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The NNLO QCD analysis of non-singlet structure function without using the orthogonal polynomials approach

Semnan University

Azam Ghaffari-tooran
Ali Khorramian
Hamed Abdolmaleki

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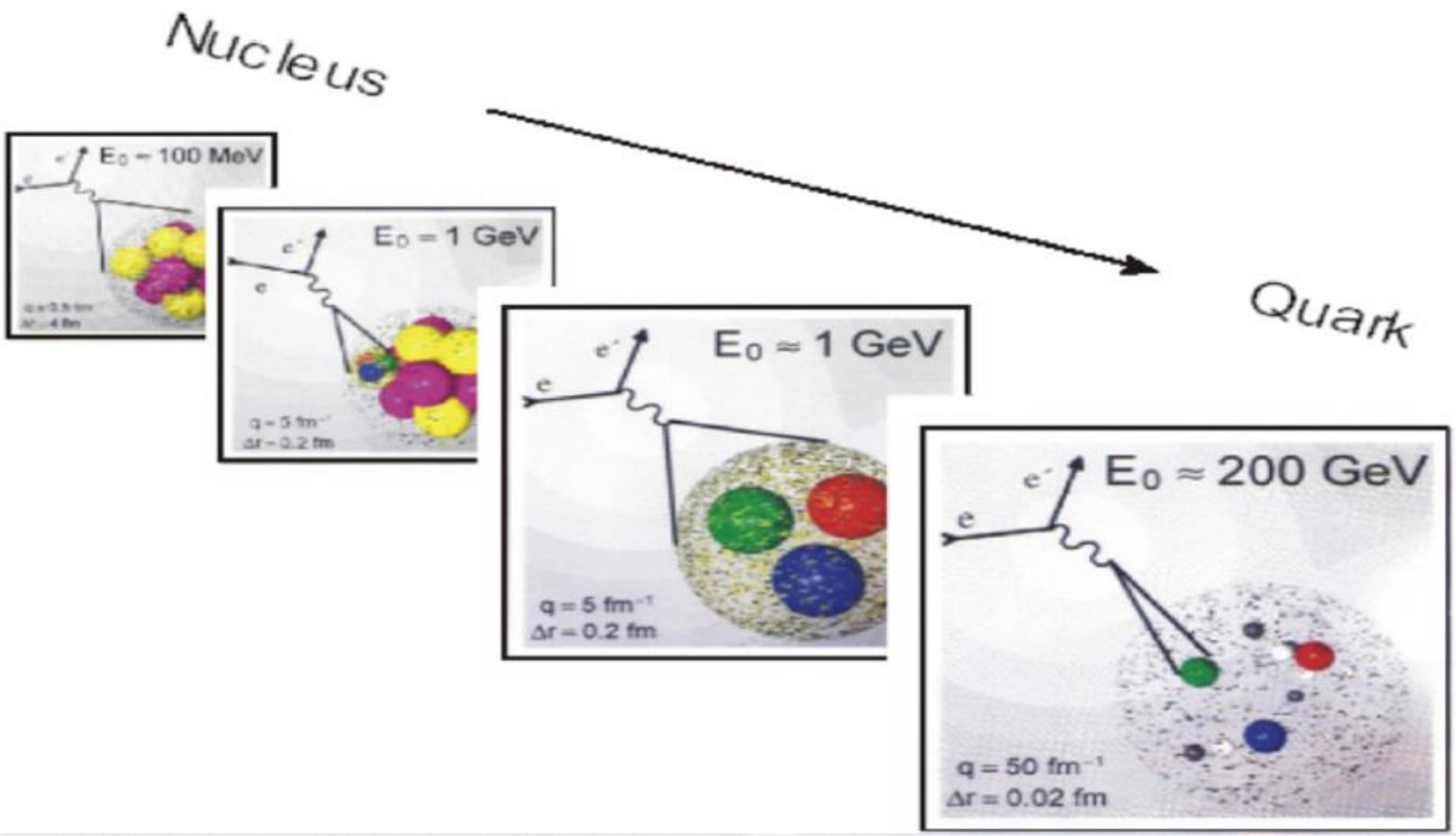
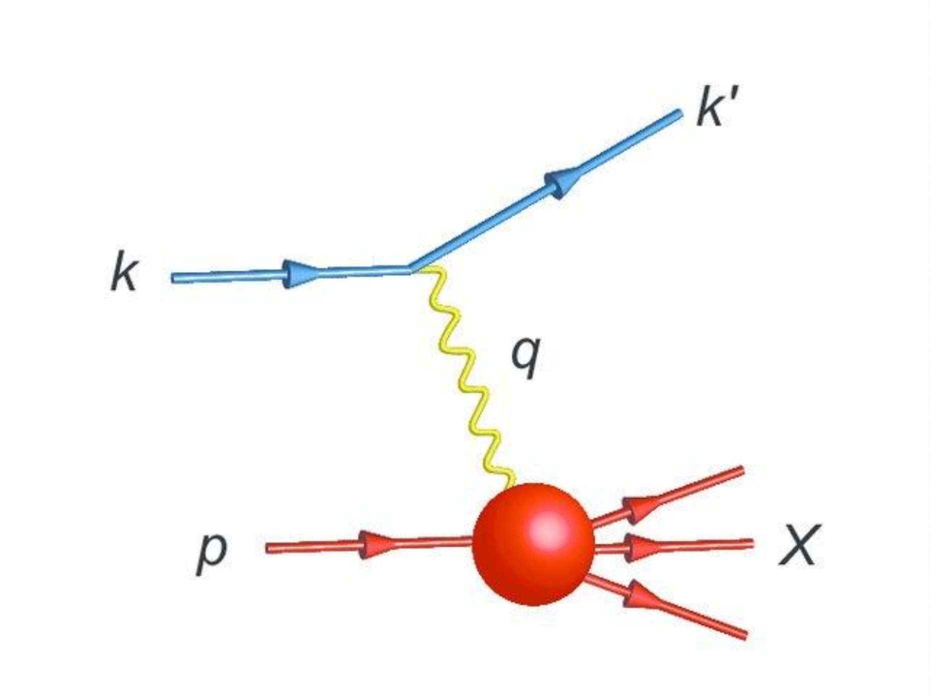
Introduction

The non-singlet structure function $xF_3(x, Q^2)$, where mainly information comes from deep inelastic neutrino-nucleon scattering, is the important input to the QCD global analysis of parton distribution function, especially at large-x, where valence quark distributions are dominant. The neutrino structure function $xF_3\,(x, Q^2)$ experimental data are the first experimental source to extract the valence quark densities x$u_v\,(x, Q^2)$ and x$d_v\,(x, Q^2)$ of the nucleon in charged current (CC) neutrino nucleon deep inelastic scattering. $\bar{\upsilon}_\mu\, p \rightarrow \mu^+ \; n$, $\bar{\upsilon}_\mu\, n \rightarrow \mu^+ \; \Lambda^- \rightarrow \mu^+ \; n\pi^-$


Nucleus
Quark
E0 = 100 MeV
E0 = 1 GeV
E0 ≈ 1 GeV
q = 5 fm-1
Δr = 0.2 fm
E0 ≈ 200 GeV
q = 50 fm-1
Δr = 0.02 fm

Deep Inelastic Scattering



Deep: $\boldsymbol{Q^2 \geq 1\ GeV^2\ and\ W^2 \geq M_p^2}$

$Q = k - k'$

$Q^2 = -q^2$ Virtuality of exchanged boson

$x = Q^2/(2p \cdot q)$ Bjorken

$W = (p+q)^2$ Invarient-mass

$Y = \frac{p.q}{p.k}$ Inelasticity

Neutrino-nucleon cross sections and parton distributions

The charged-current (CC) deep inelastic neutrino (antineutrino)-nucleon scattering differential cross sections are given by a combination of three structure functions F_1, F_2, and F_3 as

$$\frac{d^2\sigma^{\nu,\bar{\nu}}}{dxdy} = \frac{G_F^2\, M_N E}{\pi}\left(\frac{M_W^2}{M_W^2+Q^2}\right)^2\left[F_1^{\nu,\bar{\nu}}\; xy^2 + F_2^{\nu,\bar{\nu}}\;\left(1-y-\frac{M_N\, xy}{2E}\right) \pm xF_3^{\nu,\bar{\nu}}\;\left(y-\frac{y^2}{2}\right)\right].$$

However, due to the isospin symmetry, $xF_3^{(\nu+\bar{\nu})\,P} = xF_3^{(\nu+\bar{\nu})\,n}$, the average of the neutrino and antineutrino nucleon structure is

$$xF_3(x,Q^2) = \frac{1}{2}\left[\, xF_3^{\nu N}(x,Q^2) + xF_3^{\bar{\nu}N}(x,Q^2)\right] = \left[x(u_v + d_v) + x(s - \bar{s}) + x(c - \bar{c})\right](x,Q^2),$$

It should be noted that $s - \bar{s}$ and $c - \bar{c}$ are considered to be very small. Therefore, the average of the neutrino and antineutrino nucleon structure is only related to valence quark distribution as

$$xF_3(x,Q^2) = xu_v(x,Q^2) + xd_v(x,Q^2).$$

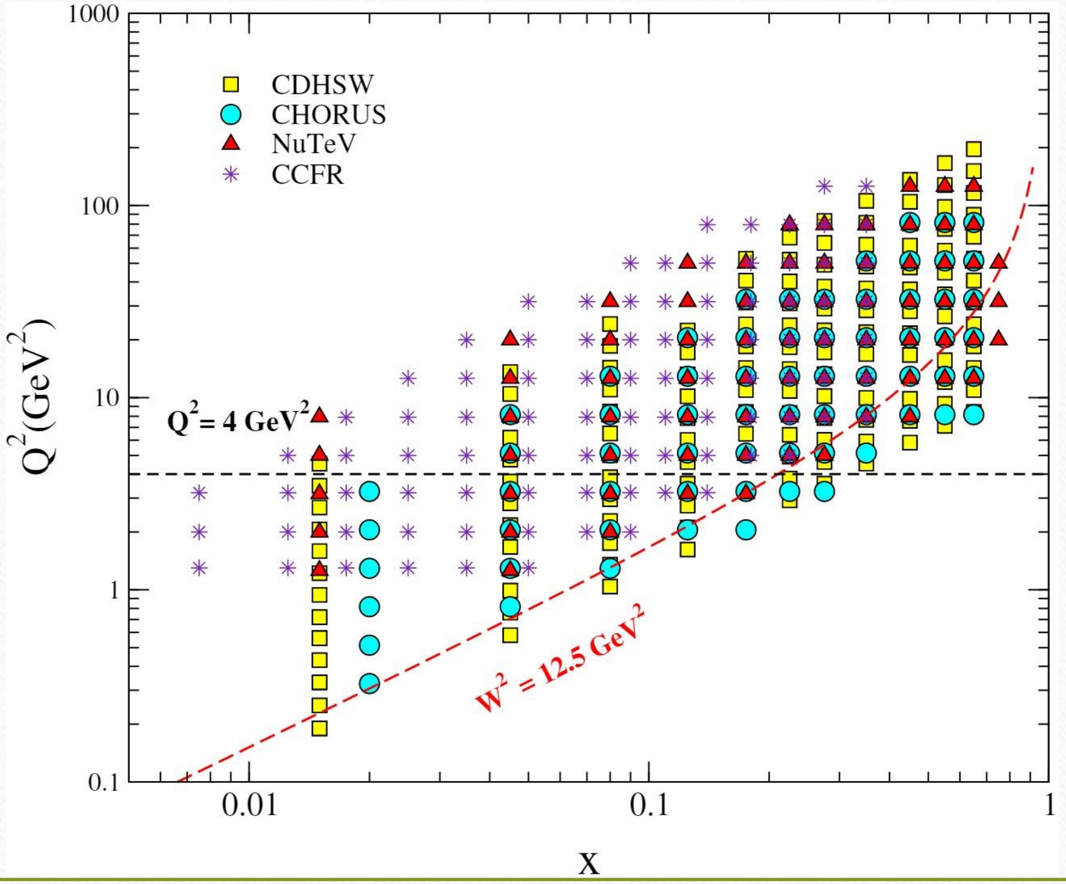
Experimental data in non-singlet structure functions

The xF_3 structure function of deep inelastic neutrino-nucleon scattering have been measured by different experimental groups, such as:

- **CCFR**: Chicago-Columbia-Fermilab-Rochester, with an iron target and $30 \leq E(\text{GeV}) \leq 360$.
- **NuTeV**: Nutrinos at the Tevatron, with an iron target and $30 \leq E(\text{GeV}) \leq 500$.
- **CHORUS**: CERN Hybrid Oscillation Research Apparat US, with a lead target and $10 \leq E(\text{GeV}) \leq 200$.
- **CDHSW**: CERN-Dortmund- Heidelberg-Saclay-Warsaw, with an iron target and $20 \leq E(\text{GeV}) \leq 212$.

These experimental data have prepared an accurate experimental origin for the valence quark densities and strong coupling constant determination.

Different experiments of DIS neutrino-nucleon data in the x and Q^2 plane. The dashed line represents the kinematic W^2 and Q^2 cuts on the data ($Q^2 \geq 4\ GeV^2$, $W^2 \geq 12.5\ GeV^2$) in this analysis. The data points lying below these lines are only excluded in the present QCD fits.



Non-singlet parametrization

$$xu_v(x,Q_0^2) = N_{uv} x^{a_{uv}} (1-x)^{b_{uv}} (1 + c_{uv} x + d_{uv}\sqrt{x}), \qquad xd_v(x,Q_0^2) = {N_{dv}}/{N_{uv}} (1-x)^{b_{dv}}\, xu_v(x,Q^2).$$

In this parametrization, xd_v distribution depends on xu_v.

Also the normalization constants N_u and N_d can be obtained from the other parameters, using conservation of the fermion number by $\int_0^1 u_v dx = 2$, $\int_0^1 d_v dx = 1$

So the normalization constants N_u and N_d are

N_{uv}= 2/[B(a_u,1 + b_u) + c_uB(1/2 + a_u,1 + b_u) + d_uB(1+ a_u,1 + b_u)] ,

N_{dv}= 1/[B(a_u,1 + b_u + b_d) + c_uB(1/2 + a_u,1 + b_u +b_d) + d_uB(1+ a_u,1 + b_u+ b_d)].

where B(a; b) is the Euler function. In above parametrization, the normalization constants N_u and N_d are very effective to determine unknown parameters via the QCD fitting procedure.

Nuclear effects

Since the detection of neutrinos always involves the heavy nuclear targets, so the nuclear effect is needed to study the DIS neutrino (antineutrino)-nucleus xF_3 structure function.

The nuclear targets are used by different neutrino experiments, such as CCFR, NuTeV, and CDHSW with the same iron target, and CHORUS with a lead target. To have the average of the neutrino and antineutrino nucleus structure functions, we require to have the nuclear PDFs.

Nuclear neutrino structure function

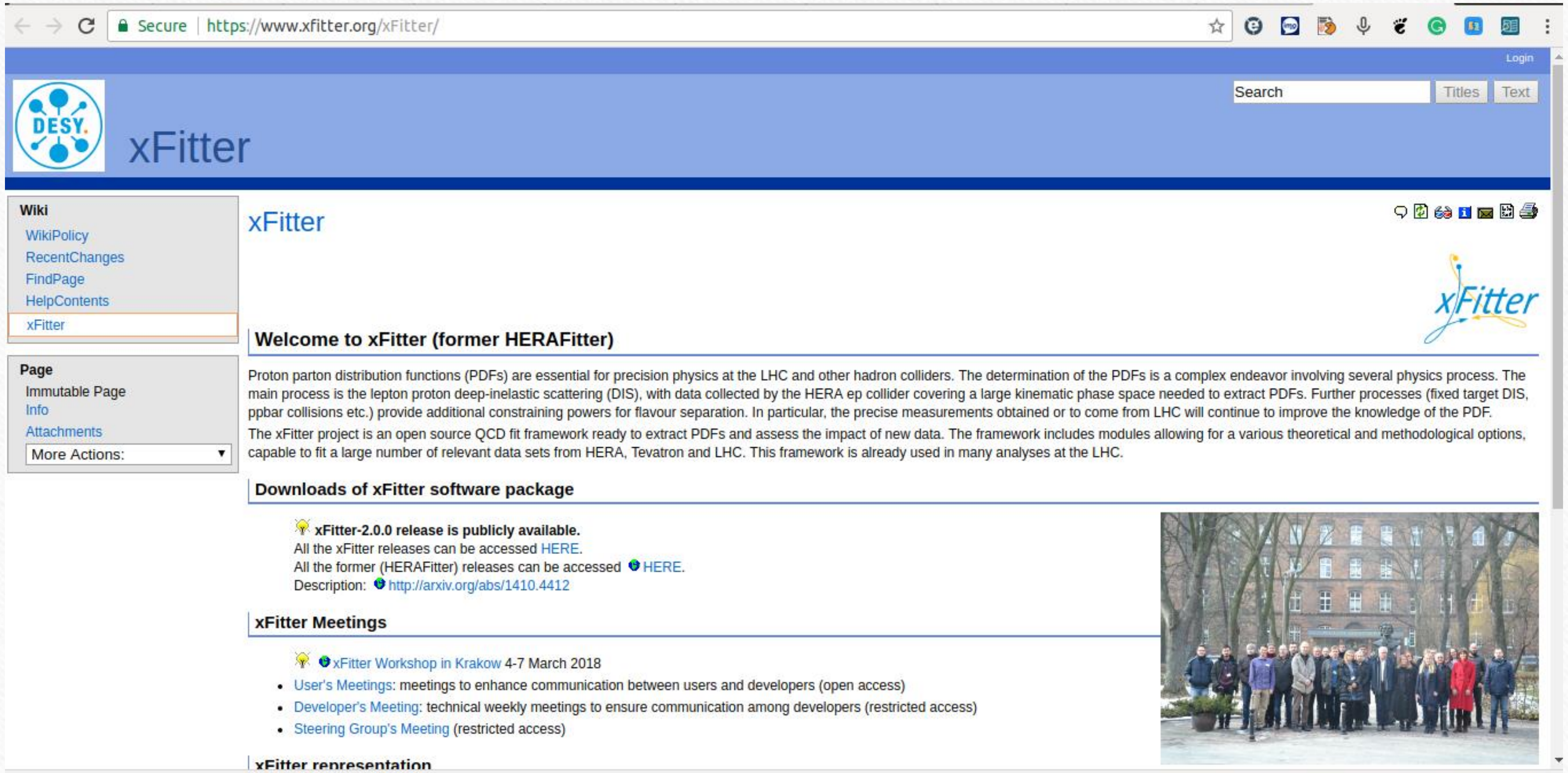
For non-singlet QCD analysis, this modification create a connection between the bounded valence PDFs in the nucleus *A* and free valence PDFs in the proton as

$$xq_v^A(x, Q_0^2) = R_v(x,A,Z)\ xq_v(x, Q_0^2)$$

$R_v(x,A,Z)$ is the nuclear weight that depends on the type of nucleus and parton flavor, $xq_v(x, Q_0^2)$ is the valence PDFs in the free parton and xq_v^A is the valence PDFs in the bounded parton , A and Z are atomic and mass numbers, respectively.

- We used the DSSZ model to consider nuclear effects.

xFitter


Secure | https://www.xfitter.org/xFitter/
Login
Search Titles Text
DESY.
xFitter
Wiki
WikiPolicy
RecentChanges
FindPage
HelpContents
xFitter
Page
Immutable Page
Info
Attachments
More Actions:
xFitter
xFitter
Welcome to xFitter (former HERAFitter)
Proton parton distribution functions (PDFs) are essential for precision physics at the LHC and other hadron colliders. The determination of the PDFs is a complex endeavor involving several physics process. The main process is the lepton proton deep-inelastic scattering (DIS), with data collected by the HERA ep collider covering a large kinematic phase space needed to extract PDFs. Further processes (fixed target DIS, ppbar collisions etc.) provide additional constraining powers for flavour separation. In particular, the precise measurements obtained or to come from LHC will continue to improve the knowledge of the PDF.
The xFitter project is an open source QCD fit framework ready to extract PDFs and assess the impact of new data. The framework includes modules allowing for a various theoretical and methodological options, capable to fit a large number of relevant data sets from HERA, Tevatron and LHC. This framework is already used in many analyses at the LHC.
Downloads of xFitter software package
xFitter-2.0.0 release is publicly available.
All the xFitter releases can be accessed HERE.
All the former (HERAFitter) releases can be accessed HERE.
Description: http://arxiv.org/abs/1410.4412
xFitter Meetings
xFitter Workshop in Krakow 4-7 March 2018
User's Meetings: meetings to enhance communication between users and developers (open access)
Developer's Meeting: technical weekly meetings to ensure communication among developers (restricted access)
Steering Group's Meeting (restricted access)
xFitter representation

Higher-Twist effects

To include the HT contribution, the average of the neutrino and antineutrino structure function may be explained as

$$xF_3(x,Q^2) = xF_3^{QCD}(x,Q^2) + \frac{h(x)}{Q^2},$$

Here, the Q^2 dependence of the first term is obtained by perturbative QCD and the HT correction term is

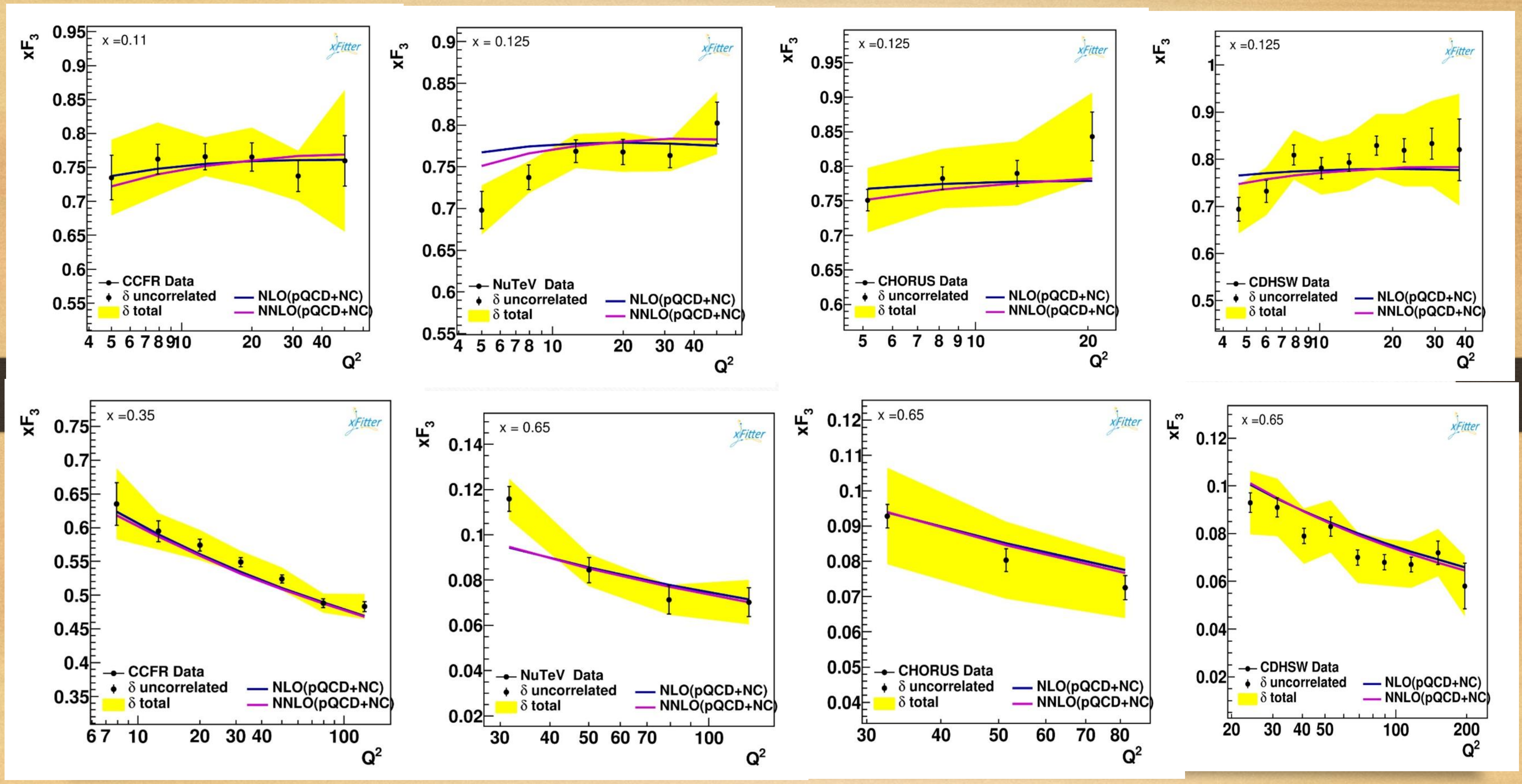
$$h(x) = \sum_{k=0}^{3} D_k\, z^k \quad , \quad z = \log(x)$$

The unknown parameters of D_k and their uncertainties can be extracted simultaneously with other unknown parameters which appeared in the valence PDFs and the strong coupling constant by fitting the experimental data.

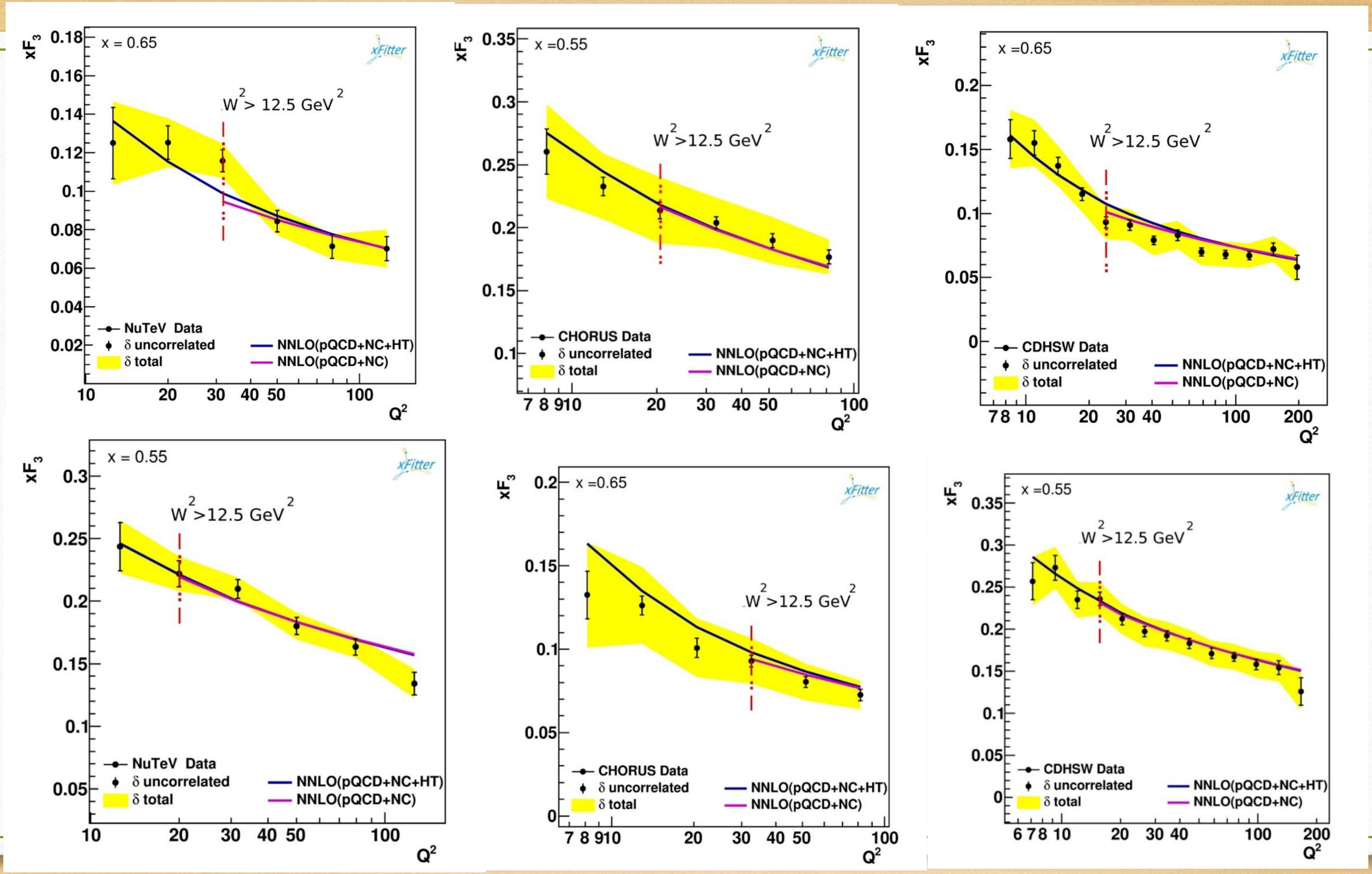
Note that, in the main xFitter package, we need to add the nuclear and higher twist effects modifications, which are not generally included in this package.

Results

Comparison of xF_3 structure function obtained from fitting as a function of Q^2 in different values of x in NLO and NNLO approximation with considering nuclear corrections, by using CCFR, NuTeV, CHORUS, and CDHSW data sets.


x =0.11
CCFR Data
δ uncorrelated
δ total
NLO(pQCD+NC)
NNLO(pQCD+NC)
x = 0.125
NuTeV Data
δ uncorrelated
δ total
NLO(pQCD+NC)
NNLO(pQCD+NC)
x =0.125
CHORUS Data
δ uncorrelated
δ total
NLO(pQCD+NC)
NNLO(pQCD+NC)
x =0.125
CDHSW Data
δ uncorrelated
δ total
NLO(pQCD+NC)
NNLO(pQCD+NC)
x =0.35
CCFR Data
δ uncorrelated
δ total
NLO(pQCD+NC)
NNLO(pQCD+NC)
x = 0.65
NuTeV Data
δ uncorrelated
δ total
NLO(pQCD+NC)
NNLO(pQCD+NC)
x =0.65
CHORUS Data
δ uncorrelated
δ total
NLO(pQCD+NC)
NNLO(pQCD+NC)
x =0.65
CDHSW Data
δ uncorrelated
δ total
NLO(pQCD+NC)
NNLO(pQCD+NC)
xF3
Q2
xFitter

The comparison of the structure function xF_3 obtained from the fit with and without higher twist corrections as a function of Q^2 in the various x, at NNLO approximation.



xF3
x = 0.65
W2 > 12.5 GeV2
NuTeV Data
δ uncorrelated
δ total
NNLO(pQCD+NC+HT)
NNLO(pQCD+NC)
Q2
x = 0.55
CHORUS Data
CDHSW Data
xFitter

The parameters values of the *u*- and *d*-valence quark densities at the input scale of $Q_0^2 = 1GeV^2$, obtained from the best fit with CCFR, NuTeV, CHORUS, and CDHSW considering pQCD , nuclear corrections and higher-twist effects at NLO and NNLO. The parameter values without error have been fixed after the first minimization in xFitter, due to the fact that the data do not constrain some parameters well enough.

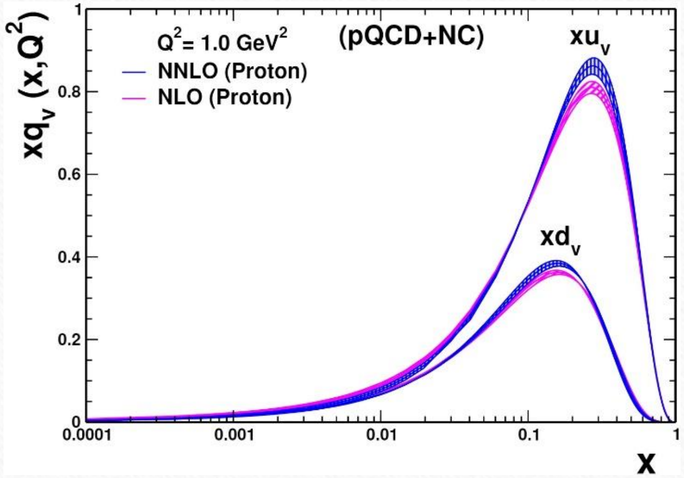
Parameter	NLO	NNLO
N_u	0.208	0.289
A_u	0.390 ± 0.038	0.455 ± 0.031
B_u	3.278 ± 0.068	3.384 ± 0.047
C_u	35.000	29.930
d_u	14.690	11.990
N_d	0.163	0.238
B_d	2.460 ± 0.360	2.700 ± 0.240
D_0	0.970 ± 0.120	0.784 ± 0.070
D_1	1.950 ± 0.220	1.545 ± 0.059
D_2	0.840± 0.110	0.672 ± 0.020
D_3	0.100 ± 0.017	0.80±0.004
$\alpha_s(M_z^2)$	0.1199 ± 0.0031	0.1185 ±0.0023

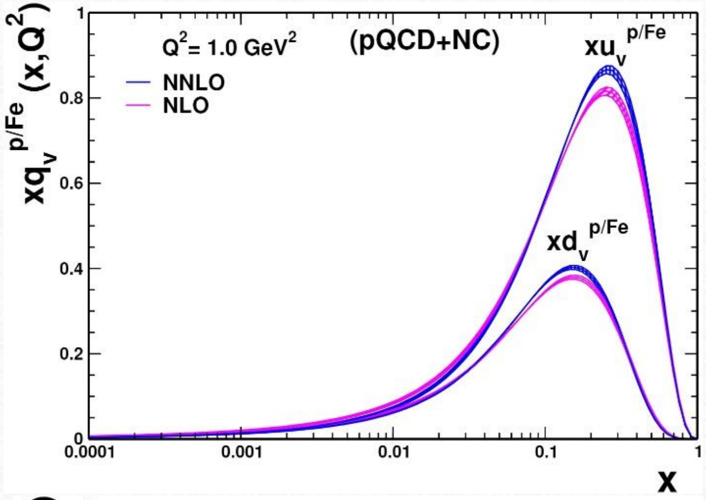
Different combinations of the subset of xF_3 data, contain the number of individual data points before and after cuts for each data set with considering $Q^2 \geq 4\ GeV^2$ cut on the data, with considering pQCD+NC+HT.

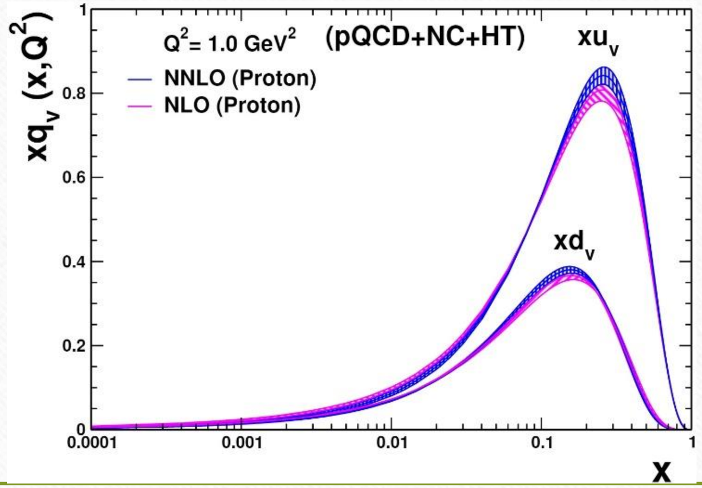
Experiment	Before cuts	After cuts	NLO (pQCD+NC+HT)	NNLO (pQCD+NC+HT)
CCFR	116	67	49	49
NuTeV	75	59	77	77
CHORUS	67	41	52	53
CDHSW	143	96	158	155
Total χ^2			360	358
Total χ^2/d.o.f.			1.290	1.283

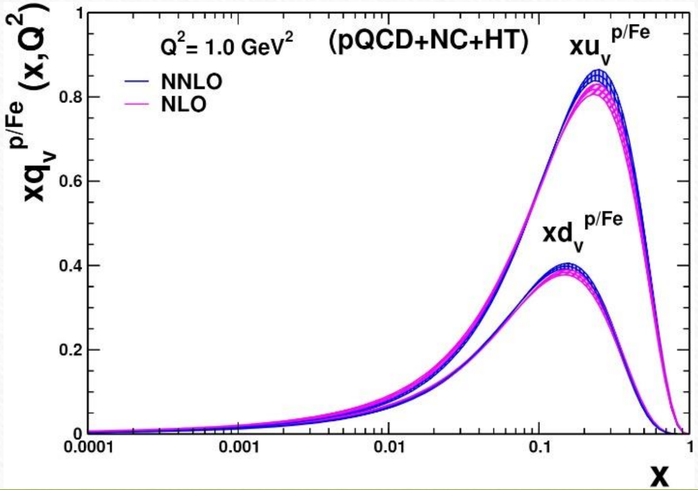
- Also, the reduction of the number of CCFR data points only by the additional cuts on this data ($x > 0.4$) due to the disagreement between CCFR and NuTeV in this region are given in these columns.

The comparison of the iron valence $xu_v^{\frac{p}{Fe}}$ and $xd_v^{\frac{p}{Fe}}$ PDFs as a function of x at $Q^2 = 1\ GeV^2$taking into account nuclear corrections and nuclear and higher twist corrections, at NLO and NNLO with their uncertainty bands.

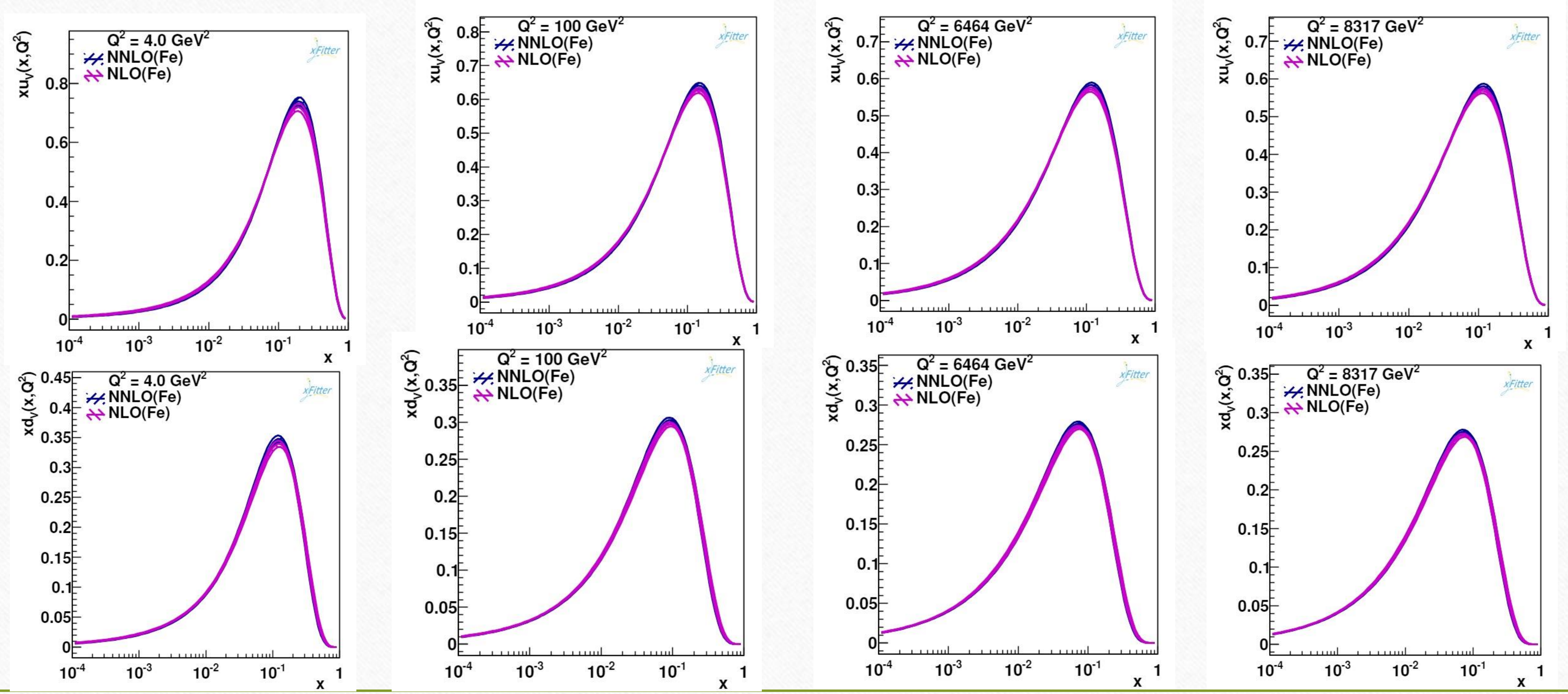








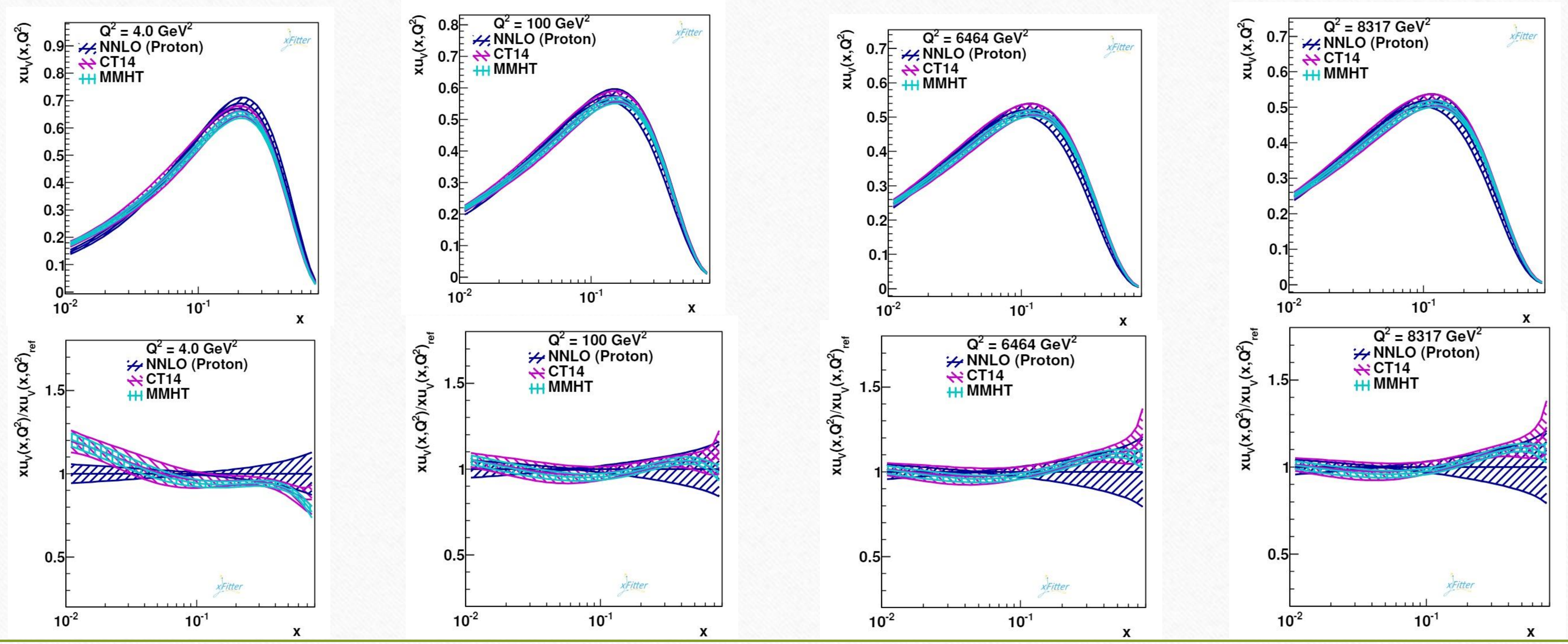
The $xu_v^{\frac{p}{Fe}}$ and $xd_v^{\frac{p}{Fe}}$ parton density distribution at the NLO and NNLO with their uncertainty bands as a function of x at different values of Q^2 = 4, 100, M_w^2 , and $M_z^2 GeV^2$.



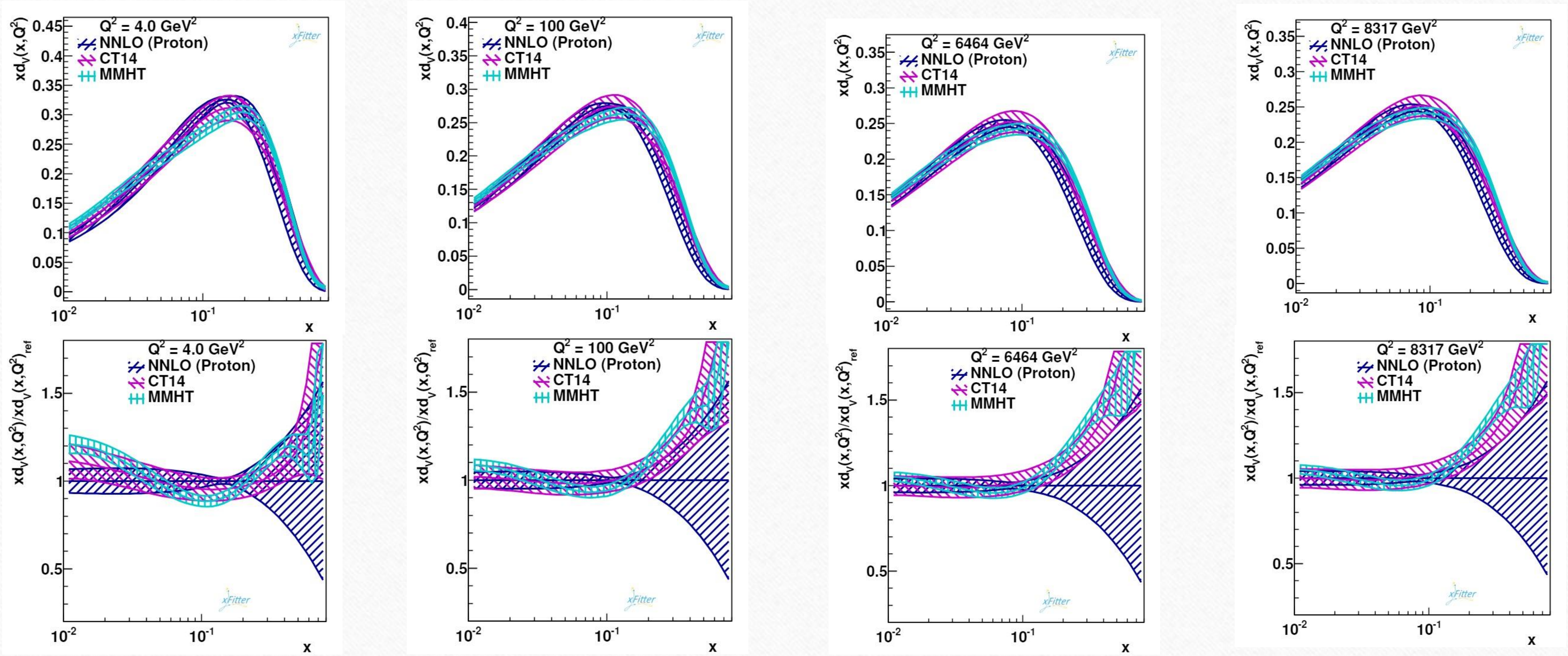
Comparison with other results

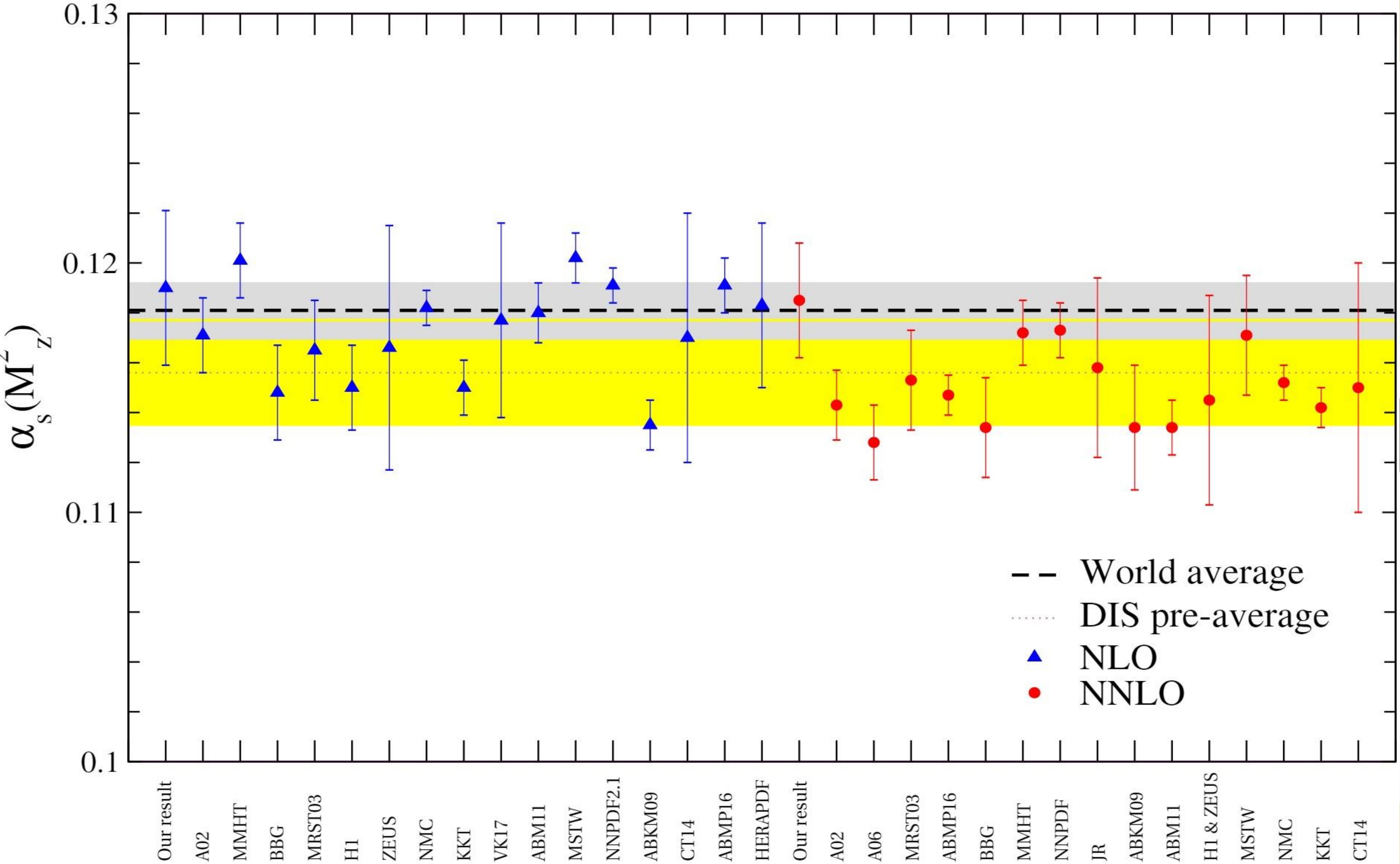
- In order to verify the accuracy of the extracted valence PDFs, comparison of the extracted results with other reported ones seems necessary.
- We have enough motivation to compare our results to CT14 and MMHT14 analyses because these PDF sets were extracted by including different combinations of data sets for the DIS, especially the neutrino-nucleon data experiments.
- The results for xu_v (x, Q^2) and xd_v (x, Q^2) valence PDFs are in good agreement with the results of CT14 and MMHT14.

xu_v (x, Q^2) valence PDF results at different values of Q^2 = 4, 100, M_w^2 , and $M_z^2 GeV^2$obtained with our QCD fits to the DIS neutrino-nucleon data, which have been compared with the results obtained by CT14 and MMHT14 as a function of *x* at the NNLO and the ratio of xu_v/ x$u_{v(ref)}$ with respect to NNLO (proton). We show our results only in the range of $x \in [10^{-2}, 0.8]$, where the data existed and were applied in the present analysis.



xd_v (x, Q^2) valence PDF results at different values of Q^2 = 4, 100, M_w^2 , and $M_z^2 GeV^2$ obtained with our QCD fits to the DIS neutrino-nucleon data, which have been compared with the results obtained by CT14 and MMHT14 as a function of x at the NNLO and the ratio of xd_v/ x$d_{v(ref)}$ with respect to NNLO (proton). We show our results only in the range of $x \in [10^{-2}, 0.8]$, where the data existed and were applied in the present analysis.





0.13
0.12
0.11
0.1
$\alpha_s(M_z^2)$
World average
DIS pre-average
NLO
NNLO
Our result
A02
MMHT
BBG
MRST03
H1
ZEUS
NMC
KKT
VK17
ABM11
MSTW
NNPDF2.1
ABKM09
CT14
ABMP16
HERAPDF
Our result
A02
A06
MRST03
ABMP16
BBG
MMHT
NNPDF
JR
ABKM09
ABM11
H1 & ZEUS
MSTW
NMC
KKT
CT14

- We obtained $\alpha_s(M_z^2)$ = 0.1199 ±0.0031 and 0.1185 ±0.0023 in the case of QCD and nuclear corrections and Higher-Twist effect, at NLO and NNLO, respectively.

- We compare our results with the reported results of different NLO and NNLO QCD analyses for $\alpha_s(M_z^2)$, the dotted line with yellow band indicates the pre-average results of the world average $\alpha_s(M_z^2)$ = 0.1156 ±0.0021.

- Many experimental observables are used to determine the average value of $\alpha_s(M_z^2)$. In fact, the central value of the world average value is determined as the strong coupling constant in the DIS subfield. It should be noted that the pre-average value of $\alpha_s(M_z^2)$ in the DIS process is smaller in comparison to the world average value of $\alpha_s(M_z^2)$= 0.1181 ± 0.0011.

- The difference of the reported results of $\alpha_s(M_z^2)$ by different groups is due to the fact that this value depends not only on the renormalization scheme, but also on different kinds of measurements in DIS, cuts on the data, and different parametrization and methodology, also this analysis is free of the correlation between strong coupling constant $\alpha_s(M_z^2)$ and the sea-quarks and gluon distributions.

Summarize

- The present QCD analysis has been performed in two approximations, NLO and NNLO, using CCFR, NuTeV, CHORUS, and CDHSW experimental data.
- A total of 279 experimental data have been used.
- In this analysis, the ZM-VFNS approach is used to consider the contribution of light quarks.
- In the future, newer experimental results will have a significant impact on QCD analysis, which can lead to the extraction of parton distribution functions with higher accuracy.
- By taking into account higher twist corrections we can get the significant improvement, about 3%, for $\alpha_s(M_z^2)$ for both NLO and NNLO as well, in comparison to when the HT terms are set to zero.

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QCD analysis of structure functions in deep inelastic neutrino-nucleon scattering without using the orthogonal polynomials approach

A. Ghaffari Tooran,[*] A. Khorramian,[†] and H. Abdolmaleki[‡]
Faculty of Physics, Semnan University, P. O. Box 35131-19111, Semnan, Iran

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A nonsinglet QCD analysis of neutrino-nucleon structure function is performed based on all the data for charged current neutrino-nucleon deep inelastic scattering (DIS) corresponds to NLO and NNLO approximations, with taking into account the nuclear and higher twist corrections. In this analysis, we extract $xu_v(x, Q^2)$ and $xd_v(x, Q^2)$ valence parton distribution functions (PDFs) in a wide range of x and Q^2, and determine their parametrization with the correlated errors using the xFitter framework. Our results regarding valence-quark densities with their uncertainties are compared to the prediction extracted using other PDF sets from different groups. We determine $\alpha_s(M_Z^2) = 0.1199 \pm 0.0031$ and 0.1185 ± 0.0023 with considering the nuclear and higher twist corrections at the NLO and NNLO, respectively, and perform a comparison with other reported results. The extracted results regarding valence-quark distributions and the value of $\alpha_s(M_Z^2)$ are in good agreement with available theoretical models.

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a.ghafary@semnan.ac.ir

Thanks for your attention

Backup

The valence nPDFs for a nucleus can be expressed as $$xq_v^A(x, Q_0^2) = \frac{Z}{A}\, xq_v^{p/A}(x, Q_0^2) + \frac{A-Z}{A}\, xq_v^{n/A}(x, Q_0^2)$$

xq_A where A and Z are mass number and atomic number, respectively, and p and n indicate proton and neutron.
In the above, $xq_{p/A\,v}$ and $xq_{n/A\,v}$ denote valence PDFs of bound protons and neutrons in the nucleus A.
By assuming isospin symmetry, the valence distributions inside a bound neutron, $xq_{n/A\,v}$, are related to the ones in a bound proton, $xq_{p/A\,v}$.
If there are no nuclear modification, the valence nPDFs, $xq_{A\,v}$, are expressed by a simple summation of free proton and neutron contributions.

Independent Parameterization

$$xu_v(x, Q_0^2) = N_u x^{a_u}(1-x)^{b_u}(1 + c_u x + d_u\sqrt{x})$$

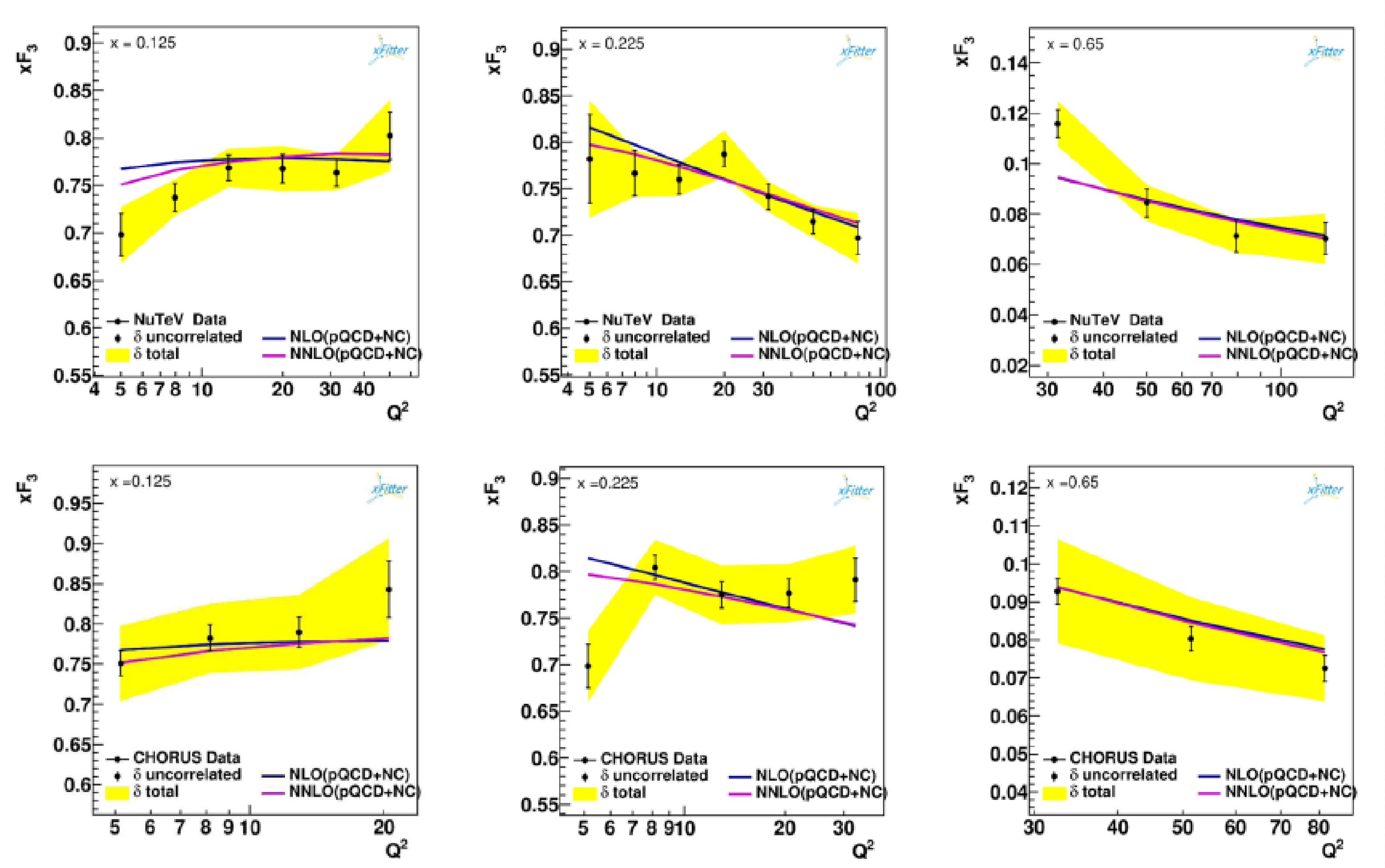
$$xd_v(x, Q_0^2) = N_d x^{a_d}(1-x)^{b_d}(1 + c_d x + d_d\sqrt{x})$$

$$\begin{aligned} N_u &= 2/\left[B(a_u, 1+b_u) + c_u B(1/2 + a_u, 1+b_u) \right. \\ &\quad \left. + d_u B(1 + a_u, 1+b_u)\right] \ , \end{aligned}$$

$$\begin{aligned} N_d &= 1/\left[B(a_d, 1+b_d) + c_d B(1/2 + a_d, 1+b_d) \right. \\ &\quad \left. + d_d B(1 + a_d, 1+b_d)\right] \ , \end{aligned}$$

				#Data		Sce. I		Sce. II	
Experiment	Reference	x	Q^2	before cuts	after cuts	NLO	NNLO	NLO	NNLO
CCFR	[20]	[0.0075-0.75]	[1.3 -125.9]	116	87	115	109	103	96
NuTeV	[21]	[0.015 - 0.75]	[1.26 - 50.12]	75	59	120	108	105	94
CHORUS	[22]	[0.02 -0.65]	[0.325 - 81.55]	67	41	86	68	72	66
CDHSW	[23]	[0.15 - 0.65]	[0.19 -196.3]	143	90	170	158	177	164
Correlated χ^2						18	15	19	17
Total χ^2/d.o.f.						500/271 =1.84	459/271 =1.67	477/269 =1.76	436/269 =1.62

Experiment	x	Q^2	xF_3	#Data		NLO		NNLO	
				xF_3(cuts)	xF_3(HT)	pQCD+NC	pQCD+NC+HT	pQCD+NC	pQCD+NC+HT
CCFR [20]	0.0075–0.75	1.3–125.9	116	67(87–20)	67	50	49	47	49
NuTeV [21]	0.015–0.75	1.26–50.12	75	59	65	96	77	86	77
CHORUS [22]	0.02–0.65	0.325–81.55	67	41	48	52	52	51	53
CDHSW [23]	0.015–0.65	0.19–196.3	143	96	107	193	158	175	155
Correlated χ^2						31	24	25	24
Total χ^2						422	360	384	358
d.o.f.						259	279	259	279
Total χ^2/d.o.f.						1.629	1.290	1.482	1.283



xF3
x = 0.125
x = 0.225
x = 0.65
NuTeV Data
CHORUS Data
δ uncorrelated
δ total
NLO(pQCD+NC)
NNLO(pQCD+NC)
Q2
xFitter

TABLE III. Comparison of low order moments at $Q^2 = 4$ GeV^2 from our nonsinglet NNLO QCD analysis with the NNLO analysis, KT08 [52], KT07 [54], BBG06 [82], MMHT14 [91], A02 [92], and A06 [93].

f	N	pQCD+NC+HT	KT08 (Jacobi poly.)	KT07 (Bernstein poly.)	BBG06	MMHT14	A02	A06
u_v	2	0.3112	0.3056	0.2934	0.2986	0.2851	0.304	0.2947
	3	0.0914	0.0871	0.0825	0.0871	0.0831	0.087	0.0843
	4	0.0346	0.0330	0.0311	0.0333	0.0322	0.033	0.0319
d_v	2	0.1019	0.1235	0.1143	0.1239	0.1202	0.120	0.1129
	3	0.0207	0.0298	0.0262	0.0315	0.0305	0.028	0.0275
	4	0.0058	0.0098	0.0083	0.0105	0.0106	0.010	0.0092