Mapping the Boundaries of the Bosonic SMEFT

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LHC EFT WG: Positivity Constraints
July 2023
SU(N) bounds
Building blocks

Let’s place IR consistency bounds on the EFT of the Standard Model.

For general SU(N), a basis of quartic dimension-8 operators is:

\[
\begin{align*}
\mathcal{O}_1^F & = (F^a F^a)(F^b F^b) \\
\mathcal{O}_2^F & = (F^a \tilde{F}^a)(F^b \tilde{F}^b) \\
\mathcal{O}_3^F & = (F^a F^b)(F^a F^b) \\
\mathcal{O}_4^F & = (F^a \tilde{F}^b)(F^a \tilde{F}^b) \\
\mathcal{O}_5^F & = d^{abc} d^{cde} (F^a F^b) (F^c F^d) \\
\mathcal{O}_6^F & = d^{abc} d^{cde} (F^a \tilde{F}^b) (F^c \tilde{F}^d) \\
\mathcal{O}_7^F & = d^{ace} d^{bde} (F^a F^b) (F^c F^d) \\
\mathcal{O}_8^F & = d^{ace} d^{bde} (F^a \tilde{F}^b) (F^c \tilde{F}^d) \\
\tilde{\mathcal{O}}_1^F & = (F^a F^a)(F^b \tilde{F}^b) \\
\tilde{\mathcal{O}}_2^F & = (F^a F^b)(F^a \tilde{F}^b) \\
\tilde{\mathcal{O}}_3^F & = d^{abc} d^{cde} (F^a F^b) (F^c \tilde{F}^d) \\
\tilde{\mathcal{O}}_4^F & = d^{ace} d^{bde} (F^a F^b) (F^c \tilde{F}^d)
\end{align*}
\]

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{a\mu\nu} + \frac{1}{M^4} \sum_i c_i \mathcal{O}_i
\]

writing

\[
(AB) = A_{\mu\nu} B^{\mu\nu}
\]
To zeroth order in the $c_i$, the Yang-Mills equation of motion is

$$D^\mu F_{\mu\nu}^a = \partial^\mu F_{\mu\nu}^a + gf^{abc} A^{\mu b} F^{c}_{\mu\nu} = 0$$

This is satisfied by the YM background solution:

$$\vec{A}_\mu^a = u_1^a \epsilon_{1\mu} \omega$$

constant vector in color space  arbitrary four-vector  coordinate, $\partial_\mu \omega = \ell_\mu$
To zeroth order in the $c_i$, the Yang-Mills equation of motion is

$$D^\mu F^a_{\mu\nu} = \partial^\mu F^a_{\mu\nu} + gf^{abc} A^b_{\mu} F^c_{\mu\nu} = 0$$

This is satisfied by the YM background solution:

$$\overline{A}^a_\mu = u^a_1 \epsilon_{1\mu} w$$

We wish to consider plane-wave perturbation around this solution:

$$A^a_\mu = \overline{A}^a_\mu + \delta A^a_\mu$$

$$\delta A^a_\mu = u^a_2 \epsilon_{2\mu} e^{i k \cdot x}$$

where $k^2 = k \cdot \epsilon_2 = 0$. Still solves the equations of motion if $f^{abc} u^b_1 u^c_2 = 0$. 

YM equation of motion
We now compute the modified dispersion relation for the plane wave, to first order in the $c_i$. The resulting speed is:

$$
v = 1 - \frac{8}{M^4 \epsilon_2^2 u_2^2 k_0^2 N} \left\{ \left[ (\epsilon_1 \cdot k)(\epsilon_2 \cdot \ell) - (k \cdot \ell)(\epsilon_1 \cdot \epsilon_2) \right]^2 A + (\epsilon_1^\mu \epsilon_2^\nu k^\rho \ell^\sigma \epsilon_{\mu \nu \rho \sigma})^2 B \\
- \left[ (\epsilon_1 \cdot k)(\epsilon_2 \cdot \ell) - (k \cdot \ell)(\epsilon_1 \cdot \epsilon_2) \right] \epsilon_1^\mu \epsilon_2^\nu k^\rho \ell^\sigma \epsilon_{\mu \nu \rho \sigma} C \right\}
$$

where

$$A = N \left[ (2c_1 + c_3)(u_1 u_2)^2 + c_3 u_1^2 u_2^2 + 2(c_5 + c_7)U^2 \right] + 2c_7 \left[ (u_1 u_2)^2 - u_1^2 u_2^2 \right]
$$

$$B = N \left[ (2c_2 + c_4)(u_1 u_2)^2 + c_4 u_1^2 u_2^2 + 2(c_6 + c_8)U^2 \right] + 2c_8 \left[ (u_1 u_2)^2 - u_1^2 u_2^2 \right]
$$

$$C = N \left[ (2\tilde{c}_1 + \tilde{c}_2)(u_1 u_2)^2 + \tilde{c}_2 u_1^2 u_2^2 + 2(\tilde{c}_3 + \tilde{c}_4)U^2 \right] + 2\tilde{c}_4 \left[ (u_1 u_2)^2 - u_1^2 u_2^2 \right]
$$

$$U^a = d^{abc} u_1^b u_2^c$$
Correction to the speed

We now compute the modified dispersion relation for the plane wave, to first order in the $c_i$. The resulting speed is:

$$v = 1 - \frac{8E^2}{M^4 N} (AX^2 + BY^2 + CXY)$$

where

- $\kappa_\mu = (k_0, 0, 0, |k|)$
- $\epsilon_1^\mu = (0, 1, 0, 0)$
- $\ell_\mu = (\cos \theta_1, \sin \theta_1 \cos \theta_2, \sin \theta_1 \sin \theta_2 \cos \theta_3, \sin \theta_1 \sin \theta_2 \sin \theta_3)$
- $\epsilon_2^\mu = E(\cos \phi_1, \sin \phi_1 \cos \phi_2, \sin \phi_1 \sin \phi_2 \cos \phi_3, \sin \phi_1 \sin \phi_2 \sin \phi_3)$

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \sin \theta_1 \cos \theta_2 & \sin \phi_1 \cos \phi_2 \\ \sin \theta_1 \sin \theta_2 \cos \theta_3 & \sin \phi_1 \sin \phi_2 \cos \phi_3 \end{pmatrix} \begin{pmatrix} \cos \phi_1 + \sin \phi_1 \sin \phi_2 \sin \phi_3 \\ \cos \theta_1 - \sin \theta_1 \sin \theta_2 \sin \theta_3 \end{pmatrix}$$
Correction to the speed

We now compute the modified dispersion relation for the plane wave, to first order in the $c_i$. The resulting speed is:

$$v = 1 - \frac{8E^2}{M^4N} (AX^2 + BY^2 + CXY)$$

Writing $X = Z \cos \psi$, $Y = Z \sin \psi$, the causality bound becomes:

$$A \cos^2 \psi + B \sin^2 \psi + C \cos \psi \sin \psi > 0$$

for all $\psi$. 
Alternatively, we can obtain this bound from the forward four-point scattering amplitude:

\[
A_{F^4}(s) = \frac{8}{M^4 N} \left\{ A(\epsilon_1 \cdot \epsilon_2)^2 s^2 + B[\epsilon_1^2 \epsilon_2^2 - (\epsilon_1 \cdot \epsilon_2)^2]s^2 + 2C(\epsilon_1 \cdot \epsilon_2)\epsilon_1^\mu \epsilon_2^\nu k_1^\rho k_2^\sigma \epsilon_{\mu\nu\rho\sigma} s \right\}
\]

\[
= \frac{8s^2}{M^4 N} \left\{ A \cos^2 \psi + B \sin^2 \psi + C \cos \psi \sin \psi \right\}
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\]

\[
= \frac{8s^2}{M^4N} \left[ A \cos^2 \psi + B \sin^2 \psi + C \cos \psi \sin \psi \right]
\]

Marginalizing over \( \psi \) gives the minimal set of independent conditions:

\[
A > 0 \\
B > 0 \\
C^2 < 4AB
\]

The third condition can be obtained by setting \( \psi = \pm \arctan \sqrt{A/B} \). Cannot be found from bounding fixed-helicity amplitudes.
We want all bosonic four-point operators that have four derivatives and/or field strengths.

- **Ingredients:**
  - **Gauge field strengths:**
    - $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$
    - $W^I_{\mu\nu} = \partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g_2 \epsilon^{IJK} W^J_\mu W^K_\nu$
    - $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_3 f^{abc} G^b_\mu G^c_\nu$
    - $\tilde{B}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} B_{\rho\sigma}/2$
    - $\tilde{W}^{I\mu\nu} = \epsilon^{\mu\nu\rho\sigma} W^I_{\rho\sigma}/2$
    - $\tilde{G}^{a\mu\nu} = \epsilon^{\mu\nu\rho\sigma} G^a_{\rho\sigma}/2$
  - **Higgs:**
    - $H_i = \frac{1}{\sqrt{2}} \left( \phi_1 + i\phi_2 \right) \left( \phi_3 + i\phi_4 \right)$
    - $D_\mu H = (\partial_\mu + \frac{1}{2} i g_1 B_\mu + i g_2 \tau^I W^I_\mu) H$
Building the bosonic SMEFT

Gauge self-quartics:

<table>
<thead>
<tr>
<th>$\mathcal{O}_1^{B^4}$</th>
<th>$(BB)(BB)$</th>
</tr>
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<tbody>
<tr>
<td>$\mathcal{O}_2^{B^4}$</td>
<td>$(B\tilde{B})(B\tilde{B})$</td>
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writing $(AB) = A_{\mu\nu}B^{\mu\nu}$
Building the bosonic SMEFT

Gauge cross-quartics:

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Higgs self-quartics:

\[
\begin{align*}
\mathcal{O}^H_1 &= (D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H) \\
\mathcal{O}^H_2 &= (D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H) \\
\mathcal{O}^H_3 &= (D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H)
\end{align*}
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Building the bosonic SMEFT

Higgs and gauge field strength cross-quartics:

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<td>$(D^\mu H^\dagger D_\mu H) B_{\rho \sigma} B^{\rho \sigma}$</td>
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<tr>
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<td>$(D^\mu H^\dagger D^\nu H) W^I_{\mu \rho} W^I_{\nu \rho}$</td>
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<tr>
<td>$\tilde{\mathcal{O}}_1^{H^2W^2}$</td>
<td>$i \epsilon^{IJK} (D^\mu H^\dagger \tau^I D^\nu H) W^J_{\mu \rho} W^K_{\nu \rho}$</td>
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On the 64 quartic bosonic operators at dimension eight in the SMEFT (39 CP-conserving and 25 CP-violating), we placed 27 independent bounds (20 positivity bounds and 7 magnitude bounds):

\[
\begin{align*}
3c_1^G + 3c_3^G + c_5^G &> 0 \\
3c_3^G + 2c_5^G &> 0 \\
3c_2^G + 3c_4^G + c_6^G &> 0 \\
3c_4^G + 2c_6^G &> 0 \\
c_1^W + c_3^W &> 0 \\
c_2^W + c_4^W &> 0 \\
c_1^B &> 0 \\
c_2^B &> 0 \\
c_1^H + c_2^H + c_3^H &> 0 \\
c_1^H + c_2^H &> 0 \\
c_2^H &> 0
\end{align*}
\]

GR, Rodd [1908.09845]
Summary of bosonic SMEFT bounds

On the 64 quartic bosonic operators at dimension eight in the SMEFT (39 CP-conserving and 25 CP-violating), we placed 27 independent bounds (20 positivity bounds and 7 magnitude bounds):

$$
(3\tilde{c}_1^G + 3\tilde{c}_2^G + \tilde{c}_3^G)^2 < 4(3c_1^G + 3c_3^G + c_5^G)(3c_2^G + 3c_4^G + c_6^G)
$$

$$
(3\tilde{c}_2^G + 2\tilde{c}_3^G)^2 < 4(3c_3^G + 2c_5^G)(3c_4^G + 2c_6^G)
$$

$$
(\tilde{c}_1^W + \tilde{c}_2^W)^2 < 4(c_1^W + c_3^W)(c_2^W + c_4^W)
$$

$$
(\tilde{c}_1^B)^2 < 4c_1^B c_2^B
$$

$$
(\tilde{c}_3^B W^2)^2 < 4c_3^B W^2 c_4^B W^2
$$

$$
(\tilde{c}_3^B G^2)^2 < 4c_3^B G^2 c_4^B G^2
$$

$$
(\tilde{c}_3 W^2 G^2)^2 < 4c_3^W G^2 c_4^W G^2
$$
Summary of bosonic SMEFT bounds

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(3\tilde{c}_2^G + 2\tilde{c}_3^G)^2 &< 4(3c_3^G + 2c_5^G)(3c_4^G + 2c_6^G) \\
(\tilde{c}_1^W + \tilde{c}_2^W)^2 &< 4(c_1^W + c_3^W)(c_2^W + c_4^W) \\
(\tilde{c}_1^B)^2 &< 4c_1^B c_2^B \\
(\tilde{c}_3^B W^2)^2 &< 4c_3^B W^2 c_4^B W^2 \\
(\tilde{c}_3^B G^2)^2 &< 4c_3^B G^2 c_4^B G^2 \\
(\tilde{c}_3^W G^2)^2 &< 4c_3^W G^2 c_4^W G^2
\end{align*}
\]

IR consistency gives factor of \(\sim 10^8\) reduction in parameter space!
UV tests
Now let’s look at a large class of one-loop completions:

Consider a massive state $\Phi$ coupled to the gauge bosons.

Generalization of how the electron couples to the photon in QED, and integrating out the electron gives $F^4$ terms (Euler-Heisenberg Lagrangian)
One-loop completions of gauge operators

Wilson coefficients:

<table>
<thead>
<tr>
<th>Operator</th>
<th>Scalar</th>
<th>Fermion</th>
<th>Vector</th>
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<tr>
<td>$c^W_1$</td>
<td>$\frac{7}{32} g^4 Q^4$</td>
<td>$\frac{1}{2} g^4 Q^4$</td>
<td>$\frac{203}{32} g^4 Q^4$</td>
</tr>
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<td>$c^W_2$</td>
<td>$\frac{7}{16} g^4 R^2$</td>
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<td>$\frac{3}{16} g^4 R^2$</td>
</tr>
<tr>
<td>$c^W_3$</td>
<td>$\frac{1}{16} \Lambda(R_2) + \frac{1}{56} I_2(R_2)$</td>
<td>$\frac{1}{8} \Lambda(R_2) + \frac{1}{448} I_2(R_2)$</td>
<td>$\frac{243}{32} \Lambda(R_2) - \frac{37}{128} I_2(R_2)$</td>
</tr>
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<td>$\frac{243}{32} \Lambda(R_2) - \frac{37}{128} I_2(R_2)$</td>
</tr>
<tr>
<td>$c^G_1$</td>
<td>$\frac{1}{32} \Lambda(R_3) + \frac{1}{8} I_2(R_3)$</td>
<td>$\frac{1}{8} \Lambda(R_3) + \frac{1}{16} I_2(R_3)$</td>
<td>$\frac{243}{16} \Lambda(R_3) + \frac{77}{128} I_2(R_3)$</td>
</tr>
<tr>
<td>$c^G_2$</td>
<td>$\frac{1}{32} \Lambda(R_3) - \frac{1}{8} I_2(R_3)$</td>
<td>$\frac{1}{8} \Lambda(R_3) - \frac{1}{16} I_2(R_3)$</td>
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</table>
All of our gauge field bounds are satisfied by arbitrary one-loop completions.
Higgs operators:

\[ \mathcal{O}_1^{H^4} = (D_\mu H^\dagger D_{\nu} H)(D^\nu H^\dagger D^\mu H) \]
\[ \mathcal{O}_2^{H^4} = (D_\mu H^\dagger D_{\nu} H)(D^\mu H^\dagger D^\nu H) \]
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Bounds:

\[ c_1^{H^4} + c_2^{H^4} + c_3^{H^4} > 0 \]
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$$c_2^{H^4} > 0$$

IR consistency bounds satisfied by example UV completions.
We notice a pattern in the SMEFT positivity bounds—there are two distinct forms:

1. \( c_1 > 0, \ c_2 > 0 \) for \( c_{1,2} \) coefficients of CP-conserving terms

2. \( \tilde{c}^2 < 4c_1c_2 \) for \( \tilde{c} \) the coefficient of a CP-violating term

Bounds form a cone:

\[
\tilde{c}^2 + c_-^2 < c_+^2, \ c_+ > 0
\]

where \( c_\pm = c_1 \pm c_2 \)
Let's take the U(1) $\mathcal{O}_i^B$ terms as an example:

$$\Delta \mathcal{L} = \frac{1}{M^4} \left[ c_1^B (B_{\mu\nu} B^{\mu\nu})^2 + c_2^B (B_{\mu\nu} \tilde{B}^{\mu\nu})^2 + \tilde{c}_1^B B_{\mu\nu} B^{\mu\nu} B_{\rho\sigma} \tilde{B}^{\rho\sigma} \right]$$

The positivity bounds $c_{1,2}^B > 0$, $(\tilde{c}_1^B)^2 < 4c_1^B c_2^B$ imply that $\Delta \mathcal{L}$ can be written as a sum of perfect squares:

$$\Delta \mathcal{L} = \frac{\alpha^2}{2M^4} \left[ (B_{\mu\nu} B^{\mu\nu} + \beta B_{\mu\nu} \tilde{B}^{\mu\nu})^2 + \gamma^2 (B_{\mu\nu} B^{\mu\nu} - \beta B_{\mu\nu} \tilde{B}^{\mu\nu})^2 \right]$$
Experimental applications
• Previously, focus on experimental signals of dimension-eight operators has been on the electroweak sector in anomalous quartic gauge-boson couplings (aQGCs).

• Induce corrections to SM couplings, e.g., $WWWW$, $WWZZ$, $WWZ\gamma$ or induce non-SM couplings like $ZZZZ$
aQGCs

• Operator basis involves:
  
  • Scalar (S-type) operators $\sim (\partial H)^4$
  • Mixed (M-type) operators $\sim (\partial H)^2 F^2$
  • Tensor (T-type) operators $\sim F^4$

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Éboli, Gonzalez-Garcia, Mizukoshi [hep-ph/0606118]
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We have identified and proven the correct minimal basis of aQGCs

<table>
<thead>
<tr>
<th>S</th>
<th>M</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{O}_1^{H^4}$</td>
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<tr>
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<td>$O_1^{W^4}$</td>
</tr>
<tr>
<td>$\mathcal{O}_1^{H^2 B^2}$</td>
<td>$i \epsilon^{IJK} (D_\mu H D_\nu H)(D^{\nu} H^{\dagger} D^{\mu} H)$</td>
<td>$O_2^{W^4}$</td>
</tr>
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LHC has already placed constraints on aQGCs:

\[ c_s,0 / M^4 \text{ [TeV}^{-4}] \]

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CMS (2019) constraint

\[ \mathcal{O}_{S,0} = \mathcal{O}^{H^4}_2, \mathcal{O}_{S,1} = \mathcal{O}^{H^4}_3 \]
LHC has already placed constraints on aQGCs:

$$\mathcal{O}_{T,6} = \frac{g_1^2 g_2^2}{8} \mathcal{O}_3 B^2 W^2, \quad \mathcal{O}_{T,7} = \frac{g_1^2 g_2^2}{32} \left( \mathcal{O}_1^B W^2 + \mathcal{O}_3^B W^2 + \mathcal{O}_4^B W^2 \right)$$
LHC has already placed constraints on aQGCs:

IR consistency constraints can sharpen bounds and motivate new places to look.

GR, Rodd [1908.09845]
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- $G\bar{G}$ operator has famously small coupling (strong CP problem)
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- Depending on details of model, dimension-8 operators can provide dominant contribution. e.g., Chemtob (1993)
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For an operator $\frac{c}{M^4} G^3 \tilde{G}$, can estimate the nEDM:

$$|d_n| \sim \frac{e c \Lambda_{\chi^0 SB}^3}{M^4 (4\pi)^2} \approx c \left( \frac{1 \text{ TeV}}{M} \right)^4 \times 10^{-28} \text{ e cm}$$

Experimental bounds require $|d_n| \lesssim 10^{-26} \text{ e cm}$, so the scale of $M$ is being probed to 100s of GeV
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\[ \Rightarrow \text{Connect different experimental measurements?} \]

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Discovery of nonzero nEDM from dimension-8 operators would lead directly to IR-consistency predictions for CP-even operators observable at the LHC.
Infrared consistency provides a powerful set of tools for constraining new physics. Causality, unitarity, and analyticity allow us to bound the couplings from physics beyond the Standard Model and connect with accessible experimental signals.

Most crucial and near-term feasible: Incorporate SMEFT bounds as priors on ATLAS and CMS bounds on anomalous quartic gauge couplings (aQGCs).
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Connecting the IR consistency program with the SMEFT represents an important new bridge between phenomenology and formal theory. It allows us to connect physics at different energy scales and offers both a test of fundamental properties of QFT up to very high energies and a sharpening of our search for new physics.
Questions