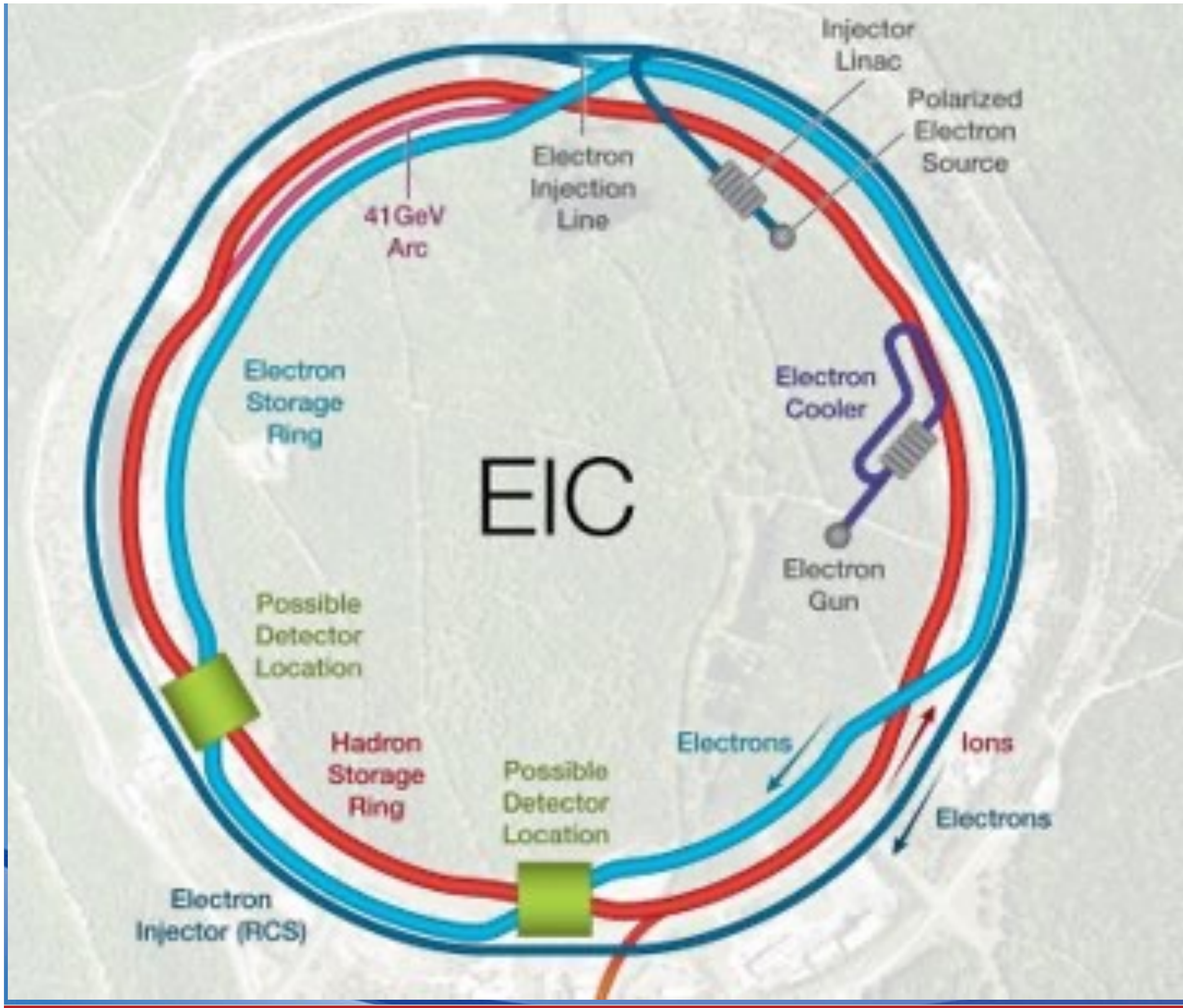


Polarized Electrons at the EIC, and lessons for the FCC

Georg Hoffstaetter de Torquat
for Cornell's ELR / EIC Group

Cornell Laboratory for Accelerator-based Sciences and Education (CLASSE)

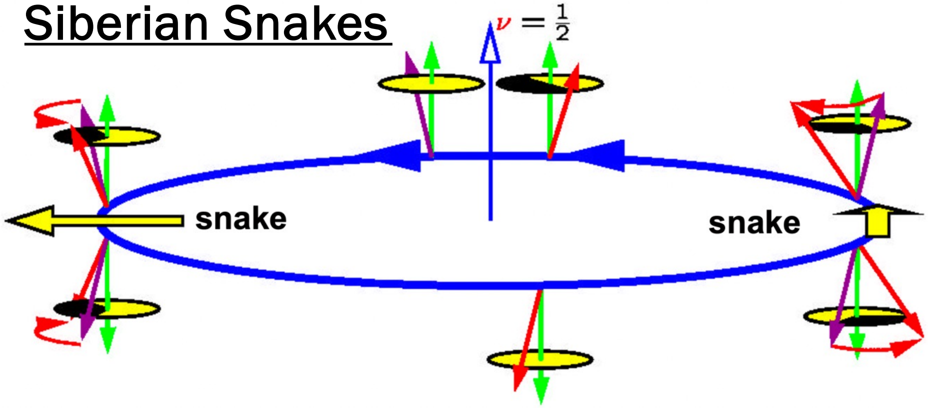
Brookhaven National Laboratory

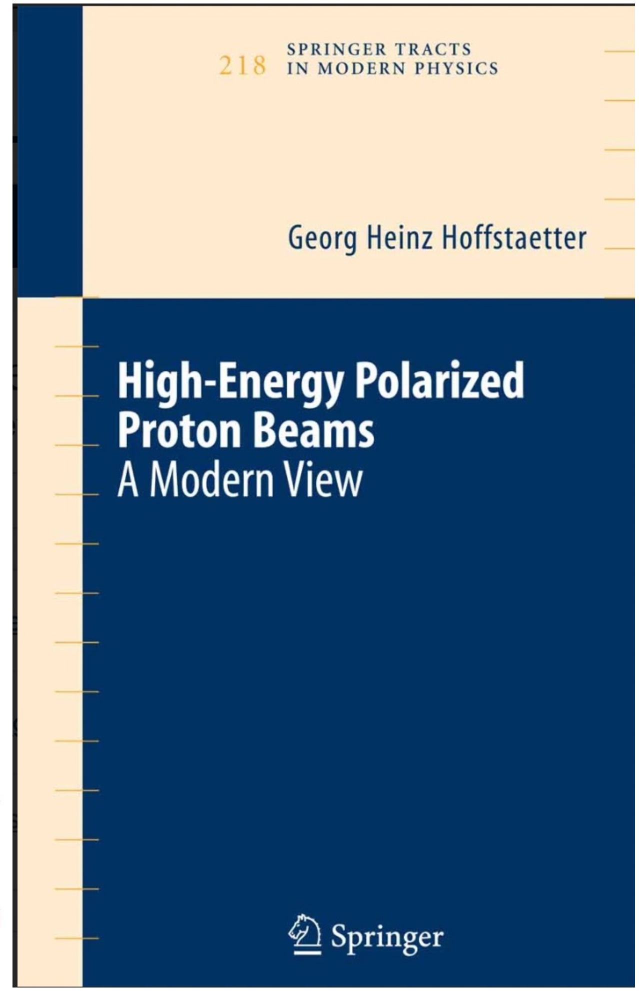
Accelerators with proton polarization

RHIC	255 GeV
AGS	25 GeV
ZGS	12 GeV
COSY	3.65 GeV
IUCS	3.6 GeV
VEPP-4	0.7 GeV
PSI Cyclotron	0.59 GeV

Polarization method:
Injection of polarized beam

Polarization preservation method:
Siberian Snakes





Cornell Laboratory for Accelerator-based Sciences and Education (CLASSE)

Modes of polarizing electrons rings

Brookhaven National Laboratory

Polarized injection

Bates (MIT)
AmPs (Nikhef)
ELSA (Bonn)
SLC
CEBAF
etc.

➔ EIC, SuperKEK-B

Time average decaying polarization

$$< P > \propto P_\infty + (P_0 - P_\infty) < e^{-t/\tau} >_t$$

Sokolov-Ternov Spin flip

HERA
LEP
VEPP
etc.

$$P_\infty \propto < \hat{b} \cdot \vec{n} >$$

Kinetic polarization

Bates (MIT)
AmPs (Nikhef)
etc.

$$P_\infty \propto < \hat{b} \cdot \frac{d\vec{n}}{d\delta} >$$

Equilibrium polarization buildup by bending fields $\hat{b}$.

Accelerators with electron polarization

VEPP	1970 vert.	80%	0.65 GeV
ACO	1970 vert.	90%	0.53 GeV
VEPP-2M	1974 vert.	90%	0.65 GeV
SPEAR	1975 vert.	90%	2 GeV
VEPP-3	1976 vert.	80%	3.7 GeV
VEPP-4	1982 vert.	80%	5 GeV
CESR	1983 vert.	30%	5 GeV
PETRA	1982 vert.	70%	16.5 GeV
DORIS	1983 vert.	80%	5 GeV
TRISTAN	1990 vert.	70% (?)	29 GeV
LEP	1993 vert.	57%	47 GeV
HERA	1993 vert.	60%	26.7 GeV
HERA	1994 long.	70%	27.5 GeV
LEP	1999 vert.	7%	60 GeV
VEPP-4M	1990 vert.	(?)	6 GeV
VEPP2000	2010 vert.	(?)	1 GeV

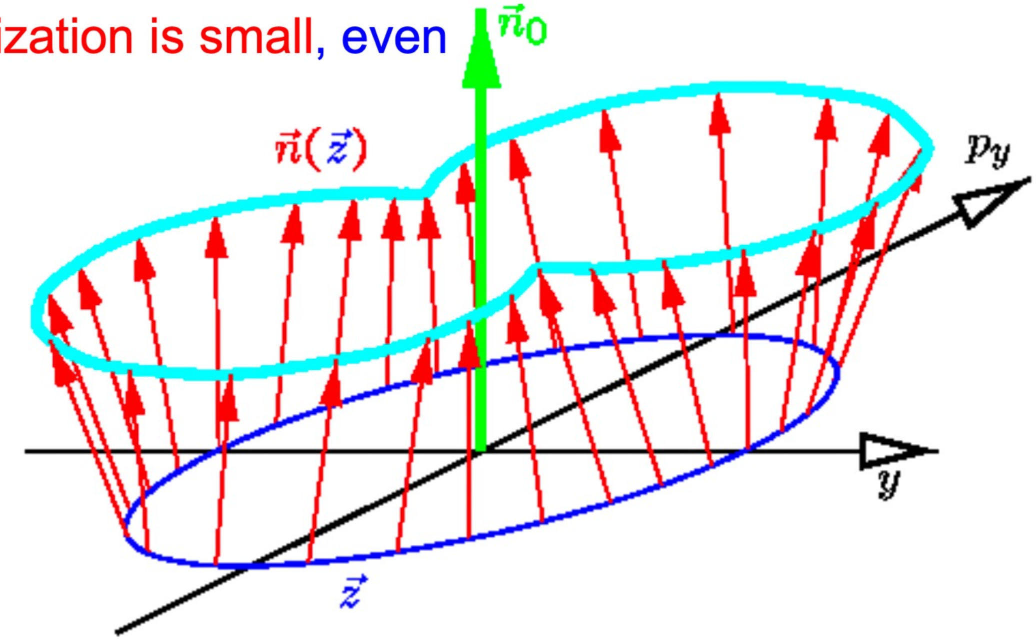
EIC	long.	<70%>	5 to 18GeV
SuperKEK-B	long.		
FCC-ee	vert.		

The invariant spin field (ISF)

A) Maximum polarization: $P_{lim} = < \vec{n}(\vec{z}) >_{\text{Phase space}}$

For a large divergence, the average polarization is small, even if the local polarization is 100%.

B) $\vec{n}(\vec{z}) \cdot \vec{S}$ is an adiabatic invariance !



The stable polarization of a beam must be parallel to the ISF at every phase space point.

Linearized $\vec{n}(\vec{z})$ can be analytically computed

The Derbenev Kondratenko equilibrium

92.4%

Unit vector of orbit rotation, i.e. B-field

$$P_{\mathrm{eq,DK}} = -\frac{8}{5\sqrt{3}} \frac{\langle \frac{1}{|\rho|^3}\hat{b}\cdot\left[\hat{n} - \frac{\partial \hat{n}}{\partial \delta}\right]\rangle}{\langle \frac{1}{|\rho|^3}\{1 - \frac{2}{9}(\hat{n}\cdot\hat{v})^2 + \frac{11}{18}\left|\frac{\partial \hat{n}}{\partial \delta}\right|^2\}\rangle}$$

Invariant spin field (ISF)

Energy dependence of the invariant spin field.

Average over phase space and around the ring

Where do these terms come from, what do they mean?

➔ *My presentation at the EPOL workshop for FCC & EIC, CERN, 09/22/2022*

Computational techniques

$$P_\infty \approx \frac{8}{5\sqrt{3}} \frac{\oint \frac{\hat{b}\cdot\hat{n}_0}{|\rho|^3} d\theta}{\oint \frac{1}{|\rho|^3}\{1-(\hat{v}\cdot\hat{n}_0)^2\} d\theta + < \frac{1}{|\rho|^3}\frac{11}{18}\left|\frac{\partial \hat{n}}{\partial \delta}\right|^2 >}$$

Compute on the closed orbit

Needs the invariant spin field ☹

- Linearize the spin-orbit equations of motion in phase space amplitudes.
  ➔ Codes: SLIM / SLICK / BMAD *(Presentation by Jacob Asimow – next Tuesday)*
- Perturbation theory nonlinear in small phase space amplitudes.
  ➔ SMILE program, did not converge in the past.
- Differential Algebra computation of $\vec{n}$ ➔ did not converge in the past, new research
- Stroboscopic averaging of $\vec{n}$ ➔ new research.
- Fourier analysis of tracking data to get $\vec{n}$ ➔ SODOM program.
- Nonlinear tracking to get depolarization time ➔ Bmad, SITROS, SITF, SLICKtrack

Polarization in the ESR of the EIC

Polarization studies have changed many important features of today's ESR

- 10 GeV Lattice Correction of 1IP/2IP **Operating Energies**
- ESR v5.3: Nonlinear Resonance Identified, **Tunes Changed**
- ESR v5.6: Partial Longitudinal Spin Match by **solenoid polarity change**

Observation: ESR v6.1: sometimes has better polarization with errors.

➔ **The Best Adjustment Groups for ELectron Spin (BAGELS) Method**

- Spin Match the ESR (+ **saving the 2-IP 18 GeV**!)
- 10 Error Seeds: **view Knobs** to **Correct the Spin Match**
- **Vertical Emittance Creation** *with minimal impact on polarization*
- **Global Coupling Compensation** *with minimal impact on polarization*

Methods

- **Bmad is used for everything**
  - All linear/nonlinear polarization calculations
  - Spin matching and optics rematching
  - 6D emittance calculations
  - Nonlinear Monte Carlo spin tracking with radiation
  - Nonlinear tune scans
  - Nonlinear calculations of τ_{dep}
  - Performing the BAGELS method
  - …

- **Map Tracking** – damped maps generated between each bend center generated by PTC

- **“Bmad” Tracking** – element-by-element damped nonlinear maps, radiation points after each element

- **PTC Tracking** – element-by-element symplectic integration, radiation points at each step

➔ **Come to the Bmad training workshop July 29 – August 2 at Brookhaven National Lab.**

10 GeV Correction of Operating Energies

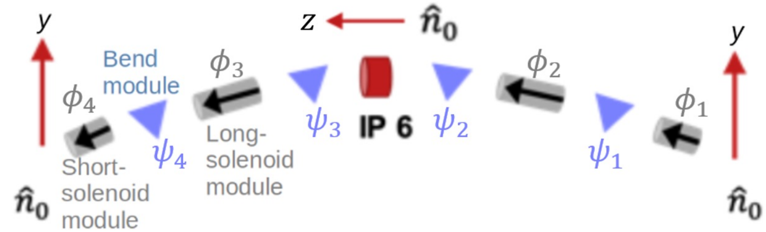
Cornell Laboratory for Accelerator-based Sciences and Education (CLASSE)

Brookhaven National Laboratory

- To avoid depolarizing spin resonances, we require the spin tune $\nu_s = \text{half integer}$

- In a perfectly flat ring, $\nu_s = a\gamma_0$, but not in general!

- Rotates $\hat{n}_0$ to longitudinal for ~5, ~10, and ~18 GeV
  - **Exact energy is chosen for $\boldsymbol{\nu_s} = \textbf{half integer}$**



- At 5 and 18 GeV, spin precession across rotator = 180°
  - **Precession through rotator unchanged if solenoids on/off**
  - **So $\boldsymbol{\nu_s = a\gamma_0}$ for 5 and 18 GeV, but NOT 10 GeV!**

	Solenoids		Bends	
	$\boldsymbol{\phi_{1,4}}$	$\boldsymbol{\phi_{2,3}}$	$\boldsymbol{\psi_{1,4}}$	$\boldsymbol{\psi_{2,3}}$
5 GeV 1IP/2IP	90°	0°	45°	45°
18 GeV 1IP/2IP	0°	90°	N/A	90°
10 GeV 1IP	53°	126°	124°	49°
10 GeV 2IP	54°	132°	130°	52°

- For 10 GeV, turning on a rotator **will change the spin tune**
  - **1IP/2IP 10 GeV lattice will have different operating energies!**
  - Exact operating energies must be numerically solved using eigenvalues of rotation matrices:

1IP: $\boldsymbol{R_{1IP}}(\gamma_0) = \boldsymbol{R_{arc}}(\gamma_0)\boldsymbol{R_{IR}}(\gamma_0) \quad \rightarrow \quad \mathbf{9.78\ GeV}$

2IP: $\boldsymbol{R_{2IP}}(\gamma_0) = \boldsymbol{R_{arc}}(\gamma_0)\boldsymbol{R_{IR}}(\gamma_0)\boldsymbol{R_{IR}}(\gamma_0) \quad \rightarrow \quad \mathbf{10.22\ GeV}$

- **Corrected the 10 GeV energies, found that different energies (and lattices!) necessary for 10GeV 1IP/2IP**

➔ **Not for the FCC: Is the spin tune really $\boldsymbol{a\gamma_0}$ when during depolarization scans?
Does it depend on the orbit or the particle's average amplitude, i.e., the emittance to the requested precision?**

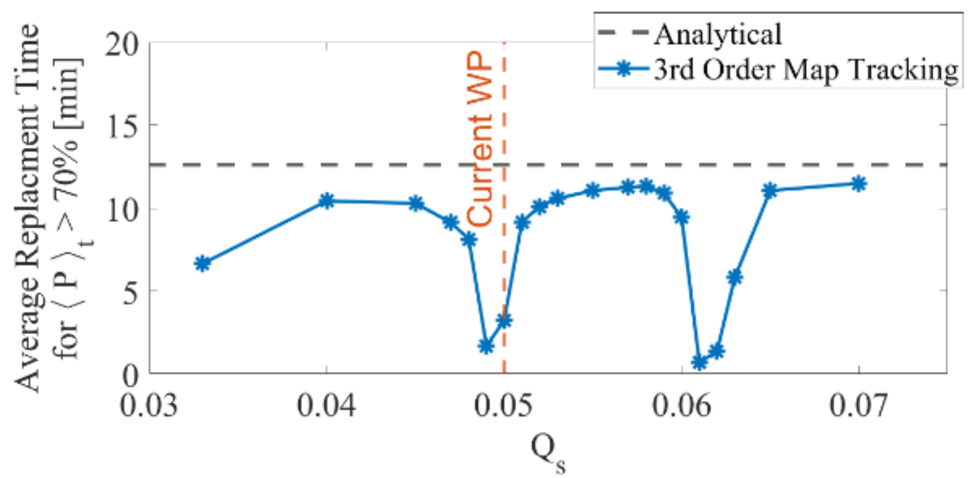
Nonlinear Resonance Identified, Tunes Changed

ESR v5.3 had better polarization without longitudinal spin match, why ?

1. Without the LSM, there is zero dispersion in the solenoid modules
2. Nonlinear tracking w/ custom element orders (in Bmad) proved blowup caused by 2nd order effects in solenoid module quads!
3. Tune scan reveals:

- **Tunes $(0.12, 0.10, 0.05)$ were directly on $\boldsymbol{Q_y - 2Q_s}$ resonance.**

- **This finding required us to change the bare lattice WP to $\boldsymbol{(Q_x, Q_y) = (0.08, 0.14)}$**

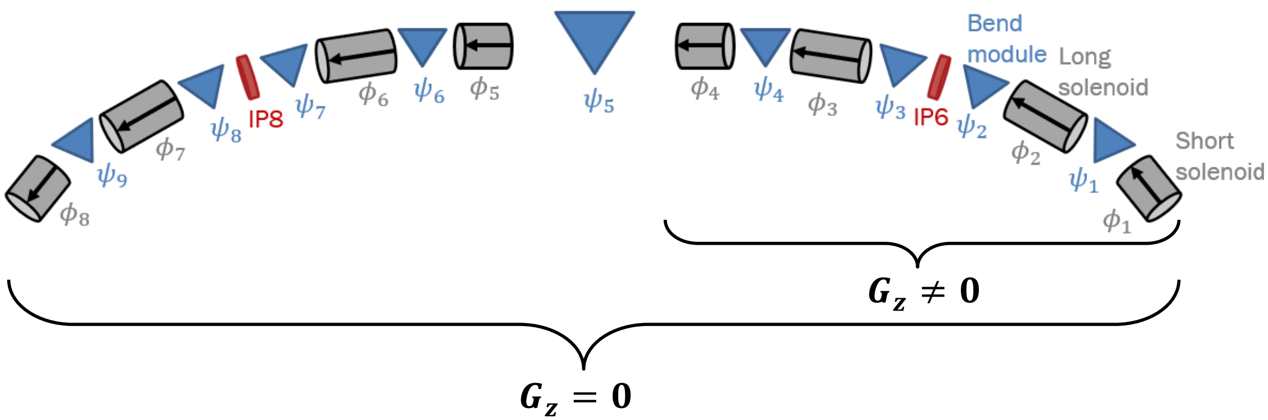


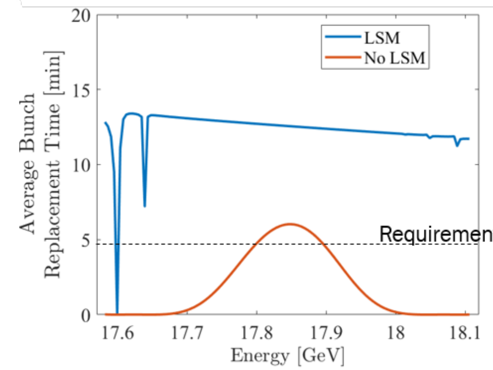
Partial Longitudinal Spin Match

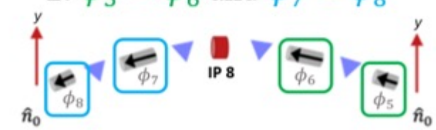
Cornell Laboratory for Accelerator-based Sciences and Education (CLASSE)

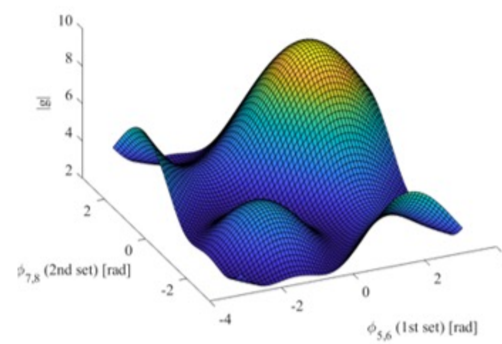
Brookhaven National Laboratory

- LSM requires 11 T solenoids and was therefore dropped.
- **ESR has 2IRs. The 2nd IR's solenoids to partially LSM !**









$|g_{min}| = 2.2561$

$\phi_{5,6} = -0.77, \phi_{7,8} = 0$ (2nd set off)

- **Calculated optimal IR-8 solenoid settings for partial LSM**
- **Implemented in Bmad: $T_{tot} = 6.0$ min → $T_{tot} = 6.4$ min**
- **Must find another way to improve the polarization…**

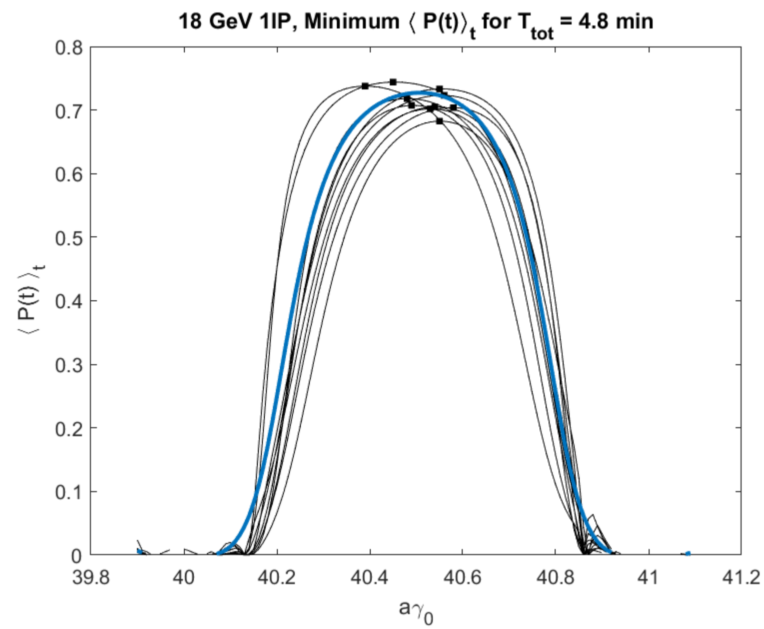
Cornell Laboratory for Accelerator-based Sciences and Education (CLASSE)

Better Polarization with Errors?

Brookhaven National Laboratory

- Added RMS errors to the lattice to check polarization robustness for 10 error seeds

RMS error	X (mm)	Y (mm)	Roll (mrad)	dB/B (%)
Dipoles	0.2	0.2	0.5	0.1
Quadrupoles	0.2	0.2	0.5	0.1
Sextupoles	0.2	0.2	0.5	0.2
High-β dipoles	0.2	0.2	0.5	0.05
FF quads	0.1	0.1	0.5	0.05



- Blue = ideal lattice
- **Black** = one of 10 error seeds

- Polarization **better than ideal** for some error seeds??

➔ This investigation lead the BAGELS method, the Best Adjustment Groups for Electron Spin

The BAGELS Method

- Radiative depolarization rate:

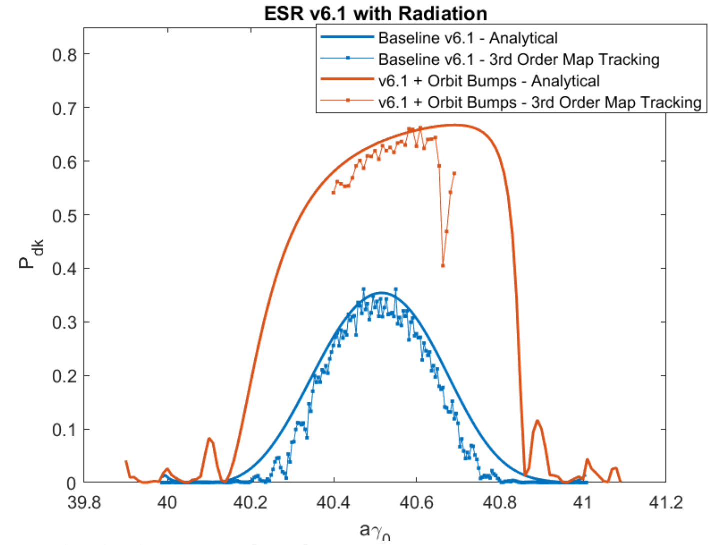
$$\tau_{dep}^{-1} \propto \oint ds \left\langle g^3 \frac{11}{18} \left| \frac{\partial \hat{n}}{\partial \delta} \right|^2 \right\rangle$$

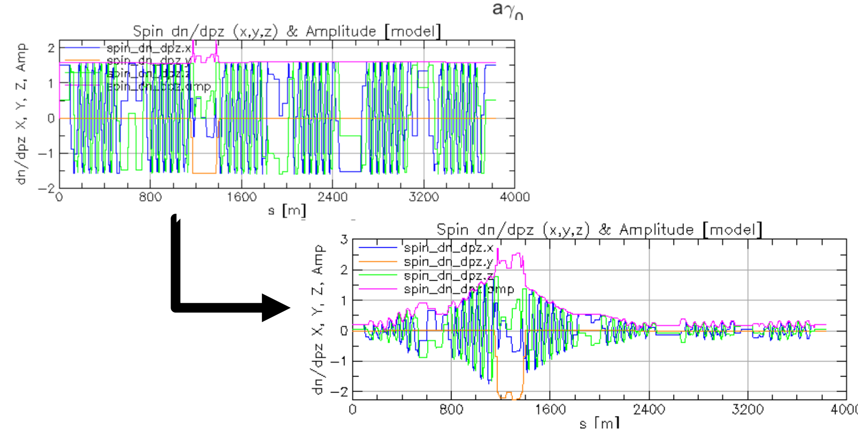
- **$\partial\hat{n}/\partial\delta$ is the bad thing!**
  - **Conventional methods:** G-matrices (spin matching), harmonics and tilts of $\hat{n}_0$, etc

- Found that vertical closed orbit bumps can individually have small impacts on $\partial\hat{n}/\partial\delta$ globally

- **Optimized all π-bumps in arcs 3,5,7,9 for minimum τ_{dep}**
  - Turned off sextupoles, then turned on + fixed coupling

- **Excellent polarization, with major problems:**
  1. Such an optimization is impossible in real life
  2. How to handle random closed orbit distortions?





The BAGELS Method

- Consider how $\partial\hat{n}/\partial\delta$ at n bends varies linearly with each m "unit" closed orbit bump θ_i in the ring

$$\begin{pmatrix}\partial\hat{n}/\partial\delta_1\\ \vdots \\ \partial\hat{n}/\partial\delta_n\end{pmatrix}_f = \begin{pmatrix}\partial\hat{n}/\partial\delta_1\\ \vdots \\ \partial\hat{n}/\partial\delta_n\end{pmatrix}_0 + \begin{pmatrix}\frac{\partial(\partial\hat{n}/\partial\delta)_1}{\partial\theta_1} & \cdots & \frac{\partial(\partial\hat{n}/\partial\delta)_1}{\partial\theta_m}\\ \vdots & \ddots & \vdots \\ \frac{\partial(\partial\hat{n}/\partial\delta)_n}{\partial\theta_1} & \cdots & \frac{\partial(\partial\hat{n}/\partial\delta)_n}{\partial\theta_m}\end{pmatrix}\begin{pmatrix}\theta_1\\ \vdots \\ \theta_m\end{pmatrix} + \ldots$$

n bends

m unit bumps

- We are looking for few orbit knobs that most effectively make $< |\partial\hat{n}/\partial\delta|^2 >$ minimal ➔ **SVD !**

- Instead for non-square matrices, we use singular value decomposition (SVD) to calculate the **Best Adjustment Groups for ELectron Spin!**

  - "Eigenvectors" with **largest** "eigenvalues" have **maximum** impact on $\partial\hat{n}/\partial\delta$

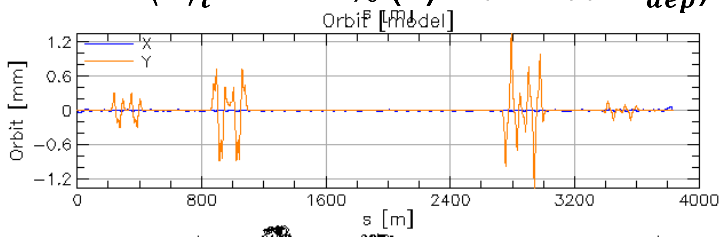
  - "Eigenvectors" with **smallest** "eigenvalues" have **minimum** impact on $\partial\hat{n}/\partial\delta$

- Eigenvectors are the knobs/groups – define the coil strengths per vertical orbit bump

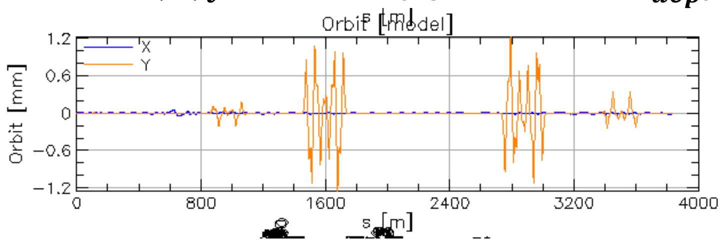
BAGELS: Spin Match the ESR

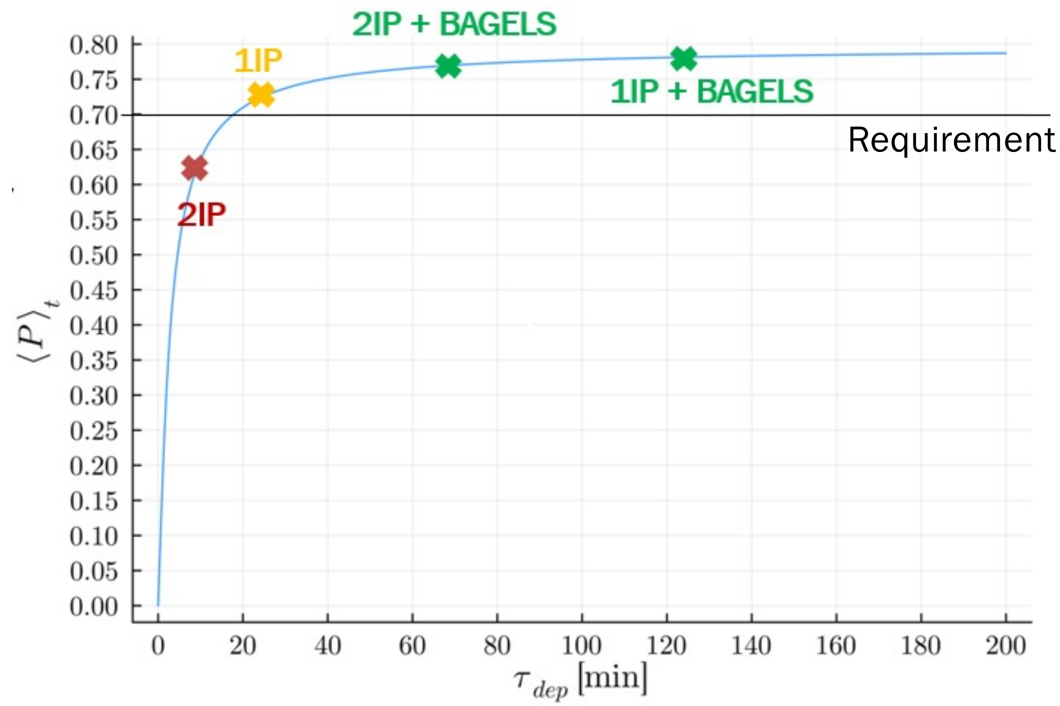
Using only **4 knobs** calculated w/ BAGELS**:**

1IP: $\langle P \rangle_t = 78.5\%$ **(w/ nonlinear** τ_{dep}**)**



2IP: $\langle P \rangle_t = 76.5\%$ **(w/ nonlinear** τ_{dep}**)**





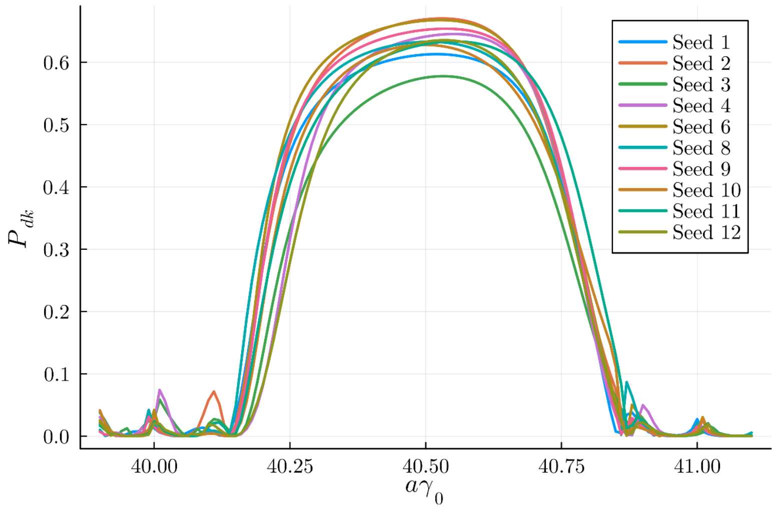
- **The BAGELS method allows polarized collisions for the 2-IP lattice**
- **Achieved highest polarizations observed in nonlinear tracking of ESR, with minimal orbits**
  - Exceeding the v5.3 (off resonance), which had a longitudinal spin match

BAGELS: 10 Error Seeds

Added RMS errors:

RMS error	X (mm)	Y (mm)	Roll (mrad)	dB/B (%)
Dipoles	0.2	0.2	0.5	0.1
Quadrupoles	0.2	0.2	0.5	0.1
Sextupoles	0.2	0.2	0.5	0.2
High-β dipoles	0.2	0.2	0.5	0.05
FF quads	0.1	0.1	0.5	0.05

After orbit correction, using the BAGELS knobs, the spin match can be fixed:

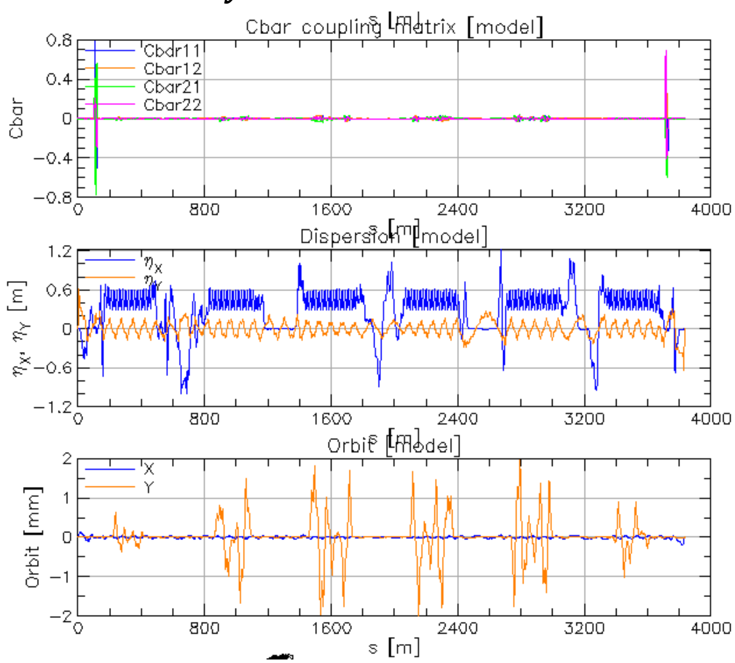


Lattice	$P_{dk,max}$ (Linear) Before	After
No Errors	35%	67.4%
Seed 1	33.0%	61.3%
Seed 2	41.8%	67.0%
Seed 3	31.2%	57.8%
Seed 4	34.6%	64.5%
Seed 6	29.7%	66.8%
Seed 8	38.3%	63.3%
Seed 9	41.8%	65.4%
Seed 10	37.7%	62.8%
Seed 11	26.6%	63.3%
Seed 12	35.7%	63.5%

- BAGELS gives a minimal number of knobs to restore polarization with realistic orbit distortions

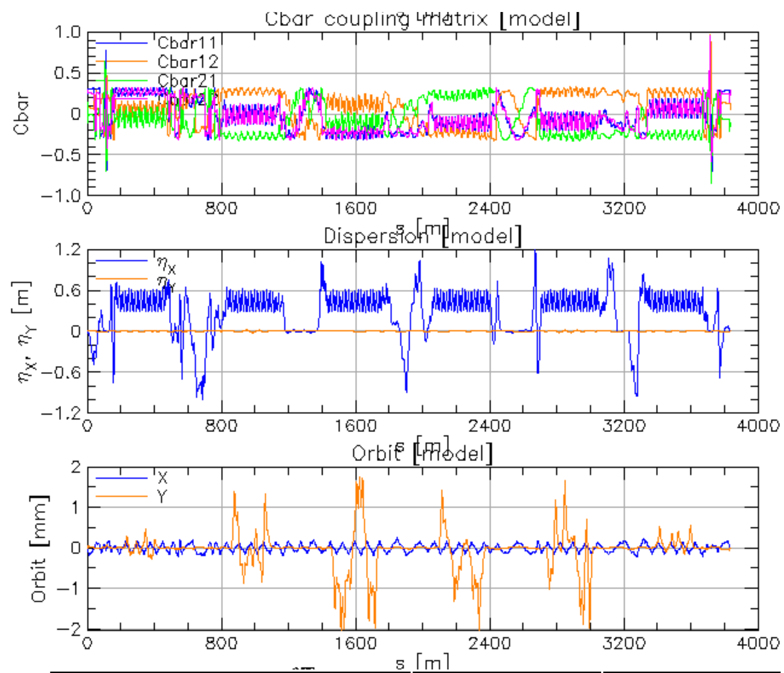
BAGELS: ϵ_y-creation

BAGELS η_y-knob: $\langle P \rangle_t = 76.0\%$



Calculation	ϵ_a [nm]	ϵ_b [nm]
Analytical	24.7	2.06
3rd Order Map Tracking	25.8	**2.96**

BAGELS Coupling knob: $\langle P \rangle_t = 75.9\%$

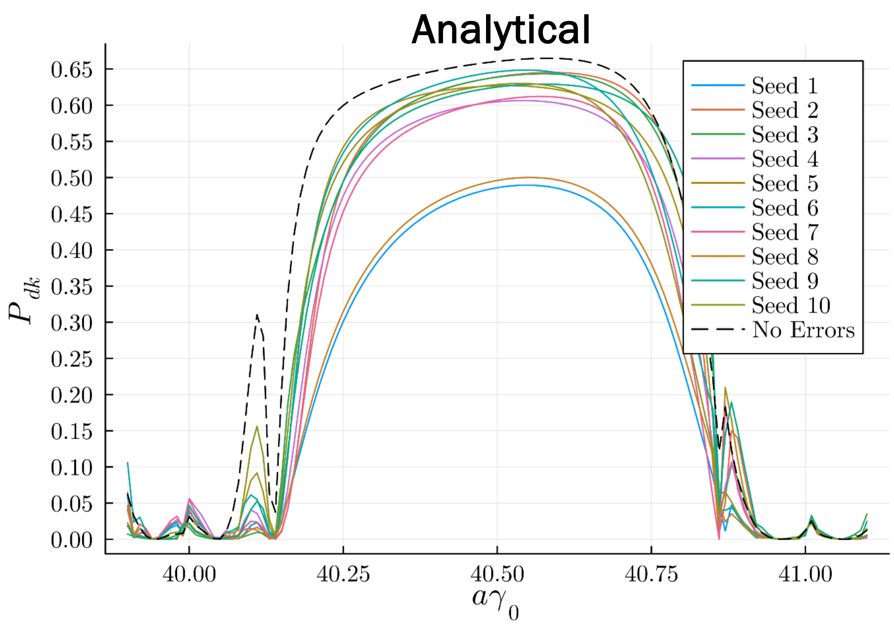


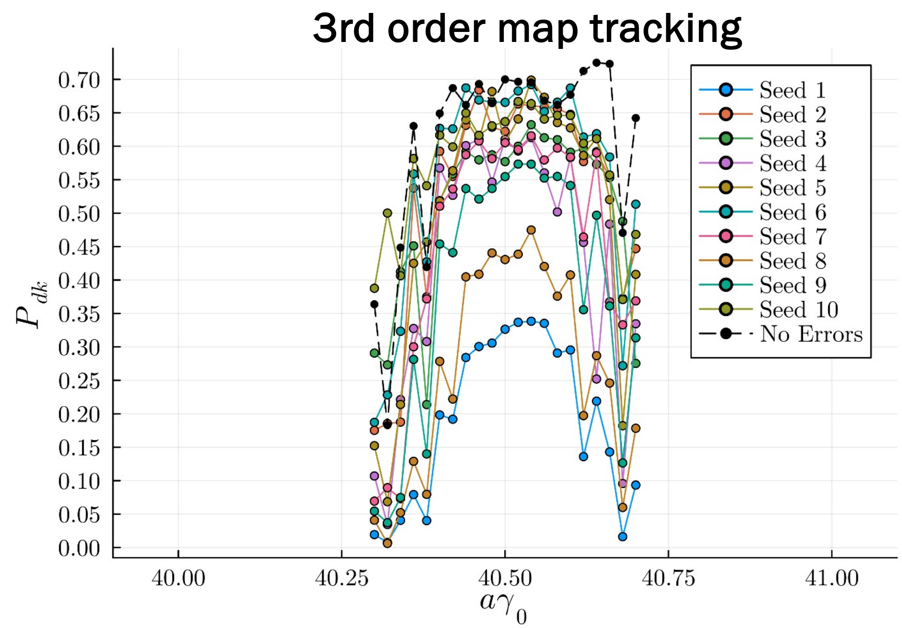
Calculation	ϵ_a [nm]	ϵ_b [nm]
Analytical	22.6	1.98
3rd Order Map Tracking	22.1	**4.60**

- Polarization requirements exceeded for both cases even with large ϵ_b

BAGELS: Global Decoupling

- Instead of SVD-ing $\partial\hat{n}/\partial\delta$, SVD $\frac{|\bar{C}|}{|\partial\hat{n}/\partial\delta|}$ where $\bar{C}$ is the coupling matrix
- **Using only 8 BAGELS global decoupling knobs:**

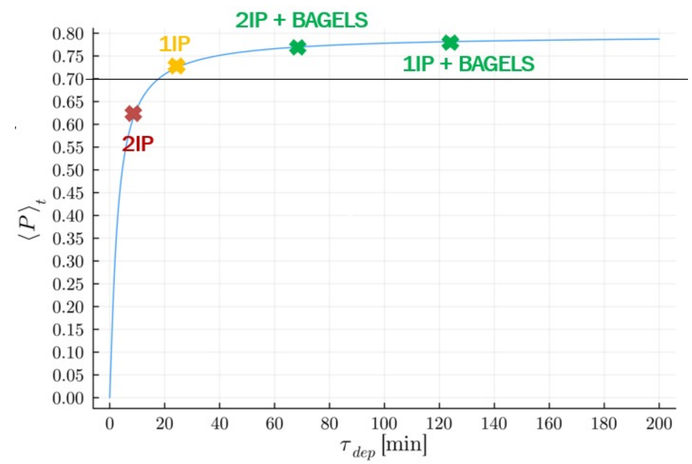




- **Coupling sufficiently under control for all error seeds**
- **Polarization requirements exceeded for all error seeds**
- **No need to buy skew quads for global coupling compensation!**

Conclusions

- ESR will provide polarized electrons at 5, 10, and 18 GeV, **both for 1-IP and 2-IP**
- **10 GeV energy corrected for half integer spin tune, 1IP and 2IP calculated**
- **Dangerous nonlinear resonance identified, tunes changed**
- Longitudinal spin match not feasible, but…
- **The Best Adjustment Groups for Electron Spin (BAGELS) method provides a novel, feasible, minimally-invasive way to minimize radiative depolarization in all scenarios**
  - 1-IP lattice far exceeds requirements (even > v5.3)
  - 2-IP lattice now exceeds requirements (even > v5.3)
  - Sufficient ϵ_b created for beam-size matching with polarization exceeding requirements
  - 8 global decoupling knobs calculated with minimal effect on polarization (no need to add skew quads!)



Georg.Hoffstaetter@Cornell.edu

Thank you!

Acknowledgements

- Matt Signorelli (Who did most of this work as his PhD project)
- David Sagan
- Desmond Barber
- Vadim Ptitsyn
- Yunhai Cai
- Jorg Kewisch
- Eliana Gianfelice-Wendt
- Daniel Marx
- Christoph Montag
- Yuri Nosochkov
- Steve Tepikian
- And the rest of the EIC-ESR team!