One Loop Effective Action up to Dimension Eight: integrating out heavy scalars and fermions

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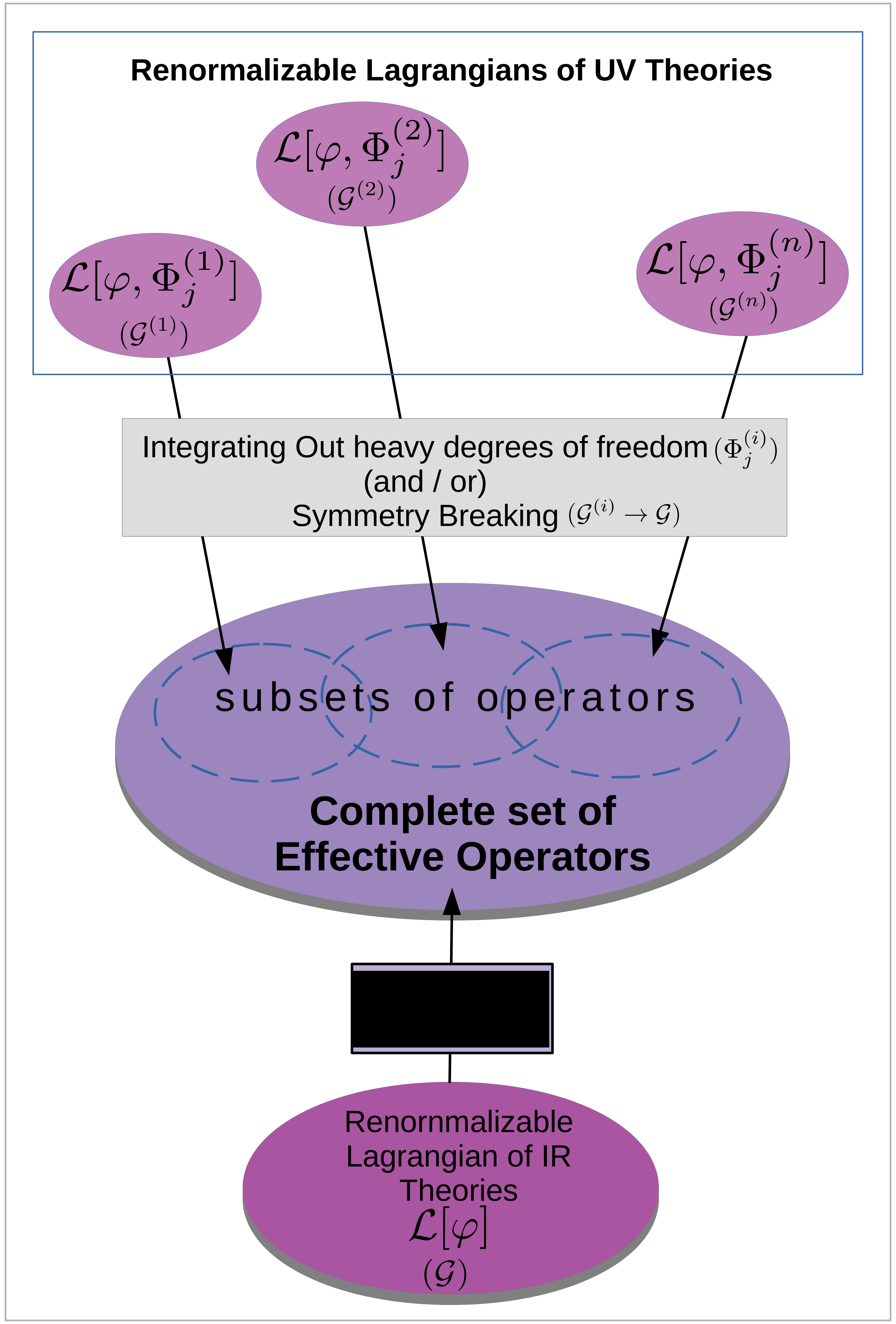
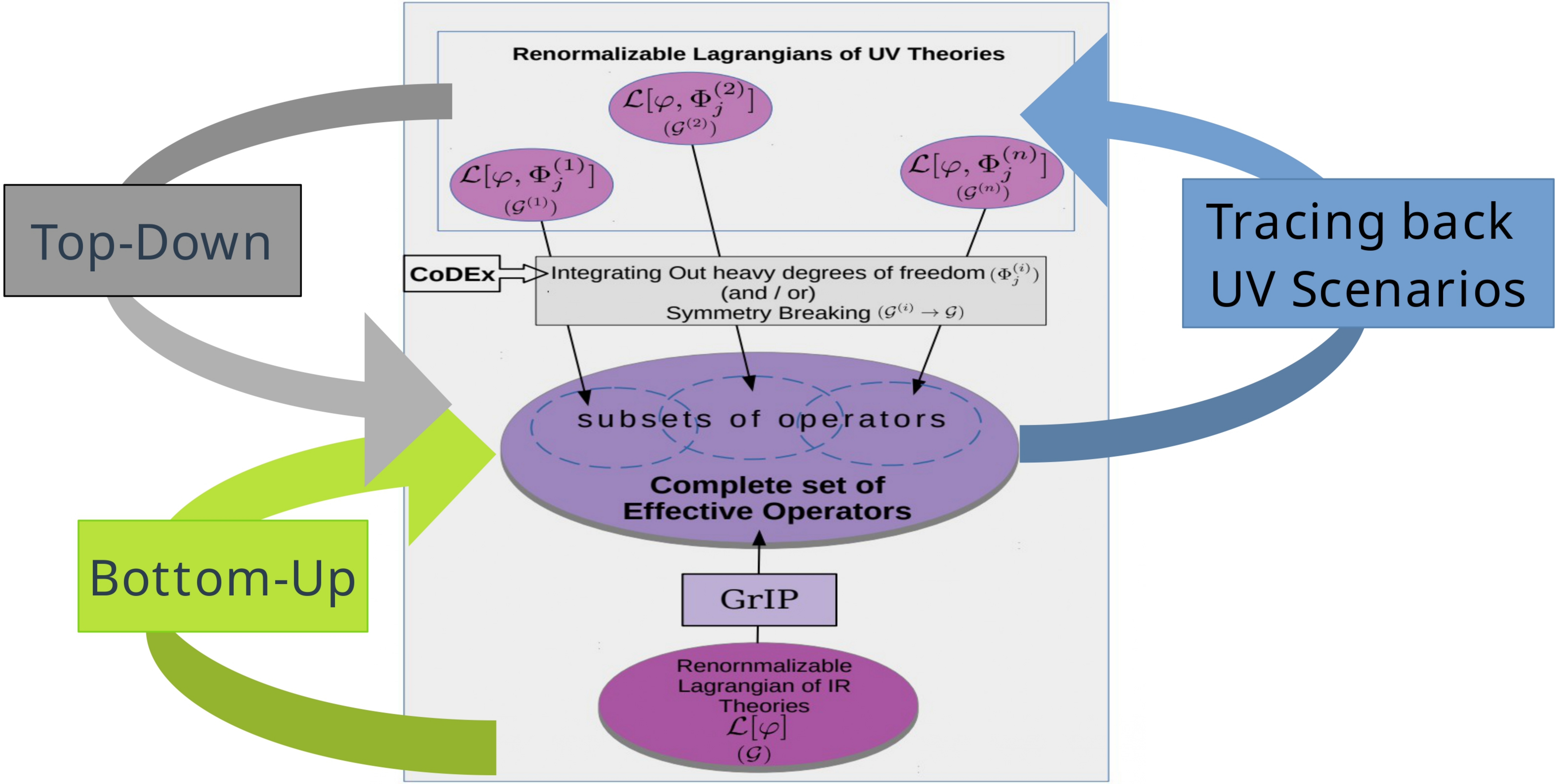
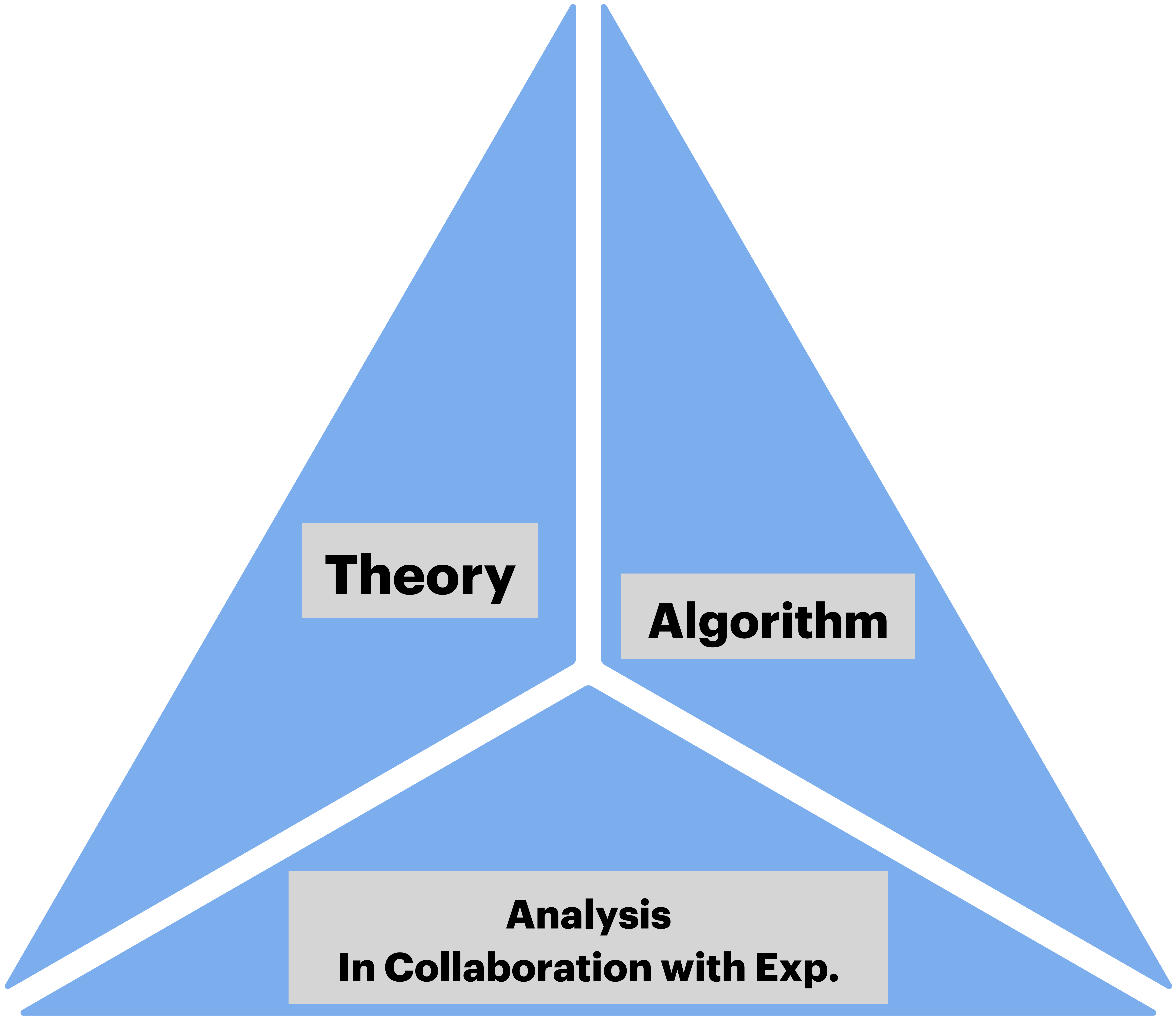
Question that we want to address!



Our plan to address!



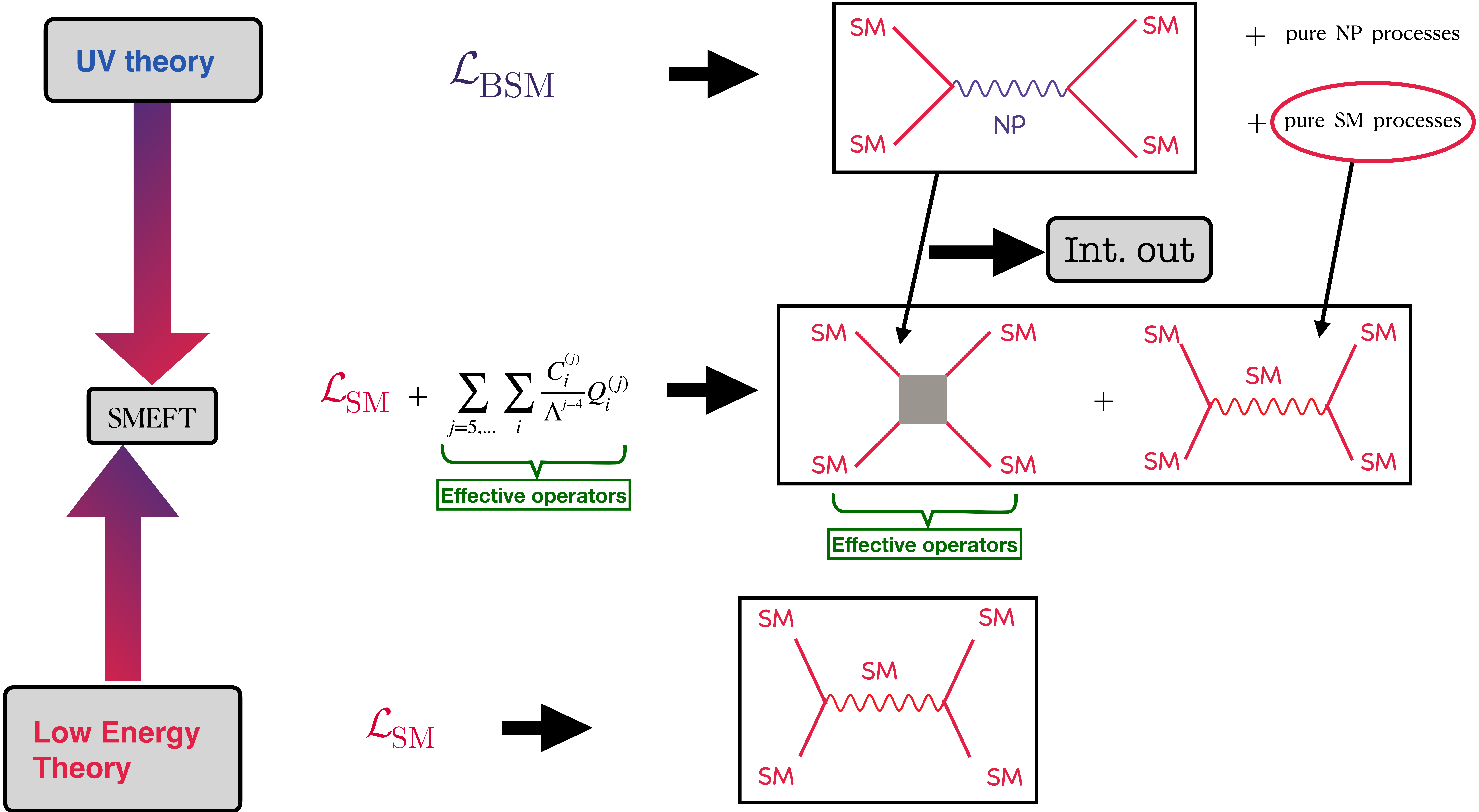
Three Important Areas : to be developed



Quick key points

- ❖ Bottom-Up vs Top-Down
- ❖ BSMs as Effective Theories
- ❖ Observables (set of operators) as "Response Screen"
- ❖ Classifications of BSMs
- ❖ Operator driven BSM construction: Reverse engineering
- ❖ CP-violation in SMEFT

Top-Down vs Bottom-Up



BSM Classifications

SM

+

Heavy Scalars

Color-singlets

BSMs	$\mathcal{S}$	$\mathcal{S}_2$	Δ	$\mathcal{H}_2$	Δ_1	Σ
$\mathcal{G}_{3,2,1}$	1,1,0	1,1,2	1,3,0	1,2,-1/2	1,3,1	1,4,1/2

Colored

BSMs	φ_1	φ_2	Θ_1	Θ_2	Ω	χ_1	χ_2	χ_3	χ_4
$\mathcal{G}_{3,2,1}$	3,1,-1/3	3,1,-4/3	3,2,1/6	3,2,7/6	3,3,-1/3	6,3,1/3	6,1,4/3	6,1,-2/3	6,1,1/3

Observable-Operator correspondence (D6)

$$\text{EWPO-LO}: \{Q_{HD}, Q_{HWB}, Q_{Hq}^{(1)}, Q_{Hq}^{(3)}, Q_{Hl}^{(1)}, Q_{Hl}^{(3)}, Q_{He}, Q_{Hu}, Q_{Hd}, Q_{ll}\}$$

$$\text{EWPO-NLO-I}: \{Q_{HB}, Q_{HW}, Q_{H\Box}\}$$

$$\text{Higgs Signal Strength (HSS)}: \text{EWPO-LO} + \text{EWPO-NLO-I} + \{Q_H, Q_{uH}, Q_{dH}, Q_{eH}, Q_G, Q_{HG}\}$$

$$\text{EWPO-NLO-II}: \{Q_{ed}, Q_{ee}, Q_{eu}, Q_{lu}, Q_{ld}, Q_{le}, Q_{lq}^{(1)}, Q_{lq}^{(3)}, Q_{qe}, Q_{uB}, Q_{uW}, Q_W, Q_{qd}^{(1)}, Q_{qq}^{(1)}, Q_{qq}^{(3)}, Q_{qu}^{(1)}, Q_{ud}^{(1)}, Q_{uu}, Q_{dd}\}$$

$$\text{Additional Operators (AdOps)}: \{Q_{ud}^{(8)}, Q_{qd}^{(8)}, Q_{qu}^{(8)}, Q_{quqd}^{(1)}, Q_{lequ}^{(1)}, Q_{quqd}^{(8)}, Q_{ledq}\}$$

B, L violating Operators : $\{Q_{qqq}, Q_{duu}, Q_{qqu}, Q_{duq}\}$

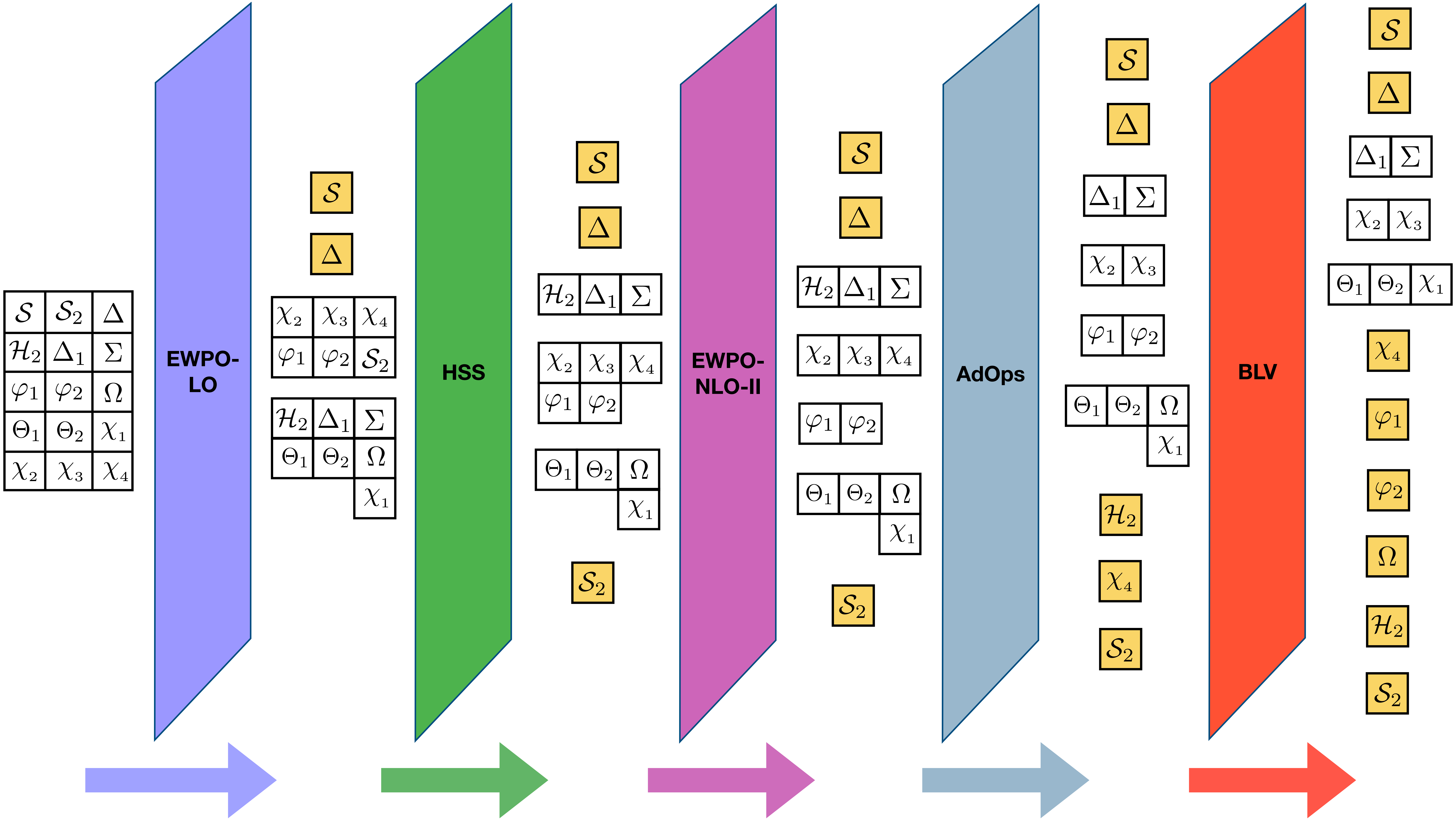
B Grzadkowski, M Iskrzynski, M Misiak, J Rosiek arXiv:1008.4884

Laure Berthier and Michael Trott, arXiv:1502.02570

S Dawson, P P Giardino arXiv:1909.02000

J Ellis, C Murphy, V Sanz, T You arXiv:1803.03252

BSM Classification based on Observables



A few queries:

Is D6 sufficient?

What are beyond D6?

How to compute beyond D6?

Decoupling!

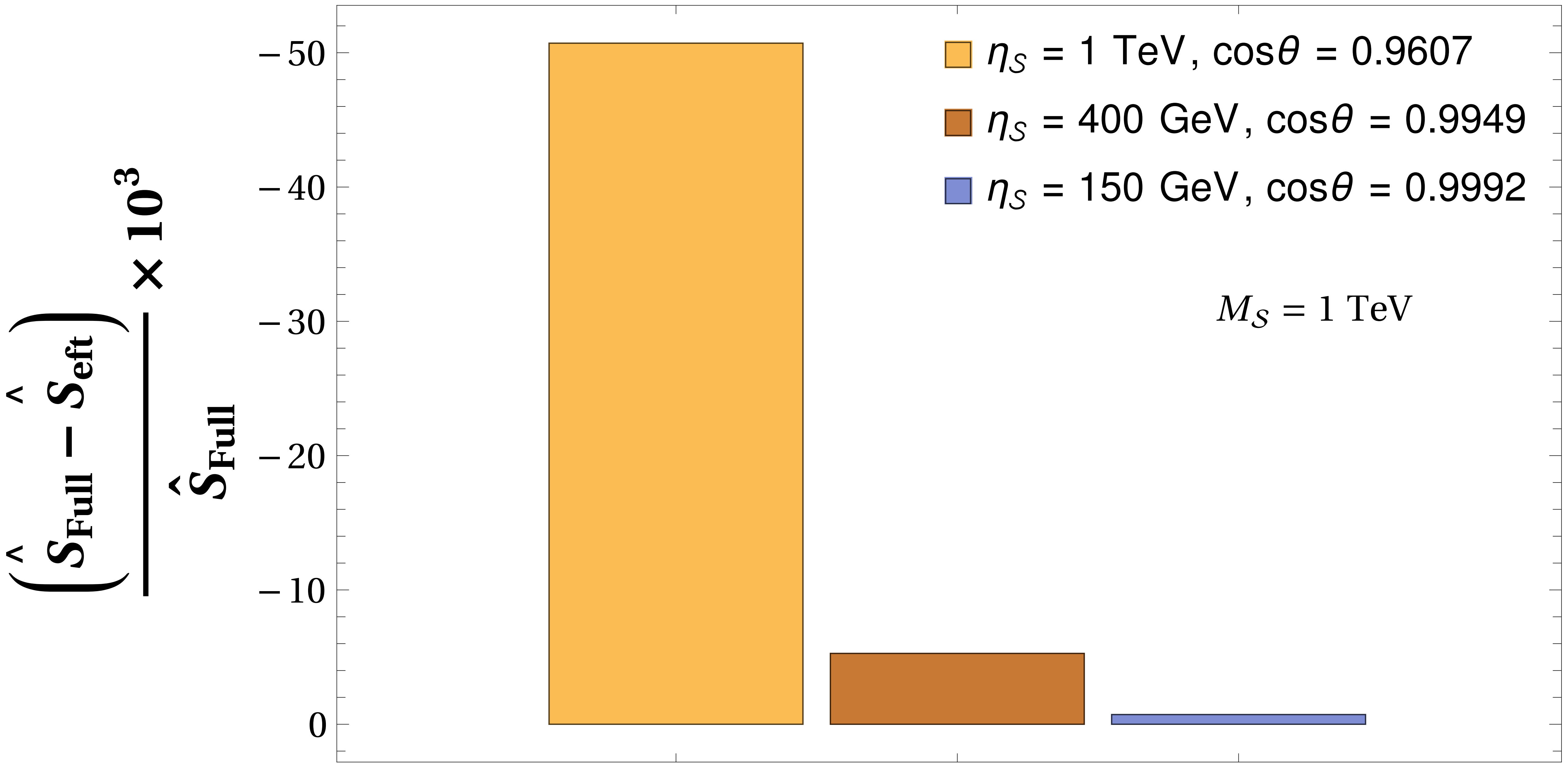
EFT vs Full Theory!

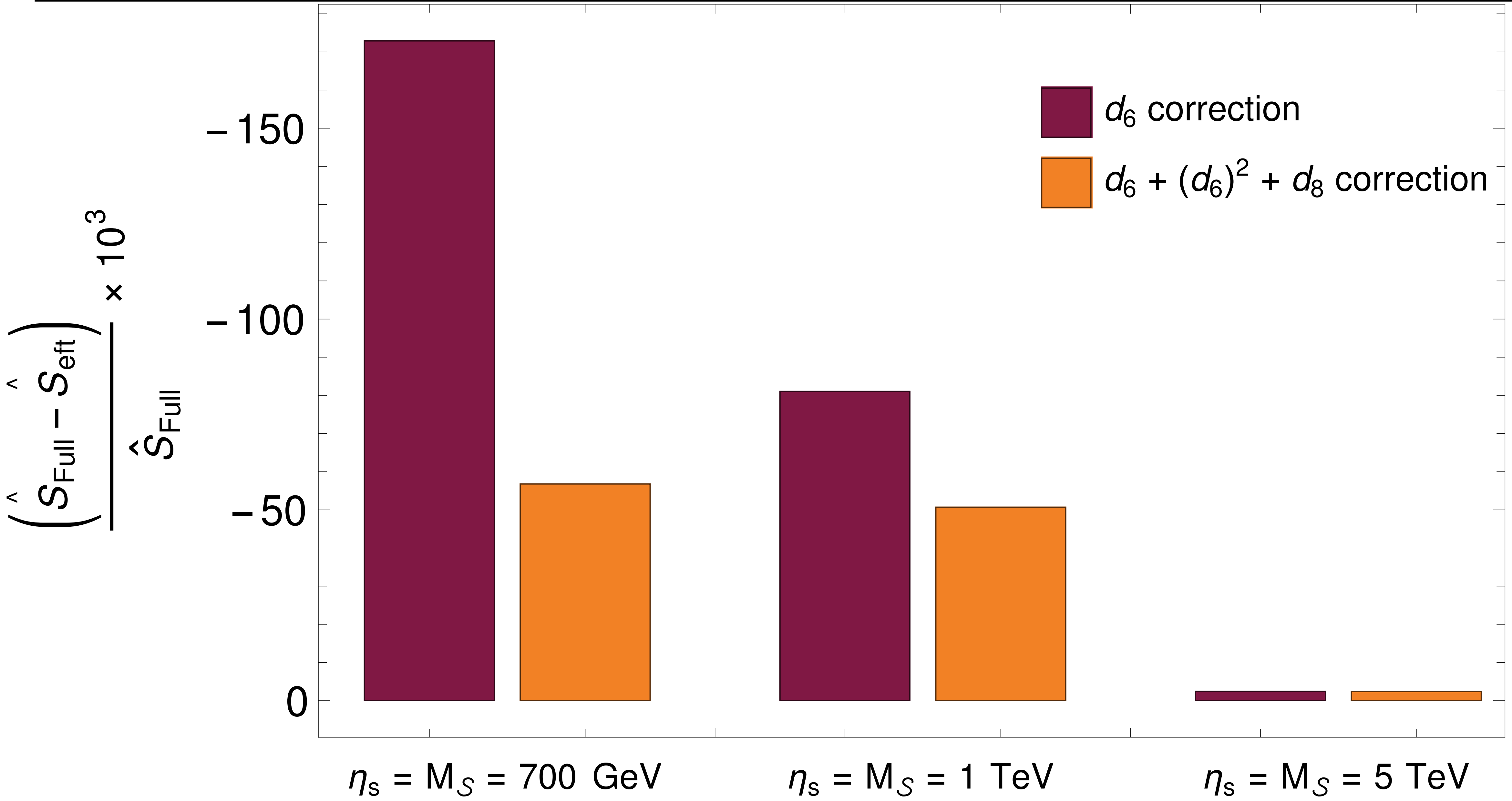
EFT Truncation!

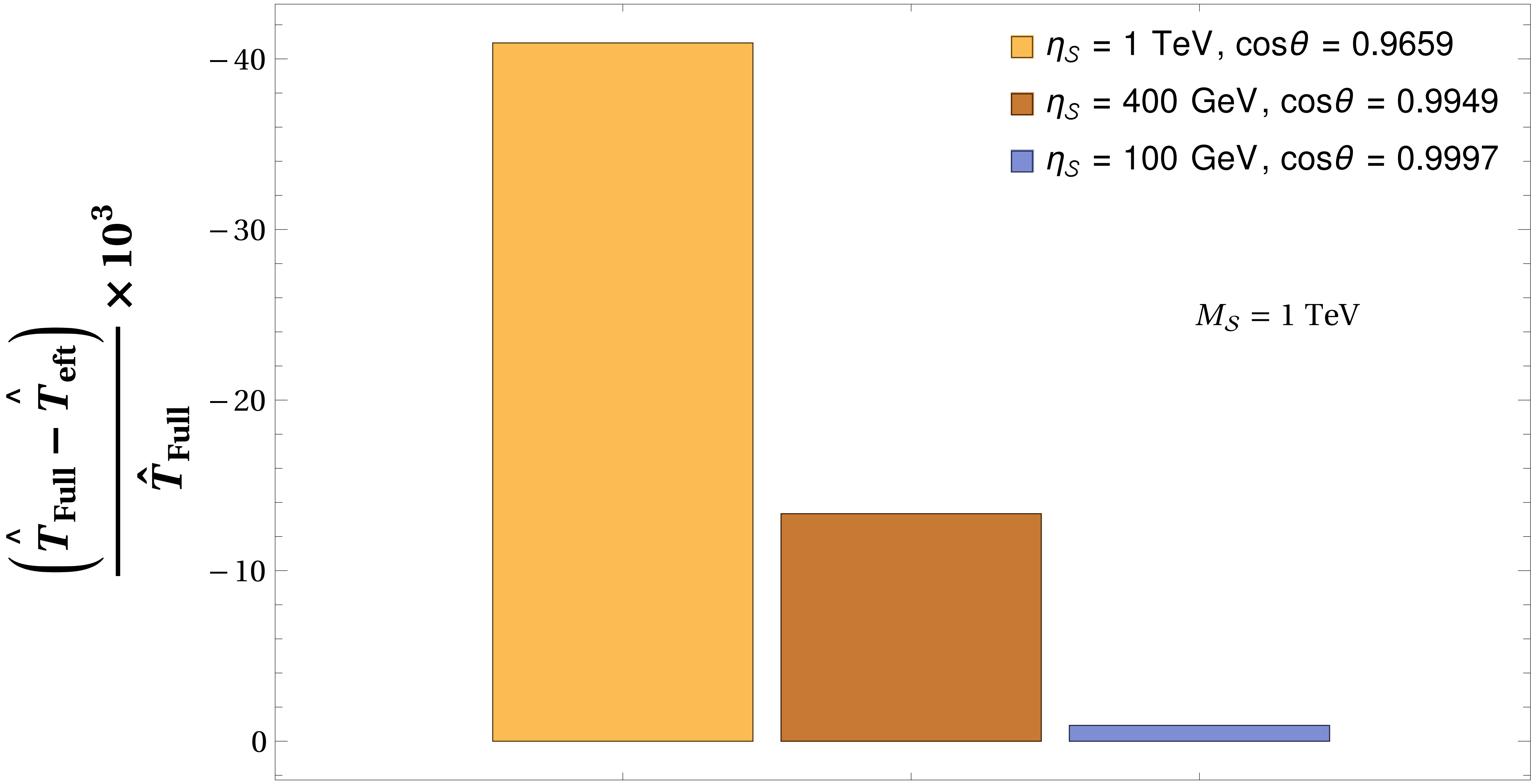
A Sample Toy Example: SM + Real Singlet scalar

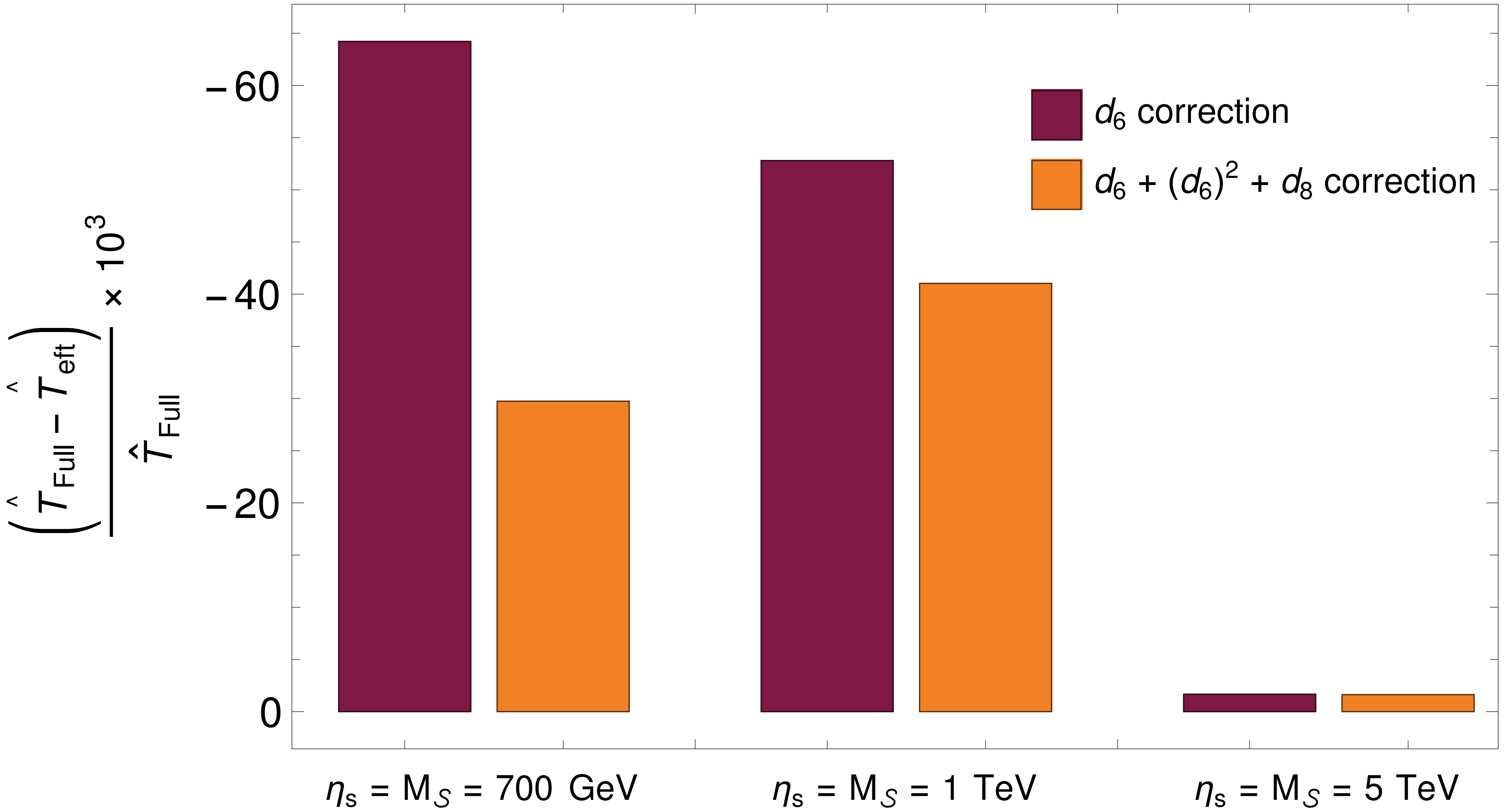
EFT, Decoupling, Higgs Mixing and All That Jazz; arXiv:2303.05224

Upalaparna Banerjee, JC, Christoph Englert, Wrishik Naskar, Shakeel Ur Rahaman, Michael Spannowsky

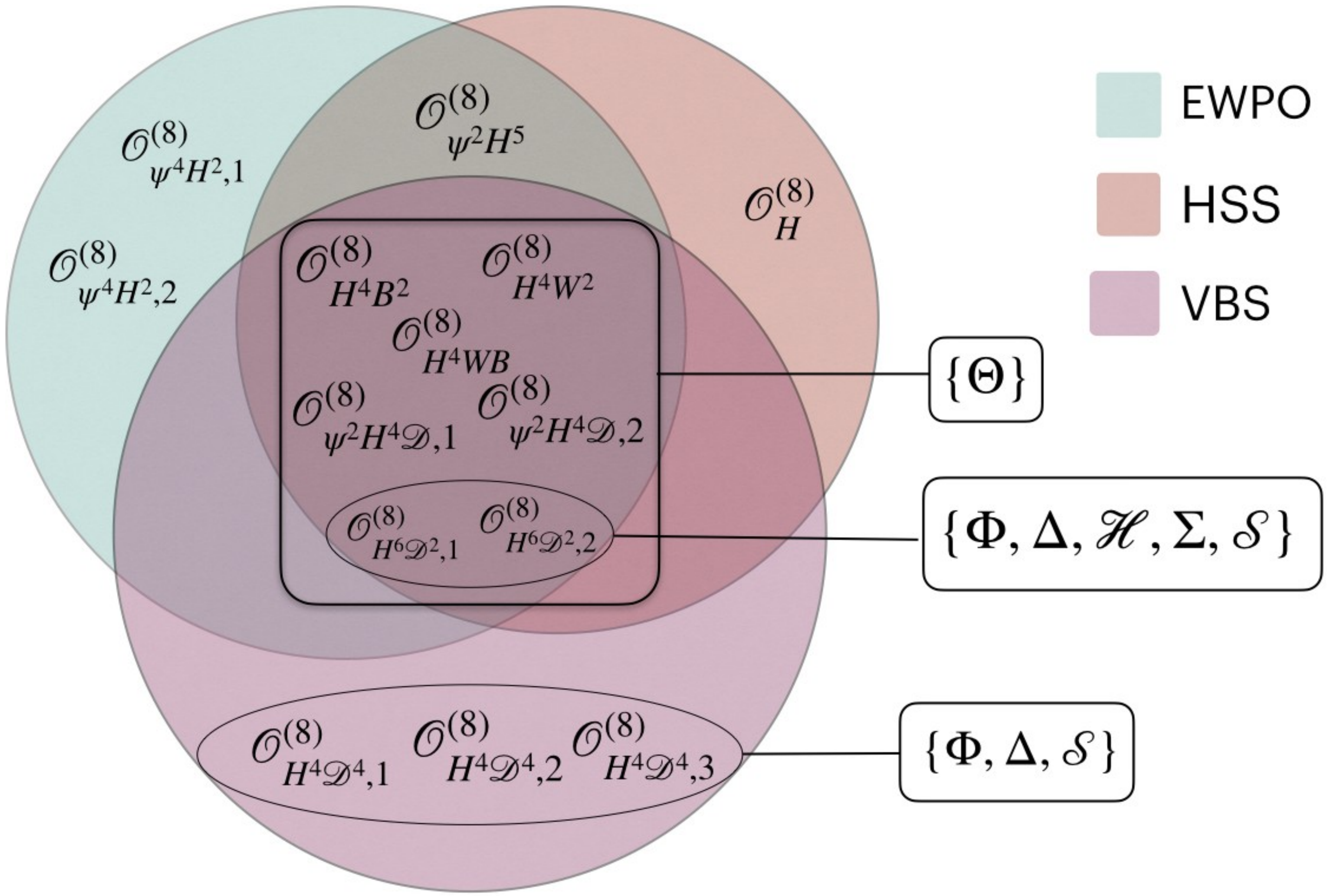








Impact of D8 on observables



Computation of D8 operators from UV theories

@ Tree-Level —trivial one, in presence of trilinear interaction e.g. HLL

From D6 part — Plugging in D6-EOM in D6-Effective action

What about D8 One-loop Effective Action?

Heat-Kernel and Effective Action

$$\mathcal{L}^{\Phi} = \Phi^{\dagger}(D^2 + U + M^2)\Phi = \Phi^{\dagger}(\Delta)\Phi,$$

Effective Action

$$\mathcal{L}_{eff} = c_s\, tr[\int_0^{\infty} \frac{dt}{t} K(t,x,x,\Delta)]$$

Heat Equation

$$(\partial_t + \Delta_x)\, K(t,x,y,\Delta) = 0,$$

Initial condition

$$K(0,x,y,\Delta) = \delta(x-y)$$

Heat-Kernel

$$K(t,x,y,\Delta) = K_0(t,x,y)\; H(t,x,y,\Delta).$$

Degenerate masses

Free HK

Interaction part

Heat-Kernel and Effective Action

Heat-Kernel

$$K(t,x,y,\Delta) = K_0(t,x,y)\; H(t,x,y,\Delta).$$

Free HK

Interaction part

$$H(t,x,y,\Delta) = \sum_k \frac{(-t)^k}{k\,!} b_k(x,y), \quad K_0(t,x,y) = (4\pi t)^{-d/2}\ \mathrm{Exp}\left[\frac{z^2}{4t} - t\,M^2\right],$$

Effective Action

$$\mathcal{L}_{eff} = \frac{c_s}{(4\pi)^{d/2}} \sum_{k=0}^{\infty} M^{d-2k} \frac{(-1)^k}{k!}\ \Gamma[k-d/2]\ tr[b_k].$$

Universal One-Loop Effective Action up to D8

$$
\begin{aligned}
\mathcal{L}_{\text{eff}}^{d\leq 8} =& \frac{c_s}{(4\pi)^2} M^4 \left[-\frac{1}{2}\left(\ln\left[\frac{M^2}{\mu^2}\right]-\frac{3}{2}\right)\right] + \frac{c_s}{(4\pi)^2}\,\mathrm{tr}\left\{ M^2\left[-\left(\ln\left[\frac{M^2}{\mu^2}\right]-1\right)U\right]\right.\\
&+ M^0\,\frac{1}{2}\left[-\ln\left[\frac{M^2}{\mu^2}\right]U^2-\frac{1}{6}\ln\left[\frac{M^2}{\mu^2}\right](G_{\mu\nu})^2\right]\\
&+\frac{1}{M^2}\frac{1}{6}\left[-U^3-\frac{1}{2}(P_\mu U)^2-\frac{1}{2}U\,(G_{\mu\nu})^2-\frac{1}{10}(J_\nu)^2+\frac{1}{15}G_{\mu\nu}\,G_{\nu\rho}\,G_{\rho\mu}\right]\\
&+\frac{1}{M^4}\frac{1}{24}\left[U^4-U^2(P^2U)+\frac{4}{5}U^2(G_{\mu\nu})^2+\frac{1}{5}(U\,G_{\mu\nu})^2+\frac{1}{5}(P^2U)^2\right.\\
&\qquad-\frac{2}{5}U\,(P_\mu U)\,J_\mu+\frac{2}{5}U(J_\mu)^2-\frac{2}{15}(P^2U)(G_{\rho\sigma})^2+\frac{1}{35}(P_\nu J_\mu)^2\\
&\qquad-\frac{4}{15}U\,G_{\mu\nu}G_{\nu\rho}G_{\rho\mu}-\frac{8}{15}(P_\mu P_\nu U)\,G_{\rho\mu}G_{\rho\nu}+\frac{16}{105}G_{\mu\nu}J_\mu J_\nu\\
&\qquad+\frac{1}{420}(G_{\mu\nu}G_{\rho\sigma})^2+\frac{17}{210}(G_{\mu\nu})^2(G_{\rho\sigma})^2+\frac{2}{35}(G_{\mu\nu}G_{\nu\rho})^2\\
&\qquad\left.+\frac{1}{105}G_{\mu\nu}G_{\nu\rho}G_{\rho\sigma}G_{\sigma\mu}+\frac{16}{105}(P_\mu J_\nu)G_{\nu\sigma}G_{\sigma\mu}\right]\\
&+\frac{1}{M^6}\frac{1}{60}\left[-U^5+2\,U^3(P^2U)+U^2(P_\mu U)^2-\frac{2}{3}U^2G_{\mu\nu}U\,G_{\mu\nu}-U^3(G_{\mu\nu})^2\right.\\
&\qquad+\frac{1}{3}U^2(P_\mu U)J_\mu-\frac{1}{3}U\,(P_\mu U)(P_\nu U)\,G_{\mu\nu}-\frac{1}{3}U^2J_\mu(P_\mu U)\\
&\qquad-\frac{1}{3}U\,G_{\mu\nu}(P_\mu U)(P_\nu U)-U\,(P^2U)^2-\frac{2}{3}(P^2U)(P_\nu U)^2-\frac{1}{7}((P_\mu U)G_{\mu\alpha})^2\\
&\qquad+\frac{2}{7}U^2G_{\mu\nu}G_{\nu\alpha}G_{\alpha\mu}+\frac{8}{21}U\,G_{\mu\nu}U\,G_{\nu\alpha}G_{\alpha\mu}-\frac{4}{7}U^2(J_\mu)^2-\frac{3}{7}(U\,J_\mu)^2\\
&\qquad+\frac{4}{7}U\,(P^2U)(G_{\mu\nu})^2+\frac{4}{7}(P^2U)U(G_{\mu\nu})^2-\frac{2}{7}U\,(P_\mu U)J_\nu G_{\mu\nu}\\
&\qquad-\frac{2}{7}(P_\mu U)U\,G_{\mu\nu}J_\nu-\frac{4}{7}U\,(P_\mu U)G_{\mu\nu}J_\nu-\frac{4}{7}(P_\mu U)U\,J_\nu G_{\mu\nu}\\
&\qquad+\frac{4}{21}U\,G_{\mu\nu}(P^2U)G_{\mu\nu}+\frac{11}{21}(P_\alpha U)^2(G_{\mu\nu})^2-\frac{10}{21}(P_\mu U)J_\nu U\,G_{\mu\nu}\\
&\qquad-\frac{10}{21}(P_\mu U)G_{\mu\nu}U\,J_\nu-\frac{2}{21}(P_\mu U)(P_\nu U)G_{\mu\alpha}G_{\alpha\nu}+\frac{10}{21}(P_\nu U)(P_\mu U)G_{\mu\alpha}G_{\alpha\nu}\\
&\qquad-\frac{1}{7}(G_{\alpha\mu}(P_\mu U))^2-\frac{1}{42}((P_\alpha U)G_{\mu\nu})^2-\frac{1}{14}(P_\mu P^2U)^2-\frac{4}{21}(P^2U)(P_\mu U)J_\mu\\
&\qquad\left.+\frac{4}{21}(P_\mu U)(P^2U)J_\mu+\frac{2}{21}(P_\mu U)(P_\nu U)(P_\mu J_\nu)-\frac{2}{21}(P_\nu U)(P_\mu U)(P_\mu J_\nu)\right]\\
&+\frac{1}{M^8}\frac{1}{120}\left[U^6-3\,U^4(P^2U)-2\,U^3(P_\nu U)^2+\frac{12}{7}U^2(P_\mu P_\nu U)(P_\nu P_\mu U)\right.\\
&\qquad+\frac{26}{7}(P_\mu P_\nu U)U\,(P_\mu U)(P_\nu U)+\frac{26}{7}(P_\mu P_\nu U)(P_\mu U)(P_\nu U)U+\frac{9}{7}(P_\mu U)^2(P_\nu U)^2\\
&\qquad+\frac{9}{7}U\,(P_\mu P_\nu U)U\,(P_\nu P_\mu U)+\frac{17}{14}((P_\mu U)(P_\nu U))^2+\frac{8}{7}U^3G_{\mu\nu}U\,G_{\mu\nu}\\
&\qquad+\frac{5}{7}U^4(G_{\mu\nu})^2+\frac{18}{7}G_{\mu\nu}(P_\mu U)U^2(P_\nu U)+\frac{9}{14}(U^2G_{\mu\nu})^2\\
&\qquad+\frac{18}{7}G_{\mu\nu}U\,(P_\mu U)(P_\nu U)U+\frac{18}{7}(P_\mu P_\nu U)(P_\mu U)U\,(P_\nu U)\\
&\qquad+\left(\frac{8}{7}G_{\mu\nu}U\,(P_\mu U)U\,(P_\nu U)+\frac{26}{7}G_{\mu\nu}(P_\mu U)U\,(P_\nu U)U\right)\\
&\qquad\left.+\left(\frac{24}{7}G_{\mu\nu}(P_\mu U)(P_\nu U)U^2-\frac{2}{7}G_{\mu\nu}U^2(P_\mu U)(P_\nu U)\right)\right]\\
&+\frac{1}{M^{10}}\frac{1}{210}\left[-U^7-5\,U^4(P_\nu U)^2-8\,U^3(P_\mu U)U(P_\mu U)-\frac{9}{2}(U^2(P_\mu U))^2\right]\\
&\left.+\frac{1}{M^{12}}\frac{1}{336}\left[U^8\right]\right\}.
\end{aligned}
$$

$$
\boxed{U_{ij}=\frac{\delta^2\mathcal{L}_{UV}}{\delta\Phi_i\delta\Phi_j}}
$$

$$
\boxed{J_\mu = P_\nu G_{\nu\mu} = [P_\nu,[P_\nu,P_\mu]].}
$$

$$
\boxed{G_{\mu\nu}=[P_\mu,P_\nu],}
$$

How to Read it

	Scalar	Fermion
c_s	1 or 1/2	-1/2
$\mathcal{P}_\mu$	P_μ	$P_\mu - i\gamma^5\gamma_\mu R$
U	U_s	$U_f = Y + 2M\Sigma,$ $Y = -\frac{1}{2}\sigma_{\mu\nu}G_{\mu\nu} + S^2 + 3R^2 - (\not{P}S) + i\gamma^5(RS + SR),\ \Sigma = S + i\gamma^5 R$
$G_{\mu\nu}$	$F_{\mu\nu}$	$F_{\mu\nu} + \Gamma_{\mu\nu},$ $\Gamma_{\mu\nu} = i\gamma^5\gamma_\mu(P_\nu R) - i\gamma^5\gamma_\nu(P_\mu R) + 2\sigma_{\mu\nu}R^2$

Generalisation of one-loop effective Lagrangian.

1. **One-loop Effective Action up to Dimension Eight: Integrating out Heavy Scalar(s)**
   Upalaparna Banerjee, Joydeep Chakrabortty, Shakeel Ur Rahaman, Kaanapuli Ramkumar
   e-Print: 2306.09103 [hep-ph].

2. **One-loop Effective Action up to Dimension Eight: Integrating out Heavy Fermion(s)**
   Upalaparna Banerjee, Joydeep Chakrabortty, Shakeel Ur Rahaman, Kaanapuli Ramkumar
   e-Print: 2308.03849 [hep-ph].

Heat-Kernel and Effective Action: Non-Degenerate Spectrum

Heat-Kernel $\quad K(t,x,y,\Delta) = K_0\cancel{(t,x,y)}\,H(t,x,y,\Delta).$

$$S_{\text{eff},1\text{-loop}} = i\,c_s\,\mathrm{Tr}\log\left(D^2 + M^2 + U\right),$$

$$\mathcal{L}_{\text{eff}} = c_s\,\mathrm{tr}\int_0^\infty \frac{dt}{t}\int \frac{d^4p}{(2\pi)^4\,t^2}\,e^{p^2}\,e^{-M^2t}\left[1+\sum_{n=1}^{\infty}(-1)^n f_n(t,\mathcal{A})\right]$$

$$f_n(t,\mathcal{A}) = \int_0^t ds_1\int_0^{s_1} ds_2\cdots\int_0^{s_{n-1}} ds_n\,\mathcal{A}(s_1)\mathcal{A}(s_2)\cdots\mathcal{A}(s_n).$$

$$\mathcal{A}(t) = e^{M^2t}(D^2 + 2i\,p\cdot D/\sqrt{t} + U)e^{-M^2t},$$

Effective Action: Non-Deg. Mass-> Deg. Mass -> Light-Heavy Mixing

$$\mathcal{L}_{\text{eff}} = c_s \,\text{tr} \int_0^\infty \frac{dt}{t} \int \frac{d^4p}{(2\pi)^4\, t^2}\; e^{p^2}\, e^{-M^2 t} \left[1 + \sum_{n=1}^{\infty} (-1)^n f_n(t, \mathcal{A})\right]$$

Features: One-Loop 1PI Effective Action up to any mass dimension

A Mathematica package will be attached.

Applicable for Scalars and Fermions both

Non-Degenerate Masses —> Sub-Degeneracy: Just take the "Limits" (Mi —> Mk)

Non-Degenerate Masses —> Light-Heavy Mixing: Just take the "Limits" (mi —> 0)

And ensure the theory is "IR"-safe

One-loop Effective Action up to any Mass-dimension for Non-degenerate Scalars and Fermions including Light-Heavy Mixing

Upalaparna Banerjee, Joydeep Chakrabortty, Shakeel Ur Rahaman, Kaanapuli Ramkumar

ArXiv: 2311.****

❖ Take home messages

HK: One-Loop Universal Effective Action up to any mass dimension

Equally Applicable for Scalars and Fermions as well.

Compact formula upto Dim-8 is provided — can be easily fed into CoDEx!

Can Deal with any number of non-degenerate fields.
Mathematica Program is also developed.

Effective Action is IR-safe — Light-Heavy mixing is easily computed from general non-degenerate result as a limiting case