

Exclusive $b \rightarrow s \mu^+ \mu^-$ decays, the current status of the theory

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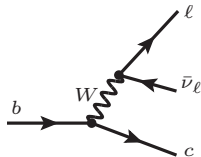
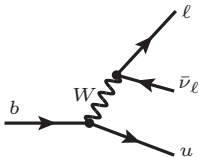
□ Flavourful semileptonic decays of b -quark in SM

- flavour-changing charged weak currents

($\ell = e, \mu, \tau$)

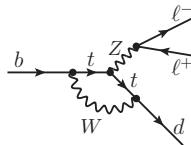
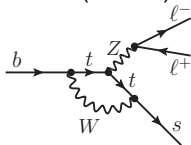
- sensitive to V_{ub}, V_{cb}

- probe beyond SM charged currents



- flavour-changing neutral currents (FCNC)

- sensitive to V_{tb}, V_{ts}, V_{td}
- trace new heavy particles in the loops



□ Switching on quark-gluon interactions (QCD) in the b -quark decay

- the $b \rightarrow sl^+l^-$ loop diagrams reduced to effective local interactions, since t, Z, W are much heavier than b

- the decaying b -quark is bounded inside the initial B meson,

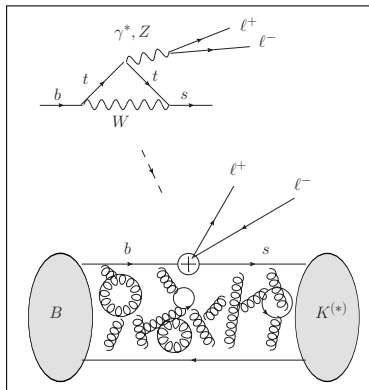
- the final-state quarks are converted into hadrons,

- exclusive channels (with a fixed final state) experimentally clean, provide specific observables

- recent measurements of $B \rightarrow K^{(*)}l^+l^-$, $B_s \rightarrow \phi l^+l^-$ (LHCb, Belle II, CMS)

revealed tensions with SM,

'**B-anomalies**', but no lepton flavour universality violation anymore !



□ $B \rightarrow K \mu^+ \mu^-$ in Standard Model

- diagrams with W, Z, t \Rightarrow ten effective interactions

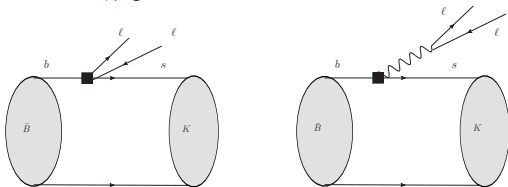
$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i O_i$$

- most important are the pointlike $b \rightarrow s \ell \ell$, $b \rightarrow s \gamma^{(*)}$ interactions (the local operators $O_{9,10}$ and $O_{7\gamma}$) generated from heavy loops

$$C_9(m_b) \simeq 4.4,$$

$$C_{10} \simeq -4.7,$$

$$C_7(m_b) \simeq -0.3$$



- the contributions of these operators factorize into leptonic part and $B \rightarrow K$ hadronic form factors

$$\langle K(p) | \bar{s} \Gamma b | B(p+q) \rangle \sim \{f^+(q^2), f^0(q^2), f^T(q^2)\}$$

□ Calculating $B \rightarrow K$ Form Factors in QCD

● $B \rightarrow K$ form factors from lattice QCD

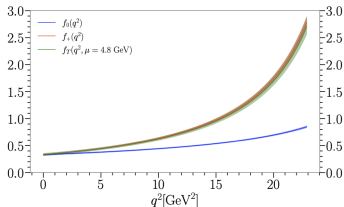


FIG. 8. Final $B \rightarrow K$ form factor results at the physical point across the full q^2 range.

● HPQCD collaboration,
W.G. Parrott et al , 2207.12468

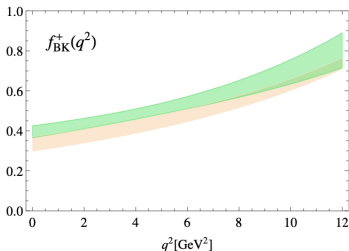
- $q^2 \sim 0$, extrapolation error ?
 $f_{BK}^+(0) = 0.332 \pm 0.012$

● QCD Light-cone sum rules

AK, A.Rusov, 1703.04765

$$f_{BK}^+(0) = 0.395 \pm 0.033$$

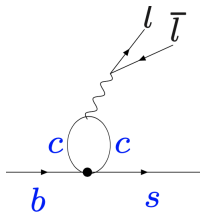
brown band - lattice QCD, Fermilab MILC Collab.1503.071



□ $B \rightarrow K \mu^+ \mu^-$, nonlocal contributions

● a chain of transitions:

weak $b \rightarrow s \bar{c} c \oplus$ electromagnetic $\bar{c} c \rightarrow \gamma^* \rightarrow \ell^+ \ell^-$
mimics the FCNC decay



● four-quark effective interactions (W -exchange),

$$O_1^{(c)} = [\bar{s}_L \gamma_\rho c_L][\bar{c}_L \gamma^\rho b_L], \quad C_1(m_b) \simeq 1.1$$

$$O_2^{(c)} = [\bar{c}_L \gamma_\rho c_L][\bar{s}_L \gamma^\rho b_L], \quad C_2(m_b) \simeq -0.25$$

● this **charm loop** effect is essentially nonlocal,
charmonium resonances at $q^2 \geq m_{J/\psi}^2$

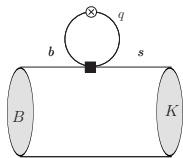
● quark-antiquark-gluon and four-quark
effective interactions generated by heavy loops

$$O_{8g} \sim \bar{s} b G, \quad C_8(m_b) \simeq 0.2$$

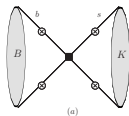
O_{3-6} - quark-penguin operators, $C_{3,4,5,6}(m_b) < 0.03$

● the u quark loops $\sim V_{ub} V_{us}^*$ part neglected due to CKM suppression

□ Diagrams of nonlocal matrix elements at $q^2 \ll 4m_c^2$

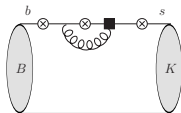
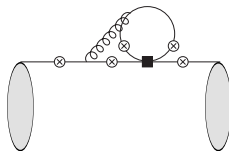
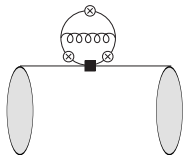


LO (factorizable)

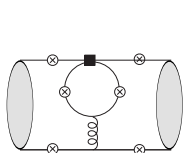


weak annihilation

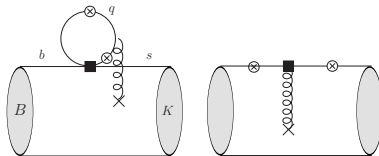
⊗ -virtual photon



NLO (factorizable) H. Asatryan, C. Greub et al. (2001)



spectator (nonfactorizable)



soft (low virtuality) gluons (nonfactorizable)

□ Amplitude of $B \rightarrow K \ell \ell$ decay

$$A(B(p+q) \rightarrow K(p) \ell^+ \ell^-) = \frac{G_F}{\sqrt{2}} \frac{\alpha_{em}}{\pi} V_{tb} V_{ts}^* \left[\bar{\ell} \gamma_\mu \ell p^\mu \left(C_9 f_{BK}^+(q^2) \right. \right. \\ \left. \left. + \frac{2(m_b + m_s)}{m_B + m_K} C_7^{eff} f_{BK}^T(q^2) + \sum_{i=1,2,\dots,6,8} C_i \mathcal{H}_i^{(BK)}(q^2) \right) + \bar{\ell} \gamma_\mu \gamma_5 \ell p^\mu C_{10} f_{BK}^+(q^2) \right]$$

- the leading contributions contain $B \rightarrow K$ form factors
- nonlocal matrix elements:

$$\langle K(p) | i \int d^4 x e^{iqx} T \{ j_\mu^{em}(x), O_i(0) \} | B(p+q) \rangle = p_\mu \mathcal{H}_i^{(BK)}(q^2)$$

$$j_{em}^\rho = \sum_{q=u,d,s,c,b} Q_q \bar{q} \gamma^\rho q, \quad \text{the hierarchy: } O_i = O_{1,2}^{(c)}, O_{8g}, O_{3,4,5,6}^{(q)}, O_{1,2}^{(u)}$$

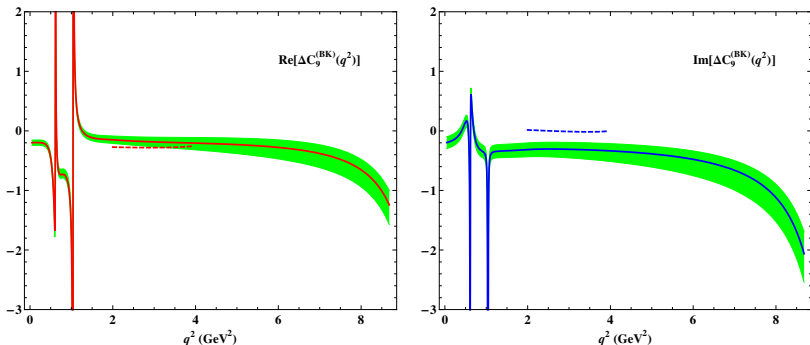
⇒ parametrized as corrections to short-distance coefficient C_9 :

$$C_9 \rightarrow C_9 + \sum_i \Delta C_9^{(BK,i)}(q^2) \quad (q^2\text{- and process-dependent})$$

- have to be estimated one by one to obtain the whole amplitude

□ $\Delta C_9(q^2)$ below charmonia region

[AK, T. Mannel, Y.M. Wang 1211.0234 [hep-ph]]

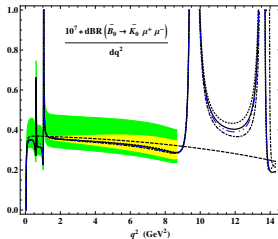


the red (blue) solid curve corresponds to the Re (Im) part obtained from the hadronic dispersion relation, fitted to the QCD calculation at $q^2 < 0$. The shaded areas indicate the uncertainties. The dashed curves - prediction of QCD.

□ Differential width of $B \rightarrow K \mu^+ \mu^-$

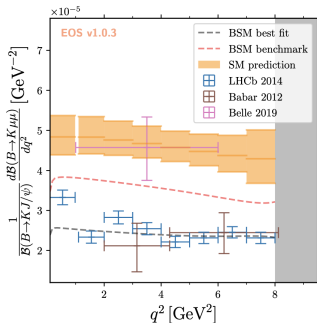
● [AK, T. Mannel, Y.M. Wang 1211.0234 [hep-ph]]

- the long-dashed line - without nonlocal contributions.
- the green (yellow) area - with (without) the uncertainties of $B \rightarrow K$ form factors



● [N. Gubernari, M. Reboud, D. van Dyk and J. Virto, 2206.03797]

- more precise $B \rightarrow K$ form factors.
- major update of NLO factor.diaqs, H.Asatryan, C.Greub, J.Virto (2020)
- updated soft nonlocal effect
- Bayesian (EOS) analysis of the uncertainty



□ Differential width of $B \rightarrow K\mu^+\mu^-$

- most recent CMS data (CMS PAS BPH-22-005)
see also the talk by G.Karathanasis (CMS) at EPS-HEP, Hamburg (2023)

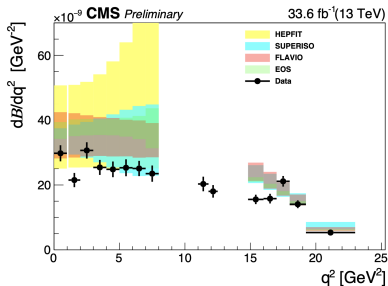


Figure 10: Comparison of the measured differential $B^+ \rightarrow K^+\mu^+\mu^-$ branching fraction with the theoretical predictions obtained using FLAVIO, SUPERISO, HEPFIT, and EOS packages.

- good agreement with LHCb,
- need more details on “theoretical predictions” ?

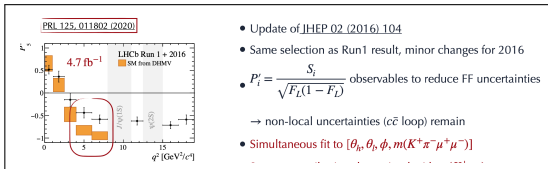
□ Is there an anomaly in $B \rightarrow K\ell\ell$?

- We observe a considerable difference between predicted in SM and measured $dBR(B \rightarrow K\mu^+\mu^-)/dq^2$,
- three possibilities:
 - there are unaccounted and large nonlocal effects
need lattice QCD calculations, update of LCSR approach is planned
 - there a shift of C_9 due to new heavy particles in the loops
see e.g. B.Capdevila, A.Crivellin, J.Matias, 2309.01311
 - both lattice and LCSR $B \rightarrow K$ form factors are unreliable...

□ Other $b \rightarrow sll$ channels

● $B \rightarrow K^* ll$

from the talk by J.Nicolini (LHCb)
at Hamburg EPS-HEP (2023)



● angular observable P'_5 independent of FFs (but not of nonlocal effects !)

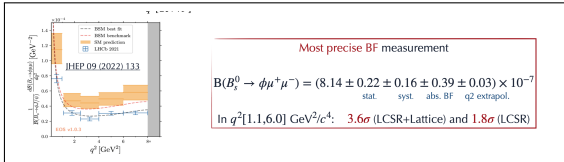
● detected $B \rightarrow K\pi ll$ state , $K^*(892)$ is a resonance,
> instability of K^* in lattice QCD:

A. Agadjanov, V. Bernard, U. G. Meissner and A. Rusetsky, 1605.03386 [hep-lat].

> LCSR calculations of $B \rightarrow K\pi$ form factors: S. Descotes-Genon, AK, J. Virto, 1908.02267

● $B_s \rightarrow \phi ll$

● the predicted width
larger than the measured



● observation of $B \rightarrow K\nu\bar{\nu}$ by Belle-II (reported at EPS-HEP in Hamburg) :
a very clean decay in SM, no nonlocal effects , $B \rightarrow K$ form factor has to be increased !