Motivation

LHC has taught us that at TeV energies nature is described by the Standard Model

\{B, W, G, L, e, Q, u, d, H\}

Disclaimer: ignore gravity throughout recent review [de Rham+ SNOWMASS 20222]
Motivation

What if we can only reach $\sqrt{\hat{s}} < \Lambda_{\text{NP}}$?
Motivation

What if we can only reach \( \sqrt{\hat{s}} < \Lambda_{NP} \)?

Apply Wilsonian EFT to the SM
Motivation

What if we can only reach $\sqrt{\hat{s}} < \Lambda_{NP}$?

Apply Wilsonian EFT to the SM

$$L = L_{SM} + L_{(5)} + L_{(6)} + L_{(7)} + L_{(8)} + \ldots$$
Motivation

What if we can only reach $\sqrt{\hat{s}} < \Lambda_{NP}$?

Apply Wilsonian EFT to the SM

$$\mathcal{L} = \mathcal{L}_\text{SM} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \mathcal{L}^{(8)} + \ldots$$

$$= c_e^{(6)}(\bar{e}\gamma e)^2 + \ldots$$

$$= c_e^{(8)}[\partial(\bar{e}\gamma e)]^2 + c_B^{(8)}B^4 + \ldots$$

Write all operators allowed by symmetry (Lorentz, gauge, …)

Assume $\Lambda_{EW} < \sqrt{\hat{s}}$
Motivation

What if we can only reach $\sqrt{\hat{s}} < \Lambda_{NP}$?

Apply Wilsonian EFT to the SM

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}(5) + \mathcal{L}(6) + \mathcal{L}(7) + \mathcal{L}(8) + \ldots$$

$$= c_e^{(6)}(\bar{e} \gamma e)^2 + \ldots$$

$$= c_e^{(8)}[\partial(\bar{e} \gamma e)]^2 + c_B^{(8)}B^4 + \ldots$$

Goal: $c_e^{(6)} \neq 0 \Rightarrow NP!$
Motivation

What if we can only reach $\sqrt{\hat{s}} < \Lambda_{NP}$?

Apply Wilsonian EFT to the SM

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}(5) + \mathcal{L}(6) + \mathcal{L}(7) + \mathcal{L}(8) + \ldots$$

$$= c_e^{(6)}(\bar{e}e\gamma)^2 + \ldots \quad = c_e^{(8)}[\partial(\bar{e}e\gamma)]^2 + c_B^{(8)}B^4 + \ldots$$

Challenge: huge unstructured space

$$[\mathcal{L}(6)] = 3,045 \quad \text{and} \quad [\mathcal{L}(8)] = 44,807$$

e.g. [Henning, Lu, Melia, Murayama 2015]
Motivation

Conventional Approach

Allowed parameter space
Motivation

Conventional Approach
Motivation

Conventional Approach

Allowed parameter space

Experimental Bounds

UV Completions

Conventional Approach
Motivation

Assume the UV obeys unitarity, locality, and causality
Motivation

Assume the UV obeys **unitarity**, **locality**, and **causality**

![Diagram showing forbidden parameter space with experimental bounds and UV completions]

- **Experimental Bounds**
- **Theory prior**
- **Forbidden parameter space**
- **UV Completions**
Motivation

Assume the UV obeys **unitarity, locality, and causality**

**Experimental Bounds**

**UV Completions**

**Forbidden parameter space**

Coefficients are correlated, \( c_1 > |c_2| \)

Connection between disparate experiments
Assume the UV obeys **unitarity, locality, and causality**
Outline

How can the UV constrain the IR?

Example: fermions and flavor violation
Outline

How can the UV constrain the IR?

Example: fermions and flavor violation
Consider a single massless scalar, invariant under
\[ \phi \rightarrow \phi + \text{constant} \quad \text{and} \quad \phi \rightarrow -\phi \]

\[ \mathcal{L} = -\frac{1}{2} (\partial \phi)^2 + c (\partial \phi)^4 \]
Unitarity, Locality, and Causality

Consider a single massless scalar, invariant under
\[ \phi \rightarrow \phi + \text{constant and } \phi \rightarrow -\phi \]

\[ \mathcal{L} = -\frac{1}{2} (\partial \phi)^2 + c (\partial \phi)^4 \]

Wilsonian Picture
\( c \) encodes short distance physics, only constraint is perturbative unitarity
Unitarity, Locality, and Causality

Consider a single massless scalar, invariant under
\[ \phi \rightarrow \phi + \text{constant} \quad \text{and} \quad \phi \rightarrow -\phi \]

\[ \mathcal{L} = -\frac{1}{2}(\partial \phi)^2 + c (\partial \phi)^4 \]

Incorrect!

Wilsonian Picture

If UV obeys unitarity, locality, and causality: \( c > 0 \)
Consider a single massless scalar, invariant under
\[ \phi \rightarrow \phi + \text{constant} \quad \text{and} \quad \phi \rightarrow -\phi \]

\[ \mathcal{L} = -\frac{1}{2} (\partial \phi)^2 + c (\partial \phi)^4 \]

Dispersion Relations

c induces \( \mathcal{A}(\phi \phi \rightarrow \phi \phi) \)

Unitarity connects amplitudes to cross-section: \( \sigma > 0 \Rightarrow c > 0 \)

[Pham, Truong 1985]
Unitarity, Locality, and Causality

2 → 2 scattering with $\mathcal{L} = -\frac{1}{2}(\partial \phi)^2 + c (\partial \phi)^4$

Conventions

$0 = p_1 + p_2 + p_3 + p_4$
$s = -(p_1 + p_2)^2$
$t = -(p_1 + p_3)^2$
$u = -s - t$
2 → 2 scattering with $\mathcal{L} = -\frac{1}{2}(\partial \phi)^2 + c (\partial \phi)^4$

**Conventions**

- $0 = p_1 + p_2 + p_3 + p_4$
- $s = -(p_1 + p_2)^2$
- $t = -(p_1 + p_3)^2$
- $u = -s - t$

**Cross 1 ↔ 3**

$s \leftrightarrow u = -s - t, t \leftrightarrow t$
Unitarity, Locality, and Causality

2 → 2 scattering with $\mathcal{L} = -\frac{1}{2} (\partial \phi)^2 + c (\partial \phi)^4$

Conventions

$0 = p_1 + p_2 + p_3 + p_4$
$s = -(p_1 + p_2)^2$
$t = -(p_1 + p_3)^2$
$u = -s - t$

$\mathcal{A}(s, t) = 2c \left( s^2 + t^2 + u^2 \right)$

Cross $1 \leftrightarrow 3$
$s \leftrightarrow u = -s - t, t \leftrightarrow t$
$\mathcal{A}(s, t) \leftrightarrow \mathcal{A}(-s - t, t) = \mathcal{A}(s, t)$
Unitarity, Locality, and Causality

$2 \to 2$ scattering with $\mathcal{L} = -\frac{1}{2}(\partial \phi)^2 + c(\partial \phi)^4$ in the forward limit

Conventions

$0 = p_1 + p_2 + p_3 + p_4$
$s = -(p_1 + p_2)^2 \rightarrow s$
$t = -(p_1 + p_3)^2 \rightarrow 0$
$u = -s - t \rightarrow -s$

$\mathcal{A}(s, 0) = 4c s^2$

Cross 1 $\leftrightarrow$ 3
$s \leftrightarrow u = -s - t, t \leftrightarrow t$
$\mathcal{A}(s, 0) \leftrightarrow \mathcal{A}(-s, 0) = \mathcal{A}(s, 0)$
Unitarity, Locality, and Causality

Study UV forward amplitude $\mathcal{A}(s)$ in the complex plane
Unitarity, Locality, and Causality

Study UV forward amplitude $A(s)$ in the complex plane

If UV is causal $A(s)$ is analytic except where $\text{Im } s = 0$

[Bogoliubov, Shirkov, Chomet 1959], [Bremermann, Oehme, Taylor 1958], [Lehmann 1958], [Hepp 1964], [Martin 1965], …
Unitarity, Locality, and Causality

Study UV forward amplitude $\mathcal{A}(s)$ in the complex plane

$$\mathcal{A}(s)$$

$$\oint_{\mathcal{C}} ds \frac{\mathcal{A}(s)}{s^3} = 4c$$

Isolate $c$ via residue theorem

$$4c = \frac{1}{2\pi i} \oint_{\mathcal{C}} \frac{ds}{s^3} \mathcal{A}(s) = \frac{4c}{2\pi i} \oint_{\mathcal{C}} \frac{ds}{s}$$

$|s| < \Lambda_{\text{NP}}^2$

work in the EFT

Can open the branch cut with an IR regulator

Work at $\mu = \Lambda_{\text{NP}}$, but can include running
Unitarity, Locality, and Causality

Study UV forward amplitude $\mathcal{A}(s)$ in the complex plane

\begin{align*}
\mathcal{C}' &= \mathcal{C} \\
\mathcal{A}(s) &= \frac{1}{2\pi i} \int_{\mathcal{C}} \frac{ds}{s^3} \mathcal{A}(s) \\
\mathcal{A}'(s) &= \frac{1}{2\pi i} \int_{\mathcal{C}'} \frac{ds}{s^3} \mathcal{A}(s)
\end{align*}

Exploit analyticity (i.e. causality)
Unitarity, Locality, and Causality

Study UV forward amplitude $\mathcal{A}(s)$ in the complex plane

Remove boundary term via Froissart bound*

$$4c = \frac{1}{2 \pi i} \oint_{\mathcal{C}} \frac{ds}{s^3} \mathcal{A}(s)$$

$$= \frac{1}{2 \pi i} \oint_{\mathcal{C}'} \frac{ds}{s^3} \mathcal{A}(s)$$

$$= \frac{1}{2 \pi i} \left( \int_{-\infty}^{-s_d} + \int_{s_d}^{\infty} \right) \frac{ds}{s^3} \text{Disc } \mathcal{A}(s)$$

$$= \lim_{\epsilon \to 0} \left[ \mathcal{A}(s + i \epsilon) - \mathcal{A}(s - i \epsilon) \right]$$

*[Froissart 1961] Unitarity + locality $\Rightarrow |\mathcal{A}(s)| < s \ln^2 s$

Requires a mass, but can also argue via causality [Camanho, Edelstein, Maldacena, Zhiboedov 2016]
Unitarity, Locality, and Causality

Study UV forward amplitude $\mathcal{A}(s)$ in the complex plane

Invoke crossing symmetry*

$$
4c = \frac{1}{2\pi i} \oint_{C} \frac{ds}{s^3} \mathcal{A}(s)
$$

$$
= \frac{1}{2\pi i} \oint_{C'} \frac{ds}{s^3} \mathcal{A}(s)
$$

$$
= \frac{1}{2\pi i} \left( \int_{-\infty}^{-s_d} + \int_{s_d}^{\infty} \right) \frac{ds}{s^3} \text{Disc} \mathcal{A}(s)
$$

$$
= \frac{1}{i\pi} \int_{s_d}^{\infty} \frac{ds}{s^3} \text{Disc} \mathcal{A}(s)
$$

* $\mathcal{A}(-s) = \overline{\mathcal{A}}(s) = \mathcal{A}(s)$

In general $\overline{\mathcal{A}}(s) \neq \mathcal{A}(s)$, result persists
Unitarity, Locality, and Causality

Study UV forward amplitude $\mathcal{A}(s)$ in the complex plane

\[ \mathcal{A}(s) \]

\[ \mathcal{C}' \]

\[ \mathcal{C} \]

Relate Disc and Im

\[ 4c = \frac{1}{2\pi i} \oint_{\mathcal{C}} \frac{ds}{s^3} \mathcal{A}(s) \]

\[ = \frac{1}{2\pi i} \oint_{\mathcal{C}'} \frac{ds}{s^3} \mathcal{A}(s) \]

\[ = \frac{1}{2\pi i} \left( \int_{-\infty}^{-s_d} + \int_{s_d}^{\infty} \right) \frac{ds}{s^3} \text{Disc} \mathcal{A}(s) \]

\[ = \frac{1}{i\pi} \int_{s_d}^{\infty} \frac{ds}{s^3} \text{Disc} \mathcal{A}(s) \]

\[ = \frac{2}{\pi} \int_{s_d}^{\infty} \frac{ds}{s^3} \text{Im} \mathcal{A}(s) \]
Unitarity, Locality, and Causality

Study UV forward amplitude $\mathcal{A}(s)$ in the complex plane

Exploit unitarity via the optical theorem

$$4c = \frac{1}{2\pi i} \oint_{\mathcal{C}} ds \frac{ds}{s^3} \mathcal{A}(s)$$

$$= \frac{1}{2\pi i} \oint_{\mathcal{C}'} ds \frac{ds}{s^3} \mathcal{A}(s)$$

$$= \frac{1}{2\pi i} \left( \int_{-\infty}^{-s_d} + \int_{s_d}^{\infty} \right) ds \frac{ds}{s^3} \text{Disc} \mathcal{A}(s)$$

$$= \frac{1}{i\pi} \int_{s_d}^{\infty} ds \frac{ds}{s^3} \text{Disc} \mathcal{A}(s)$$

$$= \frac{2}{\pi} \int_{s_d}^{\infty} ds \frac{ds}{s^3} \text{Im} \mathcal{A}(s)$$

$$= \frac{2}{\pi} \int_{s_d}^{\infty} ds \frac{ds}{s^2} \sigma(s)$$

Integral converges as $\sigma(s) < \ln^2 s$ [Froissart 1961]
Unitarity, Locality, and Causality

Study UV forward amplitude $A(s)$ in the complex plane

Cross section is positive definite

$$4c = \frac{1}{2\pi i} \oint_{C} \frac{ds}{s^3} A(s)$$

$$= \frac{1}{2\pi i} \oint_{C'} \frac{ds}{s^3} A(s)$$

$$= \frac{1}{2\pi i} \left( \int_{-\infty}^{-s_d} + \int_{s_d}^{\infty} \right) \frac{ds}{s^3} \text{Disc } A(s)$$

$$= \frac{1}{i\pi} \int_{s_d}^{\infty} \frac{ds}{s^3} \text{Disc } A(s)$$

$$= \frac{2}{\pi} \int_{s_d}^{\infty} \frac{ds}{s^3} \text{Im } A(s)$$

$$= \frac{2}{\pi} \int_{s_d}^{\infty} \frac{ds}{s^2} \sigma(s) > 0$$
Unitarity, Locality, and Causality

Study UV forward amplitude $\mathcal{A}(s)$ in the complex plane

$$\mathcal{L} = -\frac{1}{2} (\partial \phi)^2 + c (\partial \phi)^4$$

$c > 0$

Cross section is positive definite

$$4c = \frac{1}{2\pi i} \oint_{\mathcal{C}} \frac{ds}{s^3} \mathcal{A}(s) = \frac{1}{2\pi i} \oint_{\mathcal{C}'} \frac{ds}{s^3} \mathcal{A}(s) = \frac{1}{2\pi i} \left( \int_{-\infty}^{-s_d} + \int_{s_d}^{\infty} \right) \frac{ds}{s^3} \text{Disc} \mathcal{A}(s) = \frac{1}{i\pi} \int_{s_d}^{\infty} \frac{ds}{s^3} \text{Disc} \mathcal{A}(s) = \frac{2}{\pi} \int_{s_d}^{\infty} \frac{ds}{s^3} \text{Im} \mathcal{A}(s) = \frac{2}{\pi} \int_{s_d}^{\infty} \frac{ds}{s^2} \sigma(s) > 0$$

Formalism extends to fermions [Bellazzini 2016]
Looking to the SMEFT

What were the essential ingredients?

Cross section is positive definite

\[
4c = \frac{1}{2\pi i} \oint_{\mathcal{C}} \frac{ds}{s^3} \mathcal{A}(s)
\]

\[
= \frac{1}{2\pi i} \oint_{\mathcal{C}'} \frac{ds}{s^3} \mathcal{A}(s)
\]

\[
= \frac{1}{2\pi i} \left( \int_{-\infty}^{-s_d} + \int_{s_d}^{\infty} \right) \frac{ds}{s^3} \text{Disc } \mathcal{A}(s)
\]

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= \frac{1}{i\pi} \int_{s_d}^{\infty} \frac{ds}{s^3} \text{Disc } \mathcal{A}(s)
\]

\[
= \frac{2}{\pi} \int_{s_d}^{\infty} \frac{ds}{s^3} \text{Im } \mathcal{A}(s)
\]

\[
= \frac{2}{\pi} \int_{s_d}^{\infty} \frac{ds}{s^2} \sigma(s) > 0
\]
Looking to the SMEFT

What were the essential ingredients?

Four fields for $2 \rightarrow 2$ scattering
\{B, W, G, L, e, Q, u, d, H\}

Cross section is positive definite

\[
4c = \frac{1}{2\pi i} \oint_{\mathcal{C}} \frac{ds}{s^3} \mathcal{A}(s) \\
= \frac{1}{2\pi i} \oint_{\mathcal{C}'} \frac{ds}{s^3} \mathcal{A}(s) \\
= \frac{1}{2\pi i} \left( \int_{-\infty}^{-s_d} + \int_{s_d}^{\infty} \right) \frac{ds}{s^3} \text{Disc} \, \mathcal{A}(s) \\
= \frac{1}{i\pi} \int_{s_d}^{\infty} \frac{ds}{s^3} \text{Disc} \, \mathcal{A}(s) \\
= \frac{2}{\pi} \int_{s_d}^{\infty} \frac{ds}{s^3} \text{Im} \, \mathcal{A}(s) \\
= \frac{2}{\pi} \int_{s_d}^{\infty} \frac{ds}{s^2} \sigma(s) > 0
\]
Looking to the SMEFT

What were the essential ingredients?

Four fields for $2 \rightarrow 2$ scattering
\{\(B, W, G, L, e, Q, u, d, H\)\}

\[\mathcal{A}(s) \propto s^2 \sim p^4\]

Require four of \\{\(\partial_\mu, (\bar{\psi}\psi)\)\}

Cross section is positive definite

\[
4c = \frac{1}{2\pi i} \oint_{\mathscr{C}} \frac{ds}{s^3} \mathcal{A}(s)
\]
\[
= \frac{1}{2\pi i} \oint_{\mathscr{C}'} \frac{ds}{s^3} \mathcal{A}(s)
\]
\[
= \frac{1}{2\pi i} \left( \int_{-\infty}^{-s_d} + \int_{s_d}^{\infty} \right) \frac{ds}{s^3} \text{Disc } \mathcal{A}(s)
\]
\[
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\]
\[
= \frac{2}{\pi} \int_{s_d}^{\infty} \frac{ds}{s^3} \text{Im } \mathcal{A}(s)
\]
\[
= \frac{2}{\pi} \int_{s_d}^{\infty} \frac{ds}{s^2} \sigma(s) > 0
\]

Boundary circle does not vanish in general

\(s \rightarrow -s\) can introduce a sign
Looking to the SMEFT

What were the essential ingredients?

Four fields for $2 \rightarrow 2$ scattering

$\{B, W, G, L, e, Q, u, d, H\}$

$\mathcal{A}(s) \propto s^2 \sim p^4$

Require four of $\{\partial_\mu, (\bar{\psi}\psi)\}$

Cross section is positive definite

$$4c = \frac{1}{2\pi i} \oint_{\mathcal{C}} \frac{ds}{s^3} \mathcal{A}(s)$$

$$= \frac{1}{2\pi i} \oint_{\mathcal{C}'} \frac{ds}{s^3} \mathcal{A}(s)$$

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$$= \frac{1}{i\pi} \int_{s_d}^{\infty} \frac{ds}{s^3} \text{Disc} \mathcal{A}(s)$$

$$= \frac{2}{\pi} \int_{s_d}^{\infty} \frac{ds}{s^3} \text{Im} \mathcal{A}(s)$$

$$= \frac{2}{\pi} \int_{s_d}^{\infty} \frac{ds}{s^2} \sigma(s) > 0$$

Boundary circle does not vanish in general

$s \rightarrow -s$ can introduce a sign

Requires dimension eight

e.g. $B^4, [\partial(\bar{\psi}\gamma\psi)]^2$
Outline

How can the UV constrain the IR?

Example: fermions and flavor violation
Fermions and Flavor

Consider four fermion scattering mediated by

\[ \mathcal{O}_e = - c_{mnpq} \partial_\mu (\bar{e}_m \gamma^\nu e_n) \partial_\mu (\bar{e}_p \gamma^\nu e_q) \]

\[ m, n, p, q \in \{1, \ldots, N_f\} \]

\[ e = e_R \sim (1,1,-1) \]

Focus on the simplest example - physics lifts to remaining operators

[Remmen, NLR 2020]
Fermions and Flavor

Consider four fermion scattering mediated by

\[ \mathcal{O}_e = - c_{mnpq} \partial_\mu (\bar{e}_m \gamma_\nu e_n) \partial^\mu (\bar{e}_p \gamma^\nu e_q) \]

Scatter flavor superpositions:  

\[ |1\rangle = \alpha_m |\bar{e}_m\rangle \quad |2\rangle = \beta_m |e_m\rangle \]

\[ \mathcal{A}(s) = 4 c_{mnpq} \alpha_m \beta_n \beta_p^* \alpha_q^* s^2 \]

E.g.  

\[ |2\rangle = \frac{1}{\sqrt{2}} (|e\rangle + |\mu\rangle) \]
Consider four fermion scattering mediated by

\[ \mathcal{O}_e = - c_{mnpq} \partial_\mu (\bar{e}_m \gamma_\nu e_n) \partial^\mu (\bar{e}_p \gamma_\nu e_q) \]

Scatter flavor superpositions: \( |1\rangle = \alpha_m |\bar{e}_m\rangle \quad |2\rangle = \beta_m |e_m\rangle \)

\[ \mathcal{A}(s) = 4 c_{mnpq} \alpha_m \beta_n \beta^*_p \alpha^*_q s^2 \]

e.g. \( |2\rangle = \frac{1}{\sqrt{2}} (|e\rangle + |\mu\rangle) \)

\[ \Rightarrow c_{mnpq} \alpha_m \beta_n \beta^*_p \alpha^*_q > 0 \]
Fermions and Flavor

To simplify, take $\alpha_m = \delta_{1m} \Rightarrow \beta_n c_{1np1} \beta_p^* > 0 \Rightarrow c_{1np1} > 0$
Fermions and Flavor

To simplify, take $\alpha_m = \delta_{1m} \Rightarrow \beta_n c_{1np1}\beta_p^* > 0 \Rightarrow c_{1np1} > 0$

In a two flavor theory, implies three conditions

- $c_{1111} > 0$
- $c_{1221} > 0$
- $c_{1111}c_{1221} > |c_{1121}|^2$

Above simplification identified in [Banerjee, Renner, NLR in progress]
Fermions and Flavor

To simplify, take \( \alpha_m = \delta_{1m} \Rightarrow \beta_n c_{1np1}\beta_p^* > 0 \Rightarrow c_{1np1} > 0 \)

In a two flavor theory, implies three conditions

- \( c_{1111} > 0 \) \[ e\bar{e} \rightarrow e\bar{e} \]
- \( c_{1221} > 0 \) \[ \mu\bar{e} \rightarrow \mu\bar{e} \]
- \( c_{1111}c_{1221} > |c_{1121}|^2 \) \[ e\bar{e} \rightarrow \mu\bar{e} \]

Flavor violation < Flavor conservation

Above simplification identified in [Banerjee, Renner, NLR in progress]
Fermions and Flavor

Violation < Conservation ⇒ $\tilde{\Lambda} > \Lambda$
Fermions and Flavor

Violation < Conservation $\Rightarrow \tilde{\Lambda} > \Lambda$

Mu3e will probe $\text{Br}(\mu \rightarrow 3e) \sim 10^{-12} - 10^{-16}$, discovery requires

$\tilde{\Lambda} \sim 100 - 300$ GeV
Fermions and Flavor

Violation < Conservation $\Rightarrow \tilde{\Lambda} > \Lambda$

Mu3e will probe $\text{Br}(\mu \rightarrow 3e) \sim 10^{-12} - 10^{-16}$, discovery requires

$$\tilde{\Lambda} \sim 100 - 300 \text{ GeV}$$

LEP fermion pair production requires (use interference with SM)

$$\Lambda \gtrsim 500 \text{ GeV} > \tilde{\Lambda}$$

So Mu3e should not see new dimension-eight physics
Many open questions, for instance

Best starting point for the LHC? aQGCs?

What can we say at dimension-6?

How to align progress in theory & experiment?
Conclusion

UV leaves a rich structure in the EFT, which has not yet been fully determined.
Backup Slides
Completion of \((\partial \phi)^4\)

Can obtain this action from a linear sigma model

\[
V(|\Phi|) = \lambda (|\Phi|^2 - v^2)^2, \quad \Phi = (v + h)e^{i\phi/v}
\]

The action for \(\phi\) and \(h\) at tree level is then

\[
\mathcal{L} = \left(1 + \frac{h}{v}\right)^2 (\partial \phi)^2 + (\partial h)^2 + m_h^2 h^2 + \ldots
\]

Integrate out \(h\) to find

\[
\mathcal{L} = \frac{\lambda}{m_h^4} (\partial \phi)^4 + \ldots
\]
Unstable Resonances

(a) first Riemann sheet
(b) second Riemann sheet

(c) transition from first to second Riemann sheet

Pole on the unphysical sheet
Fermionic UV Completion

Completion of $\mathcal{O}_e$ with KK graviton minimally coupled to $e_R$

$$\mathcal{L} \supset \mathcal{L}_{FP} + \kappa \phi^{\mu\nu} T_{\mu\nu}$$

$$T_{\mu\nu} = -i[\bar{e}^m\gamma_{(\mu} \partial_{\nu)} e^m - \partial_{(\mu} \bar{e}^m\gamma_{\nu)} e^m] + \frac{i}{2} g_{\mu\nu} [\bar{e}^m\gamma^\rho \partial_\rho e^m - \partial_\rho \bar{e}^m\gamma^\rho e^m]$$

Integrating out $\phi^{\mu\nu}$ we obtain $\mathcal{O}_e$ with

$$c_{mnpq} = \frac{\kappa^2}{4m^2} (\delta_{mn} \delta_{pq} + 4 \delta_{mq} \delta_{np})$$

which satisfies the bounds

$$c_{mnpq} \alpha_m \beta_n \beta_p^* \alpha_q^* = \frac{\kappa^2}{4m^2} (|\alpha \cdot \beta|^2 + 4 |\alpha|^2 |\beta|^2) > 0$$

[Remmen, NLR 2020]
Bounds on the Chiral Lagrangian

Recall the form of the chiral Lagrangian, with $U = \exp\left[i\sigma \cdot \pi / f_\pi \right]

$$
\mathcal{L}_\chi = \frac{f_\pi^2}{4} \text{Tr} \left[ (D_\mu U)(D_\mu U)^\dagger \right] + L_1 \text{Tr} \left[ (D_\mu U)(D_\mu U)^\dagger \right]^2 + L_2 \text{Tr} \left[ (D_\mu U)(D_\nu U)^\dagger \right]^2 + \ldots
$$

$\pi\pi \to \pi\pi$ scattering in this theory constrains $L_{1,2} > 0$, and experimentally

$$
L_1 \approx 0.65 > 0, \quad L_2 \approx 1.89 > 0
$$
Bounds with Running

Below $\mu = \Lambda_{NP}$ running must be included
e.g. dim-6 operators cause dim-8 coefficients to run, but still positive

\[
16\pi^2 c_{\phi^4}^{(2)} = \frac{1}{3} (5c_{\phi^D}^2 + 16c_{\phi D}c_{\phi^{\Box}} + 16c_{\phi^{\Box}}^2) \log \frac{M}{\mu} > 0,
\]

\[
16\pi^2 \left[ c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} \right] = \frac{16}{3} (c_{\phi^D}^2 - c_{\phi D}c_{\phi^{\Box}} + 2c_{\phi^{\Box}}^2) \log \frac{M}{\mu} > 0,
\]

\[
16\pi^2 \left[ c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} + c_{\phi^4}^{(3)} \right] = 3(c_{\phi^D}^2 + 8c_{\phi^{\Box}}^2) \log \frac{M}{\mu} > 0;
\]

[Chala, Guedes, Ramos, Santiago 2021]

Operator definitions:

\[
(D_\mu \phi^\dagger D_\nu \phi)(D^\nu \phi^\dagger D^\mu \phi) \quad O_{\phi^4}^{(1)} \quad (D_\mu \phi^\dagger D_\nu \phi)(D^\mu \phi^\dagger D^\nu \phi) \quad O_{\phi^4}^{(2)}
\]

\[
(D^\mu \phi^\dagger D_\mu \phi)(D^\nu \phi^\dagger D_\nu \phi) \quad O_{\phi^4}^{(3)} \quad c_{\phi^{\Box}}(\phi^\dagger \phi)(\phi^\dagger \phi) + c_{\phi^D}(\phi^\dagger D^\mu \phi)^*(\phi^\dagger D_\mu \phi)
\]

In general, it is the combination of the full matching (tree+loop) and running that is positive