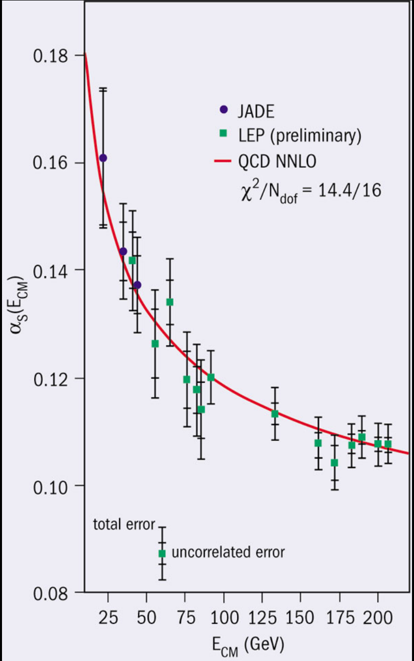
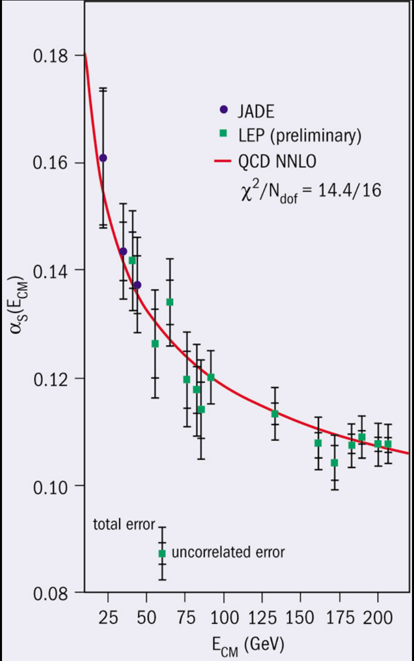
Paul Romatschke, CU Boulder

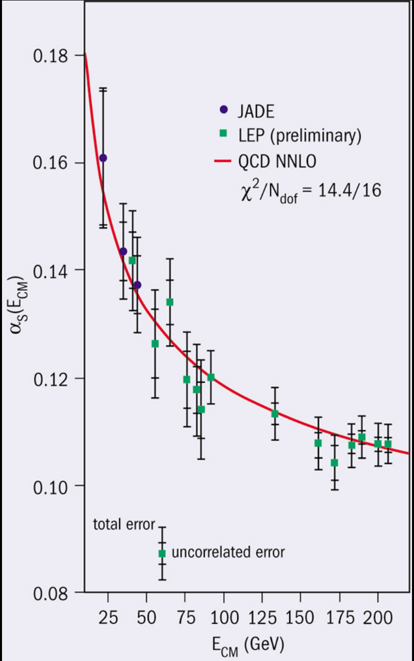


CERN Courier, November 2004



CERN Courier, November 2004

- QCD coupling decreases as function of energy



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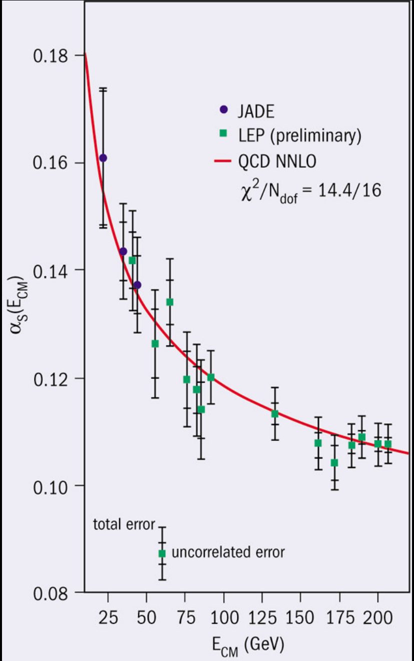
- Negative β-function:

$$\beta \equiv \frac{\partial \alpha_s(\bar{\mu})}{\partial \ln \bar{\mu}^2} < 0\,.$$



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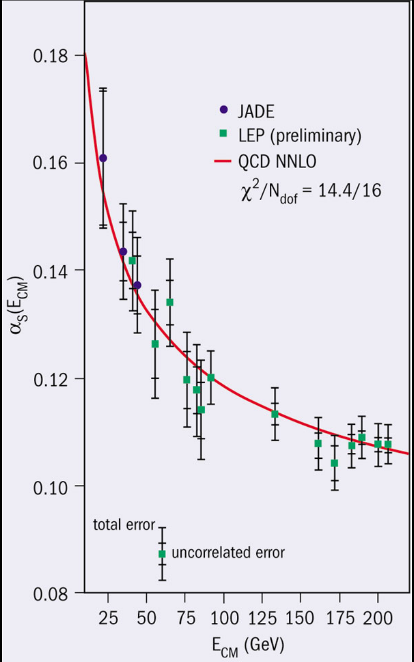

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- Implies *asymptotic freedom*: quarks and gluons do not interact for asymptotically high energy $\bar{\mu} \to \infty$

Price of Asymptotic Freedom

Sidney Coleman and David J. Gross
Phys. Rev. Lett. **31**, 851 – Published 24 September 1973

ABSTRACT

A renormalizable field theory is said to be asymptotically free if the origin of coupling-constant space is an ultraviolet-stable fixed point in the sense of Wilson. Asymptotically free theories are of great interest because they have almost-canonical light-cone singularities, and thus predict phenomena very close to Bjorken scaling. All known examples of asymptotically free theories involve non-Abelian gauge fields. We show that this is not coincidence: No renormalizable field theory without non-Abelian gauge fields can be asymptotically free.

Seemingly unrelated: Quantum Triviality

Quantum triviality

Article Talk

In a quantum field theory, charge screening can restrict the value of the observable "renormalized" charge of a classical theory. If the only resulting value of the renormalized charge is zero, the theory is said to be "trivial" or noninteracting. Thus, surprisingly, a classical theory that appears to describe interacting particles can, when realized as a quantum field theory, become a "trivial" theory of noninteracting free particles. This phenomenon is referred to as **quantum triviality**. Strong evidence supports the idea that a field theory involving only a scalar Higgs boson is trivial in four spacetime dimensions,[1][2] but the situation for realistic models including other particles in addition to the Higgs boson is not known in general. Nevertheless, because the Higgs boson plays a central role in the Standard Model of particle physics, the question of triviality in Higgs models is of great importance.

Quantum Triviality:

$$\lambda_R(m) = 0$$

in the continuum limit

2019: Proofs of Quantum Triviality in 4d

Michael Aizenman

Hugo Duminil-Copin

There is a loophole in the proofs of asymptotic freedom and quantum triviality

There is a loophole in the proofs of asymptotic freedom and quantum triviality

It is the same loophole in both proofs

(2) If we assume the theory contains *only* spinless mesons, it is easy to show it cannot be asymptotically free.[11] Let us assume the quartic form $\lambda_{ijkl}\varphi_i\varphi_j\varphi_k\varphi_l$ is positive, where the λ's are the renormalized coupling constants, and the sum on repeated indices is implied. In particular, this implies that λ_{1111} is positive. However, it is easy to compute that

$$M\,d\lambda_{1111}/dM \propto \lambda_{11rs}\lambda_{11rs} \geq 0. \quad (4)$$

Thus the theory cannot be asymptotically free. If we assume the quartic form goes to zero (asymptotic freedom) as M increases, but is not positive,[12] then an application of the methods of Coleman and Weinberg[13] shows directly that the energy of the system cannot be bounded below, and the theory is nonsense.

Coleman, Gross, 1973

with a Hamiltonian $H(\phi)$ and an a-priori measure $\rho(d\phi)$ of the form

$$H(\phi) = -\sum_{\{x,y\}\subset\Lambda_R} J_{x,y}\,\phi_x\phi_y\,, \qquad \rho(d\phi_x) = e^{-\lambda\phi_x^4 + b\phi_x^2}\,d\phi_x\,, \quad (1.12)$$

Definition 2.1 *A probability measure on $\rho(d\varphi)$ on $\mathbb{R}$ is said to belong to the Griffiths-Simon (GS) class if either of the following conditions is satisfied*

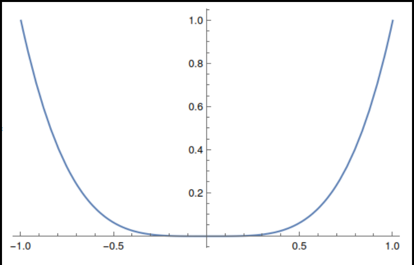
2) *ρ can be presented as a (weak) limit of probability measures of the above type, and is of sub-gaussian growth:*

$$\int e^{|\varphi|^\alpha}\rho(d\varphi) < \infty \quad \textit{for some } \alpha > 2. \quad (2.2)$$

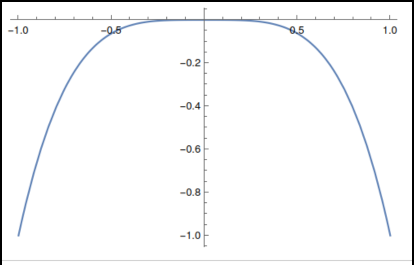
Aizenman, Duminil-Copin, 2019

Both proofs assume $\lambda_R(\bar{\mu} = \Lambda_{\mathrm{UV}}) > 0$

Classical Potentials



$\lambda > 0$: $V(x) \propto x^4$
"correct"



$\lambda < 0$: $V(x) \propto -x^4$
"nonsense"

Nobel prize winners and Fields Medalists tell you that $\lambda < 0$ QFT is nonsense. Do you want continue?

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Can field theory with $\lambda_R(\bar{\mu} = \Lambda_{\rm UV}) < 0$ make sense?

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Not in classical physics. But maybe for a quantum theory?

Reason I: Pure Math

Technical Slides Ahead!

Toy Model Field Theories

- Define

$$Z(\lambda) = \int_{-\infty}^{\infty} dx e^{-\lambda x^4}$$

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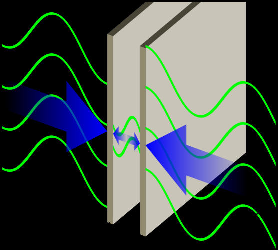
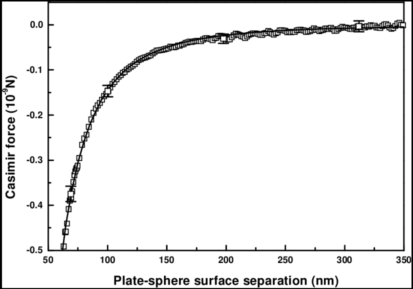
- What is $\zeta(-1) = ?$
- Riemann 1859: analytic continuation

$$\zeta(-1) = -\frac{1}{12}$$





Experimental verification of analytically continued ζ-function

Toy Model Field Theories

- Define

$$Z(\lambda) = \int_{-\infty}^{\infty} dx \, e^{-\lambda x^4}$$

- What is

$$Z(-1) = ?$$

- calculate for $\lambda > 0$

$$Z(\lambda) = 2\lambda^{-\frac{1}{4}} \Gamma\left(\frac{5}{4}\right)$$

- Analytically continue:

$$Z(-1) = \sqrt{2}(1-i)\Gamma\left(\frac{5}{4}\right)$$

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- Not unique: different branches of root
- But clearly no more "nonsense" than $\zeta(-1)$

That was tedious. And boring. What's your point?

Reason II: Symmetry

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- But is it **necessary**?

Non-Hermitian Quantum Mechanics

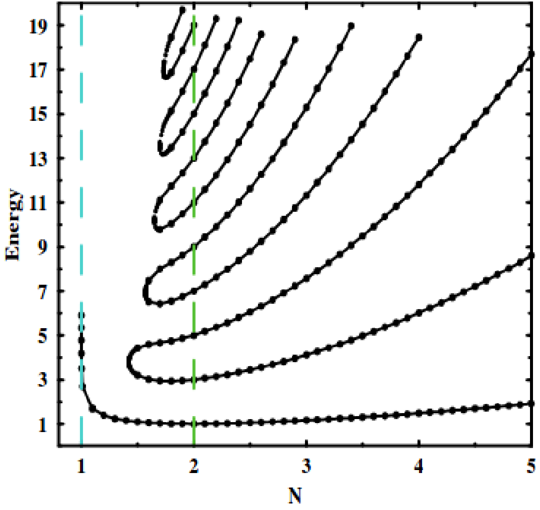

FIG. 1. Energy levels of the Hamiltonian $H = p^2 - (ix)^N$ as a function of the parameter N. There are three regions:

[Bender & Böttcher, 1997]

Non-Hermitian Quantum Mechanics

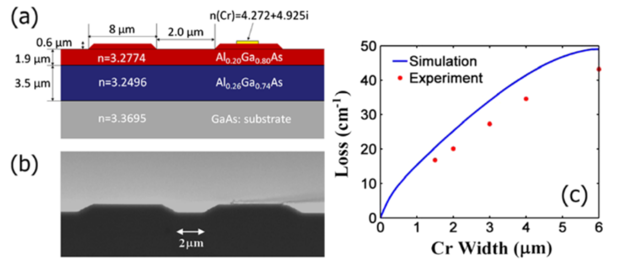

FIG. 3 (color online). Non-Hermitian dual structure. (a) Design details and complex refractive index distribution. (b) Scanning electron microscopy picture of the finalized passive $\mathcal{PT}$ device with the Cr stripe shown on the right. (c) Modal loss of isolated waveguide structure as a function of Cr width.

[needs ref]

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- Negative coupling QFT could still be “pathological” for other reasons, but $\lambda < 0$ is **not** sufficient reason to dismiss them as “nonsense”
- Proposal: calculate observables and check!

O(N) Model

Defined as

$$Z = \int \mathcal{D}\vec{\phi} e^{-S_E}\,, \quad S_E = \int_x \left[\frac{1}{2}\partial_\mu\vec{\phi}\cdot\partial_\mu\vec{\phi} + \frac{\lambda}{N}\left(\vec{\phi}^2\right)^2\right]\,,$$

with $\vec{\phi} = (\phi_1, \phi_2, \ldots, \phi_N)$.
Examples:

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Examples:

- 0+1d is quantum mechanics in N dimensions (any N)
- 2+1d: conjectured AdS_4 gravity dual for $N \to \infty$ [hep-th/0210114]
- 3+1d: N=4 is Higgs case

O($N \gg 1$) model in 4d in continuum

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- Theory is asymptotically free!

O(N$\gg$ 1) model in 4d – Physics consequence

- Standard Model Higgs Potential (2 parameters)

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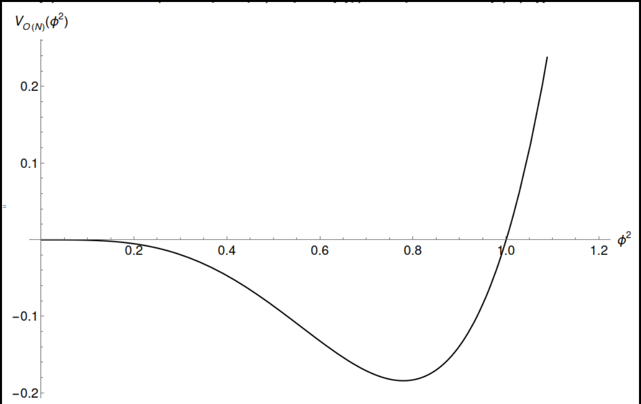
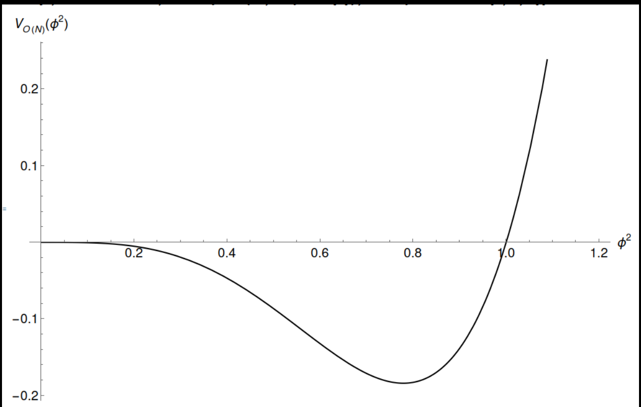
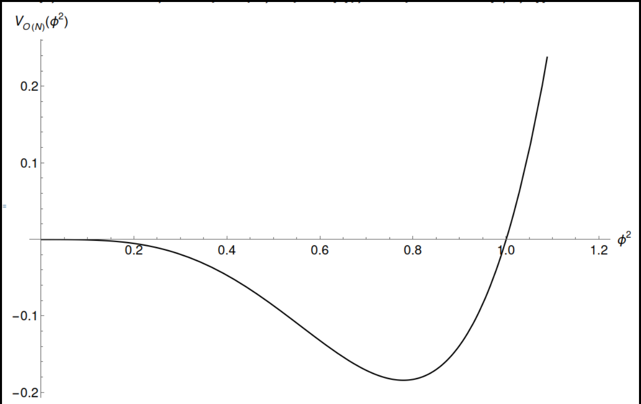
$$V_{\mathrm{O(N)}} = \lambda\phi^4 + \mathrm{rad.} - \mathrm{corr}$$



$V_{O(N)}(\phi^2)$
0.2
0.1
−0.1
−0.2
0.2
0.4
0.6
0.8
1.0
1.2
ϕ^2



Radiative corrections generate VEV – No tachyonic mass term needed!



Radiative corrections generate VEV – No tachyonic mass term needed!
Perturbative vacuum at $\phi = 0$ is unstable – agrees with EW Pheno

Standard Model Lagrangian

$$
\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{2}\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu} - \frac{1}{2}\mathrm{Tr}\, W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
& +(D_\mu\phi)^\dagger D^\mu\phi + \mu^2\phi^\dagger\phi - \frac{1}{2}\lambda\left(\phi^\dagger\phi\right)^2 \\
& +\sum_{f=1}^{3}\left(\bar{\ell}^f_L i\not{D}\ell^f_L + \bar{\ell}^f_R i\not{D}\ell^f_R + \bar{q}^f_L i\not{D} q^f_L + \bar{d}^f_R i\not{D} d^f_R + \bar{u}^f_R i\not{D} u^f_R\right) \\
& -\sum_{f=1}^{3} y^f_\ell \left(\bar{\ell}^f_L \phi \ell^f_R + \bar{\ell}^f_R \phi^\dagger \ell^f_L\right) \\
& -\sum_{f,g=1}^{3}\left(y^{fg}_d \bar{q}^f_L \phi d^g_R + (y^{fg}_d)^* \bar{d}^g_R \phi^\dagger q^f_L + y^{fg}_u \bar{q}^f_L \tilde{\phi} u^g_R + (y^{fg}_u)^* \bar{u}^g_R \tilde{\phi}^\dagger q^f_L\right),
\end{aligned}
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O(N) Model Lagrangian

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& - \sum_{f=1}^{3} y^f_\ell \left(\bar{\ell}^f_L \phi \ell^f_R + \bar{\ell}^f_R \phi^\dagger \ell^f_L \right) \\
& - \sum_{f,g=1}^{3} \left(y^{fg}_d \bar{q}^f_L \phi d^g_R + (y^{fg}_d)^* \bar{d}^g_R \phi^\dagger q^f_L + y^{fg}_u \bar{q}^f_L \tilde{\phi} u^g_R + (y^{fg}_u)^* \bar{u}^g_R \tilde{\phi}^\dagger q^f_L \right),
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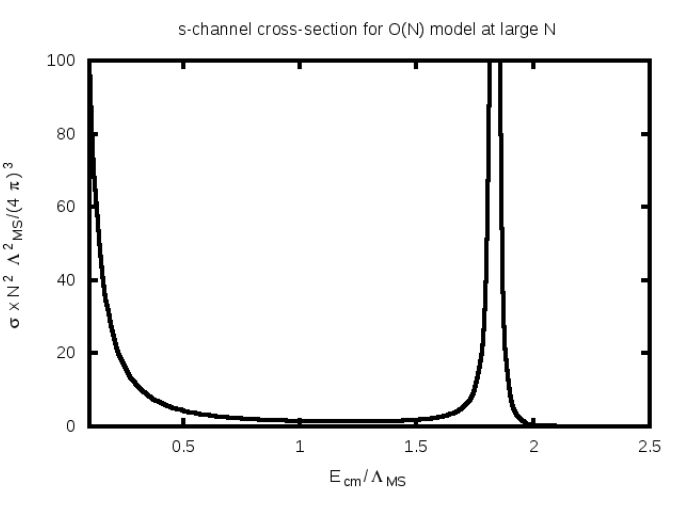
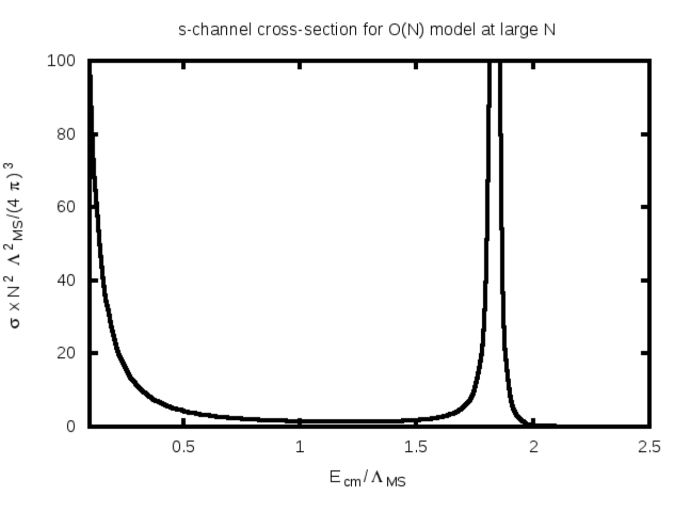
& - \sum_{f,g=1}^{3}\left(y^{fg}_d\bar{q}^f_L\phi d^g_R + (y^{fg}_d)^*\bar{d}^g_R\phi^\dagger q^f_L + y^{fg}_u\bar{q}^f_L\tilde{\phi}u^g_R + (y^{fg}_u)^*\bar{u}^g_R\tilde{\phi}^\dagger q^f_L\right),
\end{aligned}
$$

Same physics – one parameter less!



Well behaved scattering cross-section for any CM energy; prediction for scalar bound state at $m \simeq 1.84 m_{\mathrm{Higgs}}$



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This is how you kill/verify this model!

Summary and Conclusions

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- Senior management built a “mental roadblock” about scalars in 4d
- Proofs about asymptotic freedom and quantum triviality have the same loophole
- Analytic continuation to negative coupling exploits this loophole
- O(N) model is explicitly solvable theory and practical testing ground
- More checks on observables are needed

Potentially important consequences for EW Theory and QFT

Stop using classical “intuition”!
Calculate observables and check!

Thank You

Bonus Material

References & Hyperlinks

Continuum large N scalar field theory

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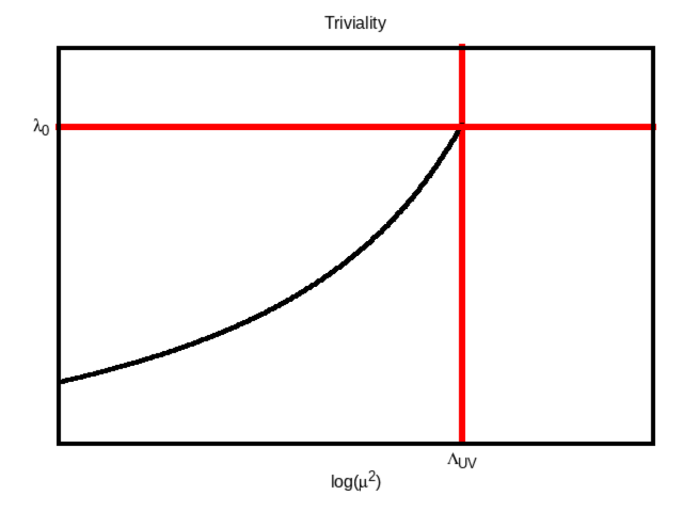
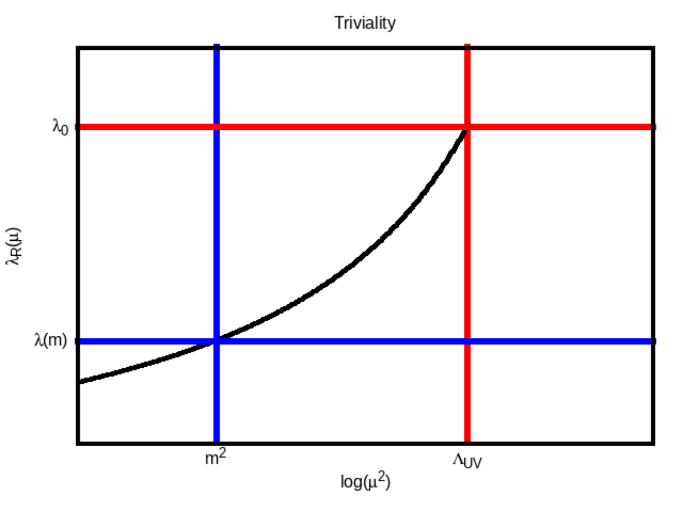
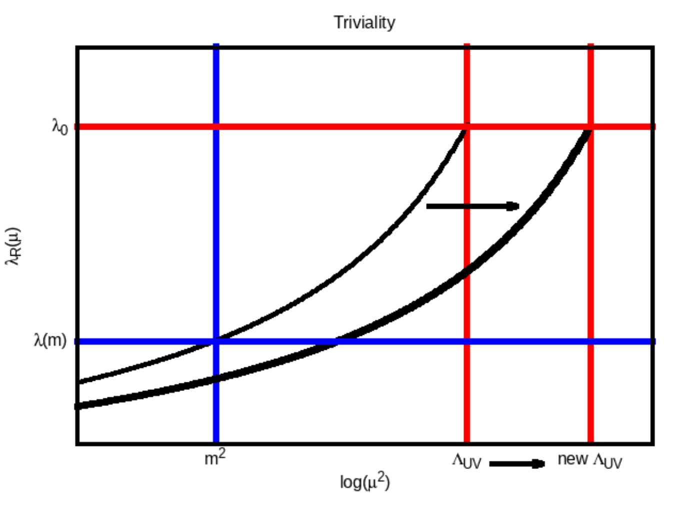
Quantum Triviality



Quantum Triviality



Quantum Triviality



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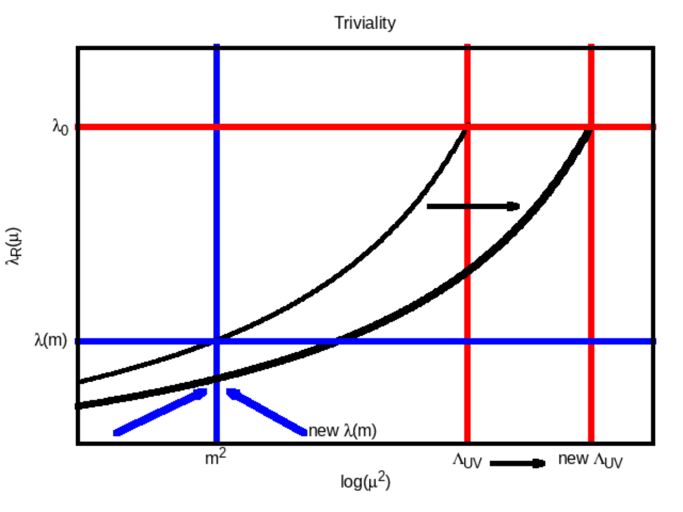


A Field Theory with Computable Large-Momenta Behaviour.

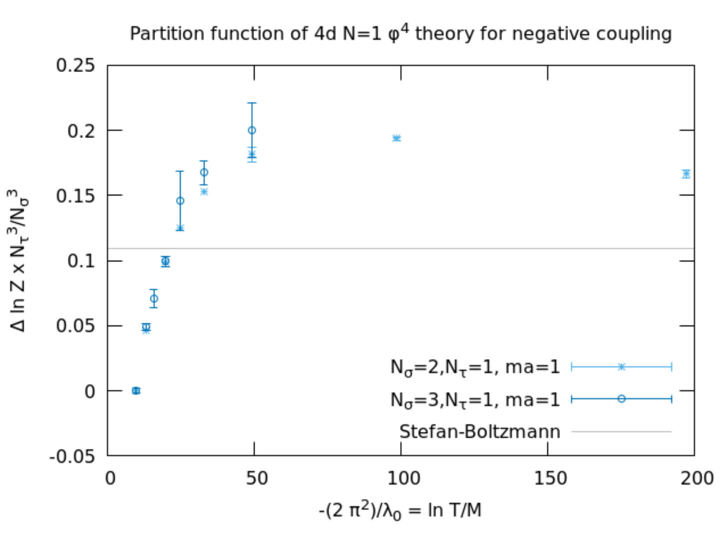
K. Symanzik

Deutsches Elektronen-Synchrotron DESY - Hamburg

(ricevuto il 12 Dicembre 1972)

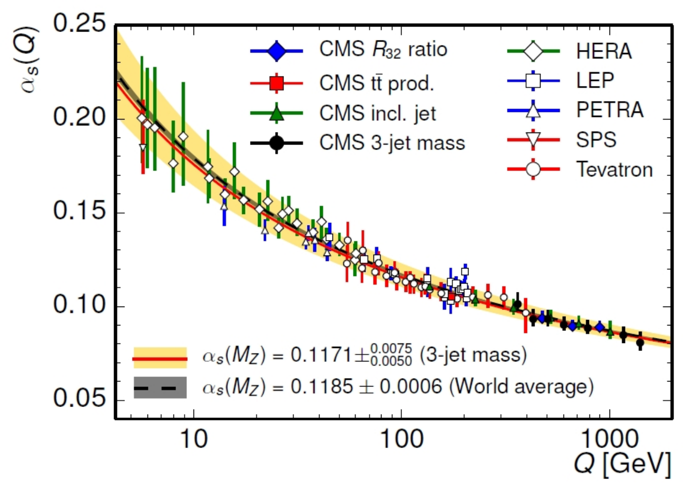
In the current extensive discussions (*) of φ^4 theory it is usually taken for granted that the renormalized coupling constant g must be positive. As emphasized previously (2) there is no known reason, axiomatic or otherwise, for $g > 0$ to be required for a physically acceptable theory. The feeling that otherwise the theory cannot have a vacuum and particles of discrete mass is not rigorously founded as discussed near the end of this letter. The interesting feature of the theory with $g < 0$, however, appears worth pointing out: If one assumes the theory to exist, the large-momenta behaviour of its Feynman amplitudes can be computed at generic momenta to arbitrary accuracy. Besides, we find that the imaginary part of the four-point vertex function in φ^4 theory should not change sign in momentum space.

Negative coupling ϕ^4 in 4d on the lattice



adapted from [2305.05678]

QCD running coupling



Somewhat misleading: really a fit of perturbation theory to experimental measurements

QCD at infinite coupling

- In pQCD, $\alpha_s(\bar{\mu})$ does diverge at $\bar{\mu} = \Lambda_{\overline{\rm MS}} \sim 0.3$ GeV
- Usually dismissed as an artifact of perturbation theory
- Non-perturbative extractions (lattice+NRQCD) exist down to $\bar{\mu} = 1.5$ GeV where

$$\alpha_s(1.5\text{GeV}) \simeq 0.336$$

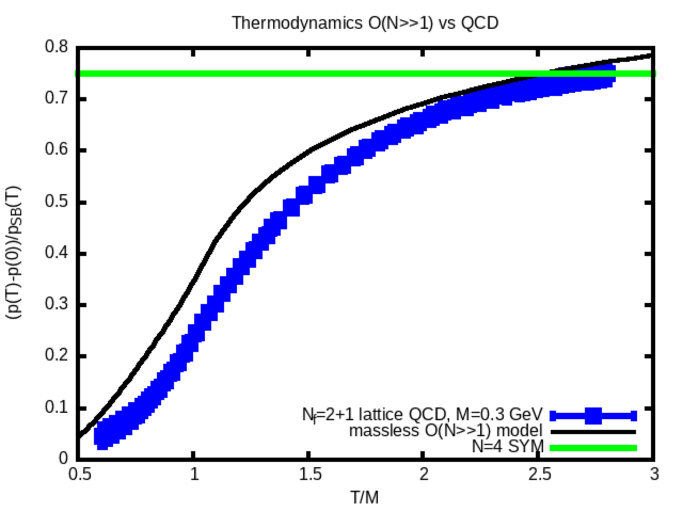
[Bazavov et al, 1407.8437]

- QCD could have a Landau pole at $\Lambda_{\overline{\rm MS}} \sim 0.3$ GeV
- No issues in QCD

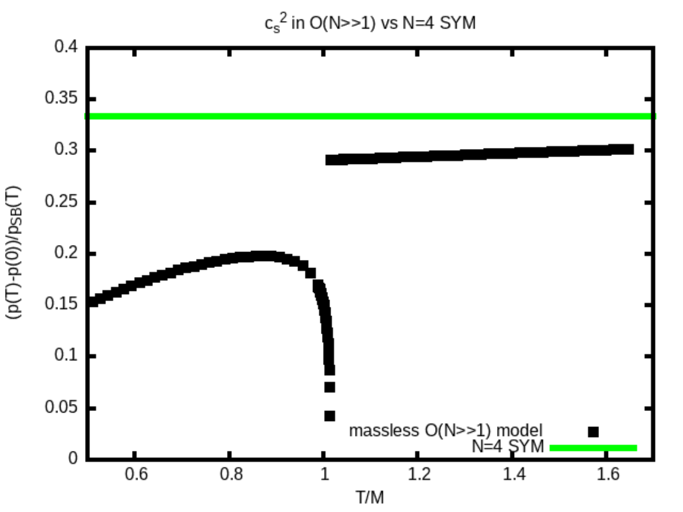
The O(N$\gg$ 1) Model as a Model for QCD

- Only one scale M
- Is M the same as $\Lambda_{\overline{\mathrm{MS}}}$ in QCD?
- Let's compare!

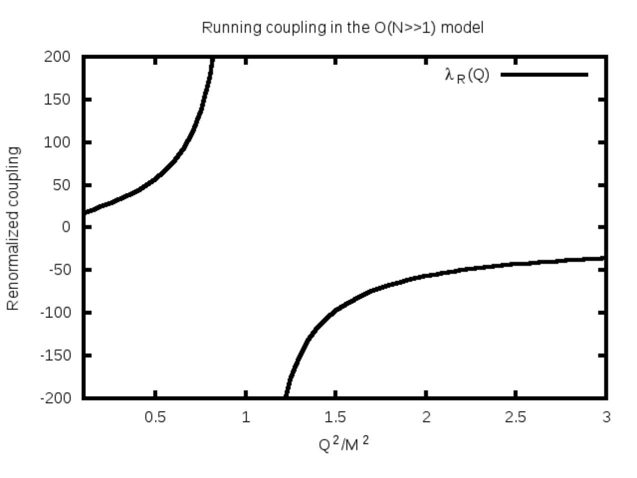
Parameter-free comparison to QCD



Parameter-free comparison to QCD



Exact Running coupling in O(N) Model



[2305.05678]

Intermezzo: Selection of Analytic Continuation

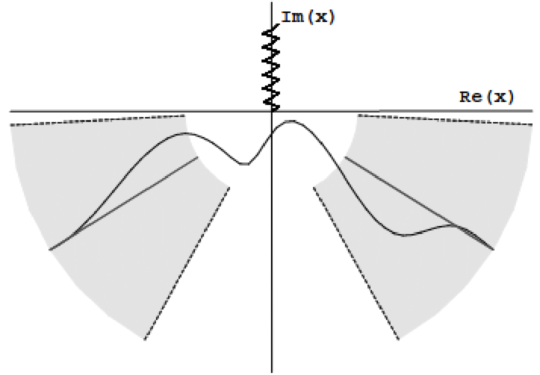
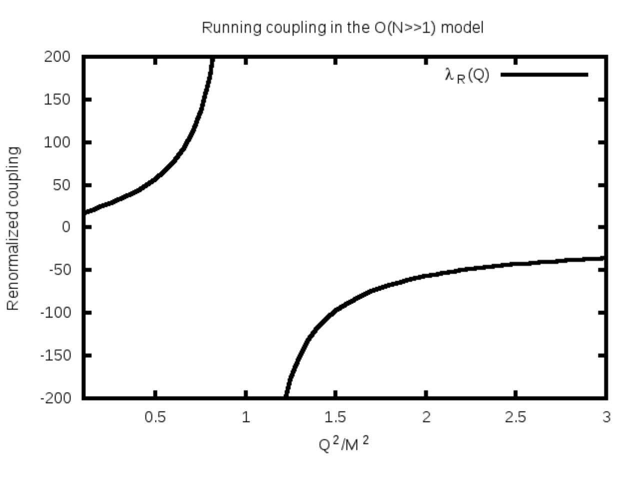


FIG. 2. Wedges in the complex-x plane containing the contour on which the eigenvalue problem for the differential

[Bender & Böttcher, 1997]

Exact Running coupling in O(N) Model



[2305.05678]

Scattering for NR fermions

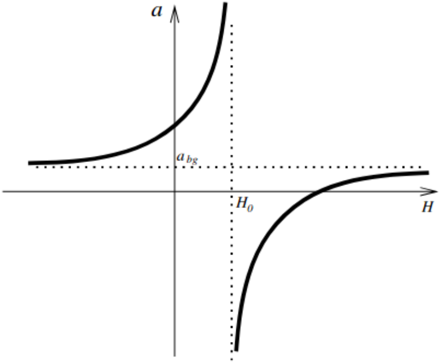


Fig. 2. A schematic of a typical, experimentally observed behavior of an s-wave scattering length $a(H)$ as a function of magnetic field H in a vicinity of a Feshbach resonance.

[Gurarie, Radzihovsky, 2007]