\widetilde{F})^2 + c_3 (FF)(F\widetilde{F}) + \ldots \qquad (FF) \equiv \frac{1}{4} F_{\mu\nu} F_{\mu\nu} \qquad (F\widetilde{F}) = \frac{1}{4} F_{\mu\nu} \widetilde{F}_{\mu\nu}$$

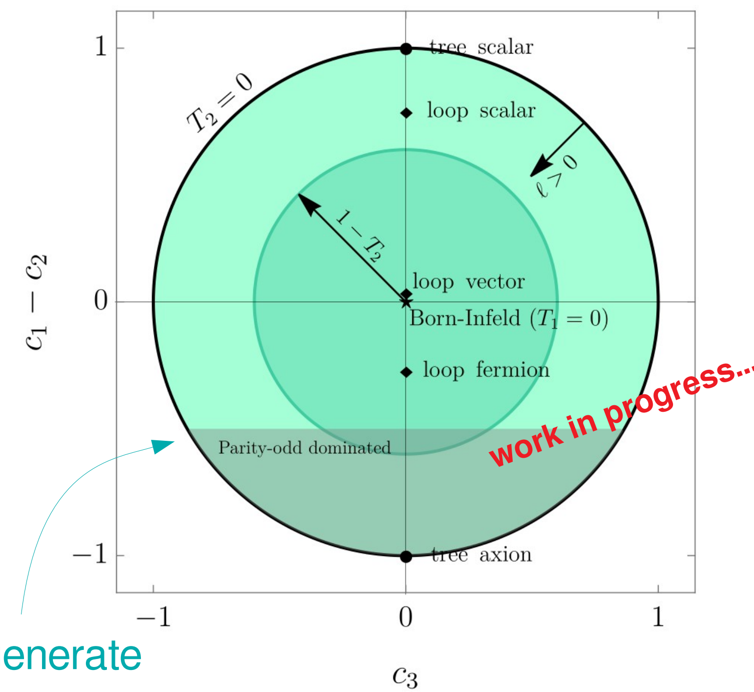
Three distinct scattering channels in the forward limit, and Wilson coeff. written as integrals of them:



$$c_1 = T^1 + T^2 + \mathrm{Re} T^3$$
$$c_2 = T^1 + T^2 - \mathrm{Re} T^3$$
$$c_3 = \mathrm{Im} T^3$$

$$\begin{pmatrix} T^1 & 0 & 0 & T^3_r - iT^3_i \\ 0 & T^2 & 0 & 0 \\ 0 & 0 & T^2 & 0 \\ T^3_r + iT^3_i & 0 & 0 & T^1 \end{pmatrix} \succ 0$$

Now the measure is a positive matrix, relating arcs with same subtractions but different channels.



Boundary made of degenerate scattering channels

Dimension 8

Generic positivity bounds apply only to dimension 8 operators.

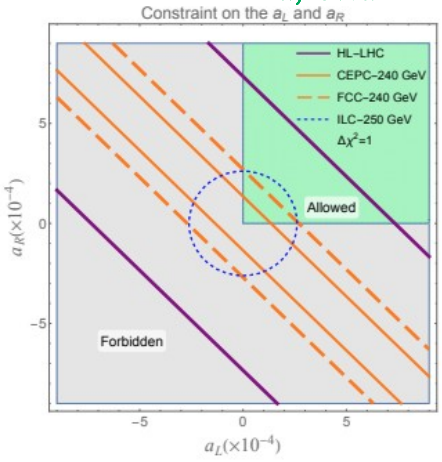
Find observables sensitive to dimension 8:

- Neutral diboson (qq → VV) [Bellazzini, Riva '18]
- Photon fusion (VV → qq) [Gu, Shu '23]
- VBS [Bi, Zhang, Zhou '19]

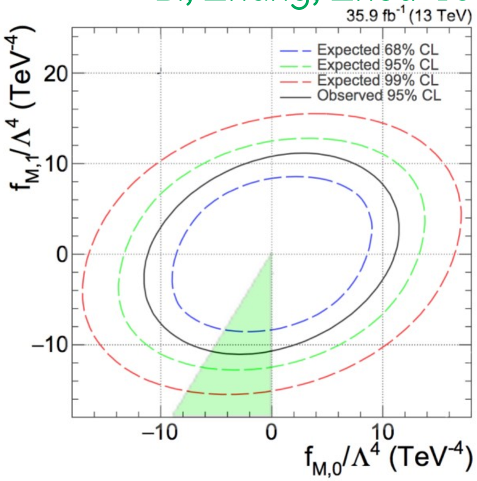
Find class of models that naturally enhance dim8 w/r dim6

- "Soft fermions" [Bellazzini, Riva, Serra, Sgarlata '17]

Gu, Shu '23



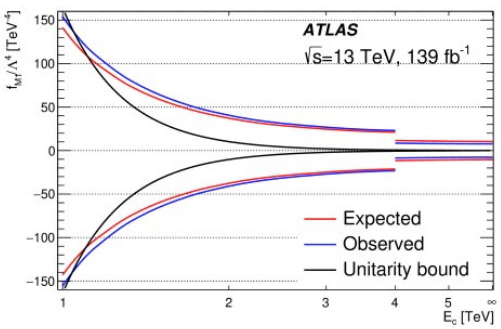
Bi, Zhang, Zhou '19



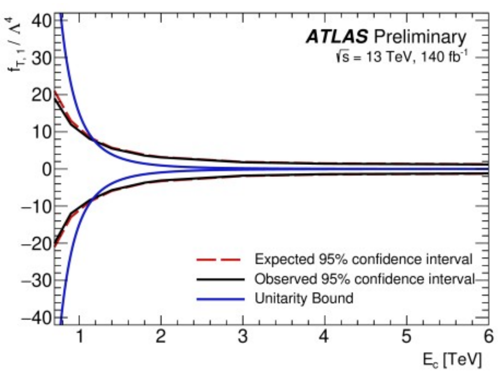
VBS

- Theoretically relevant processes with rich phenomenology
- Positivity constraints worked out in [Bi, Zhang, Zhou ‘19]
- Should be included in the fits
- However...

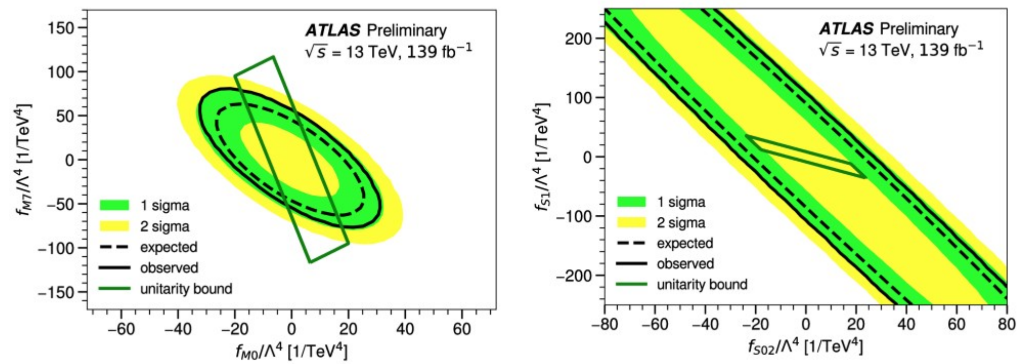
vvA [ATLAS-CONF-2023-024]



4l [ATLAS-CONF-2023-024]



WWss [ATLAS-CONF-2023-012]



Slide from Roberto Covarelli and Matteo Presilla

	Observed ($W^\pm W^\pm$) (TeV^{-4})	Expected ($W^\pm W^\pm$) (TeV^{-4})	Observed (WZ) (TeV^{-4})	Expected (WZ) (TeV^{-4})	Observed (TeV^{-4})	Expected (TeV^{-4})
f_{T0}/Λ^4	[-0.28, 0.31]	[-0.36, 0.39]	[-0.62, 0.65]	[-0.82, 0.85]	[-0.25, 0.28]	[-0.35, 0.37]
f_{T1}/Λ^4	[-0.12, 0.15]	[-0.16, 0.19]	[-0.37, 0.41]	[-0.49, 0.55]	[-0.12, 0.14]	[-0.16, 0.19]
f_{T2}/Λ^4	[-0.38, 0.50]	[-0.50, 0.63]	[-1.0 , 1.3]	[-1.4, 1.7]	[-0.35, 0.48]	[-0.49, 0.63]
f_{M0}/Λ^4	[-3.0, 3.2]	[-3.7, 3.8]	[-5.8, 5.8]	[-7.6, 7.6]	[-2.7, 2.9]	[-3.6, 3.7]
f_{M1}/Λ^4	[-4.7, 4.7]	[-5.4, 5.8]	[-8.2, 8.3]	[-11, 11]	[-4.1, 4.2]	[-5.2, 5.5]
f_{M6}/Λ^4	[-6.0, 6.5]	[-7.5, 7.6]	[-12, 12]	[-15, 15]	[-5.4, 5.8]	[-7.2, 7.3]
f_{M7}/Λ^4	[-6.7, 7.0]	[-8.3, 8.1]	[-10, 10]	[-14, 14]	[-5.7, 6.0]	[-7.8, 7.6]
f_{S0}/Λ^4	[-6.0, 6.4]	[-6.0, 6.2]	[-19, 19]	[-24, 24]	[-5.7, 6.1]	[-5.9, 6.2]
f_{S1}/Λ^4	[-18, 19]	[-18, 19]	[-30, 30]	[-38, 39]	[-16, 17]	[-18, 18]

Same limits, but cutting on unitarity violating phase space

	Observed ($W^\pm W^\pm$) (TeV^{-4})	Expected ($W^\pm W^\pm$) (TeV^{-4})	Observed (WZ) (TeV^{-4})	Expected (WZ) (TeV^{-4})	Observed (TeV^{-4})	Expected (TeV^{-4})
f_{T0}/Λ^4	[-1.5, 2.3]	[-2.1, 2.7]	[-1.6, 1.9]	[-2.0, 2.2]	[-1.1, 1.6]	[-1.6, 2.0]
f_{T1}/Λ^4	[-0.81, 1.2]	[-0.98, 1.4]	[-1.3, 1.5]	[-1.6, 1.8]	[-0.69, 0.97]	[-0.94, 1.3]
f_{T2}/Λ^4	[-2.1, 4.4]	[-2.7, 5.3]	[-2.7, 3.4]	[-4.4, 5.5]	[-1.6, 3.1]	[-2.3, 3.8]
f_{M0}/Λ^4	[-13, 16]	[-19, 18]	[-16, 16]	[-19, 19]	[-11, 12]	[-15, 15]
f_{M1}/Λ^4	[-20, 19]	[-22, 25]	[-19, 20]	[-23, 24]	[-15, 14]	[-18, 20]
f_{M6}/Λ^4	[-27, 32]	[-37, 37]	[-34, 33]	[-39, 39]	[-22, 25]	[-31, 30]
f_{M7}/Λ^4	[-22, 24]	[-27, 25]	[-22, 22]	[-28, 28]	[-16, 18]	[-22, 21]
f_{S0}/Λ^4	[-35, 36]	[-31, 31]	[-83, 85]	[-88, 91]	[-34, 35]	[-31, 31]
f_{S1}/Λ^4	[-100, 120]	[-100, 110]	[-110, 110]	[-120, 130]	[-86, 99]	[-91, 97]

VBS

- Theoretically relevant processes with rich phenomenology
- Positivity constraints worked out in [Bi, Zhang, Zhou ‘19]
- Should be included in the fits
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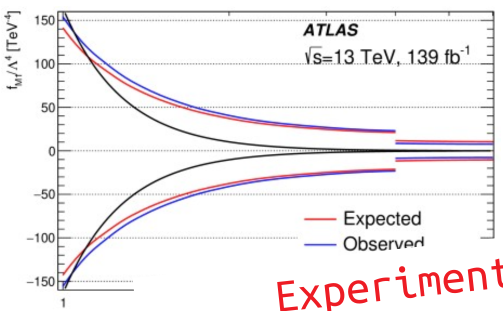
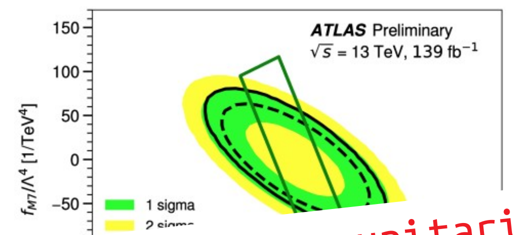
vvA [ATLAS-CONF-2023-024]





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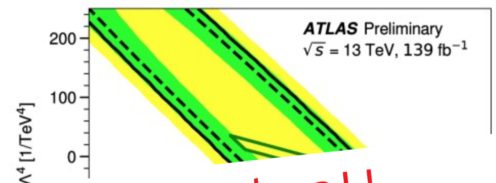

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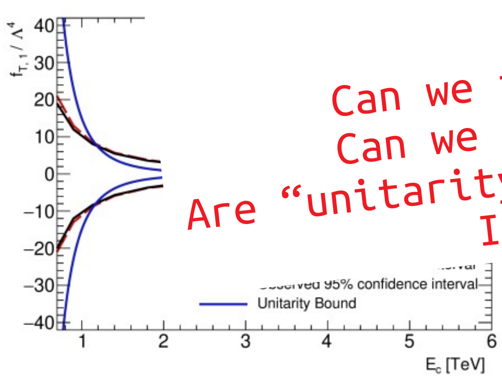

Experiments probe regions where unitarity breaks down!!
Expansion in local operators seems to make little sense.
Are we trully learning anything?
What are our questions, then?
Can we find simple “models” that saturate unitarity?
Can we “chart” the space of theories VBS is probing?
Are “unitarity-saturating” theories ruled out by other processes?
Is this issue going to go away at HL-LHC?

					[-2.7, 2.9]	[-3.6, 3.7]
			[illegible]	[-11, 11]	[-4.1, 4.2]	[-5.2, 5.5]
		[-7.5, 7.6]	[-12, 12]	[-15, 15]	[-5.4, 5.8]	[-7.2, 7.3]
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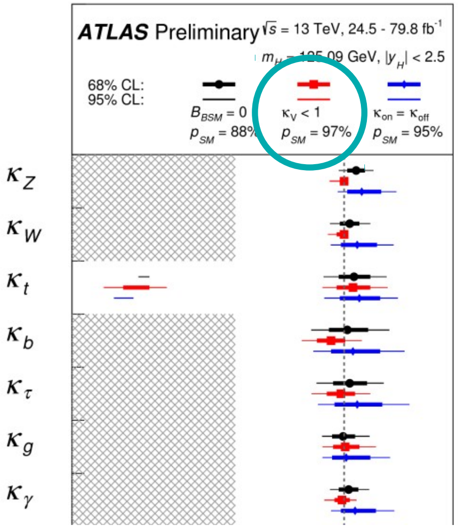
Dimension 6

Low, Rattazzi, Vichi ‘09

No doubly-charged state → $c_H > 0$ →



Remmen, Rodd ‘20

Faster convergence and spin-0/1 dominance → Definite sign in 4-fermions

Fernandez, Pomarol Riva, Sciotti ‘22

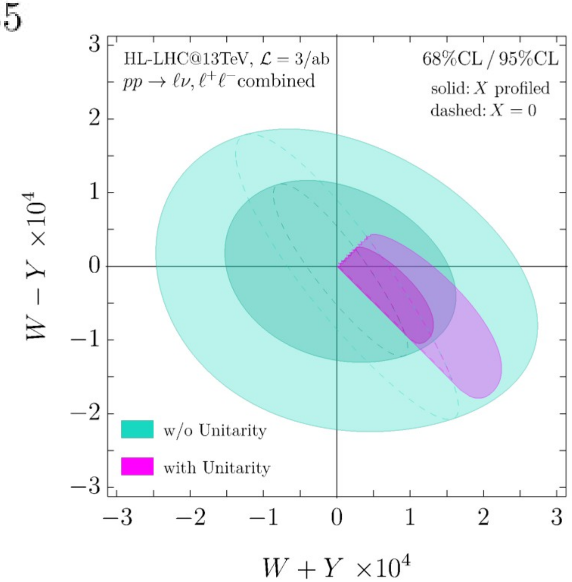
Large N → Spin 1 states dominate (vector meson dominance)

Elias Miro, Guerrieri, Gumus ‘23

S-matrix bootstrap → $-0.31 < \frac{g_H}{(4\pi)^2} \times \frac{\Lambda^2}{f^2} < 0.35$

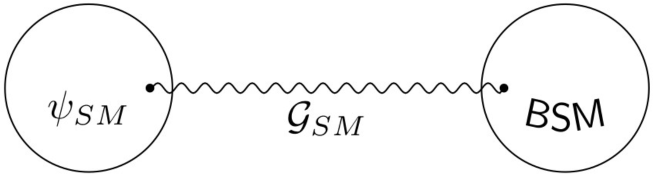
w/ McCullough, Ricci ‘next week? Who knows!

Universal UV completion → $W > 0,\, Y > 0$ →



w/ McCullough, Ricci

universal theories:



w/ McCullough, Ricci

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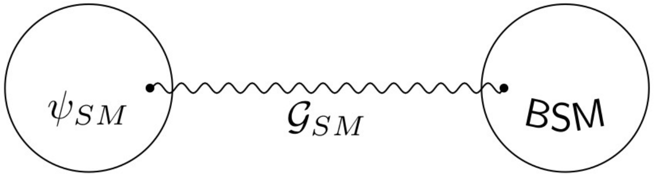
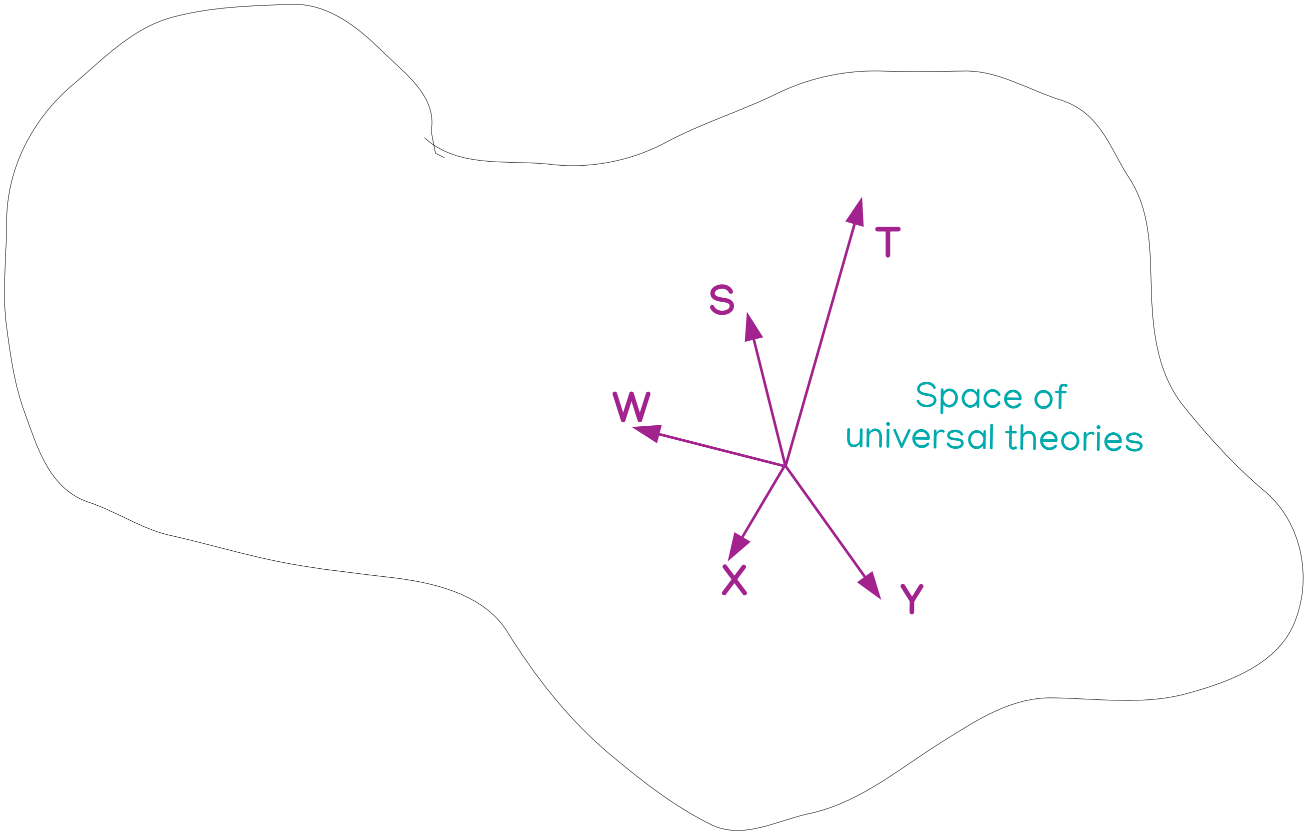
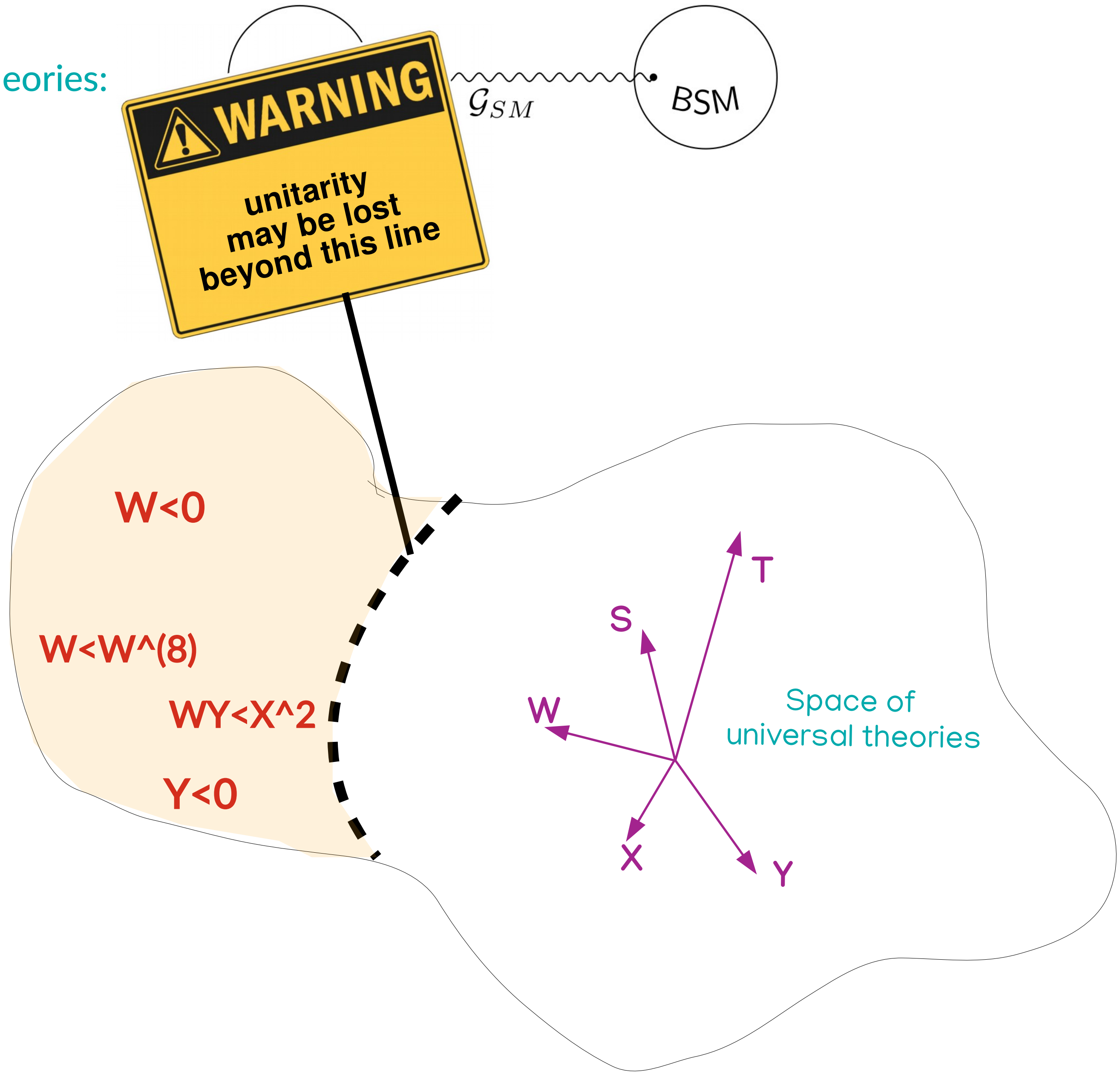
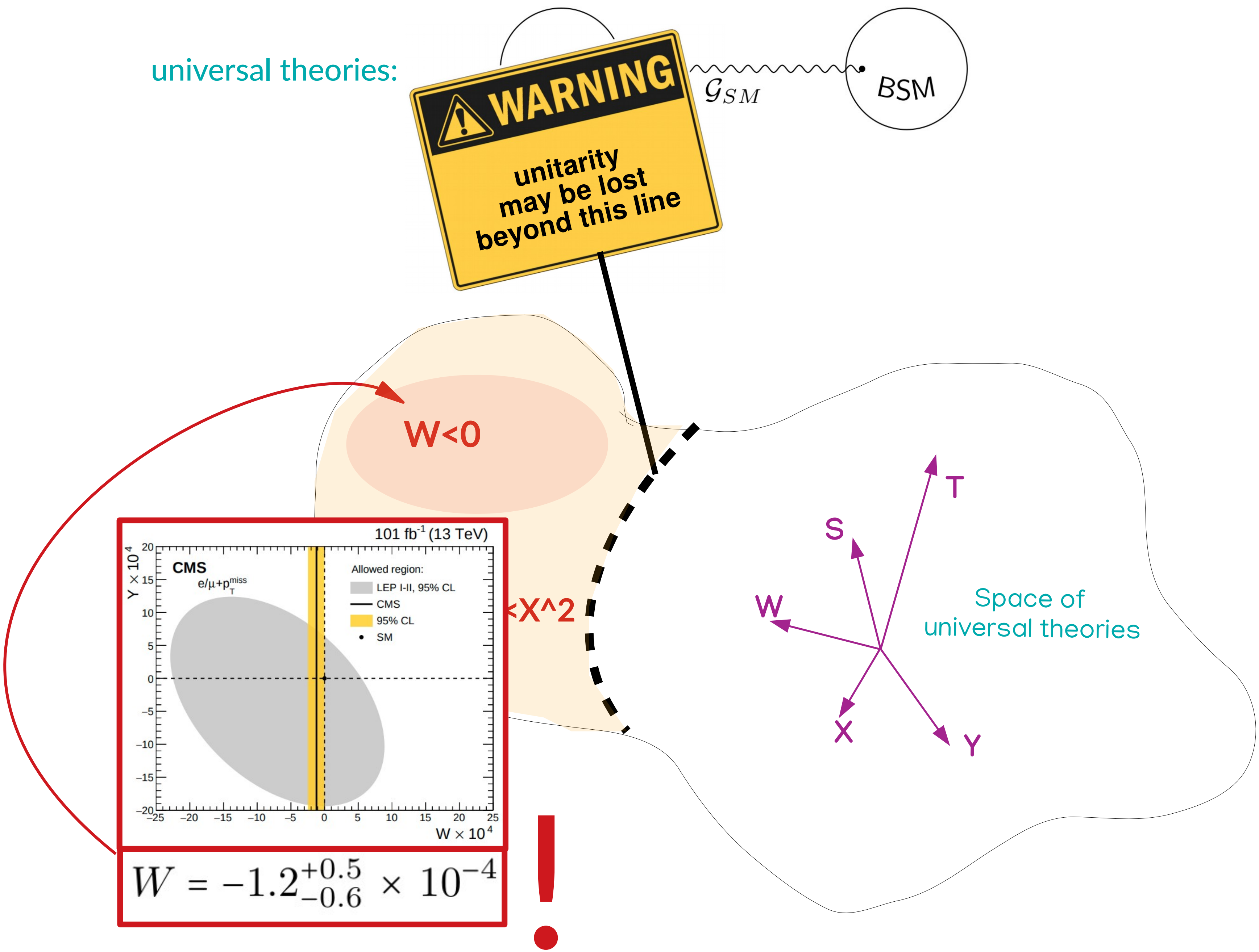


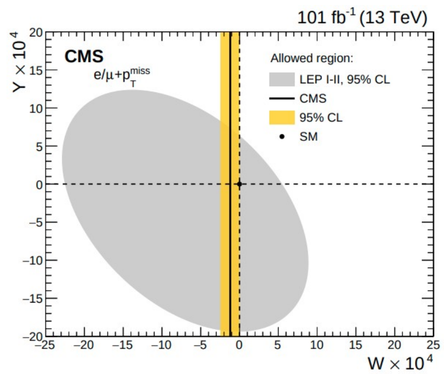


w/ McCullough, Ricci

universal theories:



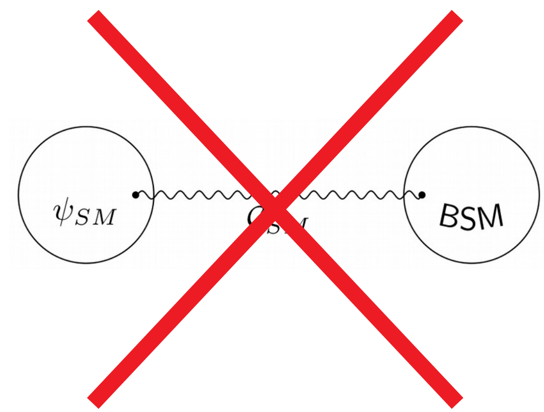

universal theories:
WARNING
unitarity
may be lost
beyond this line
$\mathcal{G}_{SM}$
BSM
W<0
<X^2
101 fb^{-1} (13 TeV)
CMS
e/μ+p$_T^{miss}$
Allowed region:
LEP I-II, 95% CL
CMS
95% CL
SM
Y × 10^4
W × 10^4
$W = -1.2^{+0.5}_{-0.6} \times 10^{-4}$
!
T
S
W
X
Y
Space of
universal theories

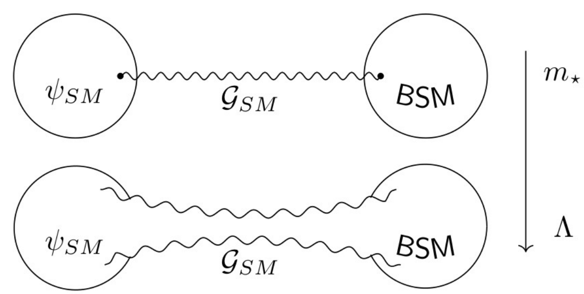


$$W = -1.2^{+0.5}_{-0.6} \times 10^{-4}$$

$$W \in [0, 0.31] \times 10^{-4} \text{ at } 95\% CL$$

$$m_\star > (g/g_\star)\, 14.3 \text{ TeV}$$





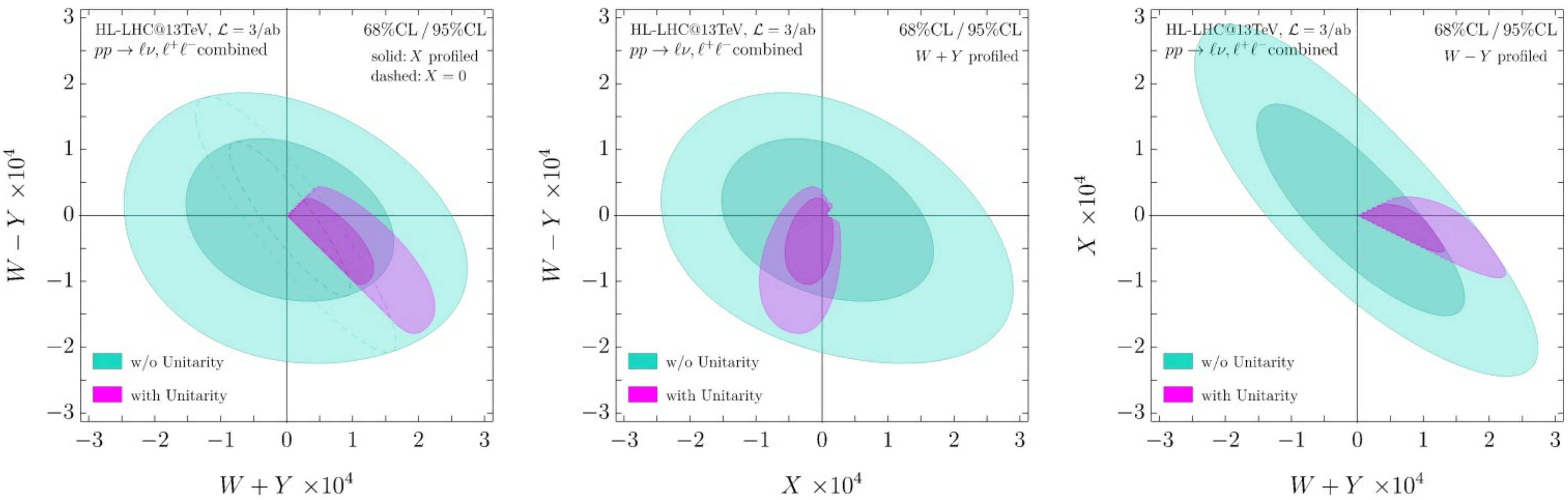
a) Bounds are *much* stronger

b) Universality assumption is wrong

c) Strong coupling at ~100TeV

Unitarity constraints have a dramatic impact on the interpretation of Drell-Yan data

$$W > 0 \quad Y > 0 \quad WY - X^2 > 0$$



Conclusions

Analyticity of amplitudes allows to connect UV and IR, therefore:

Generic UV properties

↓

Generic IR properties

Tells us what is possible and what is not in the IR,

and within what is possible,

which IR features are mapped to which UV dynamics