Holographic QCD and the anomalous magnetic moment of the muon

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Outline of the talk

- Current status of the anomalous magnetic moment of the muon
- Introduction to holographic QCD
- Predictions from holographic QCD

Anomalous magnetic moment

Angular momentum L of charged particles produces magnetic moment

$$\vec{\mu} = \mu_B \vec{L} \qquad \text{with} \quad \mu_B = \frac{e}{2m} \quad (\hbar = 1, c = 1)$$

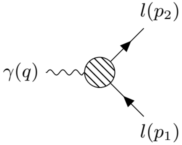
Fundamental point particles with spin S have intrinsic magnetic moment with anomalous g-factor

$$\vec{\mu} = g \mu_B \vec{S}$$

Special relativity plus quantum mechanics (Dirac equation): $g = 2$

QFT corrections parametrized by $a = \frac{1}{2}(g - 2)$

Field theory definition:



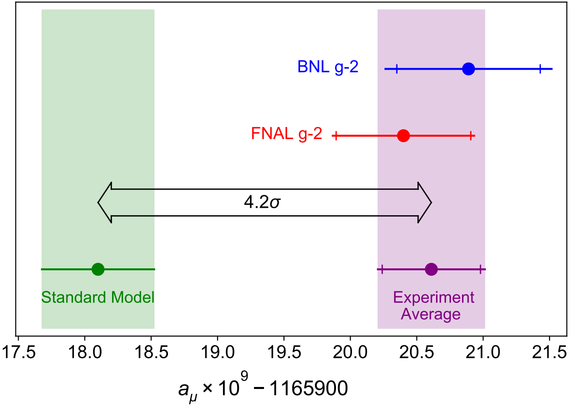
$$= (-ie)\bar{u}(p_2)\left[\gamma^\mu F_E(q^2) + i\frac{\sigma^{\mu\nu} q_\nu}{2m_\mu} F_M(q^2)\right] u(p_1)$$

with $q = p_2 - p_1$ and $a = F_M(0)$

Status of anomalous magnetic moments

- Perfect agreement for the electron to $\mathcal{O}(\alpha^5)$
- However 4.2σ discrepancy for muon [BNL 2004 and FNL 2021 vs. Aoyama et al. 2020]

$$a_\mu^{\text{exp}} = (116\,592\,061 \pm 41) \times 10^{-11}$$
$$a_\mu^{\text{SM}} = (116\,591\,810 \pm 43) \times 10^{-11}$$



- New result consistent and uncertainties halved (5.1σ) [FNL 2023 Preprint]

Standard Model prediction

[Aoyama et al., 2006.04822]

Muon 200 times heavier than electron $\Rightarrow$ more sensitive to non-QED and BSM physics

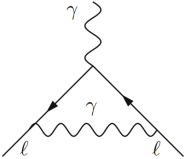
$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{HVP}} + a_\mu^{\text{HLbL}} = (116\,591\,810 \pm 43) \times 10^{-11}$$

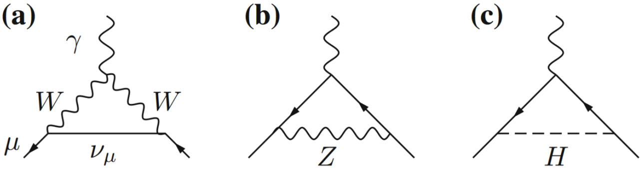
$$a_\mu^{\text{QED}} = (116\,584\,718.931 \pm 0.104) \times 10^{-11}$$

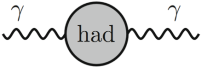
$$a_\mu^{\text{EW}} = (153.6 \pm 1.0) \times 10^{-11}$$

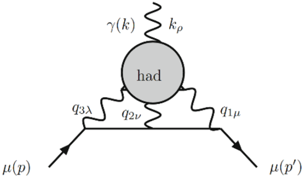
$$a_\mu^{\text{HVP}} = (6845 \pm 40) \times 10^{-11} \quad (0.6\% \text{ uncertainty})$$

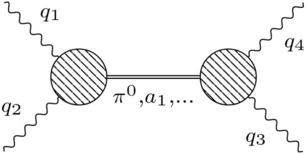
$$a_\mu^{\text{HLbL}} = (92 \pm 19) \times 10^{-11} \quad (20\% \text{ uncertainty!})$$







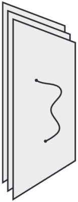


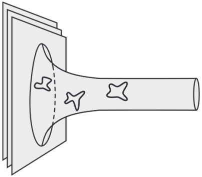


Holographic QCD

Strongly coupled gauge theory in D dimensions at large N
is dual to a weakly coupled theory of gravity in D+1 dimensions

gauge theory	**gravity dual**
degree N of the gauge group	number of branes, curvature radius
flat space time on which the gauge theory lives	boundary of higher-dimensional geometry
global symmetry	gauge symmetry
gauge invariant operators	fields acting as sources to these operators
particle mass	eigenvalue in wave equation
energy scale	radial coordinate in the AdS-space
renormalisation group flow	movement along the radial coordinate





Holographic QCD models

(Axial) vector mesons and pions are described by **5-d YM fields** $\mathcal{F}_{MN}^{L,R}$ for global $U(N_f)_L \times U(N_f)_R$ chiral symmetry of boundary theory

$$S_{\rm YM} \propto \frac{1}{g_5^2}\,\mathrm{tr}\int d^4x \int_0^{z_0} dz\, e^{-\Phi(z)}\sqrt{-g}\, g^{PR} g^{QS}\left(\mathcal{F}_{PQ}^{(L)}\mathcal{F}_{RS}^{(L)} + \mathcal{F}_{PQ}^{(R)}\mathcal{F}_{RS}^{(R)}\right),$$

where $P,Q,R,S = 0,\ldots,3,z$ and $\mathcal{F}_{MN} = \partial_M \mathcal{B}_N - \partial_N \mathcal{B}_M - i[\mathcal{B}_M,\mathcal{B}_N]$

with conformal boundary at $z=0$, confinement through either sharp cut-off of AdS$_5$ at z_0 (HW) or with nontrivial dilaton $z_0=\infty$ (SW) (SS: not asymptotically AdS$_5$, finite z_0 corresponding to point where D8 branes join)

Chiral symmetry breaking either from

- extra bifundamental scalar field [Erlich-Katz-Son-Stephanov 2005] (HW1), or
- through different boundary conditions for vector/axial-vector fields at z_0 [Hirn-Sanz 2005] (HW2), [Sakai-Sugimoto 2004] (SS)

Vector meson dominance (VMD) naturally built in: photons couple through *bulk-to-boundary propagators of vector gauge fields* whose normalizable modes give (infinite tower of!) vector mesons (ρ, ω, ϕ, ...)

Anomalous TFFs from holographic QCD

Anomalies follow uniquely from 5-dimensional *Chern-Simons* term:

$$S_{\rm CS}^L - S_{\rm CS}^R, \quad S_{\rm CS} = \frac{N_c}{24\pi^2}\int \text{tr}\left(\mathcal{B}\mathcal{F}^2 - \frac{i}{2}\mathcal{B}^3\mathcal{F} - \frac{1}{10}\mathcal{B}^5\right)$$

- The pion transition form factor is given by

$$F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2) = -\frac{N_c}{12\pi^2 f_\pi}\int_0^{z_0} dz\, \mathcal{J}(Q_1, z)\mathcal{J}(Q_2, z)\Psi(z) + \text{b.t.}$$

  with *bulk-to-boundary* propagator $\mathcal{J}$ and holographic pion profile Ψ

- The amplitude for axial-vector mesons $a_\mu^{(n)}$ decaying into two virtual photons following from the Chern-Simons action has the form

$$\mathcal{M}^a = i\frac{N_c}{4\pi^2}\text{tr}(\mathcal{Q}^2 t^a)\,\epsilon_{(1)}^\mu \epsilon_{(2)}^\nu \epsilon_A^{*\rho}\epsilon_{\mu\nu\rho\sigma}\left[q_{(2)}^\sigma Q_1^2 A_n(Q_1^2, Q_2^2) - q_{(1)}^\sigma Q_2^2 A_n(Q_2^2, Q_1^2)\right]$$

  where

$$A_n(Q_1^2, Q_2^2) \quad = \frac{2g_5}{Q_1^2}\int_0^{z_0} dz\left[\frac{d}{dz}\mathcal{J}(Q_1, z)\right]\mathcal{J}(Q_2, z)\psi_n^A(z)$$

- Landau-Yang theorem (AV$\to \gamma\gamma$ is forbidden) realized by $\mathcal{J}'(Q, z) = 0$ for $Q^2 = 0$

Short-distance constraints on TFFs

Amazingly, bottom-up models with asymptotic AdS$_5$ geometry reproduce **asymptotic momentum dependence of pQCD** [Brodsky & Lepage 1979-81] for PS and AV

- **Pseudoscalars** [Grigoryan & Radyushkin, PRD76,77,78 (2007-8)]

$$F^{\rm HW1}_{\pi^0\gamma^*\gamma^*}(Q_1^2,Q_2^2) \quad\to\quad \frac{2f_\pi}{Q^2}\sqrt{1-w^2}\int_0^\infty d\xi\, \xi^3 K_1(\xi\sqrt{1+w})K_1(\xi\sqrt{1-w})$$
$$= \frac{2f_\pi}{Q^2}\left[\frac{1}{w^2}-\frac{1-w^2}{2w^3}\ln\frac{1+w}{1-w}\right]$$

with $Q^2=\frac{1}{2}(Q_1^2+Q_2^2)$, $w=(Q_1^2-Q_2^2)/(Q_1^2+Q_2^2)$,
corresponds to the asymptotic behavior

$$F^\infty(Q^2,0)=\frac{2f_\pi}{Q^2}, \qquad F^\infty(Q^2,Q^2)=\frac{2f_\pi}{3Q^2}$$

- **Axial-vector mesons** [JL & Rebhan, 1912.01596]
(agreeing with later pQCD result [Hoferichter & Stoffer 2004.06127]):

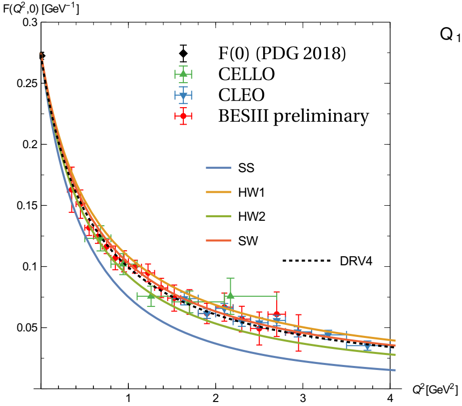
$$A_n(Q_1^2,Q_2^2)\to\frac{12\pi^2F_n^A}{N_cQ^4}\frac{1}{w^4}\left[w(3-2w)+\frac{1}{2}(w+3)(1-w)\ln\frac{1-w}{1+w}\right]$$

Holographic TFFs and experimental data

Single-virtual pion TFF:

[JL, J. Mager & A. Rebhan, 1906.11795]

(data from Danilkin et al., Prog.Part.Nucl.Phys. 107 (2019) 20)



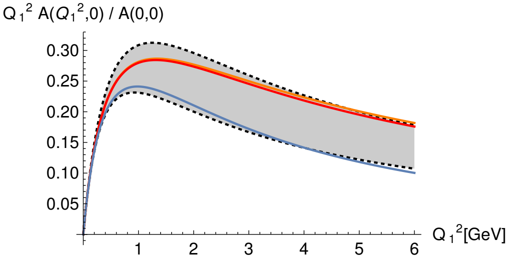
Danilkin et al. (DRV) fit below 4 GeV^2

bracketed by HW1 and HW2!

Single-virtual axial TFF:

[JL & A. Rebhan, 1912.01596]

dipole fit of L3 data for $f_1(1285)$ (gray band) vs. SS, HW1, and HW2 models:



$$A(0,0)^{\text{L3 exp.}}_{f_1(1285)} = 16.6(1.5)\,\text{GeV}^{-2}$$

Roig & Sanchez-Puertas, 1910.02881:

$$A(0,0)_{a_1(1230)} = 19.3(5.0)\,\text{GeV}^{-2}$$

hQCD results:	HW1	HW2
$\lvert A(0,0)\rvert$ [GeV^{-2}]	21.04	16.63

Hadronic light-by-light scattering

[Colangelo et al. 1506.01386]

Lorentz- and gauge invariance: interaction of four electromagnetic currents described by 12 scalar functions $\bar{\Pi}_i$

$$a_\mu^{\text{HLbL}} = \frac{2\alpha^3}{3\pi^2}\int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \sqrt{1-\tau^2} Q_1^3 Q_2^3 \sum_{i=1}^{12} T_i(Q_1, Q_2, \tau)\bar{\Pi}_i(Q_1, Q_2, \tau)$$

PS only contribute to $\bar{\Pi}_1$ (and $\bar{\Pi}_2$, $\bar{\Pi}_3$ through crossing symmetry)

$$\bar{\Pi}_1^{PS} = -\sum_{n=1}^{\infty} \frac{F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2) F_{\pi^0\gamma^*\gamma}(Q_3^2, 0)}{Q_3^2 + m_\pi^2}$$

AV contribute to all 12. $\bar{\Pi}_1$ has contribution from longitudinal component

$$\bar{\Pi}_1^{AV} = -\frac{g_5^2}{2\pi^4} \sum_{n=1}^{\infty} \int_0^{z_0} dz \left[\frac{d}{dz}\mathcal{J}(Q,z)\right] \mathcal{J}(Q,z)\psi_n^A(z) \frac{1}{(M_n^A Q_3)^2} \int_0^{z_0} dz' \left[\frac{d}{dz'}\mathcal{J}(Q_3,z')\right] \psi_n^A(z')$$

Melnikov-Vainshtein short-distance constraint

- Asymptotic (high-energy) regime not expected to give large contribution to HLbL but important for error estimate
- Short-distance constraint (SDC) derived from OPE by Melnikov & Vainshtein

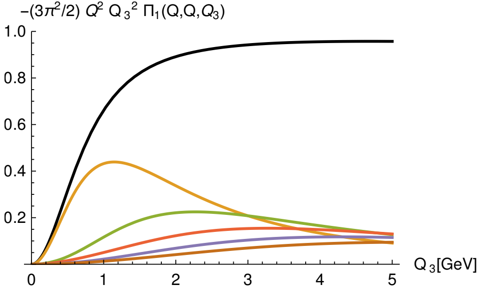
$$\lim_{Q_3\to\infty}\lim_{Q\to\infty} Q^2 Q_3^2 \bar{\Pi}_1(Q,Q,Q_3) = -\frac{2}{3\pi^2}$$

  can be used to see if a particular set of intermediate states is sufficient
- Only 3 known ways to satisfy with hadronic degrees of freedom:
  - Replacing the single-virtual TFF by hand (MV)
  - Summing an infinite number of excited PS mesons in a Regge model (Colangelo et al.)
  - Summing an infinite number of AV mesons in holographic models (LR, Cappiello et al.)

Axial-vector contributions to SDC

Infinite tower of axial-vector mesons responsible for satisfying the longitudinal SDC

- MV-SDC $\lim_{Q_3\to\infty}\lim_{Q\to\infty} Q^2 Q_3^2 \bar{\Pi}_1(Q,Q,Q_3) = -\frac{2}{3\pi^2}$: 100% for HW1 and HW2(UV-fit)



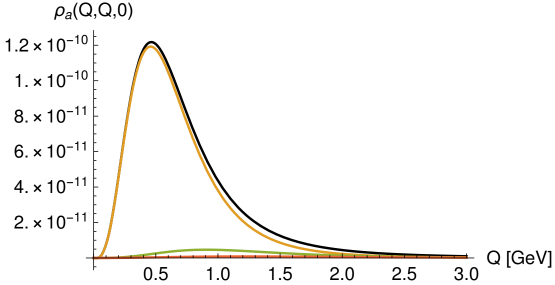
black line: infinite sum
colored lines: first 5 axial-vector modes

- SDC for symmetric limit $Q_1^2 = Q_2^2 = Q_3^2 \to \infty$ satisfied qualitatively, but quantitatively only at max. 80% level (for HW1 and HW2(UV-fit))

Contributions to muon $g-2$

$a_\mu^{\mathrm{AV}} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau\, \rho_a(Q_1, Q_2, \tau)$

E.g. at $\tau = 0$:



Strongly dominated by lowest axials, but nonnegligible (25%) contribution from higher modes

(z_0 s.t. $m_\rho = 775$ MeV, $f_\pi = 92.4$ MeV; degenerate a_1, f_1, f_1')

	HW1 (100% LSDC)	HW2 (62% LSDC)	SM
$a_\mu^{\mathrm{PS}}[\pi^0 + \eta + \eta'] \times 10^{11}$	99 [65+18+16]	84 [57+15+12]	93.8(4.0)
$a_\mu^{\mathrm{AV}}[L + T] \times 10^{11}$	41 [23+18]	29 [17+12]	21(16) [15(10)+6(6)]
$a_\mu^{\mathrm{PS+AV}} \times 10^{11}$	140	112	115(20)

(compare with MV model: longitudinal contribution estimated $\sim$ 38 $\times 10^{-11}$)

Additional holographic predictions

- Effect of finite quark masses (in HW1 model) and perturbative corrections (estimated by reducing g_5^2 by 15%) [JL & A. Rebhan, 2108.12345]
  MV-SDC is still satisfied through tower of axial-vector mesons; massive pions only have subleading contribution $\propto \log(Q_3^2)/Q_3^4 Q^2$
- Estimate of glueball contribution (in SS model) [JL, Dissertation]
  - Glueballs are dual to fluctuations of the background geometry
  - Brane-embedding determines glueball-meson interaction
  - Combined with the model's VMD this leads to surprisingly large radiative glueball decays (decay rates in keV instead of eV)
  - However glueball contribution is negligible

$$a_\mu^G \lesssim 0.16 \times 10^{-11}$$

- $U(1)_A$ anomaly and isospin-breaking effects slightly reduce AV contributions [JL, J. Mager, A. Rebhan, 2211.16562]

Conclusions

- hQCD is not QCD, but sophisticated toy model that can give clues on
  – how short-distance constraints can be implemented at the hadronic level
    - important fundamental role of axial-vector mesons $\leftrightarrow$ anomaly

  – semi-quantitative estimates of the ballparks to be expected
  (chiral HW1–HW2 models bracket experimental results for pion TFF!)
    - pion contribution from hQCD in perfect agreement with data-driven approach
    - with finite quark masses and WV mass:
      good agreement with η, η' WP results, but
      axial-vector contributions greater than estimated previously

$$a_\mu^{\rm AV}[L+T] = \mathbf{35(6)}\,[20(3)+15(3)]\times 10^{-11} \quad \text{for chiral HW1}\sim\text{HW2}$$

$$\to \mathbf{30.5}^{+\mathbf{3.5}}_{-?}\times 10^{-11} \quad \text{for HW1m+U(1)}_A\text{ (LMR)}$$

vs. WP: $a_\mu^{\rm SDC+axials} = \mathbf{21(16)}\,[15(10)+6(6)]\times 10^{-11}$

- total contributions from single PS (and PS*) and AV exchanges

$$a_\mu^{\rm PS+axials+SDC} = \to \mathbf{128}^{+\mathbf{10}}_{-?}\times 10^{-11} \quad \text{for HW1m+U(1)}_A\text{ (LMR)}$$

vs. WP: $\mathbf{115(16.5)}\times 10^{-11}$