UC San Diego

Geometry of EFTs

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6th General Meeting of the LHC EFT Working Group

November 17, 2023

Field redefinition invariance

We know physics is invariant under field redefinitions.

S-matrix elements are invariant (from LSZ formula) while correlation functions are not.

There is an ambiguity in our EFT Lagrangian description which makes this invariance at higher level unclear in intermediate steps ⇒ different operator basis give same observables but not always easy to see.

The goal of (constant) *field-space geometry* is to write the Lagrangian in such a way that intermediate quantities, i.e. scattering amplitudes, are covariant → make observable invariance manifest.

Basis choice should not matter.
For example, representations of the Goldstone are equivalent:

Linear vs Nonlinear

$$\overrightarrow{\phi} = \begin{pmatrix} \boxed{\begin{matrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{matrix}} \\ v + \mathrm{h} \end{pmatrix}$$

$$\overrightarrow{\phi} = (v+h)\,\vec{n}(\pi) \qquad \text{with} \quad \vec{n}(\pi) = \frac{1}{v}\begin{pmatrix} \boxed{\begin{matrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{matrix}} \\ \sqrt{v^2 - \vec{\pi}\cdot\vec{\pi}} \end{pmatrix}$$

Geometric interpretation

A scalar field theory can be written as: [Alonso, Jenkins, Manohar, 1605.03602]

$$\mathscr{L} = \frac{1}{2} g_{IJ}(\phi)(\partial_\mu \phi^I)(\partial^\mu \phi^J) - V(\phi) + \text{higher-derivative terms}$$

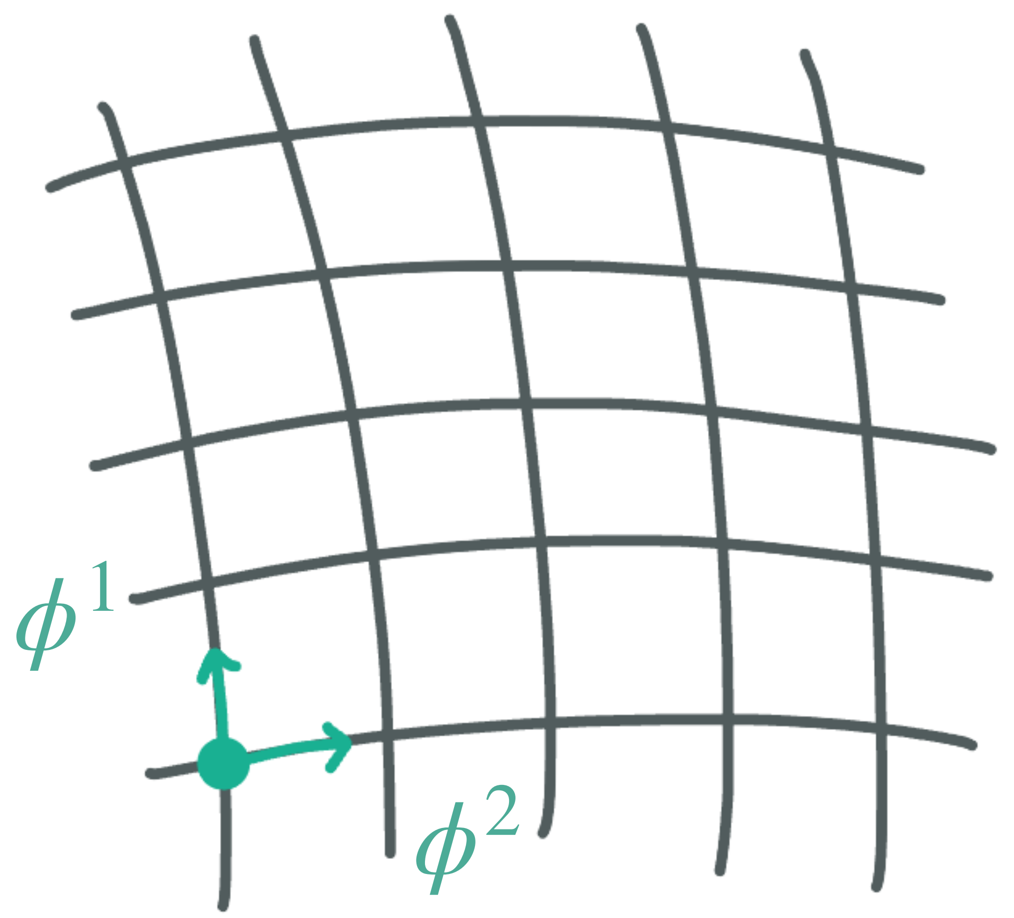
where

- field values = coordinates on a Riemannian manifold
- $g_{IJ}(\phi)$ = inner-product on the tangent space of the field manifold: metric

$$ds^2 \equiv g_{IJ}(\phi)\, d\phi^I\, d\phi^J$$

- potential $V(\phi)$ = function on the field manifold
- field redefinitions (without derivatives) = coordinate transformations

$$\phi^I \to \varphi^I(\phi)$$



SM scalar manifold is flat

Geometric interpretation

Under a coordinate transformation,

$$\phi^I \to \varphi^I(\phi)$$

- the derivative of the scalar transforms as a vector

$$\partial_\mu \phi^I \to \left(\frac{\delta \varphi^I}{\delta \phi^J}\right) \partial_\mu \phi^J$$

- the metric transforms as a tensor

$$g_{IJ} \to \left(\frac{\delta \phi^K}{\delta \varphi^I}\right)\left(\frac{\delta \phi^L}{\delta \varphi^J}\right) g_{KL}$$

so $\mathscr{L}_{\text{kin}} = \frac{1}{2} g_{IJ}(\phi)\,(\partial_\mu \phi^I)(\partial^\mu \phi^J)$ is invariant.

From the metric we can define,

- Christoffel symbols

$$\Gamma^I_{JK} = \frac{1}{2} g^{IL}\left(g_{LJ,K} + g_{LK,J} - g_{JK,L}\right)$$

- Covariant derivatives

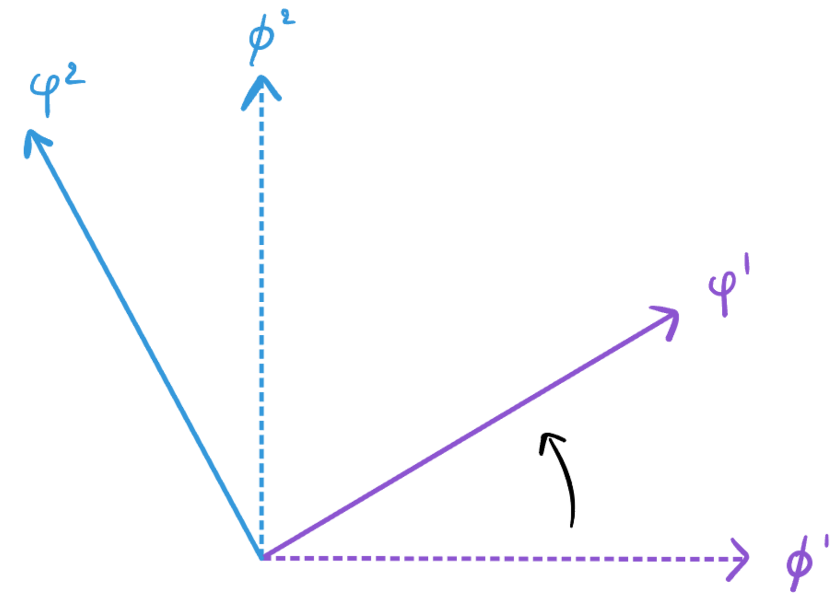
$$T_{J;I} \equiv \nabla_I T_J = \frac{\partial T_J}{\partial \phi^I} - \Gamma^K_{IJ}\, T_K$$

- Riemann curvature tensor

$$R^I_{JKL} = \partial_K \Gamma^I_{JL} + \Gamma^I_{KN}\Gamma^N_{JL} - (K \leftrightarrow L)$$

R and ∇ will appear in scattering amplitudes making them covariant.

$\Rightarrow$ $\boxed{\text{field redefinition in-/covariance} \quad = \quad \text{coordinate in-/covariance}}$



Advantages of field-space geometry

Advantages of the geometric description for EFTs :

- Resums higher dimensional operators

$$\mathscr{L}_{O(N)\ \mathrm{EFT}} \supset \frac{1}{2} \underbrace{\left(\delta_{IJ} + C_E\,(\phi\cdot\phi)\delta_{IJ} + C_2\,\phi_I\phi_J\right)}_{g_{IJ}(\phi)} (\partial_\mu\phi^I)(\partial^\mu\phi^J) - V(\phi)$$

  $\Rightarrow$ amplitudes and RGE in terms of geometric objects contain the full tower $\rightarrow$ precision   see Anke's talk

- Same geometric description can represent different EFTs (for example HEFT or SMEFT)

  $\Rightarrow$ unify picture and define more relevant quantities than Wilson coefficients $\rightarrow$ structure

  derive EFT cutoff [Cohen, Craig, Lu, Sutherland, 2108.03240]

- Covariant amplitudes and RGE

  $\Rightarrow$ more compact expressions $\rightarrow$ efficiency

see Andreas' talk

SMEFT vs HEFT from non-analyticity

see Javier's talk

SMEFT

Higgs and Goldstones are embedded into a doublet

$$H \to LH$$

Usually: linear realization

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} iG_1 + G_2 \\ v + h + iG_3 \end{pmatrix}$$

- same symmetry
  $SU(2)_L \times U(1)_Y \to U(1)_Q$
  $L \in SU(2)_L \;\; R \in U(1)_Y$
- different field content

HEFT

Higgs is a singlet

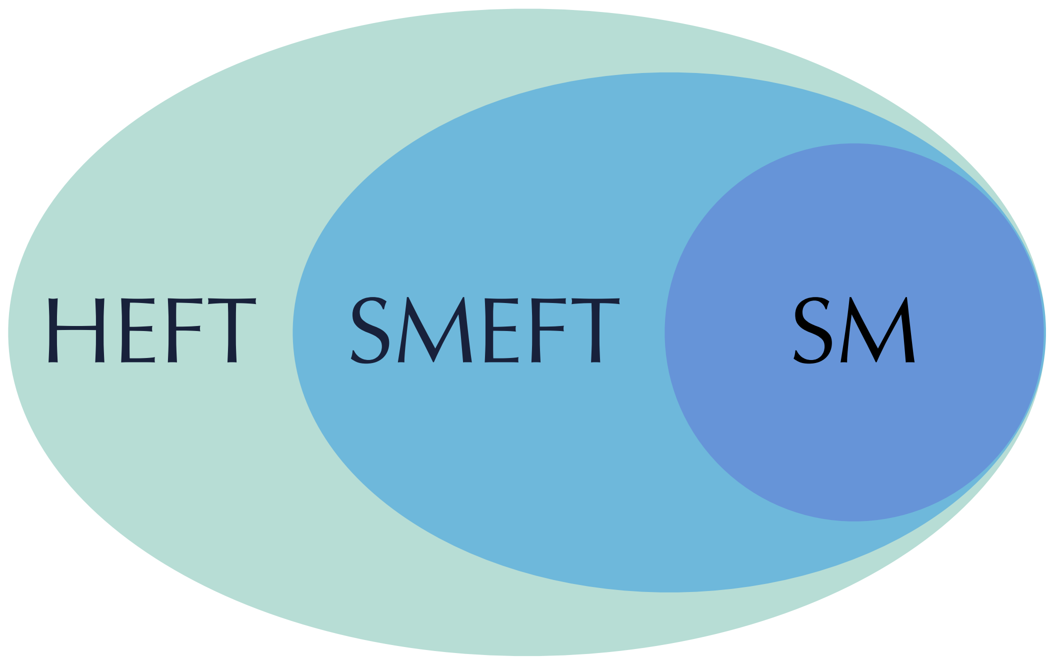
$h \to h$

and Goldstones

$U \to LUR^\dagger$

Usually: nonlinear realization

$$h,\, U = \exp\left(\frac{i\pi^a \tau^a}{v}\right)$$



Going from SMEFT-form to HEFT-form is always possible.
Going from HEFT-form to SMEFT-form is not.

$\Rightarrow$ SMEFT vs HEFT = analytic vs non-analytic Lagrangian in H
[Falkowski, Rattazzi, 1902.05936]

Some non-analyticities can be removed by field redefinitions.

see All Things EFT talk
by Xiaochuan Lu

SMEFT vs HEFT from geometry

A HEFT can be written in SMEFT-form $\Leftrightarrow$

- there exist an $O(4)$ invariant fixed point h^* on the scalar manifold
  [Alonso, Jenkins, Manohar, 1605.03602]
- metric and potential are analytic at h^*
  [Cohen, Craig, Lu, Sutherland, 2008.08597]

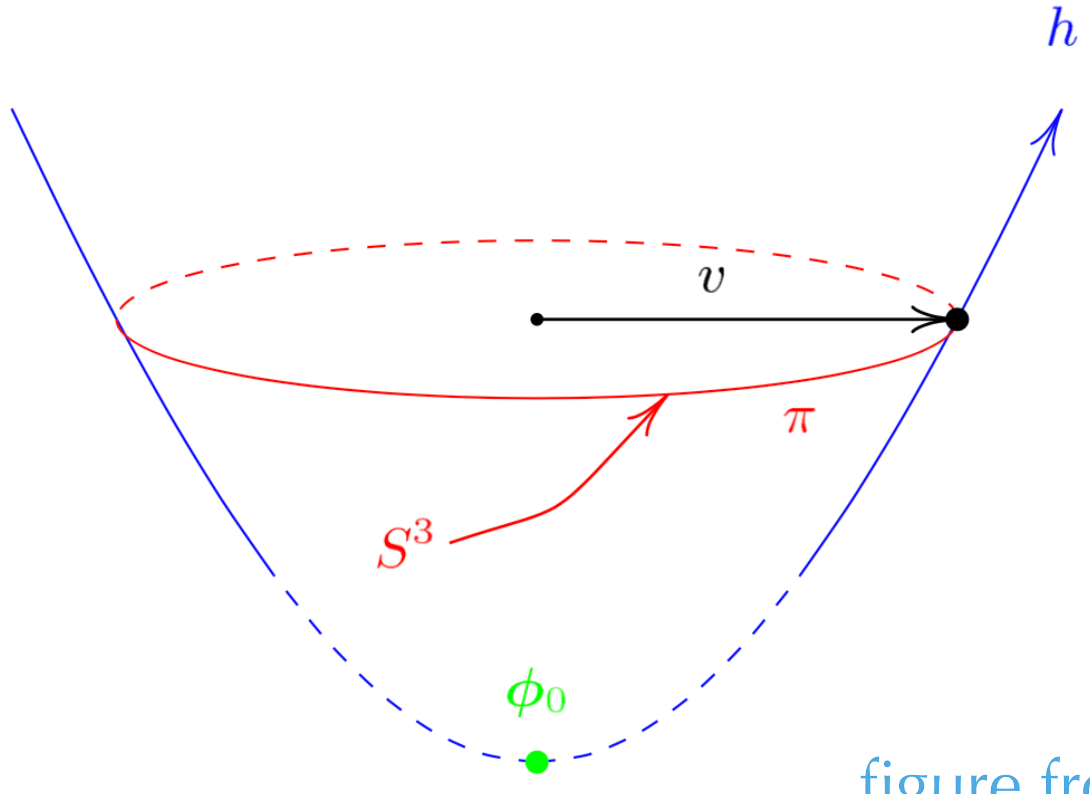
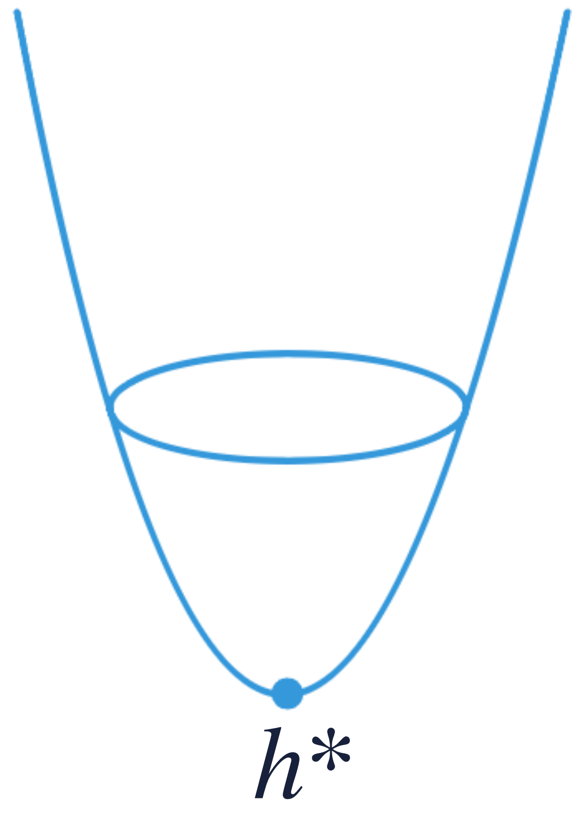
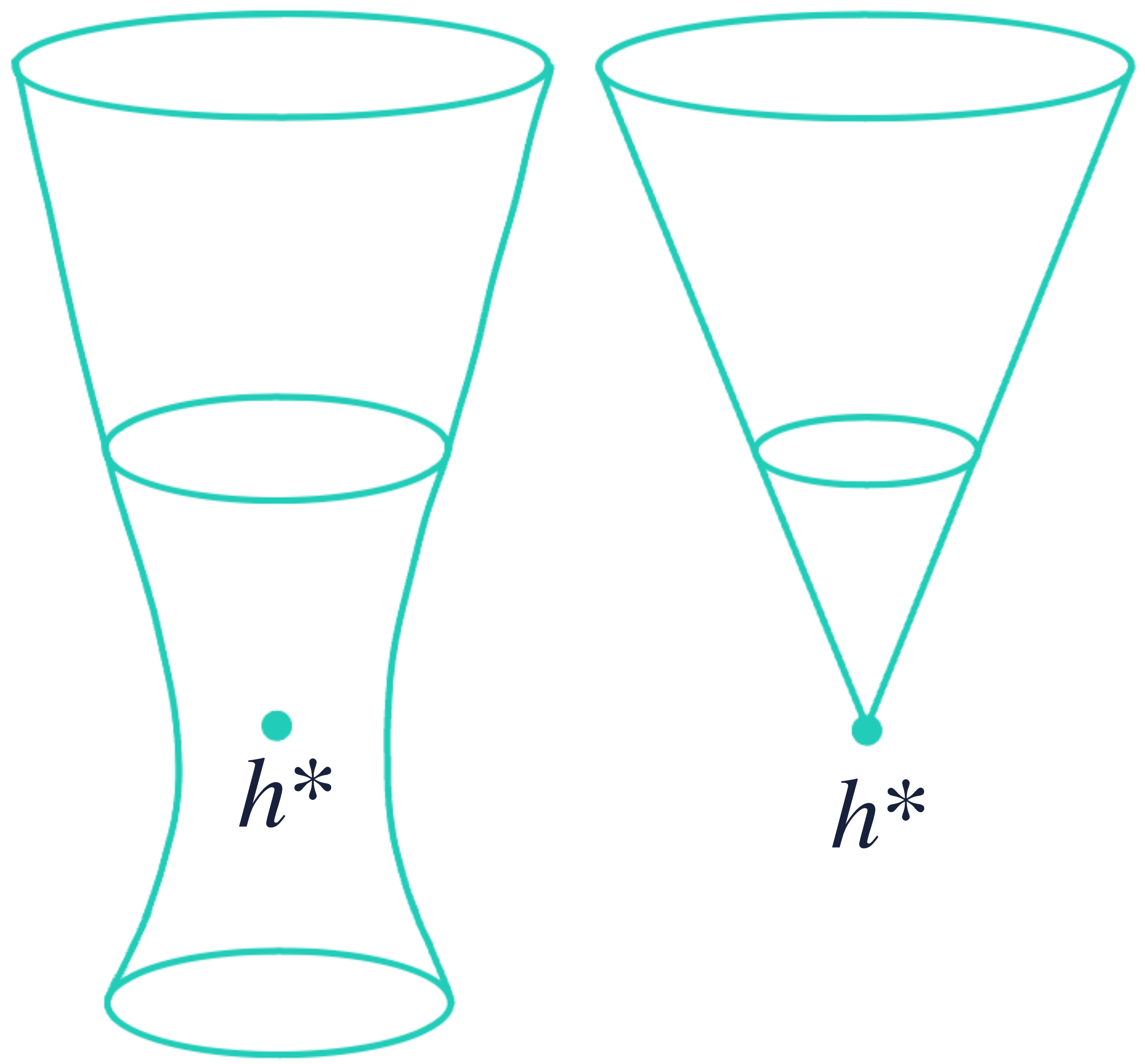


figure from
Aneesh Manohar

SMEFT



HEFT

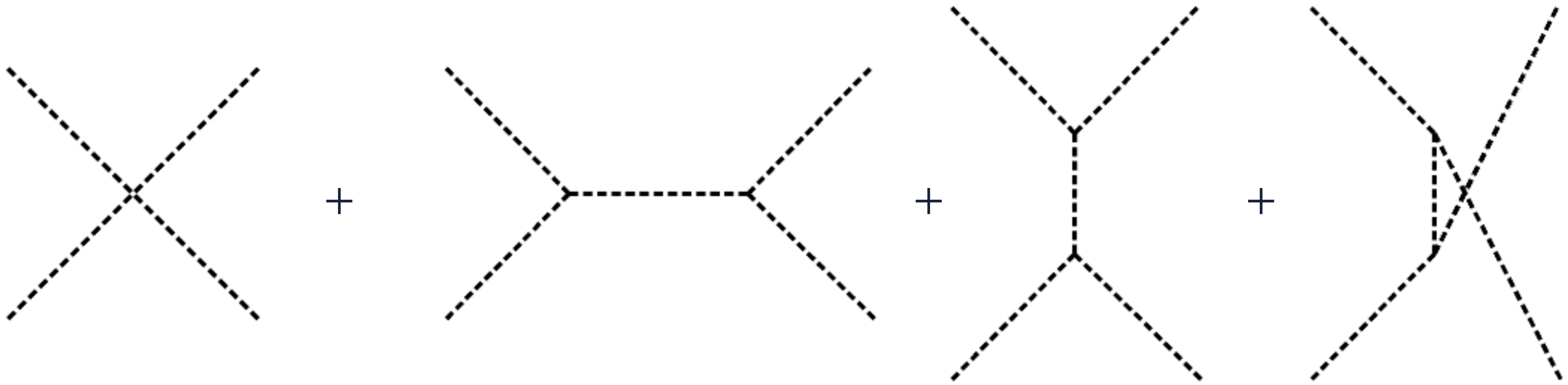


see All Things EFT talk
by Xiaochuan Lu

Amplitudes from geometry

Covariant amplitudes:

at tree-level



$$\mathscr{A}(\phi\phi \to \phi\phi) \;\sim\; (\mathrm{s}\,\partial\Gamma + \mathrm{t}\,\partial\Gamma + \mathrm{u}\,\partial\Gamma) \;+\; \mathrm{s}\,\Gamma\Gamma \;+\; \mathrm{t}\,\Gamma\Gamma \;+\; \mathrm{u}\,\Gamma\Gamma$$

$$\Rightarrow \mathscr{A}(\phi_I\phi_J \to \phi_K\phi_L) = R_{IKJL}\, s + R_{IJKL}\, t$$

Four-point amplitude depends on curvature.
Five-point amplitude depends on covariant derivative of the curvature.
...

Higgs cross-sections, W_L scattering, S parameter measurements can tell us if scalar manifold is flat or curved.
[Alonso, Jenkins, Manohar, 1511.00724]

RGE at one-loop

To obtain an algebraic formula for MS counterterms we use the background field method $\phi \to \phi + \eta$

at $\mathcal{O}(\eta^2)$: $$\delta^2 \mathscr{L} = \frac{1}{2}(\partial_\mu \eta)^T(\partial^\mu \eta) + (\partial_\mu \eta)^T N^\mu \eta + \frac{1}{2}\eta^T X \eta$$

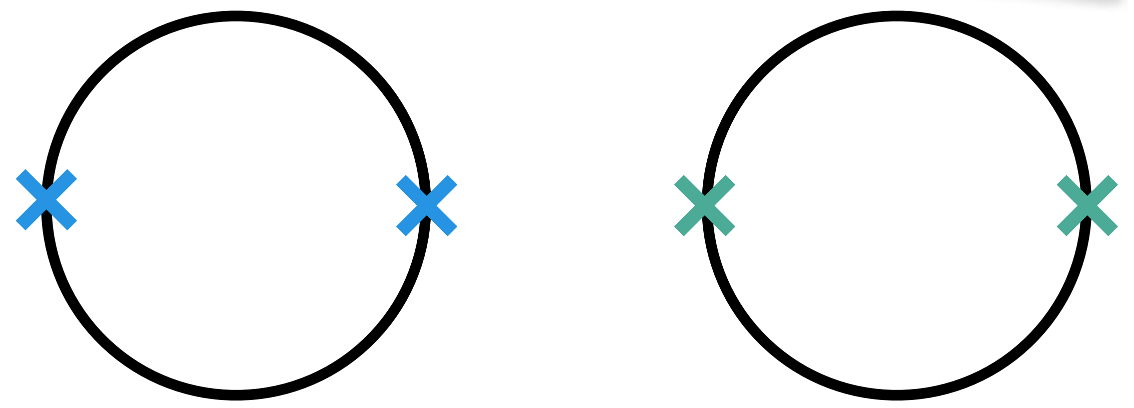
where N^μ is antisymmetric without loss of generality and X is symmetric.

With the covariant derivative $D_\mu \eta \equiv \partial_\mu \eta + N_\mu \eta$ and redefining X we have

$$\delta^2 \mathscr{L} = \frac{1}{2}(D_\mu \eta)^T(D^\mu \eta) + \frac{1}{2}\eta^T X \eta$$

Using naive dimensional analysis, the 't Hooft formula for one-loop counterterms is ['t Hooft, Nucl.Phys.B 62 (1973)]

$$\boxed{\mathscr{L}^{(1)}_{\text{c.t.}} = \frac{1}{16\pi^2 \epsilon}\text{Tr}\left[-\frac{1}{4}X^2 - \frac{1}{24}Y^2_{\mu\nu}\right]}$$ with $Y_{\mu\nu} = [D_\mu, D_\nu]$



RGE at two-loop

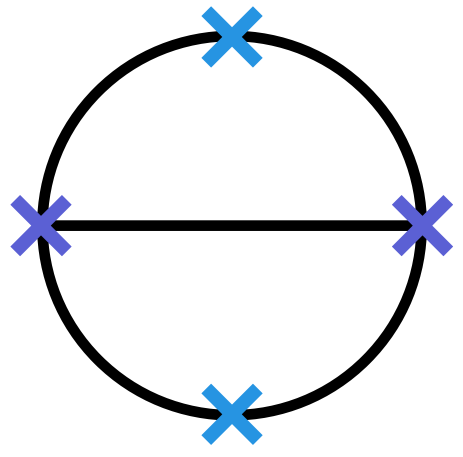
For two-loop we need the expansion of the Lagrangian in quantum fluctuations to

$\mathcal{O}(\eta^3)$: $$\delta^3\mathscr{L} = A_{abc}\eta^a\eta^b\eta^c + A^\mu_{a|bc}(D_\mu\eta)^a\eta^b\eta^c + A^{\mu\nu}_{ab|c}(D_\mu\eta)^a(D_\nu\eta)^b\eta^c$$

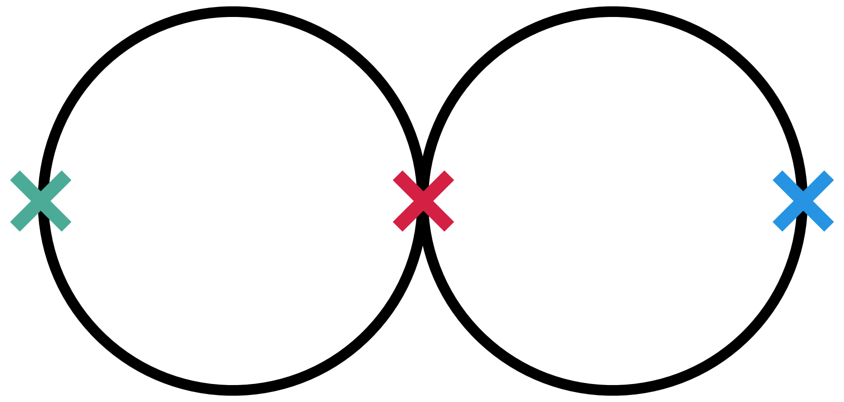
$\mathcal{O}(\eta^4)$: $$\delta^4\mathscr{L} = B_{abcd}\eta^a\eta^b\eta^c\eta^d + B^\mu_{a|bcd}(D_\mu\eta)^a\eta^b\eta^c\eta^d + B^{\mu\nu}_{ab|cd}(D_\mu\eta)^a(D_\nu\eta)^b\eta^c\eta^d$$

where A and B are symmetric and the completely symmetric parts of A^μ and B^μ vanish.

The graphs to compute for the two-loop algebraic formula are



with 0, 1 or 2 insertions of X / $Y_{\mu\nu}$



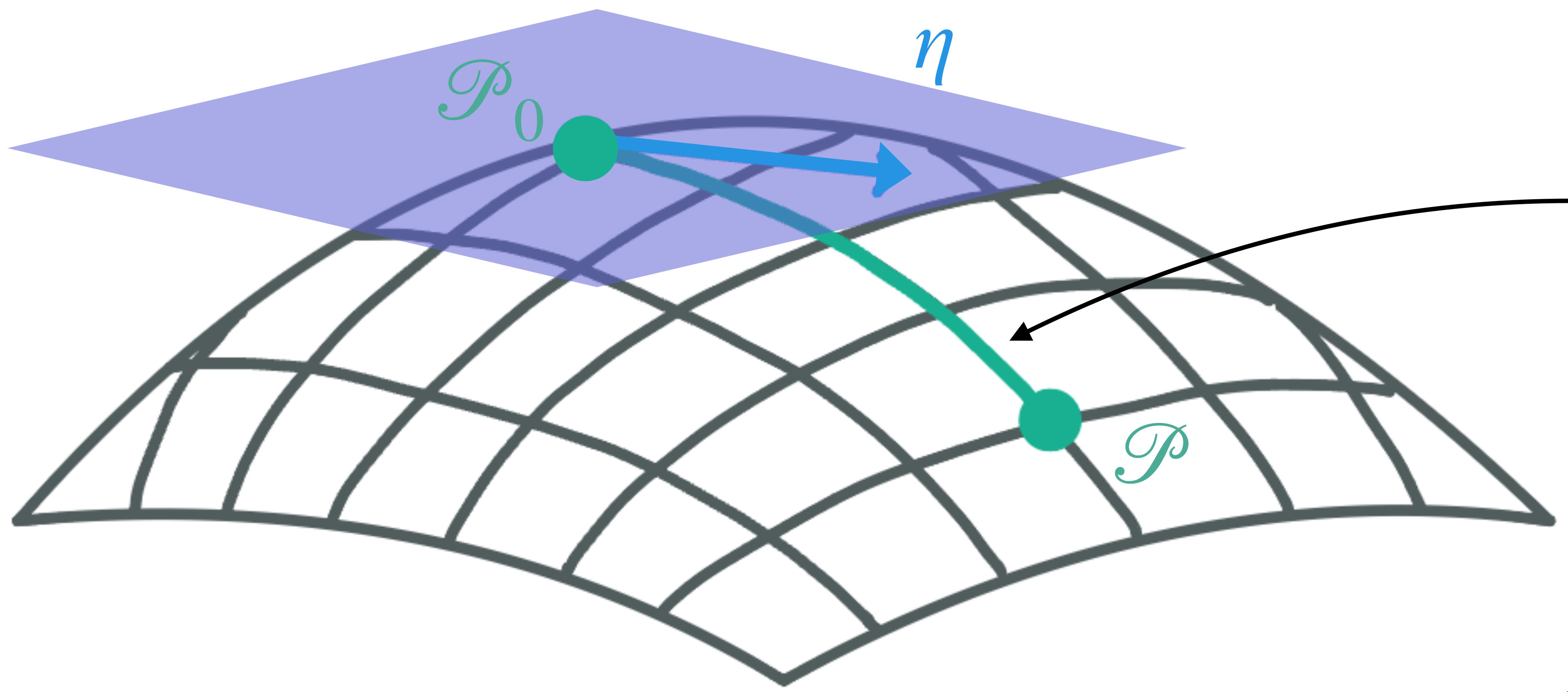
with 2 or 3 insertions of X / $Y_{\mu\nu}$

Full results in [Jenkins, Manohar, Naterop, JP, 2308.06315]

Riemannian normal coordinates

Using cartesian coordinates, we find that counterterms are not covariant.
The reason is that ϕ is a coordinate and does not transform as a tensor, but tangent vectors do.

Solution: use Riemannian normal coordinates (local coordinates obtained by applying the exponential map to the tangent space at $\mathscr{P}_0$) for the quantum fluctuation.



geodesic starting at $\mathscr{P}_0$
with tangent vector η
ending at $\mathscr{P}$ in unit time

$$g_{IJ}(\mathscr{P}_0) = \delta_{IJ} \qquad \Gamma^I_{JK}(\mathscr{P}_0) = 0 \qquad g_{IJ}(\phi) = \delta_{IJ} - \frac{1}{3} R_{IKJL}(\mathscr{P}_0)\phi^K\phi^L$$

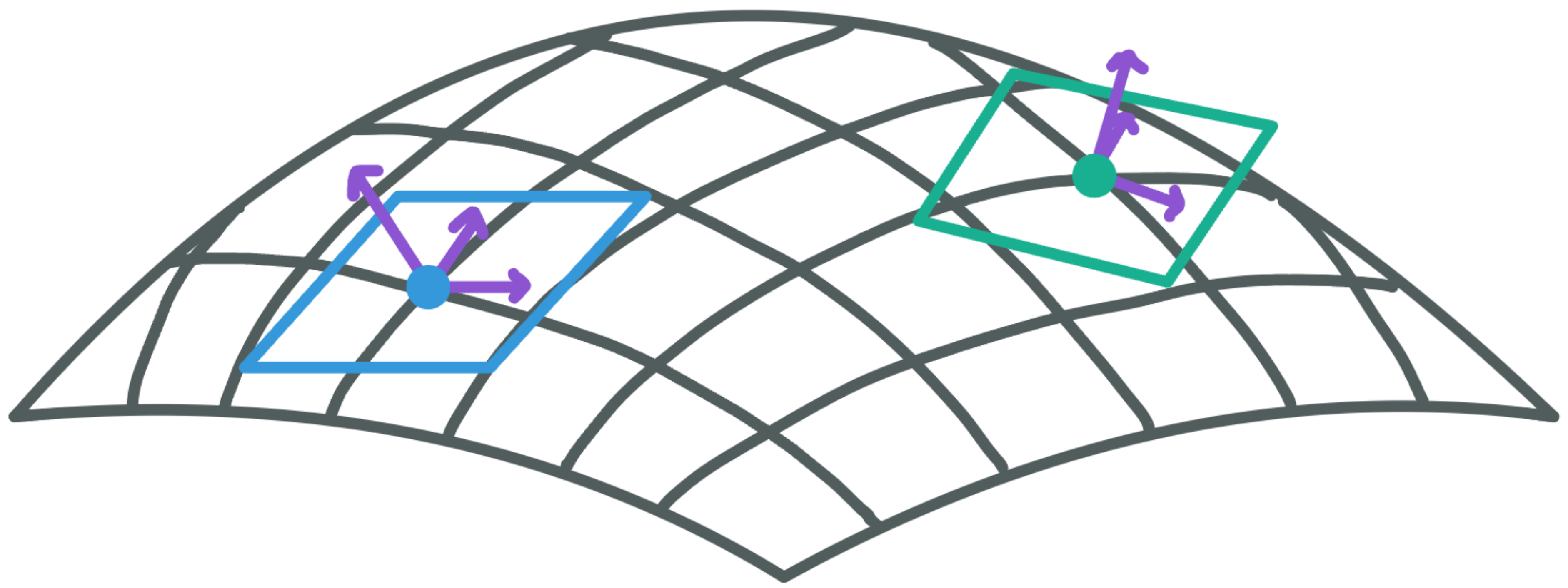
$\Rightarrow$ expand Lagrangian in

$$\boxed{\phi^I \to \phi^I + \eta^I - \frac{1}{2}\Gamma^I_{JK}\eta^J\eta^K - \frac{1}{3!}\Gamma^I_{JKL}\eta^I\eta^J\eta^K - \frac{1}{4!}\Gamma^I_{JKLM}\eta^I\eta^J\eta^K\eta^M + \mathcal{O}(\eta^5)}$$

Non-coordinate basis

Algebraic counterterm formulae were derived for renormalizable theories ⇔ for a flat field-space manifold.
So we cannot apply them to our coordinates on the curved field-space manifold.

<u>Solution:</u> go to local inertial frames using vielbeins and apply formulae there.



$$g_{IJ}(\phi) = e^a{}_I(\phi) e^b{}_J(\phi) \delta_{ab} \qquad (\mathscr{D}_\mu \eta)^I = e^I{}_a (\mathscr{D}_\mu \eta)^a \qquad R_{IJKL} = e^a{}_I e^b{}_J e^c{}_K e^d{}_L R_{abcd}$$

⇒ Since every indices are contracted, formulae are unchanged.

RGE from geometry

Using this technique we computed the RGE for:

- up to one-loop order
  - SMEFT bosonic sector to dim 8 [Helset, Jenkins, Manohar, 2212.03253]
  - SMEFT bosonic operators from a fermion loop to dim 8 [Assi, Helset, Manohar, JP, Shen, 2307.03187]

  $\rightarrow$ agree with [Chala, Guedes, Ramos, Santiago, 2106.05291]
  [Das Bakshi, Chala, Díaz-Carmona, Guedes, 2205.03301]

- up to two-loop order [Jenkins, Manohar, Naterop, JP, 2310.19883]
  - $O(N)$ scalar EFT to dim 6 $\rightarrow$ agree with [Cao, Herzog, Melia, Nepveu, 2105.12742]
  - SMEFT scalar sector to dim 6 $\rightarrow$ new!
  - χPT to $\mathcal{O}(p^6)$ $\rightarrow$ agree with [Bijnens, Colangelo, Ecker, hep-ph/9907333]

$\hookrightarrow$ directly usable for dim 8

What remains

- More RGEs
  - full one-loop RGE for SMEFT at dim 8
    - mixed scalar-fermion loops
    - four-fermion operators [Assi, Helset, JP, Shen, w.i.p]
    - contributions to fermionic operators
    - mixed vector-fermion loops
  - two-loop counterterm formula including fermions and gauge bosons
- More derivatives
  - operators with more than one derivative on each field
    - Lagrange spaces? [Craig, Lee, Lu, Sutherland, 2305.09722]
    - jet bundle geometry? [Alminawi, Brivio, Davighi, 2308.00017] [Craig, Lee, 2307.15742]
  - derivative field redefinition
    - on-shell covariance of amplitudes? [Cohen, Craig, Lu, Sutherland, 2202.06965]
    - geometry-kinematics duality? [Cheung, Helset, and Parra-Martinez, 2202.06972]
- More applications

Conclusion

Summary:

- Field-space geometry offer an alternative, more basis-independent, description of EFTs
- Scattering amplitudes and RGE are covariant and easier to compute
- RGE calculations are easily generalizable to any EFT order

Future plans:

- Some formal developments needed to generalize to arbitrary EFTs
- Develop phenomenology studies with geometry

Thank you!

Beyond scalars

Gauge fields

Fermions

$$\mathscr{L} = \frac{1}{2}h_{IJ}(\phi)(D_\mu\phi)^I(D^\mu\phi)^J - \frac{1}{4}g_{AB}(\phi)F^A_{\mu\nu}F^{B\mu\nu} - V(\phi) \quad + \quad \frac{i}{2}k_{\bar{p}r}(\phi)\left(\overline{\psi}^{\bar{p}}\gamma^\mu\overleftrightarrow{D}_\mu\psi^r\right) + i\omega_{\bar{p}rI}(\phi)(D_\mu\phi)^I(\overline{\psi}^{\bar{p}}\gamma^\mu\psi^r) - \overline{\psi}^{\bar{p}}\mathscr{M}_{\bar{p}r}\psi^r + \overline{\psi}^{\bar{p}}\sigma_{\mu\nu}\mathscr{T}^{\mu\nu}_{\bar{p}r}(\phi, F)\psi^r$$

$$(D_\mu\phi)^I = \partial_\mu\phi^I + A^B_\mu t^I_B(\phi)$$

Killing vectors

Vector-scalar field-space manifold

$$g_{ij} = \begin{pmatrix} h_{IJ} & 0 \\ 0 & \eta_{\mu_A\mu_B}g_{AB} \end{pmatrix}$$

[Helset, Jenkins, Manohar, 2210.08000]

Fermion-scalar field-space supermanifold

(with Grassmann coordinates)

$$g_{ab} = \begin{pmatrix} h_{IJ} & (\omega^-\overline{\psi})_{rI} & (\omega^+\psi)_{\bar{r}I} \\ -(\omega^-\overline{\psi})_{pJ} & 0 & k_{\bar{r}p} \\ -(\omega^+\psi)_{\bar{p}J} & -k_{\bar{p}r} & 0 \end{pmatrix} \quad \omega^{\pm}_{\bar{p}rI} = \omega_{\bar{p}rI} \pm \frac{1}{2}k_{\bar{p}r,I}$$

[Assi, Helset, Manohar, JP, Shen, 2307.03187]

[Finn, Karamitsos, Pilaftsis, 2006.05831]

A-type counterterms

$$
\begin{aligned}
\mathcal{L}^{(A,2)}_{\text{c.t.}} = \frac{1}{(16\pi^2)^2}\Big[& a_{1,1} D_\mu A_{abc} D_\mu A_{abc} + a_{2,1} A_{abc} X_{cd} A_{abd} \\
& + a_{3,1} D_\mu A^\mu_{a|bc} A_{abd} X_{cd} + a_{3,2} A^\mu_{a|bc} D_\mu A_{abd} X_{cd} + a_{4,1} D_\nu A^\mu_{a|bc} A_{abd} Y^{\mu\nu}_{cd} + a_{4,2} A^\mu_{a|bc} D_\nu A_{abd} Y^{\mu\nu}_{cd} \\
& + a_{5,1} D^2 A^\mu_{a|bc} D^2 A^\mu_{a|bc} + a_{5,2} D_\alpha D_\mu A^\mu_{a|bc} D_\alpha D_\nu A^\nu_{a|bc} \\
& + a_{6,1} D^2 A^\mu_{a|bc} A^\mu_{a|bd} X_{cd} + a_{6,2} D^2 A^\mu_{c|ab} A^\mu_{d|ab} X_{cd} + a_{6,3} D_\alpha A^\mu_{a|bc} D_\alpha A^\mu_{a|bd} X_{cd} + a_{6,4} D_\alpha A^\mu_{c|ab} D_\alpha A^\mu_{d|ab} X_{cd} \\
& + a_{6,5} D_\mu A^\mu_{a|bc} D_\nu A^\nu_{a|bd} X_{cd} + a_{6,6} D_\mu A^\mu_{c|ab} D_\nu A^\nu_{d|ab} X_{cd} + a_{6,7} D_\nu A^\mu_{a|bc} D_\mu A^\nu_{a|bd} X_{cd} \\
& + a_{6,8} D_\nu A^\mu_{c|ab} D_\mu A^\nu_{d|ab} X_{cd} + a_{6,9} D_\nu D_\mu A^\mu_{a|bc} A^\nu_{a|bd} X_{cd} + a_{6,10} D_\nu D_\mu A^\mu_{c|ab} A^\nu_{d|ab} X_{cd} \\
& + a_{7,1} D_\alpha A^\mu_{a|bc} D_\alpha A^\nu_{a|bd} Y^{\mu\nu}_{cd} + a_{7,2} D_\alpha A^\mu_{c|ab} D_\alpha A^\nu_{d|ab} Y^{\mu\nu}_{cd} + a_{7,3} D_\mu A^\alpha_{a|bc} D_\nu A^\alpha_{a|bd} Y^{\mu\nu}_{cd} \\
& + a_{7,4} D_\mu A^\alpha_{c|ab} D_\nu A^\alpha_{d|ab} Y^{\mu\nu}_{cd} + a_{7,5} D_\mu A^\alpha_{a|bc} D_\nu A^\nu_{a|bd} Y^{\mu\alpha}_{cd} + a_{7,6} D_\mu A^\alpha_{c|ab} D_\nu A^\nu_{d|ab} Y^{\mu\alpha}_{cd} \\
& + a_{7,7} D_\nu A^\alpha_{a|bc} D_\mu A^\nu_{a|bd} Y^{\mu\alpha}_{cd} + a_{7,8} D_\nu A^\alpha_{c|ab} D_\mu A^\nu_{d|ab} Y^{\mu\alpha}_{cd} + a_{7,9} A^\alpha_{a|bc} D_\mu D_\nu A^\nu_{a|bd} Y^{\mu\alpha}_{cd} \\
& + a_{7,10} A^\alpha_{c|ab} D_\mu D_\nu A^\nu_{d|ab} Y^{\mu\alpha}_{cd} + a_{7,11} D_\mu D_\nu A^\alpha_{a|bc} A^\nu_{a|bd} Y^{\mu\alpha}_{cd} + a_{7,12} D_\mu D_\nu A^\alpha_{c|ab} A^\nu_{d|ab} Y^{\mu\alpha}_{cd} \\
& + a_{8,1} A^\mu_{c|ab} A^\mu_{d|ab} X_{ce} X_{ed} + a_{8,2} A^\mu_{a|bc} A^\mu_{a|bd} X_{ce} X_{ed} + a_{8,3} A^\mu_{a|bc} A^\mu_{e|bd} X_{ae} X_{cd} + a_{8,4} A^\mu_{a|bc} A^\mu_{a|de} X_{bd} X_{ce} \\
& + a_{9,1} A^\mu_{c|ab} A^\nu_{d|ab} (X_{ce} Y^{\mu\nu}_{ed} + Y^{\mu\nu}_{ce} X_{ed}) + a_{9,2} A^\mu_{a|bc} A^\nu_{a|bd} (X_{ce} Y^{\mu\nu}_{ed} + Y^{\mu\nu}_{ce} X_{ed}) \\
& + a_{9,3} A^\mu_{a|bc} A^\nu_{e|bd} X_{ae} Y^{\mu\nu}_{cd} + a_{9,4} A^\mu_{a|bc} A^\nu_{a|de} X_{ce} Y^{\mu\nu}_{bd} + a_{9,5} A^\mu_{a|bc} A^\nu_{e|bd} X_{cd} Y^{\mu\nu}_{ae} \\
& + a_{10,1} A^\mu_{c|ab} A^\mu_{d|ab} Y^{\alpha\beta}_{ce} Y^{\alpha\beta}_{ed} + a_{10,2} A^\mu_{a|bc} A^\mu_{a|bd} Y^{\alpha\beta}_{ce} Y^{\alpha\beta}_{ed} + a_{10,3} A^\mu_{c|ab} A^\nu_{d|ab} Y^{\mu\alpha}_{ce} Y^{\nu\alpha}_{ed} \\
& + a_{10,4} A^\mu_{a|bc} A^\nu_{a|bd} Y^{\mu\alpha}_{ce} Y^{\nu\alpha}_{ed} + a_{10,5} A^\mu_{c|ab} A^\nu_{d|ab} Y^{\nu\alpha}_{ce} Y^{\mu\alpha}_{ed} + a_{10,6} A^\mu_{a|bc} A^\nu_{a|bd} Y^{\nu\alpha}_{ce} Y^{\mu\alpha}_{ed} \\
& + a_{10,7} A^\mu_{a|bc} A^\mu_{e|bd} Y^{\alpha\beta}_{ae} Y^{\alpha\beta}_{cd} + a_{10,8} A^\mu_{a|bc} A^\mu_{a|de} Y^{\alpha\beta}_{bd} Y^{\alpha\beta}_{ce} + a_{10,9} A^\mu_{a|bc} A^\nu_{e|bd} (Y^{\mu\alpha}_{ae} Y^{\nu\alpha}_{cd} + Y^{\nu\alpha}_{ae} Y^{\mu\alpha}_{cd}) \\
& + a_{10,10} A^\mu_{a|bc} A^\nu_{a|de} (Y^{\mu\alpha}_{bd} Y^{\nu\alpha}_{ce} + Y^{\nu\alpha}_{bd} Y^{\mu\alpha}_{ce}) + a_{10,11} A^\mu_{a|bc} A^\nu_{b|ed} (Y^{\mu\alpha}_{ae} Y^{\nu\alpha}_{cd} - Y^{\nu\alpha}_{ae} Y^{\mu\alpha}_{cd}) \Big] .
\end{aligned}
$$

$a_{1,1} = -\frac{3}{4\epsilon}$,	$a_{2,1} = \frac{9}{2\epsilon^2} - \frac{9}{2\epsilon}$,		
$a_{3,1} = \frac{3}{2\epsilon^2} - \frac{15}{4\epsilon}$,	$a_{3,2} = \frac{9}{2\epsilon^2} - \frac{9}{4\epsilon}$,	$a_{4,1} = -\frac{3}{2\epsilon^2} + \frac{7}{4\epsilon}$,	$a_{4,2} = -\frac{3}{2\epsilon^2} - \frac{5}{4\epsilon}$,
$a_{5,1} = \frac{1}{64\epsilon}$,	$a_{5,2} = -\frac{1}{48\epsilon}$,		
$a_{6,1} = \frac{1}{36\epsilon^2} + \frac{25}{216\epsilon}$,	$a_{6,2} = \frac{13}{72\epsilon^2} - \frac{107}{432\epsilon}$,	$a_{6,3} = -\frac{5}{36\epsilon^2} + \frac{37}{216\epsilon}$,	$a_{6,4} = \frac{2}{9\epsilon^2} - \frac{2}{27\epsilon}$,
$a_{6,5} = \frac{1}{36\epsilon^2} - \frac{5}{216\epsilon}$,	$a_{6,6} = -\frac{5}{72\epsilon^2} - \frac{65}{432\epsilon}$,	$a_{6,7} = \frac{1}{36\epsilon^2} - \frac{5}{216\epsilon}$,	$a_{6,8} = \frac{13}{72\epsilon^2} - \frac{11}{432\epsilon}$,
$a_{6,9} = -\frac{1}{9\epsilon^2} + \frac{5}{54\epsilon}$,	$a_{6,10} = \frac{1}{36\epsilon^2} - \frac{59}{216\epsilon}$,		
$a_{7,1} = -\frac{1}{48\epsilon}$,	$a_{7,2} = -\frac{13}{96\epsilon}$,	$a_{7,3} = \frac{1}{18\epsilon^2} + \frac{1}{432\epsilon}$,	$a_{7,4} = -\frac{1}{72\epsilon^2} - \frac{41}{864\epsilon}$,
$a_{7,5} = -\frac{1}{36\epsilon^2} + \frac{13}{432\epsilon}$,	$a_{7,6} = \frac{5}{72\epsilon^2} - \frac{191}{864\epsilon}$,	$a_{7,7} = \frac{1}{36\epsilon^2} - \frac{13}{432\epsilon}$,	$a_{7,8} = \frac{13}{72\epsilon^2} - \frac{61}{864\epsilon}$,
$a_{7,9} = -\frac{1}{36\epsilon^2} - \frac{17}{432\epsilon}$,	$a_{7,10} = \frac{5}{72\epsilon^2} - \frac{149}{864\epsilon}$,	$a_{7,11} = \frac{1}{36\epsilon^2} - \frac{19}{432\epsilon}$,	$a_{7,12} = \frac{13}{72\epsilon^2} - \frac{139}{864\epsilon}$,
$a_{8,1} = -\frac{5}{16\epsilon^2} + \frac{19}{96\epsilon}$,	$a_{8,2} = \frac{1}{8\epsilon^2} - \frac{11}{48\epsilon}$,	$a_{8,3} = -\frac{1}{4\epsilon^2} + \frac{5}{8\epsilon}$,	$a_{8,4} = -\frac{1}{2\epsilon^2} + \frac{1}{8\epsilon}$,
$a_{9,1} = \frac{13}{72\epsilon^2} - \frac{11}{432\epsilon}$,	$a_{9,2} = \frac{1}{36\epsilon^2} - \frac{5}{216\epsilon}$,	$a_{9,3} = -\frac{19}{36\epsilon^2} + \frac{5}{216\epsilon}$,	$a_{9,4} = \frac{11}{36\epsilon^2} + \frac{17}{216\epsilon}$,
$a_{9,5} = \frac{11}{36\epsilon^2} - \frac{145}{216\epsilon}$,			
$a_{10,1} = \frac{35}{1152\epsilon} - \frac{5}{96\epsilon^2}$,	$a_{10,2} = \frac{1}{48\epsilon^2} - \frac{25}{576\epsilon}$,	$a_{10,3} = \frac{13}{144\epsilon^2} + \frac{251}{1728\epsilon}$	$a_{10,4} = \frac{1}{72\epsilon^2} + \frac{11}{864\epsilon}$,
$a_{10,5} = \frac{13}{144\epsilon^2} - \frac{217}{1728\epsilon}$,	$a_{10,6} = \frac{1}{72\epsilon^2} - \frac{25}{864\epsilon}$,	$a_{10,7} = \frac{1}{72\epsilon^2} - \frac{67}{864\epsilon}$,	$a_{10,8} = \frac{1}{36\epsilon^2} - \frac{25}{1728\epsilon}$,
$a_{10,9} = -\frac{29}{144\epsilon}$,	$a_{10,10} = \frac{19}{288\epsilon}$,	$a_{10,11} = -\frac{1}{8\epsilon}$	

B-type counterterms

$$
\begin{aligned}
\mathcal{L}_{\text{c.t.}}^{(B,2)} = \frac{1}{(16\pi^2)^2\epsilon^2}\Bigg[& 3B_{abcd}X_{ab}X_{cd} + \frac{3}{2}B^{\alpha}_{a|bcd}(D_\alpha X)_{ab}X_{cd} + \frac{1}{2}B^{\alpha}_{a|bcd}(D_\mu Y_{\mu\alpha})_{ab}X_{cd} \\
& + \frac{1}{12}B^{\alpha\alpha}_{ab|cd}(D^2X)_{ab}X_{cd} + \frac{1}{12}B^{\mu\nu}_{ab|cd}(\{D_\mu, D_\nu\}X)_{ab}X_{cd} + \frac{1}{12}B^{\mu\nu}_{ab|cd}(D^2Y^{\mu\nu})_{ab}X_{cd} \\
& - \frac{1}{4}B^{\alpha\alpha}_{ab|cd}X_{ae}X_{eb}X_{cd} + \frac{1}{4}B^{\mu\nu}_{ab|cd}(X_{ae}Y^{\mu\nu}_{eb} + Y^{\mu\nu}_{ae}X_{eb})X_{cd} \\
& - \frac{1}{12}B^{\mu\nu}_{ab|cd}Y^{\mu\alpha}_{ae}Y^{\nu\alpha}_{eb}X_{cd} + \frac{1}{4}B^{\mu\nu}_{ab|cd}Y^{\nu\alpha}_{ae}Y^{\mu\alpha}_{eb}X_{cd} - \frac{1}{24}B^{\alpha\alpha}_{ab|cd}Y^{\mu\nu}_{ae}Y^{\mu\nu}_{eb}X_{cd} \\
& + \frac{1}{2}B^{\mu\nu}_{ab|cd}(D_\mu X)_{ac}(D_\nu X)_{bd} + \frac{1}{18}B^{\mu\nu}_{ab|cd}(D_\alpha Y^{\alpha\mu})_{ac}(D_\beta Y^{\beta\nu})_{bd} + \frac{1}{6}B^{\mu\nu}_{ab|cd}(D_\mu X)_{ac}(D_\beta Y^{\beta\nu})_{bd}\Bigg]
\end{aligned}
$$

Background coefficients in normal coordinates

$$X_{ab} = -R_{acbd}(D_\mu\phi)^c(D^\mu\phi)^d - \nabla_a\nabla_b V$$

$$[Y_{\mu\nu}]_{ab} = R_{abcd}(D_\mu\phi^c)(D_\nu\phi^d) + \nabla_b t_{a,\alpha}F^\alpha_{\mu\nu}$$

$$A_{abc} = -\frac{1}{6}\nabla_{(a}\nabla_b\nabla_{c)}V - \frac{1}{18}(\nabla_a R_{bdce} + \nabla_b R_{cdae} + \nabla_c R_{adbe})(D_\mu\phi)^d(D^\mu\phi)^e$$

$$A^\mu_{a|bc} = \frac{1}{3}(R_{abcd} + R_{acbd})(D^\mu\phi)^d$$

$$A^{\mu\nu}_{ab|c} = 0$$

$$B_{abcd} = -\frac{1}{24}\nabla_a\nabla_b\nabla_c\nabla_d V - \frac{1}{24}\nabla_a\nabla_d R_{becf}(D_\mu\phi)^e(D^\mu\phi)^f + \frac{1}{6}R_{eabf}R_{ecdg}(D_\mu\phi)^f(D^\mu\phi)^g \qquad \text{sym(bcd)}$$

$$B^\mu_{a|bcd} = \frac{1}{4}(\nabla_d R_{abce})(D^\mu\phi)^e \qquad \text{sym(bcd)}$$

$$B^{\mu\nu}_{ab|cd} = -\frac{1}{12}\eta^{\mu\nu}(R_{acbd} + R_{adbc})$$

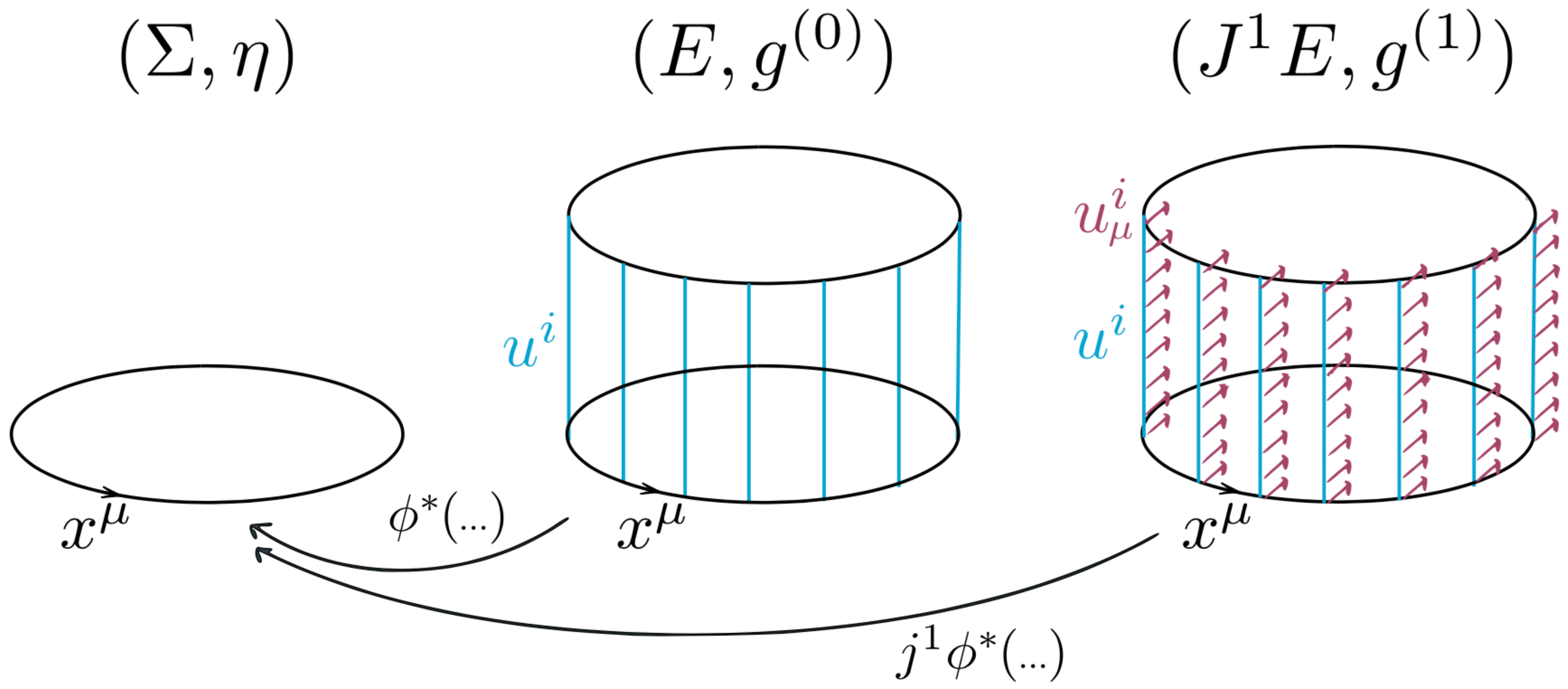
Fermion one-loop RGE

$$
\begin{aligned}
\Delta S =& \frac{1}{32\pi^2\epsilon}\int \mathrm{d}^4x \left\{\frac{1}{3}\mathrm{Tr}\left[\mathcal{Y}_{\mu\nu}\mathcal{Y}^{\mu\nu}\right] + \mathrm{Tr}\left[(\mathcal{D}_\mu\mathcal{M})(\mathcal{D}^\mu\mathcal{M}) - (\mathcal{M}\mathcal{M})^2\right]\right. \\
& - \frac{16}{3}\mathrm{Tr}[(\mathcal{D}_\mu\mathcal{T}^{\mu\alpha})(\mathcal{D}_\nu\mathcal{T}^{\nu\alpha}) - (\mathcal{T}^{\mu\nu}\mathcal{T}^{\alpha\beta})^2] \\
& \left. - 4i\mathrm{Tr}[\mathcal{Y}_{\mu\nu}(\mathcal{M}\mathcal{T}^{\mu\nu} + \mathcal{T}^{\mu\nu}\mathcal{M})] - 8\mathrm{Tr}(\mathcal{M}\mathcal{T}^{\mu\nu})^2\right\},
\end{aligned}
$$

$$
\begin{aligned}
[\mathcal{Y}_{\mu\nu}]^p{}_r =& [\mathcal{D}_\mu, \mathcal{D}_\nu]^p{}_r = \bar{R}^p{}_{rIJ}(D_\mu\phi)^I(D_\nu\phi)^J + \left(\bar{\nabla}_r t^p_A\right) F^A_{\mu\nu}\,, \\
(\mathcal{D}_\mu\mathcal{M})^p{}_r =& k^{p\bar{t}}(\mathcal{D}_\mu\mathcal{M}_{\bar{t}r}) = k^{p\bar{t}}\left[D_\mu\mathcal{M}_{\bar{t}r} - \bar{\Gamma}^{\bar{s}}_{I\bar{t}}(D_\mu\phi)^I\mathcal{M}_{\bar{s}r} - \bar{\Gamma}^{s}_{Ir}(D_\mu\phi)^I\mathcal{M}_{\bar{t}s}\right]\,, \\
(\mathcal{M}\mathcal{M})^p{}_r =& k^{p\bar{t}}\mathcal{M}_{\bar{t}q}k^{q\bar{s}}\mathcal{M}_{\bar{s}r}\,, \\
(\mathcal{D}_\mu\mathcal{T}^{\alpha\beta})^p{}_r =& k^{p\bar{t}}(\mathcal{D}_\mu\mathcal{T}^{\alpha\beta}_{\bar{t}r}) = k^{p\bar{t}}\left[D_\mu\mathcal{T}^{\alpha\beta}_{\bar{t}r} - \bar{\Gamma}^{\bar{s}}_{I\bar{t}}(D_\mu\phi)^I\mathcal{T}^{\alpha\beta}_{\bar{s}r} - \bar{\Gamma}^{s}_{Ir}(D_\mu\phi)^I\mathcal{T}^{\alpha\beta}_{\bar{t}s}\right]\,, \\
(\mathcal{T}^{\mu\nu}\mathcal{T}^{\alpha\beta})^p{}_r =& k^{p\bar{t}}\mathcal{T}^{\mu\nu}_{\bar{t}q}k^{q\bar{s}}\mathcal{T}^{\alpha\beta}_{\bar{s}r}\,.
\end{aligned}
$$

More derivatives

Jet Bundle Geometry of Scalar Field Theories



[Alminawi, Brivio, Davighi, 2308.00017]