DY PRODUCTION AT SMALL Q_T AND THE COLLINEAR ANOMALY

arXiv:1007.4005 with Matthias Neubert + upcoming with Daniel Wilhelm + upcoming with Guido Bell

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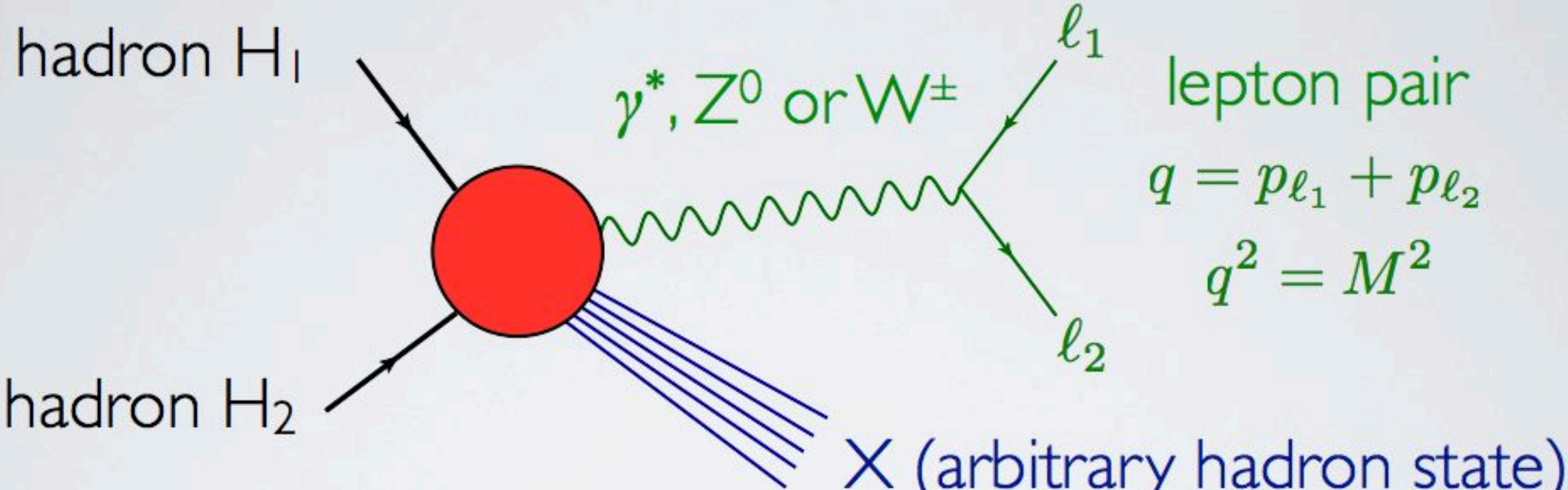
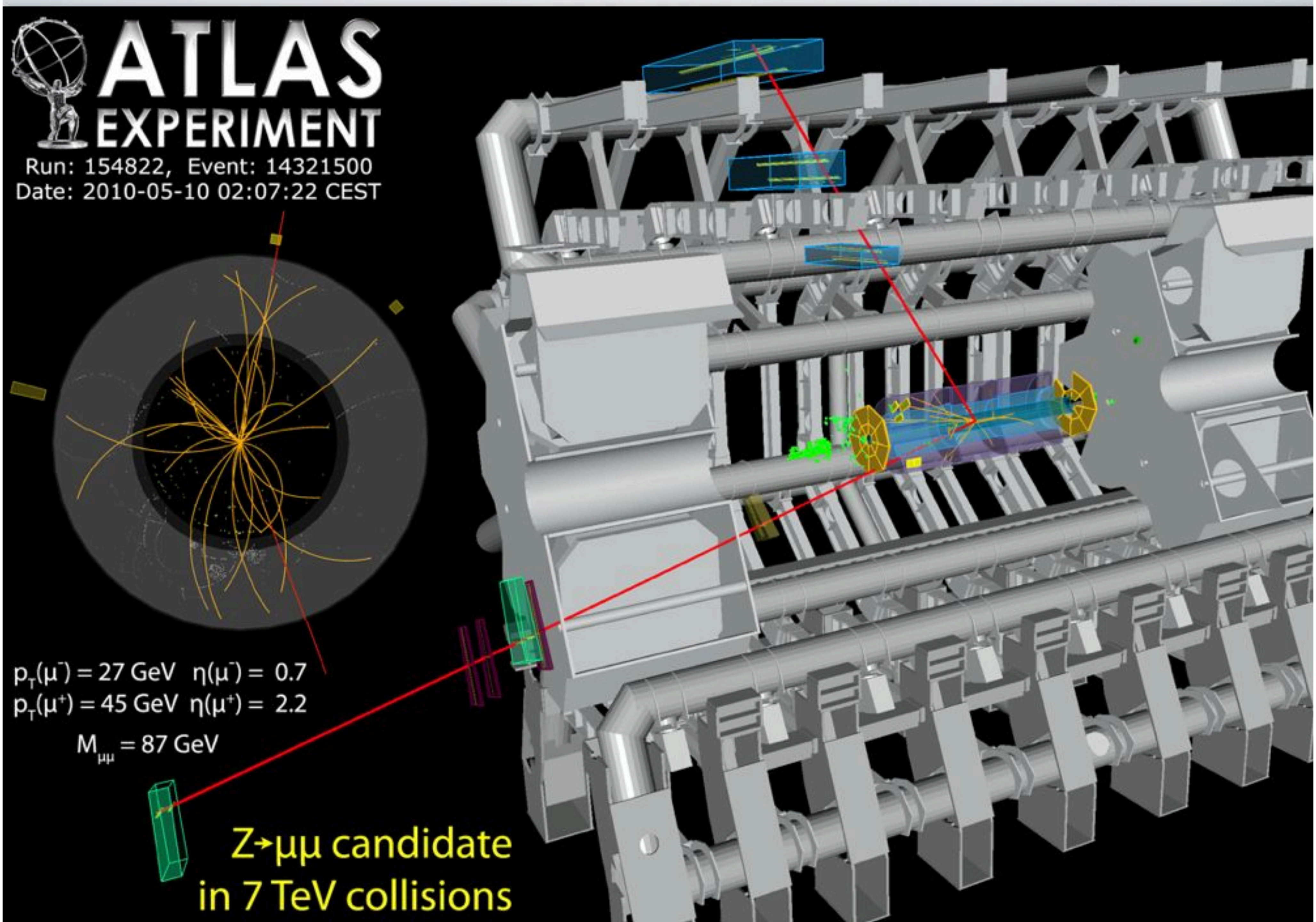
OUTLINE

- Introduction
  - Drell-Yan process
  - Soft-Collinear Effective Theory
- Factorization at low transverse momentum q_T
  - The collinear anomaly and the definition of transverse position dependent PDFs
  - Resummation of large log's, relation to CSS formalism
- Expansions from hell and non-perturbative short-distance physics at low q_T. Numerical results.
- More anomalous factorization: jet broadening

DRELL-YAN PROCESSES



The production of a lepton pair with large invariant mass is the most basic hard-scattering process at a hadron collider.


ATLAS
EXPERIMENT
Run: 154822, Event: 14321500
Date: 2010-05-10 02:07:22 CEST
$p_T(\mu^-) = 27$ GeV $\eta(\mu^-) = 0.7$
$p_T(\mu^+) = 45$ GeV $\eta(\mu^+) = 2.2$
$M_{\mu\mu} = 87$ GeV
Z→μμ candidate
in 7 TeV collisions

DRELL-YAN PROCESSES

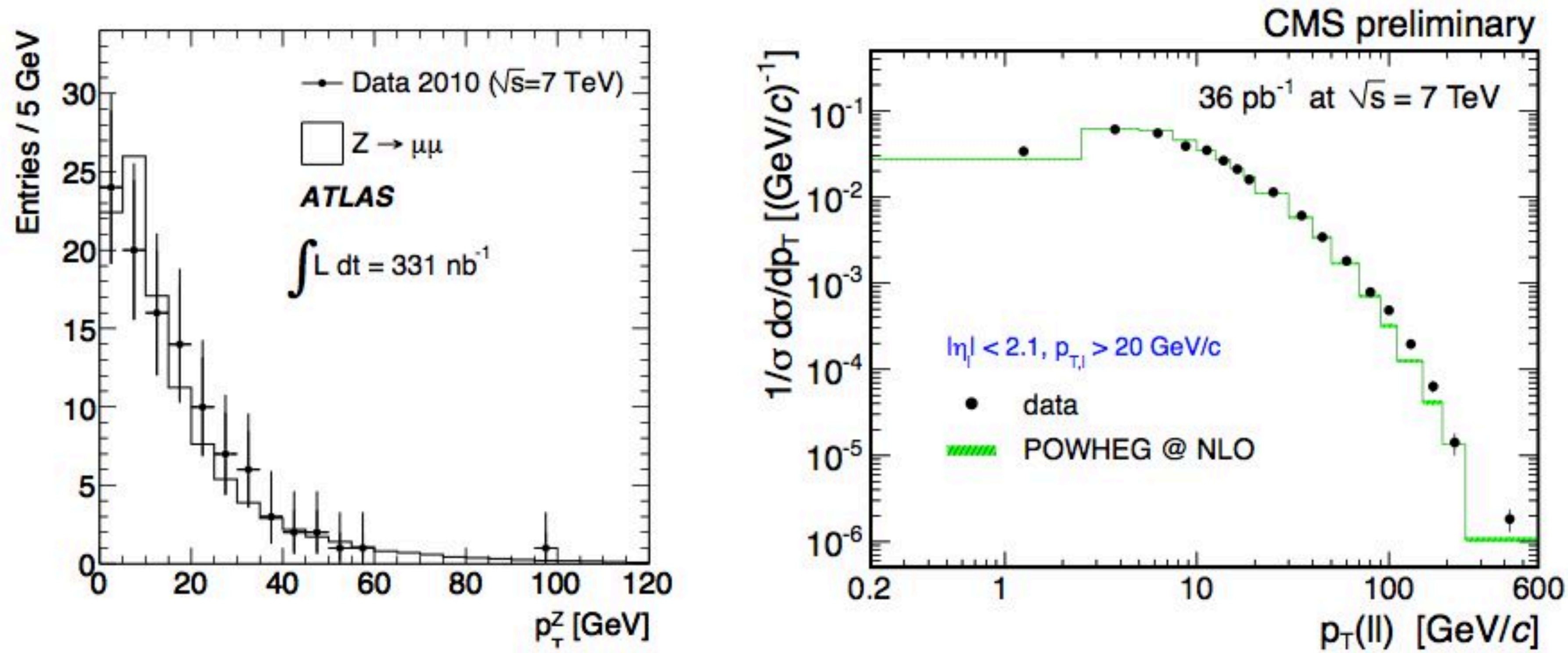
The production of a single electroweak boson $\gamma^*, Z, W^\pm, H$ is of great interest for

- W mass and width measurements,
- PDF determinations, luminosity monitoring,
- New physics searches at high q^2

Low transverse momentum q_T is particularly relevant

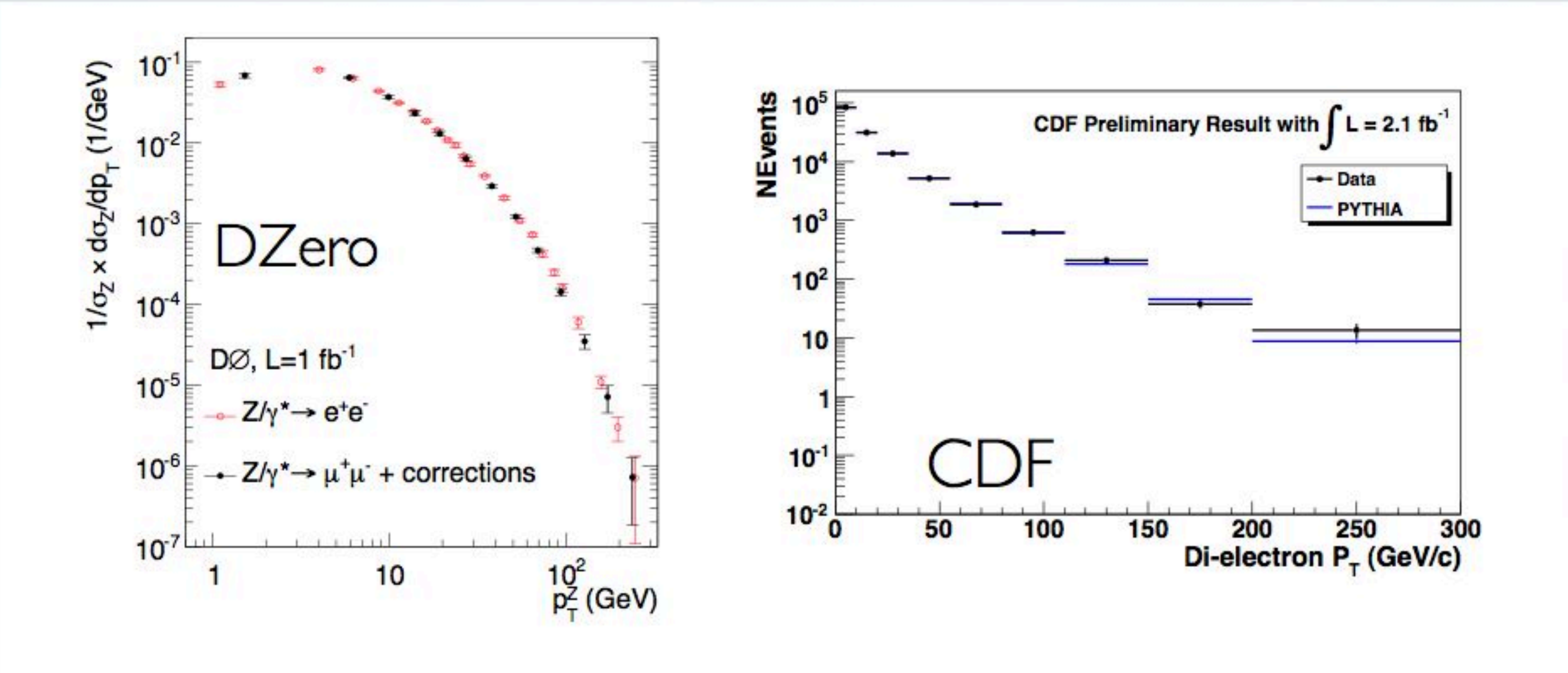
- to extract W mass
- to reduce background for Higgs search

Z-PRODUCTION AT LHC



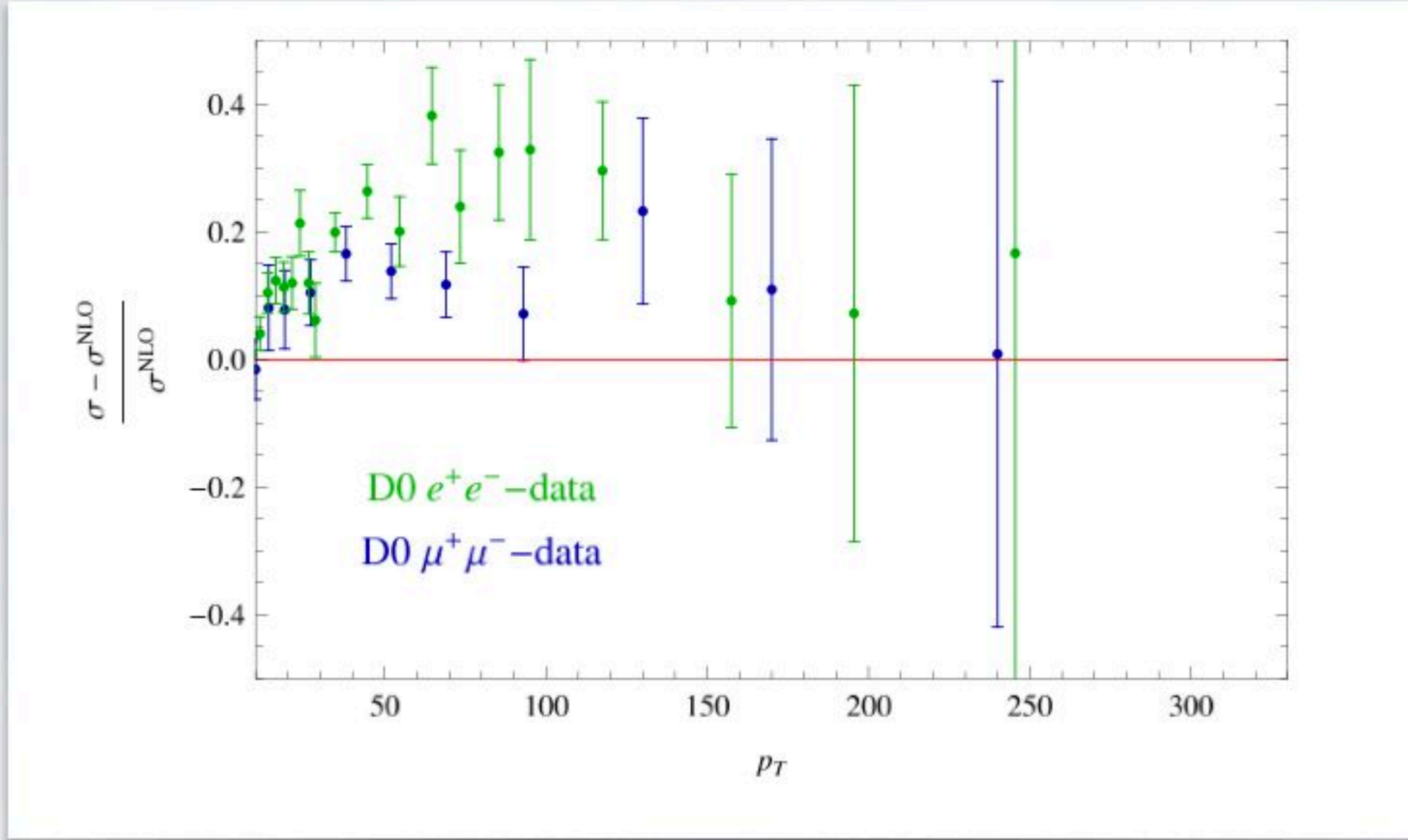
ATLAS distribution based on 300 nb^{-1}. Update?

Z-PRODUCTION AT THE TEVATRON



- DZero, arXiv:1006.0618, 0712.0803.

SIDE REMARK



- Agreement between DZero e and μ does not look as nice on a linear scale...

PERTURBATIVE EXPANSION

The perturbative expansion of the q_T spectrum contains singular terms of the form (M is the invariant mass of the lepton pair)

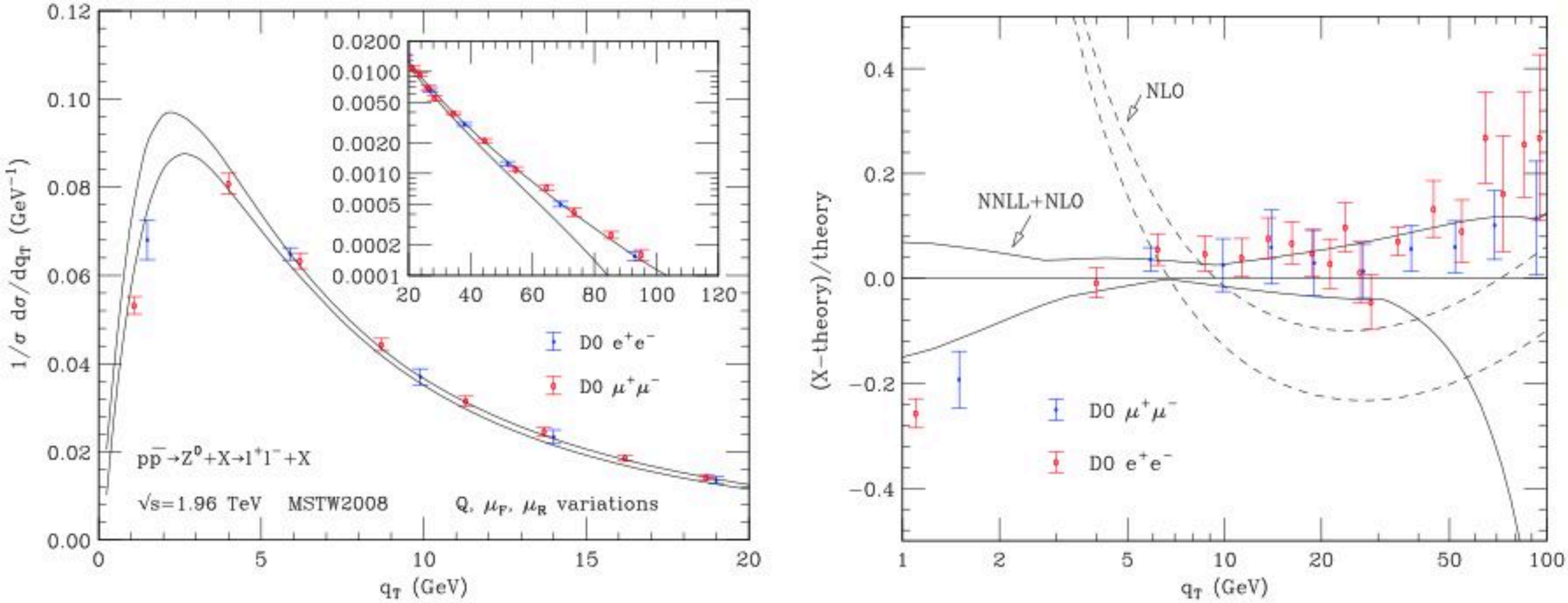
$$\frac{d\sigma}{dq_T^2} = \frac{1}{q_T^2}\Big[A_1^{(1)}\alpha_s \ln\frac{M^2}{q_T^2} + \alpha_s A_0^{(1)} + A_3^{(2)}\alpha_s^2 \ln^3\frac{M^2}{q_T^2} + \ldots$$

$$+A_{2n-1}^{(n)}\alpha_s^n \ln^{2n-1}\frac{M^2}{q_T^2} + \ldots\Big] + \ldots$$

which ruin the perturbative expansion at $q_T \ll M$ and must be resummed to all orders.

Classic example of an observable which needs resummation! Achieved by Collins, Soper and Sterman (CSS) '84.

NNLL RESUMMATION



Bozzi, Catani, Ferrera, de Florian, Grazzini '10

- Fixed order codes: QT (Gonsalves); MCFM (Campbell & Ellis), FEWZ (Melnikov & Petriello), DYNNLO (Grazzini et al.).
- NNLL resummation: RESBOS (Balazs, Nadolsky, Yuan); Bozzi et al.

SOFT-COLLINEAR EFFECTIVE THEORY

Bauer, Pirjol, Stewart et al. 2001, 2002; Beneke et al. 2002; ...

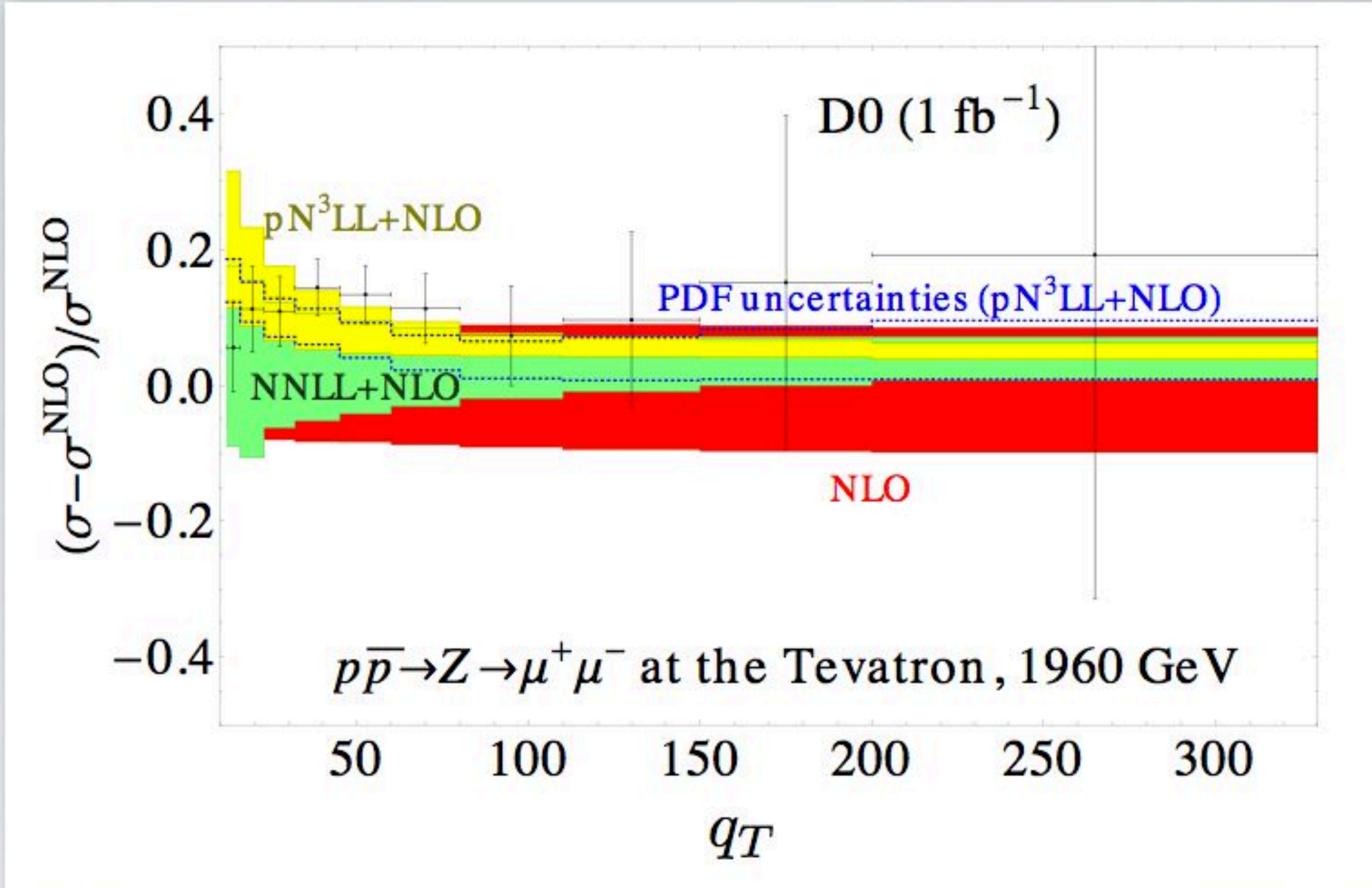
CSS used diagrammatic methods to factorize contributions with different scales, we will instead use effective field theory.

SCET has been used to perform soft gluon resummation for many processes:

- DIS at large x, Drell-Yan rapidity spectrum, inclusive Higgs production, top production, direct photon production, single top production, e^+e^- event shapes, ...

Would like to use framework to resum higher logs in multi-jet processes at hadron colliders. To do so, we first need to understand "initial state showering".

- The q_T-spectrum in DY provides simple setting to study issue

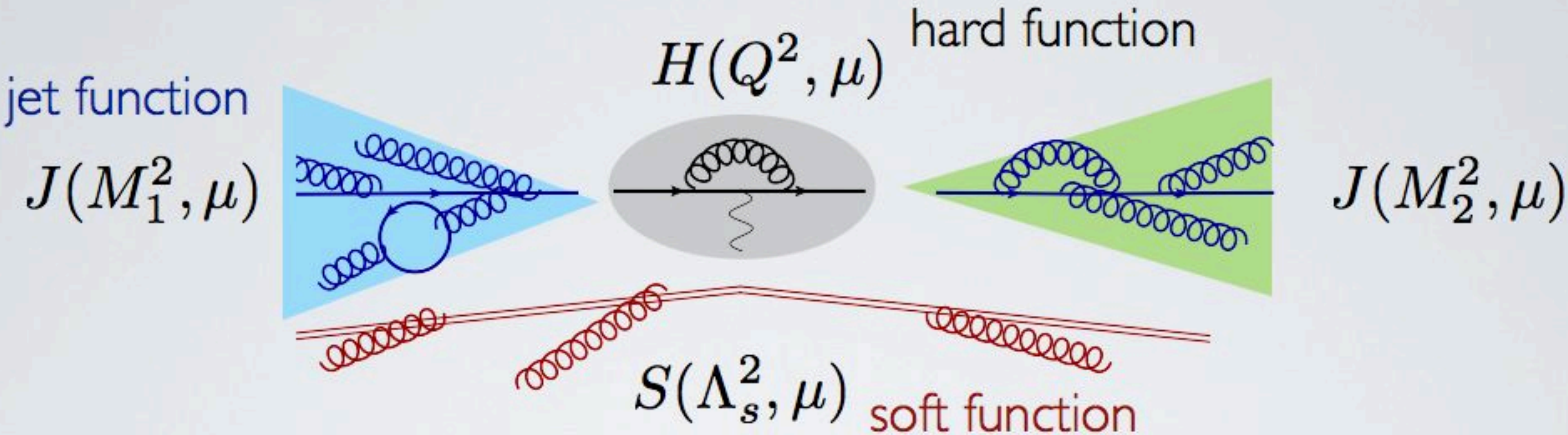


Focus here on low q_T , but we have also performed soft-gluon resummation for high-q_T spectrum.

TB, Lorentzen, Schwartz, to appear soon

FACTORIZATION

Standard soft-collinear factorization:

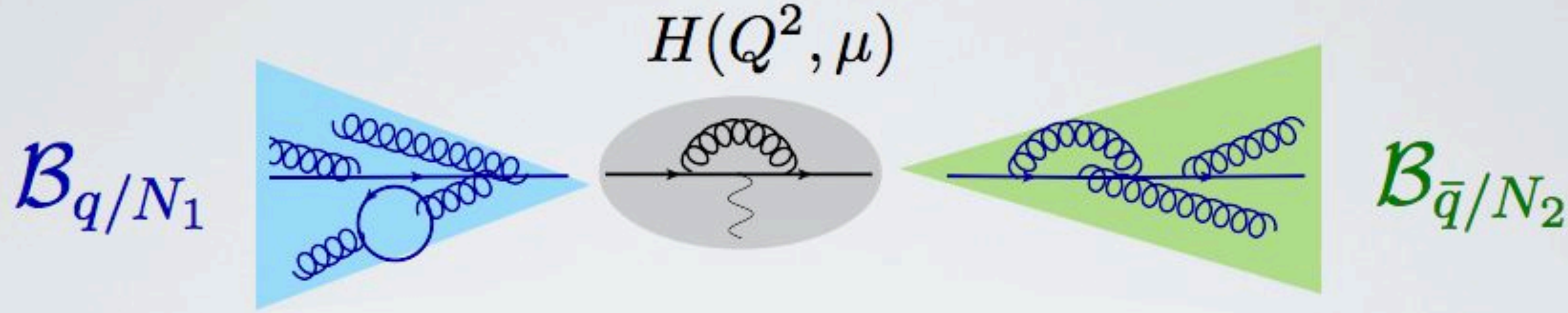


Transverse momentum of soft partons is suppressed compared to the transverse momentum of the partons inside the jets

$$q_T^{\text{soft}} \sim \frac{(q_T^{\text{jet}})^2}{Q}$$

Soft contribution to q_T spectrum can be neglected. Sum over all soft emissions: KLN cancellation.

NAIVE FACTORIZATION



"hard function" × "transverse PDF" × "transverse PDF"

Transverse PDF

light-cone vector in $\bar{q}$ direction

$$\mathcal{B}_{q/N}(z, x_T^2, \mu) = \frac{1}{2\pi} \int dt\, e^{-izt\bar{n}\cdot p} \langle N(p)|\, \bar{\chi}(t\bar{n} + x_\perp)\, \frac{\bar{\not{n}}}{2}\, \chi(0)\, |N(p)\rangle$$

this spells trouble: well known that transverse PDF not well defined w/o additional regulators

CROSS SECTION

$$\xi_{1/2} = \frac{M}{\sqrt{s}}\, e^{\pm y}$$

$$\frac{d^3\sigma}{dM^2\, dq_T^2\, dy} = \frac{4\pi\alpha^2}{3N_c M^2 s} \left| H(M^2,\mu) \right|^2 \frac{1}{4\pi} \int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp}$$

$$\times \sum_q e_q^2 \left[\mathcal{B}_{q/N_1}(\xi_1, x_T^2, \mu)\, \mathcal{B}_{\bar{q}/N_2}(\xi_2, x_T^2, \mu) + (q \leftrightarrow \bar{q}) \right] + \mathcal{O}\left(\frac{q_T^2}{M^2} \right)$$

The resummation would then be obtained by solving the RG equation

$$\frac{d}{d\ln\mu}\, H(M^2,\mu) = \left[2\Gamma_{\text{cusp}}^F(\alpha_s)\, \ln\frac{M^2}{\mu^2} + 4\gamma^q(\alpha_s) \right] H(M^2,\mu)$$

see SCET papers: Gao, Li, Liu 2005; Idilbi, Ji, Yuan 2005; Mantry, Petriello 2009

This cannot be correct! If $\mathcal{B}_{q/N_1}(\xi_1, x_T^2, \mu)\, \mathcal{B}_{\bar{q}/N_2}(\xi_2, x_T^2, \mu)$ is independent of $\boldsymbol{M}$, the above cross section is $\boldsymbol{\mu}$ dependent!

COLLINEAR ANOMALY

RG invariance of cross section implies that the product of transverse PDFs $\mathcal{B}_{q/N_1}(\xi_1, x_T^2, \mu)\, \mathcal{B}_{\bar{q}/N_2}(\xi_2, x_T^2, \mu)$ must contain hidden M^2 dependence.

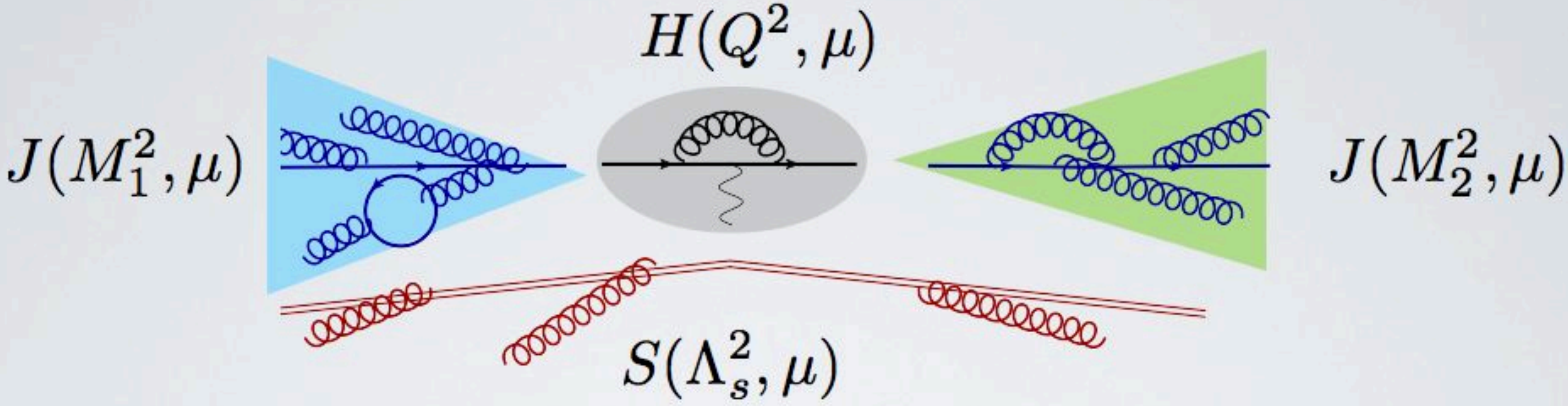
Analyzing the relevant diagrams, one finds that an additional regulator is needed to make transverse PDFs well defined. In the product of the two PDFs, this regulator can be removed, but anomalous M^2 dependence remains. Can refactorize

$$\left[\mathcal{B}_{q/N_1}(z_1, x_T^2, \mu)\, \mathcal{B}_{\bar{q}/N_2}(z_2, x_T^2, \mu)\right]_{M^2} = \left(\frac{x_T^2 M^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar{q}}(x_T^2,\mu)} B_{q/N_1}(z_1, x_T^2, \mu)\, B_{\bar{q}/N_2}(z_2, x_T^2, \mu)\,,$$

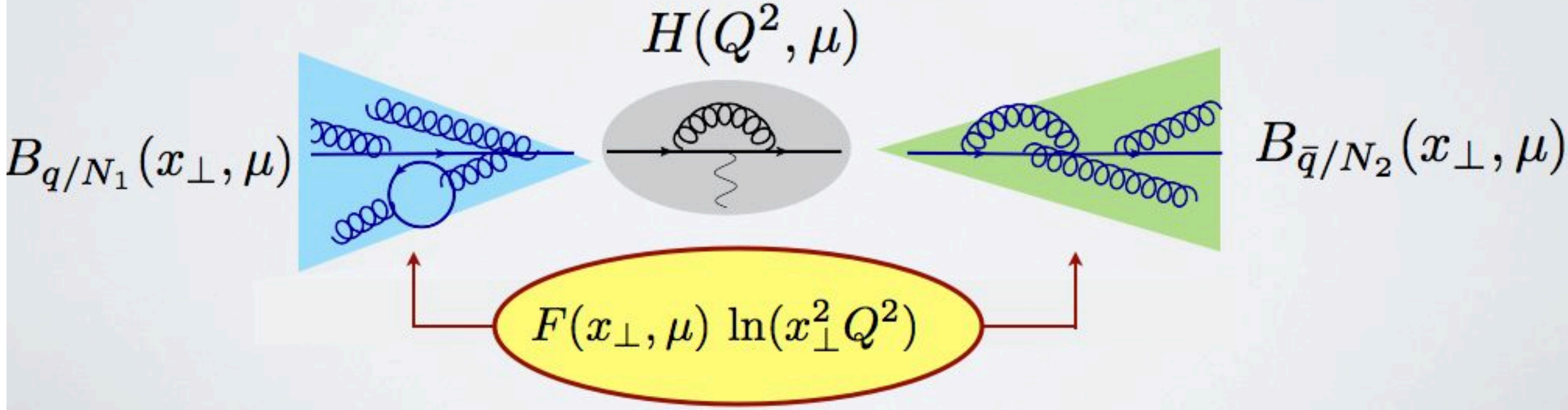
with $\dfrac{dF_{q\bar{q}}(x_T^2, \mu)}{d\ln\mu} = 2\Gamma_{\text{cusp}}^F(\alpha_s)$

Note that M^2 dependence exponentiates!

Regular soft-collinear factorization:



Anomalous factorization



TRANSVERSE PDFs

What God has joined together, let no man separate...

The "operator definition of TMD PDFs is quite problematic [...] and is nowadays under active investigation".

quote from Cherednikov and Stefanis '09. For reviews on TMD PDSs, see Collins '03, '08. For an elegant new definition by Collins, see arXiv:1101.5057.

Regularization of the individual transverse PDFs is delicate, but the product is well defined, and has specific dependence on the hard momentum transfer M^2.

Anomaly: Classically, $\langle N_1(p)|\, \bar{\chi}_{hc}(x_+ + x_\perp) \not{\bar{n}}\, \chi_{hc}(0)\, |N_1(p)\rangle$ is invariant under a rescaling of the momentum of the other nucleon N_2. Quantum theory needs regularization. Symmetry cannot be recovered after removing regulator. Not an anomaly of QCD, but of the low energy theory.

SIMPLIFICATION FOR $q_T^2 \gg \Lambda_{\rm QCD}$

For perturbative values of q_T we can perform an operator product expansion

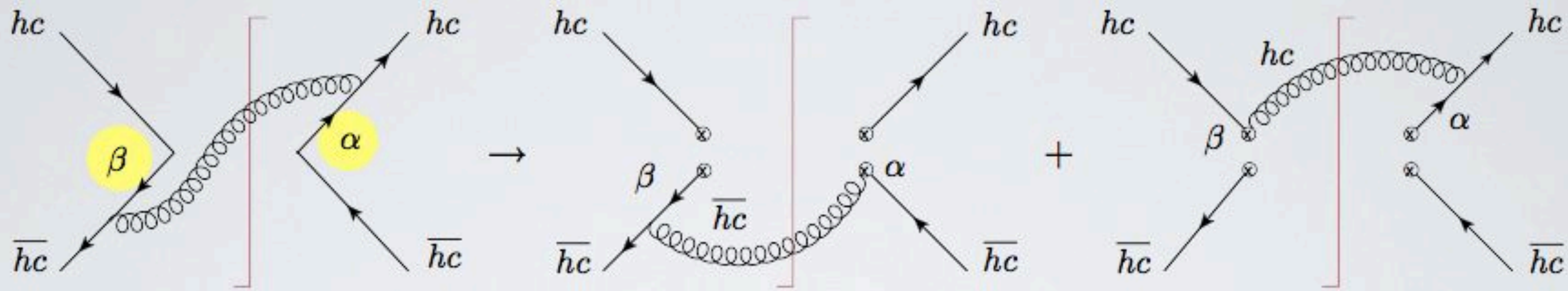
$$\mathcal{B}_{i/N}(\xi, x_T^2, \mu) = \sum_j \int_\xi^1 \frac{dz}{z}\, \mathcal{I}_{i\leftarrow j}(z, x_T^2, \mu)\, \phi_{j/N}(\xi/z, \mu) + \mathcal{O}(\Lambda_{\rm QCD}^2\, x_T^2)$$

Again, only the product of two $\mathcal{I}_{i\leftarrow j}(z, x_T^2, \mu)$ functions is well defined.

$$\left[\mathcal{I}_{q\leftarrow i}(z_1, x_T^2, \mu)\, \mathcal{I}_{\bar{q}\leftarrow j}(z_2, x_T^2, \mu)\right]_{q^2} = \left(\frac{x_T^2 q^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar{q}}(x_T^2, \mu)} I_{q\leftarrow i}(z_1, x_T^2, \mu)\, I_{\bar{q}\leftarrow j}(z_2, x_T^2, \mu)$$

Effective theory diagrams for $\mathcal{I}_{i\leftarrow j}(z, x_T^2, \mu)$ are not well-defined in dim. reg.. Following Smirnov '83, we use additional *analytical regularization*, which is very economical, since it does not introduce additional scales into the problem.

ANALYTICAL REGULARIZATION

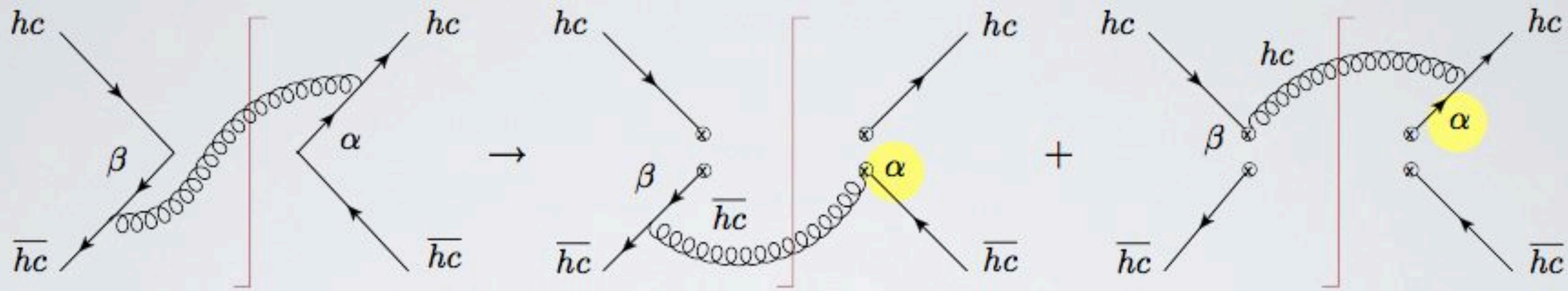


- Raise QCD propagators carrying large momentum p, $(\bar{p})$ to fractional powers α, (β):

$$\frac{1}{-(p-k)^2 - i\varepsilon} \rightarrow \frac{\nu_1^{2\alpha}}{[-(p-k)^2 - i\varepsilon]^{1+\alpha}}$$

- Limit $\alpha \rightarrow 0$, $\beta \rightarrow 0$ is trivial for QCD, but effective theory diagrams have poles, which only cancel in the sum of collinear and anti-collinear diagrams.

ANALYTICAL REGULARIZATION



- Regulators play double role. E.g. α regulates *hc* propagators and $\overline{hc}$ Wilson line

$$\frac{n^\mu}{n\cdot k - i\varepsilon} \rightarrow \frac{\nu_1^{2\alpha}\, n^\mu\, \bar{n}\cdot p}{(n\cdot k\,\bar{n}\cdot p - i\varepsilon)^{1+\alpha}}$$

- Regulator breaks invariance of anti-hard-collinear sector under a rescaling of the hard-collinear momentum $p \longrightarrow \lambda p$.

1-LOOP RESULT

Taking first $\beta \longrightarrow 0$, then $\alpha \longrightarrow 0$, one finds ($L_\perp = \ln \frac{x_T^2 \mu^2}{4e^{-2\gamma_E}}$)

$$\mathcal{I}_{q\leftarrow q}(z, x_T^2, \mu) = \delta(1-z) - \frac{C_F \alpha_s}{2\pi} \left\{ \left(\frac{1}{\epsilon} + L_\perp \right) \left[\left(\frac{2}{\alpha} - 2\ln\frac{\mu^2}{\nu_1^2} \right) \delta(1-z) + \frac{1+z^2}{(1-z)_+} \right] + \delta(1-z) \left(-\frac{2}{\epsilon^2} + L_\perp^2 + \frac{\pi^2}{6} \right) - (1-z) \right\}$$

anomalous M^2 dep.

$$\mathcal{I}_{\bar{q}\leftarrow \bar{q}}(z, x_T^2, \mu) = \delta(1-z) - \frac{C_F \alpha_s}{2\pi} \left\{ \left(\frac{1}{\epsilon} + L_\perp \right) \left[\left(-\frac{2}{\alpha} + 2\ln\frac{M^2}{\nu_1^2} \right) \delta(1-z) + \frac{1+z^2}{(1-z)_+} \right] - (1-z) \right\}$$

In the product the $1/\alpha$ divergences vanish, but anomalous M^2 dependence remains.

- For the functions I and F, one then obtains

$$F_{q\bar{q}}(L_\perp, \alpha_s) = \frac{C_F \alpha_s}{\pi} L_\perp + \mathcal{O}(\alpha_s^2)$$

$$\begin{aligned} I_{q\leftarrow q}(z, L_\perp, \alpha_s) = I_{\bar{q}\leftarrow \bar{q}}(z, L_\perp, \alpha_s) &= \delta(1-z)\left[1 + \frac{C_F \alpha_s}{4\pi}\left(L_\perp^2 + 3L_\perp - \frac{\pi^2}{6}\right)\right] \\ &\quad - \frac{C_F \alpha_s}{2\pi}\Big[L_\perp P_{q\leftarrow q}(z) - (1-z)\Big] + \mathcal{O}(\alpha_s^2) \end{aligned}$$

$$I_{q\leftarrow g}(z, L_\perp, \alpha_s) = I_{\bar{q}\leftarrow g}(z, L_\perp, \alpha_s) = -\frac{T_F \alpha_s}{2\pi}\Big[L_\perp P_{q\leftarrow g}(z) - 2z(1-z)\Big] + \mathcal{O}(\alpha_s^2)$$

- Solving its RG and using Davies, Stirling and Webber '84 and de Florian and Grazzini '01, we extract the two-loop *F*

$$F_{q\bar{q}}(L_\perp, \alpha_s) = \frac{\alpha_s}{4\pi}\Gamma_0^F L_\perp + \left(\frac{\alpha_s}{4\pi}\right)^2\left[\frac{\Gamma_0^F \beta_0}{2} L_\perp^2 + \Gamma_1^F L_\perp + d_2^q\right]$$

$$d_2^q = C_F\left[C_A\left(\frac{808}{27} - 28\zeta_3\right) - \frac{224}{27} T_F n_f\right]$$

Casimir scaling

$$\frac{F_{q\bar{q}}(L_\perp, \alpha_s)}{C_F} = \frac{F_{gg}(L_\perp, \alpha_s)}{C_A}$$

All the necessary input for NNLL resummation!

RESUMMED RESULT

$$\frac{d^3\sigma}{dM^2\,dq_T^2\,dy} = \frac{4\pi\alpha^2}{3N_cM^2s}\sum_q e_q^2 \sum_{i=q,g}\sum_{j=\bar{q},g}\int_{\xi_1}^1 \frac{dz_1}{z_1}\int_{\xi_2}^1 \frac{dz_2}{z_2}$$

$$\times\left[C_{q\bar{q}\to ij}\left(\frac{\xi_1}{z_1},\frac{\xi_2}{z_2},q_T^2,M^2,\mu\right)\phi_{i/N_1}(z_1,\mu)\,\phi_{j/N_2}(z_2,\mu) + (q,i\leftrightarrow\bar{q},j)\right]$$

The hard-scattering kernel is

$$C_{q\bar{q}\to ij}(z_1,z_2,q_T^2,M^2,\mu) = H(M^2,\mu)\frac{1}{4\pi}\int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp}\left(\frac{x_T^2M^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar{q}}(x_T^2,\mu)}$$

$$\times\, I_{q\leftarrow i}(z_1,x_T^2,\mu)\, I_{\bar{q}\leftarrow j}(z_2,x_T^2,\mu)$$

- Two sources of *M* dependence: hard function and anomaly
- Fourier transform can be evaluated numerically or analytically, if higher-log terms are expanded out.

COLLINS SOPER STERMAN FORMULA

$$\frac{d^3\sigma}{dM^2\,dq_T^2\,dy} = \frac{4\pi\alpha^2}{3N_cM^2s}\,\frac{1}{4\pi}\int d^2x_\perp\,e^{-iq_\perp\cdot x_\perp}\sum_q e_q^2\sum_{i=q,g}\sum_{j=\bar{q},g}\int_{\xi_1}^1\frac{dz_1}{z_1}\int_{\xi_2}^1\frac{dz_2}{z_2}$$
$$\times\exp\left\{-\int_{\mu_b^2}^{M^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\frac{M^2}{\bar{\mu}^2}A\big(\alpha_s(\bar{\mu})\big)+B\big(\alpha_s(\bar{\mu})\big)\right]\right\}$$
$$\times\left[C_{qi}\big(z_1,\alpha_s(\mu_b)\big)\,C_{\bar{q}j}\big(z_2,\alpha_s(\mu_b)\big)\,\phi_{i/N_1}(\xi_1/z_1,\mu_b)\,\phi_{j/N_2}(\xi_2/z_2,\mu_b)+(q,i\leftrightarrow\bar{q},j)\right]$$

- The low scale is $\mu_b = b_0/x_T$, and we set $b_0 = 2e^{-\gamma_E}$.
- Landau-pole singularity in the Fourier transform. To use the formula, one needs additional prescription to deal with this.

RELATION TO CSS

If adopt the choice $\mu = \mu_b = 2e^{-\gamma_E}/x_\perp$ in our result reduces to CSS formula, provided we identity

$$A(\alpha_s) = \Gamma^F_{\text{cusp}}(\alpha_s) - \frac{\beta(\alpha_s)}{2}\,\frac{dg_1(\alpha_s)}{d\alpha_s}\,,$$

$$B(\alpha_s) = 2\gamma^q(\alpha_s) + g_1(\alpha_s) - \frac{\beta(\alpha_s)}{2}\,\frac{dg_2(\alpha_s)}{d\alpha_s}\,,$$

$$C_{ij}\big(z, \alpha_s(\mu_b)\big) = \big[H(\mu_b^2, \mu_b)\big]^{1/2}\, I_{i\leftarrow j}\big(z, 0, \alpha_s(\mu_b)\big)\,,$$

anomaly contribution

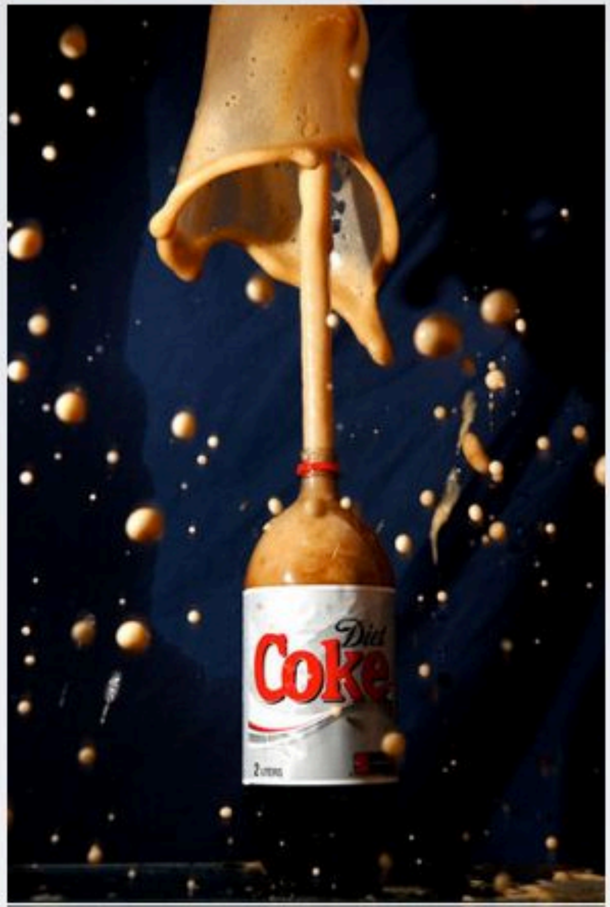
$$g_1(\alpha_s) = F(0, \alpha_s)$$

$$g_2(\alpha_s) = \ln H(\mu^2, \mu)$$

Use these relations to derive unknown three-loop coefficient, necessary for NNLL resummation

$$A^{(3)} = \Gamma_2^F + \beta_0\, g_1''(0) = 239.2 - 652.9 \neq \Gamma_2^F$$

Not equal to the cusp anom. dim. as was usually assumed!



DIVERGENT EXPANSIONS, AND OTHER SURPRISES

TRANSVERSE MOMENTUM SPECTRUM

The spectrum has a number of quite remarkable features which we now discuss in turn:

- Expansion in α_s : strong factorial divergence
- q_T-spectrum:
  - calculable, even near $q_\mathrm{T} = 0$
  - expansion around $q_\mathrm{T} = 0$: extremely divergent
- Long-distance effects associated with Λ_QCD
  - small, but OPE breaks down

LEADING MOMENTUM DEPENDENCE

Up to corrections suppressed by powers of α_s, the q_T-dependence of our formula result has the form

$$\frac{1}{4\pi}\int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp}\, e^{-\eta L_\perp - \frac{1}{4}aL_\perp^2} \equiv \frac{e^{-2\gamma_E}}{\mu^2}\, K\Big(\eta, a, \frac{q_T^2}{\mu^2}\Big)$$

with $L_\perp = \ln\dfrac{x_T^2\mu^2}{4e^{-2\gamma_E}}$, and the two quantities

$$\eta = \frac{C_F\alpha_s}{\pi}\ln\frac{M^2}{\mu^2} = \mathcal{O}(1) \qquad a = \alpha_s(\mu)\times\mathcal{O}(1)$$

Since a is suppressed one can try to expand K in it.

FACTORIAL DIVERGENCE

Unfortunately, the series in *a* is strongly factorially divergent:

$$K(\eta, a, 1)\big|_{\text{exp}} = \sum_{n=0}^{\infty} \frac{(2n)!}{n!} \left(-\frac{a}{4}\right)^n \left[\frac{1}{(1-\eta)^{2n+1}} - e^{-2\gamma_E}\right] + \ldots$$

first noted by Frixione, Nason, Ridolfi '99

Can Borel resum it, which makes the nonperturbative and highly nontrivial *a* dependence explicit

$$K(\eta, a, 1)\big|_{\text{Borel}} = \sqrt{\frac{\pi}{a}} \left\{ e^{\frac{(1-\eta)^2}{a}} \left[1 - \text{Erf}\left(\frac{1-\eta}{\sqrt{a}}\right)\right] - e^{-2\gamma_E + \frac{1}{a}} \left[1 - \text{Erf}\left(\frac{1}{\sqrt{a}}\right)\right]\right\} + \ldots$$

In practice, it is simplest, to use the exact expression and evaluate *K*-function numerically.

VERY LOW q_T

For moderate q_T, the natural scale choice is $\mu = q_T$. However, detailed analysis shows that near $q_T \approx 0$ the Fourier integral is dominated by

$$\langle x_T^{-1} \rangle = q_* = M \exp\left(-\frac{\pi}{2C_F \alpha_s(q_*)} \right) = 1.75\,\mathrm{GeV} \quad \text{for } M = M_Z$$

which corresponds to $\eta = 1$.

→ Spectrum can be computed with short-distance methods down to q_T=0!

INTERCEPT AT Q_T=0

- Dedicated analysis of $q_T \to 0$ limit yields:

$$\frac{d\sigma}{dq_T^2} \sim \frac{\mathcal{N}}{\sqrt{\alpha_s}} e^{-\#/\alpha_s} \left(1 + c_1 \alpha_s + \ldots\right)$$

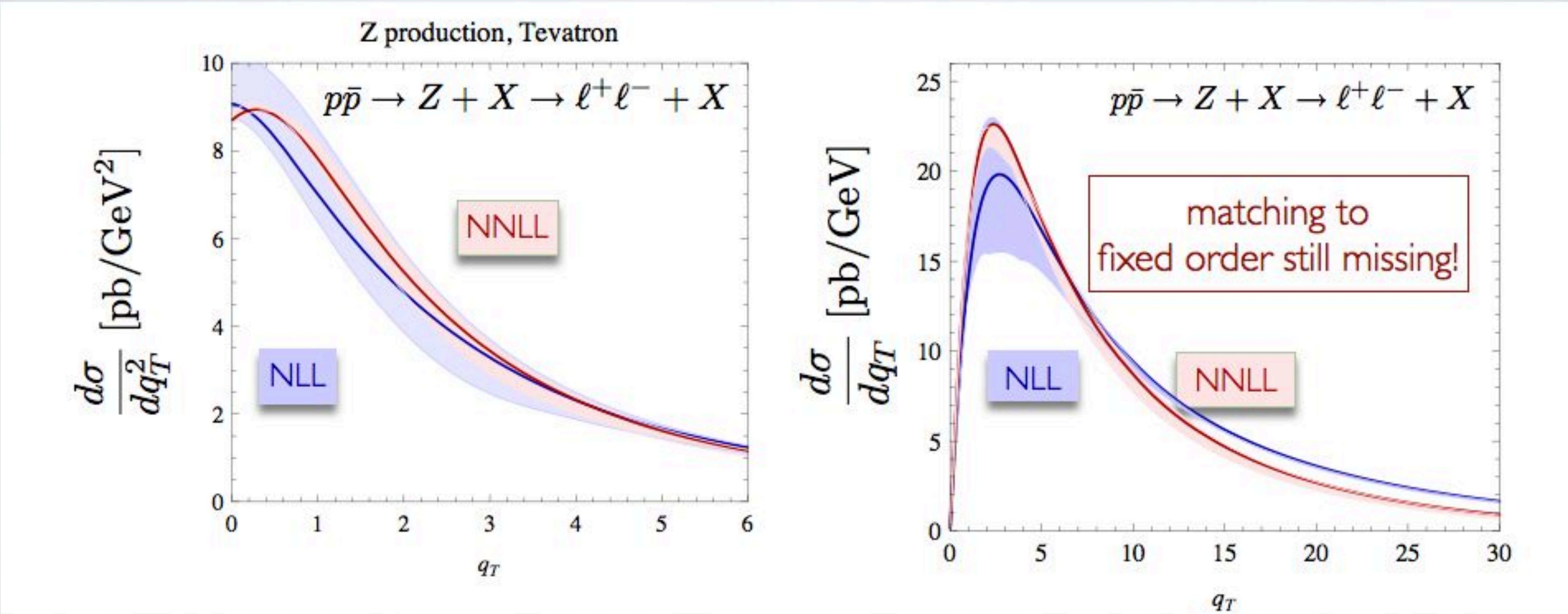
Parisi, Petronzio 1979;
Collins, Soper, Sterman 1985; Ellis, Veseli 1998

- We have computed the normalization $\mathcal{N}$ and NLO coefficient c_1

- Expression cannot be expanded about $\alpha_s = 0$ (essential singularity)

PRELIM. NUMERICAL RESULTS

- Find smooth behavior down to very small q_T



SLOPE AT Q_T=0?

Given our result for the intercept, we can also try to obtain derivatives with respect to q_T^2. Leading term is obtained by expanding

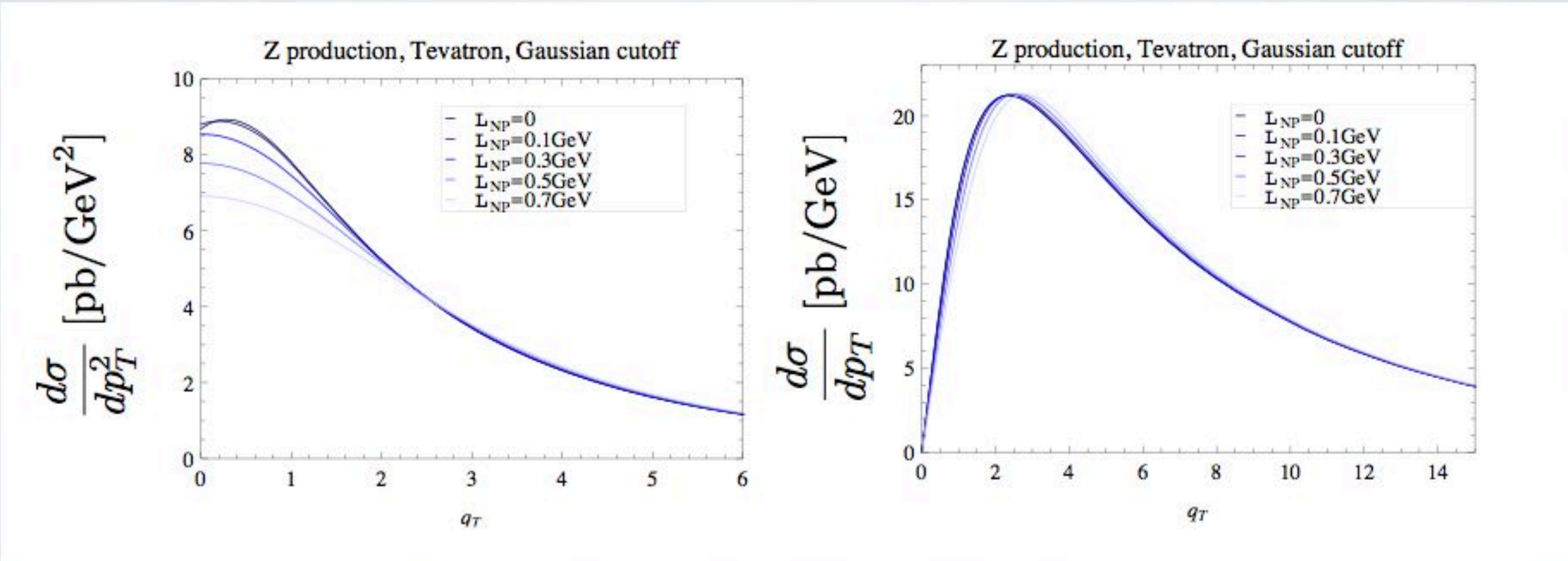
$$\frac{1}{4\pi}\int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp}\, e^{-\eta L_\perp - \frac{1}{4}aL_\perp^2} \equiv \frac{e^{-2\gamma_E}}{\mu^2}\, K\left(\eta, a, \frac{q_T^2}{\mu^2}\right)$$

Yields violently divergent series

$$K(\eta=1,a,q_T)\big|_{\mathrm{exp}} \sim \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{a}}\, e^{n^2/a} \left(\frac{q_T^2}{q_*^2}\right)^{n-1}$$

LONG-DISTANCE EFFECTS

- Can model long distance effect by cutting of Fourier integral, e.g. with a Gaussian $e^{-\Lambda^2 x_T^2}$.



- Small effects, but expansion in Λ^2/q_*^2 has the same violent divergence as expansion in q_T^2/q_*^2 .

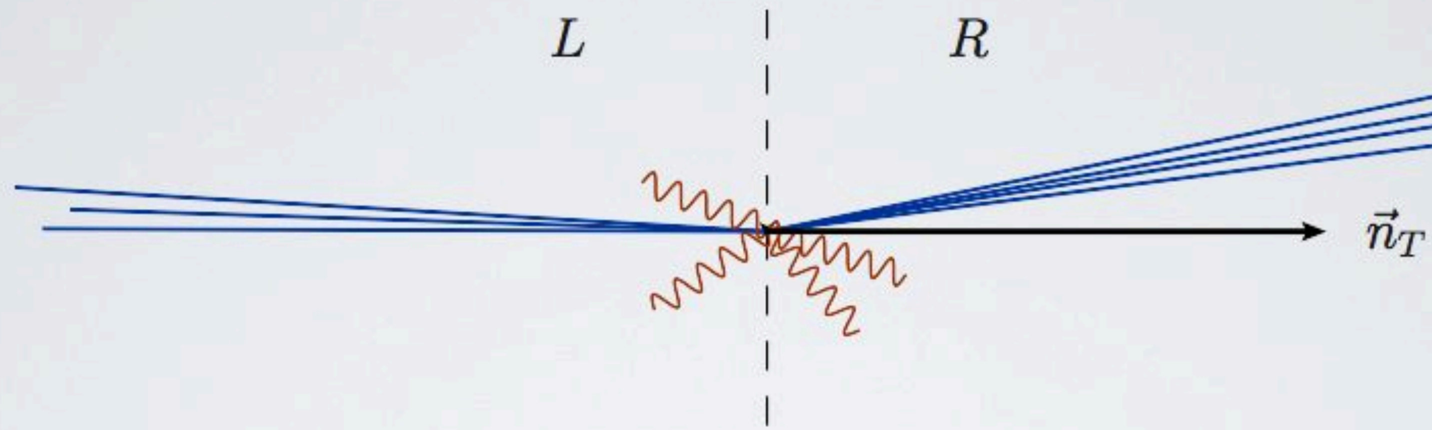
MORE ANOMALOUS FACTORIZATION: JET BROADENING

The problem that individual jet and soft functions are not well defined without additional regularization also arises in other factorization theorems, for example

- Electroweak Sudakov resummation (and any other process at high momentum transfer with small, but non-negligible masses). Chiu, Golf, Kelley, Manohar

- Other variables sensitive only to transverse momenta, such as jet broadening. TB, Bell, Neubert, to appear

JET BROADENING IN e^+e^-



- Broadening measures momentum relative to the thrust axis

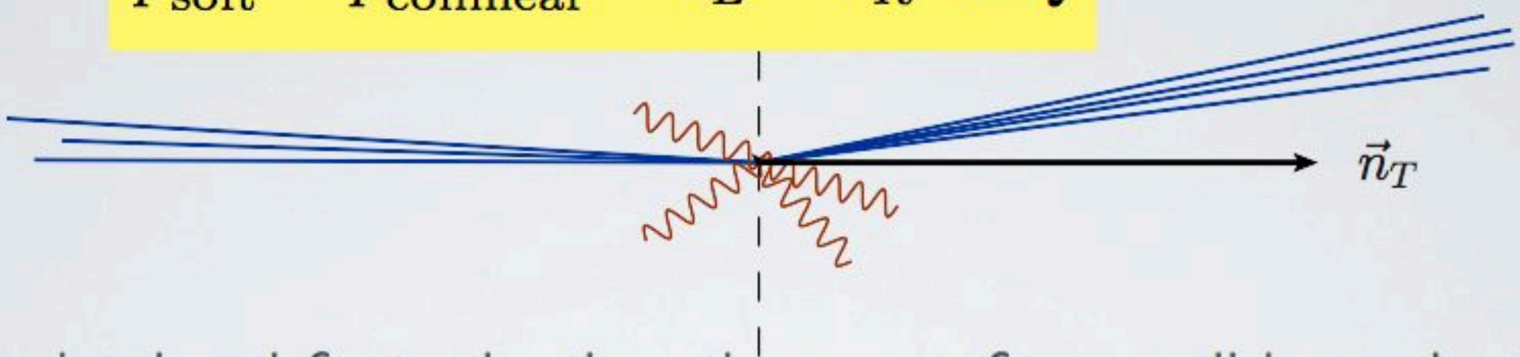
$$b_L = \frac{1}{2}\sum_i |\vec{p}_i^{\perp}| = \frac{1}{2}\sum_i |\vec{p}_i \times \vec{n}_T|$$

- Measured are the total and wide broadening

$$b_T = b_L + b_R, \qquad b_W = \max(b_L, b_R)$$

FACTORIZATION

$$p_{\text{soft}}^{\perp} \sim p_{\text{collinear}}^{\perp} \sim b_L \sim b_R \ll Q$$



- Have obtained factorization theorem for small broadening

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{db_L db_R} = H(Q^2,\mu)\int db_L^s \int db_R^s \int d^{d-2}p_L^{\perp} \int d^{d-2}p_R^{\perp}$$
$$\boxed{\mathcal{J}_L(b_L - b_L^s, p_L^{\perp}, \mu)\,\mathcal{J}_R(b_R - b_R^s, p_R^{\perp}, \mu)}\boxed{S(b_L^s, b_R^s, -p_L^{\perp}, -p_R^{\perp}, \mu)\,.}$$

  - Jet recoils against soft radiation!
  - *J* and *S* suffer again from coll. anomaly, analytic regulator

LAPLACE AND FOURIER SPACE

- Have derived all-order form of anomalous Q-dependence

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{d\tau_L d\tau_R} = H(Q^2,\mu)\int dz_L \int dz_R \,(Q^2\bar{\tau}_L^2)^{-F_B(\tau_L,z_L,\mu)}(Q^2\bar{\tau}_R^2)^{-F_B(\tau_R,z_R,\mu)}$$
$$\times\, \overline{J}(\tau_L,z_L,\mu)\,\overline{J}(\tau_R,z_R,\mu)\,\overline{S}(\tau_L,\tau_R,z_L,z_R,\mu)$$

TB, Bell, Neubert, to appear

- One-loop anomaly coefficient is

$$F_B(\tau,\mu) = \frac{C_F\alpha_s}{\pi}\left(\ln\mu\bar{\tau}_L + \ln\frac{\sqrt{1+z^2}+1}{4}\right)$$

- To NLL tree-level jet and soft functions are sufficient

$$\overline{S} = 1 \qquad\qquad \overline{J} = z/\left(1+z^2\right)^{3/2}$$

NLL RESULT

- Because of the simple τ dependence the Mellin inversion can be done analytically. Result for total broadening:

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{db_T} = H(Q^2,\mu)\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}\frac{1}{b_T}\left(\frac{b_T^2}{\mu^2}\right)^{2\eta} I^2(\eta)$$

- with

$$I(\eta) = \int_0^\infty dz\,\frac{z}{(1+z^2)^{3/2}}\left(\frac{\sqrt{1+z^2}+1}{4}\right)^{-\eta} \quad \text{and} \quad \eta = \frac{C_F\alpha_s(\mu)}{\pi}\ln\frac{Q^2}{\mu^2}$$

- Equivalent to the result of Dokshitzer, Lucenti, Marchesini and Salam '98

NNLL?

- Have operator definitions for the jet and soft functions, e.g.

$$\frac{\pi}{2}(\not{n})_{\alpha\beta}\,\mathcal{J}_L(b,p^\perp) = \sum\!\!\!\!\!\!\int_X (2\pi)^d\,\delta(\bar{n}\cdot p_X - Q)\,\delta^{d-2}(p_X^\perp - p^\perp)$$
$$\delta\Big(b - \frac{1}{2}\sum_i |p_i^\perp|\Big)\,\langle 0|\chi_\alpha(0)|X\rangle\langle X|\bar{\chi}_\beta(0)|0\rangle$$

- For NNLL we need
  - one loop jet and soft functions and
    - (already have one-loop soft function)
  - two-loop anomaly function *F* (obtained from 2-loop divergence of the soft function.

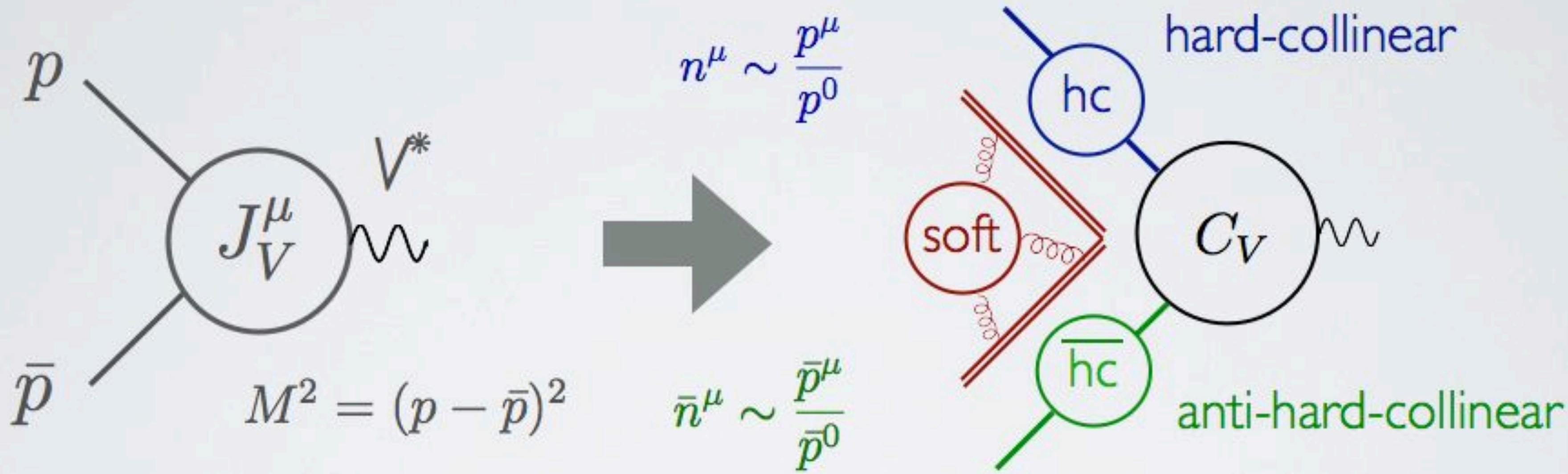
CONCLUSION

- Have derived resummed result for Drell-Yan q_T spectrum. Naive factorization broken by collinear anomaly. Only the product of two transverse PDFs is well defined, but has anomalous dependence on the large momentum transfer
- Result is equivalent to CSS a special scale choice. Obtain three-loop coefficient $A^{(3)}$, the last missing piece needed for NNLL accuracy.
- Many surprising features:
  - emergence of nonperturbative scale q_*~2GeV: spectrum is short-distance dominated, even at very low q_T
  - strongly divergent expansions in α_s, q_T/q_*, Λ_{QCD}/q_*.
- Phenomenological analysis at NNLL+NLO is in progress.
- Have all-order factorization theorem for jet broadening
  - Necessary computations for NNLL resummation look feasible

EXTRA SLIDES

SOFT-COLLINEAR FACTORIZATION

- Starting point is the factorization of the electroweak current in the Sudakov limit



- In Soft-Collinear Effective Theory (SCET) this can be written in operator form as

$$\bar{\psi}\gamma_\mu\psi \to C_V(M^2, \mu)\, \bar{\chi}_{\overline{hc}}\, S_{\bar{n}}^\dagger\, \gamma^\mu\, S_n\, \chi_{hc}$$

SCET hc quark field

The Drell-Yan cross section is obtained from the matrix element of two currents

$$(-g_{\mu\nu})\,\langle N_1(p)\,N_2(\bar{p})|\,J_V^{\mu\dagger}(x)\,J_V^{\nu}(0)\,|N_1(p)\,N_2(\bar{p})\rangle \to \frac{1}{2N_c}|C_V(M^2,\mu)|^2$$
$$\times\,\hat{W}_{\rm DY}(x)\,\langle N_1(p)|\,\bar{\chi}_{hc}(x)\,\frac{\bar{\not{n}}}{2}\,\chi_{hc}(0)\,|N_1(p)\rangle\,\langle N_2(\bar{p})|\,\bar{\chi}_{\overline{hc}}(0)\,\frac{\not{n}}{2}\,\chi_{\overline{hc}}(x)\,|N_2(\bar{p})\rangle$$

n and $\bar{n}$ are light-cone reference vectors along p and $\bar{p}$.

The soft function contains a product of four Wilson lines along the directions of large energy flow

$$\hat{W}_{\rm DY}(x) = \frac{1}{N_c}\,\langle 0|\,{\rm Tr}\big[S_n^\dagger(x)\,S_{\bar{n}}(x)\,S_{\bar{n}}^\dagger(0)\,S_n(0)\big]\,|0\rangle$$

$$S_n(x) = \mathbf{P}\exp\left[i\int_{-\infty}^{0} ds\,n\cdot A_s(x+sn)\right]$$

DERIVATIVE EXPANSION

Final step is to expand the matrix elements in small momentum components, i.e. to perform a derivative expansion.

The light-cone components $(n \cdot k, \bar{n} \cdot k, k_\perp)$ scale as

$$p_{hc} \sim M\,(\lambda^2, 1, \lambda)\,, \qquad p_{\overline{hc}} \sim M\,(1, \lambda^2, \lambda)\,.$$

expansion parameter

$$\lambda = \frac{q_T}{M}$$

$$p_s \sim M\,(\lambda^2, \lambda^2, \lambda^2)\,.$$

while the separation between the two currents scales as

$$x \sim M^{-1}(1, 1, \lambda^{-1})$$

$$\longrightarrow \quad A_s^\mu(x) = A_s^\mu(0) + x \cdot \partial A_s^\mu(0) + \dots$$

power suppressed, can be dropped

NAIVE FACTORIZATION

Dropping power suppressed *x*-dependence leads to the result

$$\underbrace{\hat{W}_{\rm DY}(0)}_{1}\,\langle N_1(p)|\,\bar{\chi}_{hc}(x_+ + x_\perp)\,\frac{\bar{n}\!\!\!/}{2}\,\chi_{hc}(0)\,|N_1(p)\rangle\,\langle N_2(\bar{p})|\,\bar{\chi}_{\overline{hc}}(0)\,\frac{n\!\!\!/}{2}\,\chi_{\overline{hc}}(x_- + x_\perp)\,|N_2(\bar{p})\rangle$$

1 × "transverse PDF" × "transverse PDF"

KLN cancellation!

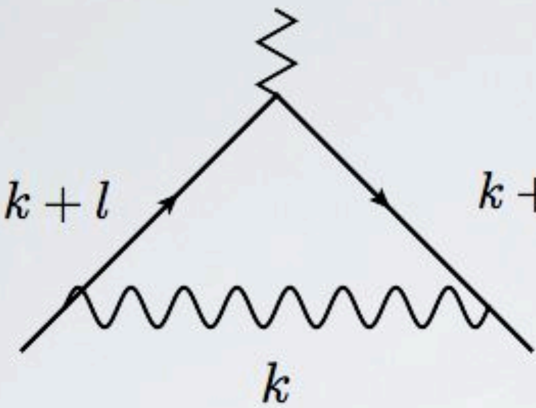
this spells trouble: well known that transverse PDF not well defined w/o additional regulators

For comparison: for soft-gluon resummation, the result is

$$\hat{W}_{\rm DY}(x_0)\,\langle N_1(p)|\,\bar{\chi}_{hc}(x_+)\,\frac{\bar{n}\!\!\!/}{2}\,\chi_{hc}(0)\,|N_1(p)\rangle\,\langle N_2(\bar{p})|\,\bar{\chi}_{\overline{hc}}(0)\,\frac{n\!\!\!/}{2}\,\chi_{\overline{hc}}(x_-)\,|N_2(\bar{p})\rangle$$

"soft" × "standard PDF" × "standard PDF"

EW SUDAKOV RESUMMATION



$$l^2 \sim p^2 \ll Q^2 = (p-l)^2$$

- For vanishing boson mass, the diagram receives a contribution from regions where the loop momentum k_μ is
    - hard, collinear to l_μ, collinear to p_μ and ultra-soft $k_\mu \sim p^2/Q$
- For boson mass $M^2 \sim p^2$ there is no ultrasoft contribution since in this region $k^2 << M^2$ can be expanded out of the boson propagator, but at the same time the two collinear regions are no longer separately well defined $\rightarrow$ collinear anomaly